2017

# COMPUTER SCIENCE AND ENGINEERING

## Paper – CSCL-0901

## (Topics in Algorithms)

**Full Marks – 70**

*The figures in the margin indicate full marks*
*Candidates are required to give their answers in their own words as far as practicable*

### Group – A

Answer ***any five*** questions 2×5

1. Solve the following recurrence equation by Master Theorem.

$$T(n)=3T(n/2)+n^2.$$

2. What is meant by Pspace?
3. State Fermat's Little Theorem. What are Pseudo-primes?
4. What is meant by Decision problem? Give an example.
5. State base-function property of the family of algorithms.
6. What is meant by SM SIMD computers?
7. What is meant by Odd-even merging network?

### Group – B

Answer ***any five*** questions

8. State and explain Master's Theorem. 1+3
9. Write a brief note on Red-Black Trees. 4
10. What are online algorithms? What do you mean by competitive analysis in regard to online algorithms? 2+2
11. Write a randomized algorithm for computing the value of $\pi$. Explain the steps. 4
12. Define primitive recursion. Show how two-variable addition function can be defined by primitive recursion. 1+3
13. Given a flow $F$ in a network, prove that the flow out of the source '$a$' equals the flow into the sink '$z$'. 4
14. Suppose that $\alpha$ is a family of algorithms in which properties 1, 2 and 4 are satisfied. Then show that the one-variable function $f$ where

$$f(x)=\begin{cases}0 & \text{if } x=0\\ 1 & \text{if } x\neq 0\end{cases}$$

is algorithmic in $\alpha$. 4

[Turn Over]

## Group – C

Answer *any four* questions

15. State and establish Chinese Remainder Theorem. 2+8

16. Explain what is meant by convex hull. Write Graham's Algorithm for computing the convex hull. Illustrate. 2+8

17. Explain, with an example, how does KMP algorithm work. 10

18. Prove that 3SAT problem is NP-complete. 10

19. Describe the twice-around-the-tree algorithm for Travelling Salesperson problem. Prove that it is a 2-approximaion algorithm with Euclidean distances. 5+5

20. Write a divide and conquer algorithm for finding the distance between a closest pair of points. 10

21. State the essential features of a parallel searching algorithm based on EREW architecture. 10

# 2018

## COMPUTER SCIENCE AND ENGINEERING

**Paper : CSCL-901**

**(Topics in Algorithms)**

**Full Marks : 70**

*The figures in the margin indicate full marks.*
*Candidates are required to give their answers in their own words as far as practicable.*

**1.** Answer ***any five*** from the following : 2×5

(a) Define approximation algorithm.

(b) Let $\prod_1$ and $\prod_2$ be two problems such that $\prod_1 \leq_P \prod_2$. Suppose that the problem $\prod_2$ can be solved in $O(n^k)$ time and the reduction can be done in $O(n^j)$ time. Show that the problem $\prod_1$ can be solved in $O(n^{jk})$ time.

(c) Compute the prefix function $\pi$ for the pattern *ababbab* when the alphabet set is $\{a, b\}$.

(d) What is the time complexity of Huffman coding algorithm? Justify.

(e) When will the approximation ratio to solve the vertex cover problem by approximation algorithm exactly equal to 2? Justify your answer.

(f) What can be the maximum length of the augmenting path in a network of bipartite graph of $n$ vertices and $m$ edges? Justify your answer.

(g) Give example of a small network where the Ford-Fulkerson algorithm can take exactly $mC$ iterations, where $m$ is the number of edges and $C$ the maximum capacity of edges.

**2.** Answer ***any five*** from the following : 4×5

(a) Given an array $\chi$ containing $n$ *unique* real numbers, design and analyze an efficient algorithm that finds out a number in $\chi$ that is neither minimum nor maximum.

(b) If module $q$ is 11, how many spurious hits does the Rabin-Karp matcher encounter in the text $T = 3141592653589793$, when looking for the pattern $P = 26$?

(c) Given a convex hull $H$ and a point $p$ exterior to the hull $H$, write an $O(\log n)$ time algorithm to compute the combined hull of $H$ and $p$.

(d) Show that satisfiability problem (SAT) is polynomial time reducible to Vertex Cover (VC).

(e) Given the following bipartite graph $G = (U \cup V, E)$ with vertices $U = \{a, c, e, g, i\}$ and $V = \{b, d, f, h, j\}$ and $E = \{(a, d),(a, f),(b, c),(b, e),(c, d),(c, f),(c, h),(d, e),(d, g),(e, h),(f, g),(h, i),(i, j)\}$. Find the maximum matching using Hungarian Tree method.

(f) Prove or disprove : Median selection algorithm does not run in linear time when the group size is 3.

**Please Turn Over**

(g) Suppose I have written a greedy algorithm to solve activity selection problem as follows : select the activity of least duration from among those that are compatible with previously selected activities. Will this algorithm always select maximum number of non-overlapping activities? Justify with an example.

**3.** Answer ***any four*** from the following :

(a) (i) Write a pseudocode for the naive string matching algorithm to find the first occurrence of a pattern $P$ of length $m$ in a text $T$ of length $n$. What is the worst-case time complexity of this algorithm?

(ii) Suppose that all characters in the pattern $P$ are different. Show how to accelerate the above naive string matcher to run in time $O(n)$ on an $n$ character text $T$. 6+4

(b) (i) What is meant by optimal substructure and overlapping subproblems in the context of dynamic programming?

(ii) The subset sum problem is : given a set of $n$ positive integers, $S = \{a_1, a_2, ...a_n\}$, and a positive integer $W$, is there a subset of $S$ whose elements sum to $W$? Derive the recurrence to decide if such a subset exists using dynamic programming. Note : $X[i, j]$ denotes true(1) iff there is a subset $\{a_1, a_2, ...a_i\}$ whose elements sum to $j$. 5+5

(c) Given a set of $n$ convex polygons. Write an efficient algorithm to find the convex hull of the set of $n$ polygons. What is the time complexity of your algorithm? Justify your answer. 6+2+2

(d) (i) Prove or disprove the following statement— "If all capacities in a network are distinct, then there exists a unique flow function that gives the maximum flow."

(ii) In a hospital, there are $n$ doctors. There are two shifts in the hospital : morning and evening. Each doctor has given his/her preference on the day and the shift when he can work. No doctor will be scheduled to work for the entire day, i.e. both morning and evening shifts. Each doctor can be engaged atmost for two days. Each shift must be covered by atmost one doctor. Design an efficient formulation to solve this problem. 4+6

(e) (i) Define P, NP, NP-Complete, NP-Hard complexity classes.

(ii) Consider the following statement. If a problem $\prod \in NP$ and $\prod \leq_P \prod'$ where $\prod'$ is an $NP$-Complete problem, then $\prod$ is $NP$-Complete. State whether the above statement is true or false with a brief explanation. You may assume that $P \neq NP$. 6+4

(f) Define Euclidean Travelling Salesman Problem (ETSP). Give a 2-approximation algorithm for ETSP. What is the time complexity of the algorithm? Justify the approximation ratio. 1+3+2+4

# 2019

## COMPUTER SCIENCE AND ENGINEERING

**Paper : CSCL-0901**

**(Topics in Algorithms)**

**Full Marks : 70**

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

**1.** Answer ***any five*** from the following : 2×5

(a) Why do we analyze the expected running time of a randomized algorithm and not its worst-case running time?

(b) Can quicksort be made to run in O(n log n) time in the worst case, assuming that all elements are distinct?

(c) Draw the decision tree for linear search on four elements.

(d) Define PSPACE.

(e) Construct an I-D range tree for the following key values: 34, 90, 12, 32, 54, 73, 29.

(f) State the properties to be satisfied by a flow in a flow network.

(g) What is the time complexity of KMP string matching algorithm?

**2.** Answer ***any five*** from the following : 4×5

(a) Show that the approximation algorithm has a ratio bound of 2 for the k-center problem.

(b) Let L1, L2, L3 are decision problems. Show that if $L1 \leq_p L2$ and $L2 \leq_p L3$, then $L1 \leq_p L3$.

(c) Given that all characters in a pattern P are different, write an algorithm to accelerate the naive string matching algorithm to run in O(n) time. Justify the time complexity.

(d) State and prove the maxflow-mincut theorem.

(e) Prove that the number of tosses required to insert in a skip list is bounded by O(log n) with high probability.

(f) Prove or disprove: if we order the characters in an alphabet set so that their frequencies are monotonically decreasing, then there exists an optimal code whose codeword lengths are monotonically increasing.

(g) Derive the average case lower bound of comparison based sorting.

(h) Let Al and A2 be two sorted arrays of n elements each. Show that at least 2n-l comparisons are required to merge Al and A2 in the worst case. Give an example of such a scenario.

**Please Turn Over**

**3.** Answer ***any four*** from the following :

(a) Give a pseudocode for the randomized quicksort algorithm. When randomized quicksort runs, how many calls are made to the random number generator in the (i) worst case and (ii) best case? Give your answer in terms of $\theta$ notation. 6+2+2

(b) Give a pseudocode for the randomized selection algorithm. Suppose we use randomized selection algorithm to select the minimum element of the array A = {3; 2; 9; 0; 7; 5; 4; 8; 6; 1}. Describe a sequence of partitions that results in a worst-case performance. 5+5

(c) Define a 3 dimensional range tree with illustration. What are the time complexities of

(i) building a 3D range tree

(ii) querying in a 3D range tree? Justify the time complexity. 10

(d) Describe the randomized closest pair algorithm. Derive a bound on the total number of insert operations. 6+4

(e) Define (i) general matching problem (ii) bipartite matching problem. Give a polynomial time reduction algorithm to reduce the bipartite matching problem to a network flow problem justifying the time complexity. 2+2+6

(f) Draw a state-transition diagram for a string-matching automation for the pattern 'ababbabbababbababbabb' over the alphabet {a,b}. Compute the prefix function for the same pattern. 5+5

(g) Define the following : 3+3+2+2

(i) approximation algorithm

(ii) approximation scheme

(iii) PTAS

(iv) FPTAS.

**Mode of Examination: Online**

**M.Tech. Semester – I Examination, 2021**

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**2020**

**Subject: Computer Science**

**Paper Code & Name: CSCL 0901 Topics in Algorithms**

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**Full Marks: 70**

**Date: 06.02.2022** **Time and Duration: 12:00 pm to 3.00 pm, 3 hours**

**Please note the following instructions carefully:**

- **Promise not to commit any academic dishonesty.**
- **Marks will be deducted if the same/similar answers are found in different answer-scripts.**
- **Candidates are required to answer in their own words as far as applicable.**
- **Each page of the answer scripts should have your University Roll # on the right-top corner. The name of the scanned copy of the answer script will be of the following format: Paper code-paper name-exam roll number.pdf (Example: CSCL-0901-Algorithm-97-CSM-201001.pdf))**
- **The subject of the mail should be the file name only.**
- **The name of the scanned answer-script is to be sent to cucse2020@gmail.com**
- **The answer-script may not be accepted after the scheduled time.**

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**Answer Question 1, 2 and any four from the rest.**

1. Answer any **five** questions from the following: 2 marks $\times 5$

   a) Find the KMP prefix function for the string $aaabbbabacab$.

   b) How is string-matching automaton computation different from the computation of prefix function in KMP algorithm?

   c) In randomized closest pair algorithm how is the notion of randomization used?

   d) What is the significance of computing the expected time $T(n)$ on the average for a randomized algorithm?

   e) If the partition procedure in quicksort divides the array into $n/3$ and $2n/3$ during each recursion of quicksort, express it as a recurrence relation and derive the running time.

   f) How is randomization used to find the $k^{th}$ smallest element in Randomized Select algorithm?

2. Answer any **five** questions from the following: 4 marks $\times 5$

   a) Arrange the following functions in the increasing order of their rates of growth:
   $(\sqrt{2})^n, 2^{\sqrt{n}}, n^2 \log n, n(\log n)^2, (n \log n)^2, n^{\log n}, n^{\sqrt{n}}, n^n, (\log n)^n$.

   b) Determine what the following functions computes. Also derive the worst-case time complexity of this algorithm. Assume that in each case the input is a sequence of $n$ positive integers $a_0, a_1, \cdots, a_{n-1}$.

```
int what ( int a[] , int n )
{
int i, j, m, b[MAXSIZE];
for (i=0; i<n; ++i) b[i]=0;
for (i=0; i<n; ++i) for (j=0; j<n; ++j) if (a[j] == a[i]) ++b[j];
m = j = 0;
for (i=0; i<n; ++i) if (b[i] > m) { m = b[i]; j = i; }
return a[j];
}
```

   c) Provide an example in which the Ford-Fulkerson algorithm makes $O(f)$ iterations, where $f$ is the maximum flow.

d) Let $n$ be a positive integer. Design an efficient randomized algorithm that generates a random permutation of the integers $1, 2, \cdots, n$. Assume that you have access to a fair coin. Analyze the time complexity of your algorithm.

e) Suppose the divide and conquer Algorithm *BinarySearch(low, high, key)* is modified as follows: Instead of halving the search interval in each iteration, select one of the remaining positions at random. Assume that every position between *low* and *high* is equally likely to be chosen by the algorithm. Compare the performance of this new randomized algorithm with that of Algorithm *BinarySearch(low, high, key)*.

f) Derive the time complexity of *Select* algorithm to find $k^{th}$ smallest number if the group size is set to 7.

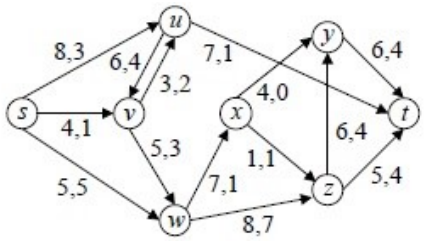
3. You are given $k$ sorted lists each of size $n$. Describe a divide-and-conquer algorithm to merge the $k$ lists into a single sorted list of size $kn$. Derive the time complexity of your algorithm. [6+4]

4. Prove that any comparison-based sorting algorithm in an array $A$ of $n$ numbers must take $\Omega(n \log n)$ running time in the worst case. Comment on the optimality of the MergeSort algorithm. [7+3]

5. (a) Give example of an ideal skip list of 16 key values such that the worst case search time complexity is $O(\log n)$. How many pointers are required to represent this skip list?

   (b) Write an algorithm to merge two skip list $s_1$ and $s_2$ storing $n_1$ and $n_2$ keys respectively to obtain a single skip list storing $n_1 + n_2$ keys. Derive the time complexity of your algorithm. [3+5+2]

6. Consider the network flow shown in the following figure. Here, $s$ is the source, and $t$ is the sink. The capacity $c(e)$ and the current flow amount $f(e)$ are shown against the edge $e$ as $c(e), f(e)$. Run the Ford-Fulkerson algorithm until the maximum flow is computed. [10]




7. (a) Two computational problems $P_1$ and $P_2$ are called polynomial-time equivalent if there exist polynomial-time reductions $P_1 \leq_P P_2$ and $P_2 \leq_P P_1$. Prove or disprove: Every two NP-Complete problems are polynomial-time equivalent.

   (b) Are Max-flow and bipartite matching problem polynomial time reducible to each other? Justify your answer. [5+5]

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# 2022

## COMPUTER SCIENCE AND ENGINEERING

**Paper : CSCL-0901**

**(Topics in Algorithms)**

**Full Marks : 70**

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

Answer ***question nos.*** **1, 2** and ***any four*** from the rest.

**1.** Answer ***any five*** questions : 2×5

(a) Suppose you have algorithms with average time complexities $n^2$, n log n and $2^n$. How much slower does each of these algorithms get when you

(i) double the input size

(ii) increase the input size by one?

(b) Define NP and NP-hard complexity classes.

(c) Give example of a flow network where Ford Fulkerson algorithm takes the maximum time.

(d) Give the node structure of a skip list.

(e) What is the significance of approximation ratio in approximation algorithm?

(f) Let A[1..95] be a sorted array of integers. In order to find the 17th smallest integer, how many calls to Select algorithm will be required?

**2.** Answer ***any five*** questions : 4×5

(a) Define universal hash function and explain how it is used in randomized closest pair algorithm.

(b) In the combined step of divide and conquer closest pair algorithm, let $Y' = \{y_i\}$ represent the sorted y coordinates of the points that lie in $2\delta$ region of the middle point 1 on x-axis, where $\delta$ is the minimum distance obtained between points either in the Left side L or Right side R of the middle point 1. Explain why the algorithm needs only to check the distance between each $y_i$ and the next eight points in the sorted sequence of Y' in order to obtain the closest pair.

(c) Draw the state transition diagram for a string matching automata for the pattern ABABBABBABABBABABBABB.

(d) Show that the first fit strategy to solve bin packing problem gives an approximation factor of 2.

(e) Show that on expectation, the search time in a randomized skip list is O(n).

(f) Give an algorithm to compute only the prefix function in KMP string matching along with time complexity.

(g) Prove the maximum flow min-cut theorem.

**Please Turn Over**

3. Suppose a pattern P to be searched in a text string T, P has gap character $\phi$ to indicate zero or more length set of characters that can be ignored in T in order to match P. For example, let P = ab $\phi$ba $\phi$c and T = cabccbacbacab. Then P occurs in T as **cabccbac**bac**a**b, i.e., **cab$\phi$ba$\phi$c**ab. Bold characters show the match of P in T. Write a polynomial time algorithm to obtain such a match and analyze the time complexity. 7+3

4. You are asked to find all the first k smallest elements from a given array A of n distinct integers. Note that k is an input.

   (a) Comment on the time complexity if Sorting is used.

   (b) Give a $\theta(n)$ algorithm to solve this problem along with time complexity analysis. 3+(4+3)

5. (a) Given vertices u and v in a flow network, where capacity c(u, v) = 5 and c(v, u) = 8, suppose that 3 units of flow are shipped from u to v and 4 units are shipped from v to u. Compute the net flow from u to v by giving a diagram.

   (b) After a devastative flood in a region, the food needs to be supplied to m different locations $R = \{r_1, r_2, r_3,...r_m\}$. The food distribution centers have been opened in a n number of places. Let the locations are represented by $L = \{l_1, l_2, l_3,...l_n\}$. The organizer knows how much supply of food can be passed through the road network N = (V, E), which are still accessible. Here V is the set of road junctions and E is the set of road segment between junctions. The amount of food supplies at every $l_i$, and the food requirement in every affected location $r_j$ is also known. Design a flow network along with the algorithm to help the organizer to decide supply from which location should go to which affected location and whether the amount of food supplies is enough to satisfy the demands at all locations. 3+7

6. (a) Give the randomized algorithm for finding the *k*th smallest element.

   (b) Derive the expected time complexity. 4+6

7. (a) Define the 3SAT and Vertex cover decision problem.

   (b) Show that 3SAT is polynomial time reducible to Vertex cover. 3+7

# 2023

## COMPUTER SCIENCE AND ENGINEERING

**Paper : CSCL-0901**

**(Topics in Algorithms)**

**Full Marks : 70**

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

Answer ***question nos.*** **1** and **2**, and ***any four*** questions from the rest.

**1.** Answer ***any five*** questions : 2×5

(a) Define the approximation ratio for a maximization problem.

(b) Compute the prefix function for the pattern ABBBACABB in a string-matching algorithm.

(c) State and prove the time complexity of a range search [a:b] in a randomized skip list.

(d) How to use randomization to find the k-th smallest element from an array with linear expected time complexity?

(e) Why do we compute expected case time complexity for randomized algorithms?

(f) Define the universal hash function.

(g) Define complexity classes NP and NP-complete.

**2.** Answer ***any five*** questions : 4×5

(a) Let $n$ be a positive integer. Design an efficient randomized algorithm that generates a random permutation of the integers 1, 2, ..., n. Assume that you have access to a fair coin. Analyze the time complexity of your algorithm.

(b) Derive the expected case time complexity of the search in the Skip list.

(c) Let f and f′ be two feasible (s, t)-flows in a flow network G, such that $|f'| > |f|$. Prove that there is a feasible (s. t)-flow with value $|f'|-|f|$ in the residual network Gf.

(d) Prove that any comparison-based sorting algorithm in an array A of n numbers must take $\Omega$ (n log n) running time in the worst case.

(e) Show that the satisfiability problem, 3SAT is polynomial time reducible to the Clique problem.

(f) Prove the approximation bound for the k-centre problem.

(g) Prove that if problem P1 is reducible to P2 in polynomial time, and P2 is reducible to P3 in polynomial time, then P1 is reducible to P3 in polynomial time.

3. (a) Define the String Matching Problem.

   (b) Give the naïve string matching algorithm and find its time complexity.

   (c) Suppose that all characters in the pattern P are different. Show how to accelerate the naïve-string matching algorithm to run in time O(n) on an n-character text T. 2+4+4

4. (a) Define flow network.

   (b) In a hospital, there are 'n' doctors and 'm' patients. Each doctor can examine a certain number of patients in a day. The hospital management would like to serve a maximum number of patients, utilizing the capacity of the doctors. Give an efficient algorithm to serve a maximum number of patients with time complexity analysis. 3+7

5. (a) State the vertex cover problem and comment on the complexity class of this problem.

   (b) Give an approximation algorithm to solve the vertex cover problem and derive the approximation bound. 3+(3+4)

6. (a) Define the closest pair problem.

   (b) Briefly discuss how randomization is used in the closest pair problem.

   (c) Derive the time complexity of the randomized closest pair algorithm. 2+4+4

7. Suppose we allow a pattern P to contain occurrences of a gap character '<>' that can match an arbitrary string of characters (one of zero length). For example, the pattern ab<>ba<> c occurs in the text 'cabccbacbacab' as 'cab<>ba<>cab'.

   Give a polynomial-time algorithm to determine whether such a pattern P occurs in a given text T, and analyze the running time of your algorithm. 10

8. (a) Define the bin packing problem.

   (b) Give the first-fit approximation algorithm for it.

   (c) State the approximation ratio for first-fit to solve a bin-packing problem and prove the approximation bound. 2+4+4

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**M. Tech, Sem I, Mid Semester Examination 2, Total marks: 20 Duration: 1 hour**

**1. Answer any *four* questions from the following:** **[2 x 4= 8]**

a) Define P and NP class.

b) State true or false with justification: a polynomial number of calls to polynomial-time subroutines may result in an exponential-time algorithm.

c) Define the prefix function used in finite automata based string matching.

d) Define the suffix function used in KMP algorithm for string matching.

e) Differentiate between randomized and deterministic algorithm.

**2. Answer any *three* questions from the following:** **[4 x 3= 12]**

a) Prove that search in a skip list is O(log n) on expectation.

b) Prove that SAT is polynomial time reducible to Vertex cover Problem.

c) In randomized closest pair algorithm, how is randomization used? State and prove the expected time complexity of randomized closest pair algorithm.

d) Why do you compute expected running time for any randomized algorithm ? Does randomization improves efficiency for Quicksort and Selection Algorithm? Justify your answer.

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**M. Tech, Semester I, Mid Semester Examination 2** **Total marks: 20** **duration: 1 hour**

**1. Answer any *four* questions from the following:** **[2 x 4= 8]**

a) State TRUE OR FALSE: Let G be an arbitrary flow network, with a source s, a sink t, and a positive integer capacity $c_e$ on every edge e. If f is a maximum s-t flow in G, then f saturates every edge out of s with flow (i.e., for all edges e out of s, we have $f(e) = c_e$).

b) Give the Dynamic Programming formulation for the all pair shortest path problem.

c) If n element input to the Select algorithm ( finding kth smallest element) is already sorted and I want to find the $n/4^{th}$ smallest element, after how many recursions will you get the n/4 th smallest element?

d) Give the reduction of circulation problem to network flow problem.

e) What is the total number of paranthesization possible for a sequence of 5 matrices $A_1$, $A_2$,$A_3$,$A_4$, $A_5$?

**2. Answer any *three* questions from the following:** **[4 x 3= 12]**

a) i) Given a Flow Network G(V,E), where V = {s,u,v,t} and edge e = (x,y, $c_e$) in E as { (s,u,1), (s,v,1), (u,t,1),(v,t, 1), (u,v,1)}. List all the minimum s-t cuts in the flow network.

ii) Prove that minimum s-t cut is maximum flow in the graph.

b) I) Rewrite the divide and conquer closest pair algorithm such that in the combined step every point to the left of vertical line L is compared with every points to the right of L.

ii) Express the time complexity of the modified closest pair algorithm in i) by a recurrence relation and give the time complexity.

c) A contiguous subsequence of a list S is a subsequence made up of consecutive elements of S. for example if S is 5,15,-30,10,-5,40,10, then 15,-30,10 is contiguous subsequence but 5,15,40 is not. You are given a list of numbers $a_1$,$a_2$, …, $a_n$ , formulate a dynamic programming formulation to compute a contiguous subsequence whose sum is maximum. For the example, 10, -5, 40, 10 with sum 55 is maximum.

d) Show the steps of divide and conquer algorithm to multiply two long binary integers 10011011 and 10111010.

# 2024

## COMPUTER SCIENCE AND ENGINEERING

**Paper : CSC-902**

**(Advanced Algorithms)**

**Full Marks : 70**

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

Answer ***question nos.*** **1 & 2,** and ***any four*** questions from the rest.

**1.** Answer ***any five*** questions : 2×5

(a) Define the Universal hash function.

(b) Find the number of possible parenthesization of matrix chain multiplication for the sequence of matrices $<M_1, M_2, M_3, M_4>$.

(c) Let $L_1$ and $L_2$ be two problems such that $L_1 \leq_p L_2$. Suppose that problem $L_2$ can be solved in $O(n^k)$ time and the reduction can be done in $O(n^j)$ time. Derive the time complexity of solving $L_1$.

(d) Prove the approximation bound of the bin-packing problem.

(e) Define *P*, *NP* complexity class.

(f) Derive the time complexity of the divide and conquer strategy-based algorithm to solve the closest pair algorithm.

(g) Define flow in flow network.

**2.** Answer ***any five*** questions : 4×5

(a) Given a flow network $G = (V, E)$, where $V = \{s, u, t, v\}$ and $E = \{(s, u, 2), (u, t, 4), (s, v, 4), (v, t, 2)\}$. Note that $(s, u, 2)$ means the capacity of edge $(s, u)$ is 2. What is the maximum flow in the flow network $G$ ? Show the steps of your solution.

(b) Give the dynamic programming formulation of all pair shortest path problems. State the time complexity of this algorithm.

(c) Suppose that flow network $G$ contains an edge $(u, v)$ and we create a new flow network $G'$ by creating a new vertex $x$ and replacing $(u, v)$ with new edges $(u, x)$ and $(x, v)$ with capacity $c(u, x) = c(x, v) = c(u, v)$. Does a maximum flow in $G'$ have the same value as the maximum flow in $G$? Justify your answer.

(d) Differentiate between the Finite state machine-based string matching algorithm and the KMP string matching algorithm.

(e) Give a polynomial time reduction algorithm to reduce the maximum bipartite matching problem to the maximum flow problem.

(f) Derive the expected search time complexity of the skip list.

(g) Derive the time complexity of randomized quicksort.

**3.** (a) Derive the expected running time of RANDOMIZED-SELECT.

(b) Suppose we use RANDOMIZED-SELECT to select the minimum element of the array $A = \{3, 2, 9, 0, 7, 5, 4, 8, 6, 1\}$. Describe a sequence of partitions that results in a worst-case performance of RANDOMIZED-SELECT. 5+5

**4.** (a) State and prove the Max-flow Min-cut Theorem.

(b) Give the time complexity of the Ford-Fulkerson Algorithm that finds the maximum flow in a flow network with justification.

(c) Give an instance of a flow network where the Ford-Fulkerson Algorithm yields the maximum number of iterations. 4+3+3

**5.** (a) Define the approximation ratio for a minimization problem for an approximation algorithm.

(b) Define $k$-center problem.

(c) Give an approximation algorithm to solve the $k$-center problem and derive the approximation bound. 2+2+(2+4)

**6.** (a) Define the closest pair problem.

(b) Briefly discuss how randomization is used in the closest pair problem.

(c) Derive the time complexity of the randomized closest pair algorithm. 2+4+4

**7.** (a) Generate the state-transition table for a string-matching automaton for the pattern *ababbabbababbababbabb* over the alphabet $\{a,b\}$.

(b) Generate the prefix function used in KMP Algorithm for the pattern *ababbabbababbababbabb* over the alphabet $\{a,b\}$. 5+5

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