

CLASS 10/05/2021

Q) Consider a universal relation

$R = \{A, B, C, D, E, F, G, H, I, J\}$ and the set of FD

$F = \{AB \rightarrow C, A \rightarrow DE, B \rightarrow F, F \rightarrow GH, D \rightarrow IJ\}$. What is the key of R? Decompose R into 2NF and 3NF relations.

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$(AB)^+ = \{AB\}$

$= \{ABC\} \quad AB \rightarrow C$

$= \{ABCDE\} \quad A \rightarrow DE$

$= \{ABCDEF\} \quad B \rightarrow F$

$= \{ABCDEFGH\} \quad F \rightarrow GH$

$= \{ABCDEFGHIJ\} \quad D \rightarrow IJ$

$= R$

So AB is the candidate key.

As the relation R contains candidate key so R is in 1NF.

2NF?

AB is candidate key and also $A \rightarrow DE$ exist. As A is a subset of candidate key AB so $A \rightarrow DE$ is a partial dependency.

AB is candidate key and also $B \rightarrow F$ exist. As B is a subset of candidate key AB so $B \rightarrow F$ is partial dependency.

Hence R is not in 2NF.

To remove the partial dependency $A \rightarrow DE$, Find

$(A)^+ = \{A\}$

$= \{ADEIJ\}$ Rule $A \rightarrow DE, D \rightarrow IJ$

To remove the partial dependency $B \rightarrow F$, Find

$(B)^+ = \{B\}$

$= \{BFGH\}$ Rule $B \rightarrow F, F \rightarrow GH$

We remove PD $A \rightarrow DE$ by creating a relation for $(A)^+$

$R1 = (A)^+ = \{ADEIJ\}$ with FD set $F1 = \{A \rightarrow DE, D \rightarrow IJ\}$

with candidate key A.

We remove PD $B \rightarrow F$ by creating a relation for $(B)^+$

$R2 = (B)^+ = \{BFGH\}$ with FD set $F2 = \{B \rightarrow F, F \rightarrow GH\}$

And a third relation

$R_3 = \{R - (R_1 \cup R_2)\} \cup AB = \{ABC\}$ with FD set $F_3 = \{AB \rightarrow C\}$

Hence R_1, R_2, R_3 is in 2NF.

3NF?

- For relation R_1

$R_1 = \{ADEIJ\}$ with FD set $F_1 = \{A \rightarrow DE, D \rightarrow IJ\}$

We have $A \rightarrow D$ a FD as A is primary key and D is a non prime attribute.

And also we have $D \rightarrow IJ$. Hence IJ is transitively dependent on primary key A . Hence R_1 is not in 3NF.

- For relation R_2

$R_2 = (B)^+ = \{BFGH\}$ with FD set $F_2 = \{B \rightarrow F, F \rightarrow GH\}$

And also we have $B \rightarrow F$. Hence GH is transitively dependent on primary key B . Hence R_2 is not in 3NF.

- For relation R_3

$R_3=\{ABC\}$ with FD set $F_3=\{AB \rightarrow C\}$. There is no transitive dependency hence it is in 3NF.

To convert relation R_1 in 3NF we are removing $D \rightarrow IJ$ which is causing transitivity,

$R_{11}=(D)^+=\{DIJ\}$ with FD set $F_{11}=\{D \rightarrow IJ\}$

$R_{12}=(R_1-R_{11}) \cup D=\{AED\}$ with FD set $F_{12}=\{A \rightarrow DE\}$

To convert relation R_2 in 3NF we are removing $F \rightarrow GH$ from R_2 .

$R_{21}=(F)^+=\{FGH\}$ with FD set $F_{21}=\{F \rightarrow GH\}$

$R_{22}=(R_2-R_{21}) \cup F=\{BF\}$ with FD set $F_{22}=\{B \rightarrow F\}$

Hence the 3NF decomposition of R is

- $R_{11}=(D)^+=\{DIJ\}$ with FD set $F_{11}=\{D \rightarrow IJ\}$
- $R_{12}=(R_1-R_{11}) \cup D=\{AED\}$ with FD set $F_{12}=\{A \rightarrow DE\}$
- $R_{21}=(F)^+=\{FGH\}$ with FD set $F_{21}=\{F \rightarrow GH\}$
- $R_{22}=(R_2-R_{21}) \cup F=\{BF\}$ with FD set $F_{22}=\{B \rightarrow F\}$
- $R_3=\{ABC\}$ with FD set $F_3=\{AB \rightarrow C\}$.

General 2NF

A relation is in 2NF if every non-key attribute is fully functionally dependent on all candidate keys.

General 3NF

A relation is in 3NF if every functional dependency of the form $X \rightarrow A$ where X is a set of attributes and A is a single attribute then

- either X is a super key
- or A is a prime attribute

- Example1: $R_{11} = (D) \twoheadrightarrow \{D, I, J\}$ with FD set $F_{11} = \{D \twoheadrightarrow I, J\}$

Is it in 3NF?

Primary key D .

$D \twoheadrightarrow I, J$ can be written as

$D \twoheadrightarrow I$

Left hand is superkey

$D \rightarrow J$

Left hand is superkey

Hence R11 is in 3NF.

- Example2: $R=(ABC)$ with FD set $F=\{AB \rightarrow C, C \rightarrow B\}$

Is it in 3NF?



Here primary key AB.

For $AB \rightarrow C$,

left hand side is super key.

$C \rightarrow B$

right hand side B is prime attribute .

Hence R is in 3NF.

Boyce Codd Normal Form(BCNF)

A relation is in BCNF if every functional dependency of the form $X \rightarrow A$ where X is a set of attributes and A is a single attribute then

- X must be a super key
- Example1: $R_{11} = (D) \rightarrow \{DIJ\}$ with FD set $F_{11} = \{D \rightarrow IJ\}$

Is it in BCNF?

Primary key D .

$D \rightarrow IJ$ can be written as

$D \rightarrow I$

$D \rightarrow J$

For both the FD LHS is superkey. Hence R_{11} is in BCNF.

- Example2: $R = (ABC)$ with FD set $F = \{AB \rightarrow C, C \rightarrow B\}$

Is it in BCNF?

Primary key AB.

For FD $AB \rightarrow C$, LHS AB is superkey

For FD $C \rightarrow B$. LHS C is not superkey.

Hence it is violating BCNF. Hence R is not in BCNF.

BCNF decomposition

- Let us consider there is a relation R
- A FD $X \rightarrow Y$ is violating BCNF property

Decompose R into

$R_1 = (XUY)$

$R_2 = (R - Y)$

For last example:

$C \rightarrow B$ is violating BCNF.

Hence $R_1 = (CB)$ with FD set $= \{C \rightarrow B\}$

and $R_2 = (AC)$ with AC is primary key.

So we have lost the FD $AB \rightarrow C$.

So this decomposition is not dependency preserving.

- **Lossless Join Decomposition or Non additive decomposition**

Definition: A decomposition $D=\{R_1, R_2, R_3, \dots, R_n\}$ of a relation schema R is lossless w.r.t a set of functional dependencies F on R if every relation r of schema R , the following holds,

$$\Pi_{R_1}(r) * \Pi_{R_2}(r) * \Pi_{R_3}(r) * \dots * \Pi_{R_n}(r) = r$$

* represents natural join.

Testing Lossless decomposition into more than 2 relations

Example:

$R=\{ssn, ename, pnumber, pname, plocation, hours\}$

There is a decomposition $D=\{R_1, R_2, R_3\}$

$R_1=EMP=\{ssn, ename\}$

$R_2=PROJ=\{pnumber, pname, plocation\}$

$R_3=Works_ON=\{ssn, pnumber, hours\}$

FD set $F=\{ssn \rightarrow ename,$

$pnumber \rightarrow pname, plocation$

ssn pnumber \rightarrow hous}

Is D lossless?

	ssn	ename	pnumber	pname	plocation	hours
R1	a1	a2	b13	b14	b15	b16
R2	b21	b22	a3	a4	a5	b26
R3	a1	b32	a3	b34	b35	a6

Apply ssn \rightarrow ename

	ssn	ename	pnumber	pname	plocation	hours
R1	a1	a2	b13	b14	b15	b16
R2	b21	b22	a3	a4	a5	b26
R3	a1	a2	a3	b34	b35	a6

pnumber \rightarrow pname plocation

	ssn	ename	pnumber	pname	plocation	hours
R1	a1	a2	b13	b14	b15	b16
R2	b21	b22	a3	a4	a5	b26
R3	a1	a2	a3	a4	a5	a6

As the last row contains all a symbols hence the decomposition is lossless.

Testing Lossless decomposition into 2 relation

- A decomposition $D=\{R_1, R_2\}$ of R is lossless w.r.t a set of FD F on R iff
 - either $(R_1 \cap R_2) \rightarrow (R_1 - R_2)$ is in F^+
 - or $(R_1 \cap R_2) \rightarrow (R_2 - R_1)$ is in F^+

Example:

$R=(ABC)$ FD set $F=\{AB \rightarrow C, C \rightarrow B\}$

Decomposition $D=\{R_1, R_2\}$ and $R_1=(CB), R_2=(AC)$.

Test whether it is lossless?

$$R_1 \cap R_2 = (CB) \cap (AC) = C$$

$$R_1 - R_2 = B$$

$$R_2 - R_1 = A$$

Here $R_1 \cap R_2 \rightarrow (R_1 - R_2)$ i.e. $C \rightarrow B$ is in F

Hence this decomposition is lossless.