Linear algebra, why?

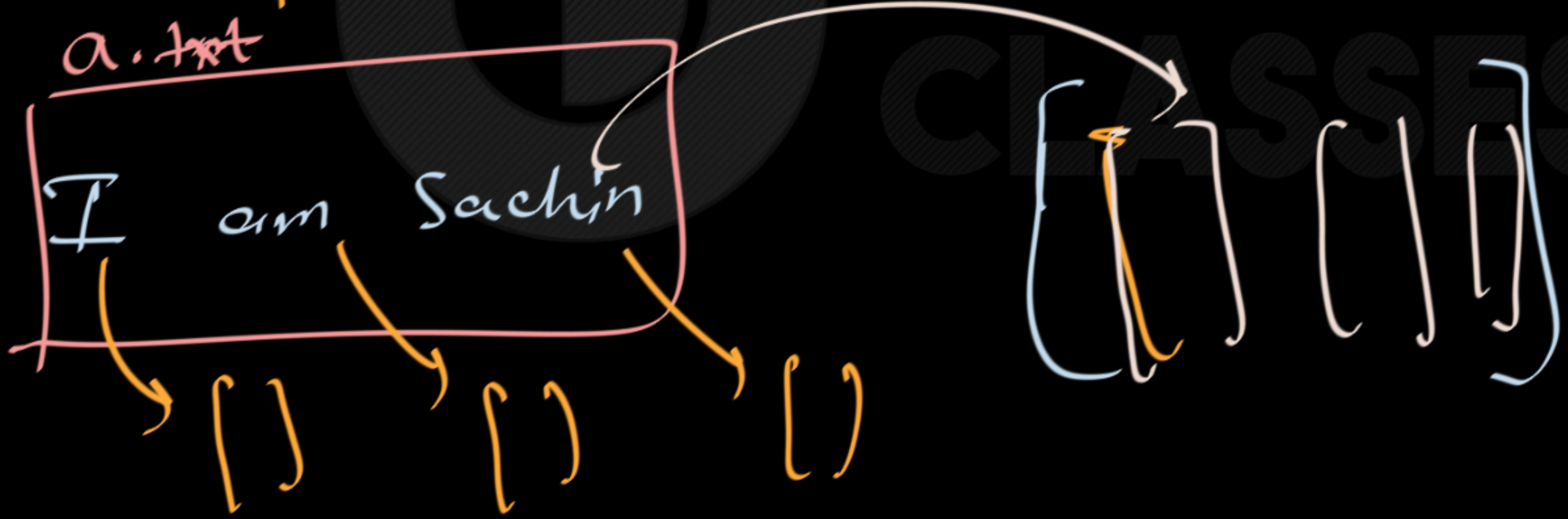
$\rightarrow$ GATE syllabus

Linear algebra, why?

→ GATE syllabus

→ helps in data interpretation

any data can be represented in the matrix.



$\Rightarrow$ LA + prob.

$\Downarrow$

ML

Chat GPT

Scalar

3

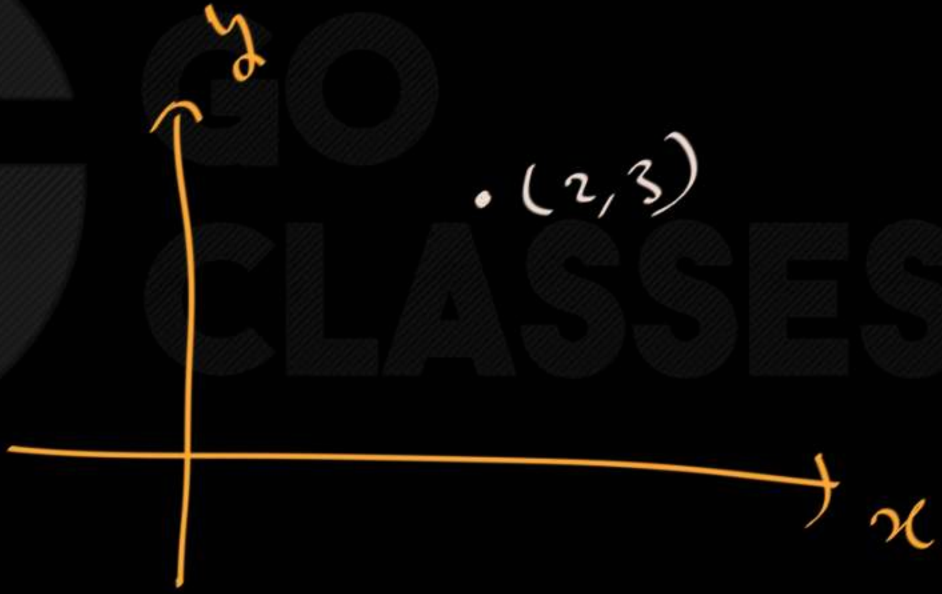
2

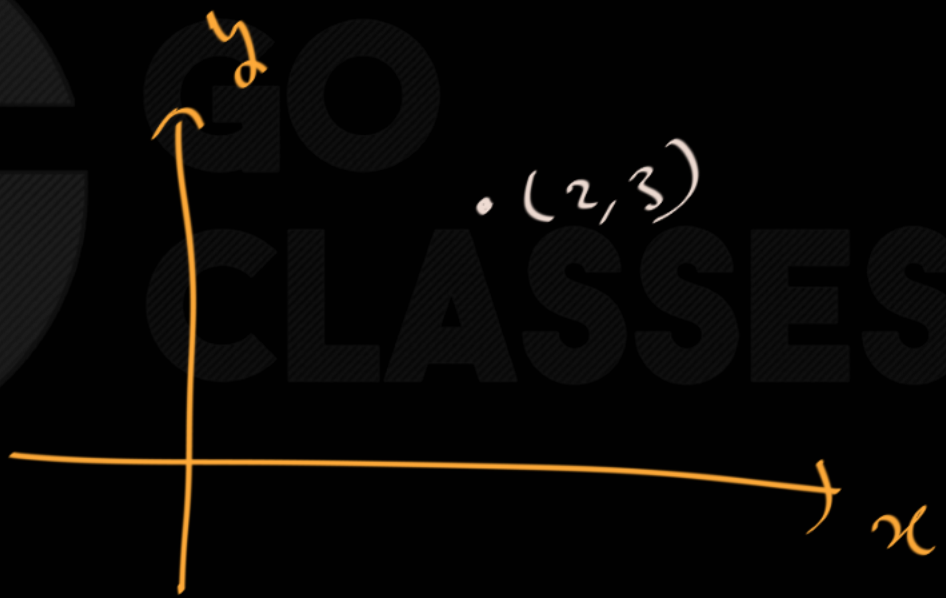
-5

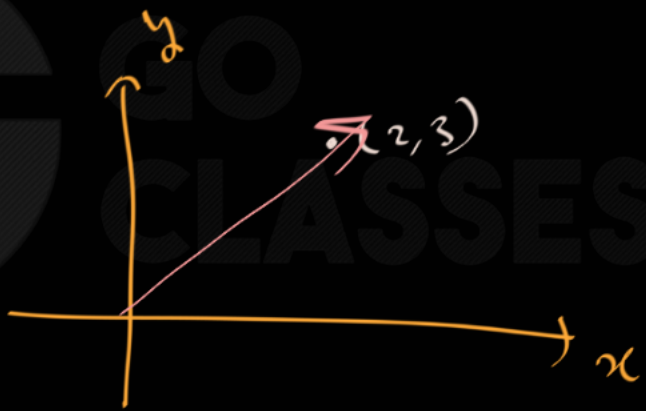
0

Vector

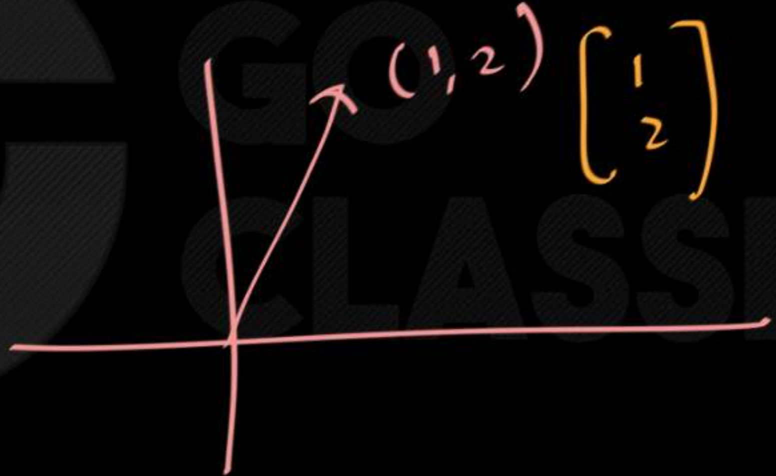
vector




y
(2,3)
x


y
(2,3)
x

$$\overbrace{\begin{bmatrix}1\\2\end{bmatrix}}^{\in R^2}$$



$$\begin{bmatrix}1\\2\end{bmatrix}$$

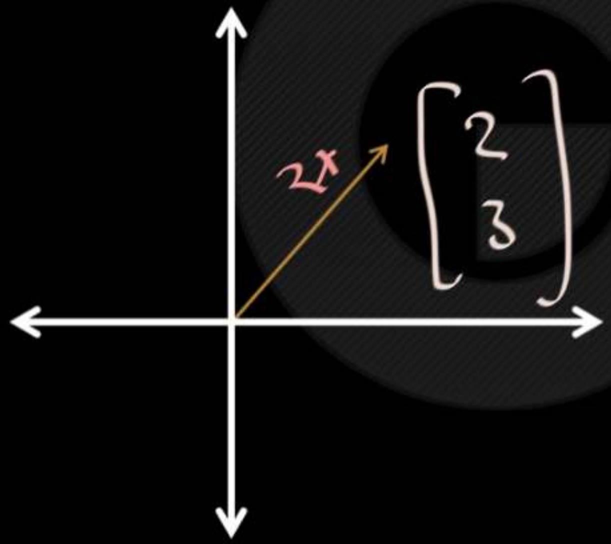
$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \in R^3$$

$$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \in R^3$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} \in R^n$$

x^n

Scalar times vector

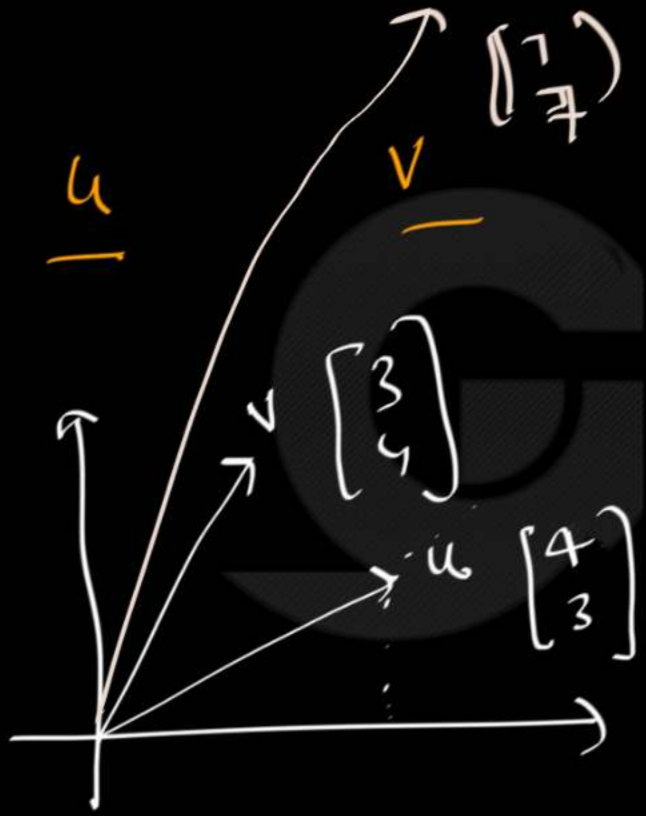


$$2\times\begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}4\\6\end{bmatrix}$$

Vector addition

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ 9 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 12 \end{bmatrix}$$

Vector addition



$$\begin{bmatrix} 4 \\ 3 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 7 \\ 7 \end{bmatrix}$$

Linear Combination of vectors

Linear Combination of vectors

$$3\begin{bmatrix}2\\3\\4\end{bmatrix} + 2\begin{bmatrix}1\\2\\3\end{bmatrix}$$

$$= \begin{bmatrix} \\ \\ \end{bmatrix}$$

linear combination

of $\begin{bmatrix}2\\3\\4\end{bmatrix}, \begin{bmatrix}1\\2\\3\end{bmatrix}$

Linear Combination of vectors

$$10\begin{bmatrix}2\\3\\4\end{bmatrix} - 105\begin{bmatrix}1\\2\\3\end{bmatrix}$$

$$= \begin{bmatrix} \\ \\ \end{bmatrix}$$

linear combination of $\begin{bmatrix}2\\3\\4\end{bmatrix}, \begin{bmatrix}1\\2\\3\end{bmatrix}$

Class 10^{th}

$$1\begin{bmatrix}1\\0\\3\end{bmatrix}+4\begin{bmatrix}1\\2\\1\end{bmatrix}-2\begin{bmatrix}2\\3\\-1\end{bmatrix}=?\quad\begin{pmatrix}1\\2\\9\end{pmatrix}$$

We will flip this question, Soon

$$? \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} + ? \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + ? \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix}$$

Vectors in $R^2, R^3, \ldots \ldots, R^n$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} 3 \\ 9 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} \vdots \\ \vdots \\ \vdots \end{bmatrix}$$

Vectors in $R^2, R^3, \ldots\ldots, R^n$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \longrightarrow \text{dim of this vector is } 2$$

Vectors in $R^2, R^3, \ldots \ldots, R^n$

$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ → dim of this vector is 2

$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ ← dim of this vector ?

$\underline{\underline{R^3}}$ 3

$$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \longleftarrow \text{dim of this vector ?}$$

$$R^3$$

$$\begin{bmatrix}1\\2\\0\end{bmatrix} \longleftarrow \text{dim of this vector ?}$$

$= 3$

This gentle introduction to scalars and vectors seems simple, but you may be surprised to learn that nearly all of linear algebra is built up from scalars and vectors. From humble beginnings, amazing things emerge. Just think of everything you can build with wood planks and nails. (But don't think of what *I* could build — I'm a terrible carpenter.)

Linearly Dependent vectors

Linearly Dependent vectors

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad 9 \quad \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

Linearly Dependent vectors

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

linearly dependent vecters

$$\begin{bmatrix} 2 \\ 4 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\left\{ \begin{bmatrix} 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \end{bmatrix} \right\}$$ are $\underline{\underline{\text{LD}}}$?

$$\left\{ \begin{bmatrix} 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \end{bmatrix} \right\}$$ are LD ?

Not linearly dependent

$$\begin{bmatrix} 7 \\ 2 \end{bmatrix} = \text{k} \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

One important thing to know about linear independence before reading the rest of this section is that independence is a property of a *set* of vectors. That is, a set of vectors can be linearly independent or linearly dependent; it doesn't make sense to ask whether a single vector, or a vector within a set, is independent.

$$\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$ is a 1D, a vector

$\{ \quad , \quad , \quad \}$

$$\left\{ \begin{array}{ccc} u & v & w \\ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, & \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}, & \begin{bmatrix} 3 \\ 6 \\ 10 \end{bmatrix} \end{array} \right\}$$

$$\left\{ \begin{array}{ccc} u & v & w \\ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, & \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}, & \begin{bmatrix} 3 \\ 6 \\ 10 \end{bmatrix} \end{array} \right\}$$

$$\begin{bmatrix} 3 \\ 6 \\ 10 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 1 \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}$$

$\boxed{u, v, w \quad \text{are} \quad L.D.}$

$$\left\{ \begin{array}{ccc} u & v & w \\ \begin{bmatrix}1\\2\\3\end{bmatrix}, & \begin{bmatrix}2\\4\\7\end{bmatrix}, & \begin{bmatrix}3\\6\\10\end{bmatrix} \end{array} \right\}$$

$$\begin{bmatrix}3\\6\\10\end{bmatrix} = 1\begin{bmatrix}1\\2\\3\end{bmatrix} + 1\begin{bmatrix}2\\4\\7\end{bmatrix}$$

$\boxed{u, v, w \text{ are LD}}$

these 3 vectors are LD.

$$\left\{ \begin{array}{ccccc} u & & v & & w \\ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} & , & \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix} & , & \begin{bmatrix} 3 \\ 6 \\ 10 \end{bmatrix} \end{array} \right\}$$

$$\begin{bmatrix} 3 \\ 6 \\ 10 \end{bmatrix} - \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\left\{ \begin{array}{ccccc} u & & v & & w \\ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} & , & \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix} & , & \begin{bmatrix} 3 \\ 6 \\ \underline{9} \end{bmatrix} \end{array} \right\}$$

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$$\left\{ \begin{array}{ccc} u & v & w \\ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, & \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}, & \begin{bmatrix} 3 \\ 6 \\ \underline{9} \end{bmatrix} \end{array} \right\}$$

$$\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 0 \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} \ \end{bmatrix} = 3\begin{bmatrix} \ \end{bmatrix} + 0\begin{bmatrix} \ \end{bmatrix}$$

Yes. Cofficients could be zero or non zero.

$$\left\{ \overset{u}{\begin{bmatrix}1\\2\\3\end{bmatrix}}, \overset{v}{\begin{bmatrix}2\\4\\7\end{bmatrix}}, \overset{w}{\begin{bmatrix}3\\6\\9\end{bmatrix}} \right\}$$

$$\begin{bmatrix}3\\6\\9\end{bmatrix} = 3\begin{bmatrix}1\\2\\3\end{bmatrix} + 0\begin{bmatrix}2\\4\\7\end{bmatrix}$$

↳ if i can repres. Just one of the vector as a linear comb. of other vectors then the set is L.D.

CLASSES

$$\left\{ \overset{u}{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}},\ \overset{v}{\begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}},\ \overset{w}{\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}} \right\}$$

at least one vector

$$w = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 0 \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}$$

$w =$

$$u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} + 0 \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}$$

$u =$

$v =$

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$\{ u, v, w, p, q, r \}$

Shubhodeep Chanda w =0u +0v

if we can represent $\boxed{\text{at least}}$ one vector as a linear comb. of other vectors then this set is LD.

AIMT - 1 question

$$\left\{ \begin{array}{ccc} u & v & w \\ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, & \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}, & \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \end{array} \right\}$$

Do you have ATLEAST one vector

$$\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 0 \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}$$

$$\left\{ \begin{array}{ccc} u & v & w \\ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, & \begin{bmatrix} 2 \\ 4 \\ 9 \end{bmatrix}, & \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{array} \right\}$$

Shubhodeep Chanda w =0u +0v

this set is a linearly dependent set?

$$\begin{array}{ccc} u & v & w \\ \begin{bmatrix}1\\2\\3\end{bmatrix}, & \begin{bmatrix}2\\4\\9\end{bmatrix}, & \begin{bmatrix}0\\0\\0\end{bmatrix} \end{array}$$

zero vector

$$\begin{bmatrix}0\\0\\0\end{bmatrix} = 0\begin{bmatrix}1\\2\\3\end{bmatrix} + 0\begin{bmatrix}2\\4\\9\end{bmatrix}$$

u, v, w this set of

vector are L.D iff

there is ATLEAST one vector s.t. the

vector is linear combination of other

vectors

coefficients could be anything

T / F

a set containing zero vector is always L.D.

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0\begin{bmatrix} \\ \\ \end{bmatrix} + 0\begin{bmatrix} \\ \\ \end{bmatrix}$$

Shikhar Mutta sir aap bhot aacha padhate ho..... Thank you so much for giving us incredible knowledge....
Ankit Singh T
stockkk yes sir
Amlan Majumdar true
Avni Singh t
Akash Debnath True
17-504 RAJESH t
Vijay Raj yes
Shyam t
Anamika Mishra true
Debarghya Adhikari t

T / F $\rightarrow$ TRUE

a set containing zero vector is always L.D.

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \underline{0}\begin{bmatrix} \vdots \end{bmatrix} + \underline{0}\begin{bmatrix} \vdots \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 1 \begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}$$

Linear Dependence

- At least one vector can be obtained by linear combination of other vectors

$$v_1 = c_2 v_2 + \cdots + c_n v_n$$

(or)

- If we **can** obtain zero vector by **nontrivial** (atleast one c_i is non-zero) linear combination of other vectors

$$c_1 v_{1+} c_2 v_2 + \cdots + c_n v_n \ = 0$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \quad , \quad \begin{bmatrix} 4 \\ 8 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix}1\\2\\3\end{bmatrix} \quad \begin{bmatrix}3\\6\\9\end{bmatrix}, \quad \begin{bmatrix}4\\8\\10\end{bmatrix}$$

$$\begin{bmatrix}3\\6\\9\end{bmatrix} = 3\begin{bmatrix}1\\2\\3\end{bmatrix} + 0\begin{bmatrix}4\\8\\10\end{bmatrix}$$

we can NOT represent $\begin{bmatrix} 4 \\ 8 \\ 10 \end{bmatrix}$ as linear combination of other vectors

$$\underbrace{\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 0\begin{bmatrix} 4 \\ 8 \\ 10 \end{bmatrix}}$$

$$v_1 = c_2 v_2 + c_3 v_3 + c_4 v_4 + \cdots c_n v_n$$

Can i represent v_2 as linear combination of other vectors?

$\Rightarrow$

$$v_1 = c_2 v_2 + c_3 v_3 + c_4 v_4 + \cdots c_n v_n$$

Can i represent v_2 as linear combination of other vectors?

$\Rightarrow$ if c_2 is not zero then YES

T/F

if a set of vectors are L. D.

then there's ATLEAST one vector which can

be represented by a linear combination

of other vectors

TRUE

T/F

if a set of vectors are L. D.

then ALL vectors can

be represented by a linear combination

of other vectors

false

T/F

if a set of vectors are <u>L. D.</u>

then ALL vectors can

be represented by a linear combination

of other vectors

Same in GATE PYQ

with WRONG interpretation

Linearly dependent

$\{u, v, w\}$

able to represent any of the vector as linear combination of other vectors

Linearly dependent

$\{u, v, w$

Not correct

able to represent any of the vector as linear combination of other vectors

Linearly dependent

correct

$\{u, v, w\}$

able to represent At least one of the vector as linear combination of other vectors

GO Classes Test Series 2023 | Mock GATE | Test 6 | Question: 46

6

Let u_i's be vectors in R^n for $i = 1, 2, 3, 4$.

A1 MT-1

Which of the following options is/are $\mathrm{CORRECT}$?

A. If $\{u_1, u_2, u_3\}$ is linearly dependent, so is $\{u_1, u_2\}$. → false

B. If u_4 is not a linear combination of $\{u_1, u_2, u_3\}$, then $\{u_1, u_2, u_3, u_4\}$ is linearly independent.

C. Any set containing the zero vector is linearly dependent.

D. If $\{u_1, u_2, u_3\}$ is linearly dependent, so is $\{u_1, u_2, u_3, u_4\}$.

goclasses2023-mockgate-6 goclasses linear-algebra vector-space multiple-selects

moderate 2-marks

A. If $\{u_1, u_2, u_3\}$ is linearly dependent, so is $\{u_1, u_2\}$.

$$\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\left\{ \begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}2\\4\\7\end{bmatrix}, \cancel{\begin{bmatrix}0\\0\\0\end{bmatrix}} \right\}$$

$u_2 = 2u_1$

B. If u_4 is not a linear combination of $\{u_1, u_2, u_3\}$, then $\{u_1, u_2, u_3, u_4\}$ is linearly independent.

$u_4 \neq u_1 + u_2 + u_2$

$u_2 = 2u_1 + 0u_3 + 0u_4$

if a subset is LD then
the superset is LD

given u_1, u_2, u_3 LD

$u_2 = \_\_ u_1 + \_\_ u_3$

if a subset is LD then the superset is LD

u_1, u_2, u_3, u_4 LD

$u_2 = \bigcirc u_1 + \bigcirc u_3 + 0 u_4$

TRUE or FALSE: If $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is linearly dependent, so is $\{\mathbf{u}_1, \mathbf{u}_2.\}$

(c) TRUE or FALSE: If $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is linearly dependent, so is $\{\mathbf{u}_1, \mathbf{u}_2.\}$

This is false in general. Consider the set $\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$, from which you can take out the zero vector and end up with two linear independent vectors.

(d) TRUE or FALSE: If $\mathbf{u}_4$ is *not* a linear combination of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$, then $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is linearly independent.

This is false: Any one of the other vectors could be the zero vector, for example.

Linear Independence

Linear Independence

a set of vectors is said to be linearly independent

- If we **can** obtain zero vector **only** by **trivial** linear combination of other vectors

$$c_1 v_{1+} c_2 v_2 + \cdots + c_n v_n = 0 \; \boldsymbol{iff} \; c_i = 0 \; \forall i$$

$$\left[c_1 v_1 + c_2 v_2 + c_3 v_3 + \cdots c_n v_n = 0 \right]$$

$c_1 \neq 0$

$$v_1 = \quad v_2 + \quad v_3 + \cdots$$

$$\left\{ c_1 v_1 + c_2 v_2 + c_3 v_3 + \cdots c_n v_n = 0 \right]$$

$c_1 \neq 0$

$$v_1 = \quad v_2 + \quad v_3 + \cdots$$

at least one of the $c_i \neq 0$

$v_i =$

Can you represent

[1]

LD

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$$\left\{ \bigcirc u + \overset{1/2}{\bigcirc} v + \bigcirc w + \bigcirc x + \bigcirc y + \bigcirc z = 0 \right]$$

is this set LD ?

Imp.

$$c_1 v_1 + c_2 v_2 + c_3 v_3 + \cdots\cdots c_n v_n = 0$$

can you give me at least one $c_i \neq 0$

Yes → 1 LP

NO → 2 LI

all of the $c_i = 0$

Imp.

$c_1 v_1 + c_2 v_2 + c_3 v_3 + \cdots c_n v_n = 0$

$\square =$ ___ can you give me atleast one $c_i \neq 0$

Yes → LD

NO → LI

$$\overset{\to 0}{c_1} v_1 + \overset{\to 0}{c_2} v_2 + \overset{\to 0}{c_3} v_3 + \cdots \overset{\to 0}{c_n} v_n = 0$$

these set of vectors are LI

T/F

Not telling anything.

a set of vectors $v_1, v_2 \cdots v_n$ satisfy

$$c_1 v_1 + c_2 v_2 + c_3 v_3 + \cdots c_n v_n = 0$$

where all $c_i = 0$.

false

then $\{v_1, v_2 \cdots v_n\}$ is Linearly indep.

T/F

a set of vectors $v_1, v_2 \ldots v_n$ satisfy

$$c_1 v_1 + c_2 v_2 + c_3 v_3 + \cdots c_n v_n = 0$$

only when all $c_i = 0$.

then $\{v_1, v_2 \ldots v_n\}$ is Linearly indep.

T/F

a set of vectors $v_1, v_2 \ldots v_n$ satisfy

$$c_1 v_1 + c_2 v_2 + c_3 v_3 + \cdots c_n v_n = 0$$

only when all $c_i = 0$.

$c_1 v_1 + c_2 v_2 + c_3 v_3 + \cdots c_n v_n = 0$

Do you have one non zero c_i ?

yes

no

$v_i =$

LT

$$c_1 v_1 + c_2 v_2 + c_3 v_3 + \cdots c_n v_n = 0$$

values of $c_1, c_2, \ldots$

the only sol^n to this equation is trivial $\Rightarrow$ linearly independent

$$c_1 v_1 + c_2 v_2 + c_3 v_3 + \cdots c_n v_n = 0$$

the sol^n to this equation is $c_i = 0$

$\Rightarrow$ we don't know

Linear Independence

a set of vectors are LI

iff they are NOT LD

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$$c_1 v_1 + c_2 v_2 + c_3 v_3 + \cdots c_n v_n = 0$$

मुझे ये बताओ कि

Non Zero है कि नहीं

(directly or indirectly)

$$\underbrace{c_1 v_1 + c_2 v_2 + c_3 v_3 + \cdots c_n v_n = 0}$$

only trivial combination is possible

zeros

L.I.

$$c_1 v_1 + c_2 v_2 + c_3 v_3 + \cdots c_n v_n = 0$$

Can you give me atleast one c_i such that $c_i \neq 0$

$\leftarrow c_1 \neq 0$

$$v_1 = -\frac{1}{c_1}\left(c_2 v_2 + c_3 v_3 + \cdots c_n v_n\right)$$

$$c_1 v_1 + c_2 v_2 + c_3 v_3 + \cdots c_n v_n = 0$$

$c_1 \neq 0$ $\quad v_1 =$ as linear comb. of other vectors.

$c_2 \neq 0$ $\quad v_2 \Rightarrow$ ✓

$$c_1 \overset{\to 0}{} v_1 + c_2 \overset{\to 0}{} v_2 + c_3 \overset{\to 0}{} v_3 + \cdots c_n \overset{\to 0}{} v_n = 0$$

(Do you have at least one $c_i \neq 0$)

sorry, i can not give any $c_i \neq 0$

all of the $c_i = 0$

these vectors are LI

if you can not represent any vector as a linear combination of other vectors $\Rightarrow$ Linearly indep.

$$\overset{\neq 0}{c_1} v_1 + \overset{\to 0}{c_2} v_2 + \overset{\to 0}{c_3} v_3 + \cdots \overset{\to 0}{c_n} v_n = 0$$

(Do you have at least one $c_i \neq 0$)

$$v_1 = \frac{1}{c_1} \left(\qquad \right)$$