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Counting Techniques - Part IV

Course on Engineering Mathematics for GATE - CSE

Rahul • Lesson 4 • June 19, 2024

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Engineering Mathematics

Probability & Statistics

counting principles

16-12

Lecture Number- 03

By- Rahul Sir

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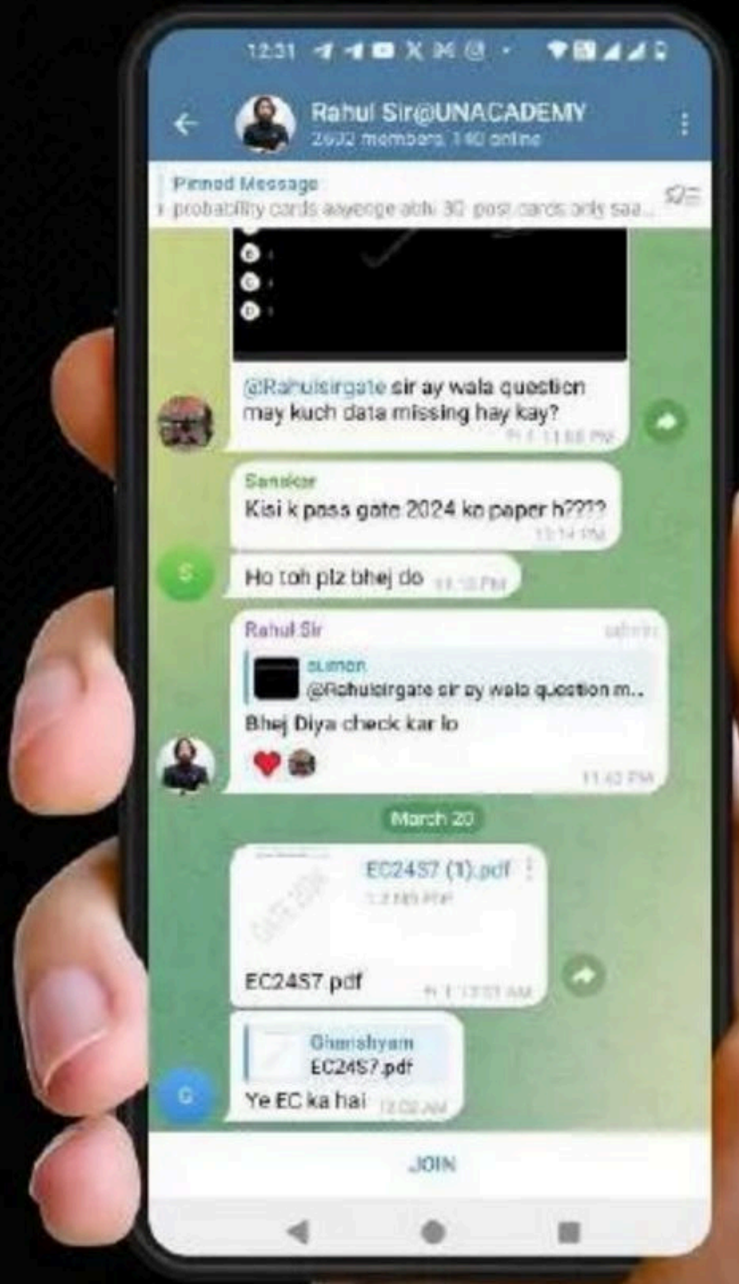

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Topics

to be covered

1. counting principles_III

✓ 10-12 Question

QUESTION

Illustration:

Q. How many 3 digit numbers can be formed by the digit 1, 2, 3, 4, 5 without repetition

QUESTION

Illustration:

Q. In an examination of 10 T/F question, How many sequence of answers are possible.

QUESTION

Illustration:

Q. 10 students complete in a swimming race. In how many ways can they occupy the first 3 positions.

QUESTION

Illustration:

Q. There are 7 flags of diffrent colour. Find the number of diffrent signals that can be transmitted by the use of 2 flags one above the other.

QUESTION

Illustration:

Q. A letter lock consists of 3 rings each marked with 10 different letters. In how many ways, its is possible to make an unsuccessful attempt to open the lock?

QUESTION

Illustration:

Q. How many 6 digits odd number greater than 6,00,000 can be formed from the digits 5,6,7, 8, 9, 0 if repetition of digit is allowed?

QUESTION

Illustration:

Q. How many integers greater than 5000 can be formed with the digit 7, 6, 5, 4 & 3 using each digit at most once.

QUESTION

Illustration:

Q. How many natural number less than 30000 can be made from the digits 0, 1, 2, 3, 4, 5, 6.

QUESTION

Illustration:

Q. Consider the word DAUGHTER. How many 4 letter word can be formed from the letter of above word so that each word contain letter G

QUESTION

Illustration:

Q. How many different words can be formed using all the letters in the word "MIRACLE".

(a) If vowels may accupy the even position.

(b) If vowels may occupy odd position.

QUESTION

Illustration:

Q. There are m men and n monkey. Number of ways in which every monkey has a master, if a man can have any number of monkey.

QUESTION

Illustration:

Q. Number of ways in which m different toys can be distributed in "n" children if every child may receive any number of toys

QUESTION

Illustration:

Q. Find the number of ways in which we can post 5 letters in 10 letter boxes.

QUESTION

Illustration:

Q. In a car plate number containing only 3 or 4 digits not containing the digit 0. What is the maximum numbers of cars that can be numbered?

QUESTION

Q. There are 6 single choice questions in an examination. How many sequence of answers are possible, if the first three questions have 4 choices each and the next three have 5 each?

A 15625

B 8000

C 4000

D 4096

Lot Type	Nu. of Permutation
n Diff. Items Taken all at a time (Repetation Not allowed	$n!$
n Diff. Items Taken r at a time.	^nP_r

n Diff. Items select all at a time

$= 1$

n Diff. Items Select 'r' at a time

$= {}^nC_r$

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n Diff. Items p alike, q alike, r alike.

✓ A A A A B B B B C C C C D D D E E — Taken all at a time

n Different

P alike q alike r alike s alike t alike.

✓ CALCULUS

V V

C C, L L, A S, V V

p alike r alike q alike

✓ ALLAHABAD

n Diff. { p alike, q alike, r alike }

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n Diff. Items p alike, q alike, r alike (Taken all at a time)

No. of Arrangement / No. of ways to arrange It.

STEP 01

CALCULUS

① ② ③ ④ ⑤ ⑥ ⑦ ⑧

Total No. of letters = 8

No. of ways = 8!

STEP 02

CC, LL, UU, A, S

Remove the overcounting

p alike q alike r alike.

$$= \frac{8!}{2!\,2!\,2!\,1!\,1!}$$

Order change

CC LL UU

CC LL UU

Overcounting

$$= \frac{8!}{2!\,2!\,2!}$$

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MOM

Total No. of ways to arrange It

MOM

2M, O

$\Rightarrow 3!/2! = 3$ ✓

$M_1 O M_2$
$M_1 M_2 O$
$O M_1 M_2$

ONLY THREE arrangEMENT

$M_2 O M_1$
$M_2 M_1 O$
$O M_2 M_1$

X (overcounting) Remove It

Ans = 3 ✓

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BABY No. of Total arrangEMENT

$= 4!_{a} / 2! = 12$

"

n Diff. Items p_{alike}, q_{alike}, r_{alike}.

No. of arrangEMENTS $= \dfrac{n!_{e}}{p!_{e}\, q!_{e}\, r!_{e} \cdots}$

No. of arrangements $= \dfrac{(p+q+r+\cdots)!_{e}}{(p!_{e}\, q!_{e}\, r!_{e} \cdots)}$

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ALLAHABAAD

AAAAA, LL, B, H, D

No. of arrangements

$$= \frac{10!}{5!\,2!\,1!\,1!\,1!} = \left\{\frac{10!}{5!\,2!}\right\} = \frac{10!}{2!\,2!\,3!}$$

PROPORTION

No. of arrangements

n Diff. Items p alike, q alike, r alike.

No. of Selection / group / committee.

(Taken all at a time)

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n Diff. p alike, q alike, r alike. Taken all at a time.

A A A A B B B C C

No. of selection = 1

| A A A A | B B B | C C |

| C C | B B B | A A A A |

| B B B | A A A A | C C |

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n Different Items p alike, q alike, r alike.

Taken 'R' at a time

✓ No. of arrangements
✓ No. of Selection | group.

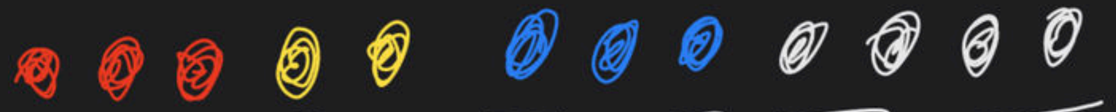
Task

AAABBCCCDDDD

palike qalike ralike

Taken 3 At a time

Taken 3 ball at a time



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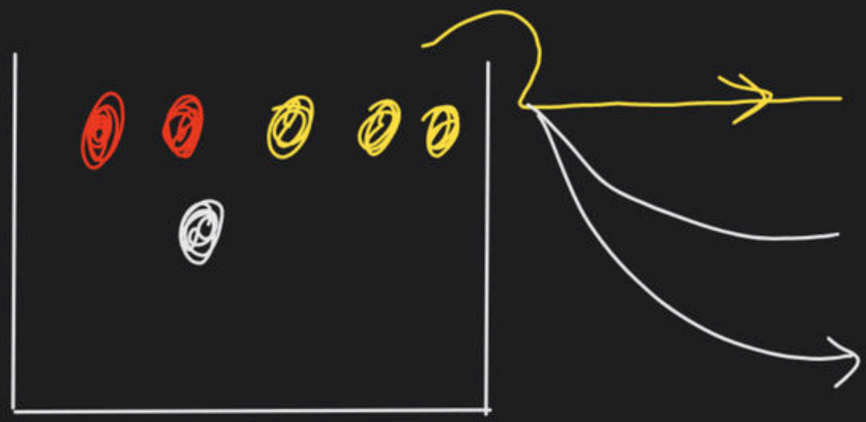
Taken 3 at a time

"Find the Number of arrangements



] 3 balls Different

] 2 ball same + one Diff.

] all are same

3 balls

a b c

a a a	all alike.
a a b	2 alike 1 Diff.
a b c	3 Different

CASE

Taken 3 at a Time

✓ **No. of arrangements = Selection X ORDER**

⇒ 3 alike  ⇒ $^1C_1 \times 1 = 1$

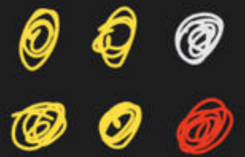
group

order :- n Diff. Items p alike, q alike, r alike

$= \frac{3!}{3!} = 1$

→ 2 alike, 1 diff.

$\frac{3!}{2!}$



3 alike (Not)

2 alike, 1 diff. $= {}^2C_1 \times {}^2C_1 \times \frac{L3}{L2}$

$= 2 \times 2 \times 3 = 12$

$=$ Select $\times$ order.

$= \frac{L3}{L2}$

$= \frac{L3}{L2}$

CASE 03 :- all are Different

$= {}^1C_1 \times L3$

$= 6$

n Diff. Taken all at a time.

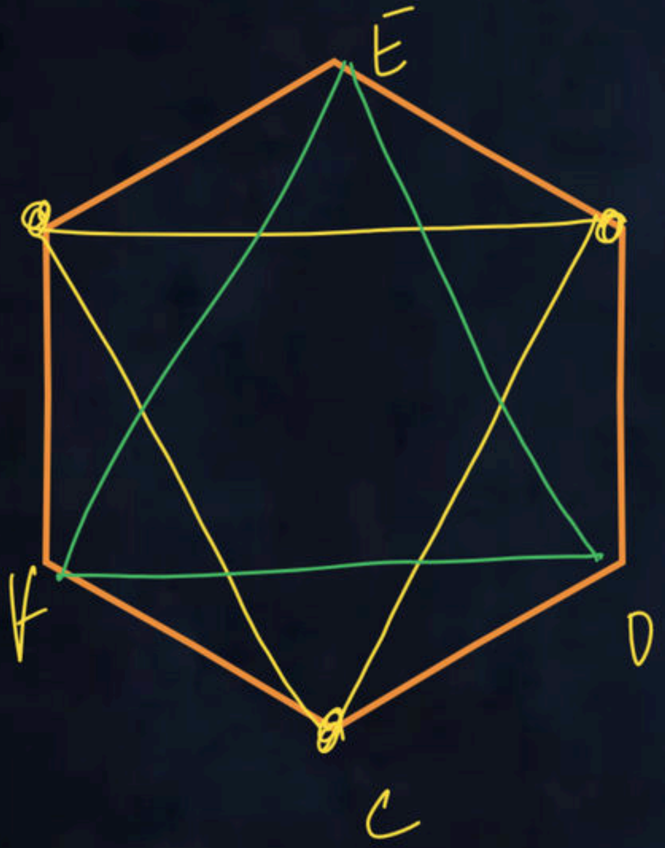
$=$ Total No. of ways $= 1 + 12 + 6 = 19$ ways.

a b c d :-

- a a a a — all SAME, all alike
- a a b b — 2 alike, 2 alike
- a a a b — 3 alike, 1 diff
- a a b c — 2 alike, 2 diff
- a b c d — 4 different

QUESTION

Q. How many triangles can be formed by joining the vertices of a hexagon?



A, B, C, D, E F

No. of Triangles

$= {}^{6}C_{3}$

$= \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20$

A 10

B 20

C 30

D 60

QUESTION

Q. How many diagonals are these in a polygon with n sides?

A $\frac{n(n-1)}{2}$

B $\frac{n(n+1)}{2}$

C $\frac{n(n-3)}{2}$

D $\frac{n(n+3)}{2}$

QUESTION

Q. In how many ways we can arrange letter A, A, B, B, B, C taken 3 at a time.

A A, B B B, C Taken 3 at a time

= 3 alike BBB ${}^1C_1 \times 1 = 1$

2 alike + 1 diff. BBC, AAC ${}^2C_1 \times {}^2C_1 \times \frac{L3}{L2} = 12$

3 different = ${}^1C_1 \times L3$

= 6

✓ = 1 + 12 + 6 = 19

4 Red 3 yellow 2 blue 1 green

"4 balls Are Taken"

"NO. of arrangEMENTS"

Number of arrangEMENTS = Selection X Ordering.

CASE (A)

= 4 alike

group.

$= {}^{!}C_1 \times \frac{L4}{L4} = 1$

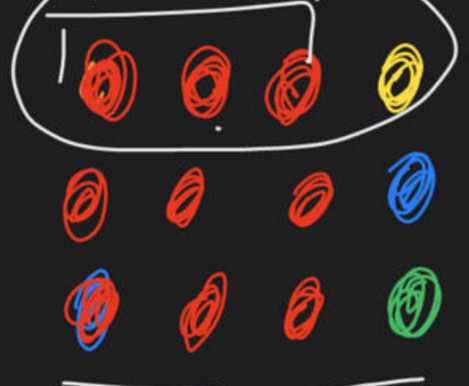
Order = alike, alike Taken $= \frac{n!}{1!\,2!\,1!}$

CASE 02 :- 3 alike + 1 diff

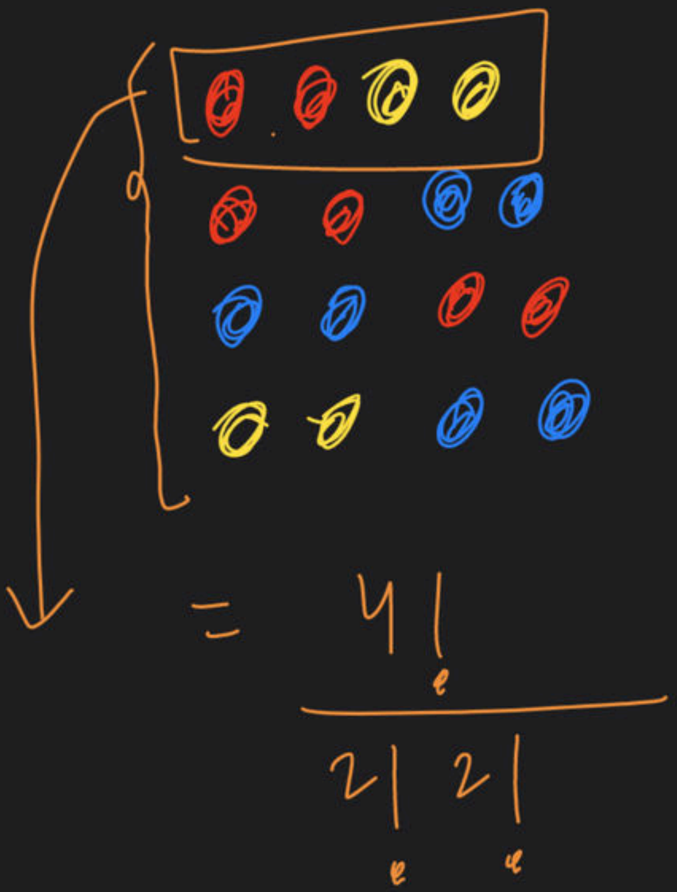
$\Rightarrow$ Selection $\times$ order

$\Rightarrow {}^{2}C_1 \times {}^{3}C_1 \times \frac{4!}{3!} = 2 \times 3 \times 4 = 24$

Problem:



n Diff. Items p alike, q alike, r alike $= \frac{4!}{3!} = 4$



$$= \frac{4!}{2!\,2!}$$

CASE D 3

2 alike, 2 alike

1 group + 1 group

Total group = 2

$$= {}^{3}C_{2} \times \frac{4!}{2!\,2!}$$

$\Rightarrow$ 18 ✓

3 groups

$= {}^{3}C_{1}$

order $= \frac{L4}{L2}$

selection

CASE 04 2 like + 2 different

selection

$= {}^{3}C_{1} \times {}^{3}C_{2} \times \frac{L4}{L2}$

$= 108$

CASE 05 all are different

$= {}^{1}C_{1} \times 4! = 24$

Total No. of ways = 175 answer

QUESTION

Q. How many four-letter words can be formed using the letters of the word 'INEFFECTIVE'?

A 840

B 1380

C 1422

D None of these

INEFFECTIVE

EEE, FF, II, N, C, T, V

✓ CASE 01 :- 4 alike (0)

CASE 02 :- 3 alike + 1 Diff.

EEE | F, I, N, C, T, V

$= {}^{1}C_{1} \times {}^{6}C_{1} \times \frac{\lfloor 4}{\lfloor 3} = 24$

✓ CASE 03

2 alike + 2 alike.

1 group + 1 group

EE | FF
FF | II
II | EE

3 group

$= {}^{3}C_{2} \times \frac{\lfloor 4}{\lfloor 2 \lfloor 2} = 18$

CASE 04

2 alike + 2 diff.

EE | F, I, N, C, T, V

$= {}^{3}C_{1} \times {}^{6}C_{2} \times \frac{\lfloor 4}{\lfloor 2} \Rightarrow 540$

CASE 05

all are Diff.

E, F, I, N, C, T, V

$= {}^{7}C_{4} \times 4! = 840$

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Total No. of arrangements

$= 0 + 24 + 18 + 540 + 840$

$= 1422$

QUESTION

Q. In how many ways 3 letters can be selected from letter A, A, B, B, B, C?

DONE

QUESTION

Selection

Q. In how many ways 4 letters can be selected from letter of the word 'INEFFECTIVE'?

arrangEMENT = Selection × ordering

(A) 80

(B) 89

= only. Selection

(C) 51

(D) None of these ✓

Ans = 89

#Hard Many 5 lettered words can be formed using The word

INDEPENDENCE

"Find the No. of arrangements"

INDEPENDENCE

AAAAA
AAAAB
AAABC
AABBC
AABCD
AABBC
ABCDE

all CASES

⇒ 5 alike = 0
⇒ 4 alike + 1 diff. = 25
⇒ 3 alike + 2 diff. = 400
⇒ 3 alike + 2 alike of Diff. kind = 40
⇒ 2 alike + 3 diff. = 1800
⇒ 2 alike + 2 alike + 1 diff. = 360
⇒ all diffrent = 720 = $^6C_5 \times 5!$

Total No. of arrangements = 25 + 400 + 40 + 1800 + 360 + 720 = 3345 ✓

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Ques: find the No. of words. Each consisting 5 letters of the word

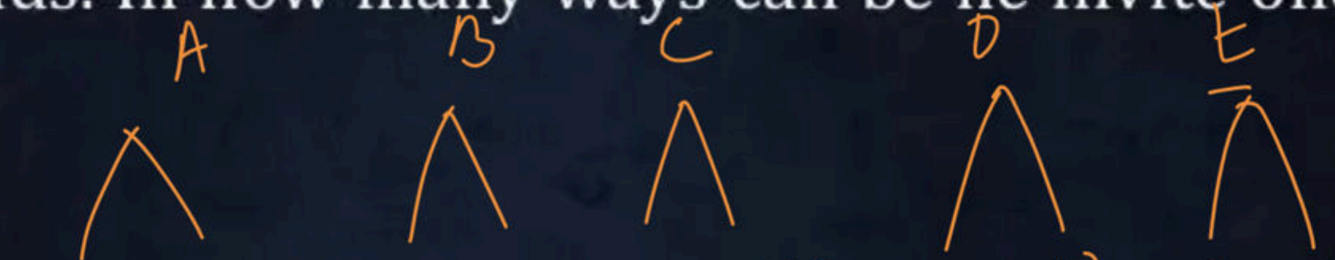
= MISS ISSIPPI ✓

✓ Ans = 1350 answer

QUESTION

ONE or more.

Q. A man has 5 friends. In how many ways can be he invite one or more of them to a party?



$2 \times 2 \times 2 \times 2 \times 2$

$= 2^5 - 1$

$= 31$ ways.

$^5C_0 + {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$

$= 2^5$

$= 2^5 - 1$

$^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n = 2^n$

A 32

B 31

C 30

D 16

QUESTION

Q. The question paper in the examination contain three sections – A, B, C. Therefore are 6, 4, 3 questions in sections A, B, C respectively. A student has the freedom to answer any number of question attempting at least one from each section. In how many ways can the paper be attempted by a student?

A, B, C

6, 4, 3

↓

Ans $= (2^6 - 1)(2^4 - 1)(2^3 - 1)$

$\Rightarrow$

(A) 8192

(B) 6615

(C) 7168

(D) None of these

SECTION A	SECTION B	SECTION C
6 question	4 question	3 question
$(2^6 - 1)$	$(2^4 - 1)$	$(2^3 - 1)$
W_1	W_2	W_3

Total No. of ways $= (2^6 - 1)(2^4 - 1)(2^3 - 1)$

$= 6615$

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(A), (B), (C), (D) question { A, B, C, D } = 4ways.

(A) (B) (C) (D) (MSQ) ONE or more can be correct

$^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 =$ Total

Q_1 Q_2 Q_3 Q_4 Q_5 Q_6

2 2 2 2 2 2 $= 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 2^6 - 1$

$$^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n = 2^n$$

$$^nC_1 + {}^nC_2 + \cdots + {}^nC_n = 2^n - 1$$

QUESTION

4 alike, 3 alike + 1 diff, 2 alike + 2 alike
2 alike + 2 diff, all Diff.

Q. In a box there are 10 balls, 4 red, 3 black, 2 white and 1 yellow. In how many ways can a child select 4 balls out of these 10 balls? (Assume that the balls of the same color are identical)

A 20

B 18

C 19

D None of these

Select 4 balls.

= 20

n Diff. Items
p alike, q alike
r alike.

Selection 'R' at a time
= Selection (only)

QUESTION

Q. In how many ways a team of 11 players be selected from a list of 16 players where two particular players should always be included in the team.

QUESTION

Q. In how many ways a team of 11 players can be selected from a list of 16 players such that 2 particular player should be included in the selection.

QUESTION

Q. How many three-letter words can be made using the letters of the words 'SOCIETY', so that.

(i) 'S' is included in each word?

(ii) 'S' is not included in any word?

(iii) To include S in every word, we have two cases.

QUESTION

Q. How many words can be formed using the letters of the word "TRIANGLE" so that

(i) 'A' and 'N' are always together?

(ii) 'T', 'R', 'I' are always together?

QUESTION

Q. How many five-letter words containing 3 vowels and 2 consonants can be formed using the letters of the word 'EQUATION' so that the two consonants occur together in every word?

A 240

B 1440

C 720

D 480

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Orientation session

10 PM

THANK YOU!

Here's to a cracking journey ahead!

#Revise - Pride

↳ "problem solving session"