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Random Variables Part-V

Course on Engineering Mathematics for GATE - CSE

Rahul • Lesson 18 • June 30, 2024

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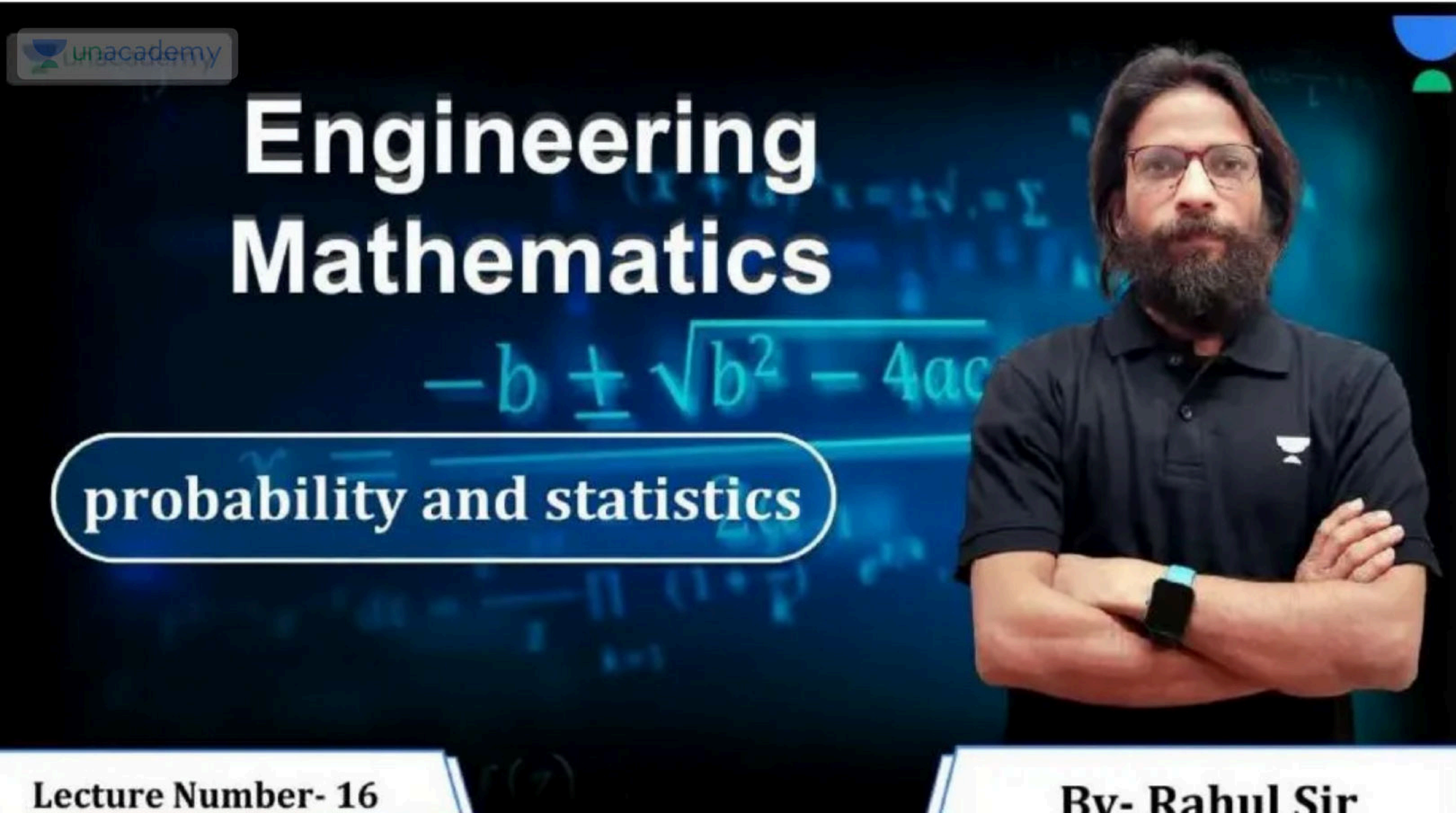
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Engineering Mathematics
probability and statistics
Lecture Number- 16
By- Rahul Sir

Topics

to be covered

1. Problem solving class

Expected value $= \sum x_i p_i$

$\mu = E(X) =$ "X is DISCRETE Random variable"

Expected value $= \int_a^b x\, f(x)\, dx$

$E(X) = \mu$

variance :- DISPERSION + Distance about MEAN

1	
2	3.5
3	
4	
5	
6	

$= (1-3.5)^2$ $(6-3.5)^2$

$= (2-3.5)^2$

$= (3-3.5)^2 = \sum f_i x_i^2 - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2$

$= (4-3.5)^2$

$= (5-3.5)^2$

$$\boxed{\text{variance} = E(x^2) - [E(x)]^2}$$

x_0	x_1	x_2	x_3	$\cdots$	x_n
p_0	p_1	p_2	p_3		p_n

$$E(x^2) = x_0^2 p_0 + x_1^2 p_1 + x_2^2 p_2 + \cdots + x_n^2 p_n$$

$$\boxed{\begin{aligned} E(x^2) &= \sum x_i^2 p_i \\ \text{variance} &= \sum x_i^2 p_i - \left[\sum x_i p_i\right]^2 \end{aligned}}$$

standard deviation $= \sqrt{\text{variance}}$

variance can't be Negative

In Continuous Random Variable

$$Var(X) = E(X^2) - [E(X)]^2$$

$$\boxed{E(X^2) = \int_a^b x^2 f(x)\,dx}$$

$a \le X \le b$

$$E(X^3) = \int_a^b x^3 f(x)\,dx$$

$$\boxed{E(X^n) = \int_a^b x^n f(x)\,dx}$$

$$\operatorname{var}(X) = \int_a^b x^2 f(x)\, dx - \left[\int_a^b x f(x)\right]^2$$

$$\text{standard deviation} = \sqrt{\text{variance}}.$$

Throwing A Die

X	$\boxed{1}$	2	3	4	5	6
P	1/6	1/6	1/6	1/6	1/6	1/6

$$E(X) = 3.5$$

$$E(X^2) = (1)^2 \times \frac{1}{6} + (2)^2 \frac{1}{6} + (3)^2 \frac{1}{6} + (4)^2 \frac{1}{6} + (5)^2 \frac{1}{6} + (6)^2 \frac{1}{6}$$

$$E(X^2) = \boxed{91/6.} = \underline{\underline{15.16}}$$

$var(X) = E(X^2) - [E(X)]^2$

variance $= 15.16 - (3.5)^2$

$\sigma_X^2 =$ variance $= 15.16 - 12.25 = 2.91$

$\sigma_X =$ standard deviation $= \sqrt{2.91}$

$= 1.70$

$\Downarrow$

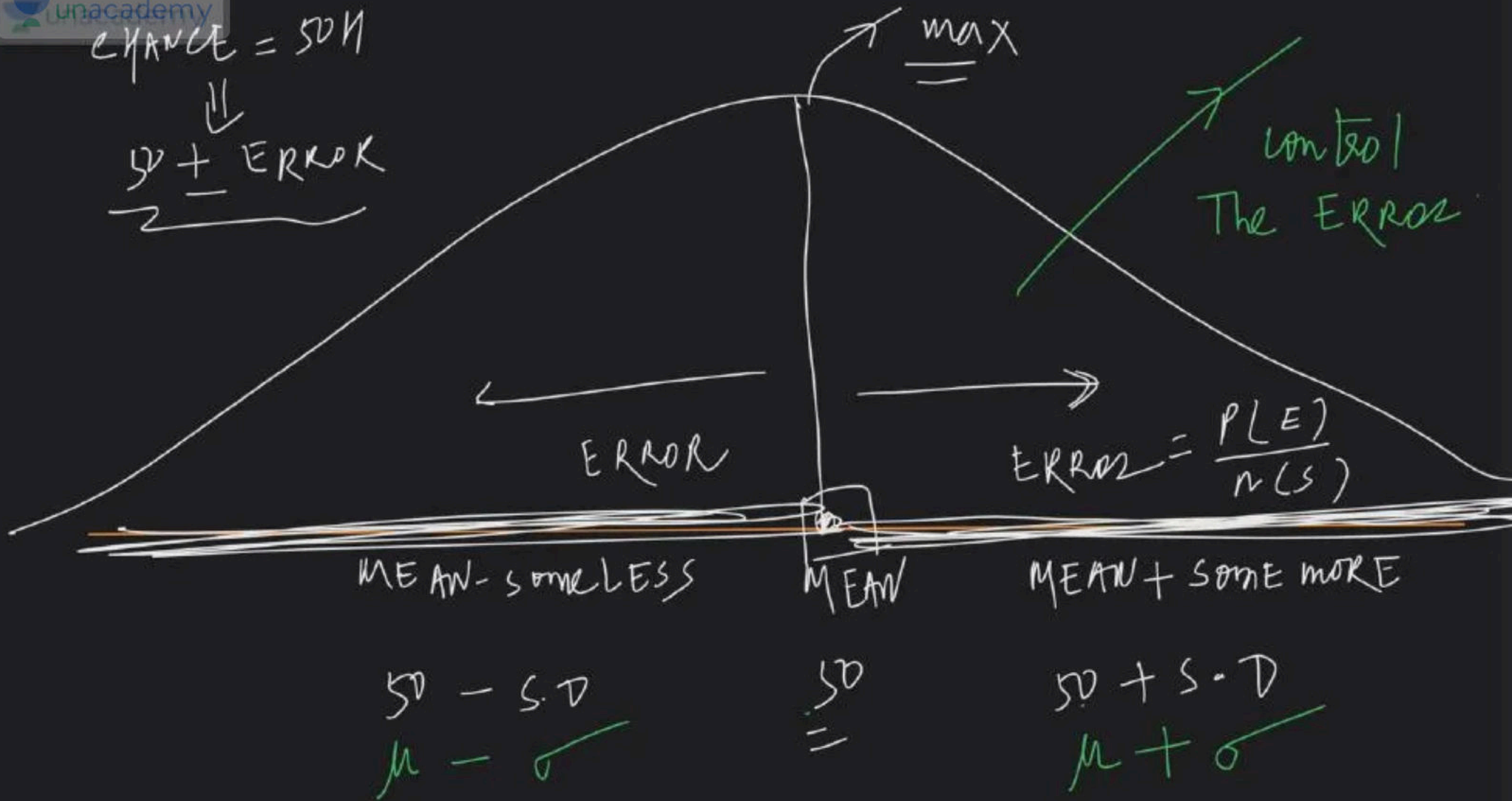
standard ERROR large no. of Trials

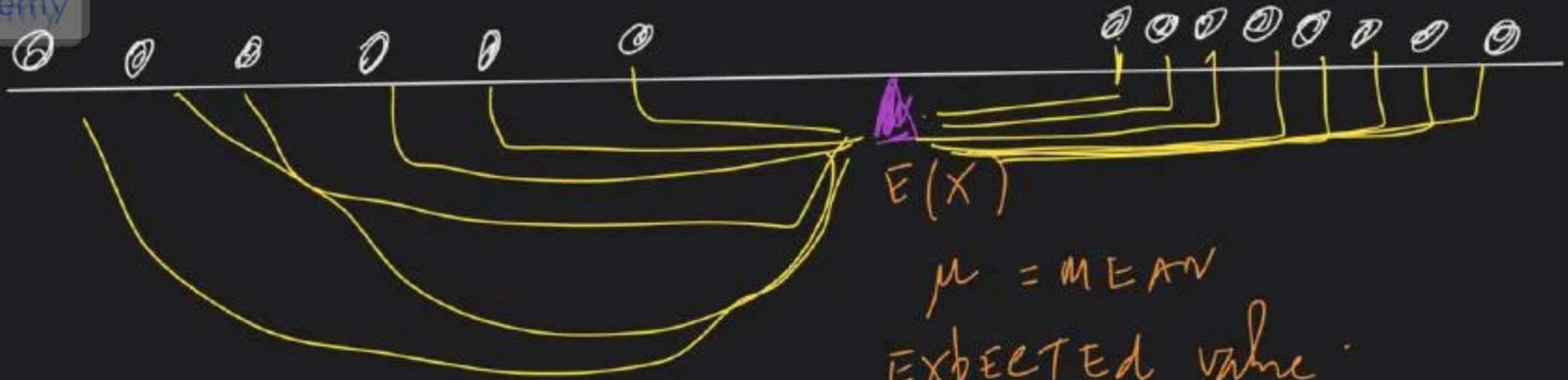
ERROR $= 1.70$ MEAN

CHANCE = 50%

⇓

50 ± ERROR





μ = MEAN

Expected value

$\boxed{\text{MEAN} = \text{max likelihood Number}}$

Tossing A coin 100 times

$P(H) = \frac{1}{2}$ 50%

$P(T) = \frac{1}{2}$ 50%

Head	
47	→ 50
51	→ 50
43	→ 50
44	→ 50

Error ↑

↓ No Error

	Read	Diff
50	50 =	(Diff)
50	41 =	(Diff)
50	42	
50	43	
50	46	
50	54	
50	52	
50	49	
50	50	
50	52	
50	52	
50	51	

QUESTION

Q. Let $f(x)=\dfrac{k|x|}{(1+|x|)^4}, -\infty < x < \infty$

Then the value of k for which f(x) is a probability density function is

(A) 1/6

(B) 1/2

(C) 3

(D) 6

QUESTION

Q. A continuous random variable X has density function

$$f(x)=\begin{cases} 2x & 0<x<\frac{1}{2} \\ \frac{4-2x}{3} & \frac{1}{2}\leq x<2 \\ 0, & \text{elsewhere} \end{cases}$$

Find $P[0.25 < x \leq 1.25]$

QUESTION

Q. Let X be a continuous random variable with probability density function

$$f(x) = \frac{1}{2}e^{-|x-1|}, -\infty < x < \infty$$

Find the value of $P(1 < |X| < 2)$

QUESTION

GATE

X is DISCRETE Random variable

Q. A machine produces 0, 1 or 2 defective pieces in a day with associated probability of 1/6, 2/3 and 1/6, respectively. Then mean value and the variance of the number of defective pieces produced by

X	0	1	2
$P(X=x)$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$

$$\text{MEAN} = \sum x_i p_i$$

$$= 0 \times \frac{1}{6} + 1 \times \frac{2}{3} + 2 \times \frac{1}{6}$$

$$\boxed{\text{MEAN} = 1}$$

$$\text{var} = E(X^2) - [E(X)]^2$$

$$= (0)^2 \frac{1}{6} + (1)^2 \frac{2}{3} + (2)^2 \frac{1}{6} - (1)^2$$

$$\boxed{\text{Variance} = \frac{1}{3}}$$

$$\text{S.D} = \frac{1}{\sqrt{3}}$$

answer

(A) 1 and 1/3

(B) 1/3 and 1

(C) 1 and 4/3

(D) 1/3 and 4/3

QUESTION

GATE

Q. In the following table, x is a discrete random variable and P(x) is the probability density.
The standard deviation of x is:

standard deviation $= \sqrt{var(x)}$ $\quad var(x) > 0$

X	1	2	3
P(x)	0.3	0.6	0.1

$var(X) = E(X^2) - [E(X)]^2$

$V(X) = E(X^2) = 1^2 \times 0.3 + (2)^2\, 0.6 + (3)^2\, 0.1 - [1 \times 0.3 + 2 \times 0.6 + 3 \times 0.1]^2$

A 0.18

$= 0.3 + 2.4 + 0.9 - [0.3 + 1.2 + 0.3]^2$

B 0.36

$= 3.6 - [1.8]^2 = 3.6 - 3.24$

C 0.54

$= .36$

D 0.6

$S.D = \sqrt{.36} = .6$

QUESTION

Q. A random variable X has probability density function f(x) as given below:

$$f(x)=\begin{cases} a+bx & \text{for } 0<x<1 \\ 0 & \text{otherwise} \end{cases}$$

valid pdf $\int_0^1 (a+bx)\,dx = 1$

If the expected value $\boxed{E|X| = 2/3}$ then Pr[X < 0.5] is_____.

= ⓐ ⓑ

ⓐ ⓑ

0.5

0    $\underbrace{a+b}x$ 1

$E(X) = \frac{2}{3}$

$0<x<1$

$P(X<0.5) = \int_0^{0.5} (a+bx)\,dx$

X is a continuous Random variable.

= a or b term involve

If X is a valid prob Density function

$$\int_0^1 (a+bx)\,dx = 1$$

$$\Rightarrow \quad a + \frac{b}{2} = 1 \quad -①$$

$E(X) = \frac{2}{3}$ — X is a CRV

$$\int_a^b x f(x)\,dx = E(X)$$

$$\int_0^1 x(a+bx)\,dx = \frac{2}{3}$$

$$\int_0^1 ax + bx^2\,dx = \frac{2}{3}$$

$$\left[a\frac{x^2}{2} + b\frac{x^3}{3} \right]_0^1 = \frac{2}{3}$$

$$\frac{a}{2} + \frac{b}{3} = \frac{2}{3} \quad -②$$

Solve The Eqn ① and ②

$a = 0$

$b = 2$

$$P(X < 0.5)$$

$$= \int_0^{0.5} (a+bx)\,dx$$

$$= \int_0^{0.5} (0+2x)\,dx$$

$$= (0.25) \text{ ans}$$

QUESTION

6pm

Q. Consider the following probability mass function (p.m.f) of a random variable X.

$$p(x,q)=\begin{cases} q & \text{if } x=0 \\ 1-q & \text{if } x=1 \\ 0 & \text{otherwise} \end{cases}$$

DISCRETE Random variable

X	0	1
$P(X=x)$	q	$1-q$

if q = 0.4, the variance of X is ______.

$\text{var}(X) = (0)^2 \times q + (1)^2 \times (1-q) - [0 \times q + 1 \times (1-q)]^2$

$q=0.4$

$= (1-q) - [1-q]^2$

$= [1-0.4] - [1-0.4]^2 = 0.24$

variance $= 0.24$

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QUESTION

GATE CSE

Q. Each of the nine words in the sentence "The Quick brown fox jumps over the lazy dog" is written on a separate price of paper. These nine pieces of paper ~~p~~are kept in a box. One of the pieces is drawn at random ~~for~~ from the box. $\boxed{\text{The expected length of the word drawn is}}$____.
(The answer should be rounded to one decimal place)

THE QUICK BROWN FOX JUMPS OVER THE LAZY DOG

Expected length of word

$$\boxed{E(X) = 3.88}$$

X = Random variable

X = No. of letters

THE QUICK BROWN FOX JUMPS OVER THE LAZI DOG
3 5 5 3 5 4 3 4 3

EXPECTED length of word

$X = 3, 4, 5$ (arrival / DRV / countable)

X = Random variable

X = No. of letters

X	3	4	5
$P(X=x)$	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{3}{9}$

#

$$E(X) = 3\times\frac{4}{9} + 4\times\frac{2}{9} + 5\times\frac{3}{9} = \frac{35}{9}$$

answer

$$\boxed{E(X) = 3.88}$$

QUESTION

(GATE)

Q. The variance of the random variable X with probability density function $f(x) = \frac{1}{2}|x|e^{-|x|}$ is ____.

$\checkmark\ f(x) = \frac{1}{2}|x|e^{-|x|}$

variance $= E(X^2) - [E(X)]^2$

$f(x) = \frac{1}{2}|x|e^{-|x|}$

simple function

x, x^2, x^3

modulus function $y = |x|$

$|x| \rightarrow |-5| = 5$

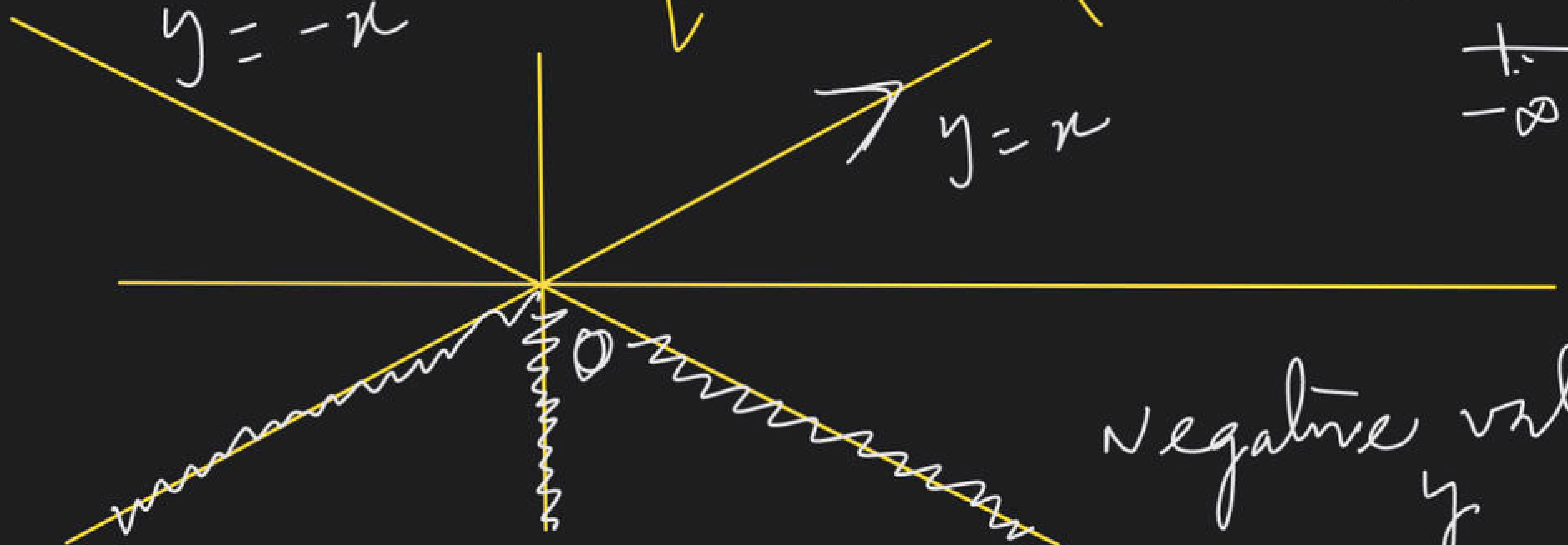
– value
– input

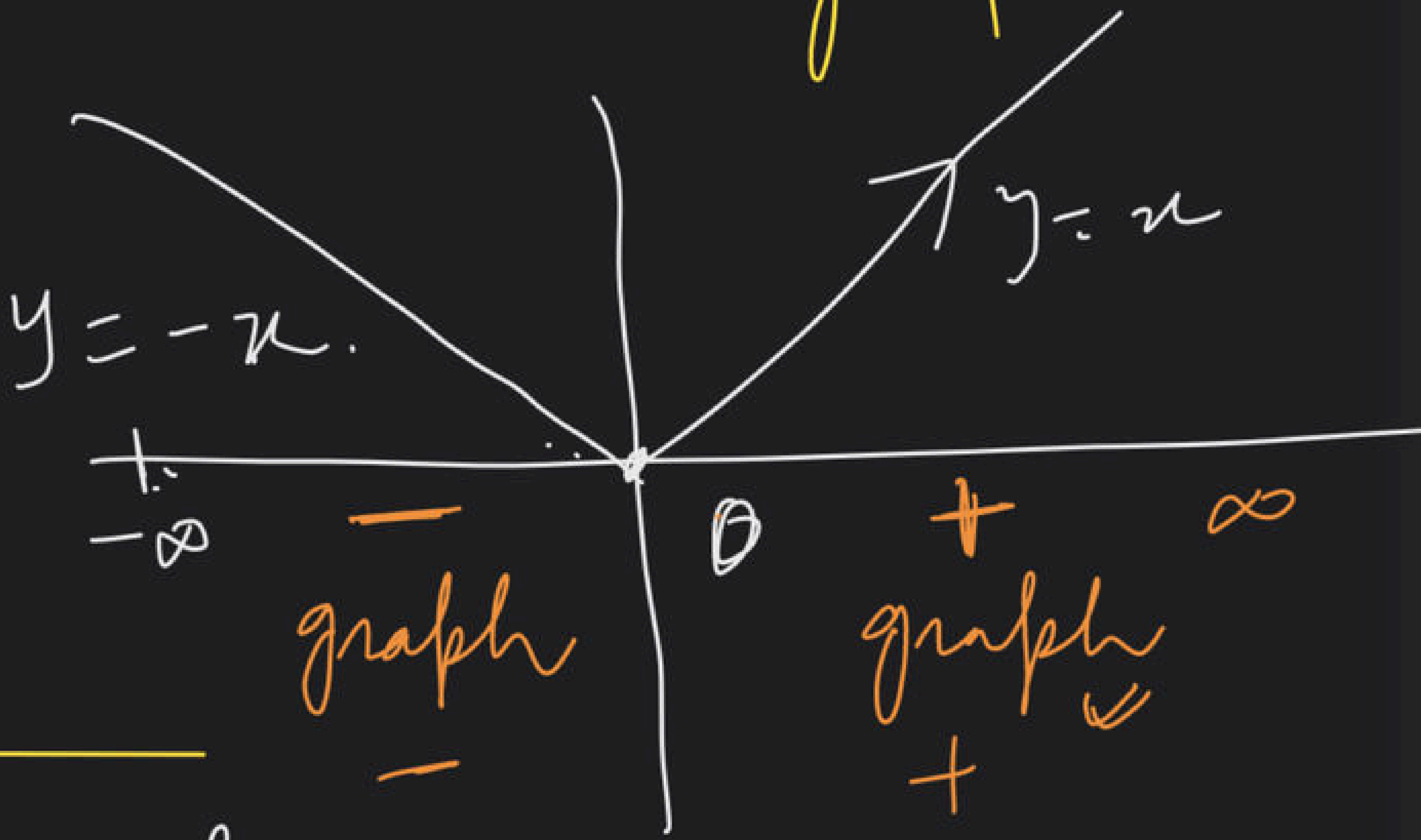
positive $|5| = 5$

+

"modulas function is always positive"

$$y = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$





Negative value

y

$$f(x) = \frac{1}{2}|x| e^{-|x|} \qquad -\infty \le X \le \infty \quad \text{Prob}$$

$$\operatorname{Var}(X) = E(X^2) - [E(x)]^2$$

$$E(X) = \int_{-\infty}^{\infty} x \cdot \frac{1}{2}|x| e^{-|x|}\, dx$$

$$= \int_{-\infty}^{0} x \cdot \frac{1}{2}|x| e^{-|x|}\, dx + \int_{0}^{\infty} x \cdot \frac{1}{2}|x| e^{-|x|}\, dx$$

$-\infty$ to 0 block | $+\,0$ to ∞ block 1

$$= \int_{-\infty}^{0} x\, \frac{1}{2}(-x)\, e^{-(-x)}\, dx + \int_{0}^{\infty} x \cdot \frac{1}{2}(+x)\, e^{-(+x)}\, dx$$

$$= \int_{-\infty}^{0} -\frac{x^2}{2} e^{x}\, dx + \int_{0}^{\infty} \frac{x^2}{2} e^{-x}\, dx$$

$$= 0 \quad \text{Ans.}$$

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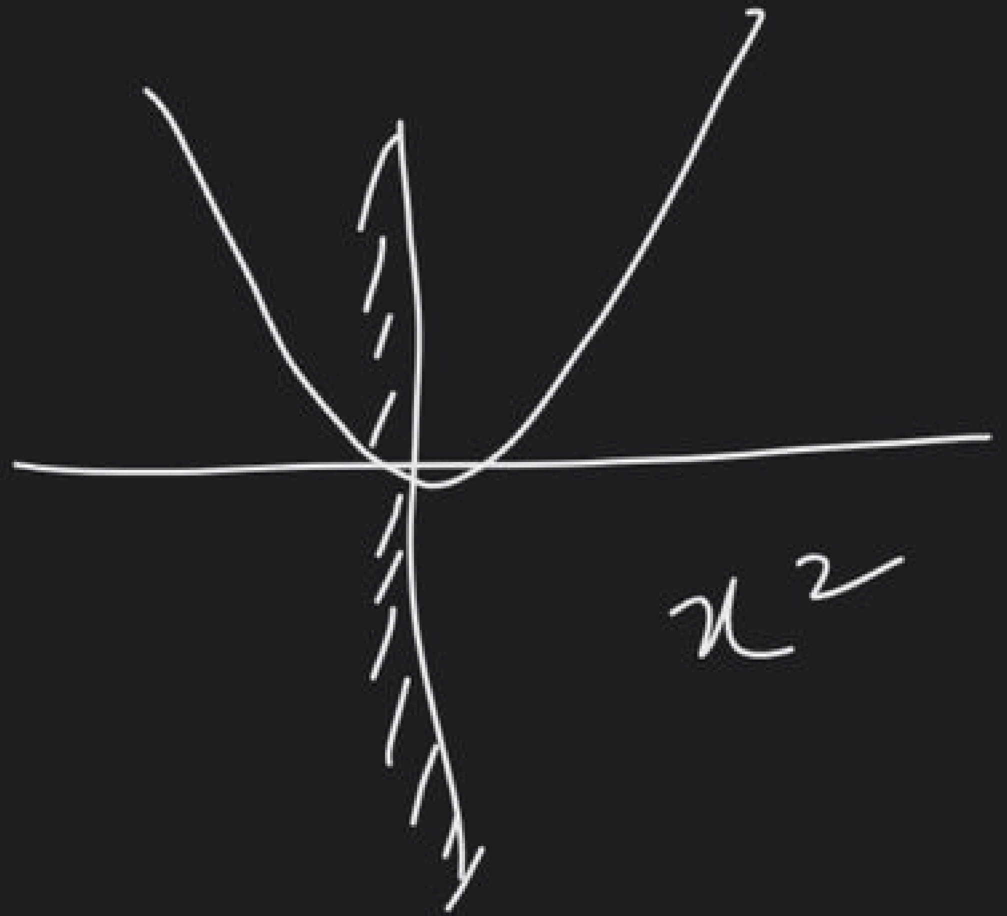
$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

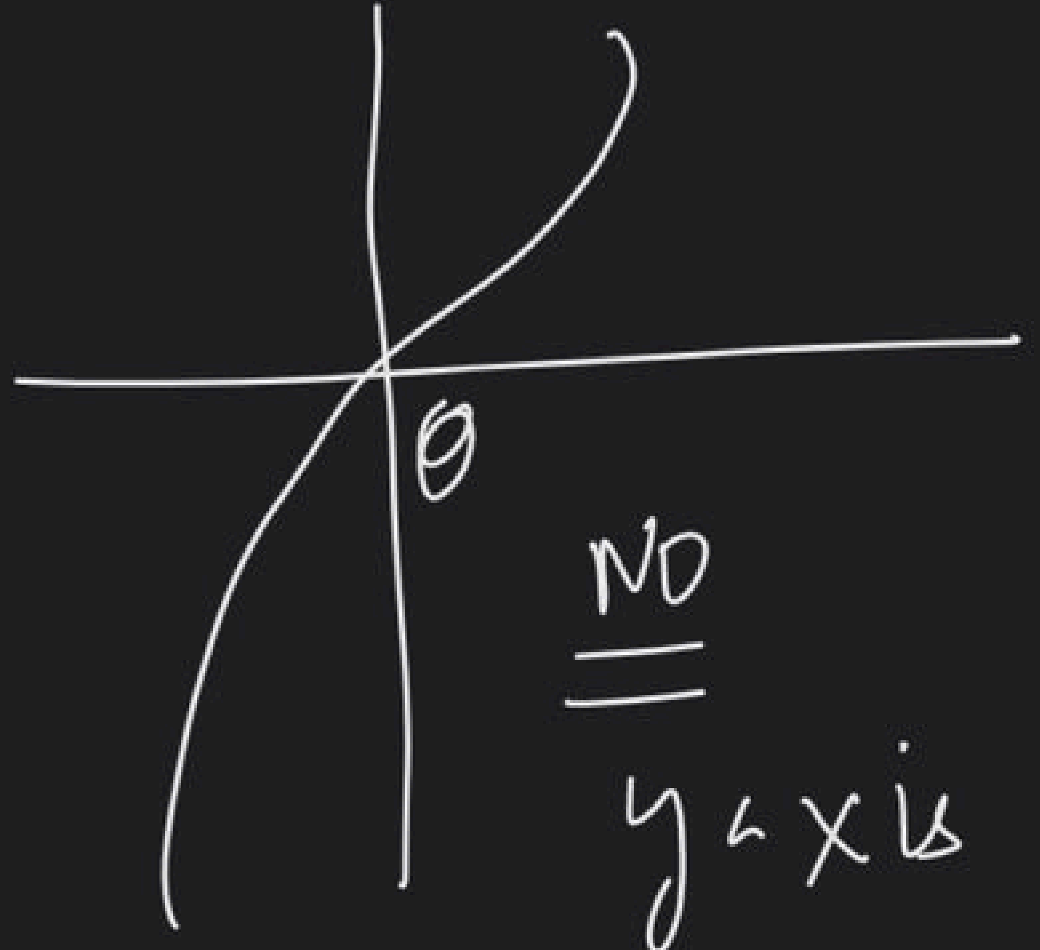
$$= \underbrace{\int_{-\infty}^{\infty} x^2 \frac{1}{2} |x| e^{-|x|}\, dx}$$

$$= 6$$

$$\boxed{\begin{aligned} V(X) &= E(X^2) - [E(X)]^2 \\ &= 6 - 0 \\ &= 6 \end{aligned}}$$

Ans

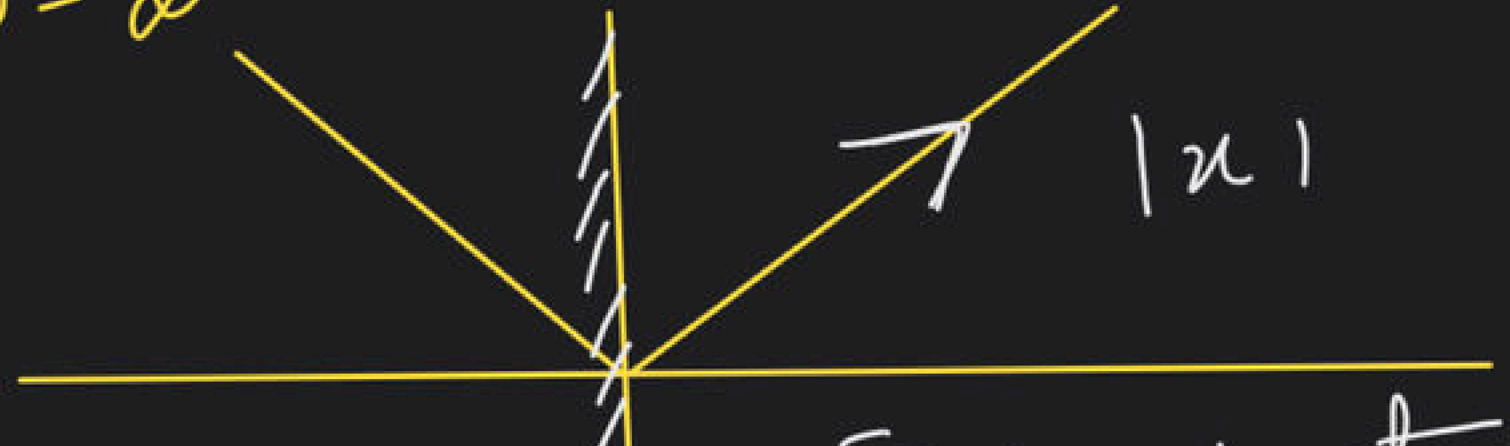




method 2 :

$$E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$$

$$= \int_{-\infty}^{\infty} \frac{1}{2} |x|\, e^{-|x|}\, dx$$



[Even function]

Symmetric about y axis

$$\begin{cases} f(-x) = f(x) & \text{even function} \\ f(-x) = -f(x) & \text{odd function} \end{cases}$$

$|x|$ even function

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx \quad f(x) \text{ is even}$$

$$= 0 \quad f(x) \text{ is odd function}$$

$$E(X) = \int_{-\infty}^{\infty} x \cdot \frac{1}{2}|x| e^{-|x|}$$

$$= 2\int_{0}^{\infty} x \cdot \frac{1}{2} x e^{-x}\,dx$$

$$= \cancel{2}\int_{0}^{\infty} \frac{x^2}{\cancel{2}} e^{-x}\,dx = \int_{0}^{\infty} x^2 e^{-x}\,dx$$

$$I = \int_0^{\infty} x^2 e^{-x}\, dx$$

algebraic exponential

$$= \left[-x^2 e^{-x} - 2x e^{-x} + 2e^{-x} \right]_0^{\infty}$$

D		I
x^2	$+$	e^{-x}
$2x$	$-$	$-e^{-x}$
2	$+$	$+e^{-x}$
0		e^{-x}

$$= \underline{0}$$

$$\int x^2 e^{x}\, dx$$

algebraic exponential

Diff. Integration

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QUESTION

Q. A fair die with faces {1, 2, 3, 4, 5, 6} is thrown repeatedly till '3' is observed for the first time. Let X denote the number of times the dice is thrown. The expected value of X is______.

$p(3) = \frac{1}{6}$ $p(\bar{3}) = 5/6$

$3 = p$

$\bar{3}\,3 = qp$

$\bar{3}\,\bar{3}\,3 = q^2 p$

$\bar{3}\,\bar{3}\,\bar{3}\,3 = q^3 p$

$\bar{3}\,\bar{3}\,\bar{3}\,\bar{3}\,3 = q^4 p$

$\boxed{3}$ $n = 1$ ✓

$\boxed{\bar{3}}$ $\boxed{3}$ $n = 2$

$\bar{3}\ \bar{3}\ 3$ $n = 3$

$\bar{3}\ \bar{3}\ \bar{3}\ 3$ $n = 4$

$\bar{3}\ \bar{3}\ \bar{3}\ \bar{3}\ 3$ $n = 5$

Sucess (S)

S

F S

F F S

F F F S

F F F F S

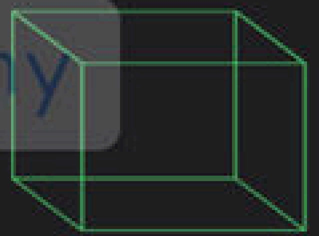
F F F F F S

⋮

✓ $X = n$ success

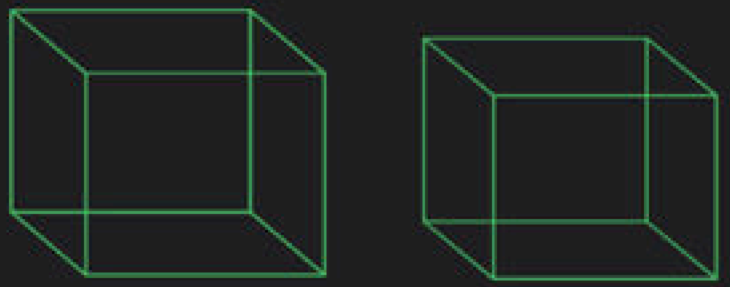
STOP: success achieve (only one success)

Insurance companies - Geometric Distribution

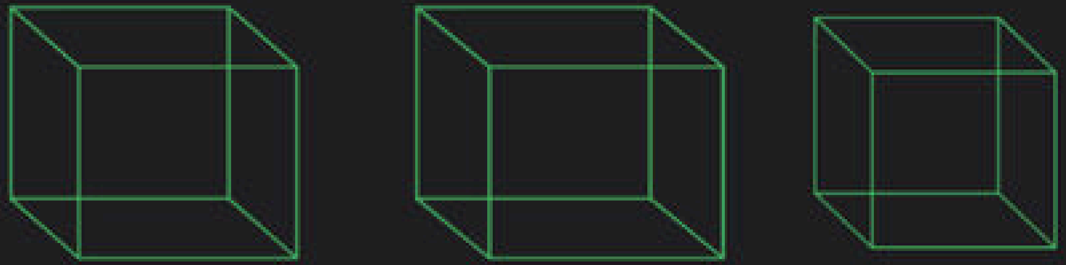


$3 = \boxed{1/6}$ $n = 1$

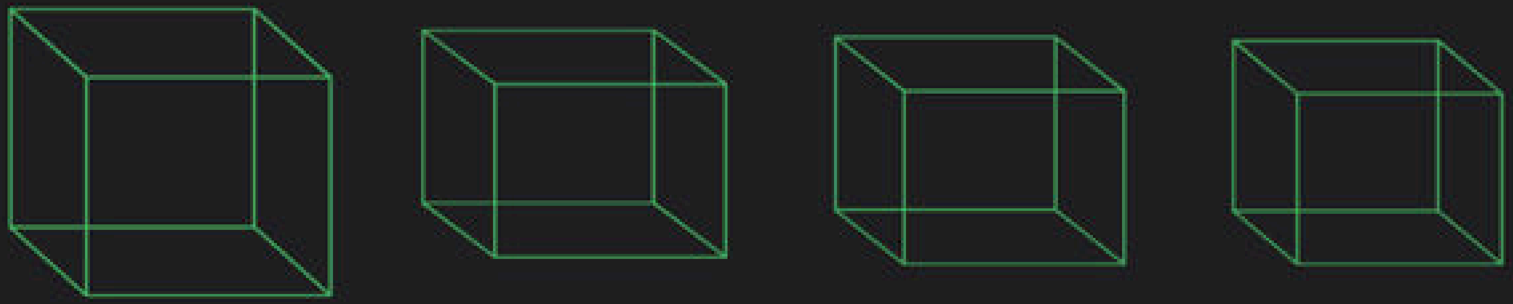
$X =$ discrete Random variable



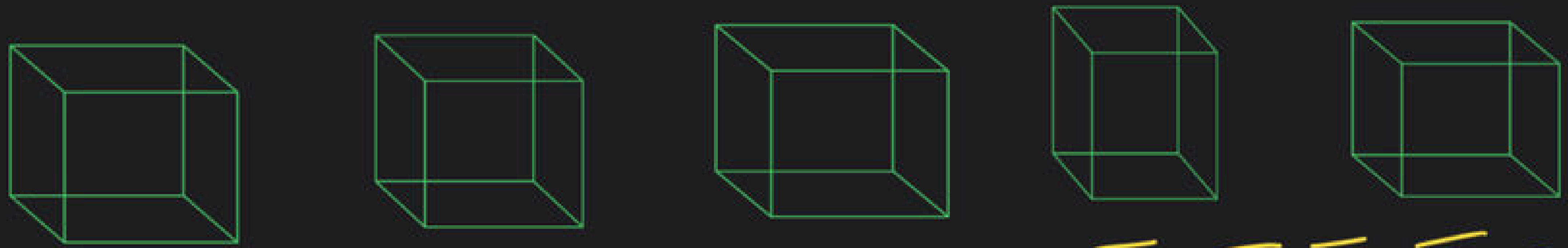
$= \bar{3}\ 3 = \boxed{\frac{5}{6}\ \frac{1}{6}}$ $n = 2$



$\bar{3}\ \bar{3}\ 3 = \boxed{\frac{5}{6}\ \frac{5}{6}\ \frac{1}{6}}$ $n = 3$



$\bar{3}\ \bar{3}\ \bar{3}\ 3 = \boxed{\left(\frac{5}{6}\right)^3 \frac{1}{6}}$ $n = 4$



$= \boxed{\left(\frac{5}{6}\right)^4 \frac{1}{6}}$

$\bar{3}\ \bar{3}\ \bar{3}\ \bar{3}\ 3$

$X = 5$

X	1	2	3	4	5	6	$---$
$P(X=x)$	$\frac{1}{6}$	$\frac{5}{6}\frac{1}{6}$	$\left(\frac{5}{6}\right)^2\frac{1}{6}$	$\left(\frac{5}{6}\right)^3\frac{1}{6}$	$\left(\frac{5}{6}\right)^4\frac{1}{6}$	$\left(\frac{5}{6}\right)^5\frac{1}{6}$	

$$E(X) = \sum_{i=1}^{n} x_i \, p_i$$

$$E(X) = 1\cdot\frac{1}{6} + 2\left(\frac{5}{6}\right)\left(\frac{1}{6}\right) + 3\left(\frac{5}{6}\right)^2\frac{1}{6} + 4\left(\frac{5}{6}\right)^3\frac{1}{6} + ---$$

(Rearrangement)

$$E(X) = 1\cdot\frac{1}{6} + 2\left(\frac{5}{6}\right)\left(\frac{1}{6}\right) + 3\left(\frac{5}{6}\right)^2\frac{1}{6} + 4\left(\frac{5}{6}\right)^3\frac{1}{6} + \cdots$$

A.P.

(Rearrangement)

G.P.   A.G.P.

$$-\ \frac{5}{6}E(X) = +1\cdot\left(\frac{5}{6}\right)\left(\frac{1}{6}\right) + 2\left(\frac{5}{6}\right)^2\frac{1}{6} + 3\left(\frac{5}{6}\right)^3\frac{1}{6} + \cdots$$

$$\frac{1}{6}E(X) = 1\cdot\frac{1}{6} + \left(\frac{5}{6}\right)\frac{1}{6} + \left(\frac{5}{6}\right)^2\frac{1}{6} + \left(\frac{5}{6}\right)^2\frac{1}{6} + \cdots$$

$$= \frac{1}{6} + \frac{1}{6}\left[\frac{5}{6} + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^3 + \cdots\right]$$

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{1}{6} + \frac{1}{6}\left[\frac{5/6}{1-5/6}\right]$$

$$\frac{E(X)}{6} = \frac{1}{6} + \frac{1}{6}\left[\frac{5/6}{1/6}\right]$$

$$\boxed{E(X) = 6}$$

Ans

$S = p$

$FS = qp$

$FFS = q^2 p$

$FFFS = q^3 p$

$FFFFS = q^4 p$

$FFFFFS = q^5 p$

$S = p$

$F = q$

X	1	2	3	4	5	6

X	1	2	3	4	5	6	- - -
$P(X=x)$	p	qp	q^2p	q^3p	q^4p	q^5p	- - -

$$E(X) = 1 \times p + 2qp + 3q^2p + 4q^3p + 5q^4p + 6q^5p + \cdots$$

$$E(x) = 1 \times p + 2qp + 3q^2 p + 4q^3 p + 5q^4 p + 6q^5 p + \cdots$$

$$qE(x) = qp + 2q^2 p + 3q^3 p + 4q^4 p + 5q^5 p + \cdots$$

$$(1-q)E(x) = p + qp + q^2 p + q^3 p + q^4 p + q^5 p + \cdots$$

$$(1-q)E(x) = p + p\left[q + q^2 + q^3 + \cdots\right] = p + p\left[\frac{q}{1-q}\right]$$

$$\boxed{E(x) = \frac{1}{p}}$$

Ans. $p = \frac{1}{6}$

$$E(x) = \frac{1}{1/6} = 6 \quad \text{Ans.}$$

quick

QUESTION

Q. A player tosses two unbiased coins. He wins Rs 5 if 2 heads appear, Rs 2 if one head appears and Rs 1 if no head appears. Find the expected value of the amount won by him.

H.W

QUESTION

Q. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the expected value for the number of aces.

H.W

QUESTION

Q. If it rains, a rain coat dealer can earn Rs 500 per day. If it is a dry day, he can lose Rs 100 per day. What is his expectation, if the probability of rain is 0.4?

H.W

QUESTION

Q. You toss a fair coin. If the outcome is head, you win Rs 100. if the outcome is tail, you win nothing. What is the expected amount won by you?

H.W

QUESTION

Q. A fair coin is tossed until a tail appears. What is the expectation of number of tosses?

H.W

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QUESTION

0 1 2

Q. The distribution of a continuous random variable X is defined by

(1)

$$f(x)\begin{cases} x^3, & 0<x\leq 1 \\ (2-x)^3, & 1<x\leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

obtain the expected value of X.

$$f(x) = \begin{cases} x^3 & 0 < x \leq 1 \\ (2-x)^3 & 1 < x \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

E(X) = Expected value of X

$$E(X) = \int_0^1 x \cdot \underbrace{x^3}_{f(x)}\, dx + \int_1^2 x \cdot (2-x)^3\, dx$$

$(A-B)^3$

$2-x = t$

Expected value =

$$\int x^n \, dx = \frac{x^{n+1}}{n+1}$$

$$E(X) = \int_0^1 x \cdot x^3 \, dx + \int_1^2 x(2-x)^3 \, dx$$

$$= \underbrace{\int_0^1 x^4 \, dx}_{I_1} + \underbrace{\int_1^2 x(2-x)^3 \, dx}_{I_2}$$

$$I_1 = \frac{1}{5}$$

$$I_2 = \int_1^2 x(2-x)^3 \, dx = -\int_1^0 (2-t)\, t^3 \, dt$$

$$2 - x = t \qquad 2 - t = x$$

$$-dx = dt$$

$$\int_a^b = -\int_b^a$$

$$= \int_0^1 t^3 (2-t) \, dt$$

$$\boxed{I_2 = \frac{1}{10}}$$

$$E(X) = \frac{1}{5} + \frac{1}{10} = \frac{3}{10}$$

QUESTION

2

Q. For a continuous distribution, whose probability density function is given by:

$$f(x)=\frac{3x}{4}(2-x), 0 \le x \le 2,$$

find the expected value of X.

$$E(X) = \int_0^2 x \cdot \frac{3x}{4}(2-x)\,dx$$

$$f(x) = \frac{3x}{4}(2-x) \quad 0 \le x \le 2$$

$= \underline{\underline{\text{Ans}}}$

<u>H.W</u>

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QUESTION

2

Q. Given the following probability distribution

X	–2	–1	0	1	2
P(x)	0.15	0.30	0	0.30	0.25

Find (i) E(X)
(ii) E(2X + 3)
(iii) E(X^2)
(iv) E(4X – 5)

$E(X) = -2 \times 0.15$
$\cancel{-1 \times 0.30} + 0$
$\cancel{+0.30} + 2 \times 0.25$
$= -0.30 + 0.50$

$\boxed{E(X) = 0.20}$

$E(2x+3) = 2E(X) + 3$
$= 2 \times 0.20 + 3$
$= 0.4 + 3 = 3.4$

$E(x^2) =$ ✓ H.W

$E(4X-5) = 4E(X) - 5 = 4 \times 0.2 - 5$
$=$ ✓ H.W

QUESTION

Q. For each of the following, determine whether the given values can serve as the probability distribution of a random variable with the given range:

(A) $f(x) = \frac{x-2}{5}$ For $x = 1, 2, 3, 4, 5$;

(B) $f(x) = \frac{x^2}{30}$ For $x = 1, 2, 3, 4$;

(C) $f(x) = \frac{x}{5}$ For $x = 1, 2, 3, 4, 5$;

QUESTION

Q. Verify that $f(x)=\dfrac{2x}{k(k+1)}$ for x = 1, 2, 3,.........., k can serve as the probability distribution of a random variable with the given range.

M·W

QUESTION

Q. For each of the following, determine c so that the function can serve as the probability distribution of a random variable with the given range:

A $f(x) = cx$ for $x = 1, 2, 3, 4, 5$;

B $f(x) = c\left(\frac{5}{x}\right)$ for $x = 1, 2, 3, 4$;

C $f(x) = c\left(\frac{1}{4}\right)^{x}$ for $x = 1, 2, 3$____

D $f(x) = cx^2$ for $x = 1, 2, 3$_____k

QUESTION

Q. A random variable X has the following probability function:

x	0	1	2	3	4	5	6
f(x)	k	3k	5k	7k	9k	11k	13k

(i) Find k,

(ii) Find $P(X \geq 5)$, $P(3 < X \leq 6)$, $P(X < 4)$

11   33k   16   16k

5+6 $\frac{33}{49}$ $\frac{16}{49}$

$\frac{11}{49}$

H.W

QUESTION

Q. If $P(x)=\begin{cases} x/15\,; & x=1,2,3,4,5 \\ 0\,; & \text{otherwise} \end{cases}$

Find

(i) $P(X=1 \text{ or } 2)$

(ii) $P\left(\frac{1}{2}<X<\frac{5}{2} / X>1\right)$

X	1	2	3	4	5
$P(X=x)$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

$$P(X=1 \text{ or } 2) = P(1)+P(2) = \frac{1}{15}+\frac{2}{15} = \frac{3}{15}$$

$$P\left(\frac{1}{2}<X<\frac{5}{2} / X>1\right) = \frac{P\left(\frac{1}{2}<X<\frac{5}{2} \cap X>1\right)}{P(X>1)}$$

H.W

QUESTION

Q. For each of the following, determine whether the given values can serve as the values of a distribution function of a random variable with the range $x = 1, 2, 3$ and 4;

(A) $F(1) = 0.3$, $F(2) = 0.5$, $F(3) = 0.8$, and $F(4) = 1.2$;

(B) $F(1) = 0.5$, $F(2) = 0.4$, $F(3) = 0.7$, and $F(4) = 1.0$;

(C) $F(1) = 0.25$, $F(2) = 0.61$, $F(3) = 0.83$, and $F(4) = 1.0$;

QUESTION

Q. Given that the discrete random variable X has the distribution function

$$f(x)=\begin{cases} x/6\,;\ x=1,2,3 \\ 0 \quad \text{elsewhere} \end{cases} \text{ Find } F(x)$$

Probability mass function

$= PMF = P(X = x)$

cdf $P[X \le x] = F_X(x)$

$\Downarrow$

PMF prob at a time

cdf $\rightarrow$ pdf

X is continuous Random variable.

pdf $\rightarrow$ cdf

X is continuous. Random

QUESTION

Q. If X has the distribution function

$$\cancel{f(x)} = \begin{cases} 0 & \text{for } x < 1 \\ \frac{1}{3} & \text{for } 1 \le x < 4 \\ \frac{1}{2} & \text{for } 4 \le x < 6 \\ \frac{5}{6} & \text{for } 6 \le x < 10 \\ 1 & \text{for } x \ge 10 \end{cases}$$

Find

(a) $P(2 < X \le 6)$;

(b) $P(X = 4)$

(c) $P(X \ge 10)$

(d) $P(X < 4)$

(e) $P(X > 4)$

(f) $P(X \ge 4)$

done

QUESTION

Q. If X has the distribution function

$$f(x)=\begin{cases}0 & \text{for } x<-1\\ \frac{1}{4} & \text{for } -1\leqq x<1\\ \frac{1}{2} & \text{for } 1\leqq x<3\\ \frac{3}{4} & \text{for } 3\leqq x<5\\ 1 & \text{for } x\geqq 5\end{cases}$$

cdf

Find

(a) $P(X \leq 3)$;

(b) $P(X = 3)$;

(c) $P(X < 3)$;

(d) $P(X \geq 5)$;

(e) $P(-0.4 < X < 4)$;

(f) $P(X = 5)$;

(g) $P(3 < X < 5)$;

(i) $P(3 \leq X < 5)$;

(j) $P(3 \leq X \leq 5)$

done

QUESTION

Q. Find distribution fx^n of the random variable that has the prob. distribution

$$f(x) = \frac{x}{15}; x = 1,2,3,4,5$$

QUESTION

Q. Let $X_1, X_2 \ldots\ldots\ldots X_n$ be random sample from the following density function

$$f(x;\theta) = \frac{kx}{\theta^2}; 0 < x < \theta, \theta > 0$$

Find k such that above is a valid density function.

If This is a valid prob. density function

$$\int f(x)\,dx = 1$$

$$\int_0^\theta \frac{kx}{\theta^2}\,dx = 1$$

$$\int_0^\theta \frac{kx}{\theta^2}\,dx = 1$$

$$= \frac{k}{\cancel{\theta^2}} \times \frac{\cancel{\theta^2}}{2} = 1$$

$$= \frac{k}{2} = 1$$

$$\boxed{k = 2}$$

QUESTION

Q. Let X be a continuous random variable with p.d.f:

$$f(x)=\begin{cases} ax; 0<x<1 \\ a; 1\le x\le 2 \\ -ax+3a; 2\le x\le 3 \\ 0;\ \text{elewhere} \end{cases}$$

(i) Determine constant a

pdf = 1

(ii) P (X ≤ 1.5)

Key Point

$$\int_0^{1.5} = \int_0^{1} + \int_1^{1.5}$$

H.W

QUESTION

H.W

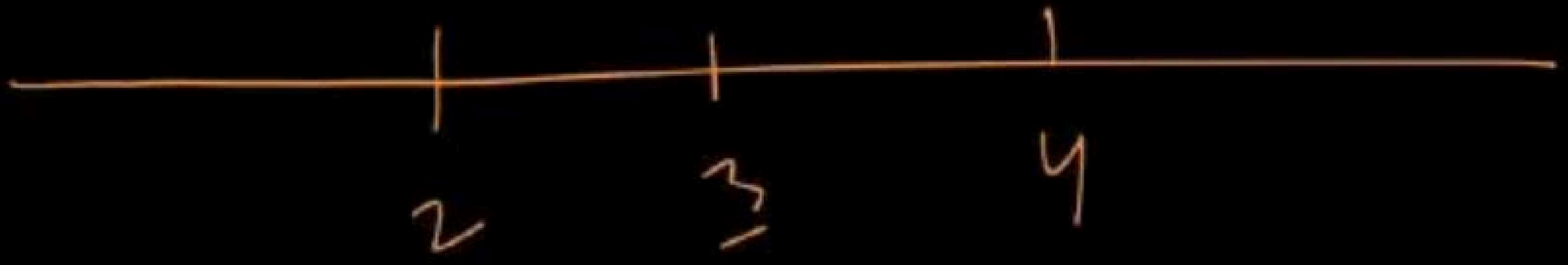
Q. The probability density of the random variable Y is given by

$$f(y) = \begin{cases} \frac{1}{8}(y+1) & \text{for } 2 < y < 4 \\ 0 & \text{elsewhere} \end{cases}$$

$P(Y < 3.2) =$

$P(2.9 < Y < 3.2) =$

Find P (Y < 3.2) and P (2.9 < Y < 3.2).



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QUESTION

Q. The p.d.f of the random variable X is given by

$$f(x)=\begin{cases}\dfrac{c}{\sqrt{x}} & \text{for } 0<y<4\\ 0 & \text{elsewhere}\end{cases}$$

H.W

Find (a) The value of c;

(b) $P\left(X<\frac{1}{4}\right)$ and $P(X>1)$

QUESTION

Q. The density function of the random variable X is given by

$$g(x)=\begin{cases}6x(1-x) & \text{for } 0<x<1\\ 0 & \text{elsewhere}\end{cases}$$

Find $P\left(X<\frac{1}{4}\right)$ and $P\left(X>\frac{1}{2}\right)$

NUMBER LINE

Draw

$$g(x)=\begin{cases}6x(1-x) & 0<x<1\\ 0 & \text{elsewhere}\end{cases}$$

$$P\left(X<\frac{1}{4}\right)=\int_0^{1/4}6x(1-x)\,dx$$

$$P\left(X>\frac{1}{2}\right)=\int_{1/2}^{1}6x(1-x)\,dx$$

QUESTION

done

Q. (a) Show that $f(x) = 3x^2$ for $0 < x < 1$ represents a density function.

(b) Calculate $P(0.1 > X < 0.5)$

H.W

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QUESTION

Q. If X has the prob. density fx^n

$$f(x)\begin{cases} ke^{-3x} & ; \quad x>0 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

Find k and P $(0.5 \leq X \leq 1)$

H.W

$f(x) = \begin{cases} Ke^{-3x} & x>0 \\ 0 & \text{elsewhere} \end{cases}$

find K

valid prob density function

$P(0.5 \leq X \leq 1) = \int_{0.5}^{1} Ke^{-3x}\, dx$

$=$ answer

QUESTION

Q. The probability density of the continuous random variable X is given by

$$f(x)\begin{cases}\frac{1}{5} & \text{for } 2<x<7\\ 0 & \text{elsewhere}\end{cases}$$

$$f(x) = \begin{cases}\frac{1}{5} & 2<x<7\\ 0 & \text{elsewhere}\end{cases}$$

$P(3<X<5)$

H.W

Find P(3 < X< 5)

$$P(3<X<5) = \int_3^5 \frac{1}{5}\,dx = \frac{2}{5}$$

Ans

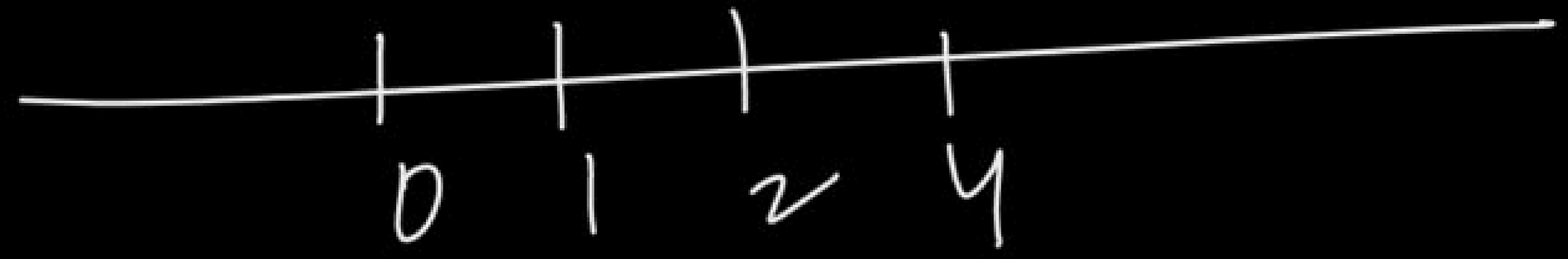
QUESTION

Q. Find the distribution function of the random variable X whose Probability density is given by

$$f(x)\begin{cases} \frac{1}{3} & \text{for } 0<x<1 \\ \frac{1}{3} & \text{for } 2<x<4 \\ 0 & \text{elsewhere} \end{cases}$$

Distribution $F(x)$ cdf

given $f(x)$



H.W

QUESTION

Q. The distribution fx^n of the random Variable X is given by

$$F(x) = \begin{cases} 1-(1+x)^{e^{-x}} & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases}$$

Find

(i) $P(X \le 2)$

(ii) $P(1 < X < 3)$

(iii) $P(X > 4)$

QUESTION

Q. Find the distribution function of the random variable X whose probability density is given by

$$f(x)=\begin{cases} x & \text{for } 0<x<1 \\ 2-x & \text{for } 1\le x\le 2 \\ 0 & \text{elsewhere} \end{cases}$$

prob. Density Function given, find out Distribution function

cdf and pdf = Relation

$$F_X(x) = \int_{\text{Region START (upto)}}^{x} f(x)\,dx$$

$$F_X'(x) = f(x)$$

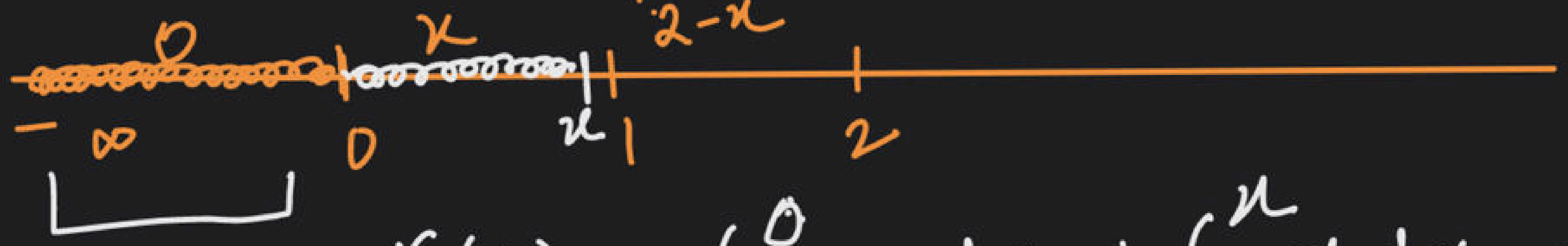
DRAW THE NUMBER LINE

$$f(x)=\begin{cases} x & 0<x<1 \\ 2-x & 1\le x<2 \\ 0 & \text{elsewhere} \end{cases}$$

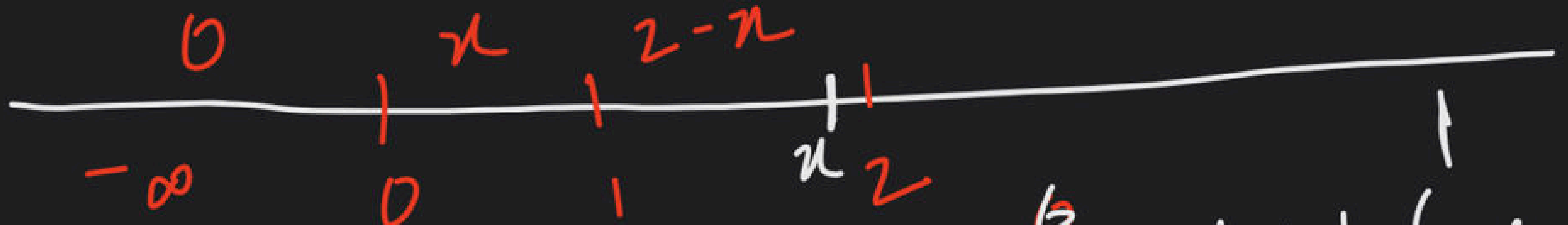
$$F(x)=\int_{\text{START}}^{x} f(x)\,dx$$

X is continuous Random variable.

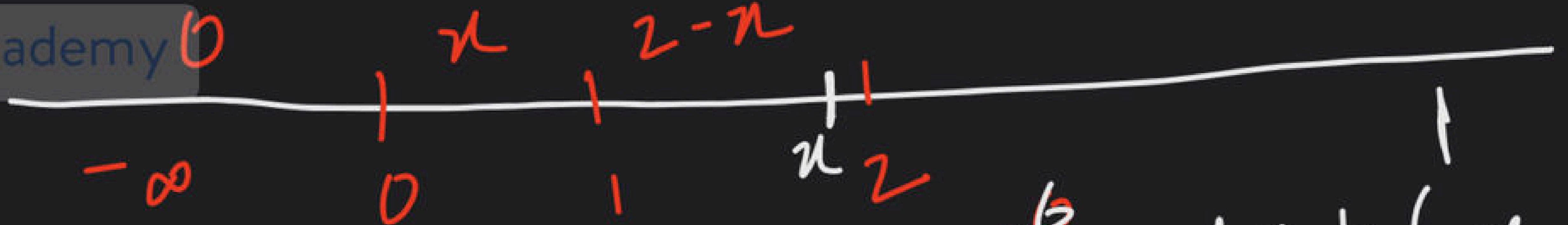
$$F(x)=\int_{-\infty}^{x} 0\,dx = 0$$



$$F(x)=\int_{-\infty}^{0} 0\,dx+\int_{0}^{x} x\,dx=\frac{x^2}{2}$$



$$F(x)=\int_{-\infty}^{0} 0\,dx+\int_{0}^{1} x\,dx+\int_{1}^{x}(2-x)\,dx$$



$$F(x) = \int_{-\infty}^{0} 0\,dx + \int_{0}^{1} x\,dx + \int_{1}^{x} (2-x)\,dx$$

$$= 0 + \frac{1}{2} + \left[2x - \frac{x^2}{2}\right]_1^x$$

$$= \frac{1}{2} + \left[2x - \frac{x^2}{2} - \left(2 - \frac{1}{2}\right)\right]$$

$$= \frac{1}{2} + 2x - \frac{x^2}{2} - 2 + \frac{1}{2}$$

$$= 2x - \frac{x^2}{2} - 1$$

$$\begin{array}{ccccccc} & 0 & & x & & 2-x & \\ \hline -\infty & & 0 & & 1 & & 2 \quad x \end{array}$$

$$F(x) = \int_{-\infty}^{0} 0\,dx + \int_{0}^{1} x\,dx + \int_{1}^{2} (2-x)\,dx + \int_{2}^{x} 0\,dx$$

$F(x) =$ Ans

$$F(x) = \begin{cases} 0 & x < 0 \\ \dfrac{x^2}{2} & 0 \le x < 1 \\ \boxed{} & 1 \le x < 2 \\ \boxed{} & x > 2 \end{cases}$$

cdf

Cumulative Distribution function

QUESTION

05 min

Do It # Draw

Q. Find the distribution function of the random variable X whose probability density is given by

$$f(x)=\begin{cases}\dfrac{x}{2} & \text{for } 0<x\leqq 1\\[2mm] \dfrac{1}{2} & \text{for } 1<x\leqq 2\\[2mm] \dfrac{3-x}{2} & \text{for } 2<x<3\\[2mm] 0 & \text{elsewhere}\end{cases}$$

$-\infty \quad 0 \quad 0 \quad \frac{x}{2} \quad 1 \quad \frac{1}{2} \quad 2 \quad \frac{3-x}{2} \quad 3 \quad 0$

$$F(x)=\int_{-\infty}^{x} 0\,dx = 0$$

$$F(x)=\int_{-\infty}^{0} 0\,dx+\int_{0}^{x}\frac{x}{2}\,dx=\frac{x^2}{4}$$

$$F(x)=\int_{-\infty}^{0} 0\,dx+\int_{0}^{1}\frac{x}{2}\,dx+\int_{1}^{x}\frac{1}{2}\,dx$$

$$=\frac{x+1}{2}$$

$-\infty \quad 0 \quad 0 \quad \frac{x}{2} \quad 1 \quad \frac{1}{2} \quad 2 \quad \frac{3-x}{2} \quad 3 \quad 0$

$$F(x) = \int_{-\infty}^{x} 0\,dx = 0$$ ✓

$$F(x) = \int_{-\infty}^{0} 0\,dx + \int_{0}^{x} \frac{x}{2}\,dx = \frac{x^2}{4}$$ ✓

$$F(x) = \int_{-\infty}^{0} 0\,dx + \int_{0}^{1} \frac{x}{2}\,dx + \int_{1}^{x} \frac{1}{2}\,dx$$

$$= \left[\frac{x^2}{4}\right]_0^1 + \frac{1}{2}\left[x\right]_1^x$$

$$\boxed{F(x) = \frac{x}{2} - \frac{1}{4}}$$ ✓ =

$$F(x) = \int_{-\infty}^{0} 0\,dx + \int_{0}^{1} \frac{x}{2}\,dx + \int_{1}^{2} \frac{1}{2}\,dx + \int_{2}^{3} \frac{3-x}{2}\,dx + \int_{3}^{x} 0\,dx$$

$$= \frac{x}{2}$$ ✓

DONE

QUESTION

Q. Find a prob. density fx^n for the random variable whose distribution fx^n is given by

$$f(x)=\begin{cases}0 & \text{for } x\le 0\\ x & \text{for } 0<x<1\\ 1 & \text{for } x>1\end{cases}$$

Prob. Density Function

$$F(x)=\begin{cases}0 & x\le 0\\ x & 0<x<1\\ 1 & x>1\end{cases}$$

H.W

QUESTION

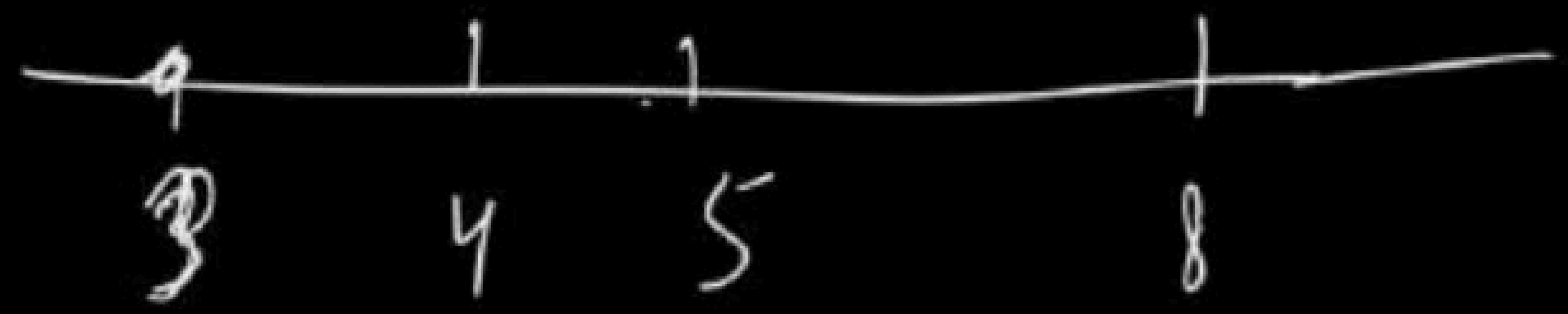
H.W

Q. The ~~distribution~~ density function of the random variable Y is given by

$$f(y)=\begin{cases}1-\dfrac{9}{y^2} & \text{for } y>3\\ 0 & \text{elsewhere}\end{cases}$$

Find P(Y ≤ 5) and P(Y > 8)

$$f(y)=\begin{cases}1-\dfrac{9}{y^2} & y>3\\ 0 & \text{elsewhere}\end{cases}$$



$$P(Y\le 5)=\int_3^5\left(1-\frac{9}{y^2}\right)dy$$

$$P(Y>8)=\int_8^\infty 1-\frac{9}{y^2}\,dy$$

QUESTION

Q. A random variable X which can be used in certain circumstances as a model for claim sizes has ~~cumulative distribution function~~

DENISITY function

$$f(x)=\begin{cases}0 & ,x<0\\ 1-\left(\dfrac{2}{2+x}\right)^3 & ,x>0\end{cases}$$

Calculate the value of the conditional probability P (X > 3/X >1)

$$P\left(X>3/X>1\right) = \frac{P(X>3 \cap X>1)}{P(X>1)}$$

H.W DONE

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QUESTION

Q. The probability density of the random variable Z is given by

$$f(z)=\begin{cases} kze^{-z^2} & \text{for } z>0 \\ 0 & \text{for } z\leq 0 \end{cases}$$

find The value of k.

If this is a valid pdf

$$\int_0^\infty kze^{-z^2}\,dz = 1$$

$$z^2 = t$$

$$z\,dz = \frac{dt}{2}$$

$$= \frac{k}{2}\int_0^\infty e^{-t}\,dt = 1$$

$$= \frac{k}{2}\left[-e^{-\infty}+e^{0}\right] = 1$$

$$\frac{k}{2}\left[+1\right] = 1$$

$$\frac{k}{2} = 1$$

$$\boxed{k=2}$$

QUESTION

Q. A random variable X has the following probability distribution

X	−2	−1	0	1	2
P(X)	1/6	p	1/4	p	1/6

(i) Find the value of p.

(ii) Calculate E (X + 2), E (2X² + 3X + 5)

$$\frac{1}{6} + p + \frac{1}{4} + p + \frac{1}{6} = 1$$

pmf = 1

$$E(x+2) = E(x) + 2$$

$$E[(2x^2) + 3x + 5]$$

$$= 2E(x^2) + 3E(x) + 5$$

=

H.W

$\int_0^\infty \frac{e^{-\frac{x}{2}}}{k}\,dx = 1$

$$\frac{1}{k}\int_0^\infty e^{-\frac{x}{2}}\,dx = 1$$

$$\frac{1}{k}\int_0^\infty e^{-t}\cdot 2\,dt = 1 \qquad \frac{x}{2} = t$$

$$\frac{2}{k}\int_0^\infty e^{-t}\,dt = 1 \qquad \frac{2}{k}\left[-e^{-t}\right]_0^\infty = 1$$

$$\frac{2}{k} = 1 \quad \boxed{k=2} \qquad \frac{2}{k}\left[0+1\right] = 1$$

$\frac{\text{shivansh}}{\text{problem}}$

QUESTION

Q. If X is the number of points rolled with a balanced die, find the expected value of

$g(X) = 2X^2 + 1.$

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QUESTION

Q. Let X be a random variable with the following probability fxn

x:	-3	6	9
$P(X = x)$	$1/6$	$1/2$	$1/3$

Find $E(X)$ and $E(X^2)$ and evaluate $E(2X + 1)^2$

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QUESTION

Q. Find the expected value of the random variable Y whose probability density is given by

$$f(y)=\begin{cases}\frac{1}{8}(y+1) & \text{for} -1<y<1 \\ 0 & \text{elsewhere}\end{cases}$$

Expected value

$$\boxed{E(Y)=\int_{-1}^{1} y\cdot\frac{1}{8}(y+1)\,dy}$$

QUESTION

Q. If X has the probability density

$$f(x)=\begin{cases} e^{-x} & \text{for } x>0 \\ 0 & \text{elsewhere} \end{cases}$$

Find the expected value of $g(X) = e^{3X/4}$.

QUESTION

Q. If the probability density of X is given by

$$f(x) = \begin{cases} 2x^{-3} & \text{for } x > 1 \\ 0 & \text{elsewhere} \end{cases}$$

check whether its mean and its variance exist.

H.W

QUESTION

Q. If the probability density of X is given by

$$f(x)=\begin{cases}2(1-x) & \text{for } 0<x<1\\ 0 & \text{elsewhere}\end{cases}$$

(a) Show that $E(X^r)=\dfrac{2}{(r+1)(r+2)}$

(b) and use this result to evaluate $E[(2X+1)^2]$

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QUESTION

Q. A continuous random variable X has the p.d.f,

$$f(x)=\begin{cases} a(1-x^2) & 2\le x\le 5 \\ 0 & \text{elsewhere} \end{cases}$$

(i) find a

(ii) Find E(X)

$$f(x)=\begin{cases} a(1-x^2) & 2\le x\le 5 \\ 0 & \text{elsewhere} \end{cases}$$

$$\int_2^5 a(1-x^2)\,dx = 1 \quad \boxed{a=?}$$

$$E(X) = \int_2^5 x\cdot f(x)\,dx = \underline{\underline{\text{answer}}}$$

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QUESTION

Q. Certain coded measurements of the pitch diameter of threads of a fitting have the probability density

$$f(x)=\begin{cases}\dfrac{4}{\pi(1+x^2)} & \text{for } 0<x<1\\ 0 & \text{elsewhere}\end{cases}$$

Find the expected value of this random variable.

$$f(x)=\frac{4}{\pi(1+x^2)}$$

$$0<x<1$$

expected value $= \int_0^1 x \cdot f(x)\, dx$

done

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THANK YOU!

Here's to a cracking journey ahead!

✓ 9 o'clock