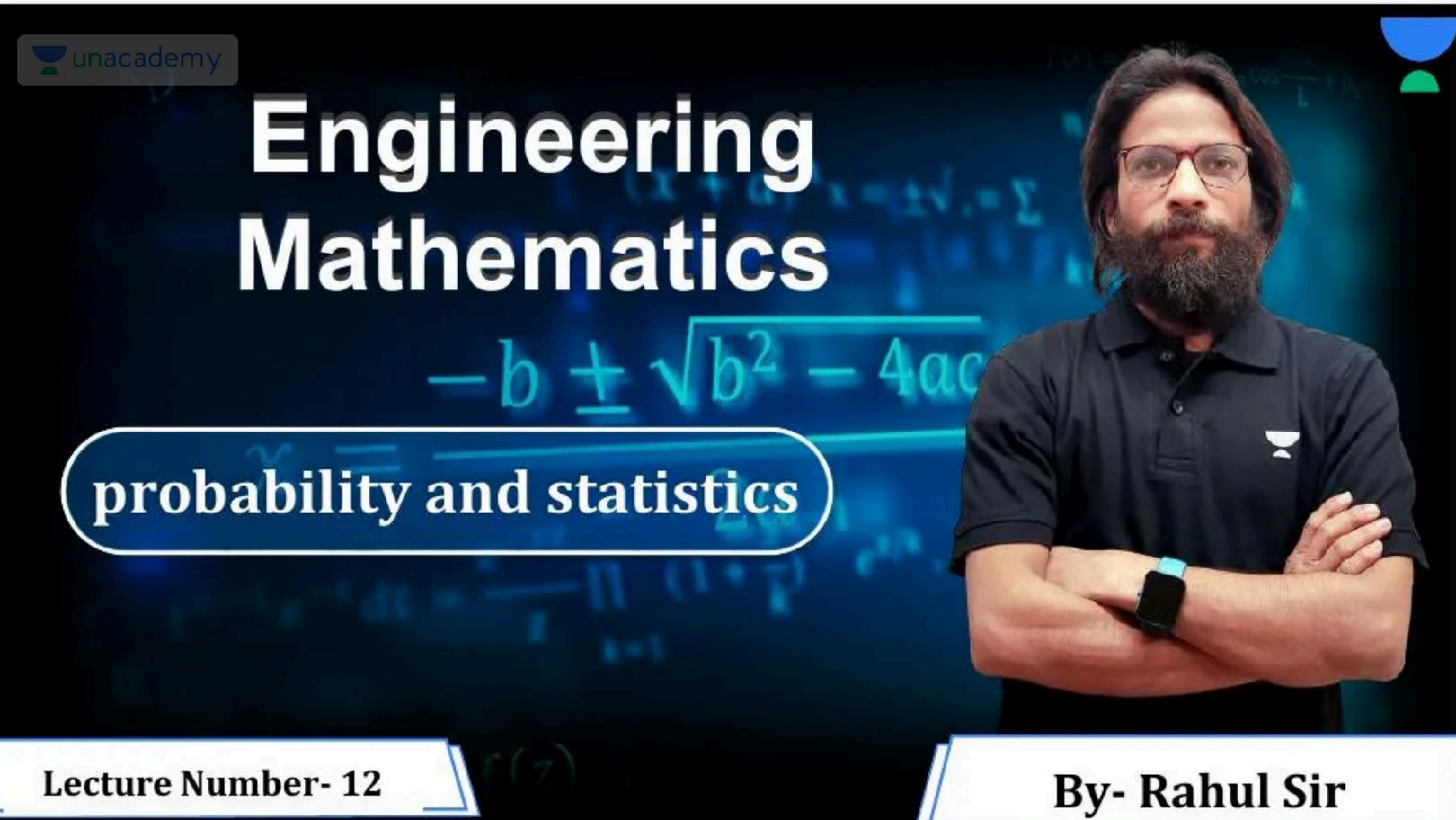
Probability Theory - Part VII

Course on Engineering Mathematics for GATE - CSE

Rahul • Lesson 13 • June 26, 2024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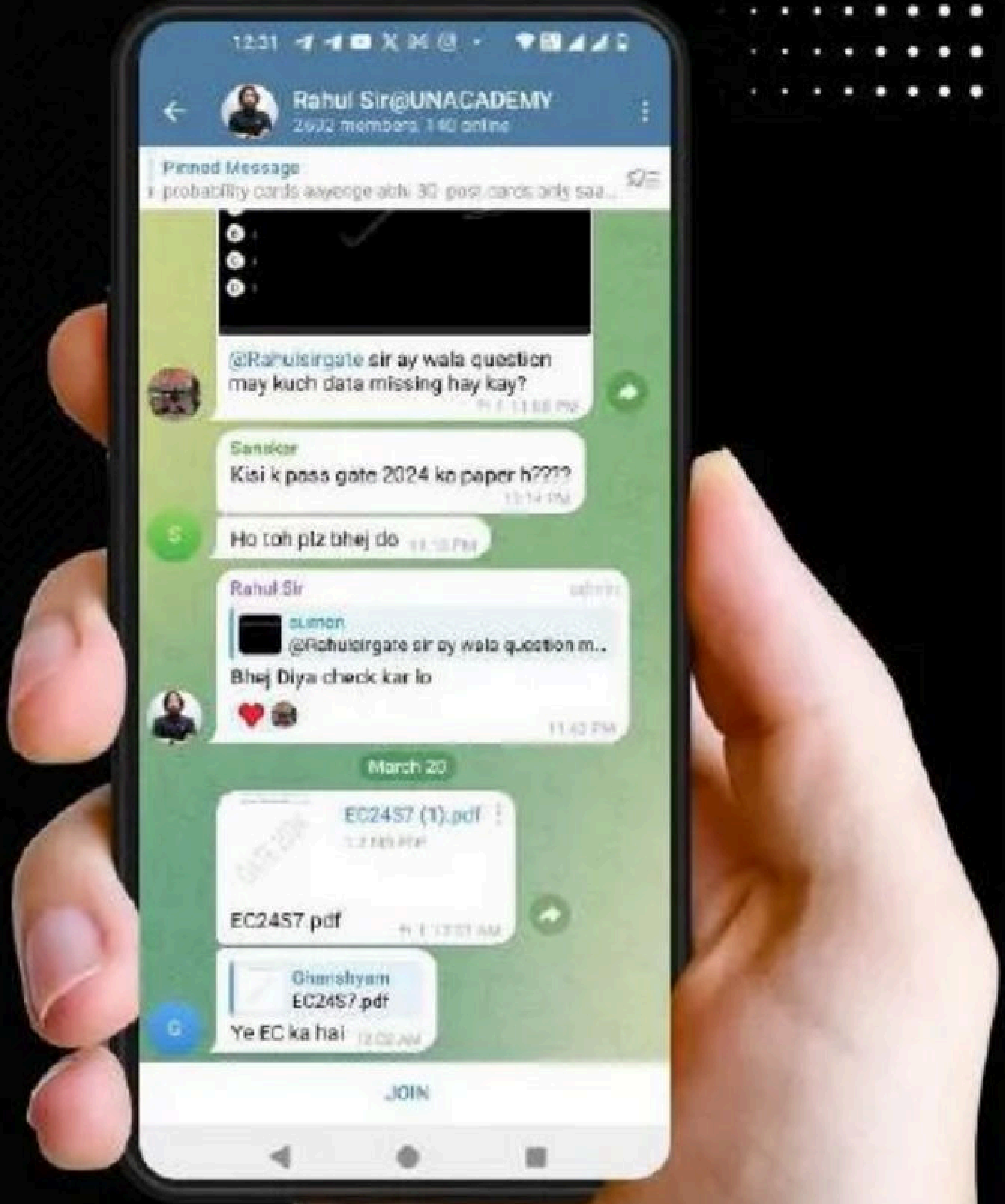
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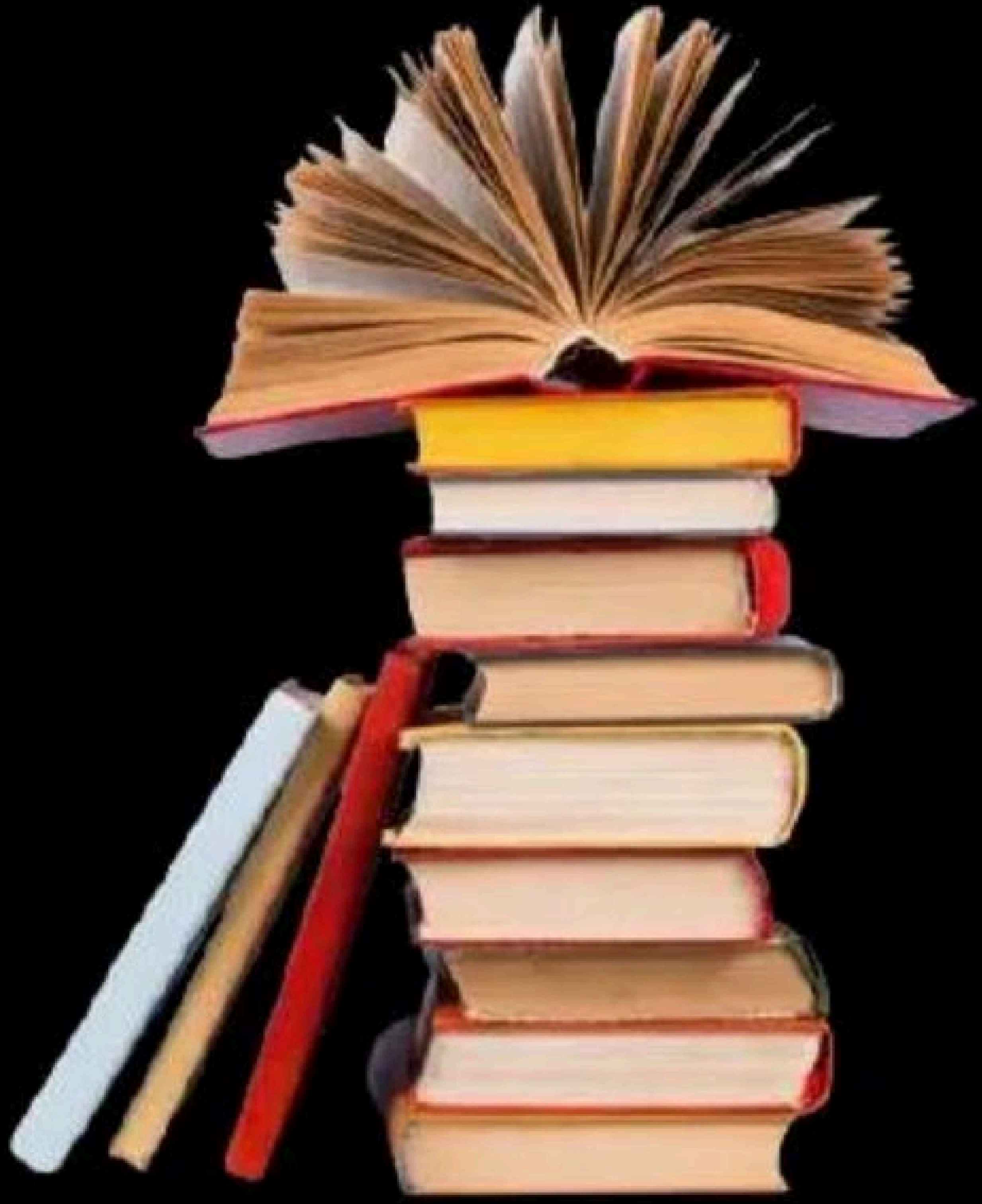
Topics

to be covered



1. Problem solving class_II

QUESTION

Q. Parcels from sender S to receiver R pass sequentially through two post - offices.
Each post - office has a probability 1/5 of losing an incoming parcel, independently of all other parcels. Given that a parcel is lost, the probability that it was lost by the second post - office is ________.

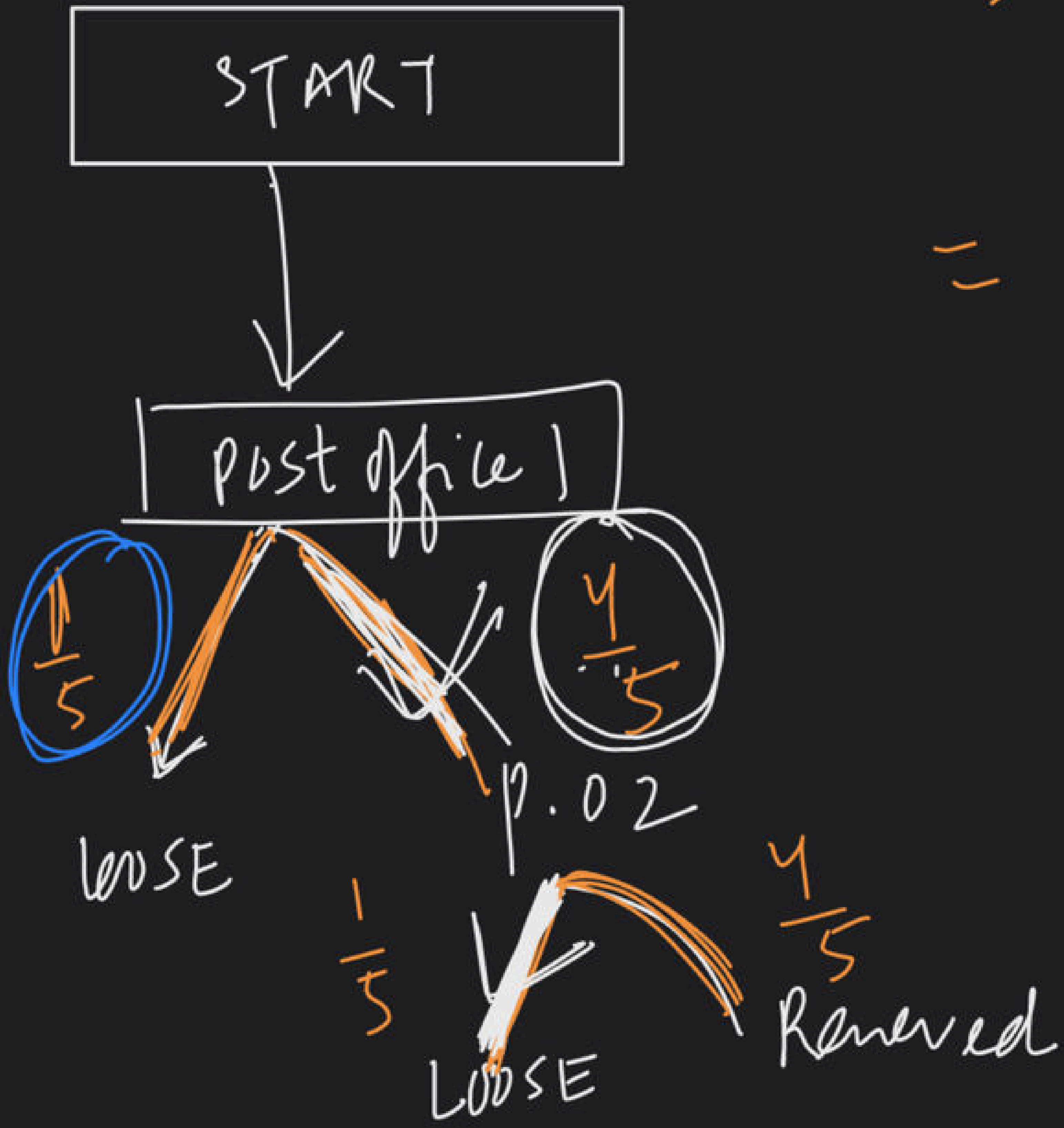
SENDER $\longrightarrow$ Receiver

Through Two post offices.

$P(\text{Loosing}) = \frac{1}{5}$

SENDER → post office 1 → post office 2 / (lost)

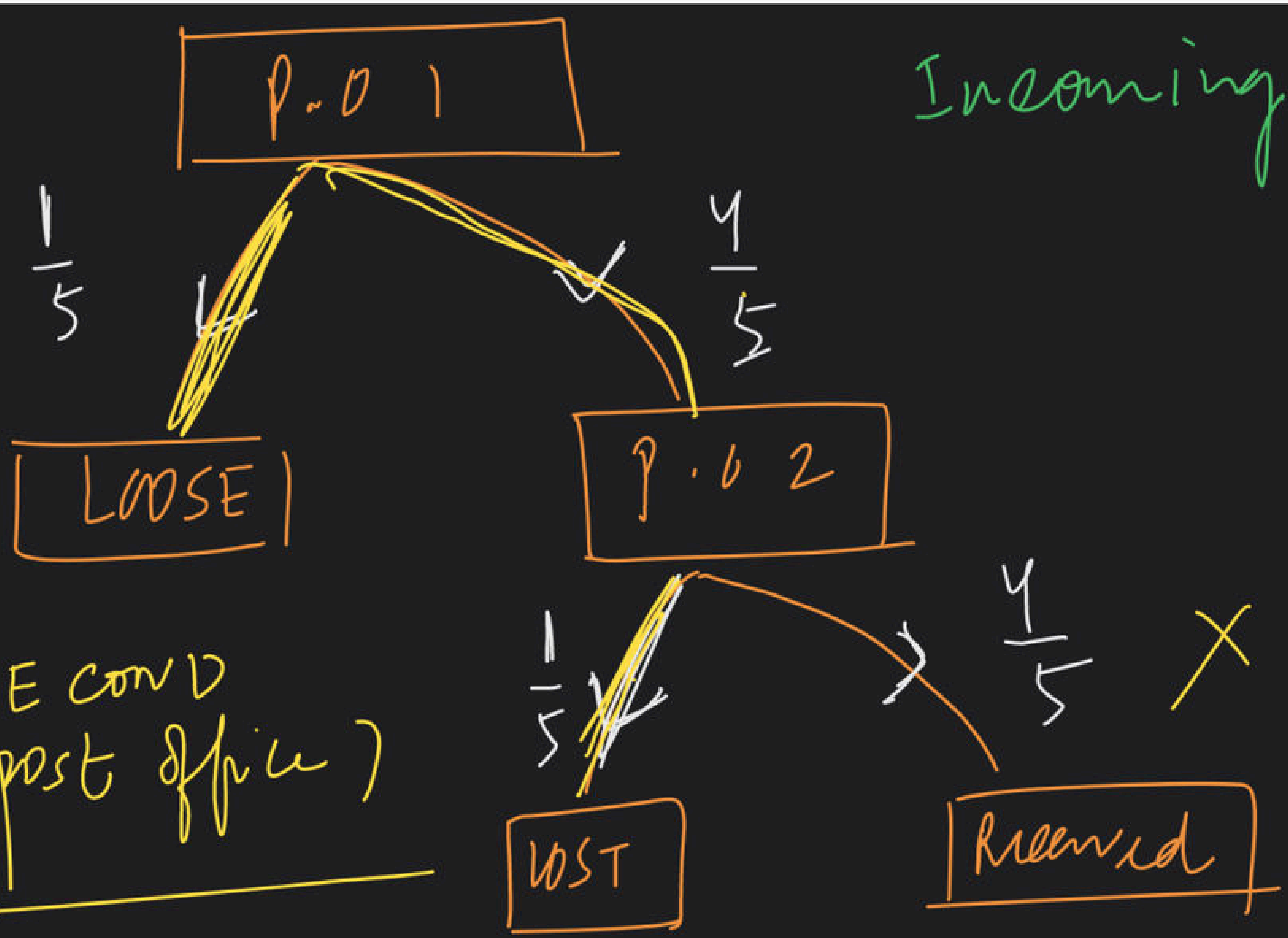
SENDER → loosing



$$= P\left(\frac{\text{second post office}}{\text{parcel is lost}}\right)$$

$$\Rightarrow \frac{\frac{4}{5} \times \frac{1}{5}}{\frac{1}{5} + \frac{4}{5} \times \frac{1}{5}}$$

$$= \frac{\frac{4}{25}}{9/25} = \frac{4}{9}$$



Incoming lost $= \frac{1}{5}$

$$= \frac{\frac{4}{5} \times \frac{1}{5} \ \text{(SECOND post office)}}{\frac{1}{5} + \frac{4}{5} \times \frac{1}{5} \ \text{(prob of lost)}}$$

$$= \frac{4}{9}$$ answer

QUESTION

Q. An anti-aircraft gun can take a maximum of four shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first, second, third and fourth shot are 0.4, 0.3, 0.2, and 0.1, respectively. What is the probability that the gun hits the plane?

H.W

QUESTION

Q. A box contains 2 black, 4 white and 3 red balls. One ball is drawn at random from the box and kept aside. From the remaining balls in the box, another ball is drawn at random and kept beside the first. This process is repeated till all the balls are drawn from the box. Find the probability that the balls drawn are in

H.W

QUESTION

Q. Of the three independent events E_1, E_2 and E_3, the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability P that none of events E_1, E_2 or E_3 occurs satisfy the equations $(\alpha - \beta).P = \alpha\beta$ and $(\beta - 3\alpha).P = 2\beta\gamma$. All the given probability are assumed to lie in the interval (0, 1).

Then, $\dfrac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$ is equal to:

E_1, E_2, E_3 Independent events

\# $P(\text{only } E_1) = \alpha$

$P(\text{only } E_2) = \beta$

$P(\text{only } E_3) = \gamma$

$P(\text{None of THEM}) = P$

$$\boxed{\begin{aligned} E_1(1-E_2)(1-E_3) &= \alpha \\ E_2(1-E_1)(1-E_3) &= \beta \\ E_3(1-E_2)(1-E_1) &= \gamma \\ (1-E_1)(1-E_2)(1-E_3) &= P \end{aligned}}$$

E_1, E_2, E_3 ARE independent.

$(\alpha - \beta)P = \alpha\beta$ — ①

$(\beta - 3\alpha)P = 2\beta\gamma$ — ②

$\dfrac{P(E_1)}{P(E_3)} = \text{Ratio}$

H.W

QUESTION

Q. Let $H_1, H_2, \ldots H_n$ be mutually exclusive with $P(H_i) > 0$, $i = 1, 2, \ldots, n$. Let E be any other event with $0 < P(E) < 1$.

Statement – 1: $P(H_i / E) > P(E / H_i) . P(H_i)$ for $i = 1, 2, \ldots, n$

Statement- 2: $\sum_{i=1}^{n} P(H_i) = 1$

A Statement-1 is true, Statement-2 is true: Statement-2 is a correct explanation for Statement-1.

B Statement-1 is true, Statement-2 is true: Statement-2 is not a correct explanation for Statement-1.

C Statement-1 is true, Statement-2 is false.

D statement -1 is false, statement-2 is true.

QUESTION

✓ "challenging Problem"

Q. Consider three sets $E_1 = \{1, 2, 3\}$, $F_1 = \{1, 2, 3\}$ and $G_1 = \{2, 3, 4, 5\}$. Two elements are chosen at random, without replacement, from the set E_1, and let S_1 denote the of these chosen elements. Let $E_2 = E_1 - S_1$ and $F_2 = F_1 \cup S_1$. Now two elements are chosen at random, without replacement, from the set F_2 and let S_2 denote the set of these chosen elements.

Let $G_2 = G_1 \cup S_2$. Finally, two elements are chosen at random, without replacement, from the set G_2 and let S_3 denote the set of these chosen elements.

Let $E_3 = E_2 \cup S_3$. Given that $E_1 = E_3$, let p be the conditional probability of the event $S_1 = \{1, 2\}$. Then the value of p is

(A) 1/5

(B) 3/5

(C) 1/2

(D) 2/5

QUESTION

Challenging problem

Q. Suppose that

Box-I contains 8 red, 3 blue and 5 green balls,

Box-II contains 24 red, 9 blue and 15 green balls,

Box-III contains 1 blue, 12 green and 3 yellow balls,

Box-IV contains 10 green, 16 orange and 6 white balls.

A ball is chosen randomly from Box-I; call this ball b. If b is red then a ball is chosen randomly from Box-II, if b is blue then a ball is chosen randomly from Box-III, and if b is green then a ball is chosen randomly from Box-IV. The conditional probability of the event 'one of the chosen balls is white' given that the event 'at least one of the chosen balls is green' has happened, is equal to

(A) $\frac{15}{256}$

(B) $\frac{3}{16}$

(C) $\frac{5}{52}$

(D) $\frac{1}{8}$

QUESTION

Q. In a test, an examinee either guesses or copies or knows the answer to a multiple choice question with four choices (more than one choice is correct). The probability that he makes a guess is 1/3 and the probability that he copies the answer is 1/6. The probability that his answer is correct, given that he copied it, is 1/8, find the probability that he knew the answer to the question, given that he correctly answered it.

A) 0.85

B) 0.92

C) 0.95

D) 0.90

A B C D

ONE option is correct $= \frac{1}{4}$

A B C D

MSQ GATE / IIT advanced

4 options

$1 + 2 + 3 + 4$

Total No. of choices

$= {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$

$= 4 + 6 + 4 + 1$

$= 15$ choices

1	2	3	4
A	A, B	A B C	A B C D
B	B, C	B C D	
C	C, D	D B A	
D	D A	⋮	
	A D		
	B D		

${}^4C_1$ OR ${}^4C_2$ OR ${}^4C_3$ OR ${}^4C_4$

= answer is correct

P(ONE question is correct guess) $= \frac{1}{15}$

4 options $\rightarrow {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$

n options $\rightarrow {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n$

$= \boxed{2^n - 1}$ atleast one Perform

A B C D - leave

$\Downarrow$

Not Performed $= {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_2 + \cdots + {}^nC_n$

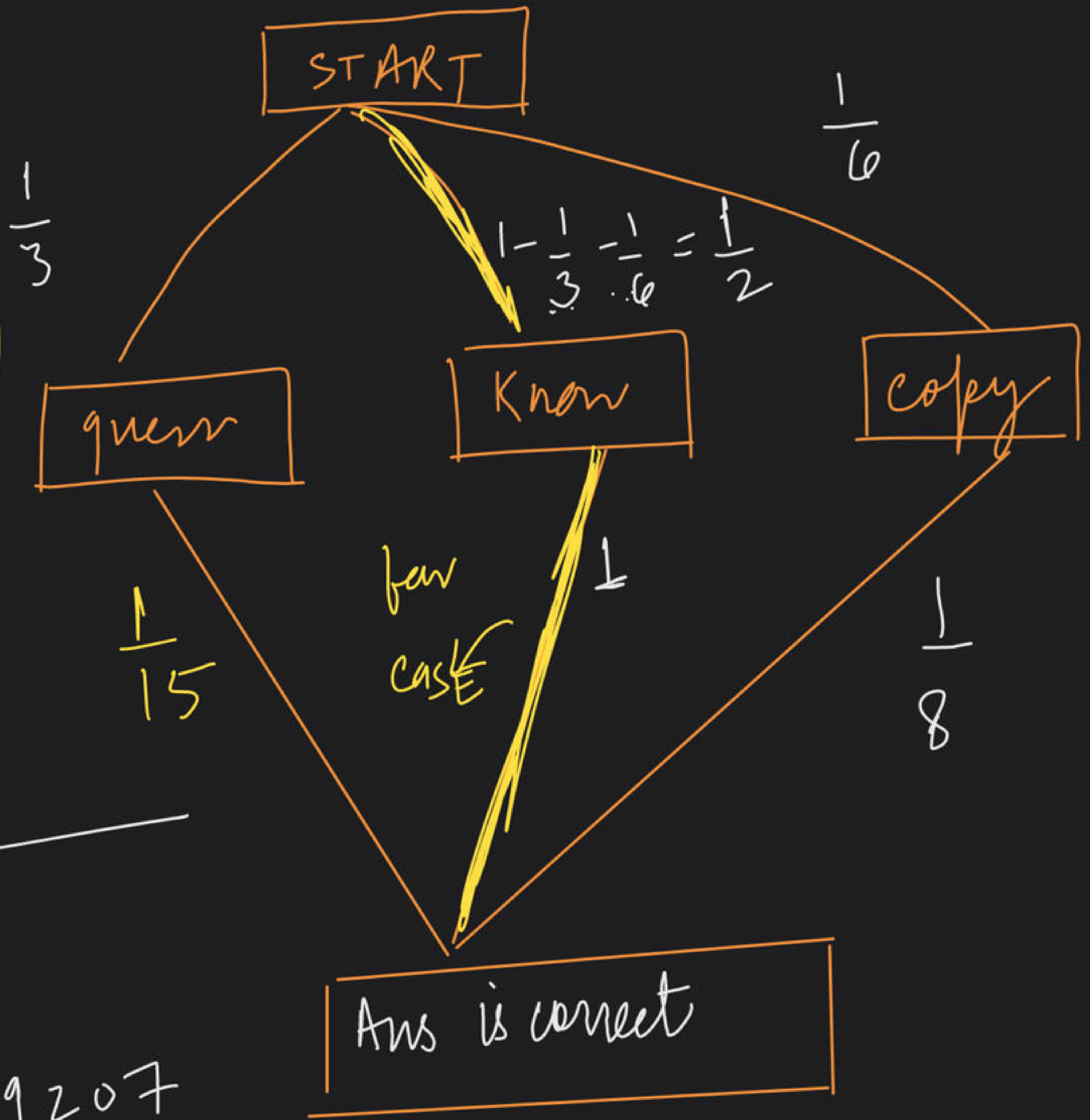
$= \boxed{2^n}$

$$P\left(\frac{\text{Know}}{\text{Ans is correct}}\right) = \frac{P(\text{Know})\, P\left(\frac{\text{Ans is correct}}{\text{Know}}\right)}{\text{Total}}$$

→ bayes THEOREM

$$\Rightarrow \frac{1 \times \frac{1}{2}}{1 \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{15} + \frac{1}{6} \times \frac{1}{8}}$$

$\Rightarrow 0.9207$ ✓ done ✓



QUESTION

Q. Cards are drawn one by one without replacement from a pack of 52 cards till all the aces are drawn out. What is the probability that only two cards are left unturned when all aces are out?

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QUESTION

IIT - advanced

Q. There are three bags B_1, B_2 and B_3. The bag B_1 contains 5 red and 5 green balls, B_2 contains 3 red and 5 green balls, and B_3 contains 5 red and 3 green balls. Bags B_1, B_2 and B_3 have probabilities $\frac{3}{10}$, $\frac{3}{10}$ and $\frac{4}{10}$ respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is(are) correct?

MSQ

(MSQ)

(A) Probability that the selected bag is B_3 given that the chosen ball is green, equals $\frac{5}{13}$

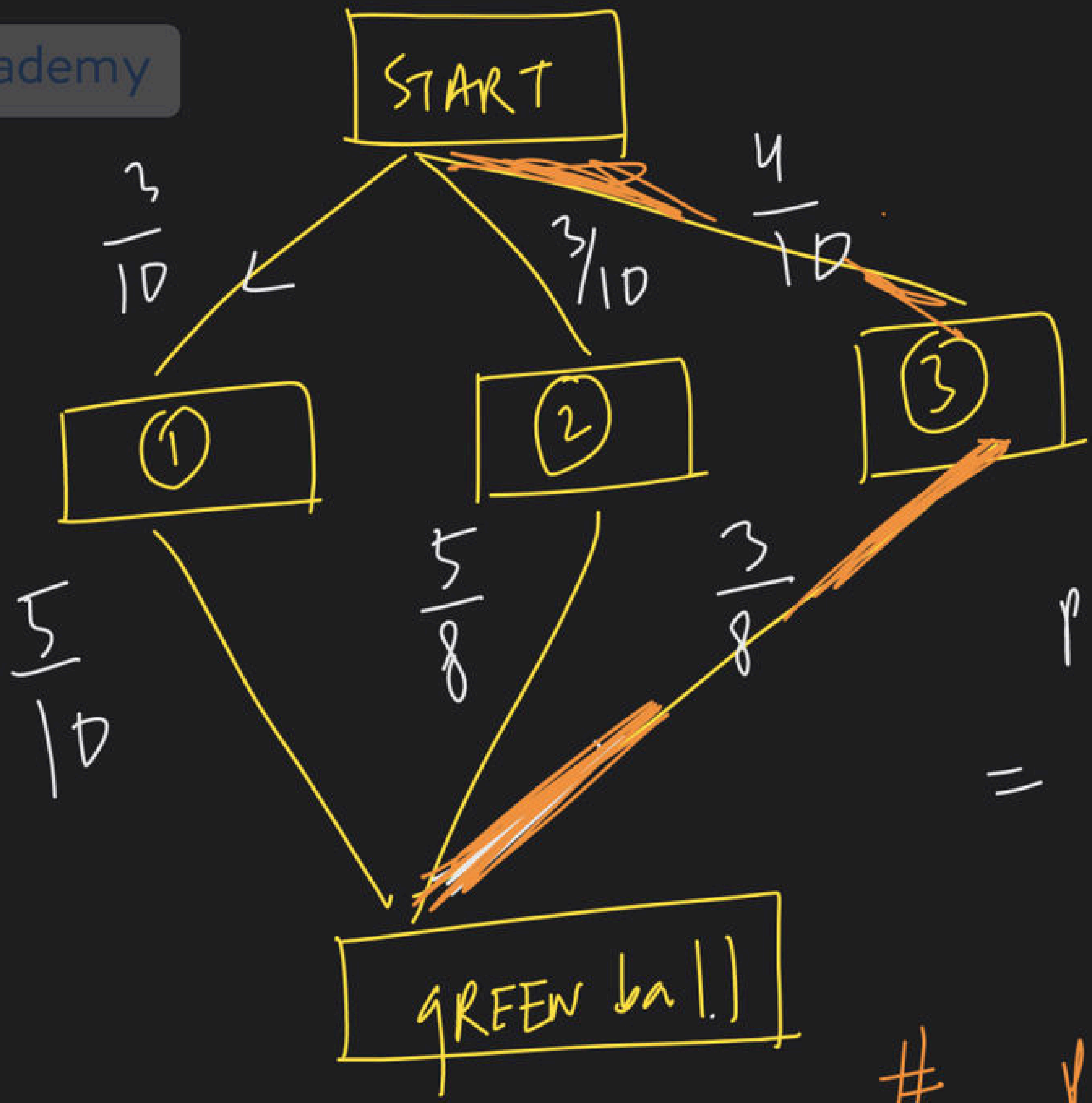
(B) Probability that the chosen ball is green equals $\frac{39}{80}$

Total prob.

g g g

(C) Probability that the selected bag is B_3 and the chosen ball is green equals $\frac{3}{10}$

(D) Probability that the chosen ball is green, given that the selected bag is B_3, equals $\frac{3}{8}$



#

Bag 1	Bag 2	Bag 3
5R	3R	5R
5G	5G	3G

$P(B_1) = \frac{3}{10}$ $\quad P(B_2) = \frac{3}{10}$ $\quad P(B_3) = \frac{4}{10}$

$P(\text{GREEN ball})$

$$= \frac{3}{10} \times \frac{5}{10} + \frac{3}{10} \times \frac{5}{8} + \frac{4}{10} \times \frac{3}{8} = \frac{39}{80}$$

(B) done

$P\left(\frac{\text{GREEN ball}}{\text{bag } 3}\right) = \frac{3}{8}$ (D)

QUESTION

Q. Let S be the sample space of all 3×3 matrices with entries from the set $\{0, 1\}$.

Let the events E_1 and E_2 be given by $E_1 = \{A \in S : \det A = 0\}$ and $E_2 = E_2 = \{A \in S : \text{Sum of entries A is} 7\}$.

If a matrix is chosen at random from S, then the conditional probability $P(E_1/E_2)$ equals

QUESTION

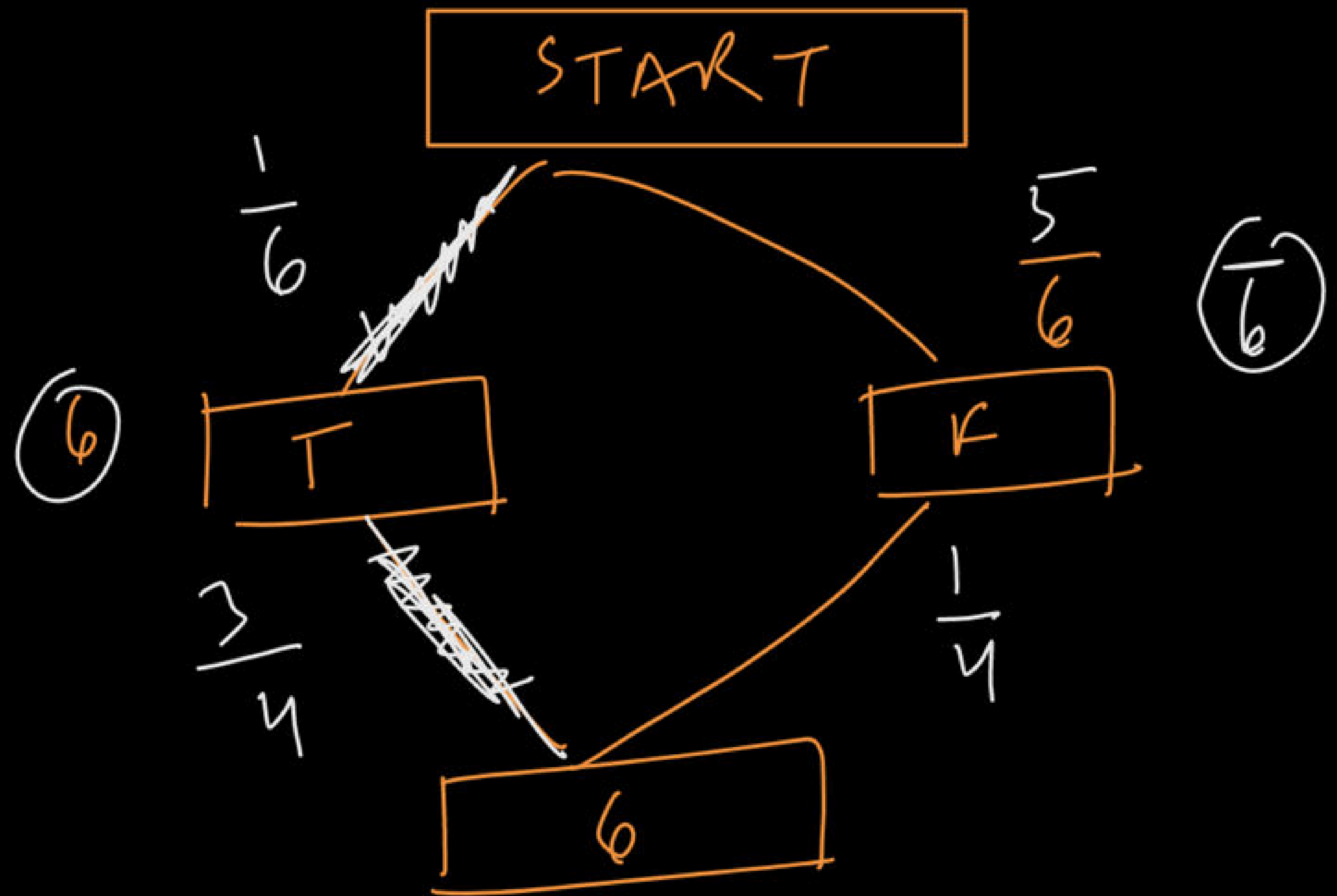
Answer = $\frac{3}{8}$.

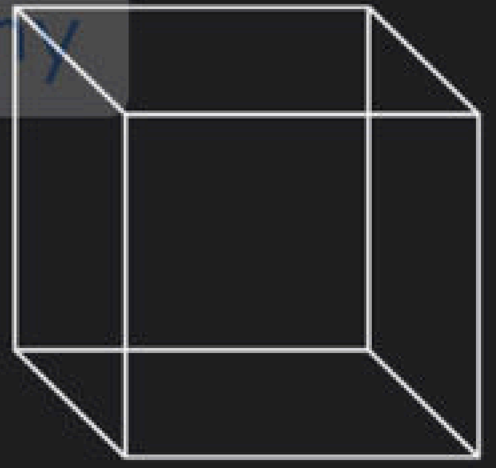
Q. A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is a six. The probability that it is actually a six is

Problems Based on Speaking Truth

$$= \frac{\frac{1}{6} \times \frac{3}{4}}{\frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4}}$$

$$= \frac{\frac{3}{24}}{\frac{8}{24}} = \frac{3}{8}$$





A Die is THROWN

$S = \{1, 2, 3, 4, 5, 6\}$

\# Akash $= \boxed{6}$ (Truth)

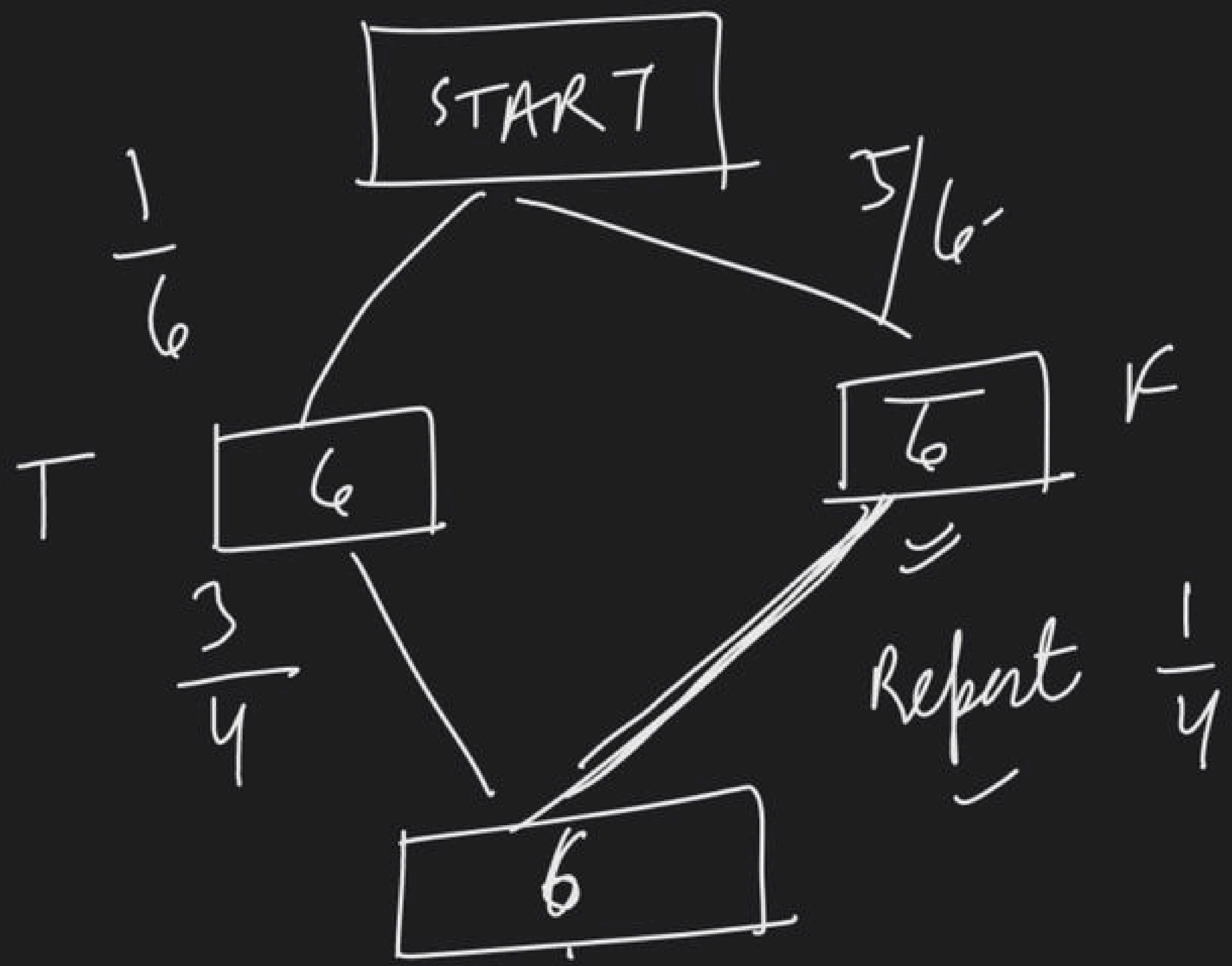
1, 2, 3, 4, 5 (False)

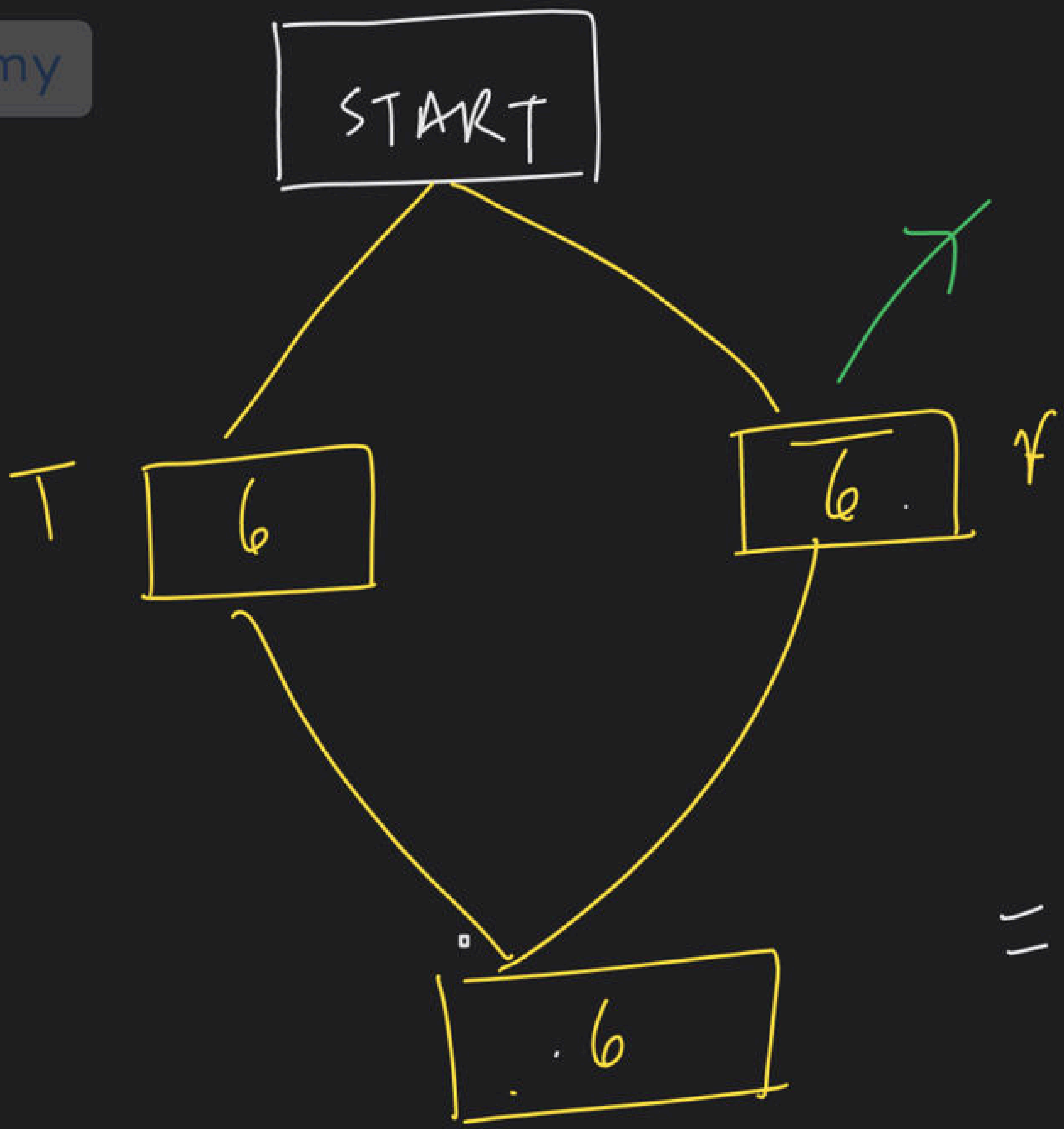
Krishan - Report

6

$= \dfrac{3}{8}$

$P\left(\dfrac{\text{Report } 6}{\text{Actually } 6}\right)$





wrong - single wrong

$$= P\left(\frac{\text{Repeat } 5}{\text{Actually } 6}\right)$$

$$= P\left(\frac{\text{Repeat } 1}{\text{Actually } 6}\right)$$

$\longrightarrow$ $\underbrace{1, 2, 3, 4, 5,}_{(1-p)} \;\Big|\; \underset{p}{6}$

1 2 3
4 5

Prob. of correct letter $= p$.

particular wrong letter

Prob. of wrong letter $= (1-p)$

Prob. of particular wrong letter $= \underline{\underline{x}}$

$1, 2, 3, 4, 5, 6$ = Mutually exclusive events

$$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$$

$$x + x + x + x + x + p = 1$$

$$5x + p = 1$$

$$\checkmark \quad \boxed{x = \frac{1-p}{5}}$$

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$$P(\text{particular wrong letter}) = \frac{1-p}{(n-1)}$$

1 2 3 4 5 | 6

$\underbrace{1 \quad 2 \quad 3 \quad 4 \quad 5}_{1-p}$ $\quad$ 6: p

$$(1-p) = \frac{1-p}{5} + \frac{1-p}{5} + \frac{1-p}{5} + \frac{1-p}{5} + \frac{1-p}{5}$$

$\boxed{\text{Prob. of particular wrong letter} = \frac{1-p}{(n-1)}}$

$E_1, E_2, \ldots E_n$

$P(E_1) + P(E_2) + \ldots + P(E_n) = 1$

$p + x + x + \ldots + (n-1)$ times $= 1$

$P(E_1) = p$

$P(E_2) = x$

$P(E_3) = x$

$\vdots$

$x(n-1) = 1-p$

$$\boxed{x = \frac{1-p}{(n-1)}}$$

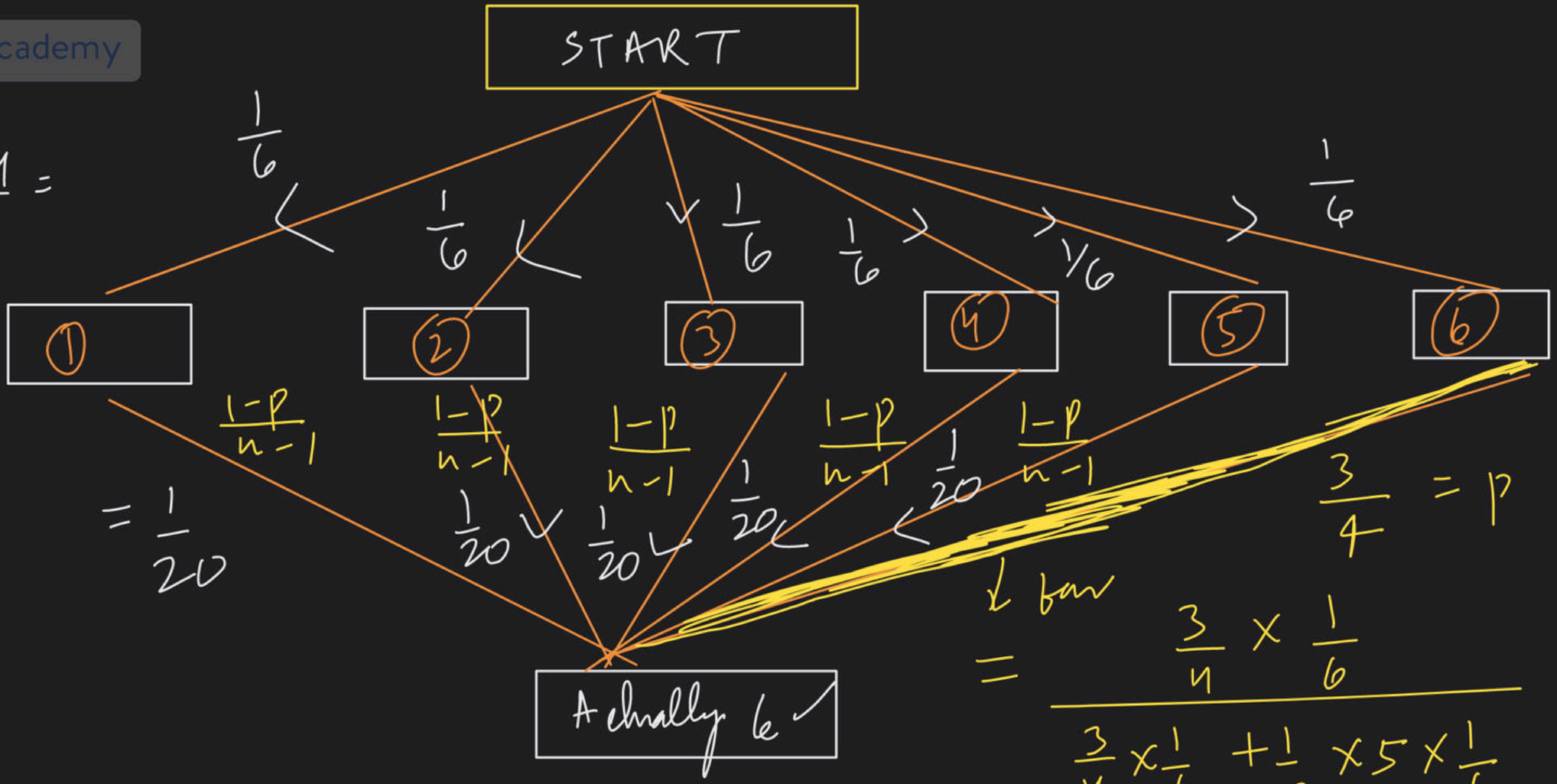
particular wrong letter / Dots

Right / T

wrong / F

$\Rightarrow$ Silence w × k × ×

$$\frac{1-p}{n-1} = \frac{1-3/4}{6-1} = \frac{1}{20}$$

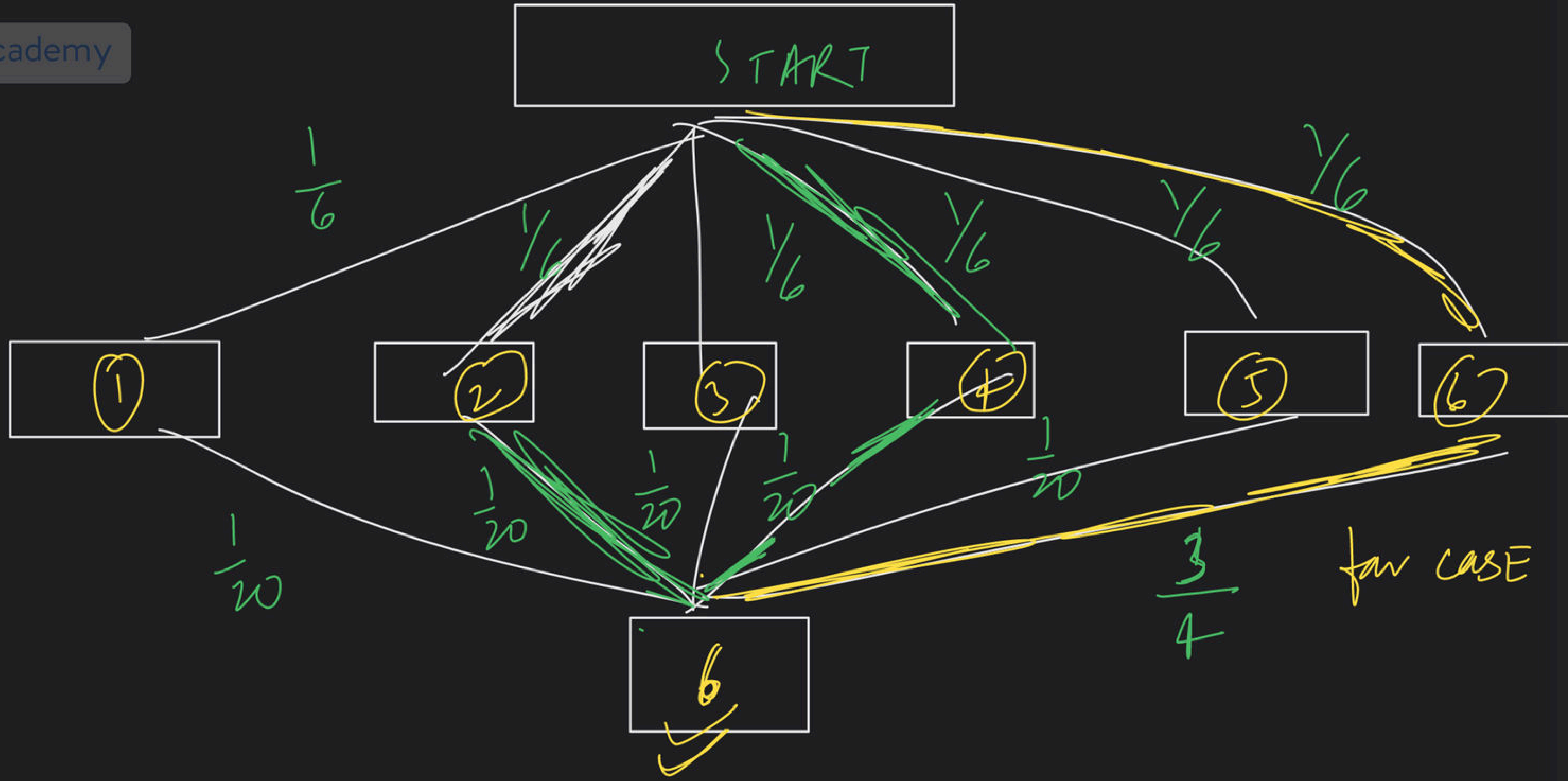


$$\frac{3}{4} = p$$

↓ fair

$$= \frac{\frac{3}{4} \times \frac{1}{6}}{\frac{3}{4} \times \frac{1}{6} + \frac{1}{20} \times 5 \times \frac{1}{6}}$$

$$\Rightarrow \quad 3/4$$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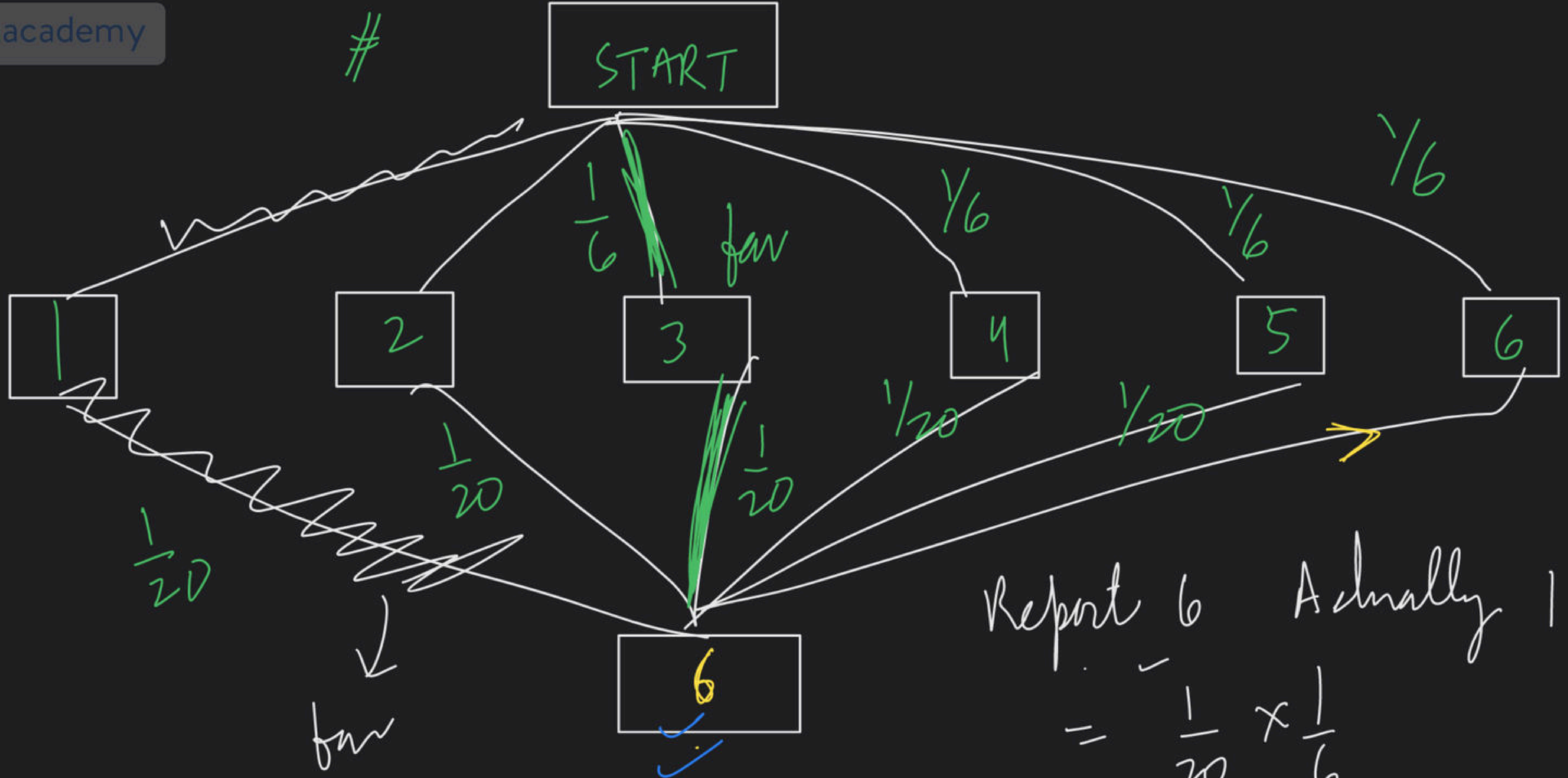
fav case
6



Report 6 Actually 1

$$= \frac{\frac{1}{20} \times \frac{1}{6}}{\text{Total}}$$

QUESTION

Q. A box contains three coins, one coin is fair, one coin is two-headed, and one coin is weighted so that the probability of head appearing is $1/3$. A coin is selected at random and tossed. Find the probability that

(i) head

(ii) tail appears.

QUESTION

Q. A letter is to come from either LONDON or CLIFTON. The postal mark on the letter legibly shows consecutive letters "ON". The probability that the letter has come from LONDON is

(A) $\frac{12}{17}$

(B) $\frac{13}{17}$

(C) $\frac{5}{17}$

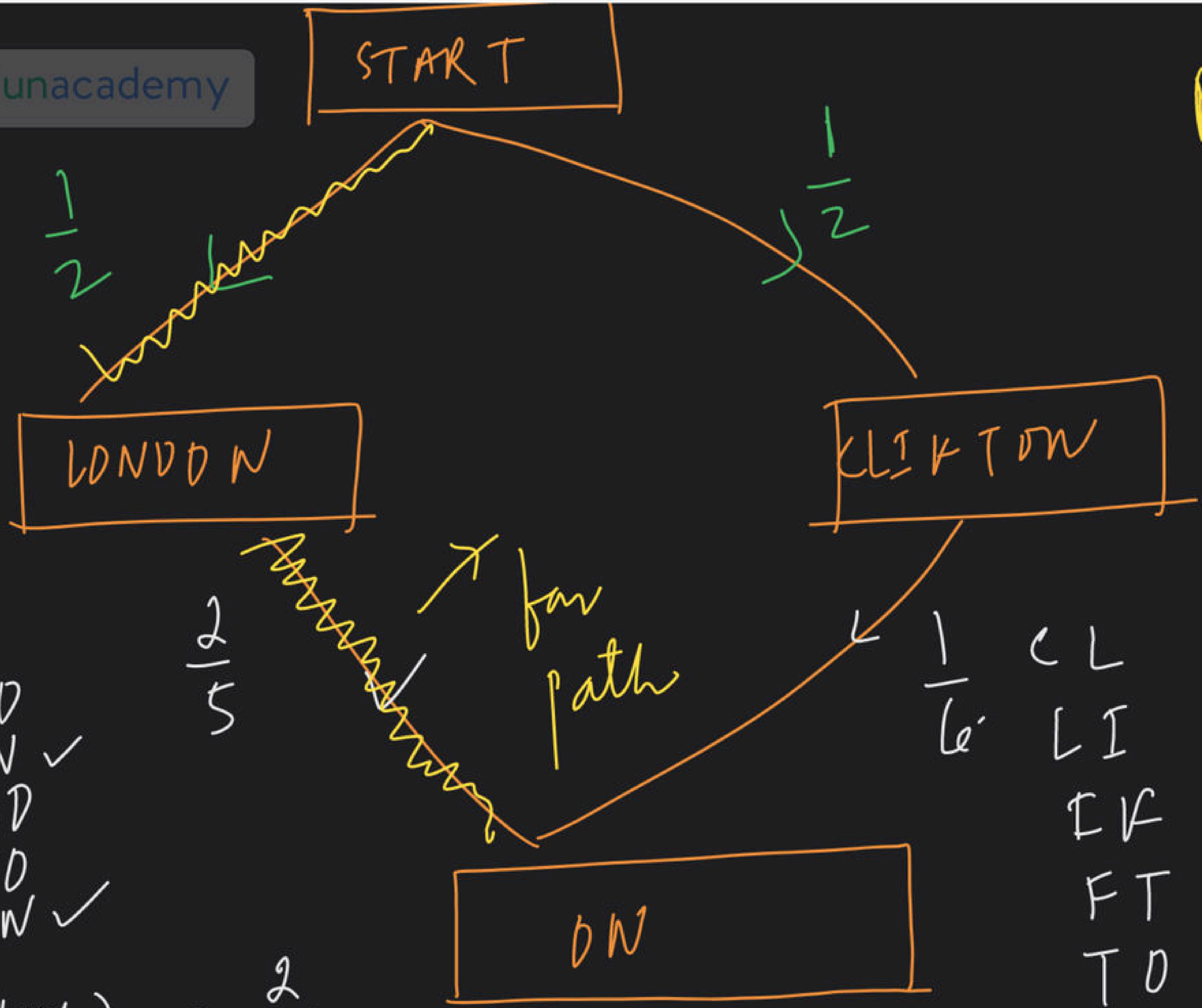
(D) $\frac{4}{17}$

"LONDON" OR "CLIFTON"

$\boxed{\text{CLIF}}$ ON

LOND

$P\left(\frac{\text{LONDON}}{\text{ON}}\right)$



LO
ON ✓
ND
DO
ON ✓

$P(ON) = \frac{2}{5}$

CL
LI
IF
FT
TO
ON

$P\left(\frac{LONDON}{ON}\right)$ = Inverse prob.

bayes THEOREM

$$= \frac{\frac{1}{2} \times \frac{2}{5}}{\frac{1}{2} \times \frac{1}{6} + \underbrace{\frac{1}{2} \times \frac{2}{5}}} \checkmark$$

$$= \frac{12}{17}$$ Ans.

QUESTION

CSE + DATA SCIENCE (Bayes)

Q. A bag contains 4 balls of unknown colours. A ball is drawn at random from it and is found to be white. The probability that all the balls in the bag are white is

(A) 4/5

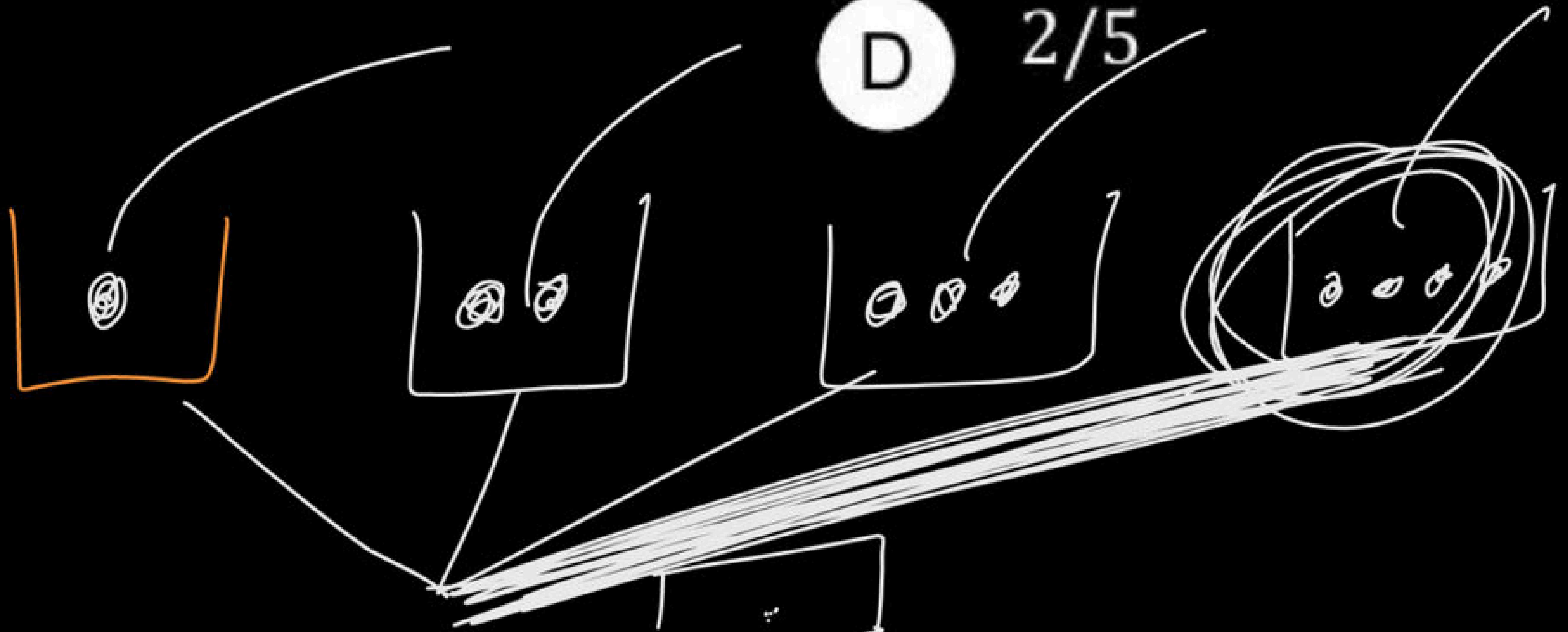
(B) 1/5

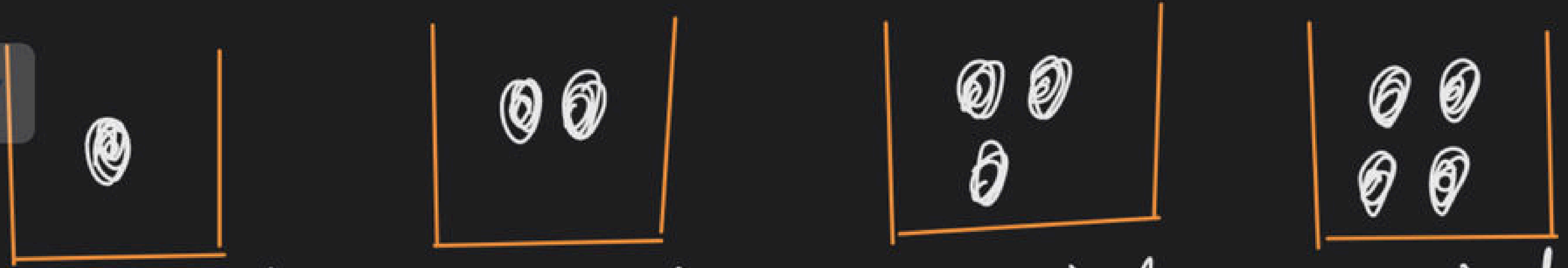
(C) 3/5

(D) 2/5

white ball

1 ball W
2 ball W
3 ball W
4 ball W

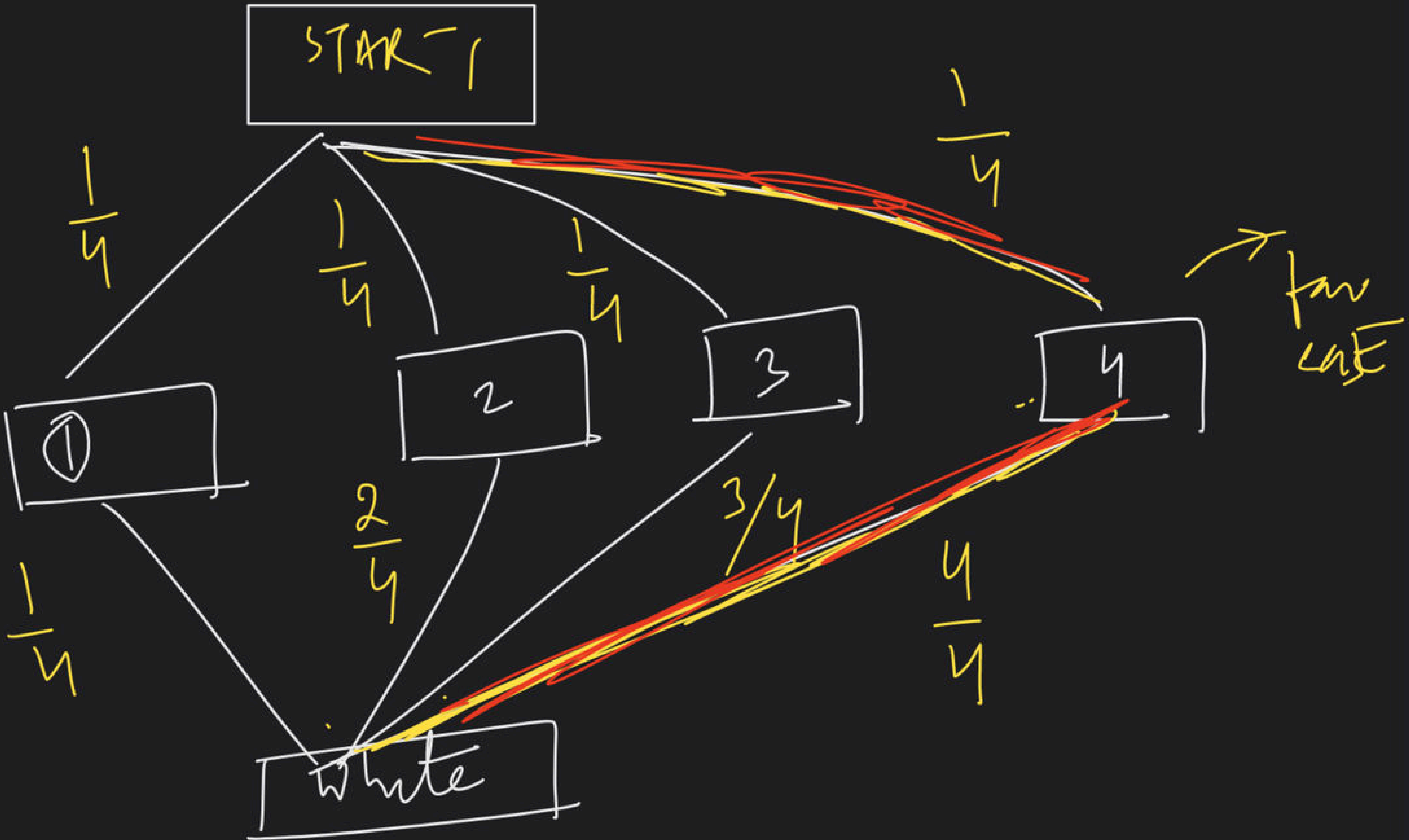




1 white 2 white 3 white 4 white

$$= \frac{\frac{1}{4} \times \frac{4}{4}}{\frac{1}{4}\times\frac{1}{4} + \frac{1}{4}\times\frac{2}{4} + \frac{1}{4}\times\frac{3}{4} + \frac{1}{4}\times\frac{4}{4}}$$

$$= \frac{2}{5}$$



QUESTION

Q. Box I contains 4 red, 5 white balls and box II contains 3 red, 2 white balls. Two balls are drawn from box I and are transferred to box II. One ball is then drawn from box II. Find the probability that:

(a) ball drawn from box II is white, (Total prob)

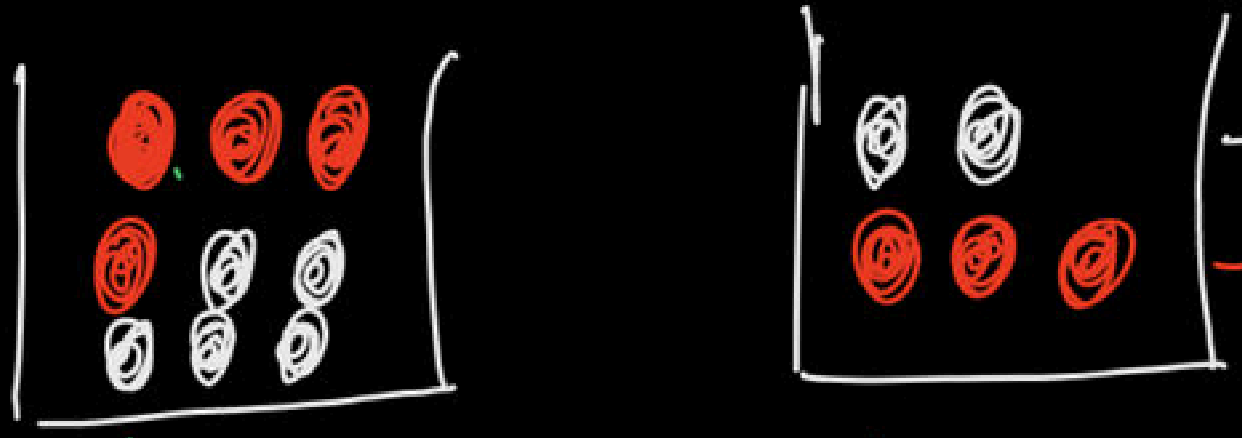
(b) the transferred balls were both red given that the ball drawn from box II is white.

$= P\left(\frac{\text{both Red}}{\text{box II is White}}\right)$

= (INVERSE / BAYES THEOREM)

4 Red

5 White



→ 2 White

→ 3 Red

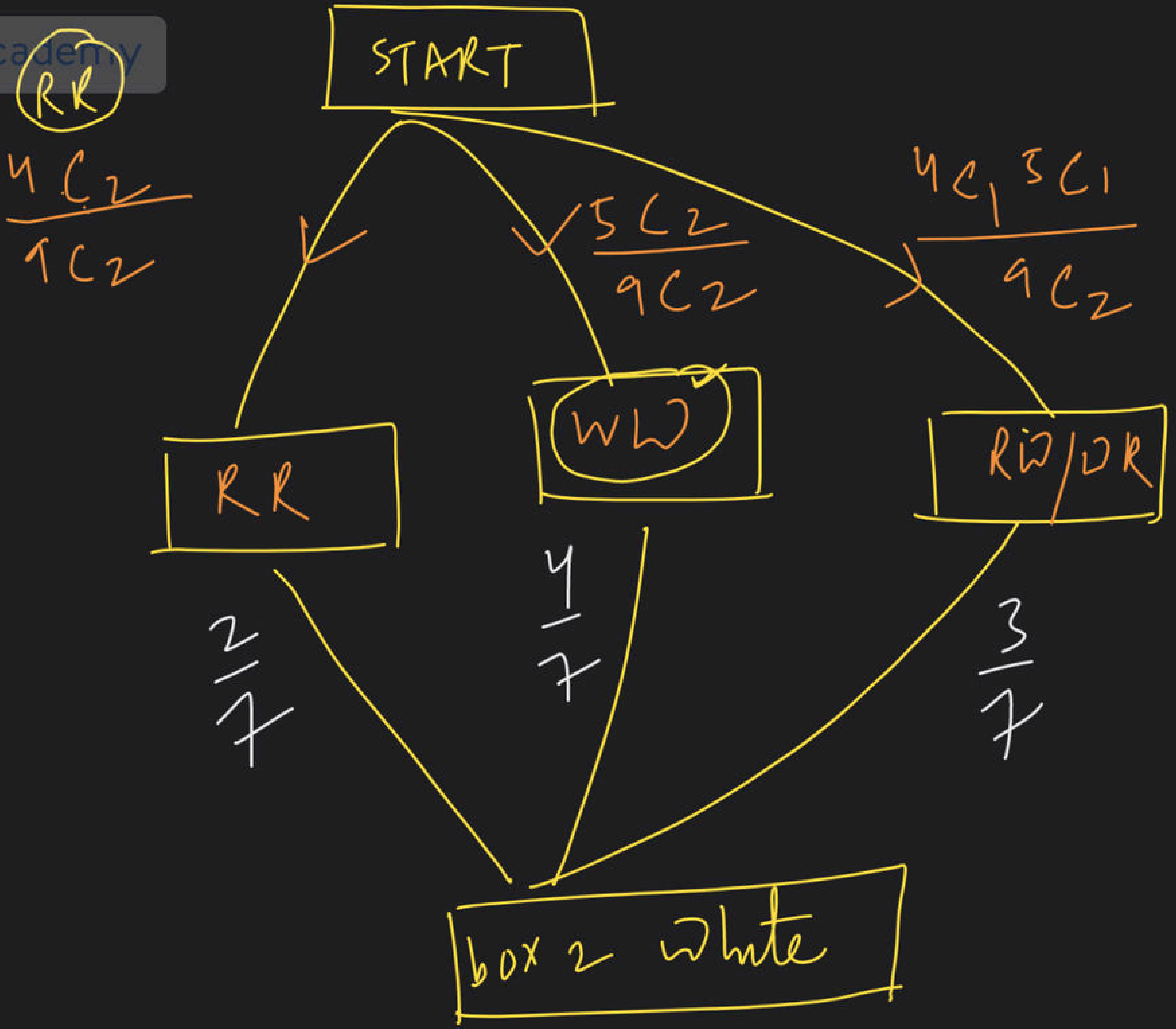
box 1

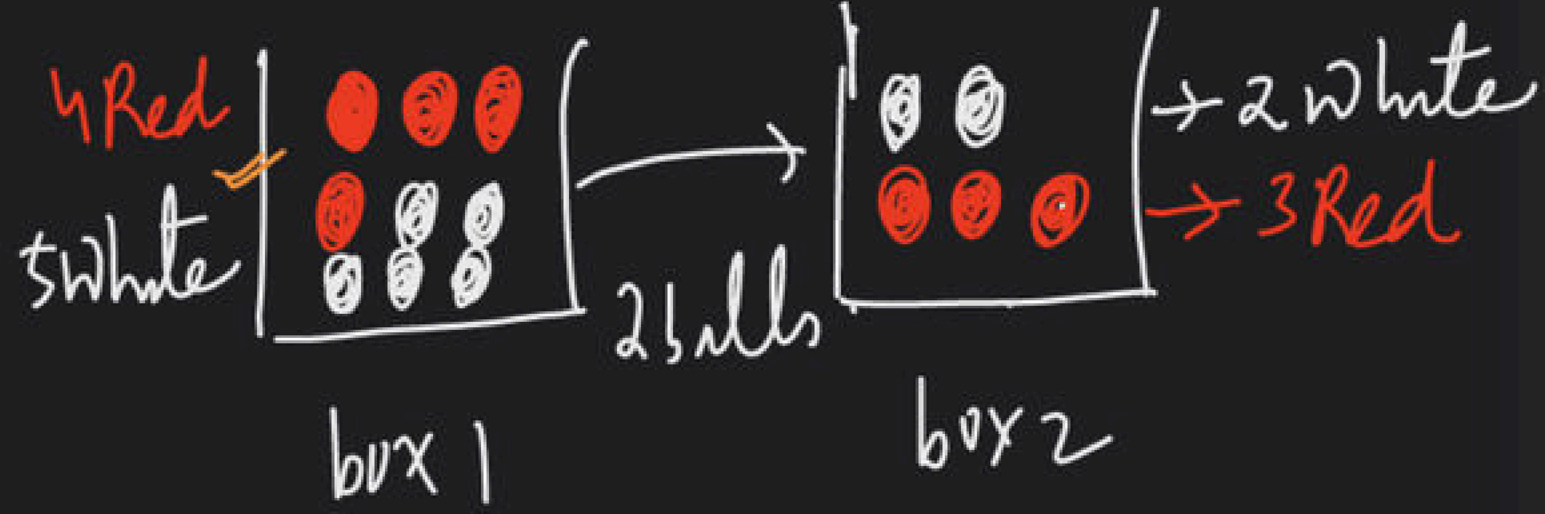
box 2

2 balls: RR

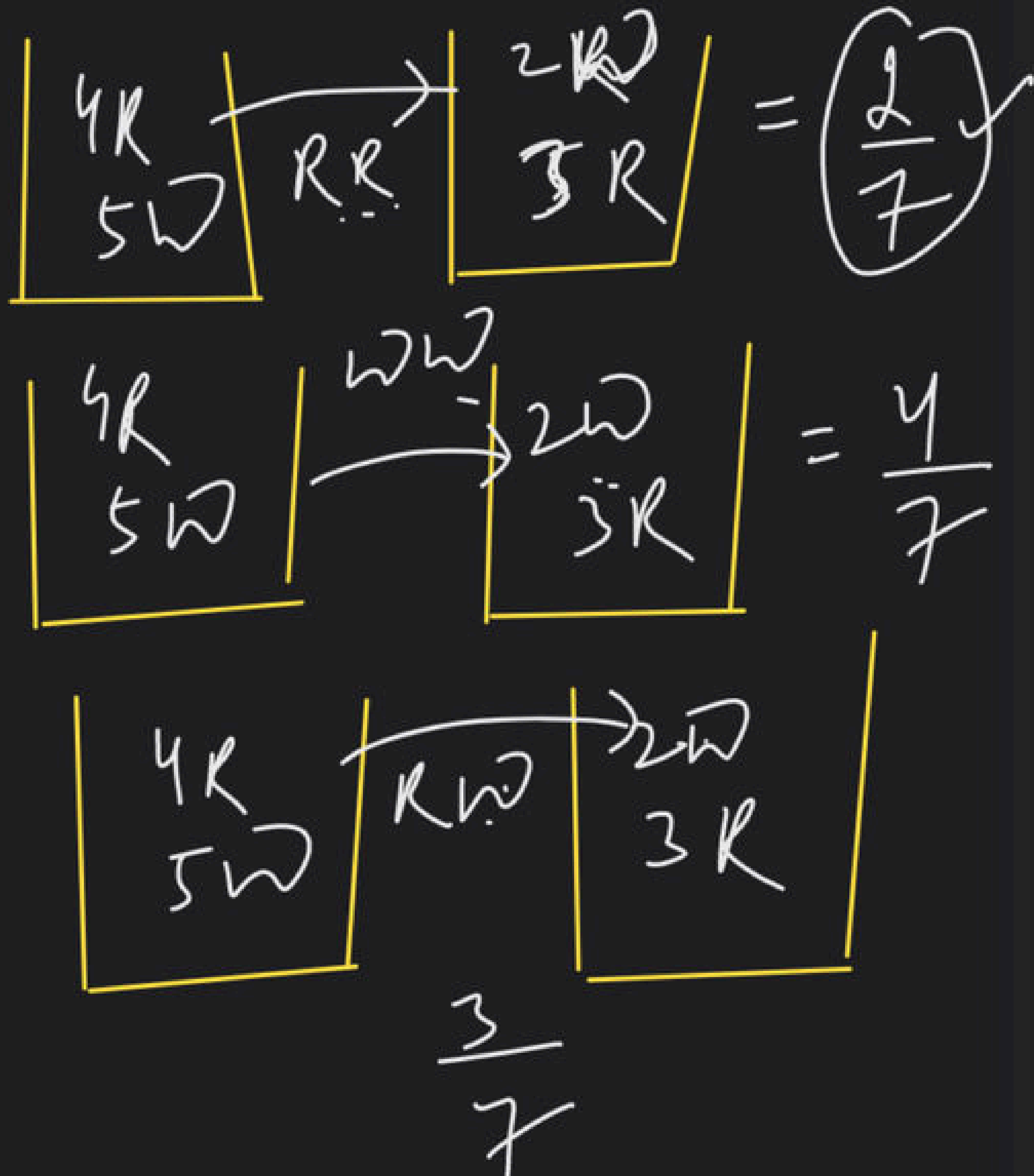
WW

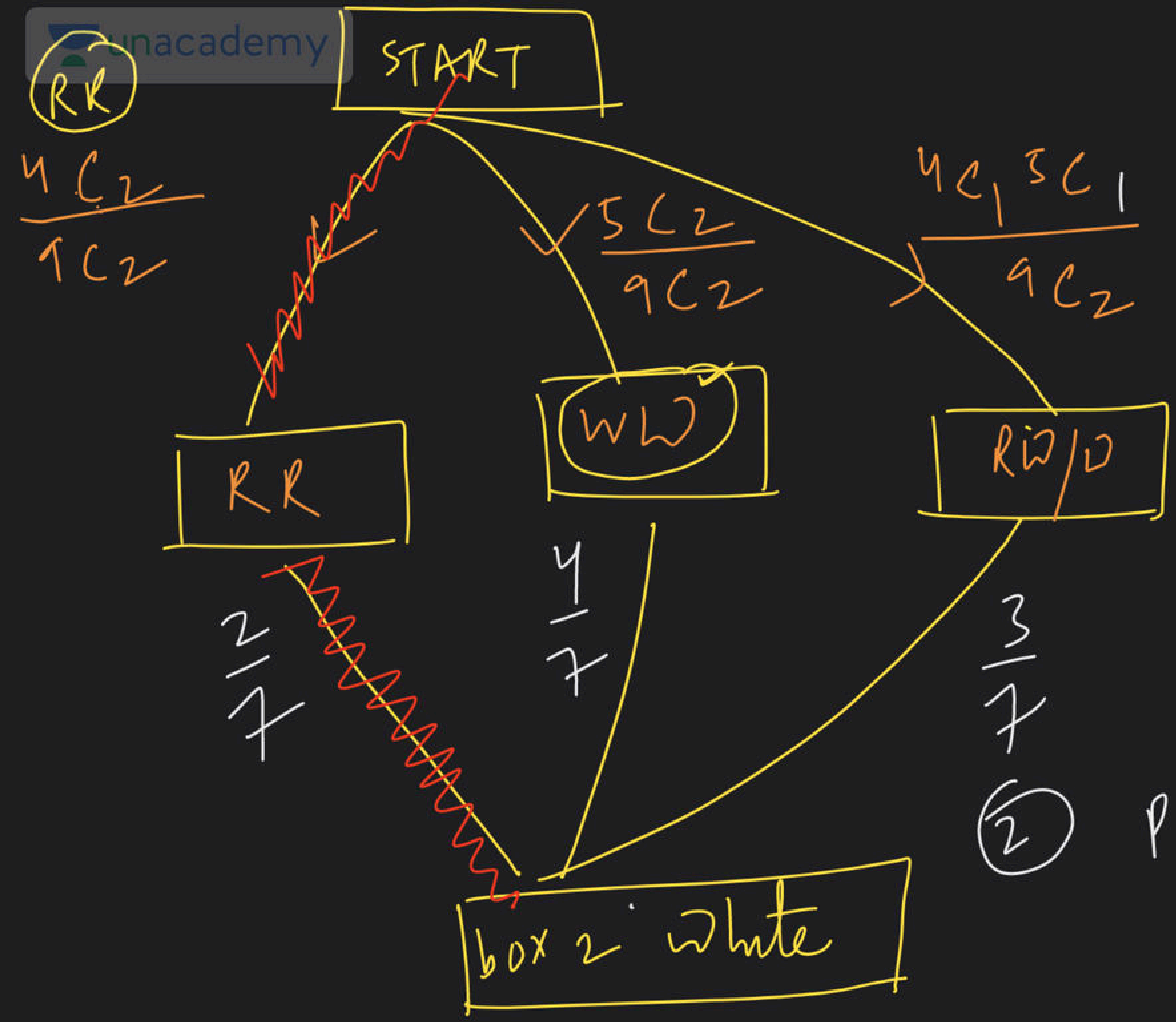
RW OR WR





box 2 → White ball.





①

$\Rightarrow P(\text{White})$

$$\Rightarrow \frac{^4C_2}{^9C_2}\times\frac{2}{7}+\frac{^5C_2}{^9C_2}\times\frac{4}{7}+\frac{^4C_1\,^5C_1}{^9C_2}=\boxed{\frac{4}{9}}$$

②

$$P\left(\frac{RR}{\text{box2 white}}\right)=\frac{\frac{^4C_2}{^9C_2}\times\frac{2}{7}}{\text{Total}}$$

$$\Rightarrow \boxed{\frac{3}{28}}$$

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THANK YOU!

Here's to a cracking journey ahead!

P and C

Prob. + DPP