More about

Linearly Dependence

Can we have more than 2 independent vectors in R^2 ?

$$\left\{ \underbrace{\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix}}_{LI}, \begin{bmatrix} 3 \\ 8 \end{bmatrix} \right\}$$

$$\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 8 \end{bmatrix} \right\}$$

is this set is linearly dependent or indep.?

Question :

LD

$$\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \end{bmatrix} \right\}$$ are LI or LD

$$\begin{bmatrix} 3 \\ 5 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 5 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Hit and trial

LD

Question :

$$\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 10 \\ y_2 \end{bmatrix} \right\}$$ are LI or LD

$$\begin{bmatrix} 10 \\ y_2 \end{bmatrix} = 10 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Hit and trial

LD

Question :

$$\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \right\}$$ are LI or LD

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Hit and trial

LD

Question :

$$\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3.1 \\ 57 \end{bmatrix} \right\}$$ are LI or LD

$$\begin{bmatrix} 3.1 \\ 57 \end{bmatrix} = 3.1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 57 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Hit and trial

Convenient vectors in R^2 $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \underline{\alpha} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \underline{\beta} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

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[Optional]

Vector Space

Collection of vectors (should always have zero vector)

example: R^2, R^3, R^4

R^2 vector space

technically

filling the Space (span)

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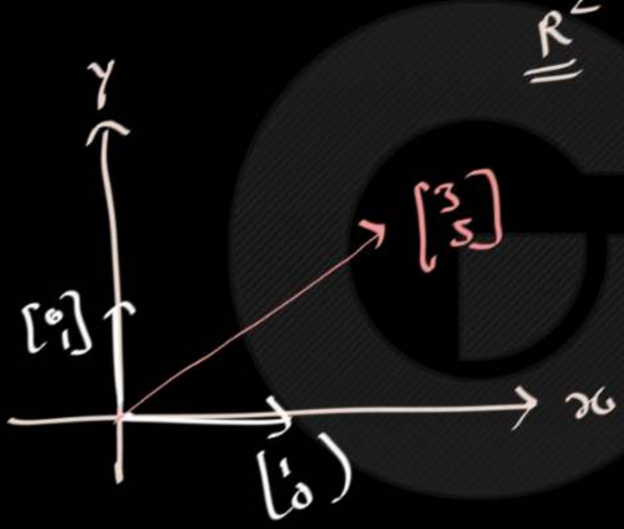
filling the space



$\left\{ \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix} \right\}$ can fill R^2

$$\begin{bmatrix}3\\5\end{bmatrix} = 3\begin{bmatrix}1\\0\end{bmatrix} + 5\begin{bmatrix}0\\1\end{bmatrix}$$

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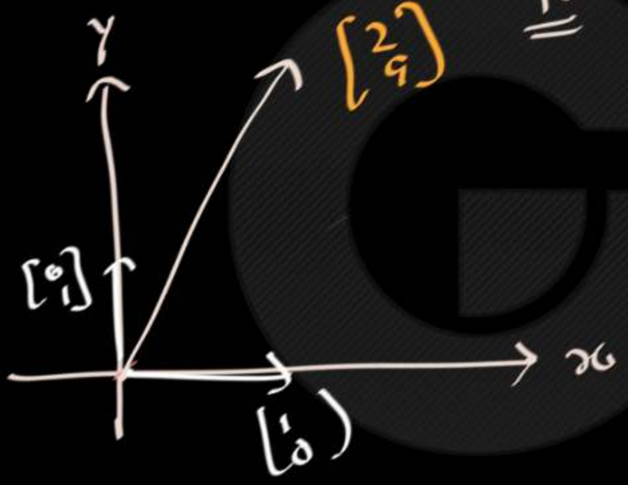
filling the Space

$\underline{\underline{R^2}}$



$\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ can fill R^2

$$\underline{\begin{bmatrix} 2 \\ 9 \end{bmatrix}} = 2\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 9\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha \underline{\underline{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}} + \beta \underline{\underline{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}}$$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ are filling the space of R^2

$$\begin{bmatrix}-3\\-5\end{bmatrix} = -3\begin{bmatrix}1\\0\end{bmatrix} + (-5)\begin{bmatrix}0\\1\end{bmatrix}$$

$$\begin{bmatrix}3\\-10\end{bmatrix} = 3\begin{bmatrix}1\\0\end{bmatrix} - 10\begin{bmatrix}0\\1\end{bmatrix}$$

* $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ fills R^2

Do we have any other set that fills R^2?

$\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix} \right\}$ fills R^2 ?

$$\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 3 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix} \right\} \rightarrow \text{Linearly indep. vectors}$$

$$\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \beta \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} \alpha + 2\beta \\ 2\alpha + 5\beta \end{bmatrix}$$

fill R^2

Why these 2 vectors filling the R^2

Ans: bcoz these 2 vectors are L.I.

Any 2 linearly indep. vectors in R^2 fills the space.

will prove it later using rank

$\begin{bmatrix}2\\5\end{bmatrix}$
$\begin{bmatrix}3\\4\end{bmatrix}$
$\begin{bmatrix}1\\2\end{bmatrix}$

$$\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\} \leftarrow LI$$

arbitrary

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \end{bmatrix} \right\}$$

does this fill the space of R^2?

NO

$$\begin{bmatrix} 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 6 \end{bmatrix} ✓$$

$$\cancel{\begin{bmatrix} 4 \\ 7 \end{bmatrix}} = \alpha \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \beta \begin{bmatrix} 3 \\ 6 \end{bmatrix} \rightarrow \alpha \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 3\beta \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$(\alpha + 3\beta) \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$\{ u, v \}$ are Linearly dep.

then they do NOT fill the

Space of R^2

Conclusion :

* Any 2 LI vectors in R^2 fills the space.

Q. 1

Does this set fill the space of R^2?

$$\left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$$

Q. 1

Does this set fill the space of R^2?

$$\left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$$

Yes

Q. 2

Does this set fill the space of R^2?

$$\left\{ \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 6 \\ 11 \end{bmatrix} \right\}$$

Yes

Q.3

Does this set fill the space of R^2?

$$\left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \end{bmatrix} \right\}$$

Arbitrary

$$\begin{bmatrix} 10 \\ 27 \end{bmatrix} = 10 \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 27 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0 \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\left\{ \begin{bmatrix} 1 \\ 9 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \end{bmatrix} \right\}$$

↳ they are filling the space R^2

Q: Whether given set is

$$\left\{ \begin{bmatrix} 1 \\ 9 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix} \right\}$$

Linearly dep. or Indep.

Q: whether given set is

$$\left\{ \begin{bmatrix} 1 \\ 9 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix} \right\}$$ L.D.

Linearly dep. or Indep.

filling R^2

$$\begin{bmatrix} 2 \\ 4 \end{bmatrix} = [\ ] + [\ ]$$

Can we have more than 2 independent vectors in R^2 ?

$$\left\{ \underbrace{\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix}}_{LI}, \begin{bmatrix} 3 \\ 8 \end{bmatrix} \right\}$$

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Can we have more than 3 independent vectors in R^3 ?

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 5\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 7\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 7 \\ 23 \end{bmatrix} \right\}$$

$$\begin{bmatrix} -2 \\ 7 \\ 23 \end{bmatrix} = -2 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 7 \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix} + 3 \begin{bmatrix} 6 \\ 0 \\ 1 \end{bmatrix}$$

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \right\}$$

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix} + \gamma \begin{bmatrix} 6 \\ 0 \\ 1 \end{bmatrix}$$

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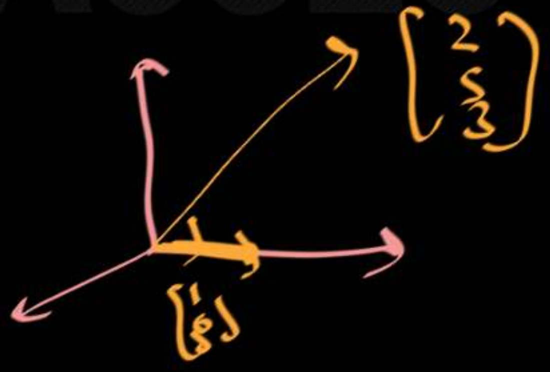
$$\underbrace{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix} \text{ Can fill the space of } R^3}$$

$\downarrow$

any vector in R^3, Can be represented as a linear combination of above 3 vectors



Any 3 linearly indep. vectors in R^3 can fill the space of R^3

$$\underbrace{\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix} \right\}}_{\text{can fill } R^3} \rightarrow LI$$

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix} \right\} \leftarrow \text{LI in } \mathbb{R}^3$$

$$\begin{bmatrix} 2 \\ 9 \\ 11 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \beta \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix} + \gamma \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}$$

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix} \right\} \quad LI$$

Can they fill R^3 ?

$$\begin{bmatrix} 3 \\ 6 \\ 7 \end{bmatrix} = \alpha \underline{\begin{bmatrix} \\ \\ \end{bmatrix}} + \beta \underline{\begin{bmatrix} \\ \\ \end{bmatrix}}$$

* any 2 LI vectors in R^2 fill the Space of R^2

* any 3 LI vectors in R^3 fill the Space of R^3

$\{u, v, w\}$

ex: $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \leftarrow LI$

$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ 7 \end{bmatrix} \right\} \leftarrow LI$

Can we have more than 3 independent vectors in R^3 ?

$$\{ \underbrace{u, v, w}_{LI}, z \}$$

No.

→ they will fill the space

Can we have more than n independent vectors in R^n ?

No.

$$\{\underbrace{v_1, v_2 \cdots v_n}_{LI}, 0\}$$

filling the space.

There are Atmost n linearly independent vectors in R^n

How to check if given vectors are dependent or independent ?

-We will come to this later

(Hint – just put all vectors in matrix and find rank)

Conclusion

* if you have 2 vectors then you can easily check if they are LD or LI.

* if a set containing zero vector then the set is LD set

* 5 linearly indep. vectors in R^5 can fill the space of R^5

* 6 vectors in R^5 can never be linearly indep.

$$\underbrace{x_1 , x_2 , x_3 , x_4 , x_5}_{LI} , x_6$$

Question

Determine if the following pairs of vectors are linearly independent:

(a) $\begin{pmatrix} -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ -6 \end{pmatrix}$ (b) $\begin{pmatrix} 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ (c) $\begin{pmatrix} -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

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https://www.dr-eriksen.no/teaching/GRA6035/2011/problems3.pdf

Question

Determine if the following pairs of vectors are linearly independent:

(a) $\begin{pmatrix} -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ -6 \end{pmatrix}$ (b) $\begin{pmatrix} 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ (c) $\begin{pmatrix} -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

LD LI LD

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https://www.dr-eriksen.no/teaching/GRA6035/2011/problems3.pdf

Question

Determine, if the following set of vectors are linearly independent and justify your answers:

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

https://sites.math.washington.edu/~naehrn/teaching/2014_fall_quarter_lin_alg_midterm_practice_with_solutions.pdf

Question

Determine, if the following set of vectors are linearly independent and justify your answers:

$$\underbrace{\left\{\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}\right\}.}$$

↳ LD

https://sites.math.washington.edu/~naehrn/teaching/2014_fall_quarter_lin_alg_midterm_practice_with_solutions.pdf

Question

Determine, if the following set of vectors are linearly independent and justify your answers:

$$\underbrace{\left\{ \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}}.$$

we have not studied how to check

https://sites.math.washington.edu/~naehrn/teaching/2014_fall_quarter_lin_alg_midterm_practice_with_solutions.pdf

what we know about

checking LD or LD

* if you have 2 vectors then you can easily check if they are LD or LI.

* if a set containing zero vector then the set is LD set

ERn

* if you have more than n vectors in the set then definately LD

Question

In each case, determine whether the given set of vectors is linearly dependent or linearly dependent.

(i) $\left\{\begin{pmatrix}1\\2\\3\end{pmatrix},\begin{pmatrix}1\\0\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix},\begin{pmatrix}1\\-1\\2\end{pmatrix}\right\}$

(ii) $\left\{\begin{pmatrix}1\\1\\0\end{pmatrix},\begin{pmatrix}3\\3\\0\end{pmatrix}\right\}.$

(iii) $\left\{\begin{pmatrix}-1\\1\end{pmatrix},\begin{pmatrix}3\\9\end{pmatrix}\right\}.$

https://chrisj.math.gatech.edu/23s/1553/materials/m2-sample-B-solutions.pdf

Question

In each case, determine whether the given set of vectors is linearly dependent or linearly dependent.

(i) $\left\{\begin{pmatrix}1\\2\\3\end{pmatrix},\begin{pmatrix}1\\0\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix},\begin{pmatrix}1\\-1\\2\end{pmatrix}\right\} \longrightarrow$ LD

(ii) $\left\{\begin{pmatrix}1\\1\\0\end{pmatrix},\begin{pmatrix}3\\3\\0\end{pmatrix}\right\}. \longrightarrow$ LD

(iii) $\left\{\begin{pmatrix}-1\\1\end{pmatrix},\begin{pmatrix}3\\9\end{pmatrix}\right\}. \longrightarrow$ LI

https://chrisj.math.gatech.edu/23s/1553/materials/m2-sample-B-solutions.pdf

Question True/False

If A is a 3×5 matrix, then the columns of A are linearly dependent.

https://web.ma.utexas.edu/users/sadun/S12/340L/2010sol1.pdf

Question

True/False

If A is a 3×5 matrix, then the columns of A are linearly dependent.

$\in \mathbb{R}^3$

$\{u, v, w, x, y\}$

2D

https://web.ma.utexas.edu/users/sadun/S12/340L/2010sol1.pdf

Question

True/False

If a set contains more vectors than there are entries in each vector, then the set is linearly dependent. That is, any set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ *in* $\mathbb{R}^n$ *is linearly dependent if* $p > n$.

https://math.berkeley.edu/~arash/54/notes/01_07.pdf

Question

True/False

If a set contains more vectors than there are entries in each vector, then the set is linearly dependent. That is, any set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ *in* $\mathbb{R}^n$ *is linearly dependent if* $p > n$.

4 2

$v_1 \; - - - - \; v_p = 4$

$10 > 3$

$\{[\;], [\;], [\;], [\;]\}$

https://math.berkeley.edu/~arash/54/notes/01_07.pdf

Question

True/False

If a set $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ *in* $\mathbb{R}^n$ *contains the zero vector, then the set is linearly dependent.*

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Question

Determine if the following vectors are linearly dependent or linearly independent.

(a) $\left\{\begin{pmatrix}-4\\0\\1\\5\end{pmatrix}, \begin{pmatrix}0\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\4\\3\\6\end{pmatrix}\right\}$, (b) $\left\{\begin{pmatrix}4\\4\end{pmatrix}, \begin{pmatrix}-1\\3\end{pmatrix}, \begin{pmatrix}2\\5\end{pmatrix}, \begin{pmatrix}8\\1\end{pmatrix}\right\}$,

(c) $\left\{\begin{pmatrix}-8\\12\\-4\end{pmatrix}, \begin{pmatrix}2\\-3\\-1\end{pmatrix}\right\}$.

H.W.

http://math.bu.edu/people/popovic/MA242/midterm1_sol.pdf

Solution.

(a) The set is linearly dependent, since it contains the zero vector $\mathbf{0}$. (Any set containing the zero vector is linearly dependent.)

(b) The set is linearly dependent, since it consists of four vectors in $\mathbb{R}^2$. (Any set containing more vectors than each vector has entries is linearly dependent.)

(c) The set is linearly independent, since it contains two vectors neither of which is the zero vector, and since these vectors are not multiples of each other.

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Question

True/False

If there are six linearly dependent vectors in R^6, then these vectors can not fill R^6

to fill R^6 we need Exactly 6 L I vectors

Question

True/False

If there are six linearly dependent vectors in R^6, then these vectors can not fill R^6

to fill R^6 we need Exactly 6 LI vectors which is sufficient to fill R^6 -

× a) 6 LD vectors

× b) 5 LI vectors

× c) 7 any vectors

✓ d) 6 LI vectors

Question

Given that below two vectors are linearly independent, does they fill R^3 ?

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

Question

Given that below two vectors are linearly independent, does they fill R^3 ?

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

NO

Question

If four vectors in R^4 are linearly independent, then they fill R^4.

https://web.ma.utexas.edu/users/sadun/S12/340L/2010sol1.pdf

Question

If four vectors in R^4 are linearly independent, then they fill R^4.

Yes

https://web.ma.utexas.edu/users/sadun/S12/340L/2010sol1.pdf

Question

In R^4,

Any four vectors fill the space

Any five vectors fill the space

Any four linearly dependent vectors fill the space

Any three linearly independent vectors fill the space

Any ten vectors fill the space

None of the above

Question

In R^4, → I need 4 LI vectors

- ✗ Any four vectors fill the space
- ✗ Any five vectors fill the space
- ✗ Any four linearly dependent vectors fill the space
- ✗ Any three linearly independent vectors fill the space
- ✗ Any ten vectors fill the space
- ✓ None of the above

Question

In R^4, → I need 4 LI vectors

- ✗ Any four vectors fill the space
- ✗ Any five vectors fill the space
- ✗ Any four linearly dependent vectors fill the space
- ✗ Any three linearly independent vectors fill the space
- ✗ Any ten vectors fill the space

any 20 vectors s.t. 4 out of 20 are LI.

Summary so far...

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Imp.

$$c_1 v_1 + c_2 v_2 + c_3 v_3 + \cdots c_n v_n = 0$$

can you give me at least one $c_i \neq 0$

Yes → **LD**

NO → **LI**

to fill R^n, we need n L.I vectors.

in other words, n L.I vectors can fill the space of R^n.

Linearly indep. & dep.

multiplying a matrix with a vector

$Ax = 0, \quad Ax = b$

End of Linearly dependent and Independent Vectors

Next: Multiplying a matrix with a vector

End of Linearly dependent and Independent Vectors

Next: Multiplying a matrix with a vector

Matrix $\times$ Vector = ?

Matrix × Vector = ?

$$\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Matrix × Vector = ?

$$\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$1 \begin{bmatrix} 2 \\ 3 \end{bmatrix} + 0 \begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

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$$\begin{bmatrix} 4 & 7 \\ \underline{6} & \underline{0} \end{bmatrix} \underline{\begin{bmatrix} 1 \\ 2 \end{bmatrix}} = \begin{bmatrix} 18 \\ 6 \end{bmatrix}$$

$$1\begin{bmatrix} 4 \\ 6 \end{bmatrix} + 2\begin{bmatrix} 7 \\ 0 \end{bmatrix} = \begin{bmatrix} 18 \\ 6 \end{bmatrix}$$

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$$\begin{bmatrix} 4 & 7 \\ 6 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 18 \\ 6 \end{bmatrix}$$

$$1 \begin{bmatrix} 4 \\ 6 \end{bmatrix} + 2 \begin{bmatrix} 7 \\ 0 \end{bmatrix} = \begin{bmatrix} 18 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 5 & 9 \\ 2 & 6 & 10 \\ 3 & 7 & 11 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} =$$

$$1\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 0\begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix} + 1\begin{bmatrix} 9 \\ 10 \\ 11 \end{bmatrix} = \begin{bmatrix} 10 \\ 12 \\ 14 \end{bmatrix}$$

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$Ax = b$

b is linear combination of columns of A.

$$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}$$

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$$\begin{array}{c} \begin{matrix} 2 & 1 & 0 \end{matrix} \\ \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \end{array} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \\ 12 \end{bmatrix}$$

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$$\overset{A}{\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}}\overset{x}{\begin{bmatrix} \\ \\ \end{bmatrix}} = \overset{b}{\begin{bmatrix} \\ \\ \end{bmatrix}}$$

b is Always linear combination of columns of A.

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Best answer

Answer A,C, D

$Ax = \lambda x$

Learn very powerful and important way to multiply matrix and vector. Ax is just linear combination of columns of A.. if you can understand this then solution of system of linear equation is just cakewalk for you. And complete linear algebra will come very intutive.

A.

$$\begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = -1 \begin{bmatrix} -9 \\ -8 \\ 20 \\ 32 \end{bmatrix} + 1 \begin{bmatrix} -6 \\ -6 \\ 15 \\ 21 \end{bmatrix} + 0 \begin{bmatrix} -2 \\ -3 \\ 8 \\ 7 \end{bmatrix} + 1 \begin{bmatrix} -4 \\ -1 \\ 5 \\ 12 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

B.

$$\begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} = 1 \begin{bmatrix} -9 \\ -8 \\ 20 \\ 32 \end{bmatrix} + 0 \begin{bmatrix} -6 \\ -6 \\ 15 \\ 21 \end{bmatrix} - 1 \begin{bmatrix} -2 \\ -3 \\ 8 \\ 7 \end{bmatrix} + 0 \begin{bmatrix} -4 \\ -1 \\ 5 \\ 12 \end{bmatrix} = \begin{bmatrix} -7 \\ -5 \\ 12 \\ 25 \end{bmatrix}$$

C.

$$\begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 2 \\ 2 \end{bmatrix} = -1 \begin{bmatrix} -9 \\ -8 \\ 20 \\ 32 \end{bmatrix} + 0 \begin{bmatrix} -6 \\ -6 \\ 15 \\ 21 \end{bmatrix} + 2 \begin{bmatrix} -2 \\ -3 \\ 8 \\ 7 \end{bmatrix} + 2 \begin{bmatrix} -4 \\ -1 \\ 5 \\ 12 \end{bmatrix} = 3 \begin{bmatrix} -1 \\ 0 \\ 2 \\ 2 \end{bmatrix}$$

D.

$$\begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -3 \\ 0 \end{bmatrix} = 0 \begin{bmatrix} -9 \\ -8 \\ 20 \\ 32 \end{bmatrix} + 1 \begin{bmatrix} -6 \\ -6 \\ 15 \\ 21 \end{bmatrix} - 3 \begin{bmatrix} -2 \\ -3 \\ 8 \\ 7 \end{bmatrix} + 0 \begin{bmatrix} -4 \\ -1 \\ 5 \\ 12 \end{bmatrix} = 3 \begin{bmatrix} 0 \\ 1 \\ -3 \\ 0 \end{bmatrix}$$

answered Feb 15, 2022 • edited Feb 15, 2022 by Sachin Mittal 1

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#c14749

Sachin Mittal 1

$$\begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = -1 \begin{bmatrix} -9 \\ -8 \\ 20 \\ 32 \end{bmatrix} + 1 \begin{bmatrix} -6 \\ -6 \\ 15 \\ 21 \end{bmatrix} + 0 \begin{bmatrix} -2 \\ -3 \\ 8 \\ 7 \end{bmatrix} + 1 \begin{bmatrix} -4 \\ -1 \\ 5 \\ 12 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

GO CLASSES

T/F

$$\begin{bmatrix} \ \ \ \end{bmatrix} \begin{bmatrix} x \end{bmatrix} = \begin{bmatrix} b \end{bmatrix}$$

b is a linear combination of columns of A. ?

True

True/False

If $Ax = 0$ has some nontrivial solution then columns of A are linearly dependent.

$$\begin{bmatrix} \uparrow & \uparrow & \uparrow \\ a_1 & a_2 & a_3 \\ \downarrow & \downarrow & \downarrow \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Downarrow$

True

$$c_1 a_1 + c_2 a_2 + c_3 a_3 =$$

$$\begin{bmatrix} \uparrow & \uparrow & \uparrow \\ a_1 & a_2 & a_3 \\ \downarrow & \downarrow & \downarrow \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Updownarrow$$

$$\underbrace{c_1 \begin{bmatrix} \uparrow \\ a_1 \\ \downarrow \end{bmatrix} + c_2 \begin{bmatrix} \uparrow \\ a_2 \\ \downarrow \end{bmatrix} + c_3 \begin{bmatrix} \uparrow \\ a_3 \\ \downarrow \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$$

$\hookrightarrow$ do you have some nonzero c_i ✓

a_1, a_2, a_2 are LD.

Ax same as linear combination of columns of A.

GO CLASSES

$Ax = 0$ has some sol^n then what can you say about linear dependecy of columns of A ?

$$c_1 a_1 + c_2 a_2 + c_3 a_3 = 0$$

then i don't know

$Ax = 0$ has some non trivial sol^n then what can you say about linear dependecy of columns of A ?

$$c_1 a_1 + c_2 a_2 + c_3 a_3 = 0$$

$\Downarrow$

LD

what we know

1. what is <u>LD</u> or <u>LI</u> $c_1 v_1 + c_2 v_3 + c_3 v_3 = 0$

2. we can not have more than n LI vectors in R^n

3. How to multiply <u>A matrix with vector</u>

$$\begin{bmatrix} 1 & 4 & 3 & 5 \\ 2 & 2 & 9 & 6 \\ 3 & 7 & 4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 10 \end{bmatrix}$$

$$Ax = b$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

GATE only & mainly 2 things

1. $Ax = b$

2. Eigen values / Eigen vectors

Remember we have talked about Linear combination of vectors ?

$$\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} + 4 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} - 2 \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} = ?$$

Now, Lets ask reverse question

$$? \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} + ? \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + ? \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix}}_{b}$$

$$Ax = b$$

Solving system of linear equations

Ax=0

Ax=b