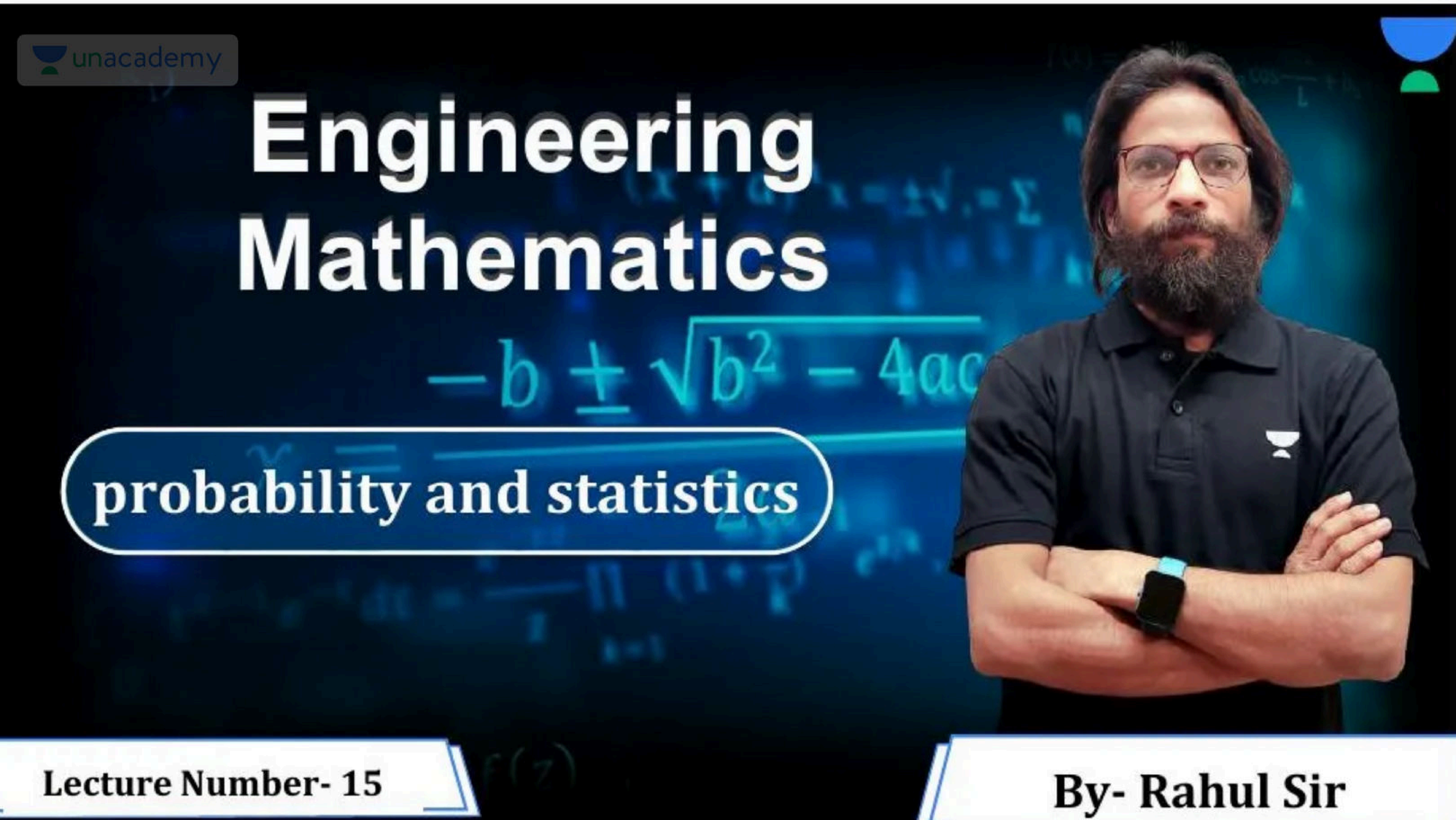
Random Variables Part-III

Course on Engineering Mathematics for GATE - CSE

Rahul • Lesson 16 • June 29, 2024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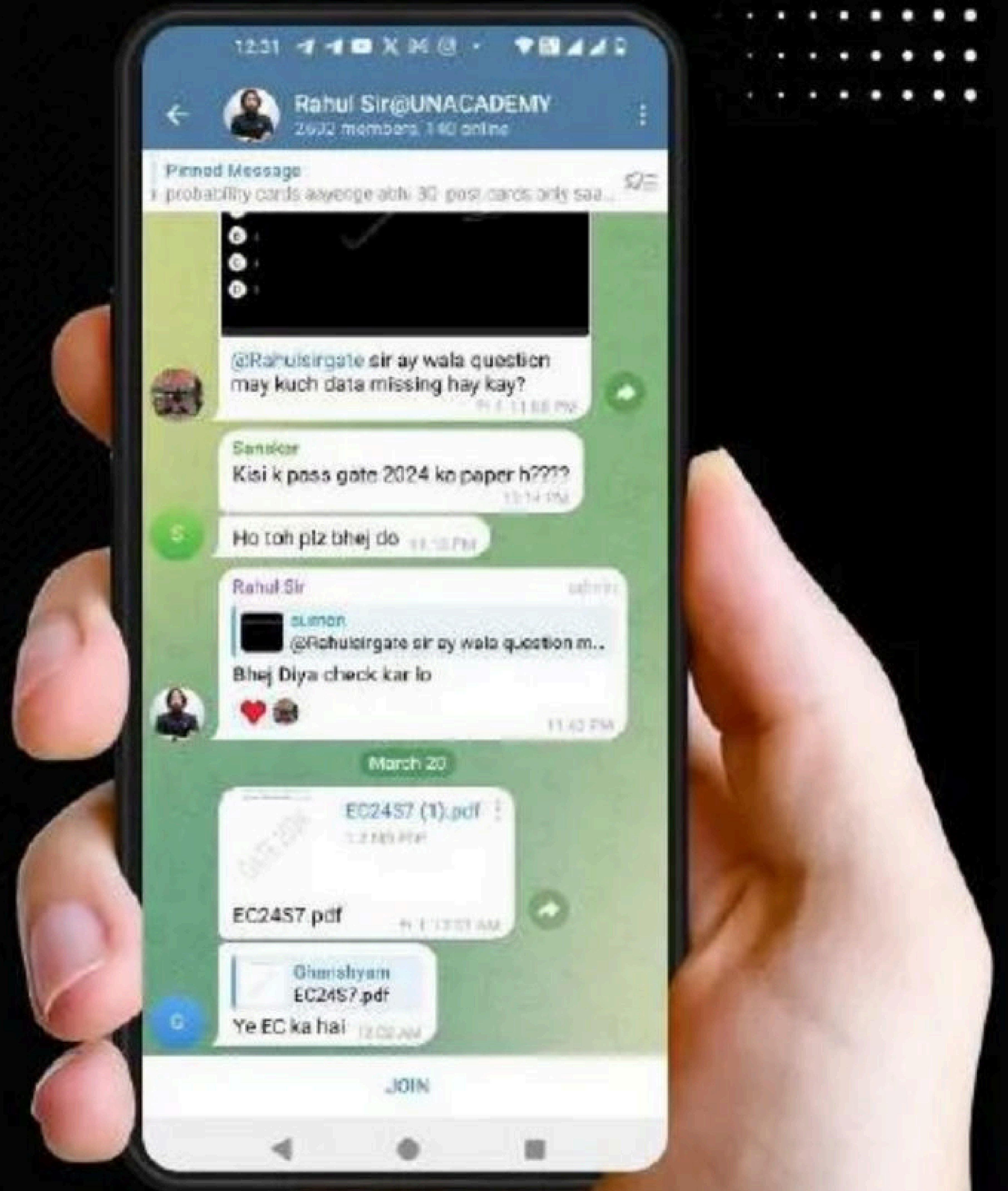
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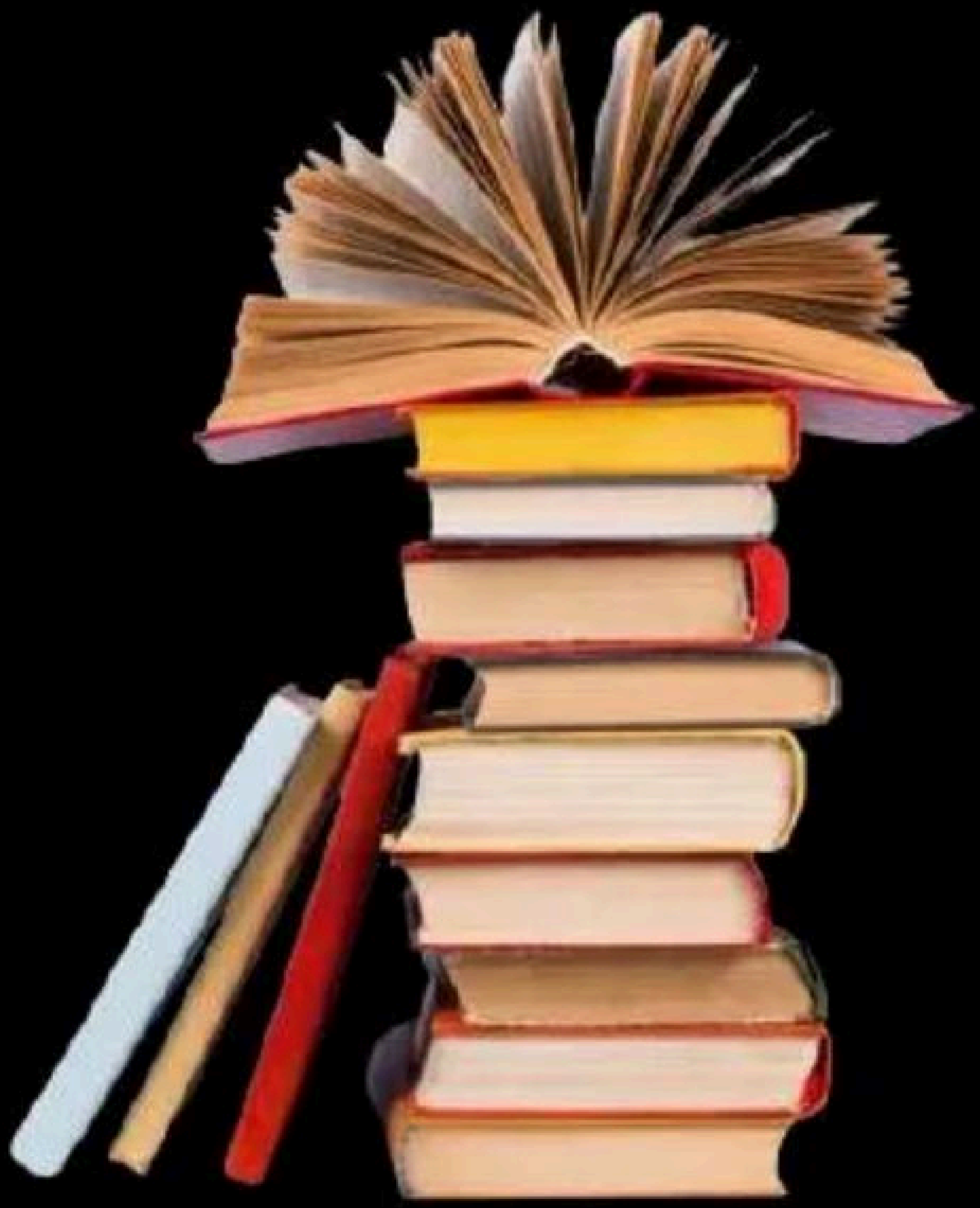
Topics

to be covered



1. Random Variables_III

7v31250

QUESTION

Q. The life (in hours) X of a certain type of light bulb may be supposed to be a continuous random variable with p.d.f.:

$$f(x)=\begin{cases}\frac{A}{x^3} & 1500<x<2500\\ 0, & \text{elsewhere}\end{cases}$$

$$f(x)=\begin{cases}\frac{A}{x^3} & 1500<x<2500\\ 0 & \text{elsewhere}\end{cases}$$

Determine the constant A and compute the probability that

$1600 \le x \le 2000.$

Determine the constant A

If this is valid prob. density function

$$\int_a^b f(x)\,dx = 1$$

$$\int_{1500}^{2500} \frac{A}{x^3}\,dx = 1$$

$$\# \int x^n\,dx = \frac{x^{n+1}}{n+1}$$

$$A \int_{1500}^{2500} \frac{1}{x^3}\,dx = 1$$

$$\Rightarrow A\left[\frac{x^{-3+1}}{-3+1}\right]_{1500}^{2500} = 1$$

$$\Rightarrow A\left[-\frac{1}{2x^2}\right]_{1500}^{2500} = 1$$

$$A\left\{-\frac{1}{2\times(2500)^2} + \frac{1}{2(1500)^2}\right\} = 1$$

$$\boxed{A = 7031250}$$

$$P(\underbrace{1600 \le X \le 2000}) =$$

X is a Random (CRV)

$$= \int_{1600}^{200} \frac{7031250}{x^3}\, dx$$

$$\Rightarrow \underline{0.494} \quad \underline{\underline{\text{Ans}}}$$

QUESTION

Q. The diameter 'X' of a cable is assumed to be a continuous random variable with p.d.f.

$$f(x)=\begin{cases}6x(1-x), & 0\le x\le 1\\ 0, & \text{elsewhere}\end{cases}$$

Obtain the c.d.f. of x.

QUESTION

Q. A random variable x has the following probability function:

x	0	1	2	3	4	5	6	7
p(x)	0	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{1}{100}$	$\frac{1}{50}$	$\frac{1}{100}$

Determine the distribution function of X.

QUESTION

Q. A random variable X has the following probability distribution:

x	0	1	2	3	4	5	6	7	8
p(x)	k	3k	5k	7k	9k	11k	13k	15k	17k

(i) Determine the value of k.

(ii) Find the distribution function of X.

H.W

$\Downarrow$

$F_X(x)$

QUESTION

Q. If X is a continuous random variable whose probability density function is given by

$$f(x)=\begin{cases}k\left(5x-2x^2\right), & 0\le x\le 2.\\ 0, & \textit{otherwise}\end{cases}$$

Then $P(x>1)$ is

(A) 3/14

(B) 4/5

(C) 14/17

(D) 17/28

DONE

QUESTION

A continuous random variable X has a probability density function $f(x) = e^{-x}, 0 < x < \infty$. Then $P\{X > 1\}$ is .

A 0.368

B 0.5

C 0.632

D 1.0

DONE

QUESTION

Q. Let X be a random variable with probability density function

$$f(x)=\begin{cases}0.2 & for\,|x|\le 1\\ 0.1 & for\,1<|x|\le 4\\ 0 & othewise\end{cases}$$

The probability $P(0.5 < x < 5)$ is ______.

DONE

QUESTION

M.M. M-Imp.

{GATE / OTHER EXAMS

Q. A normal random variable X has the following probability density function

$$f_x(X) = \frac{1}{\sqrt{8\pi}} e - \left\{ \frac{(x-1)^2}{8} \right\}, -\infty < x \, \infty$$

Then $\int_1^{\infty} f_x(X)dx =$

$$fx(X) = \frac{1}{\sqrt{8\pi}} e^{-\left(\frac{(x-1)^2}{8}\right)}$$

$-\infty < x < \infty$

then $\int_1^{\infty} fx(X)\,dx$

(A) 0

(B) $\frac{1}{2}$

(C) $1 - \frac{1}{e}$

(D) 1

$$f_X(X) = \frac{1}{\sqrt{8\pi}} e^{-\left(\frac{(x-1)^2}{8}\right)}$$

$$-\infty < x < \infty$$

then $\int_1^\infty f_X(X)\,dx$

$$\int_1^\infty \frac{1}{\sqrt{8\pi}} e^{-\frac{(x-1)^2}{8}} dx$$

$\#$ $x-1 = t$

Trans form t domain

$t = 0 \quad t = \infty$

both sides Diff. It

$$\boxed{dx = dt}$$ ✓

$$= \frac{1}{\sqrt{8\pi}} \int_0^\infty e^{-\frac{t^2}{8}} dx$$

$$= \frac{1}{\sqrt{8\pi}} \int_0^\infty \underbrace{e^{-t^2/8}} \, dt$$

Not integrable form.

e^{-t^2}

e^{-t^2}

Not integrable

$$= \frac{1}{\sqrt{8\pi}} \underbrace{\int_0^\infty e^{-t^2/8}}$$

GAMMA function.

complex No. addition

$$\boxed{\text{Gamma function} = \int_0^\infty e^{-t} \, t^{n-1} \, dt}$$

gamma function $\int_0^{\infty} e^{-t}\, t^{n-1} = \Gamma n$

$\Gamma n = (n-1)!$ $\qquad \Gamma 6 = 5!$

$\Gamma(n+1) = n\,\Gamma n$ $\qquad \Gamma 7 = 6!$

$\Gamma 1 = 0!$

$\Gamma \frac{1}{2} = \sqrt{\pi}$

M. Imp.

$$\Gamma \frac{5}{2} = \frac{3}{2}\,\frac{1}{2}\,\Gamma\frac{1}{2}$$

$$= \frac{3}{4}\sqrt{\pi}$$

$$\Gamma \frac{11}{2} = \frac{9}{2} \times \frac{7}{2} \times \frac{5}{2} \times \frac{3}{2} \times \frac{1}{2}\,\Gamma\frac{1}{2}$$

GAMMA function

$$\frac{t^2}{8} = z$$

$$2t\,dt = 8\,dz$$

$$dt = \frac{8\,dz}{2t} = \frac{8\,dz}{2\sqrt{8z}}$$

$$= \frac{\sqrt{2}\,z^{-1/2}}{2\sqrt{2}}\,dz$$

$$\boxed{dt = \sqrt{2}\,z^{-1/2}\,dz}$$

$$\frac{1}{\sqrt{8\pi}}\int_0^\infty e^{-\frac{1}{8}t^2}\,dt$$

LINEAR

$$\int_0^\infty e^{-t}\,t^{n-1}\,dt$$

$$\frac{1}{\sqrt{8\pi}}\int_0^\infty e^{-z}\sqrt{2}\,z^{-1/2}\,dz$$

$$\frac{1}{2}\frac{1}{\sqrt{\pi}}\underbrace{\int_0^\infty e^{-z}z^{-1/2}\,dz}_{} = \frac{1}{2}\frac{1}{\sqrt{\pi}}\Gamma\left(\frac{1}{2}\right)$$

look like gamma Function

$$= \frac{1}{2}\times\frac{1}{\cancel{\sqrt{\pi}}}\times\cancel{\sqrt{\pi}}$$

$$= \frac{1}{2}$$ answer

Compare It $\int_0^\infty e^{-t}\,t^{n-1}\,dt$

Γn

$$n-1 = -\frac{1}{2}$$

$$n = 1/2$$

QUESTION

Q. The density function for the continuous random variable X is

$$f_x(X) = \begin{cases} e^{-x} \text{ for } x > 0 \\ 0 \text{ for } x \le 0 \end{cases}$$

Find the probability $P[x \le 2 | x > 1]$

DONE

QUESTION

Q. Suppose the random variable X has a probability density function

$$f(x)=\begin{cases} kx^3\ e^{-x/2}, \text{ for } x>0 \\ 0 \qquad \text{otherwise} \end{cases}$$

The value of k is______.

(A) 1/96

(B) 96

(C) 8/3

(D) 1/4

$f(x) = \begin{cases} kx^3 e^{-x/2} & x>0 \\ 0 & \text{otherwise} \end{cases}$

GAMMA $\int_0^\infty e^{-t}\, t^{n-1}\, dt$

$\Downarrow$ exponetial

$\Downarrow$ algebraic

limit 0 to ∞

$$\int_0^\infty kx^3 e^{-x/2}\, dx = 1$$

0 otherwise

$$\int_0^\infty Kx^3 e^{-x/2}\,dx = 1$$

$$\frac{x}{2} = t \qquad \boxed{dx = 2dt}$$

$$\int_0^\infty k(2t)^3 e^{-t}\cdot(2dt) = 1$$

compare It

$n-1 = 3$

$\boxed{n = 4}$

$$2^4\int_0^\infty K\,t^3 e^{-t}\,dt = 1$$

$$= 16k\int_0^\infty t^3 e^{-t}\,dt = 1 \qquad = 16\times 3!\,k = 1$$

$\Gamma 4 = 3!$

$16\times 6(k) = 1$

$96k = 1$

$\boxed{k = 1/96}$ answer

QUESTION

Q. Lex X be a continuous random variable with pdf

$$f_x(X) = \begin{cases} cx^2, \text{ for } 0 < x \le 1, \\ 0 \quad \text{otherwise} \end{cases}$$

For some positive constant c. The value of $P\left(x \le \frac{2}{3} \middle| x > \frac{1}{3}\right)$

A. 3/26

B. 5/26

C. 7/26

D. 11/26

$$f(x) = \begin{cases} cx^2 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$ c is constant

$$P\left(x \le 2/3 \mid x > 1/3\right) = \frac{P\left(x \le 2/3 \cap x > 1/3\right)}{P\left(x > 1/3\right)}$$

$=$

$$f(x) = \begin{cases} cx^2 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$ c is constant

$$P\left(X \le 2/3 \mid X > 1/3\right) = \frac{P\left(X \le 2/3 \cap X > 1/3\right)}{P\left(X > 1/3\right)} = \frac{7}{26}$$

DONE

QUESTION

Q. Suppose the random variable X has the probability density function

$$f(x)=\begin{cases} ce^{x/3}, & x \le 0, \\ ce^{-x/3}, & x > 0 \end{cases}$$

For some positive constant c. The value of P [x > 6/x > 0] is

(A) e^{-2}

(B) ce^{-2}

(C) 0

(D) $1-e^{-2}$

$$P\left(\frac{x>6}{x>0}\right) = P\left(\frac{x>6 \cap x>0}{x>0}\right)$$

DONE H.W

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QUESTION

M.M. Imp.

Q. Let X be a discrete random variable with probability function $P(X = x) = \frac{2}{3^x}$ for x = 1, 2, 3, ... What is the probability that X is even?

X is a DISCRETE Random variable.

X = Arrival 1, 2, 3 - - - -

Prob. at a Point $P[X=x] = \frac{2}{3^x}$

(A) 1/4

(B) 2/7

(C) 1/3

(D) 2/3

$P(X=2) = \frac{2}{3^2}$ OR $P(X=6) = \frac{2}{3^6}$

OR $P(X=4) = \frac{2}{3^4}$ OR $P(X=8) = \frac{2}{3^8}$

$P(X \text{ is EVEN}) = P(X=2) + P(X=4) + P(X=6) + \text{----}$

DISCRETE ~~Infinite~~ Infinite countable

$$P(x \text{ is even}) = \frac{2}{3^2} + \frac{2}{3^4} + \frac{2}{3^6} + \text{---}$$

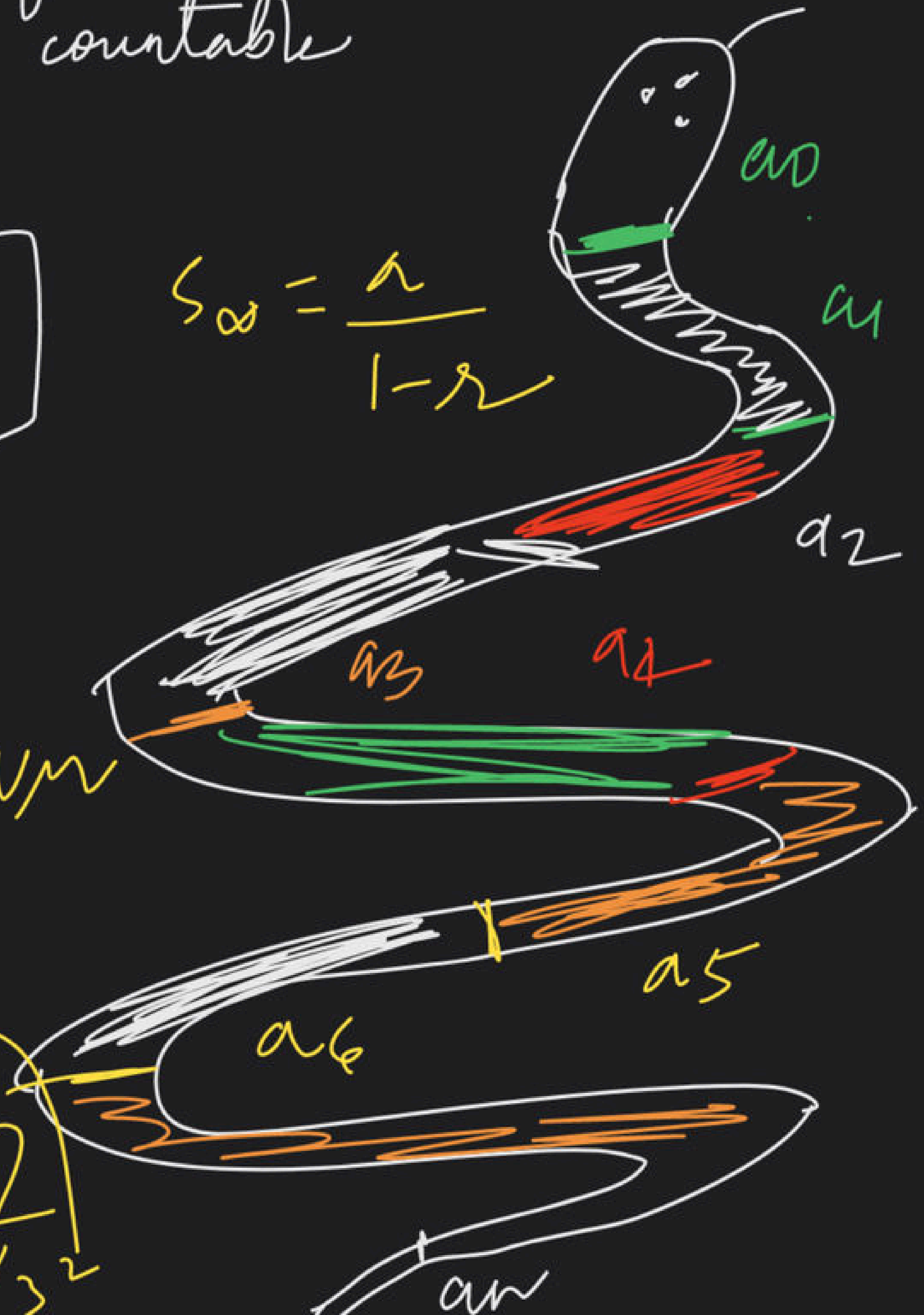
$$= 2\left[\underbrace{\underset{a_0}{\frac{1}{3^2}} + \underset{a_1}{\frac{1}{3^4}} + \underset{a_2}{\frac{1}{3^6}} + \text{---}}\right] = \text{SUM}$$

G.P. Partial

$$= 2\left[\frac{1/3^2}{1 - \frac{1}{3^2}}\right]$$

$$= \frac{1}{4} \quad \underline{\text{Ans.}}$$

$$S_\infty = \frac{a}{1-r}$$



$$r = \frac{1/3^4}{1/3^2}$$

M. Imp. **AVERAGE** :-

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

$$\bar{x} = \frac{\text{sum of all observation}}{\text{Total No. of observation}}$$

class Performance 30 STUDENTS / Large no. of students

STUDENT	Marks out of 100	
A	81	I_1
B	80	I_2
C	35	I_3
D	75	I_4
E	80	I_4
F	71	
G	69	

average = $\boxed{\text{SINGLE Number}}$

PERFormace - measure

✓ CENTER of gravity

$\boxed{\text{Data balance}}$

balance point

✓ Average / MEAN $\longrightarrow$ balance point / center of mass / mean / center of gravity

Median (Mid value)

- n is even: $\frac{n}{2}$, $\frac{n}{2}+1$
- n is odd $\longrightarrow$ $\frac{n+1}{2}$ th Term

Arrange the value

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1, 2, 3, 4, 5, 6.

$P(E) = \frac{n(E)}{n(S)}$

Median n is even   1 2 3 4 5 6

$n = 6$ $\frac{n}{2}$ th Term , $\frac{n}{2}+1$ th Term.

$\frac{6}{2} = 3^{th}$ Term $\frac{6}{2}+1^{th}$ Term.

4^{th} Term.

Mode = more Frequent value.

frequency 2, 1, 1, 1, 2, 2, 2, 3, 3, 3, 4, 4, 5, 5, 5, 1, 1, 1, $\boxed{\text{frequency}}$

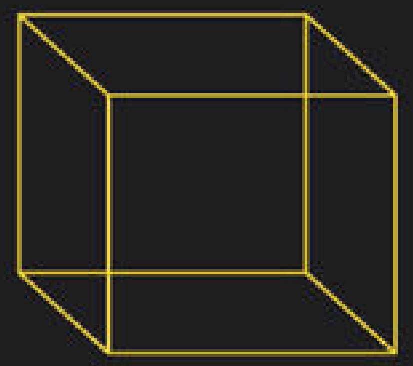
mode = 1

$X = 1, 2, 3, 4, 5$

X	1	2	3	4	5
$P(X=x)$					

$\sum x_i p_i$

Playing A GAME in Casino :- Casino always win

✓  $\Rightarrow \{1, 2, 3, 4, 5, 6\}$

$\left.\begin{matrix} 1 \\ 6 \end{matrix}\right]$ -3 Rupees. $-$ LOSE

$\left.\begin{matrix} 2 \\ 3 \\ 4 \end{matrix}\right]$ $+1$ win

$5\,]$ $+2$ win

Playing GAME

Should WE Play This GAME OR NOT ?

$$(+1) \times n \times \left(\frac{3}{6}\right) + (+2) \times \left(\frac{1}{6}\right) \times n$$

$$+ (-3) \times n \times \frac{2}{6}$$

$$= -\frac{n}{6} \checkmark \quad \underline{\underline{\text{Neg}}}$$

$$x_1 \times n \times P[X = x_1] + x_2 \times n \, P[X = x_2] + n \times_3 P[X = x_3]$$

$$= -\frac{n}{6}$$

$$\sum_{i=1}^{n} x_i p_i = -\frac{1}{6} \checkmark$$

$$\frac{\sum f_i x_i}{\sum f_i} = \underline{\underline{\text{Average}}}$$

X	1	2	3	4	5	6
Pay off	-3	+1	+1	+1	+2	-3

X = Random variable / Discrete

X = +1, +2, -3 (frequency)

X	+1	+2	-3
$P(X=x)$	$\frac{3}{6}$	$\frac{1}{6}$	$\frac{2}{6}$

SINGLE Trial.

$P(\text{winning}) = \frac{4}{6} = \frac{2}{3}$

$P(\text{losing}) = \frac{2}{6} = \frac{1}{3}$

prob of winning more than P(losing)

for large Number of Trials:

$$\underbrace{n \times (+1) \times \left(\frac{3}{6}\right)}_{} + \underbrace{n \times (+2) \times \left(\frac{1}{6}\right)}_{} + \underbrace{n \times (-3) \times \frac{2}{6}}_{}$$

n large Total Pay off

Total Pay off / Total money

$$= \frac{n}{2} + \frac{n}{3} - n = \underbrace{\boxed{-}\frac{n}{6}}_{\text{Total}}$$

Large no. of Trials (Neg)

quit this game

For Large Number of Trials:

$$n \times (+1) \times \left(\frac{3}{6}\right) + n \times (+2) \times \left(\frac{1}{6}\right) + n \times (-3) \times \frac{2}{6} = -\frac{n}{6}$$

$$n \cdot x_1 P[X = x_1] + n\, x_2 P[X = x_2] + n x_3 P[X = x_3] = -\frac{n}{6}$$

$$\cancel{n} \left[x_1 P[X = x_1] + x_2 P[X = x_2] + x_3 P[X = x_3] \right] = -\frac{\cancel{n}}{6}$$

$$\boxed{x_1 P[X = x_1] + x_2 P[X = x_2] + x_3 P[X = x_3] = -\frac{1}{6}}$$

average

Expected value / average

$$\boxed{\sum_{i=1}^{n} x_i P[X = x_i] = -\frac{1}{6}}$$

Expected value Neg

Average - Neg

Average

expected value $= \sum_{i=1} x_i P[X - x_i]$

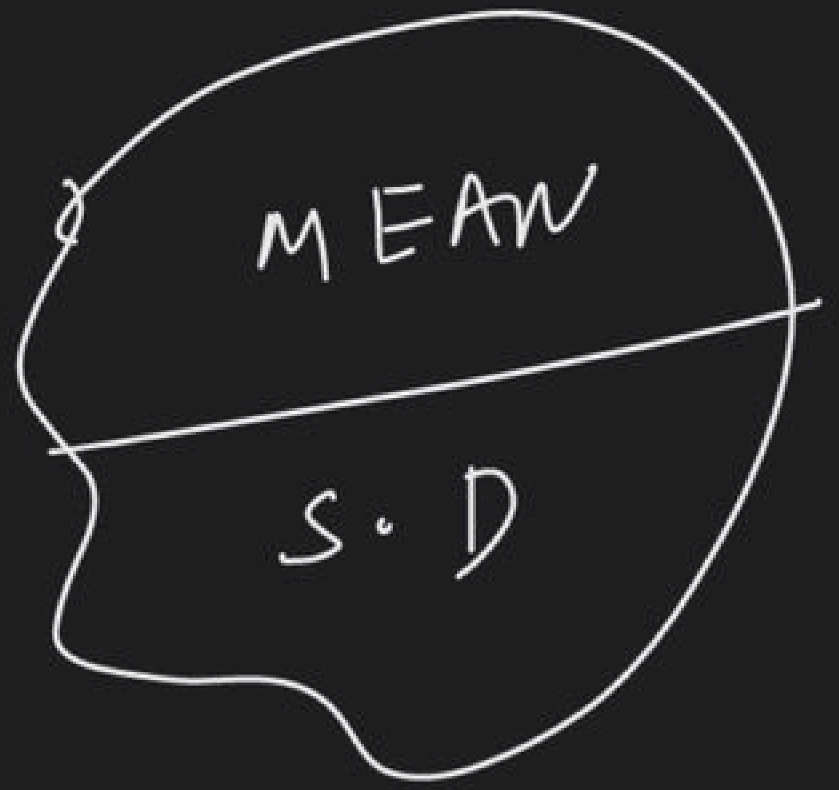
X	x_1	x_2	x_3	x_4	x_n
$P[X=x]$	p_1	p_2	p_3	p_4	p_n

E(X) = expected value / MEAN / average / center of mass.

$$E(X) = \frac{x_1 p_1 + x_2 p_2 + x_3 p_3 + \cdots + x_n p_n}{\underbrace{p_1 + p_2 + p_3 + \cdots + p_n}_{}} \quad = 1$$

$$\boxed{E(X) = x_1 p_1 + x_2 p_2 + x_3 p_3 + \cdots + x_n p_n}$$

$$E(X) = \sum_{i=1}^{n} x_i p_i$$



Throwing A Die

X = No. of Dots → LARGE NUMBER of Trials.

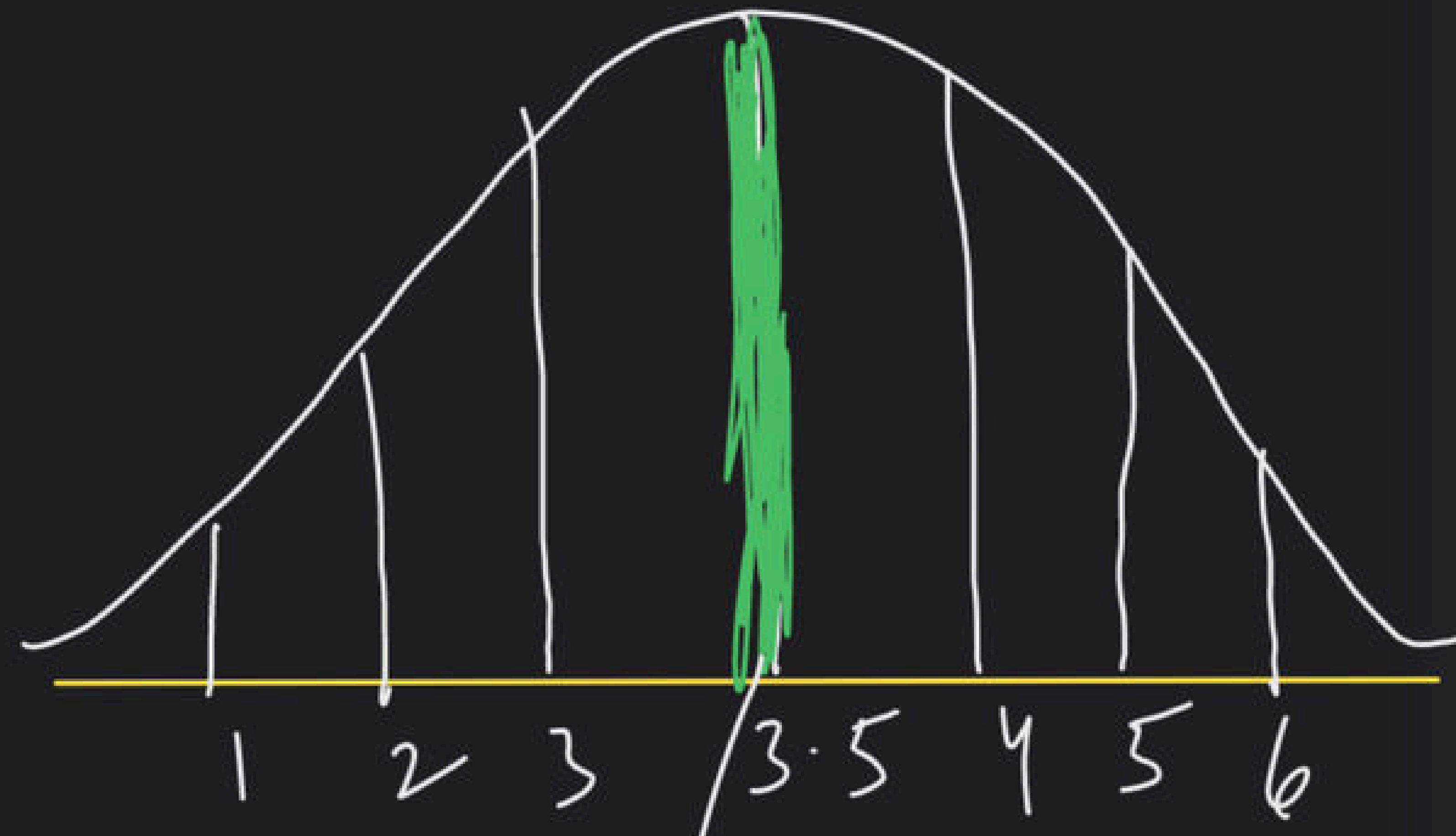
$X = 1, 2, 3, 4, 5, 6$

X	1	2	3	4	5	6
$P(X=x)$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$

$$E(X) = 1\times\frac{1}{6} + 2\times\frac{1}{6} + 3\times\frac{1}{6} + 4\times\frac{1}{6} + 5\times\frac{1}{6} + 6\times\frac{1}{6}$$

$$E(X) = \frac{(1+2+3+4+5+6)}{6} = \underline{\underline{3.5}}$$

$$\boxed{E(X) = 3.5}$$



Data balance (balance Point)

expected value.

$$\boxed{E(X) = \mu = \sum_{i=1}^{n} x_i \cdot P[X = x_i]}$$

X is DISCRETE Random variable.

" When X is a continuous Random variable.

$a \leq X \leq b$

$$\boxed{E(X) = \mu = \int_a^b x f(x)\, dx}$$

X is CRV

where $f(x)$ is a prob. Density Function.

PROPERTIES of EXPECTED value :-

(A) If X and Y ARE independent Random variables.

$$E(X+Y) = E(X) + E(Y)$$

Joint both Random variable ARE Added

$E(X)$ ⇓ Tossing A coin

$E(Y)$ ⇓ Throwing A Die

(LINEAR / superposition Rule)

$$L(ax+by) = aL(x) + bL(Y)$$

X is DISCRETE Random variable.

X = No. of Heads

X	0 T	1 H	
$P(X=x)$	$1/2$	$1/2$	

$$E(x) = 0 \times \frac{1}{2} + 1 \times \frac{1}{2}$$

$$E(X) = \frac{1}{2} = 0.5$$

Y is a also discrete random vari

Y = No. of Dots

Y	1	2	3	4	5	6
	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$E(Y) = 3.5$$

$$E(X) + E(Y) = 0.5 + 3.5 = 4.0$$

X, Y both ARE work together (X + Y)

+ Y	1	2	3	4	5	6
X						
0 (T)	$\boxed{1}$	2	3	4	5	6
1 (H)	2	3	4	5	6	$\boxed{7}$

X + Y

frequency

$$X + Y = 1, 2, 3, 4, 5, 6, 7$$

$$P(0T \cap 1) = P(0T)\,P(1) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

$$P(0T \cap 2) = P(0T)\,P(2) = \frac{1}{12} + \frac{1}{12} = \frac{2}{12}$$

$X+Y$	1	2	3	4	5	6	7
$P(X+Y = x_i + y_j)$	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{1}{12}$

$$\text{Expected value} = \sum_{i=1}^{n} x_i p_i$$

$$= (1+7)\frac{1}{12} + (2+3+4+5+6) \times \frac{2}{12}$$

$$= \frac{48}{12} = 4$$

Verified

✓ $$\boxed{E(X+Y) = E(X) + E(Y)}$$

If $X_1, X_2, X_3 -- X_n$ ARE indep. Random variable THEN

$$\boxed{E(X_1 + X_2 + X_3 + -- + X_n) = E(X_1) + E(X_2) + -- E(X_n)}$$

X_1 = Tossing A coin

X_2 = Throwing A Die

X_3 = picking A card

X_4 = drawing A ball (red)

$\#$ $\boxed{E(x+y) = E(x) + E(y)}$

$\#$ $\boxed{E(ax+b) = aE(x) + b}$

$$E(X) = 3.5$$

$$E(X+5) = 3.5 + 5 = \underline{\text{Ans}}$$

$$E(2X+3) = 2E(X) + 3$$

$$= 2 \times 3.5 + 3 = 10 \checkmark$$

$\#$ $\boxed{E(aX-b) = aE(X) - b}$

$\#$ $\boxed{E(aX) = aE(X)}$

$\#$ $\boxed{E(ax+by) = aE(x) + bE(y)}$

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$E\left(\frac{1}{X}\right) \neq \frac{1}{E(X)}$

$E\left(\frac{1}{X^2}\right) \neq \frac{1}{E(X^2)}$

$E\left(\frac{1}{\log x}\right) \neq \frac{1}{E(\log x)}$

$E(X^3) \neq [E(X)]^3$

Remember It

QUESTION

Q. Let $f(x)=\dfrac{k|x|}{(1+|x|)^4}, -\infty<x<\infty$

Then the value of k for which f(x) is a probability density function is

(A) 1/6

(B) 1/2

(C) 3

(D) 6

QUESTION

Q. A continuous random variable X has density function

$$f(x)=\begin{cases}2x & 0<x<\frac{1}{2}\\ \frac{4-2x}{3} & \frac{1}{2}\le x<2\\ 0, & \text{elsewhere}\end{cases}$$

Find $P[0.25 < x \le 1.25]$

QUESTION

Q. Let X be a continuous random variable with probability density function

$$f(x) = \frac{1}{2}e^{-|x-1|}, -\infty < x < \infty$$

Find the value of $P(1 < |X| < 2)$

6 PM to 8 PM

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THANK YOU!

Here's to a cracking journey ahead!