Random Variables - Part II

Course on Engineering Mathematics for GATE - CSE

Rahul • Lesson 15 • June 28, 2024

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Engineering Mathematics

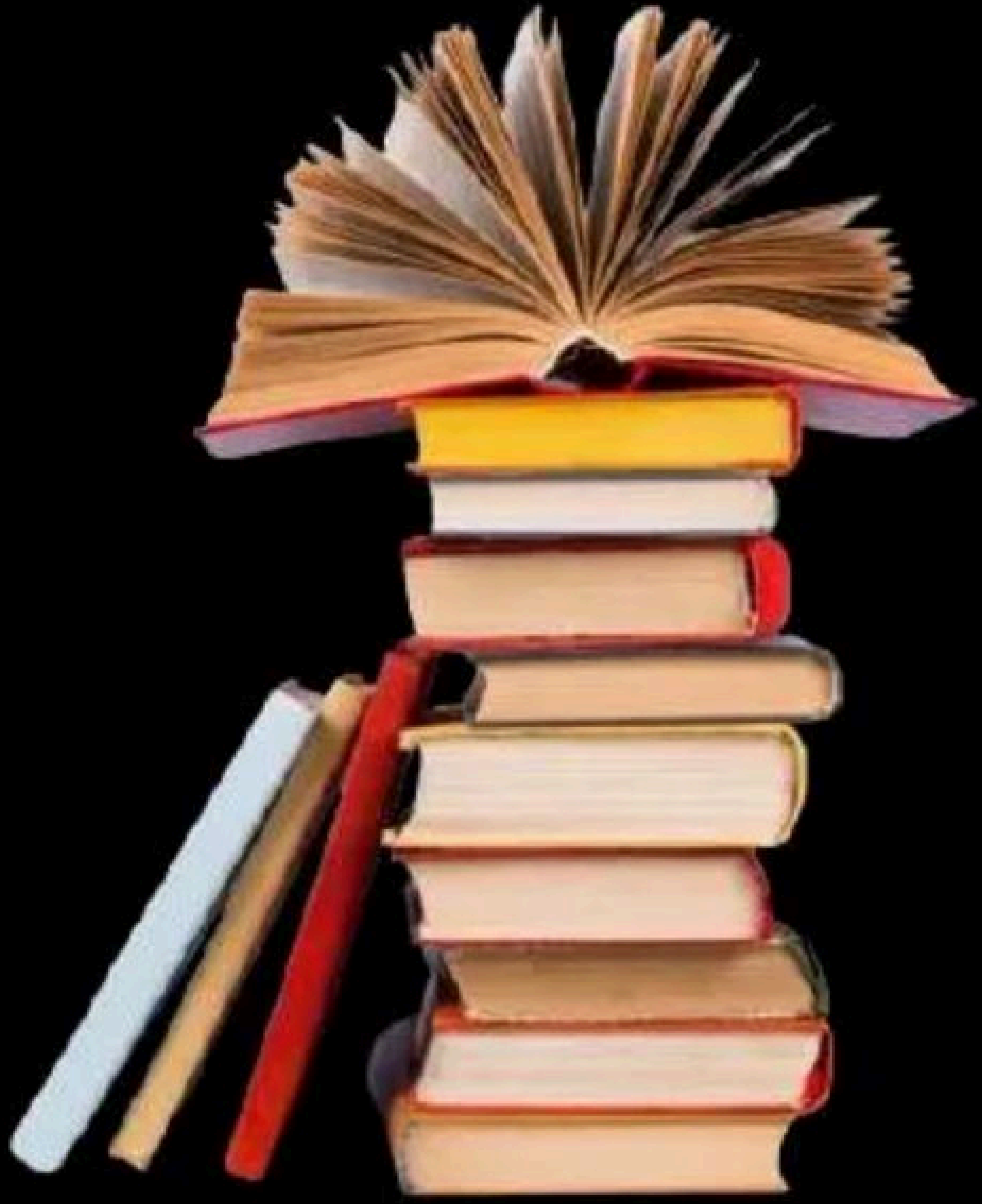
probability and statistics

Lecture Number- 14

By- Rahul Sir

Topics

to be covered



1. Random Variables_II

Prob. MASS Function PMF = Prob. at a Point

$$PMF = P[X = x]$$

Integer value. (Arrival)

cumulative distribution function

$$F_X(x) = \text{upto to } x$$

$$F_X(x) = \sum_{START}^{x} P(X \le x)$$

upto x (Arrival)

conversion between PMF to cdf OR cdf to PMF

Prob. at a point $\longrightarrow$ cdf $\longrightarrow$ PMF

QUESTION

Q. If X has the distribution function (data science / CSE)

$$F(X)=\begin{cases}0 & \text{for } X<1\\ \frac{1}{3} & \text{for } 1\le X<4\\ \frac{1}{2} & \text{for } 4\le X<6\\ \frac{5}{6} & \text{for } 6\le X<10\\ 1 & \text{for } X\ge 10\end{cases}$$

Find

(A) $P(2<X\le 6)$; (B) $P(X=4)$

(C) $P(X\ge 10)$ (D) $P(X<4)$

(E) $P(X>4)$ (F) $P(X\ge 4)$

Distribution function $F(x)$

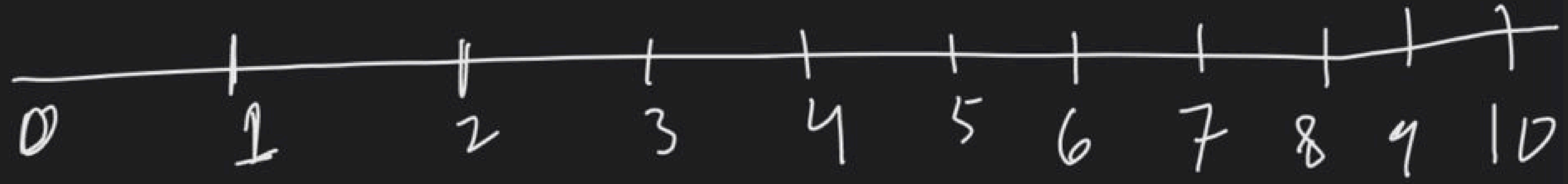
(prob. at a point)

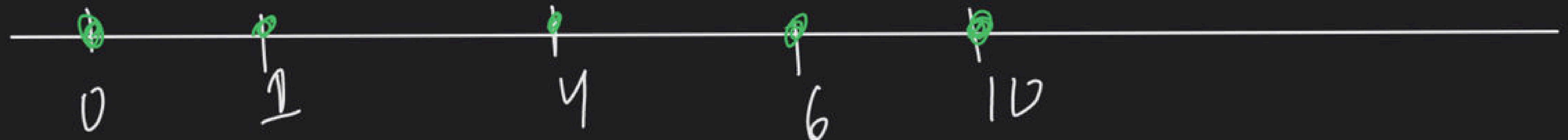
X has a distribution function

$$F(X) = \begin{cases} 0 & \text{for } x < 1 \\ \frac{1}{3} & 1 \le X < 4 \\ \frac{1}{2} & 4 \le X < 6 \\ 5/6 & 6 \le X < 10 \\ 1 & X \ge 10 \end{cases}$$

//STEP 01 Draw The Number Line

STEP 02 Marking - Points





$\left(\frac{1}{3}-0\right)$ $\left(\frac{1}{2}-\frac{1}{3}\right)$ $\left(\frac{5}{6}-\frac{1}{2}\right)$ $\left(1-\frac{5}{6}\right)$

0 1 $\frac{1}{3}$ 4 $\frac{1}{2}$ 6 $\frac{5}{6}$ 10 1

$\frac{1}{3}$ $\frac{1}{6}$ $\frac{1}{3}$ $\frac{1}{6}$

1 2 4 6 10

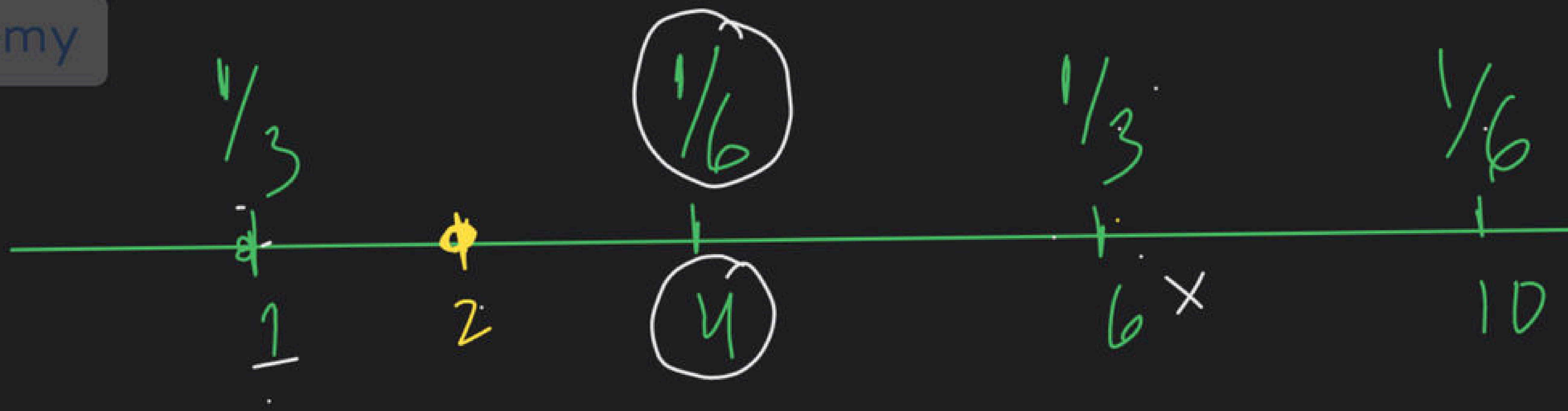
$P(2<X<6)$ Ans $=\frac{1}{6}$

$P(2\le X\le 6)$ $\qquad \frac{1}{6}+\frac{1}{3}=\frac{9}{18}=\frac{1}{2}$

2 6

$P(X\ge 4)=$

$\frac{1}{6}+\frac{1}{3}+\frac{1}{6}=\frac{2}{6}+\frac{2}{6}=\frac{4}{6}=\frac{2}{3}$



closed Add

open leave

$$P(1 \le X \le 6) = \frac{1}{3} + \frac{1}{6} + \frac{1}{3}$$

$$P(1 \le X < 6) = \frac{1}{3} + \frac{1}{6} =$$

$$P(X > 4) = \frac{1}{3} + \frac{1}{6} =$$

$$P(X < 4) = \frac{1}{3}$$

done

QUESTION

0 1/4 1/2 3/4 1

-1 1 3 0 5 cdf

-1 1 3 5 pmf

1/4 1/4 1/4 1/4

Q. If X has the distribution function

$$F(X)=\begin{cases}0 \text{ for } X<-1\\ \frac{1}{4} \text{ for } -1\le X<1\\ \frac{1}{2} \text{ for } 1\le X<3\\ \frac{3}{4} \text{ for } 3\le X<5\\ 1 \text{ for } X\ge 5\end{cases}$$

Find

TRUE
false

(A) $P(X \le 3)$; $= 3/4$

(B) $P(X = 3)$; $= 1/4$

(C) $P(X < 3)$; $2/4$

(D) $P(X \ge 1)$; $3/4$

(E) $P(-0.4 < X < 4)$; $= 2/4$

(F) $P(X = 5)$; $= 1/4$

(G) $P(3 < X < 5)$; $= 0$

(H) $P(3 \le X < 5)$; $= 1/4$

(I) $P(3 \le X \le 5)$; $\frac{2}{4}$

QUESTION

Q.

$$F_x(X) = \begin{cases} 0 & x < 0 \\ \frac{1}{8} & 0 \le x < 1 \\ \frac{4}{8} & 1 \le x < 2 \\ \frac{7}{8} & 2 \le x < 3 \\ 1 & 3 \le x \end{cases}$$

$1 \quad x \geq 3$

0 0 1/8 1 4/8 2 7/8 3 1

$\frac{1}{8} \quad \frac{3}{8} \quad \frac{3}{8} \quad \frac{1}{8}$

Calculate probability

(A) $P(X<2)$;

(B) $P(X \geq 2)$

(C) $P(X > 2)$

(D) $P(1 \leq X<3)$

(E) $P(X \leq 4)$

1/8 3/8 3/8 1/8

0 1 2 3

$P(X<2) = \frac{4}{8} = \frac{1}{2}$

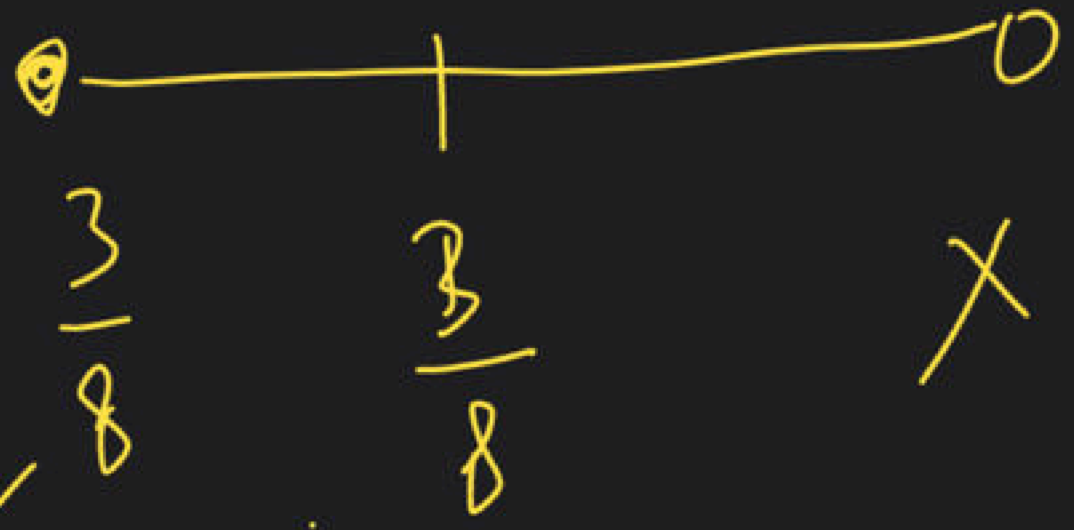
x	0	1	2	3
P	$1/8$	$3/8$	$3/8$	$1/8$

$$P(X \geq 2) = \frac{4}{8} = \frac{1}{2}$$

$$\frac{3}{8} + \frac{1}{8}$$

$$P(X > 2) = \frac{1}{8}$$

$$P(1 \leq X < 3) = \frac{6}{8} = \frac{3}{4}$$



$$P(X \leq 4) = 1 \checkmark$$

QUESTION

Q.

$$F_x(X) = \begin{cases} 0 & x < -1 \\ \frac{1}{4} & -1 \le x < 1 \\ \frac{1}{2} & 1 \le x < 3 \\ \frac{3}{4} & 3 \le x < 5 \\ 1 & 5 \le x \end{cases}$$

Find the probability

(A) $P(X \le 3)$;

(B) $P(X = 3)$ Last

(C) $P(X < 3)$

(D) $P(X \ge 1)$

(E) $P(3 < X < 5)$

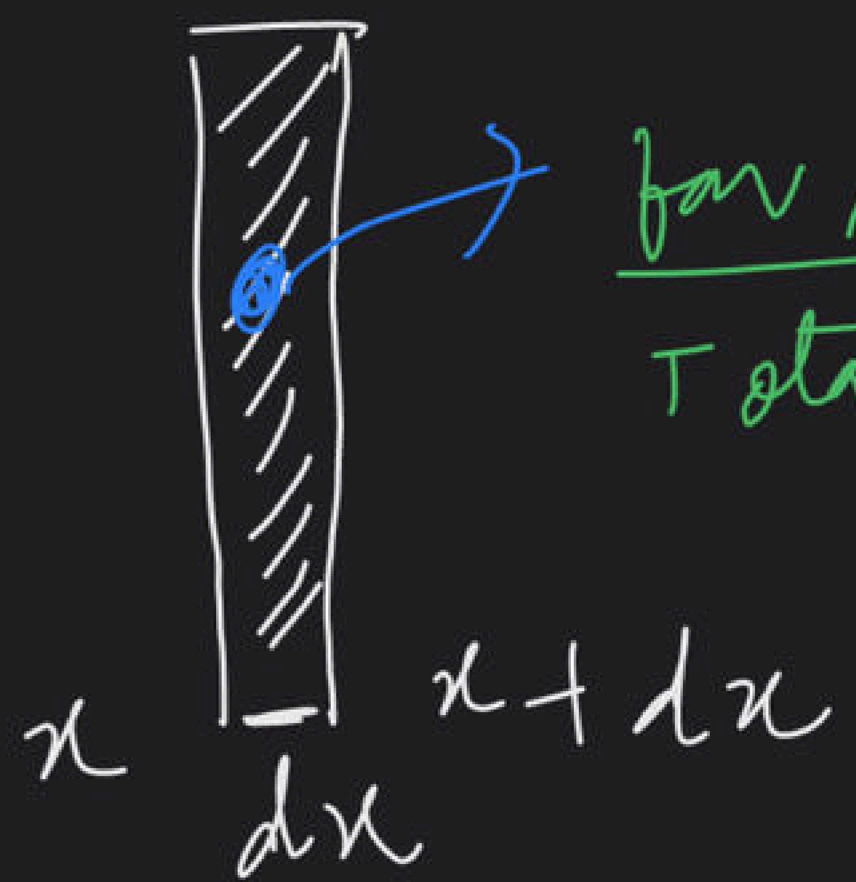
(D) $P(-0.4 < X < 4)$

DONE

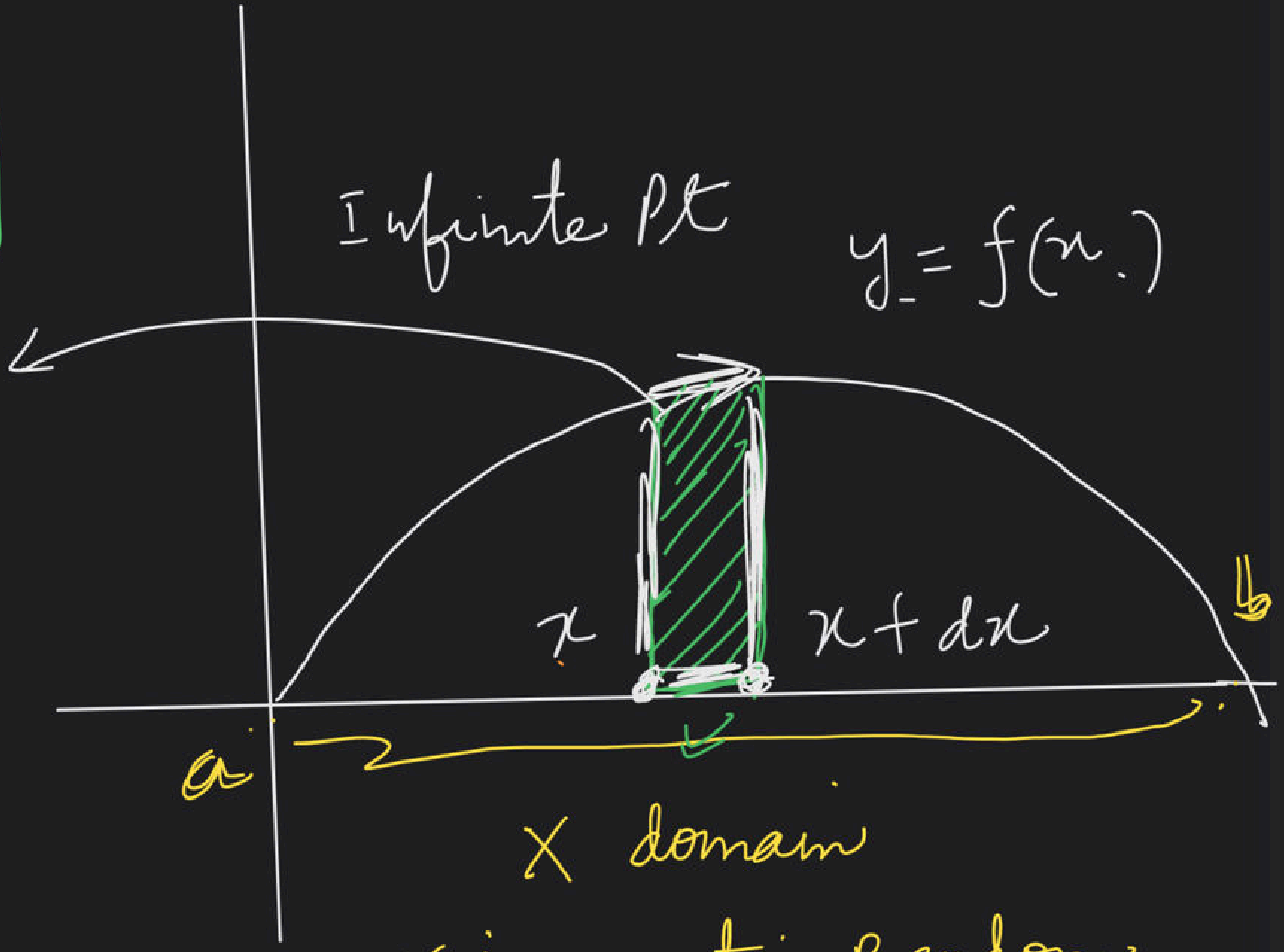
H.W

Home work

X is a continuous Random variable.



$$\frac{\text{fav region}}{\text{Total region}}$$



$P[x \le X \le x + dx]$

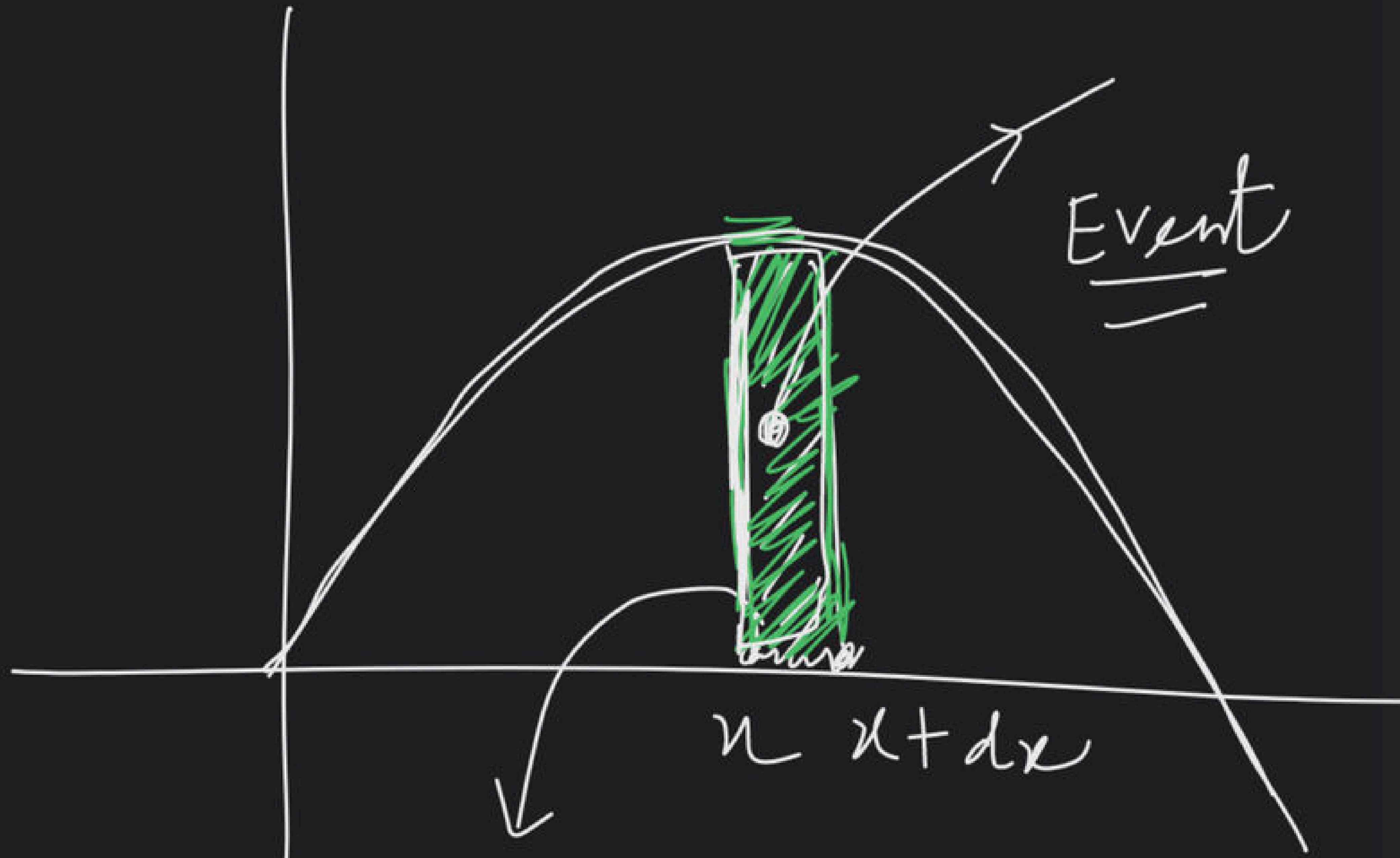
$$\boxed{\text{Prob} = \text{AREA}}$$

X is conti Random

$a \le X \le b$

$\int_{\text{Region}} f(x)\, dx$

$$\frac{\text{Prob. (AREA)}}{\text{Total AREA}}$$



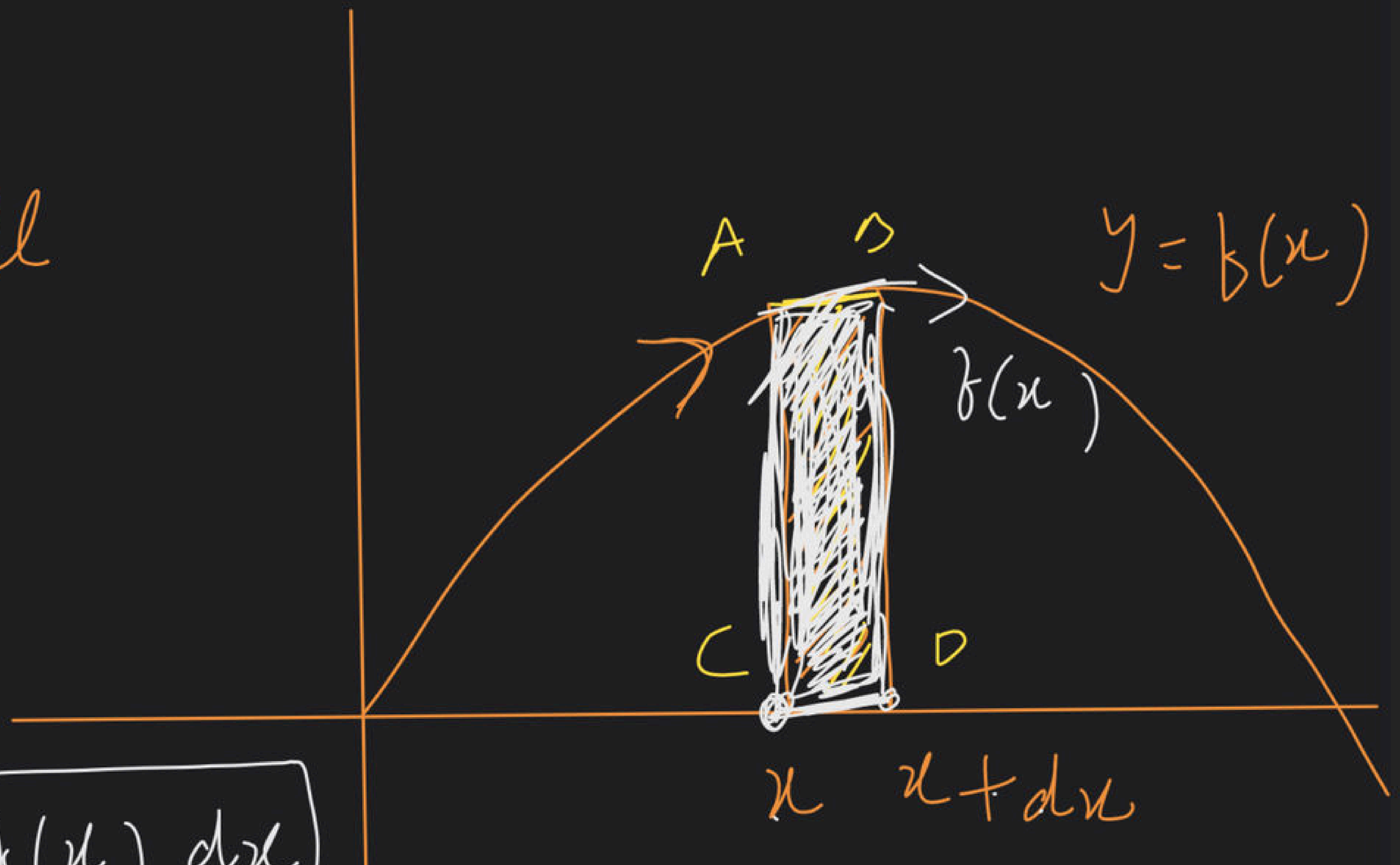
Infinite Points

$$P(E) = \frac{\text{small AREA}}{\text{Total AREA}}$$

$P[x \le X \le x + dx]$

$= \text{AREA of small Rectangle}$

$= f(x)\,dx$



$$\boxed{P(x \le X \le x + dx) = f(x)\,dx}$$

Prob. of event = AREA of Rectangle.

$x \le X \le x + dx$

continuous.

$$P(x \le X \le \underbrace{x + dx}) = f(x)\,dx$$

Using DEfination of cdf

$$F_X(x) = P(X \le x)$$

$$F_X(x + dx) - F_X(x) = f(x)\,dx$$

divide via dx

$$\underbrace{\lim_{dx \to 0} \frac{F_X(x + dx) - F_X(x)}{dx}} = f(x)$$

$$F_X'(x) = f(x)$$

$$F_X'(x) = f(x)$$

cdf

X - C.R.V Interval

$$\boxed{F_X(x) = \int_{\text{Region}} f(x)\,dx}$$

X is a continuous Random variable.

$$\boxed{F_X(x) = \int_{\text{Region}} f(x)\,dx}$$

Integration

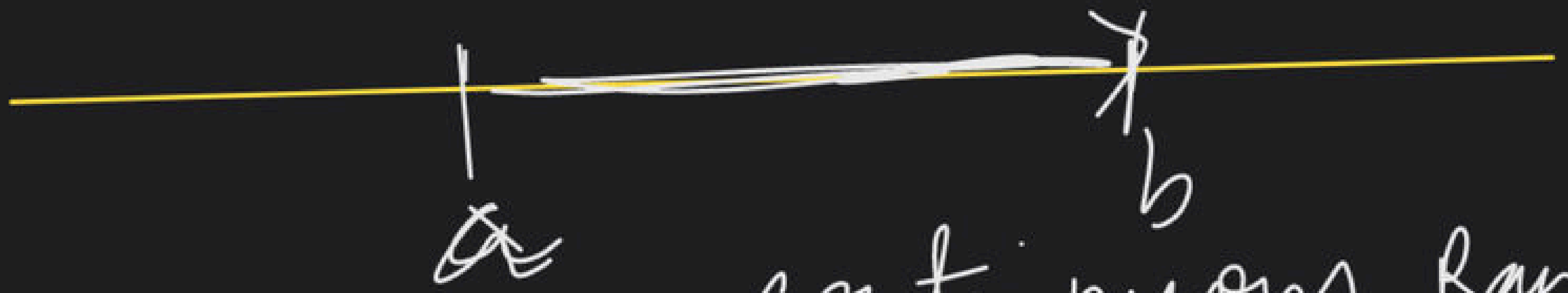
If X is a continuous Random variable.

$$F_X(x) = \int_{\text{Region}} f(x)\, dx \qquad a \le X \le b.$$

$\displaystyle\int_a^b f(x)\, dx = F_X(b) - F_X(a)$

$\displaystyle\int_{-\infty}^{\infty} f(x)\, dx = 1$ prob. density function

$f(x)$ = Density function



continuous Random

Integ

$$F_X(x) = \int_a^b f(x)\,dx$$

x

X is Discrete Random variable

$$F_X(x) = \sum P(X \le x)$$

START

X = integer

SUM

X is continuous Random variable.

$$P(X \geq a) = \int_a^{\infty} f(x)\, dx$$

$$P(X \leq a) = \int_{-\infty}^{a} f(x)\, dx$$

$$P(a \leq X \leq b) = \int_a^{b} f(x)\, dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

X is conti

$$P(a \leq X < b) = \int_a^b f(x)\,dx$$

$$P(a < X \leq b) = \int_a^b f(x)\,dx$$

$$P(a \leq X \leq b) = \int_a^b f(x)\,dx$$

$P(a < X < b) = \int_a^b f(x)\,dx$ X is continuous (Interval)

$P(a \le X \le b) = \underbrace{P(X=a)}_{0} + \int_a^b f(x)\,dx + P(X=b)$

$0 + \int_a^b f(x)\,dx + 0$

$= \int_a^b f(x)\,dx$

Prob. at Point $= 0$

a X is continuous Random variable b.

$P(a \le X < b) = \underbrace{P(X=a)}_{0} + \int_a^b f(x)\,dx = \int_a^b f(x)\,dx$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

slope $\dfrac{dy}{dx} = \lim_{dx \to 0} \dfrac{f(x+dx) - f(x)}{dx}$

QUESTION

Q. An urn contains 3 white and 4 red balls. 3 balls are drawn one by one with replacement. Find the probability distribution of the number of red balls.

QUESTION

$F(x)$ cdf $<$ Discrete / continuous. conti $\to$ pdf

Q. A continuous random variable X has the probability density function:

$$f(x) = Ax^3, 0 \le x \le 1.$$

(i) $A = 4$ value of A

(ii) $P[0.2 < X < 0.5]$

(iii) $P\left[X > \frac{3}{4} \text{given } X > \frac{1}{2}\right]$

X Has a prob. Density function (pdf) $f(x)$

$f(x) = Ax^3 \quad 0 \le x \le 1$

value of A.

$P(X > 3/4 \text{ given } X > \frac{1}{2})$

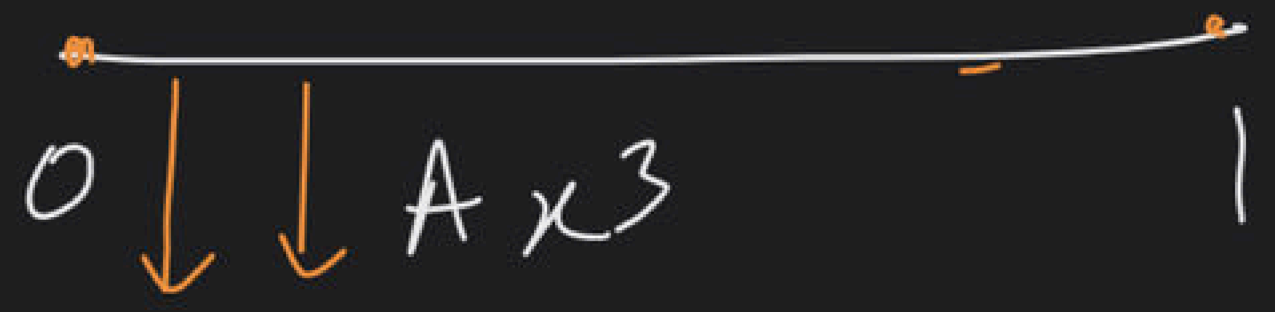
Constant finding :- f(x) is valid prob. Density function

∴ all prob. sum = 1

Total AREA = 1

$$\int_0^1 A x^3 \, dx = 1$$

$$0 \le x \le 1$$



$$A\left[\frac{x^4}{4}\right]_0^1 = 1$$

$$\boxed{A = 4}$$ ✓

$P(0.2 < X < 0.5) = \int_{0.2}^{0.5} Ax^3 = \int_{0.2}^{0.5} 4x^3$

✓ Interval

✓ Infinite uncountable number.

✓ continuous

Random variable.

$$= 4\left[\frac{x^4}{4}\right]_{0.2}^{0.5}$$

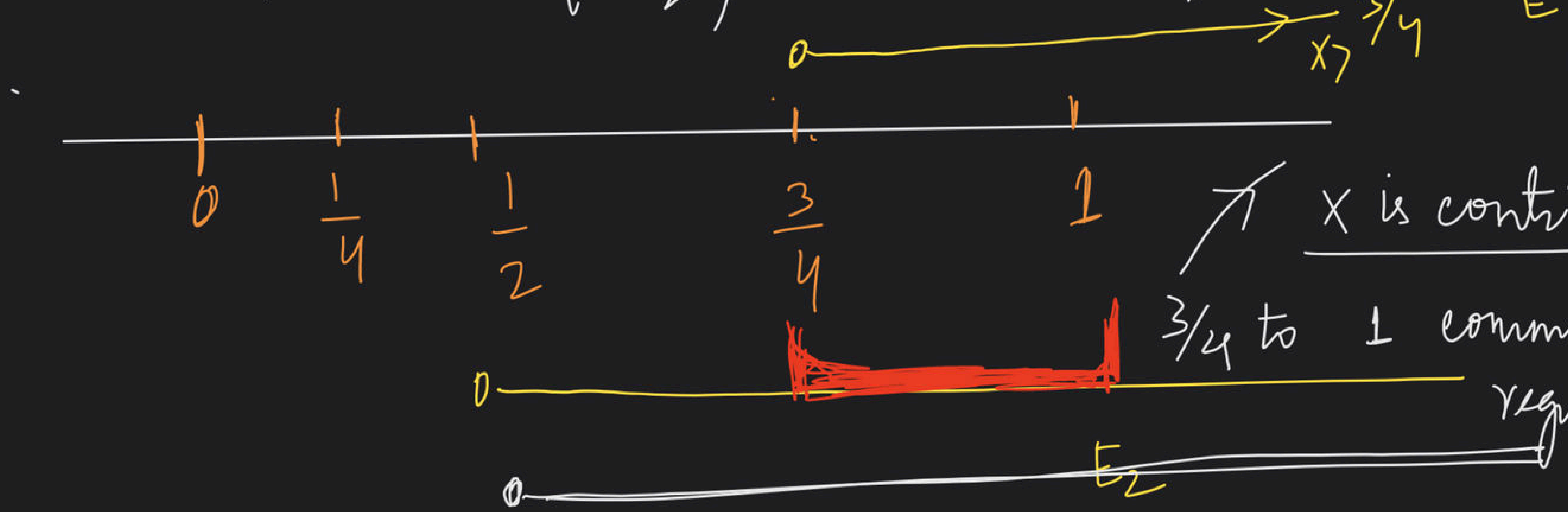
$$= \underbrace{(0.5)^4 - (0.2)^4}$$

Ans.

$P(X > 3/4 \text{ given } X > \frac{1}{2}) = P(X > 3/4 \text{ given } X > \frac{1}{2})$

conditional prob.

$$P\left(\frac{X > 3/4}{X > 1/2}\right) = \frac{P(\overrightarrow{X > 3/4} \wedge X > 1/2)}{P(X > \frac{1}{2})} = \frac{\overset{\text{common}}{\text{INTERSECTION}}}{P(X > 1/2)}$$



X is conti R.V

3/4 to 1 common region

$X > \frac{1}{2} = \frac{1}{2}$ to 1

$$P\left(\frac{X > 3/4}{X > 1/2}\right) = \frac{P\left(X > 3/4 \cap X > 1/2\right)}{P\left(X > \frac{1}{2}\right)}$$

$$= \frac{\int_{\frac{3}{4}}^{1} 4x^3 \, dx}{\int_{\frac{1}{2}}^{1} 4x^3 \, dx}$$

$$P\left(X > \frac{3}{4} \cap X > \frac{1}{2}\right) = \int_{\frac{3}{4}}^{1} 4x^3 \, dx$$

$$= (1)^4 - \left(\frac{3}{4}\right)^4 =$$

$$P\left(X > \frac{1}{2}\right) = (1)^4 - \left(\frac{1}{2}\right)^4 =$$

$\Rightarrow$

$$P\left(X > \frac{3}{4} \wedge X > \frac{1}{2}\right) = \int_{\frac{3}{4}}^{1} 4x^3 \, dx$$

$$= \cancel{4}\left[\frac{x^4}{\cancel{4}}\right] = \left[x^4\right]_{\frac{3}{4}}^{1} = (1)^4 - \left(\frac{3}{4}\right)^4$$

$$= \frac{175}{256}$$

$$P\left(X > \frac{1}{2}\right) = \int_{1/2}^{1} (4x^3)\, dx = \left[\frac{x^4}{\cancel{4}} \times \cancel{4}\right]_{1/2}^{1}$$

$$= 1 - \frac{1}{16} = \frac{15}{16} \checkmark$$

$$\frac{P(X > 3/4 \wedge X > 1/2)}{P(X > 1/2)} = \frac{\frac{175}{256}}{\frac{15}{16}} = \frac{35}{48} \checkmark = \underline{\underline{0.729}}$$

QUESTION

Q. The life (in hours) X of a certain type of light bulb may be supposed to be a continuous random variable with p.d.f.:

$$f(x)=\begin{cases}\dfrac{A}{x^3} & 1500 < x < 2500 \\ 0, & \text{elsewhere}\end{cases}$$

Determine the constant A and compute the probability that

$1600 \le x \le 2000$.

QUESTION

Q. The diameter 'X' of a cable is assumed to be a continuous random variable with p.d.f.

$$f(x) = \begin{cases} 6x(1-x), & 0 \le x \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

Obtain the c.d.f. of x.

QUESTION

Q. A random variable x has the following probability function:

x	0	1	2	3	4	5	6	7
p(x)	0	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{1}{100}$	$\frac{1}{50}$	$\frac{1}{100}$

Determine the distribution function of X.

QUESTION

Q. The p.d.f. of the different weights of a "1 litre pure ghee pack" of a company is given by:

$$f(x) = \begin{cases} 200(x-1) & \text{for } 1 \leq x \leq 1.1 \\ 0, & \textit{otherwise} \end{cases}$$

Examine whether the given p.d.f. is a valid one. If yes, find the probability that the weight of any pack will lie between 1.01 and 1.02.

QUESTION

Q. A random variable X has the following probability distribution:

x	0	1	2	3	4	5	6	7	8
p(x)	k	3k	5k	7k	9k	11k	13k	15k	17k

(i) Determine the value of k.

(ii) Find the distribution function of X.

QUESTION

Q. If X is a continuous random variable whose probability density function is given by

$$f(x) = \begin{cases} k(5x - 2x^2), & 0 \le x \le 2. \\ 0, & otherwise \end{cases}$$

Then $P(x > 1)$ is

(A) 3/14

(B) 4/5

(C) 14/17

(D) 17/28

$$f(x) = \begin{cases} k(5x - 2x^2 & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$P(X>1) = \int_1^2 k(5x - 2x^2)\,dx$$

If $f(x)$ is a valid pdf

$$f(x) = \begin{cases} k(5x - 2x^2) & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$P(X>1) = \int_1^2 \underbrace{k(5x-2x^2)}\,dx$$

$$\int_0^2 k(5x-2x^2)\,dx = 1$$

= k – form of const

k – remove

$$k\left(\frac{5x^2}{2} - \frac{2x^3}{3}\right)_0^2 = 1$$

$$\boxed{k = \frac{3}{14}}$$

$$P(X>1) = \int_1^2 \frac{3}{14}(5x-2x^2)\,dx$$

$$\Rightarrow \frac{17}{28}$$

QUESTION

Q. $P_x(X) = Me^{(-2|x|)} + Ne^{(-3|x|)}$ is the probability density function for the real random variable X, over the entire x-axis, M and N are both positive real numbers. The equation relating M and N is.

(A) $M + \frac{2}{3}N = 1$

(B) $2M + \frac{1}{3}N = 1$

(C) $M + N = 1$

(D) $M + N = 3$

QUESTION

✓ A continuous random variable X has a probability density function $f(x) = e^{-x}, 0 < x < \infty$. Then $P\{X > 1\}$ is .

(A) 0.368

(B) 0.5

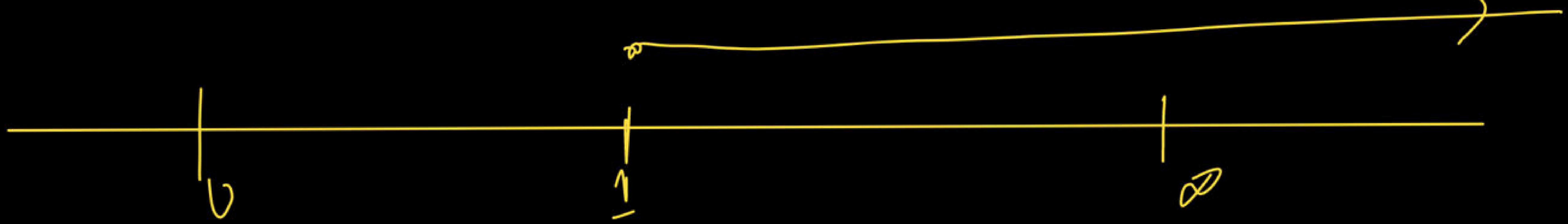
(C) 0.632

(D) 1.0

A conti Random vari X

$f(x) = e^{-x} \quad 0 < x < \infty$

$P(X > 1) = \int_1^\infty e^{-x}\,dx = $ ✓



$$P(X > 1) = \int_1^{\infty} e^{-x}\, dx \qquad f(x) = e^{-x}$$

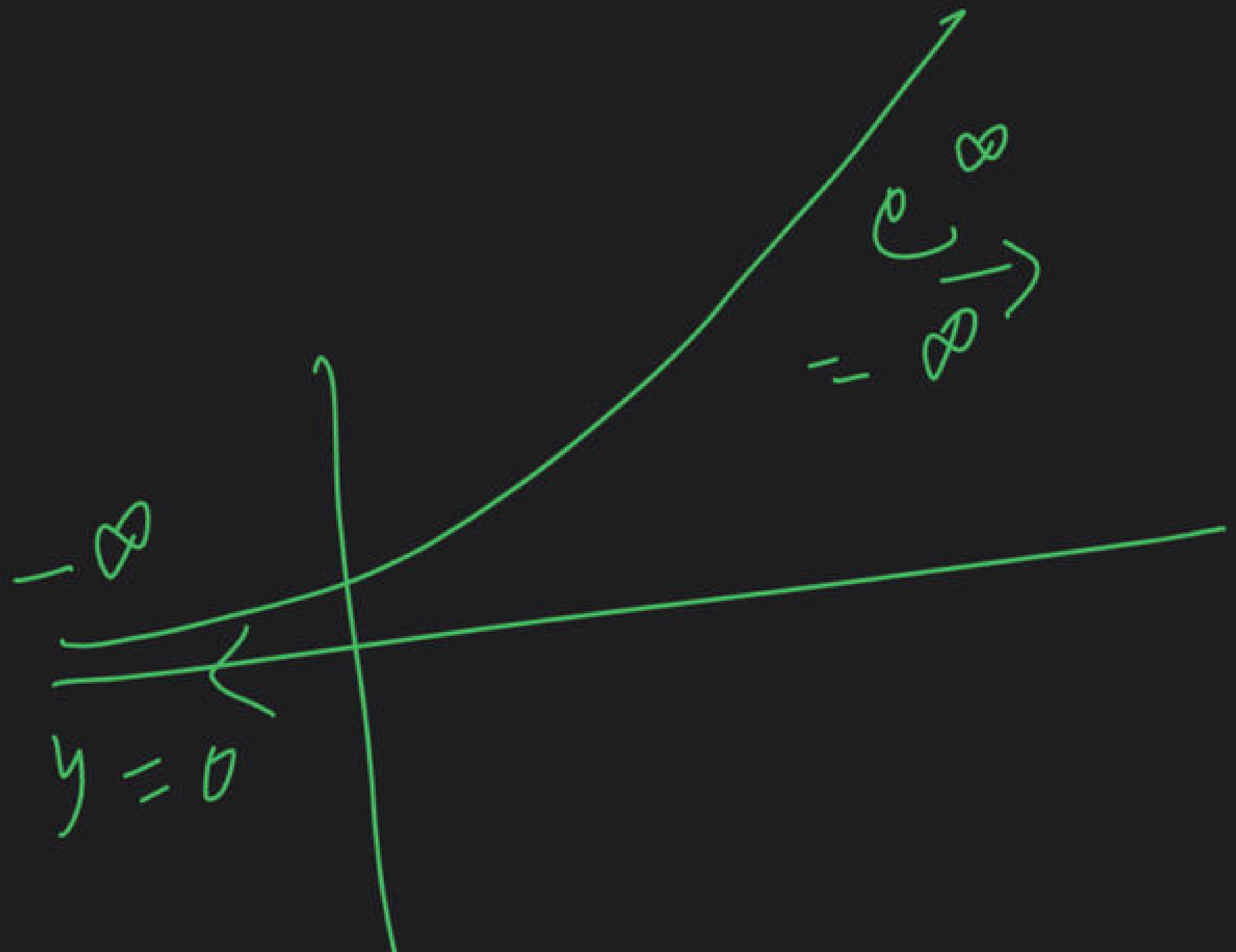
$$0 < x < \infty$$

$$= \left[-e^{-x}\right]_1^{\infty}$$

$$= -e^{-\infty} + e^{-1}$$

$$= 0 + e^{-1}$$

$$= \frac{1}{e} = 0.368$$



QUESTION

Q. Find the value of λ such that the function f(x) is a valid probability density function_____.

$$f(x) = \lambda(x-1)(2-x) \quad for\ \underbrace{1 \le x \le 2}$$
$$= 0 \quad otherwise$$

$\lambda = 6$

A) 2

B) 0

C) −6

D) 6

valid prob. density function

given: Range $\longrightarrow$ AREA $= 1$

$$\int_a^b f(x)\,dx = 1$$

$$\int_1^2 \lambda (x-1)(2-x)\,dx = 1$$

$$\lambda \int_1^2 (2x - x^2 - 2 + x)\,dx = 1$$

$$= \lambda \int_1^2 (-x^2 + 3x - 2)\,dx = 1$$

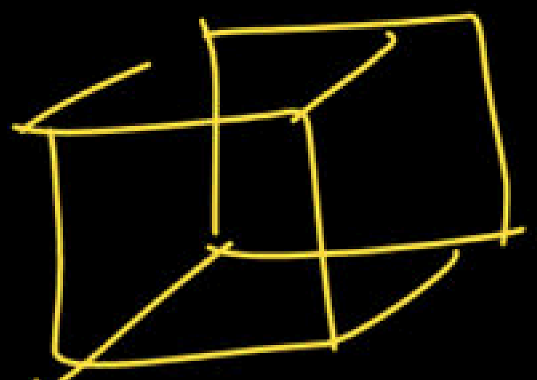
$$= \lambda \left[-\frac{x^3}{3} + \frac{3x^2}{2} - 2x \right]_1^2 = 1$$

$$= \boxed{\lambda = 6}$$

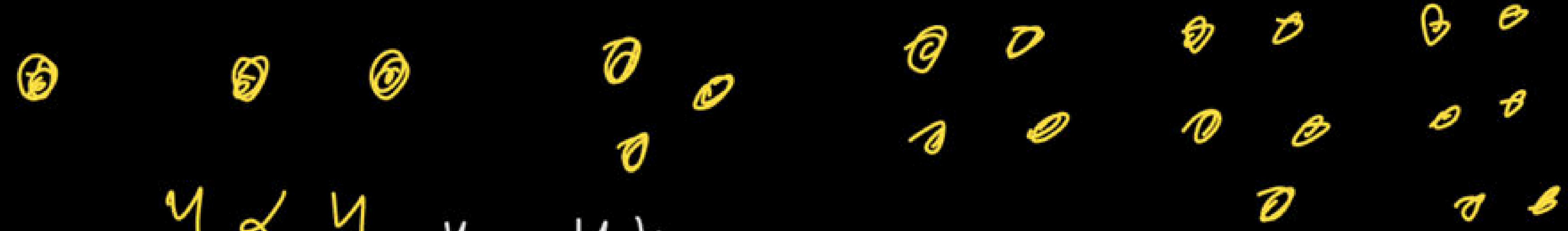
answer

QUESTION

Q. Consider a die with the property that the probability of a face with 'n' dots showing up is proportional to 'n'. The probability of the face with three dots showing up is______.



6 Dots (n dots)



$1 \propto 1$

$1 = k$

$2 \propto 2 \quad 2 = 2k$

$3 \propto 3 \quad 3 = 3k$

$4 \propto 4 \quad 4 = 4k$

$5 \propto 5 \quad 5 = 5k$

$6 \propto 6 \quad 6 = 6k$

$1 \propto 1$

$1 = k \cdot 1$

X is a Random variable

X = No. of dots / discrete / arm ival

Prob. Mass Table (PMF)

$X =$	1	2	3	4	5	6
$P(X=x)$	k	$2k$	$\boxed{3k}$	$4k$	$5k$	$6k$

If X is a discrete random value.

$P(3 \text{ dots}) = 3k$

k is unknown

Σ all prob. $= 1$

$$k+2k+3k+4k+5k+6k = 1$$

$$21k = 1$$

✓ $$\boxed{k = \frac{1}{21}}$$

$P(3 \text{ dots}) = 3k$

$= \frac{3}{21} = \frac{1}{7}$

QUESTION

Q. Let X be a random variable with probability density function

$$f(x)=\begin{cases}0.2 & for\,|x|\leq 1\\ 0.1 & for\,1<|x|\leq 4\\ 0 & othewise\end{cases}$$

The probability $P(0.5 < x < 5)$ is ______.

QUESTION

GATE

Q. Lifetime of an electric bulb is a random variable with density $f(x) = kx^2$, where x is measured in years. If the minimum and maximum lifetimes of bulb are 1 and 2 years respectively, then the value of k is ______.

$f(x) = kx^2$

x MEASURED in YEARS

$1 \leq X \leq 2$ YEARS

X is continuous Random variable

valid PDF

$$\int_1^2 kx^2\,dx = 1$$

$$\boxed{k = \frac{3}{7}}$$

QUESTION

GATE

Q. Given that x is a random variable in the range $(0, \infty)$ with a probability density function $\frac{e^{-\frac{x}{2}}}{k}$, the value of the constant k is.

valid pdf = 1

$$\int_0^{\infty} \frac{e^{-x/2}}{k}\, dx = 1$$

$$\boxed{k = 2}$$

answer

QUESTION

Q. A normal random variable X has the following probability density function

$$f_x(X) = \frac{1}{\sqrt{8\pi}} e - \left\{ \frac{(x-1)^2}{8} \right\}, -\infty < x \infty$$

Then $\int_1^{\infty} f_x(X)dx =$

A) 0

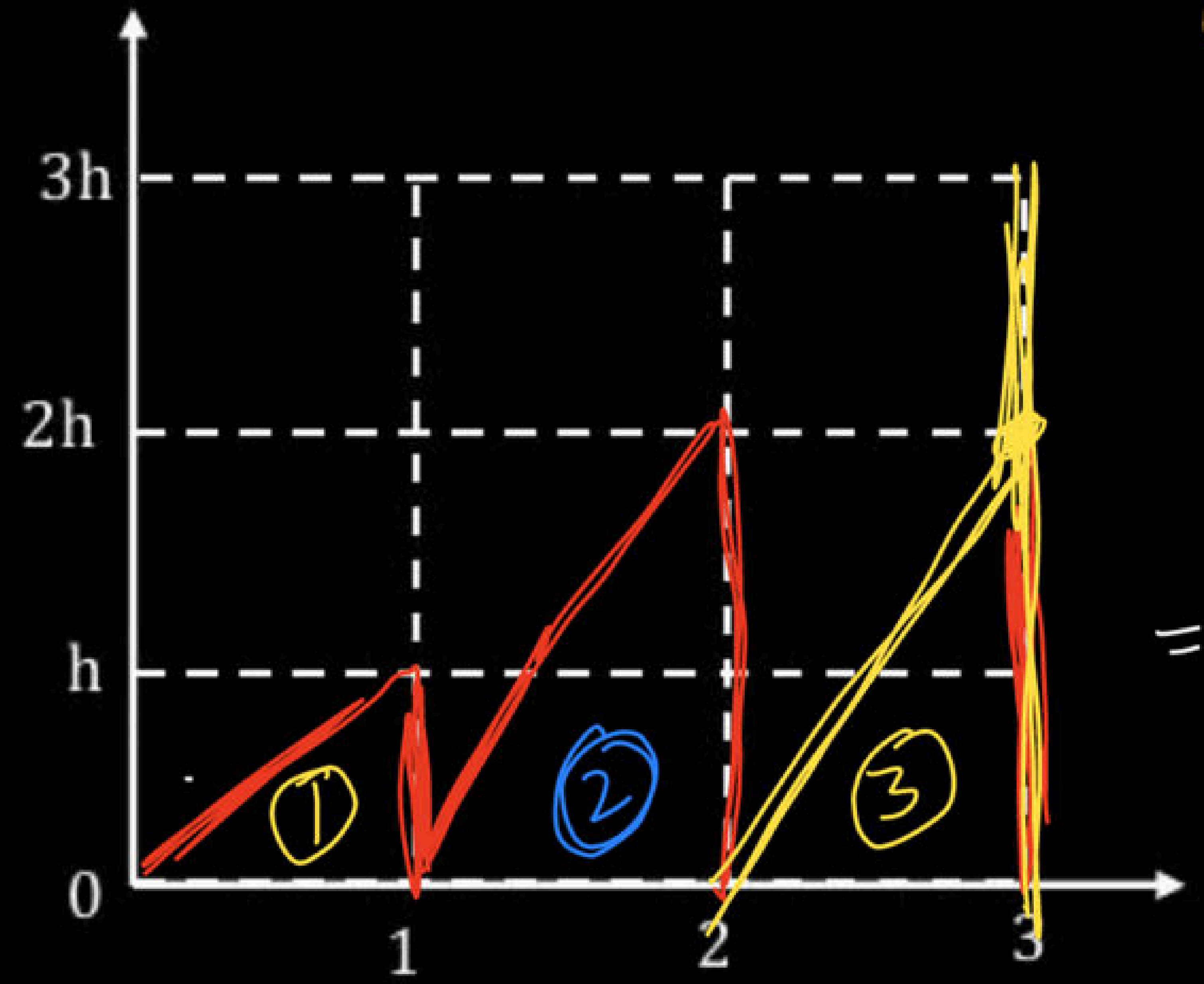
B) $\frac{1}{2}$

C) $1 - \frac{1}{e}$

D) 1

QUESTION

Q. The graph of function f(x) is shown in figure for f(x) to be valid probability density function, the value of h is



A) 1/3

B) 2/3

C) 1

D) 3

f(x) is valid PDF

AREA ① + ② + ③ = 1

$= \frac{1}{2} \times 1 \times h$

$+ \frac{1}{\cancel{2}} \times 1 \times \cancel{2}h$

$+ \frac{1}{\cancel{2}} \times 1 \times \cancel{2}h$

$= 1$

$\frac{h}{2} + h + h = 1$

$\frac{5h}{2} = 1$

$\boxed{h = \frac{2}{5}}$

$$\text{AREA}_{1} = \frac{1}{2} \times h \times 1 = \frac{h}{2}$$

$$\text{AREA}_{2} = \frac{1}{2} \times 2h \times 1 = h$$

$$\text{AREA} = \frac{1}{2} \times 3h \times 1$$

$$= \frac{3h}{2}$$

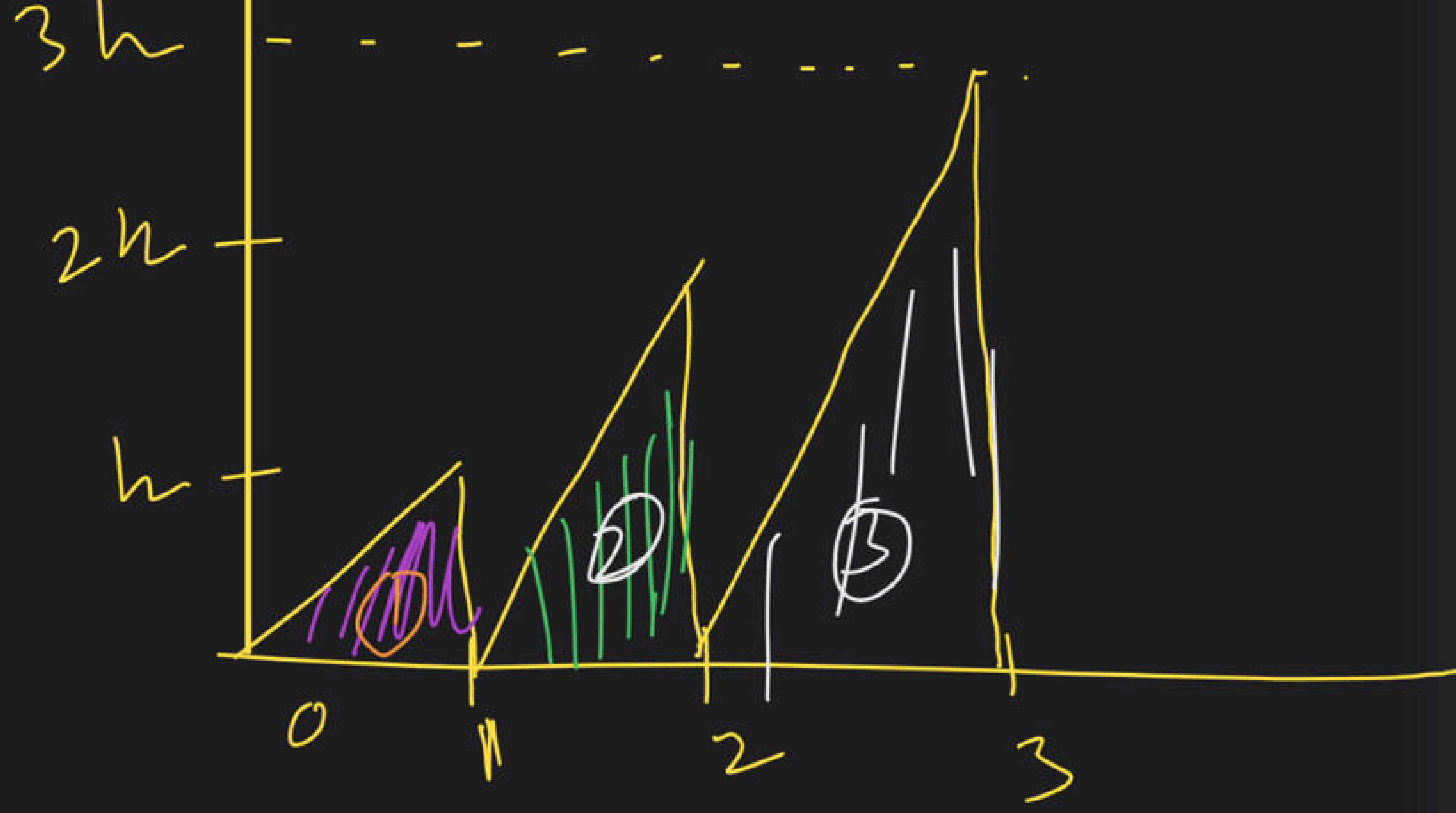
$$\frac{h}{2} + h + \frac{3h}{2} = 1$$

$$\frac{h + 2h + 3h}{2} = 1$$

$$\frac{6h}{2} = 1$$

$$\boxed{h = \frac{1}{3}}$$

A



QUESTION

Q. The random variable x takes on the value 1, 2 (or) 3 with probability $\frac{2+5P}{5}$, $\frac{1+3P}{5}$ and $\frac{1.5+2P}{5}$ respectively the values of P.

A 0.05

B 1.90

C 0.05

D 0.25

[X is a DISCRETE Random variable

X	1	2	3
$P(X=x)$	$\frac{2+5p}{5}$	$\frac{1+3p}{5}$	$\frac{1.5+2p}{5}$

all prob. sum = 1

$$\frac{2+5p}{5}+\frac{1+3p}{5}+\frac{1.5+2p}{5}=1$$

$$p=\frac{1}{20}=0.05$$

QUESTION

Q. The function p(x) is given by $p(x) = A/x^{\mu}$ where A and μ are constants with $\mu > 1$ and $1 \le x < \infty$ and $p(x) = 0$ for $-\infty < x < 1$. For p(x) to be a probability density function, the value of A should be equal to.

(A) $\mu - 1$

(B) $\mu + 1$

(C) $1/(\mu - 1)$

(D) $1/(\mu + 1)$

$P(x) = \begin{cases} \dfrac{A}{x^{\mu}} & \\ 0 & \end{cases}$

A and μ ARE constants

$\mu > 1$

$1 \le x < \infty$

otherwise

P(x) is valid prob. function

the value of A.

$P(x) = \begin{cases} \dfrac{A}{x^{\mu}} & 1 \le x < \infty \\ 0 & \text{otherwise} \end{cases}$

A and μ ARE constants

$\mu > 1$

$P(x)$ is valid prob. function

the value of A.

$$\int_1^{\infty} \frac{A}{x^{\mu}}\, dx = 1 \qquad \mu > 1$$

$$A \int_1^{\infty} x^{-\mu}\, dx = 1$$

$$A \left[\frac{x^{-\mu+1}}{-\mu+1} \right]_1^{\infty} = 1$$

$$A \left[0 - \left(\frac{1}{-\mu+1} \right) \right] = 1$$

$$\boxed{A = \mu - 1}$$

$$\boxed{\mu - 1 = A}$$

QUESTION

Q. A fair coin is tossed 3 times. Let the random variable X denote the number of heads in 3 tosses of the coin. Find the probability density function of X.

(A) $\left(\frac{3}{x}\right)\left(\frac{1}{2}\right)^{2x}\left(\frac{1}{2}\right)^{2-x}$

(B) $\left(\frac{3}{2x}\right)\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{1-x}$

(C) $\left(\frac{3}{x}\right)\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{3-x}$

(D) $\left(\frac{3}{x}\right)\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{4-x}$

QUESTION

Q. If the probability of a random variable X is given by $f(x) = k\,(2x - 1)$, $x = 1, 2, 3 \ldots, 12$. Find k.

QUESTION

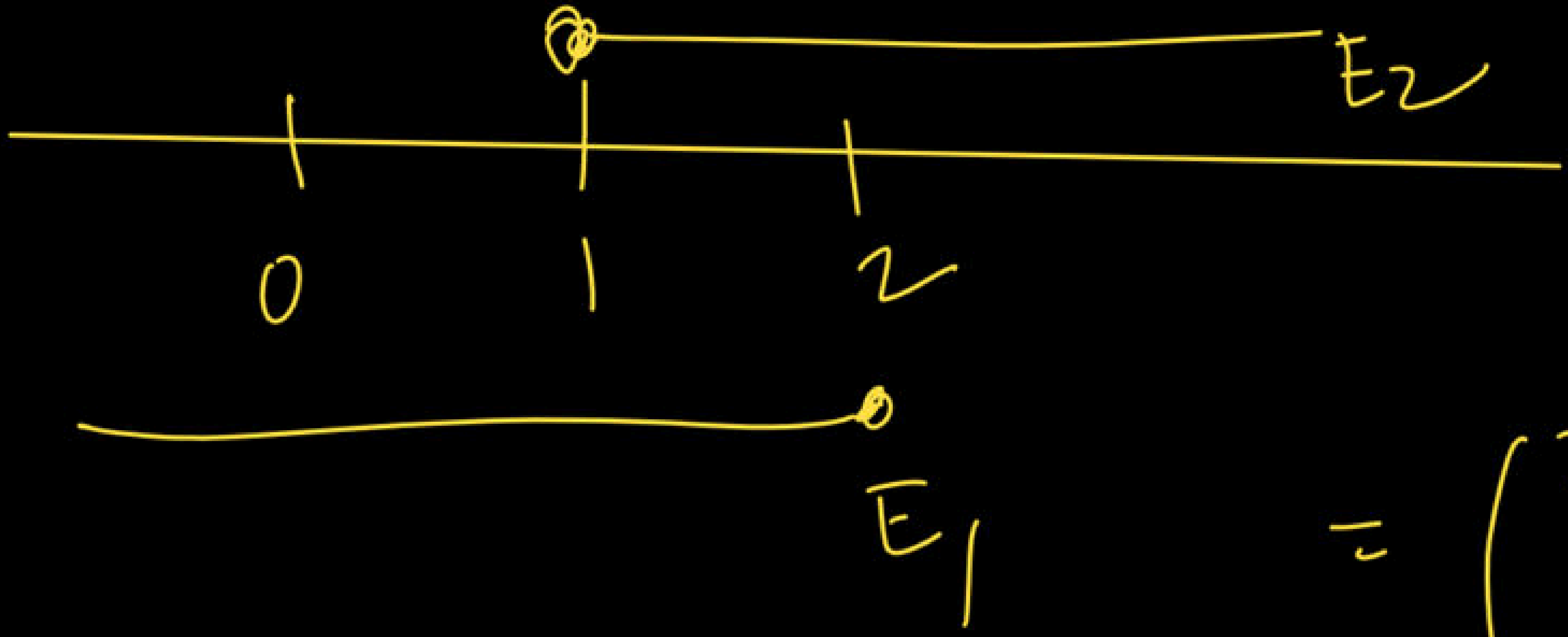
H.W

Q. The density function for the continuous random variable X is

$$f_x(X)=\begin{cases} e^{-x} & \text{for } x>0 \\ 0 & \text{for } x \le 0 \end{cases}$$

$$P\left(\frac{X\le 2}{X>1}\right)$$

Find the probability $P[x \le 2 | x > 1]$



$$= P\left(\frac{X \le 2 \cap X>1}{X>1}\right)$$

$$= \underbrace{\int_1^2 e^{-x}\,dx \Big/ \int_1^\infty e^{-x}\,dx}$$

QUESTION

Q. Suppose the random variable X has a probability density function

$$f(x) = \begin{cases} kx^3 \, e^{-x/2}, \text{ for } x > 0 \\ 0 \qquad\qquad \text{otherwise} \end{cases}$$

The value of k is______.

A 1/96

B 96

C 8/3

D 1/4

QUESTION

Q. Lex X be a continuous random variable with pdf

$$f_x(X) = \begin{cases} cx^2, \text{ for } 0 < x \le 1, \\ 0 \quad\quad \text{otherwise} \end{cases}$$

For some positive constant c. The value of $P\left(x \le \frac{2}{3} \middle| x > \frac{1}{3}\right)$

A 3/26

B 5/26

C 7/26

D 11/26

QUESTION

Q. Suppose the random variable X has the probability density function

$$f(x)=\begin{cases} ce^{x/3}, x \le 0, \\ ce^{-x/3}, x > 0 \end{cases}$$

For some positive constant c. The value of P $[x > 6/x > 0]$ is

A e^{-2}

B ce^{-2}

C 0

D $1-e^{-2}$

QUESTION

Q. Let X be a discrete random variable with probability function $P(X = x) = \frac{2}{3^x}$ for $x = 1, 2, 3, \ldots$. What is the probability that X is even?

(A) 1/4

(B) 2/7

(C) 1/3

(D) 2/3

QUESTION

Q. Let $f(x)=\dfrac{k|x|}{(1+|x|)^4}, -\infty < x < \infty$

Then the value of k for which f(x) is a probability density function is

A) 1/6

B) 1/2

C) 3

D) 6

QUESTION

Q. A continuous random variable X has density function

$$f(x)=\begin{cases} 2x & 0<x<\frac{1}{2} \\ \frac{4-2x}{3} & \frac{1}{2}\leq x<2 \\ 0, & \text{elsewhere} \end{cases}$$

Find $P[0.25 < x \leq 1.25]$

QUESTION

Q. Let X be a continuous random variable with probability density function

$$f(x) = \frac{1}{2}e^{-|x-1|}, -\infty < x < \infty$$

Find the value of $P(1 < |X| < 2)$

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THANK YOU!

Here's to a cracking journey ahead!