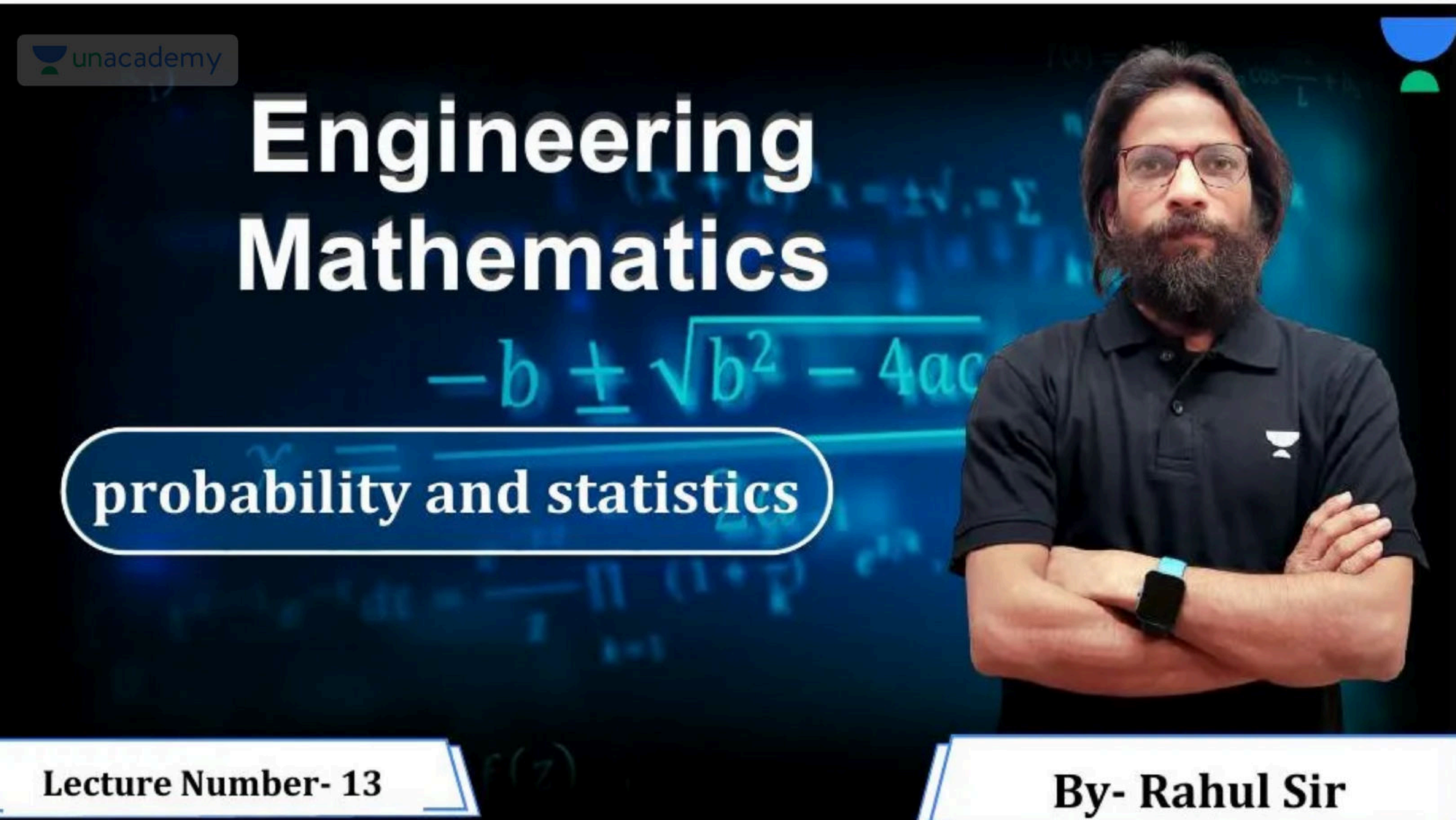
Random Variables - Part I

Course on Engineering Mathematics for GATE - CSE

Rahul • Lesson 14 • June 27, 2024



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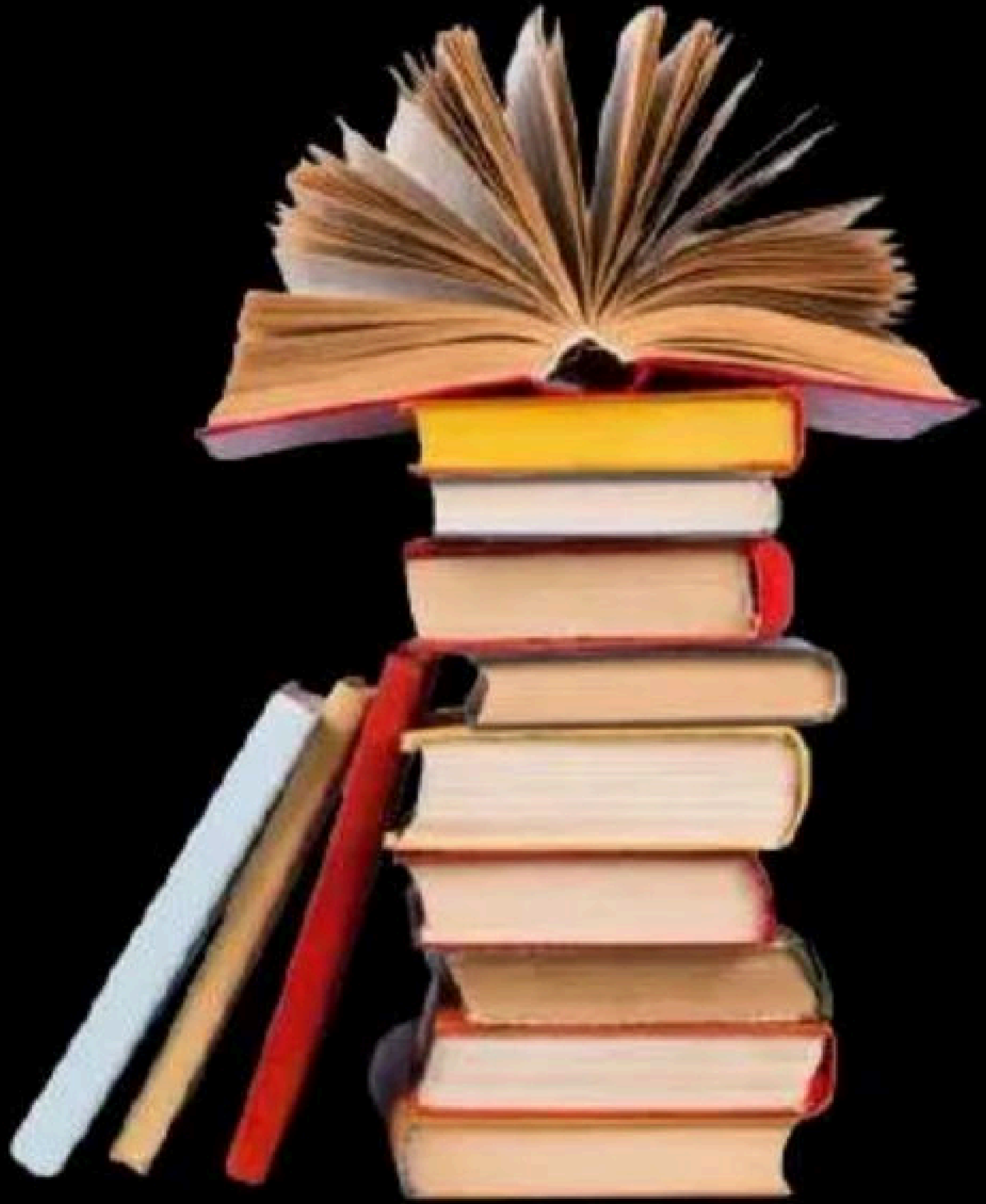
Engineering Mathematics
−b ± √b² − 4ac
probability and statistics
Lecture Number- 13
By- Rahul Sir

Topics

to be covered



1. Random Variables

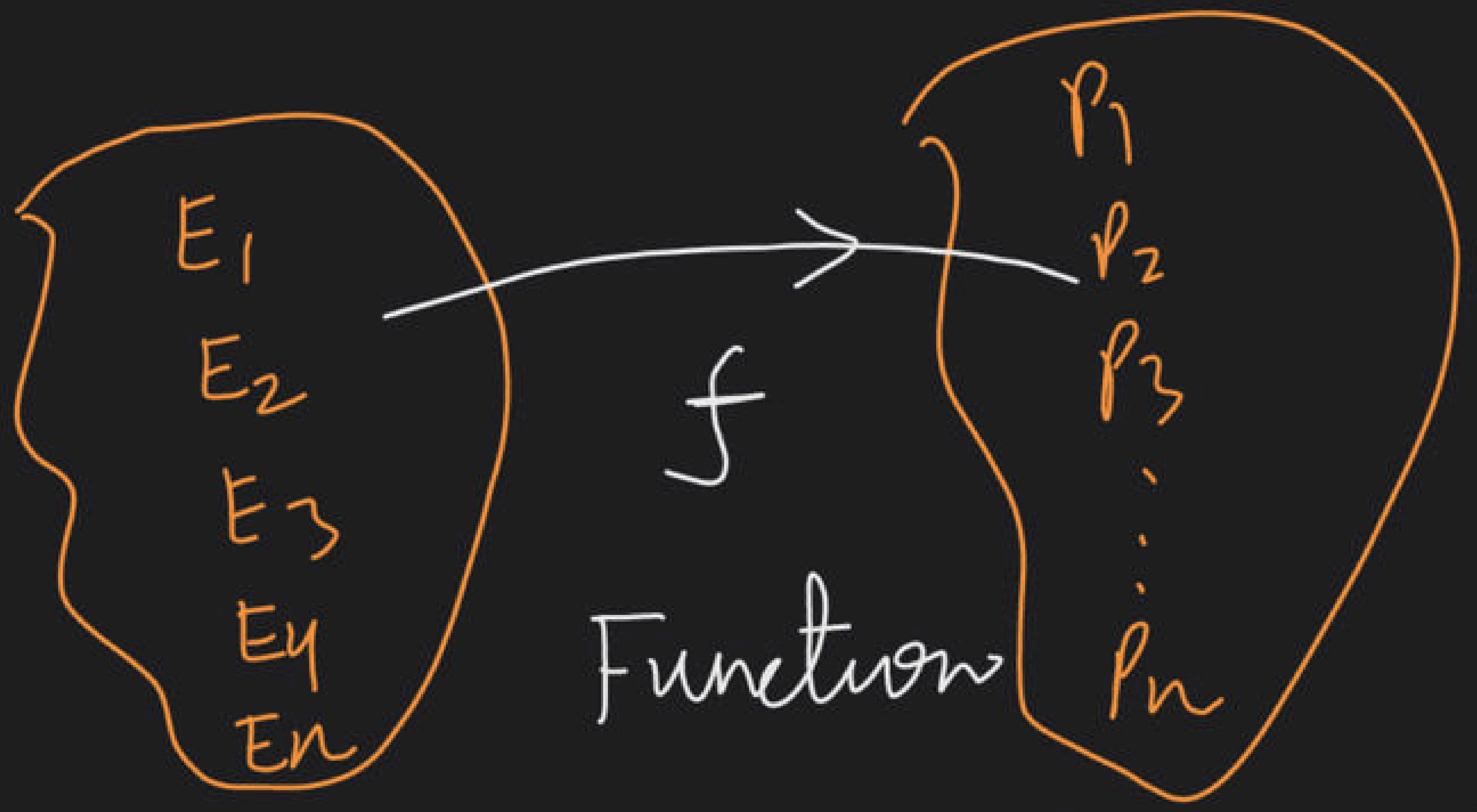
PYQ + Revision

Random Variable :- $P(\text{Event}) = \dfrac{n(E)}{n(S)}$ = frequency

⇓

more Than 2 events

$P(\text{more Than 2 events}) = \dfrac{\text{No. of fav}}{\text{Total events}}$



Event and prob ⟶ Relationship.

Function = probability

Random Variable is a Mathematical function

(Input) — Function PROCESS — output

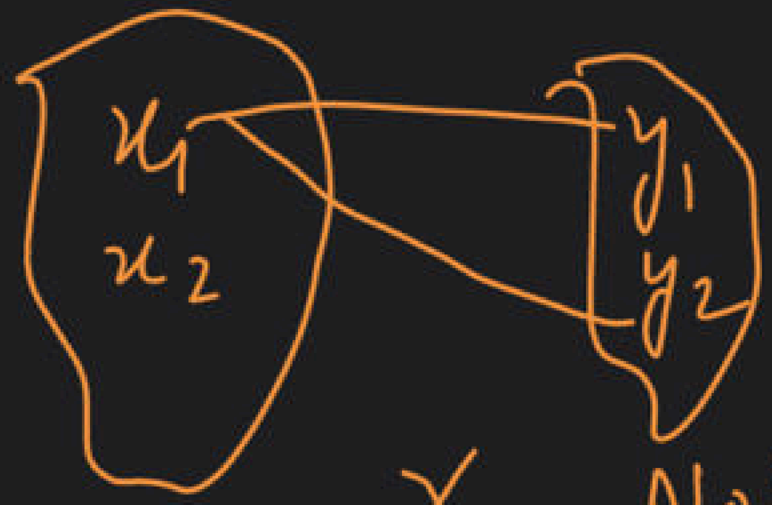
x f y

$f : X \to Y$

output is always in case of prob. THEORY

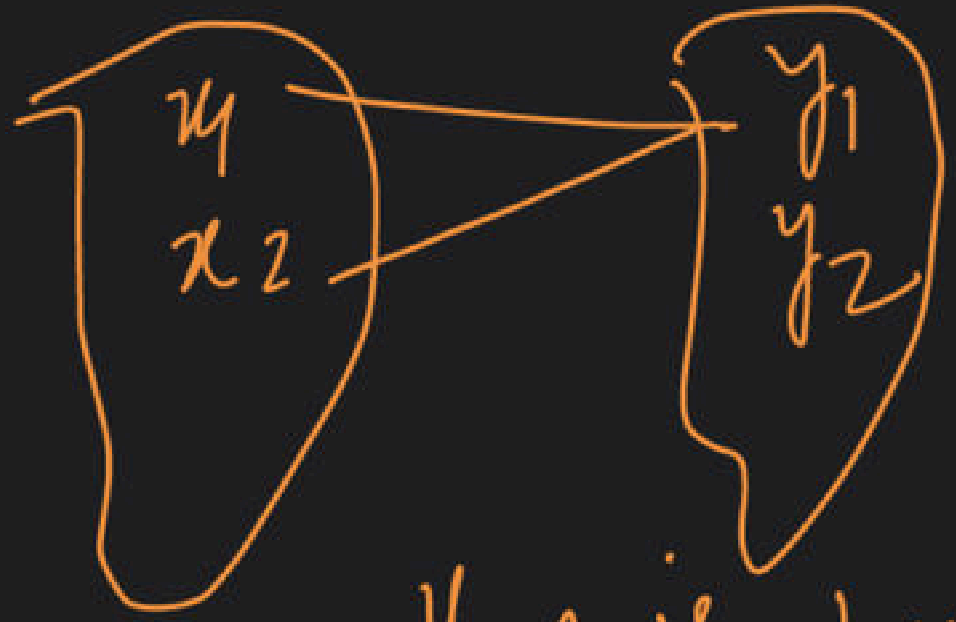
$$\boxed{0 \le P(E) \le 1}$$ ✓

#



X Not a function

#



ONTO

This is function

Input

(sample space)

(sample point)

PROCES

output

$0 \le P(E) \le 1$

Prob ← Random ①, sample point ②, Event ③

(Ω, S, E)

Tossing A Single coin

$S = \{H, T\}$

(H, t)

PROCESS

output

Tossing A Single coin

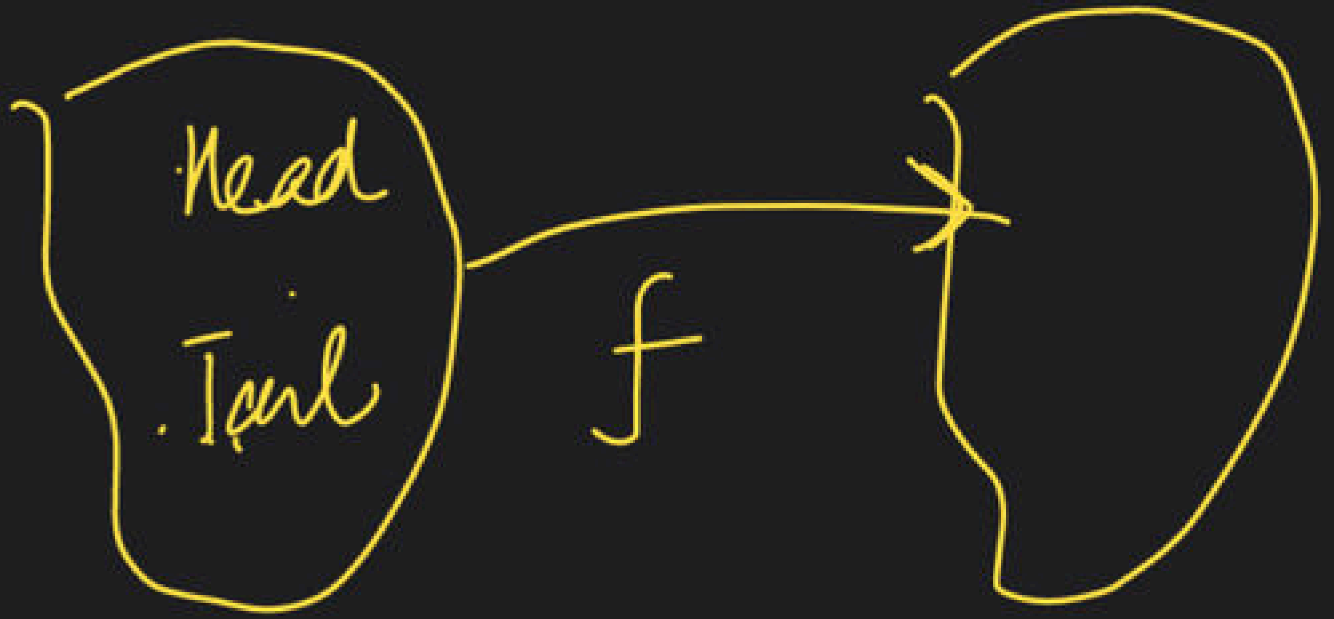
$S = \{H, T\}$

PROCESS

(H, t)

output

Sample Point	Head	Tail



We Define a variable

X = No. of Heads / No. of Tails

(YOUR DESIRE)

$X = 0H, 1H$

OR

$X = 0T, 1T$

$\{0, 1\}$ Head $\rightarrow$ Function $\rightarrow$ output

X = chance $\longrightarrow$ Function process $\longrightarrow$ output (unique value)

$P[\underbrace{X = 0}] = \frac{1}{2}$ $\quad$ $P[\underbrace{X = 1}] = \frac{1}{2}$

'X' is a Random Variable.

Tossing A Two coin

Re 1, Re 2

$H < \begin{matrix} H & = HH \\ T & = HT \end{matrix}$

$T < \begin{matrix} H & = TH \\ T & = TH \end{matrix}$

Sample Pt   HH   HT   TH   TT

WE DEFINE a Random variable X

$\sum$ All prob. = 1

X = No. of Heads / No. of Tails. (you DEcide / depend on problem)

Sample Pt	HH	HT	TH	TT
X	2	1	1	0

= Frequency

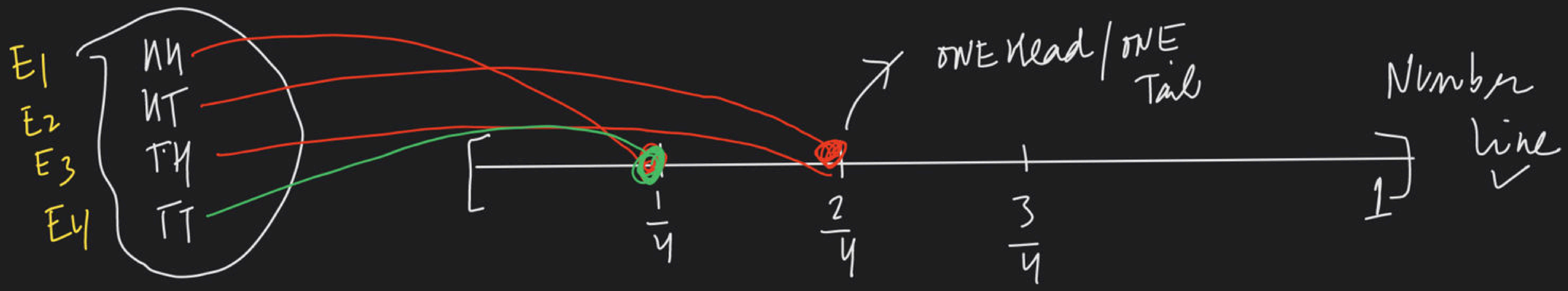
$X = 0, 1, 2$

$\xrightarrow[X=0,1,2]{}$ $\boxed{\text{Function}}$ $\longrightarrow$

$P(X=0) = \frac{1}{4}$

$P(X=1) = \frac{2}{4}$

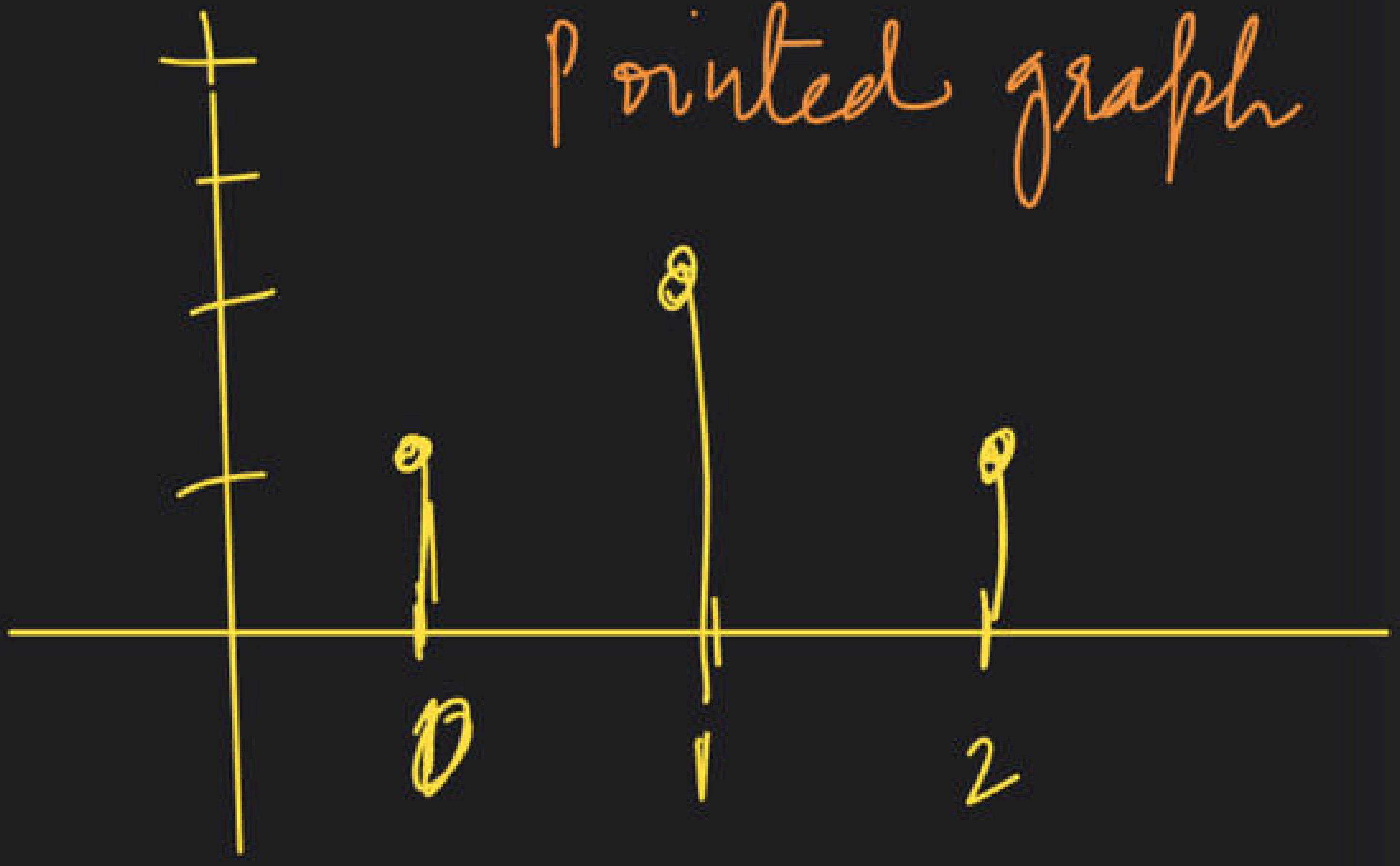
$P(X=2) = \frac{1}{4}$



Sample pt	HH	HT	TH	TT
X	2	1	1	0

= Frequency

Pointed graph



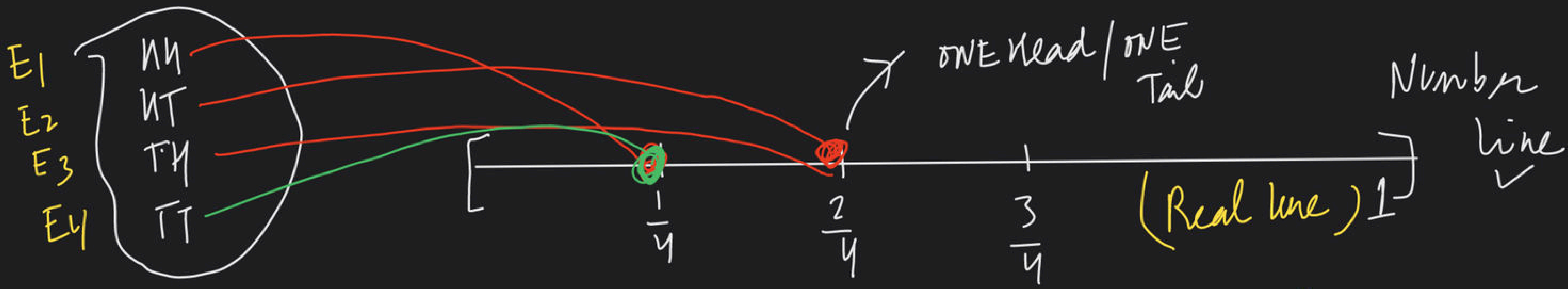
$X = 0, 1, 2$

$\xrightarrow[x=0,1,2]{}$ $\boxed{\text{Function}}$ $\longrightarrow$

$P(X=0) = \frac{1}{4}$

$P(X=1) = \frac{2}{4}$

$P(X=2) = \frac{1}{4}$



1) $P(E_1) + P(E_2) + P(E_3) + P(E_4) = 1$

2) all prob (sum) $= 1$

Tossing A THREE coins

H < H < H, T; T < H, T

T < H < H, T; T < H, T

= HHH – 3H

= HHT = 2H

= HTH = 2H

= HTT = 1H

= THH = 2H

= THT = 1H

= TTH = input 1H

= TTT = 0 H

Re1 Re2 Re3

H T H T H T

X is a Random variable

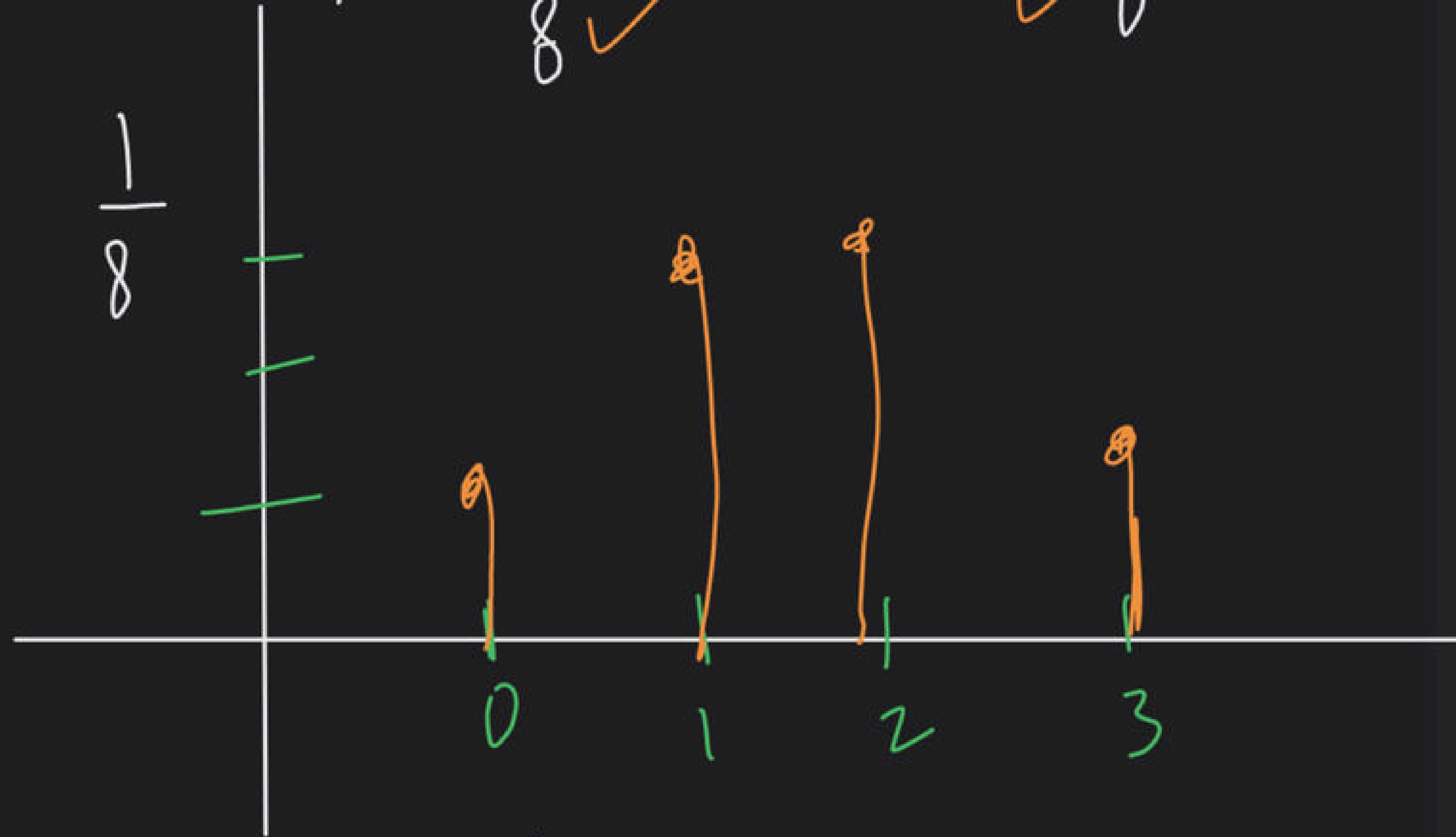
X = No. of Heads/ Tails.

X = 0, 1, 2, 3 → Function → prob.

$X = 0, 1, 2, 3$ → Function — prob.

$P(X=0) = \frac{1}{8}$ $\quad P(X=1) = \frac{3}{8}$ $\quad P(X=2) = \frac{3}{8}$

$P(X=3) = \frac{1}{8}$



A) $\sum_{i=0}^{n} P[X = x_i] = 1$

B) ONTO function

C) Input → Integer form.

$\left.\begin{matrix} \cdot\, 1 \\ \cdot\, 2 \end{matrix}\right\rangle \frac{3}{8}$

Input $\longrightarrow$ PROCESS $\longrightarrow$ output

$X = \underbrace{0, 1, 2, 3}$

$X = \underbrace{\text{Integer Form}}$ (prob)

X = Integer / Arrival pattern ($\underbrace{\text{आना / SUCCESS}}$)

Ex:- A) Die 1, 2 3 4 5 6 — Arival

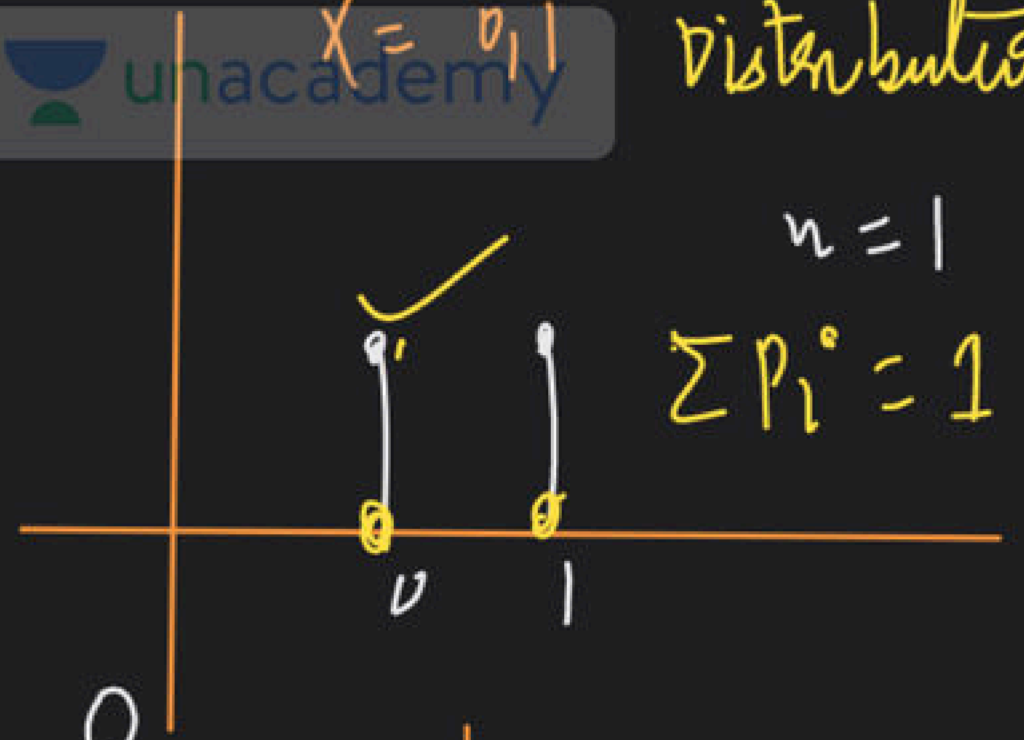
B) shop: 2 3 4 5 6 7 — Arival

C) No. of Insurance in any Day — 12 insurance ✓ — 12.5 ✗

D) No. of vehicles ✓ $\underbrace{\text{Road}}$ — 1.5 vehicle ✓

✓ $\boxed{\text{Arrival / integer / } X = \text{Random variable.}}$

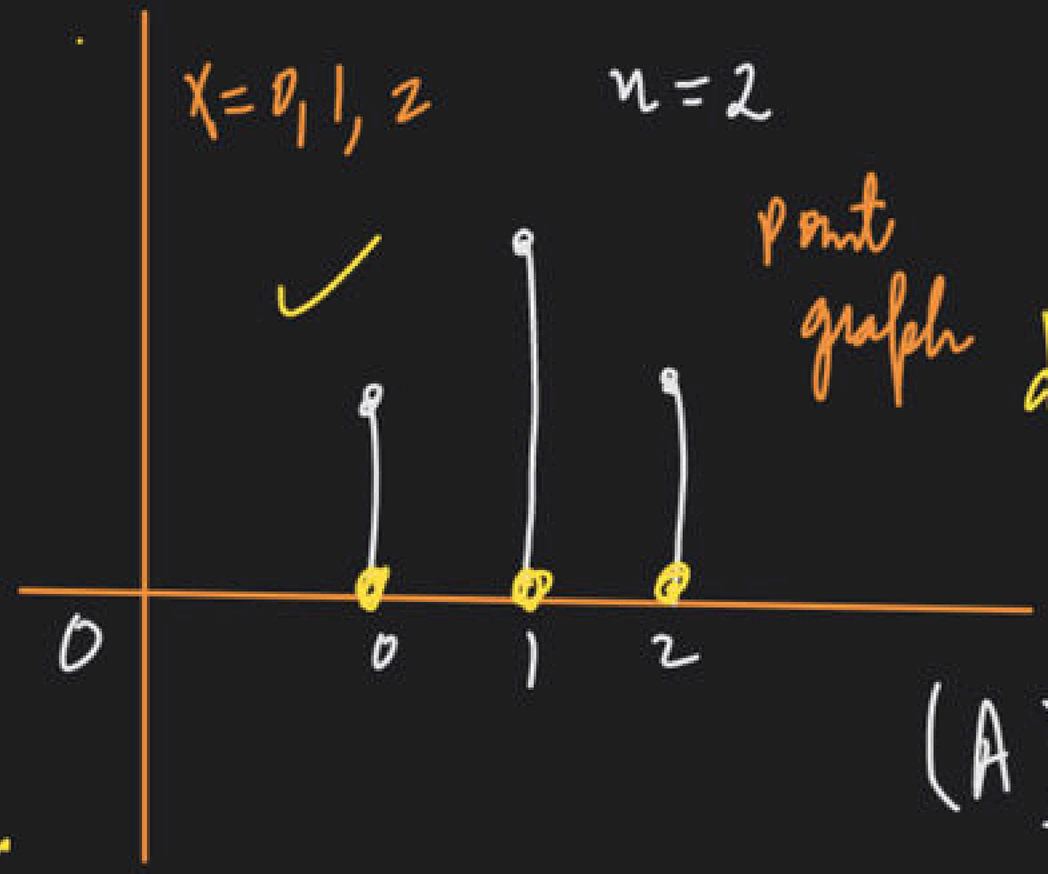
$X = 0, 1$ Distribution



$n = 1$

$\sum P_i = 1$

$X = 0, 1, 2$ $n = 2$



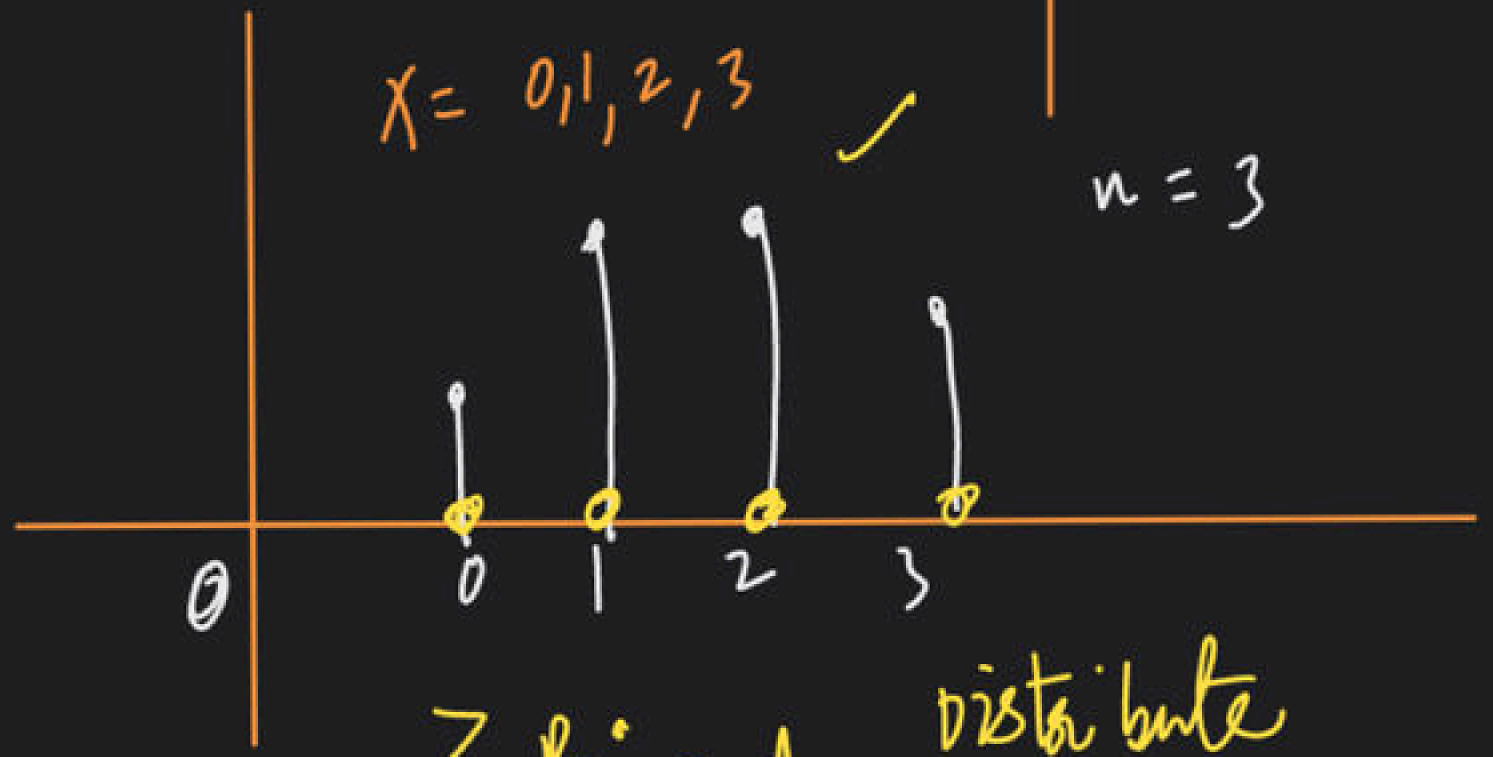
point graph

$\sum P_i = 1$

distribute

$X = 0, 1, 2, 3$

$n = 3$



$\sum P_i = 1$ distribute

(A) all prob. sum (arrival/integer) $= 1$

Prob. Distribution = valid

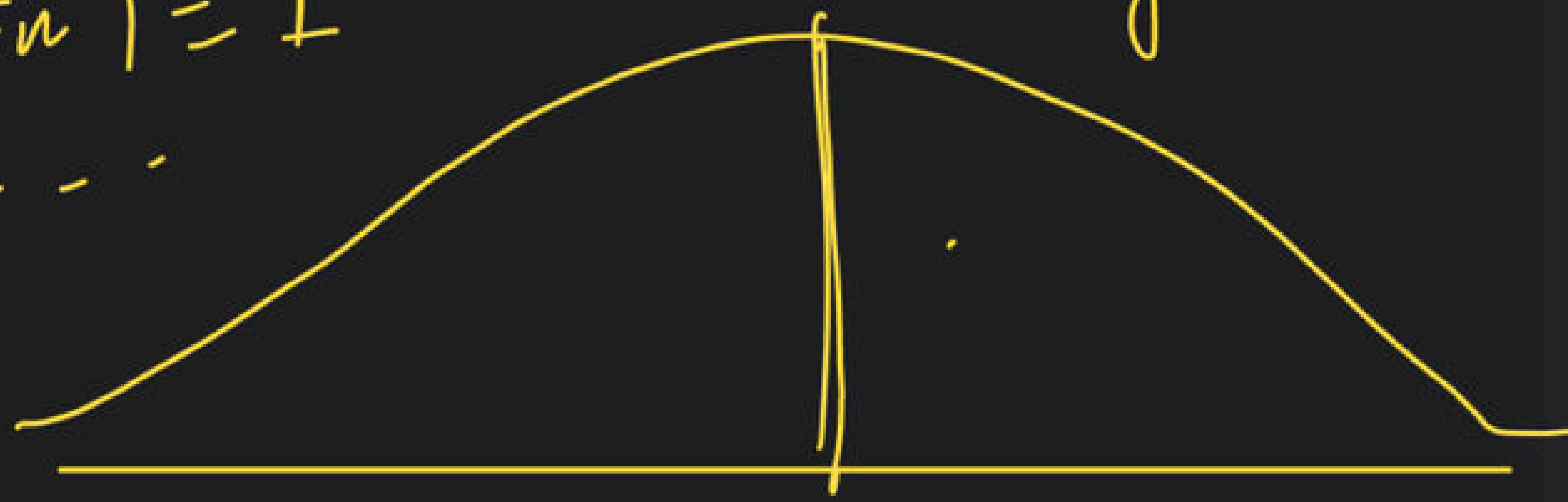
$P(E_1) + P(E_2) + \dots + P(E_n) = 1$

$X = 0, 1, 2, 3 \dots$

n Large No. of Trials

Normal Random / Guassian Random variable.

symmetric



bell shape

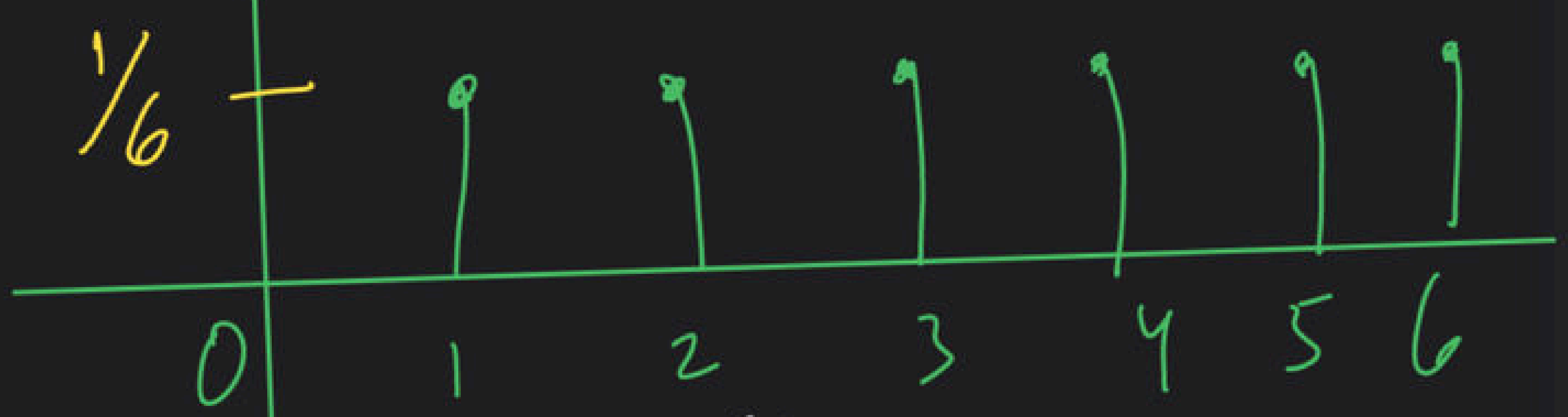
Throwing A Die

X = No. of Dots / Arrival

X	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

1) all prob. are Equal

(UNIform Random variable)



Function = ✓ ?

2) $\sum_{i=1}^{n} p_i = 1$ ✓ valid

GREEN

X = No. of white ball

X = 0, 1, 2, 3, 4, 5 (arrival)

0, 1, 2, 3, 4, 5 $\longrightarrow$ PROCESS $\longrightarrow$ output

✓ Prob. Mass Table

X = 0 1 2 3 4 5

X	0	1	2	3	4	5
P(X=x)	$\frac{^5C_0}{^9C_0}$	$\frac{^5C_1}{^9C_1}$	$\frac{^5C_2}{^9C_2}$	$\frac{^5C_3}{^9C_3}$	$\frac{^5C_4}{^9C_4}$	$\frac{^5C_5}{^9C_5}$

Homework :-

SAQ

SELF assessment

4 Ace
52 cards

No. of Ace card

prob. Mass Table

graph

This is valid prob. Distribution OR NOT

Types of Random Variable :-

Types of function in basis of Probability Function < domain, Type

Countable (count)

MEASURE (Measurement)

Arrival

{ ∋ No. of Head / No. of Tail / After retake GATE appear / No. of balls / No. of Heads / customer / Insurance. Integer = countable. }

"Infinite countable Number"

Infinite countable Number

~~X~~ is a countable Number

GEOMETRIC SERIES (Infinite form)

$0.1 + 0.11 + 0.111 + 0.1111 + \cdots\cdots$

$= \text{sum} = \text{finite} = \frac{a}{1-r}$

Infinite countable SET

Add

0.99
0.999
0.9999
0.99999

0.999999
$\vdots$
$= 1$ ✓

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MEASURABLE (MEASURE)

1) HEIGHT of a Person $\longrightarrow$ $1.2 \le X \le 1.3$

2) swiggy order - TimE $\longrightarrow$ $0 < X < 30$ START

3) Discharging A battery $\longrightarrow$ X = Interval

4) waiting TimE $\longrightarrow$ X = Interval.

DISCRETE

0 1

Hrs.

NO of STUDENTS

Arrival.

#

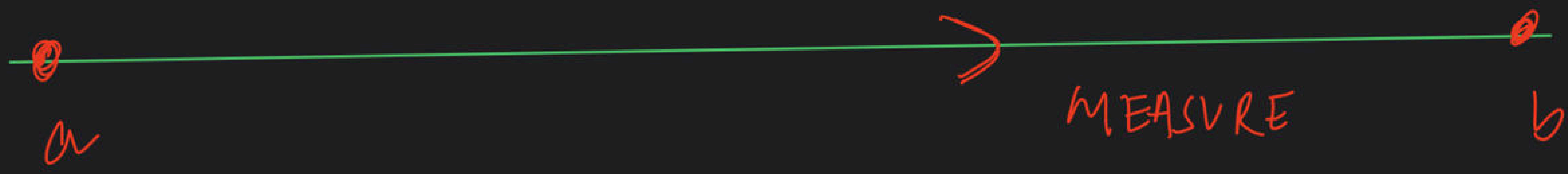
2	3	4	5	2	3	5	2

DISCRETE / counting / arrival

\# Inverter battery ON → Discharging battery

2 - 3 Hrs - discharging

Interval (time - waiting TimE)



A B

Infinite Points

'X' is Infinite uncountable number

Real line.

A B

✓Infinite uncountable number (add)

Tool $\Rightarrow$ Integration $\int_A^B$ = integrate

Add

Random var

(Integration)

(waiting Time)

(Interval)

MEASURable

Integer

(arrival)

Infinite

Infinite

Uncountable

Number

Discrete

countable

Number

continuous

No. of balls

No. of Heads

No. of customer

No. of dots

Discharge A battery

Weight of a Person

waiting time

Temp.

Tossing a four coins

$P(X=0) = 1/16$

$P(X=1) = \frac{4}{16}$

$P(X=2) = 6/16$

$P(X=3) = 4/16$

$P(X=4) = 1/16$

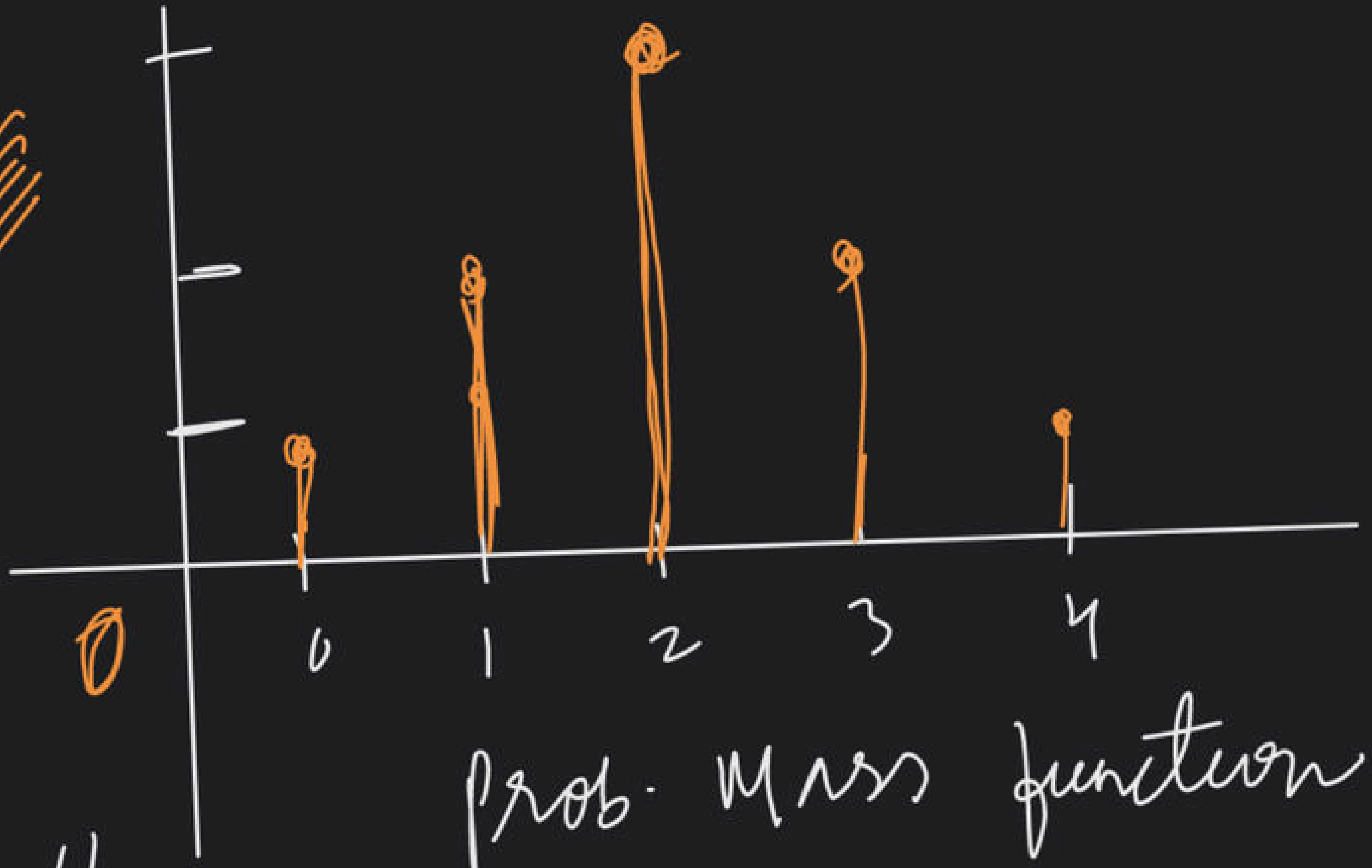
1

1 1

1 2 1

1 3 3 1

1 4 6 4 1



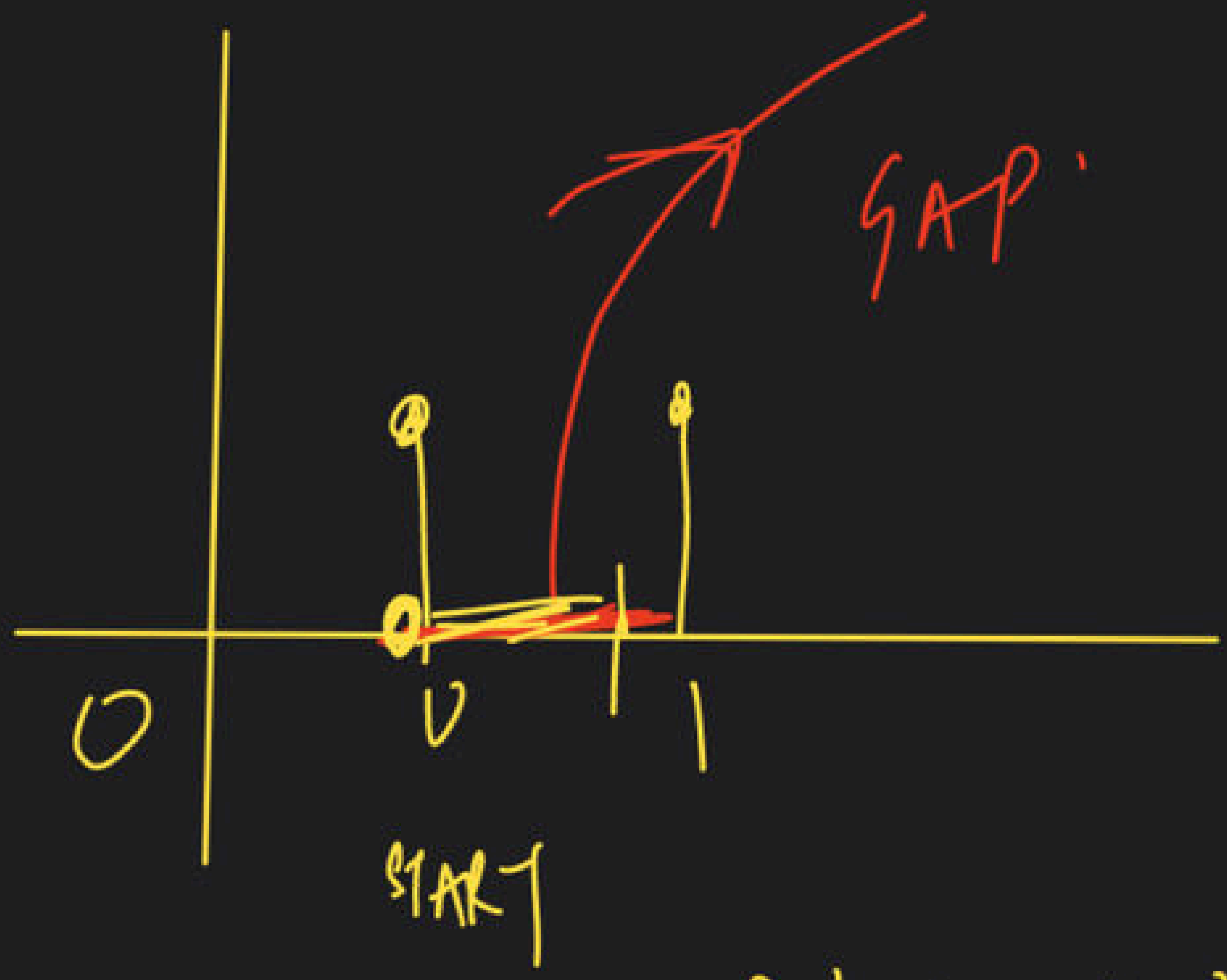
"prob. mass function"

$P[X=x]$ = prob. at a point"

Cumulative distribution Function :- cdf

We Know That prob. Mass function

$P[X = x]$ = Prob. at a point



$P(0.1) = 0$ ✓

Cumulative Distribution function : up to

$$\text{cdf} \quad F_X(x) = \sum_{\text{START}}^{x} P[X \le x]$$

Throwing A Die   X = No. of dots (discrete / Arrival)

$X =$	1	2	3	4	5	6
	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Throwing A Die X = No. of dots (DISCRETE / Arrival)

$X =$	1	2	3	4	5	6
	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$F_X(x) = \sum P(X \le x)$$

$$F_X(1) = P(X \le 1) = P_1$$

$$F_X(2) = P(X \le 2) = P_1 + P_2$$

$$F_X(3) = P(X \le 3) = P_1 + P_2 + P_3$$

$$F_X(4) = P(X \le 4) = P_1 + P_2 + P_3 + P_4$$

$$F_X(5) = P(X \le 5) = P_1 + P_2 + \cdots + P_5$$

$$F_X(6) = P_1 + P_2 + P_3 + P_4 + P_5 + P_6$$

$$F_X(1) = P(X \le 1) = \frac{1}{6}$$

$$F_X(2) = \frac{2}{6}$$

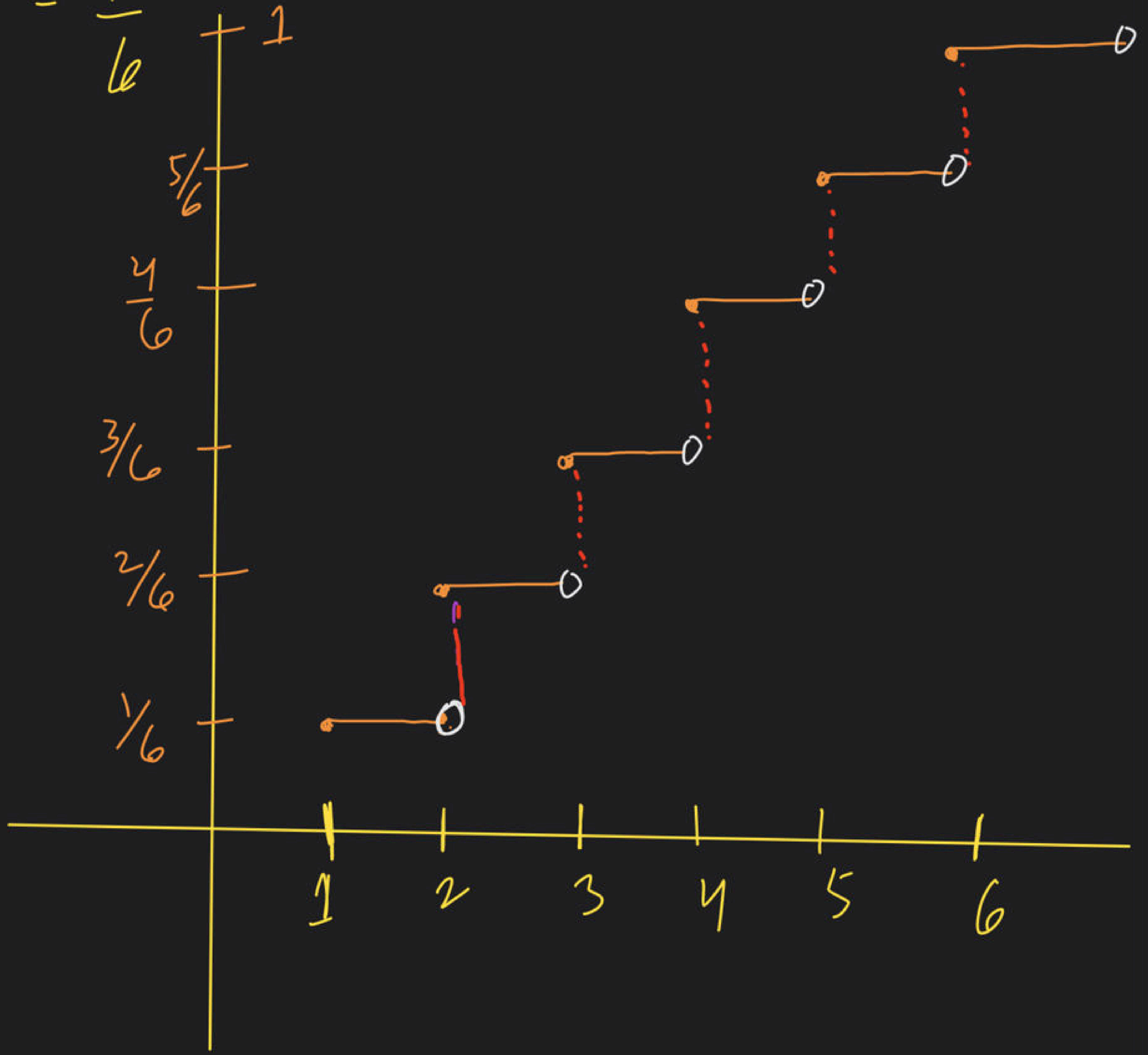
$$F_X(3) = \frac{3}{6}$$

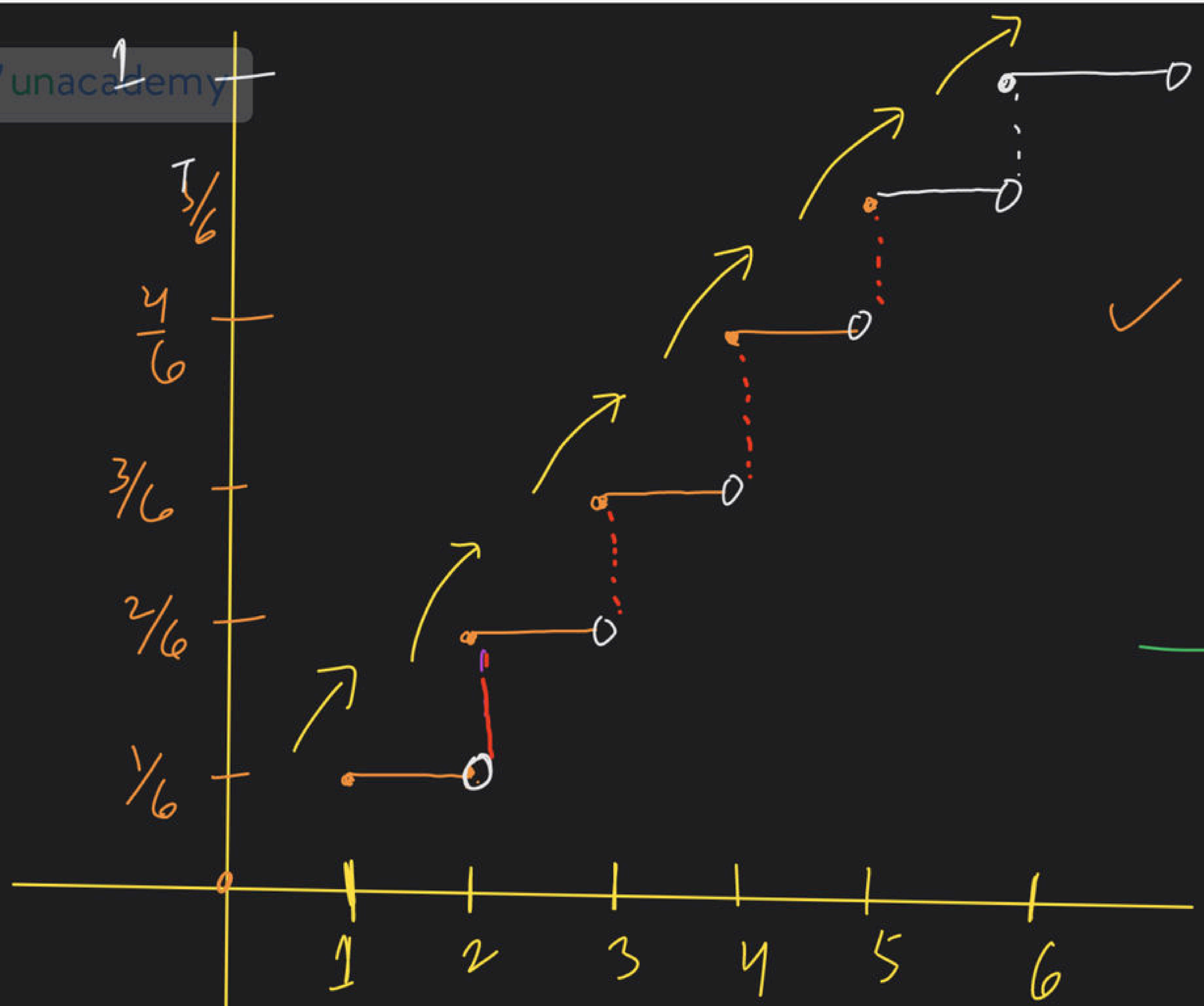
$$F_X(4) = \frac{4}{6}$$

$$F_X(5) = \frac{5}{6}$$

$$F_X(6) = 1$$

$$F_X(1.1) = 1/6$$





✓ (A) STEP function

✓ (B) ladder function

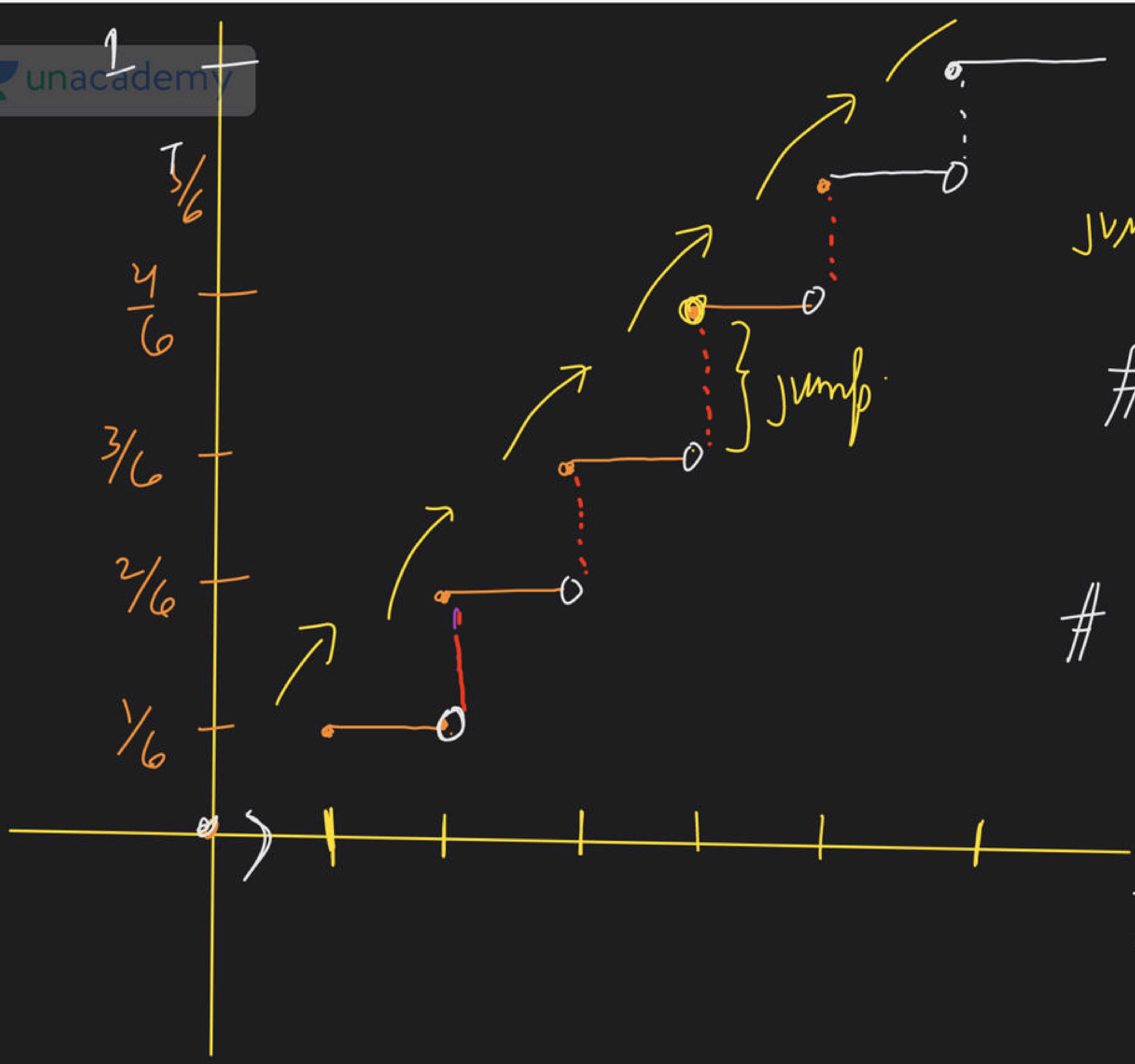
✓ c) Monotonic increasing

strictly increasing

D) $F_X(x_i) \geq 0$

prob always positive

E) $P(X_i) \geq 0$



upper value − lower value

jump = RHL − LHL

$F_X(\infty) = 1$

$F_X(-\infty) = 0$

diff. $= F_X(x^+) - F_X(x^-)$

step function

Tossing A THREE coins.

Re 1, Re 2, Re 3

X = No. of HEADS / cdf

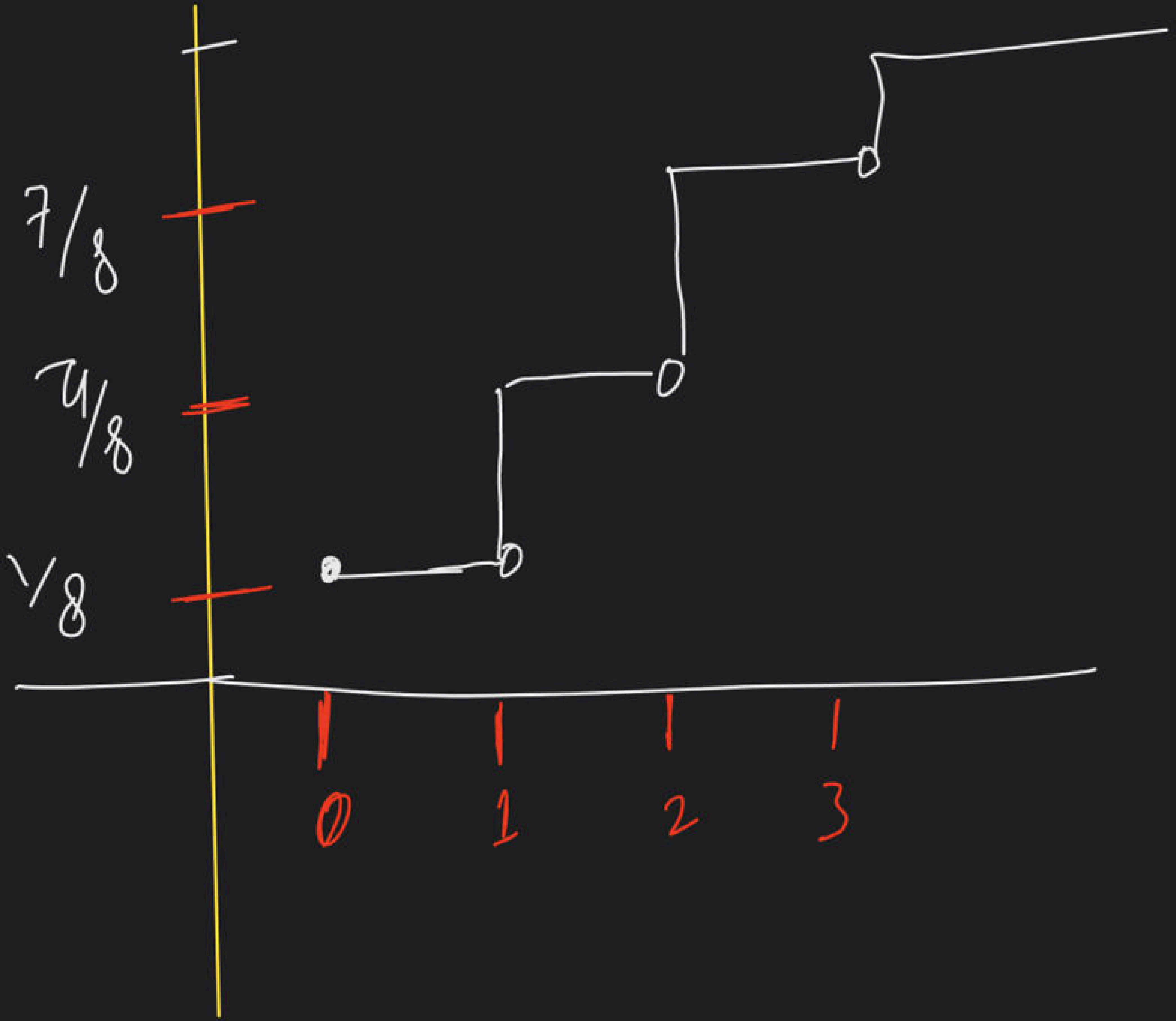
✓A) ladder / step / monotonically

✓B) $F_X(-\infty) = 0$ $F_X(\infty) = 1$ ✓

✓C) $P_X(x) \geq 0$ $F_X(x) \geq 0$

✓D) jump = $F_X(x^+) - F_X(x^-)$

E) strictly increasing.

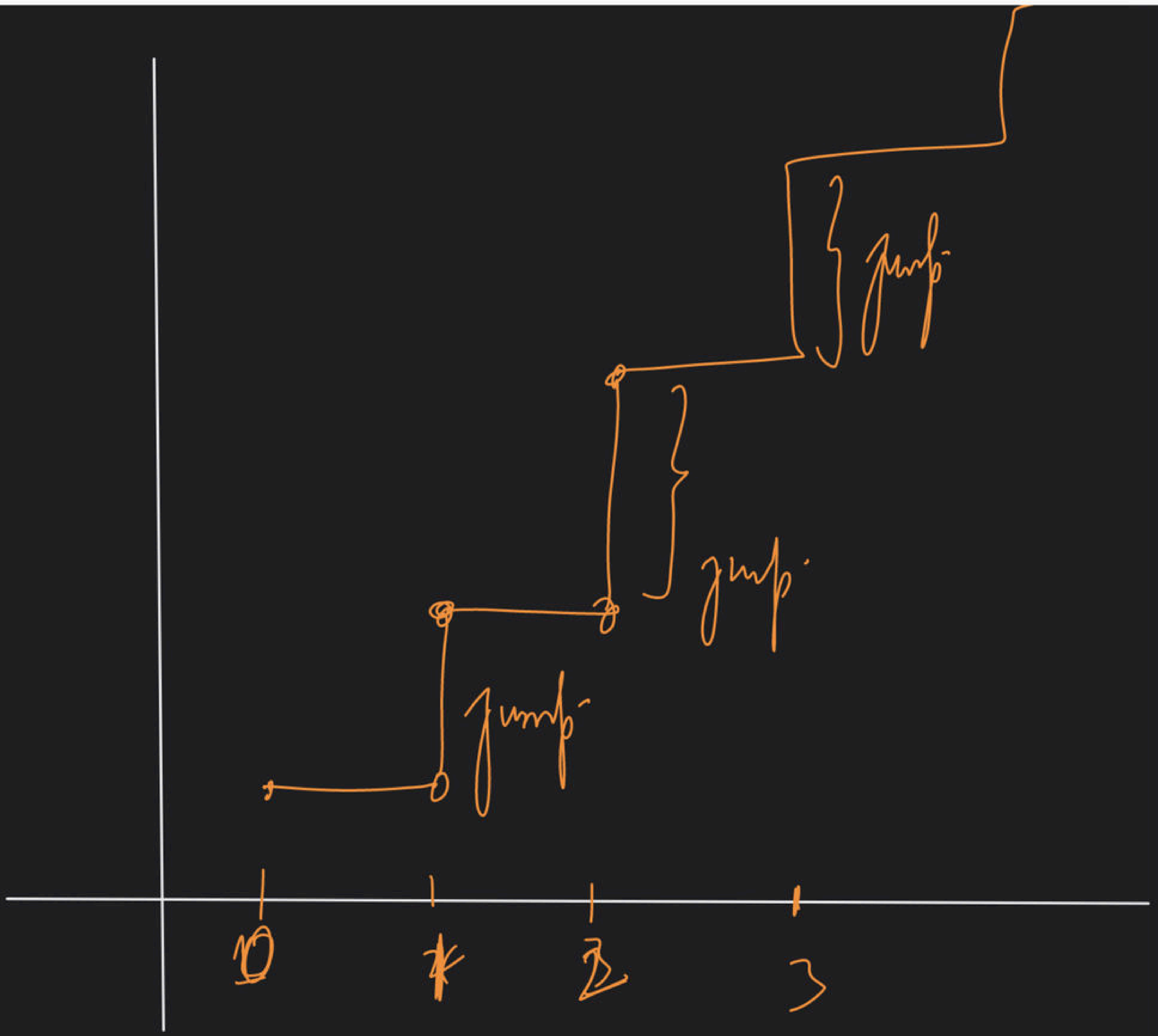


$$F_X(0) = \frac{1}{8}$$

$$F_X(1) = \frac{4}{8}$$

$$F_X(2) = \frac{7}{8}$$

$$F_X(3) = 1$$



QUESTION

Q. If X has the distribution function

$$F(X) = \begin{cases} 0 \text{ for } X<1 \\ \frac{1}{3} \text{ for } 1 \le X<4 \\ \frac{1}{2} \text{ for } 4 \le X<6 \\ \frac{5}{6} \text{ for } 6 \le X<10 \\ 1 \text{ for } X \ge 10 \end{cases}$$

Find

(A) $P(2< X \le 6)$; (B) $P(X = 4)$

(C) $P(X \ge 10)$ (D) $P(X<4)$

(E) $P(X > 4)$ (F) $P(X \ge 4)$

QUESTION

Q. If X has the distribution function

$$F(X)=\begin{cases}0 \text{ for } X<-1\\ \frac{1}{4} \text{ for } -1\leq X<1\\ \frac{1}{2} \text{ for } 1\leq X<3\\ \frac{3}{4} \text{ for } 3\leq X<5\\ 1 \text{ for } X\geq 5\end{cases}$$

Find

(A) $P(X\leq 3)$;

(B) $P(X=3)$;

(C) $P(X<3)$;

(D) $P(X\geq 1)$;

(E) $P(-0.4<X<4)$;

(F) $P(X=5)$;

(G) $P(3<X<5)$;

(H) $P(3\leq X<5)$;

(I) $P(3\leq X\leq 5)$;

QUESTION

Q.

$$F_x(X) = \begin{cases} 0 & x < 0 \\ \frac{1}{8} & 0 \le x < 1 \\ \frac{4}{8} & 1 \le x < 2 \\ \frac{7}{8} & 2 \le x < 3 \\ 1 & 3 \le x \end{cases}$$

Calculate probability

A. $P(X<2)$;

B. $P(X \ge 2)$

C. $P(X > 2)$

D. $P(1 \le X < 3)$

E. $P(X \le 4)$

QUESTION

Q.

$$F_x(X)=\begin{cases}0 & x<-1\\ \frac{1}{4} & -1\le x<1\\ \frac{1}{2} & 1\le x<3\\ \frac{3}{4} & 3\le x<5\\ 1 & 5\le x\end{cases}$$

Find the probability

(A) $P(X\le 3)$;

(B) $P(X=3)$

(C) $P(X<3)$

(D) $P(X\ge 1)$

(E) $P(3<X<5)$

(D) $P(-0.4<X<4)$

QUESTION

prob. Distribution $\boxed{\text{sum} = 1}$

Q. State, giving reasons, which of the following are not probability distributions:

X

(c) prob. Distribution = sum = 1

(A)

x	0	1
p(x)	$\frac{1}{2}$	$\frac{3}{4}$

$\frac{1}{2} + \frac{3}{4} \neq 1$

(B)

x	0	1	2
p(x)	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{3}{4}$

$\frac{3}{4} + \frac{1}{2} + \frac{3}{4} \neq 1$

(C)

x	0	1	2
p(x)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$\frac{1}{4} + \frac{1}{2} + \frac{1}{4}$

✓ YES

(D)

x	0	1	2	3
p(x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{8}$

X NO $\neq 1$

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QUESTION

Q. Find the probability distribution of the number of heads when three fair coins are tossed simultaneously.

done

QUESTION

Q. 2 bad articles are mixed with 5 good ones. Find the probability distribution of the number of bad articles, if 2 articles are drawn at random.

QUESTION

Q. Given the probability distribution:

x	0	1	2	3
p(x)	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{1}{2}$	$\frac{1}{10}$

Let $Y = X^2 + 2X$. Find the probability distribution of Y?

QUESTION

Q. An urn contains 3 white and 4 red balls. 3 balls are drawn one by one with replacement. Find the probability distribution of the number of red balls.

QUESTION

Q. A continuous random variable X has the probability density function:

$$f(x) = Ax^3, 0 \le x \le 1.$$

(i) $A = 4$

(ii) $P[0.2 < X < 0.5]$

(iii) $P\left[X > \frac{3}{4} \text{ given } X > \frac{1}{2}\right]$

QUESTION

Q. The life (in hours) X of a certain type of light bulb may be supposed to be a continuous random variable with p.d.f.:

$$f(x) = \begin{cases} \dfrac{A}{x^3} & 1500 < x < 2500 \\ 0, & \text{elsewhere} \end{cases}$$

Determine the constant A and compute the probability that

$1600 \le x \le 2000.$

QUESTION

Q. The diameter 'X' of a cable is assumed to be a continuous random variable with p.d.f.

$$f(x)=\begin{cases}6x(1-x), & 0\le x\le 1\\ 0, & \text{elsewhere}\end{cases}$$

Obtain the c.d.f. of x.

QUESTION

Q. A random variable x has the following probability function:

x	0	1	2	3	4	5	6	7
p(x)	0	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{1}{100}$	$\frac{1}{50}$	$\frac{1}{100}$

Determine the distribution function of X.

QUESTION

Q. The p.d.f. of the different weights of a "1 litre pure ghee pack" of a company is given by:

$$f(x)=\begin{cases}200(x-1) & \text{for } 1 \le x \le 1.1\\ 0, & \textit{otherwise}\end{cases}$$

Examine whether the given p.d.f. is a valid one. If yes, find the probability that the weight of any pack will lie between 1.01 and 1.02.

QUESTION

Q. A random variable X has the following probability distribution:

x	0	1	2	3	4	5	6	7	8
p(x)	k	3k	5k	7k	9k	11k	13k	15k	17k

(i) Determine the value of k.

(ii) Find the distribution function of X.

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THANK YOU!

Here's to a cracking journey ahead!