Trees - Part VIII

Course on Data Structure

Pankaj Sharma • Lesson 29 • Aug 11, 2024

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CS & IT Engineering

Data Structure

Tree

Lecture Number- 29

By- Pankaj Sir

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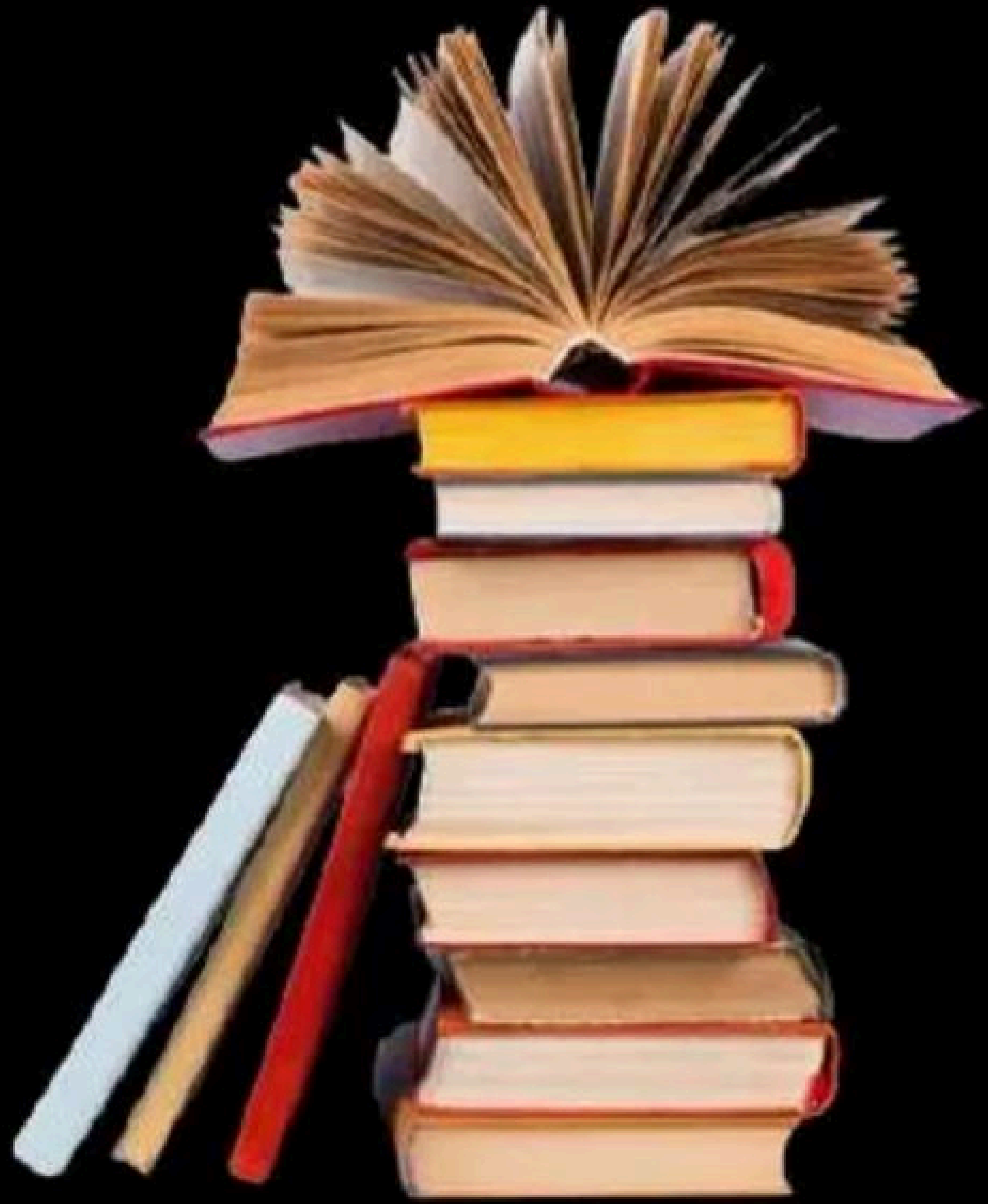
Topics

to be covered



1. Tree-VIII

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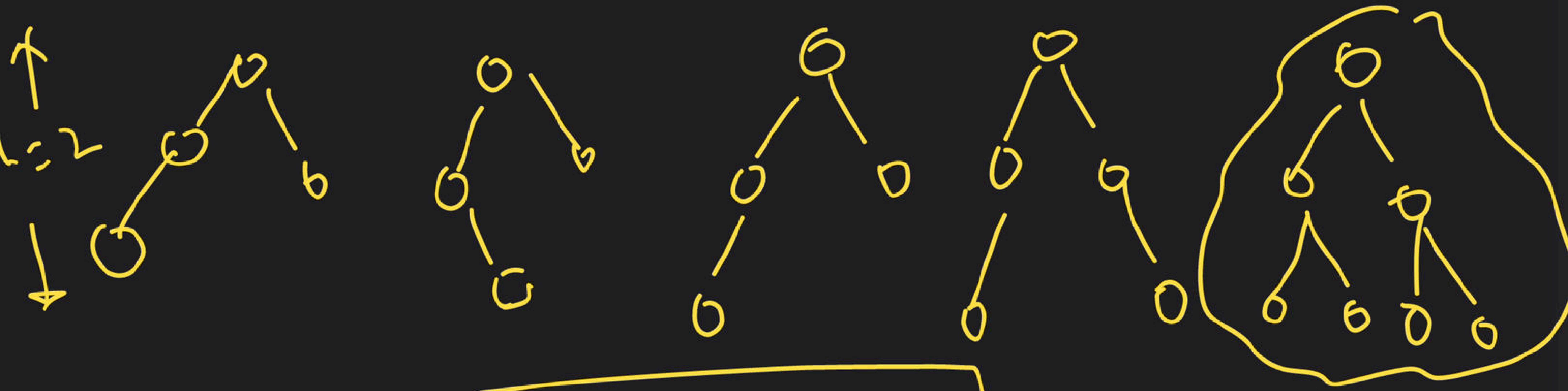
Q) What is the max. no. of nodes in an AVL tree of height h?



$$\boxed{n_{max} = 2^{h+1} - 1}$$

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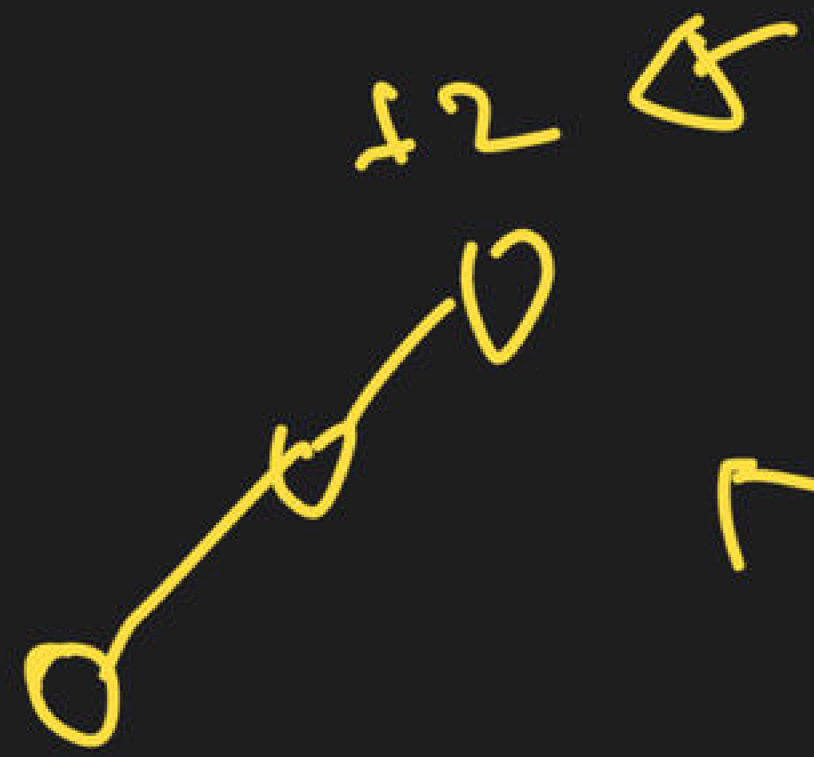
Q) What is the min. no. of nodes in an AVL tree of height h?



Not a AVL tree

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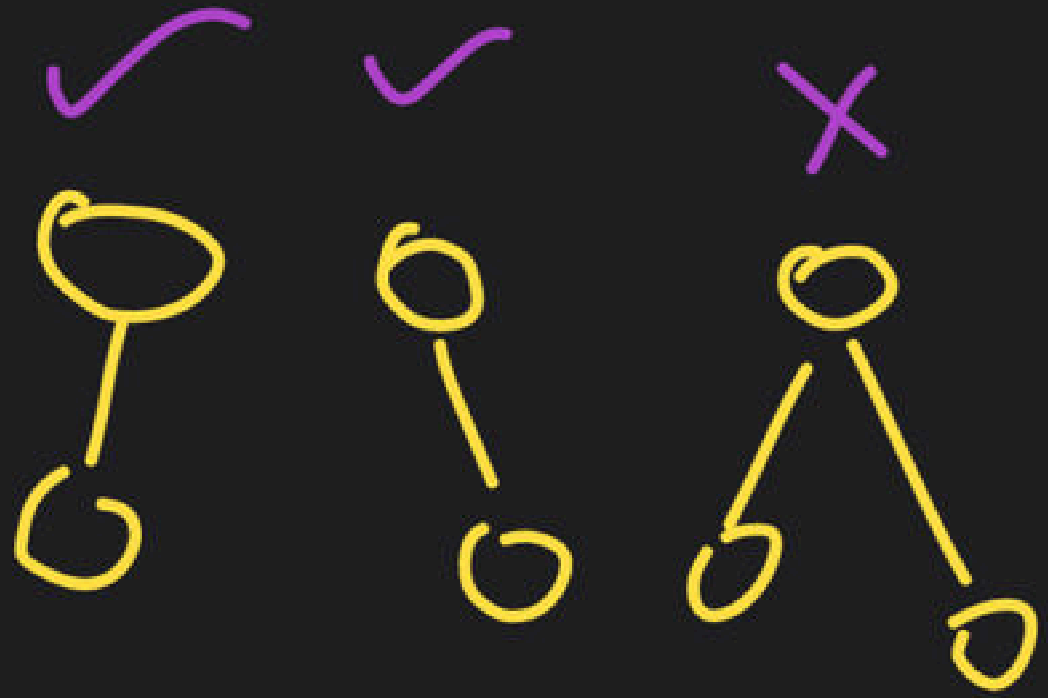
h = 0

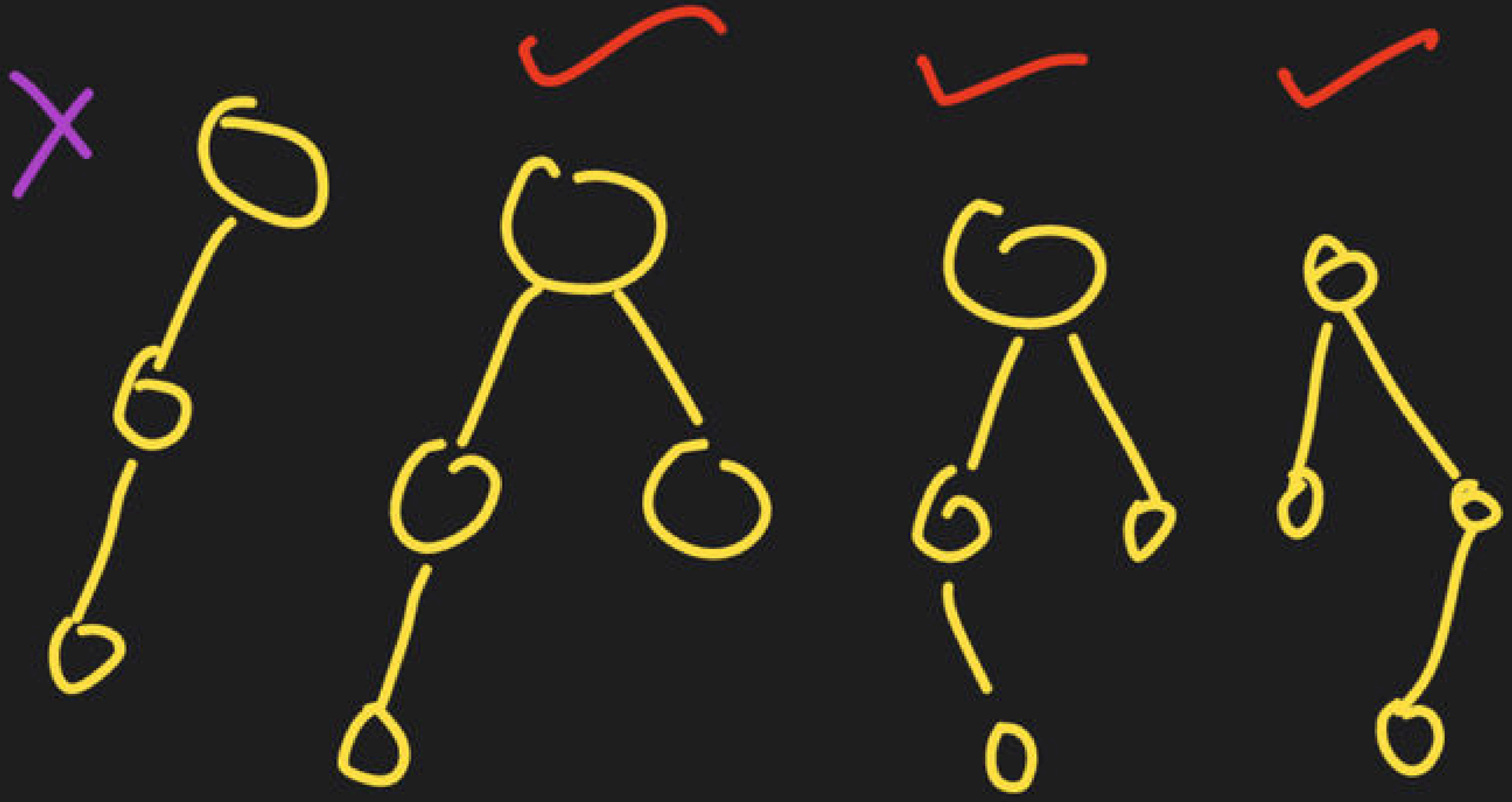
1 node

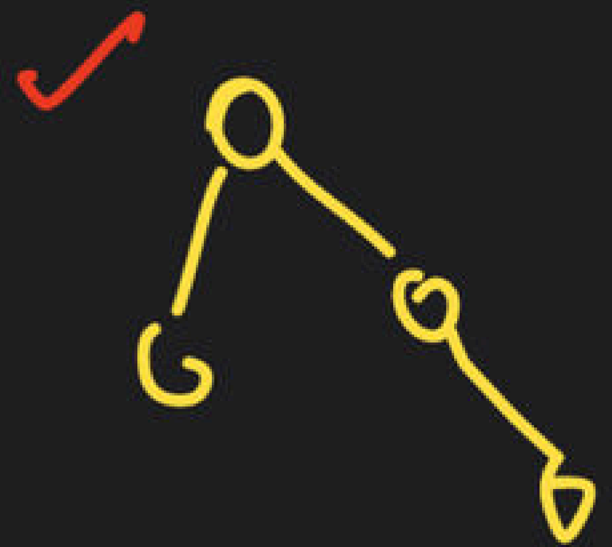
h = 1



2 node

h = 2





4 node

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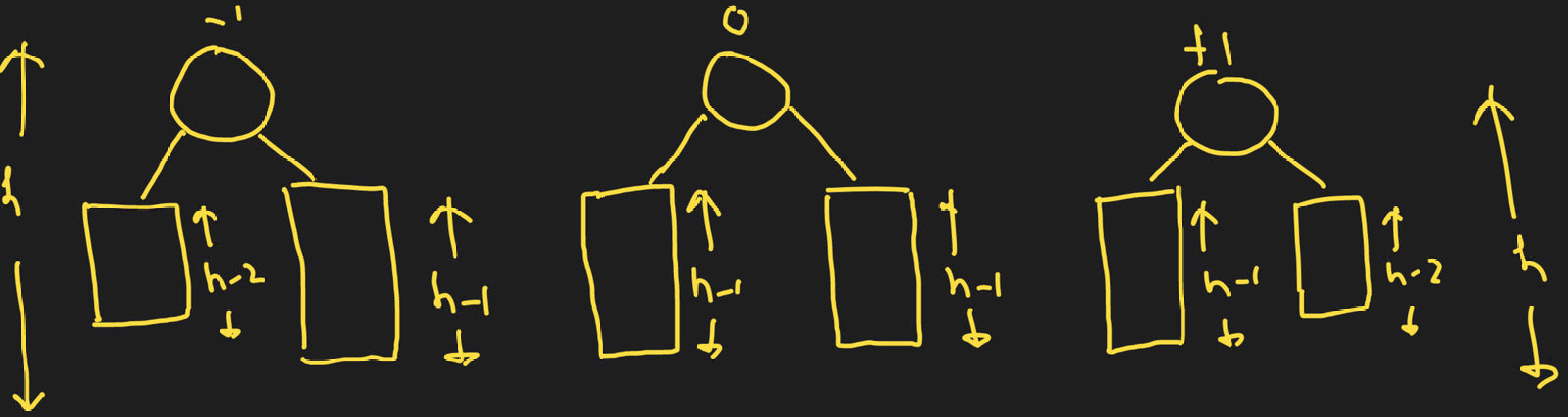
AVL tree of h height



let $n(h)$: min. no. of nodes in an AVL tree of height h.

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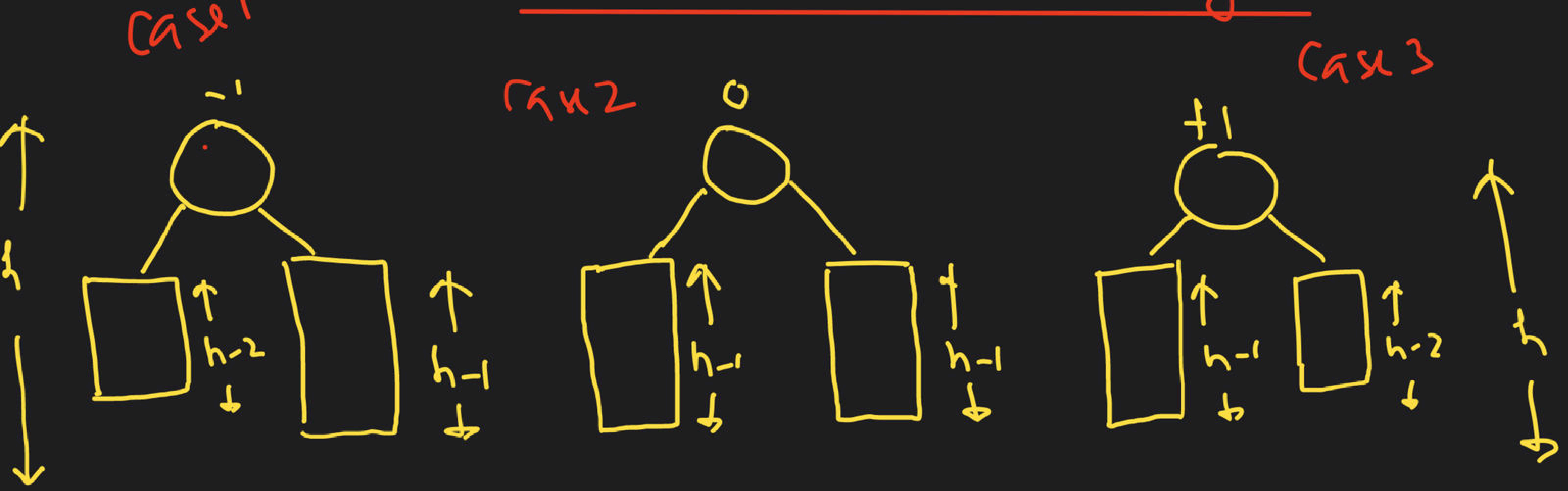
AVL tree of h height



let $n(h)$: min. no. of nodes in an AVL tree of height h.

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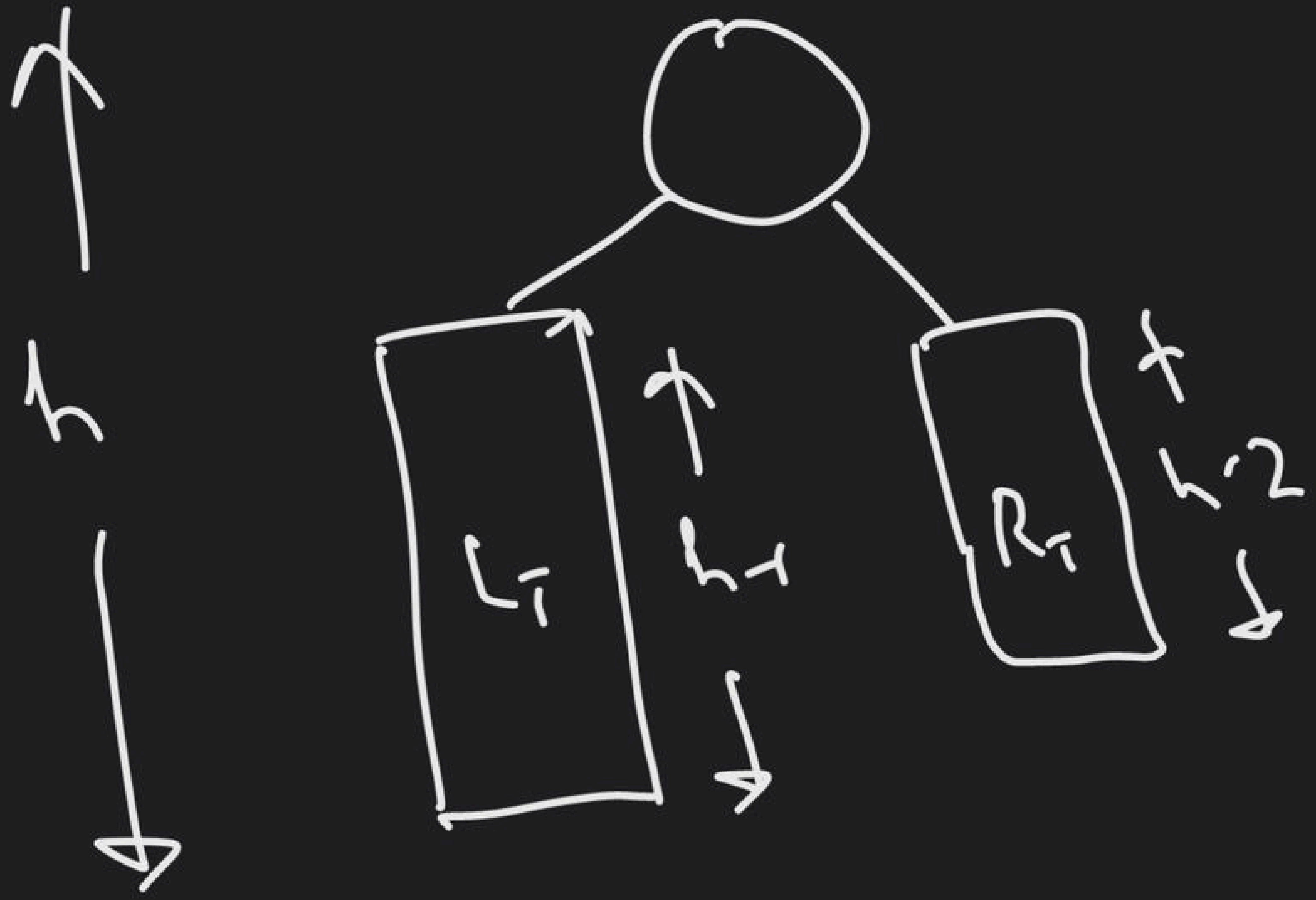
For minimum $\Rightarrow$ Either case 1 or case 3



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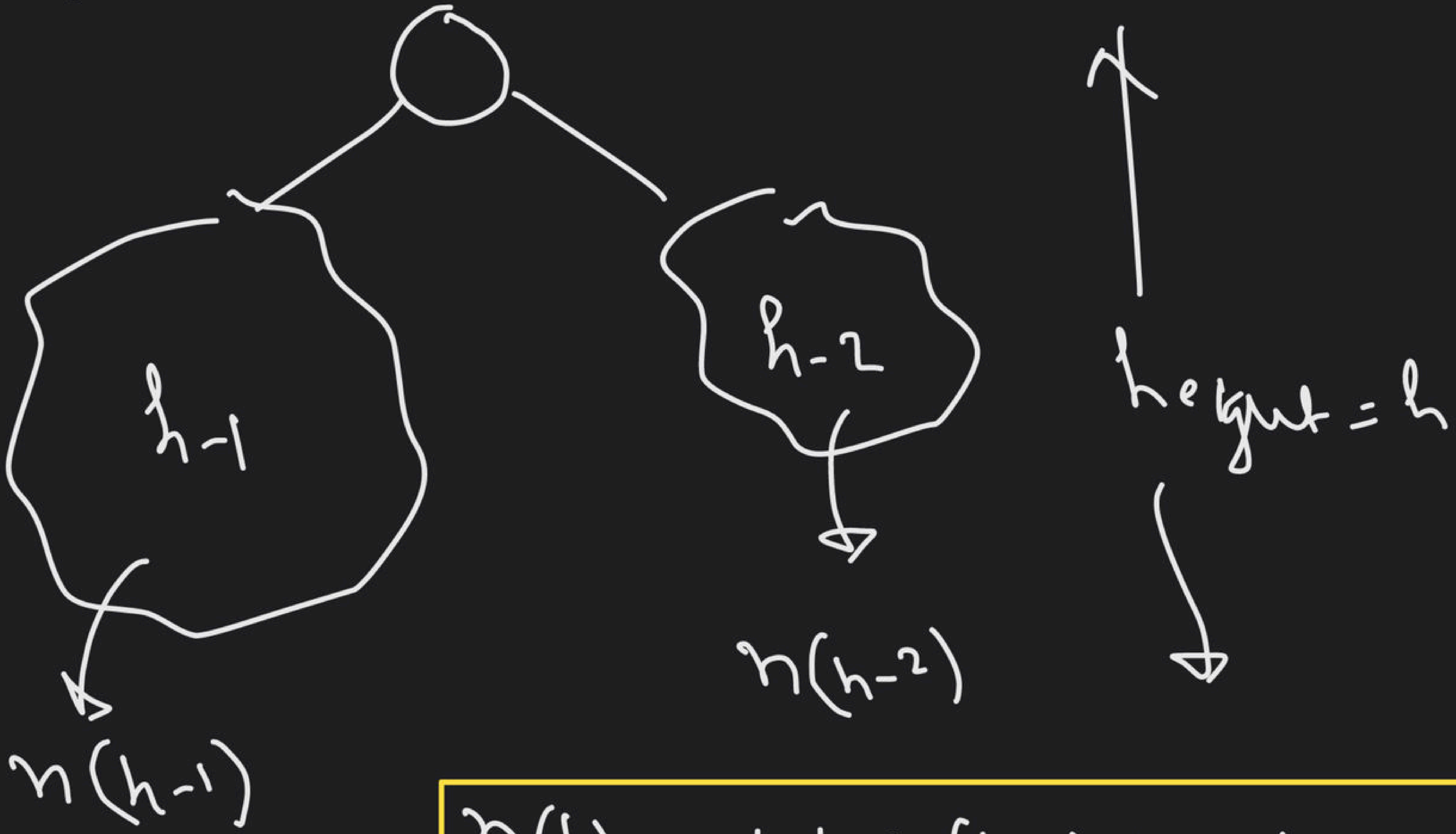
$n(h)$: min. no. of nodes in an AVL tree of height h.



$$\boxed{n(h) = 1 + n(h-1) + n(h-2)}$$

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$$\boxed{n(h) = 1 + n(h-1) + n(h-2)} \qquad h \geq 2$$

$n(0) = \underline{1}$

$n(1) = 2$

$n(0) = 1$

$n(1) = 2$

$n(h) = 1 + n(h-1) + n(h-2)$

$n(2) = 1 + n(1) + n(0) = 1 + 2 + 1 = 4$

$n(3) = 1 + n(2) + n(1)$

$= 1 + 4 + 2 = 7$

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h	0	1	2	3	4	5	6	7	8	...
$h(h)$	1	2	4	7	12	20	33	54	88	

$1 + 2 + 4$

+1

+1

...

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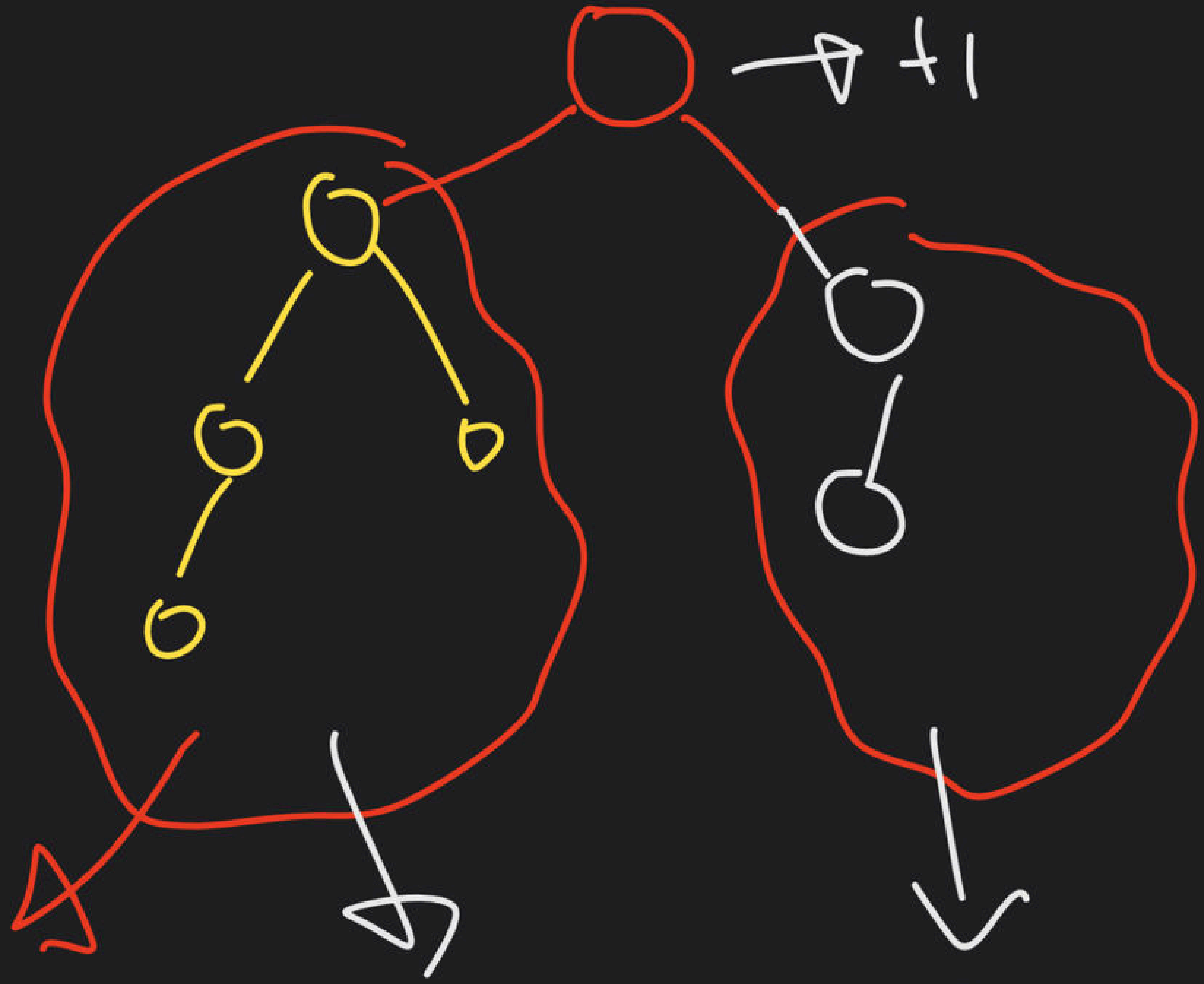
$h = 3$

$h = 2$



$4 + 2$

$\Rightarrow 4 + 2 + 1 = 7$

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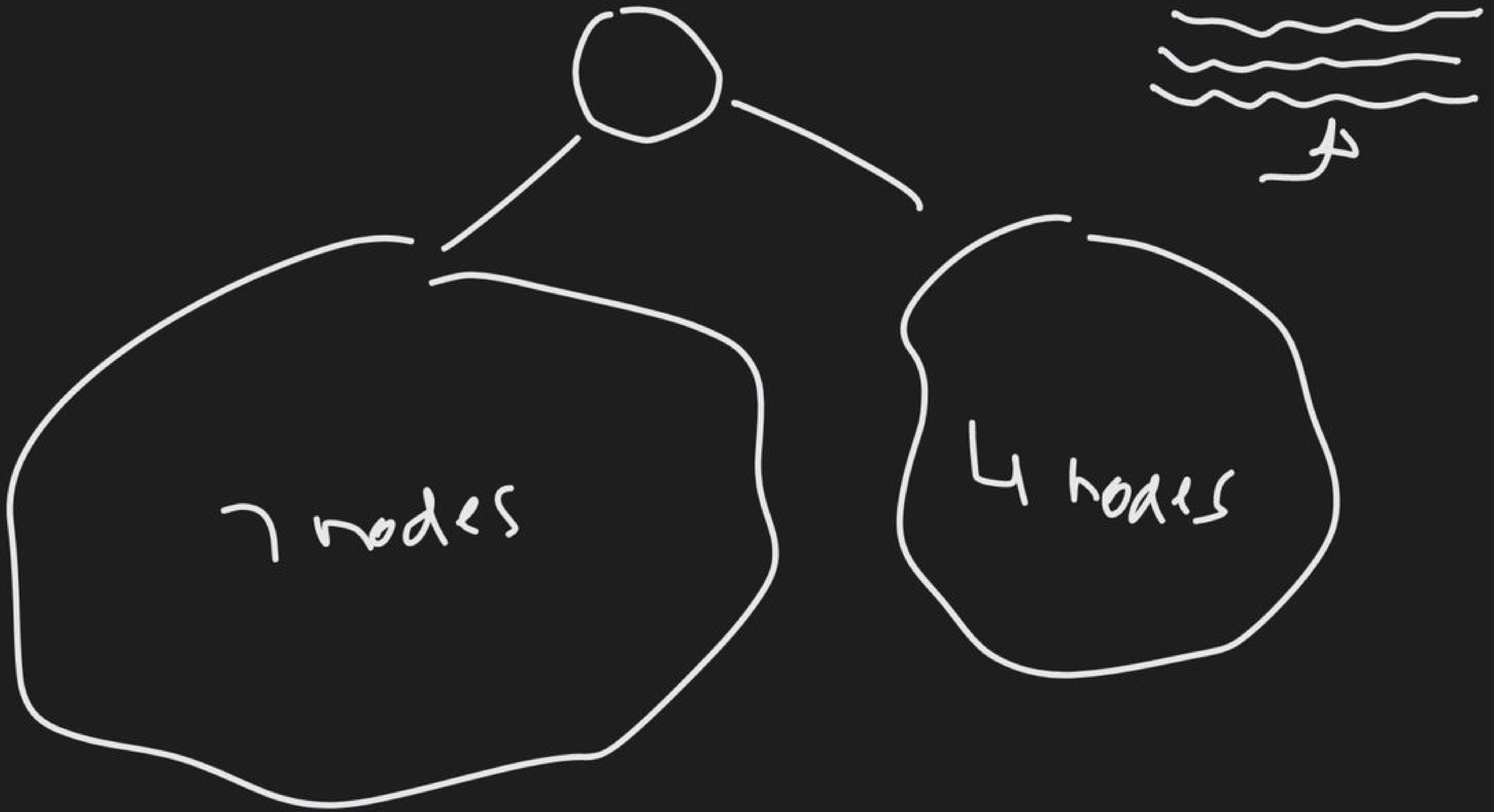
$h = 4$



Q) Consider a binary tree in which every node satisfies the property :

the diff. b/w no. of nodes in L_T of a node & no. of nodes in R_T of node is atmost 1

Node balancing

$$\boxed{|h(L_T) - h(R_T)| <= 1}$$

height

What is the min. no. of nodes in such a tree of 5?

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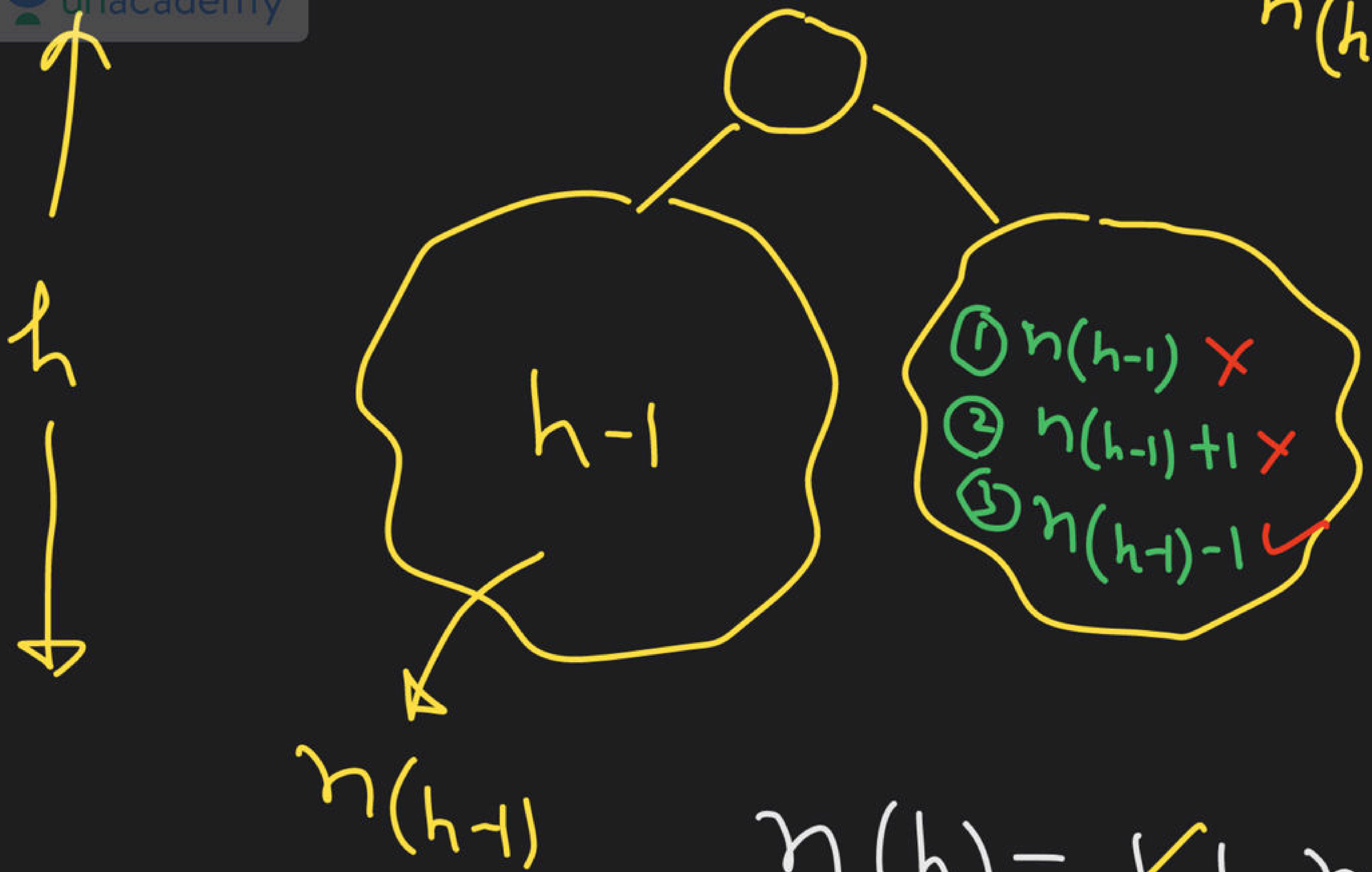

$n(h)$: Min no. of nodes in such tree of height h

$$n(h) = \cancel{1} + n(h-1) + n(h-1) - \cancel{1}$$

$$\boxed{n(h) = 2\,n(h-1)}$$

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$$\boxed{n(h) = 2n(h-1)} \qquad h \geq 1$$

$n(0) = 1$

$n(1) = 2n(0) = 2^1$

$n(2) = 2^2$

$n(3) = 2^3$

$\cdots$

$$\boxed{n(h) = 2^h}$$

$n(5) = 2^5 = 32$

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Q) What is the $\boxed{\text{max. height}}$ possible of an AVL tree with 7 nodes.

h	0	1	2	3	4
$n(h)$	1	2	4	7	12

7, 8, 9, 10, 11

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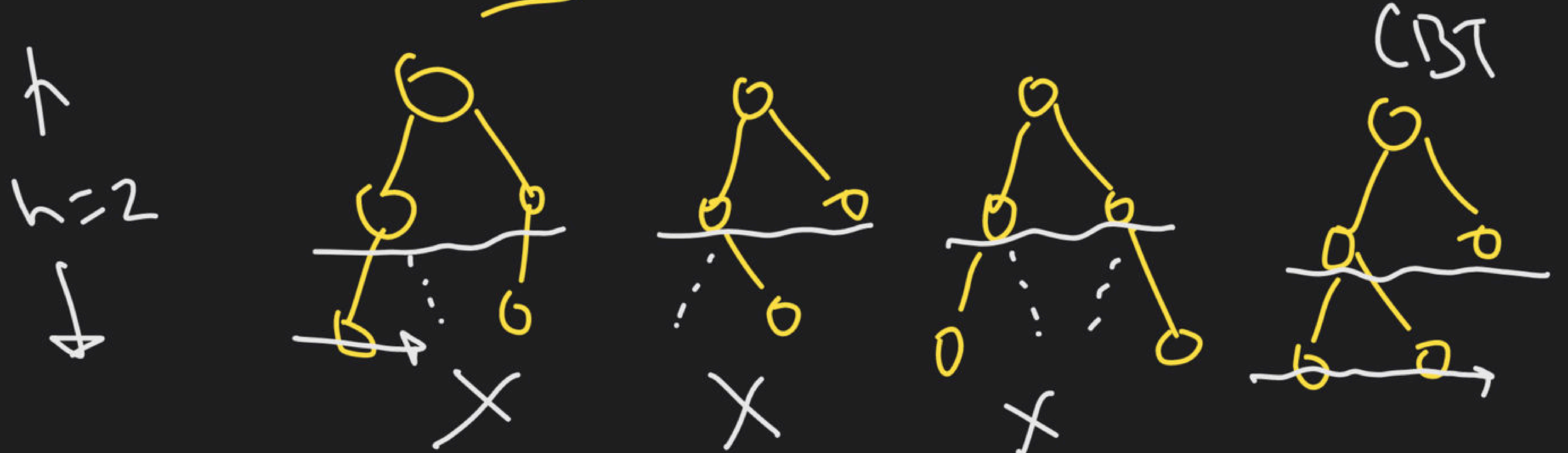
Heap

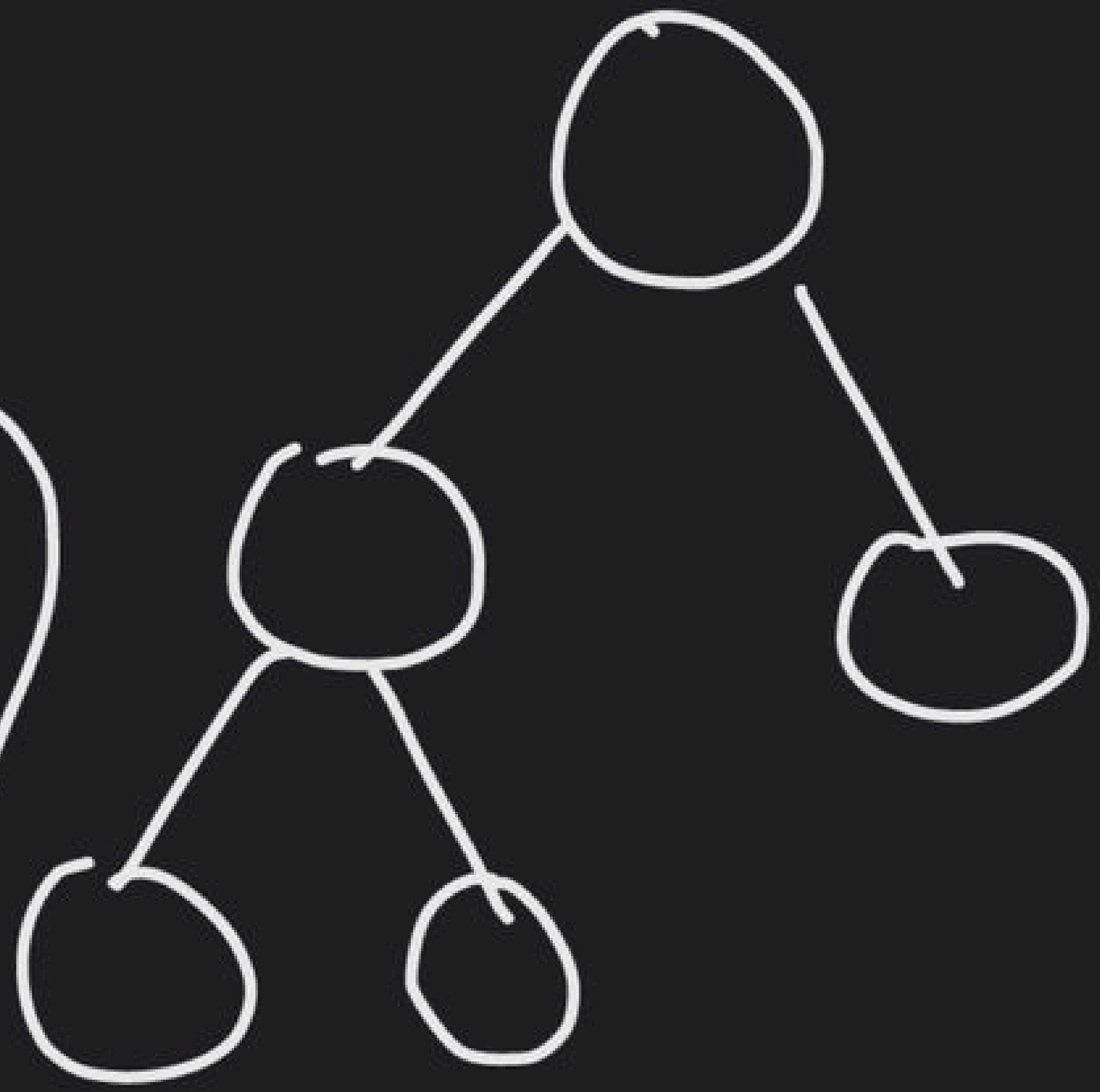
Heap is a CBT.

Complete Binary Tree



CBT with 5 nodes ⇒

Only 1 structure is possible



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Heap

Max heap

Min heap

A CBT in which every node satisfies the property: The value of a node is greater than its children.

A CBT in which every node satisfies the property: The value of a node is smaller than its children.

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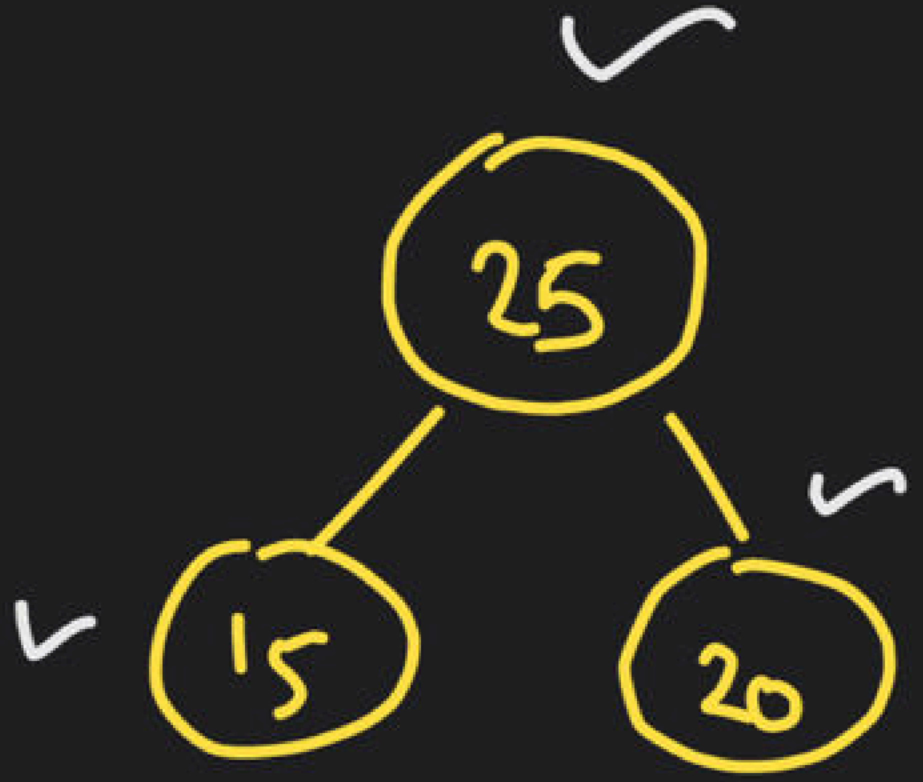
Single Node:

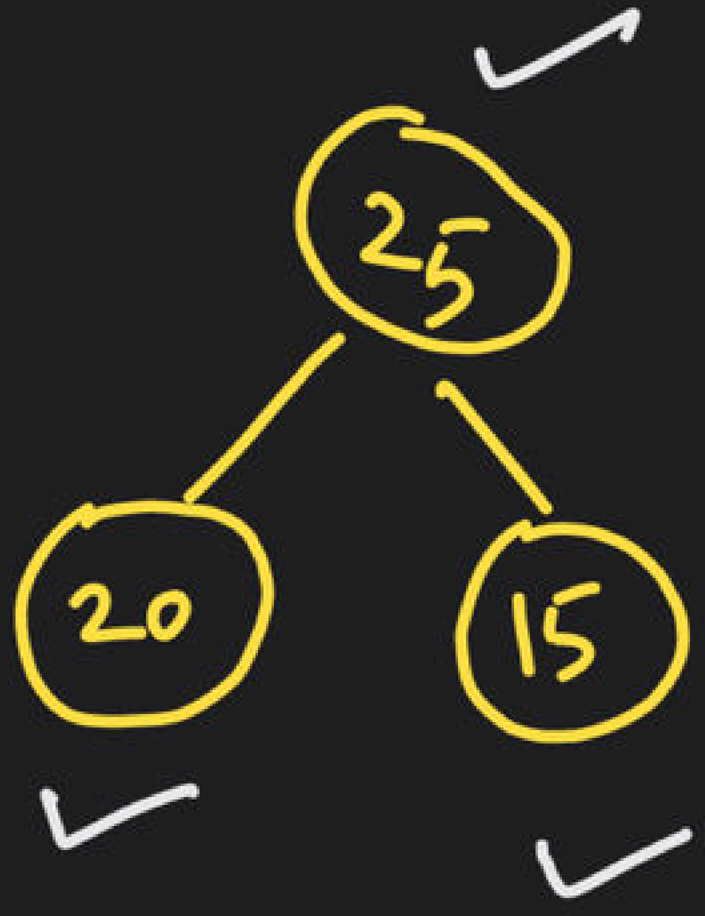
Every leaf node

↳ satisfies



Max-heap ✓

Min-heap ✗



Max-heap

Min-heap

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FBT → Structure

CBT

FBT

data X

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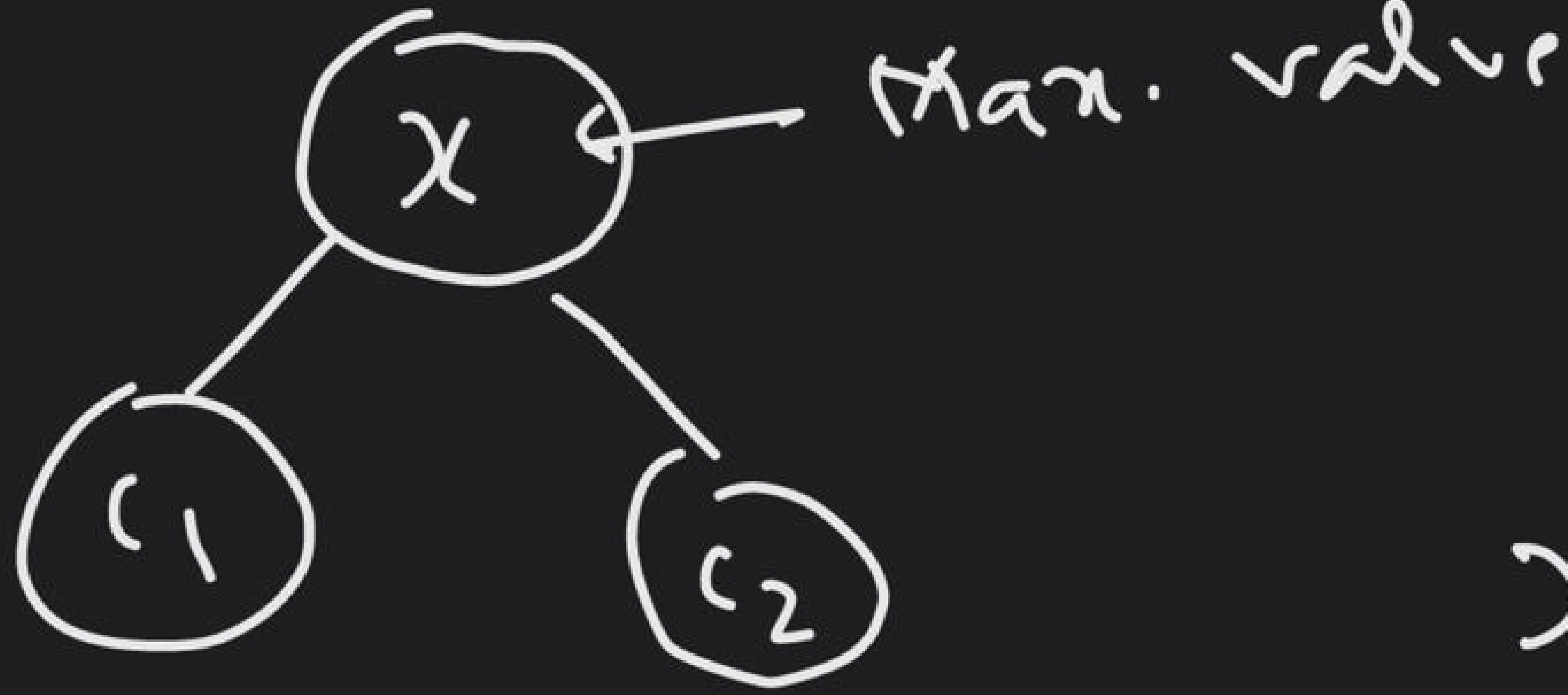
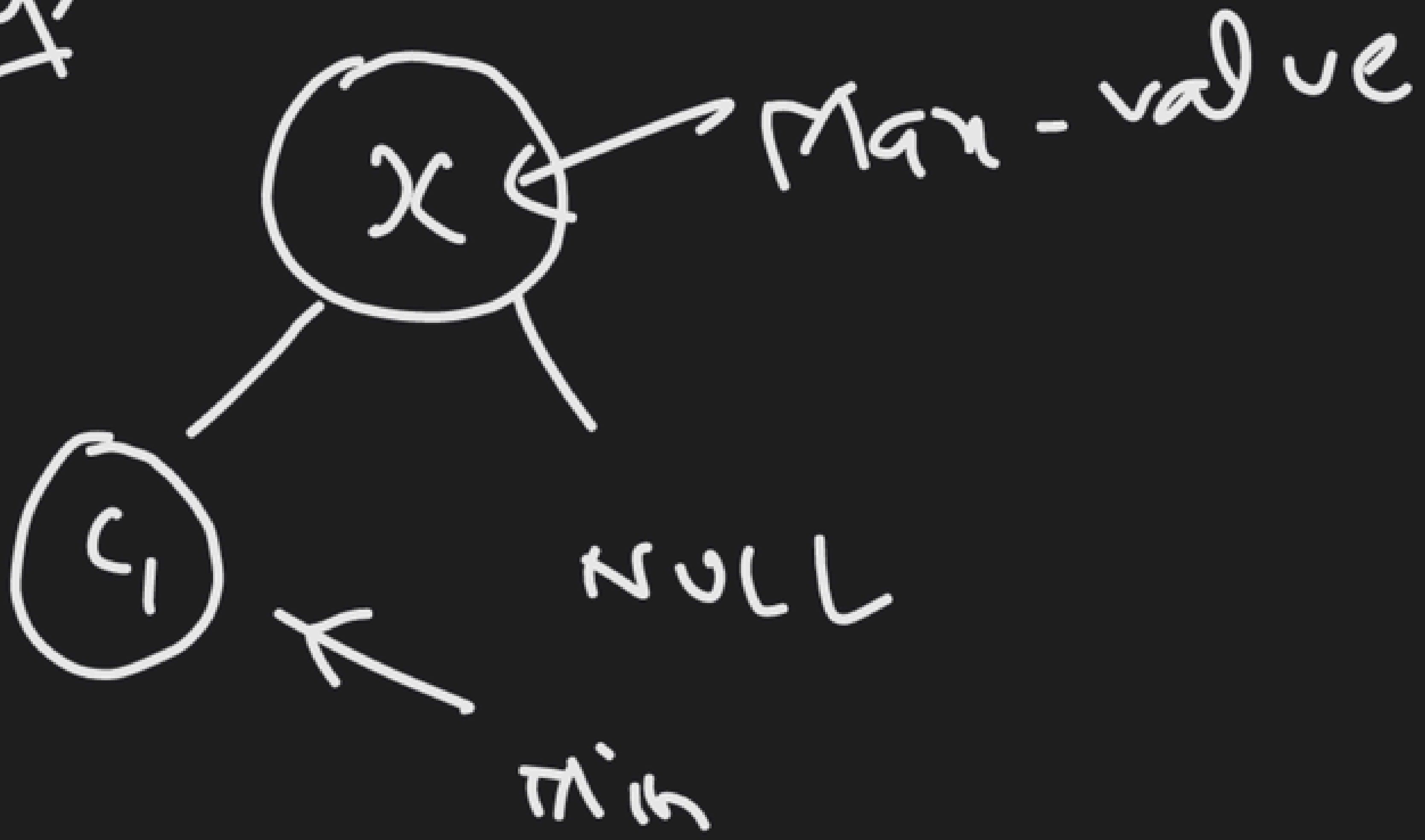
Max-heap



$x > c_1, c_2$

Min-value $\Rightarrow$ c_1 or c_2

Max-heap



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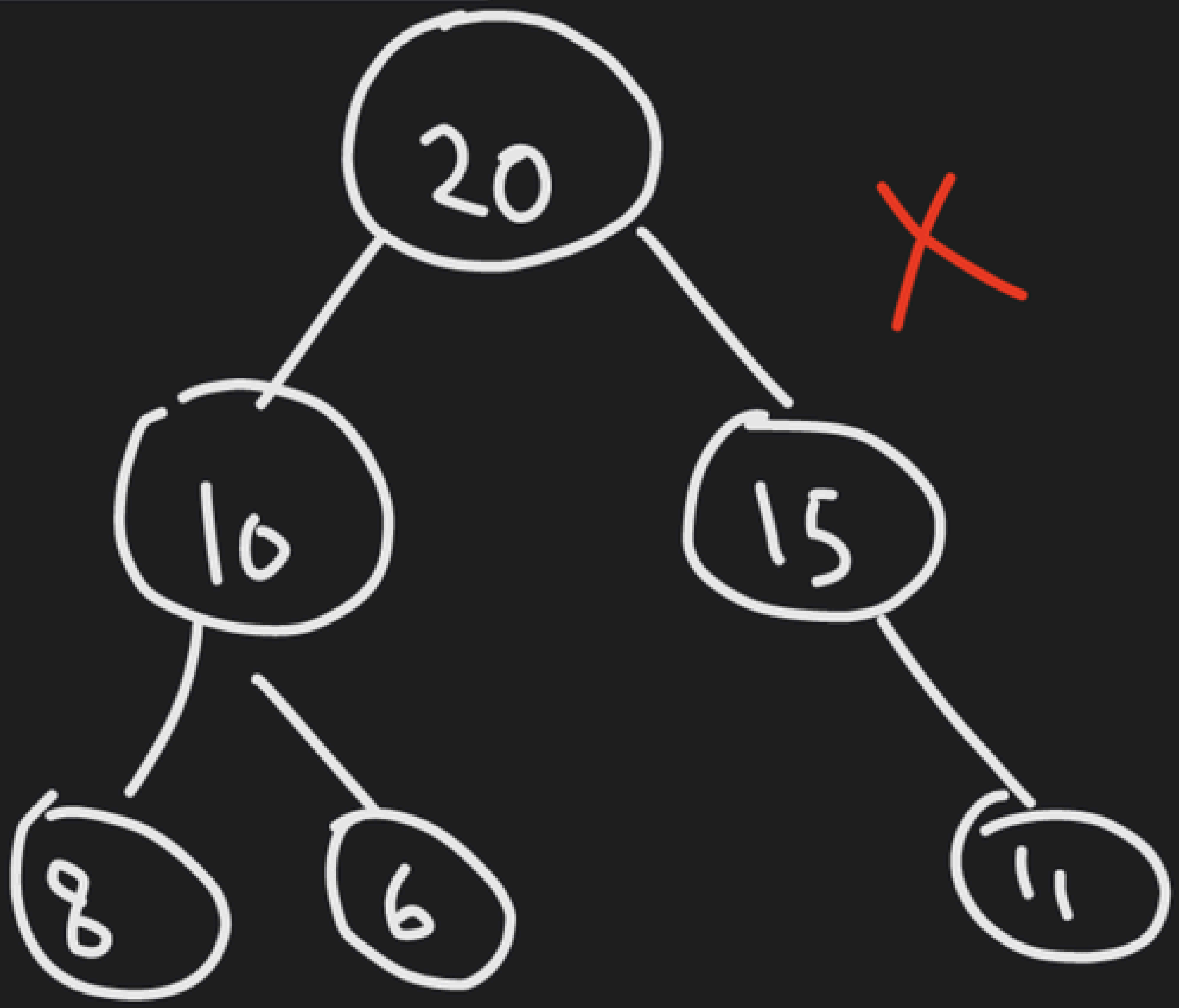
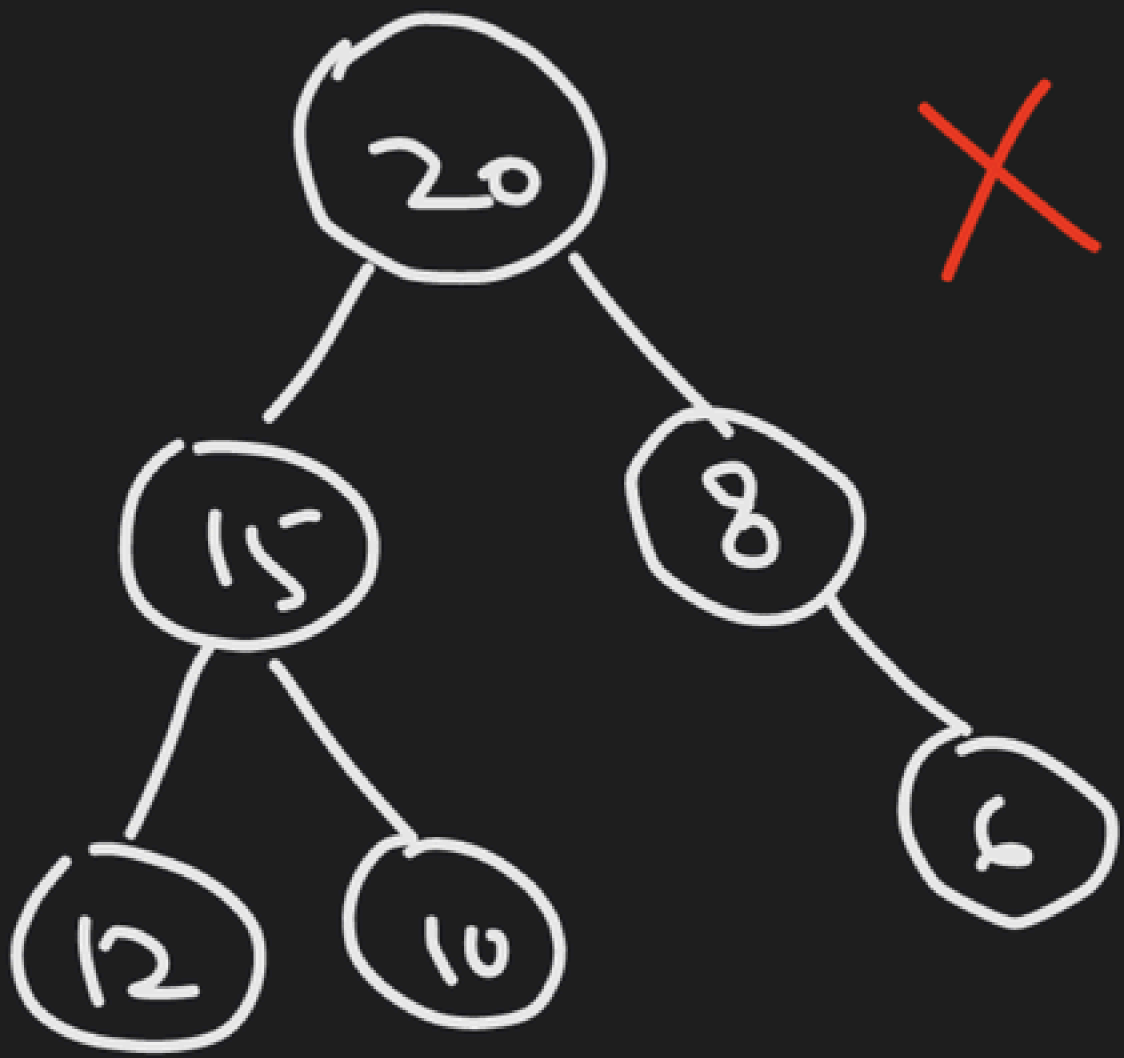
Max-heap ⇒ Min value ⇒ where ?

Can be present in some leaf node.

Max-heap ⇒ Where is the maximum value?

⇒ Root node.





Not a CBT

↳ Not a max-heap

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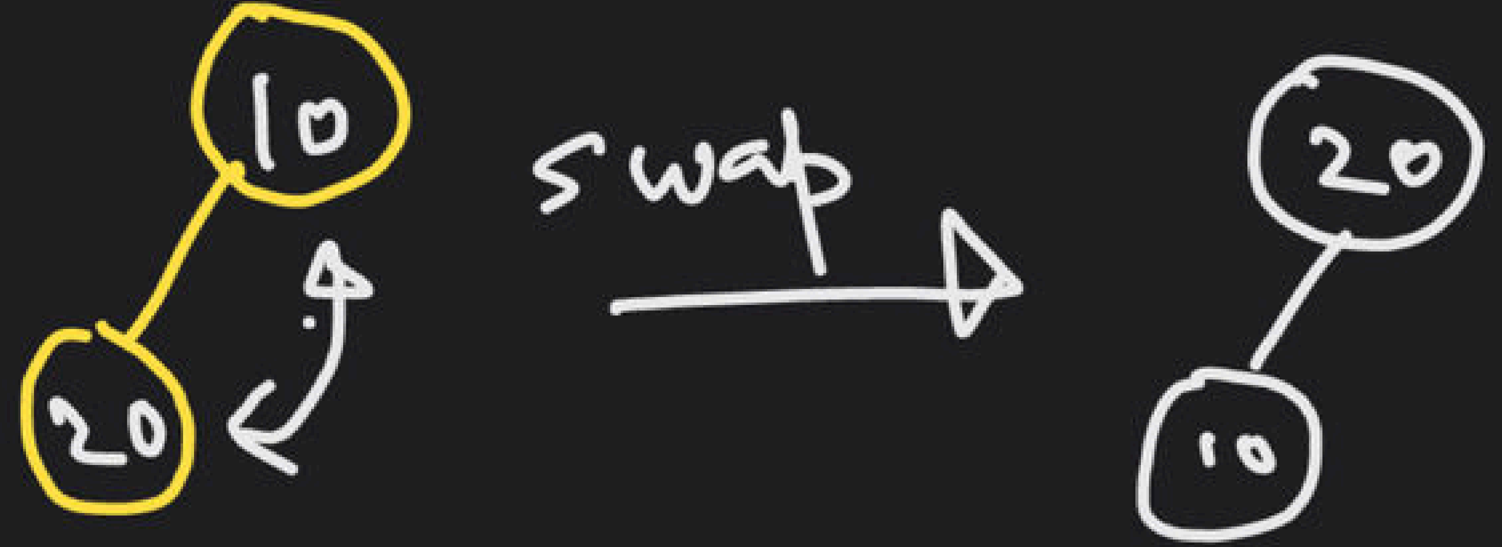
A CBT in wh

Const. of heap by inserting keys in order one after another.

Const. max-heap by inserting keys 10, 20, 30, 40, 50, 60, 70 one after another respectively.

(i) 10 (ii) Insert 20



Is it a max-heap? .

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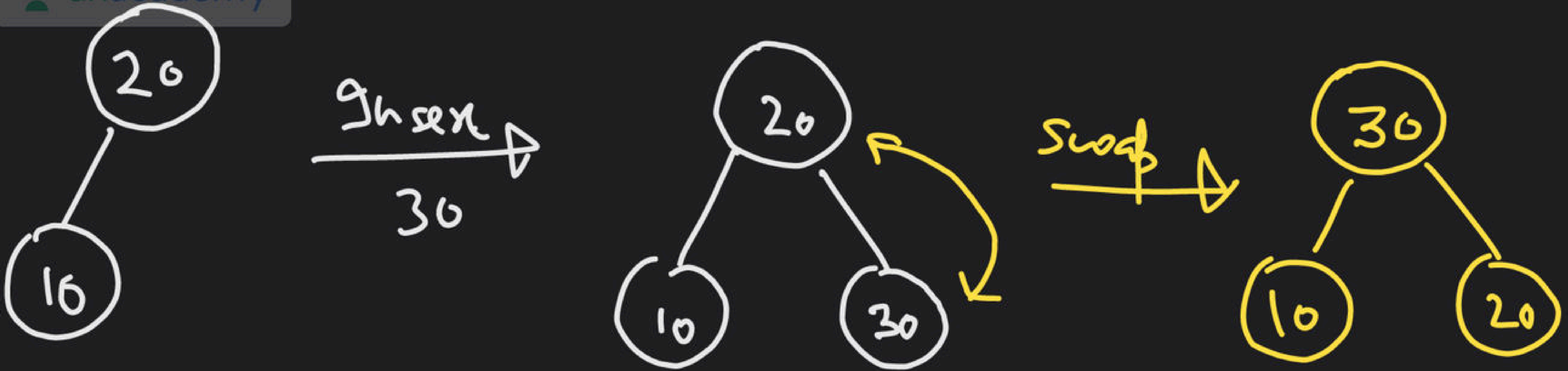
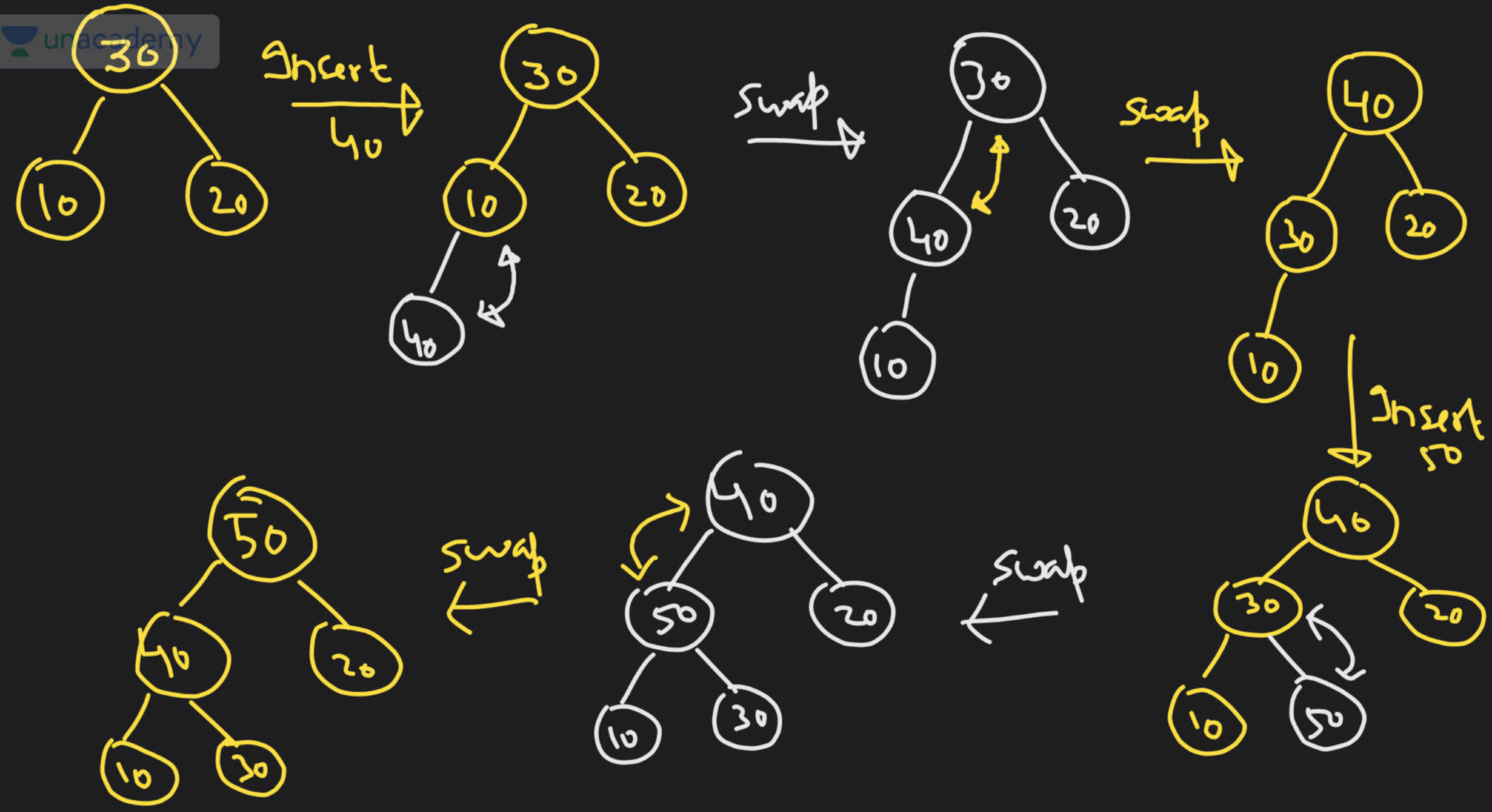
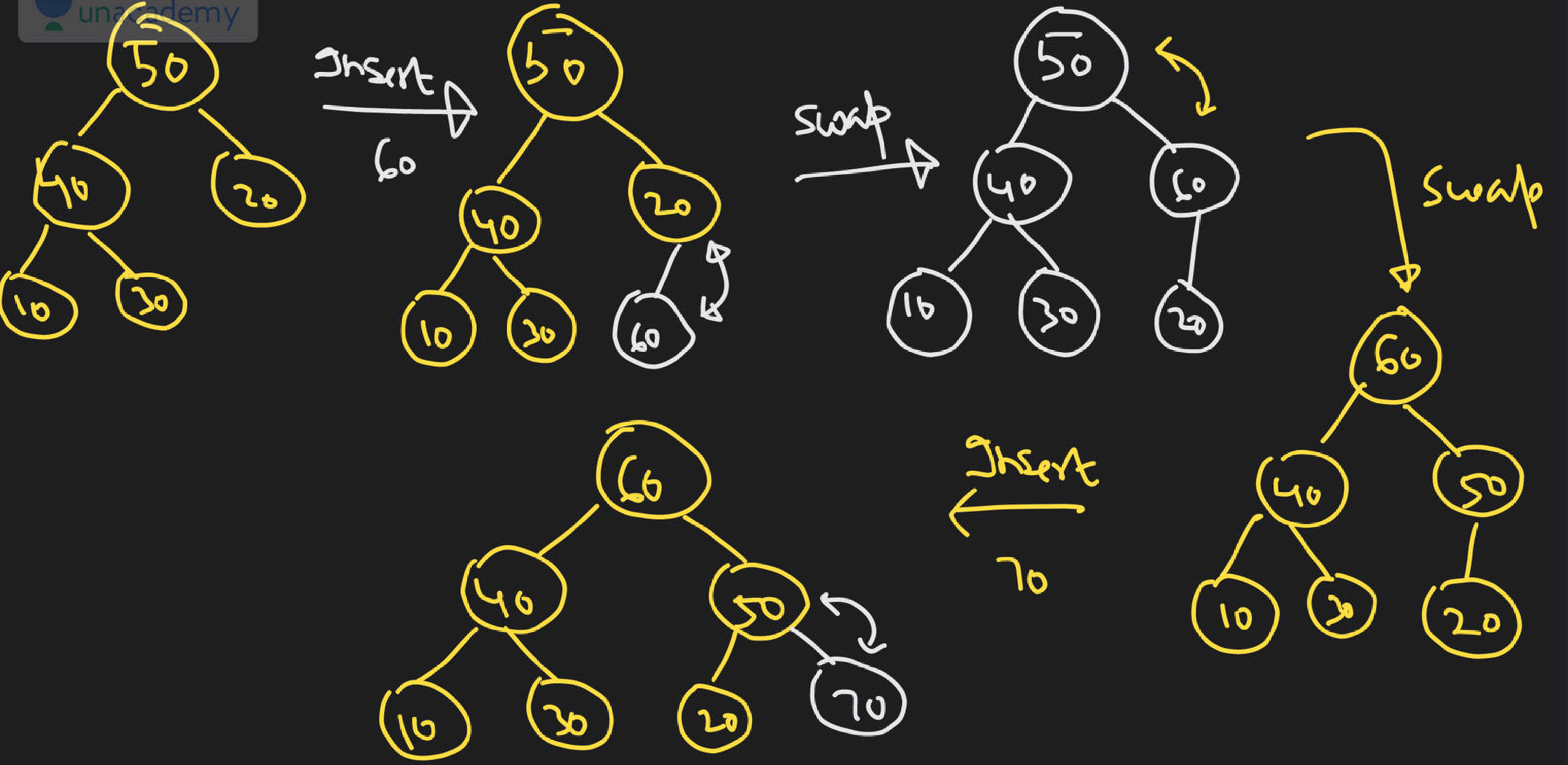


Is it a max-heap?


Insert
40
swap
swap
Insert
50
swap
swap



Insert
60
swap
swap
Insert
70

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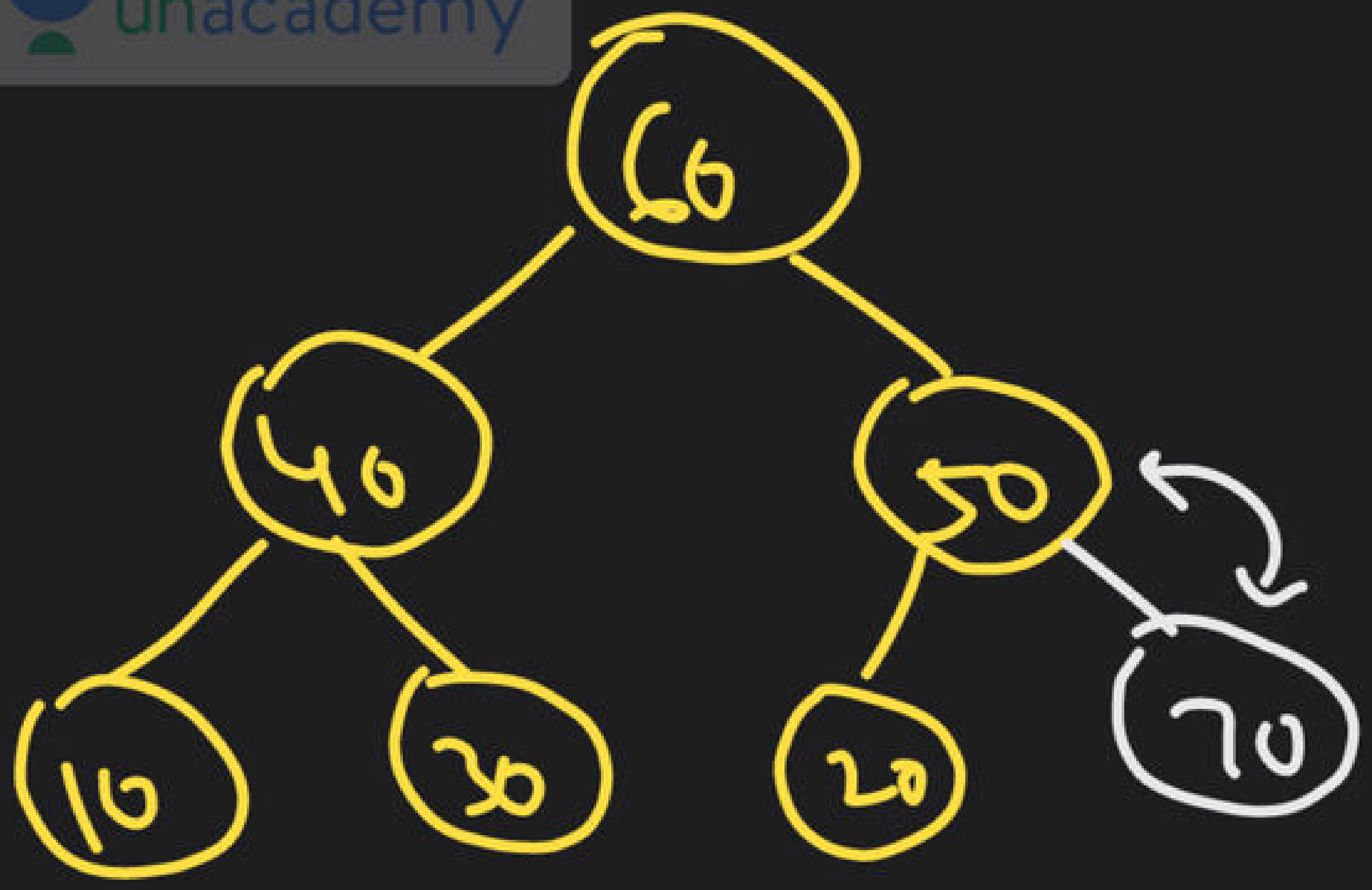
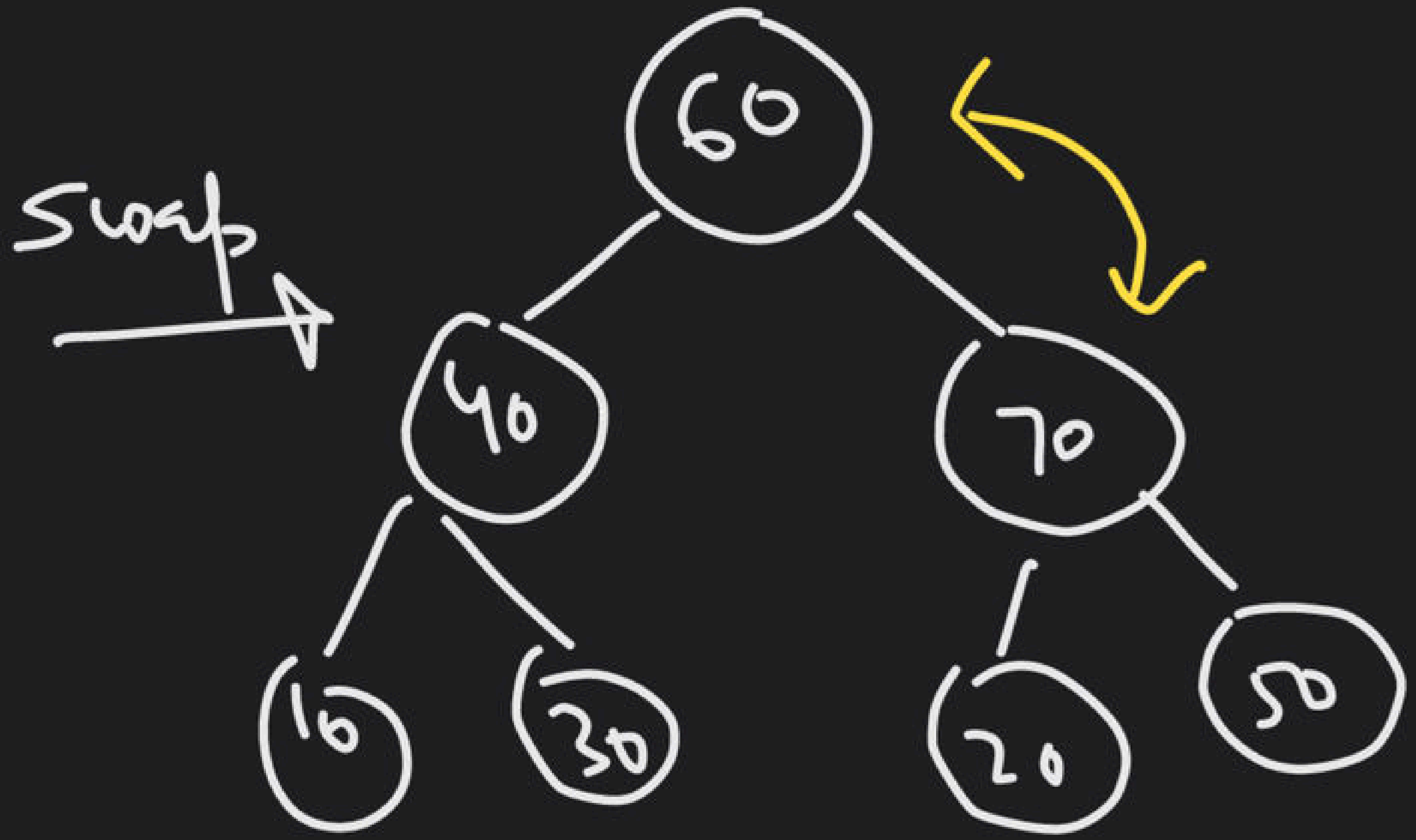

60
40
50
10
30
20
70


Swap
60
40
70
10
30
20
50


Swap
70
40
60
10
30
20
50

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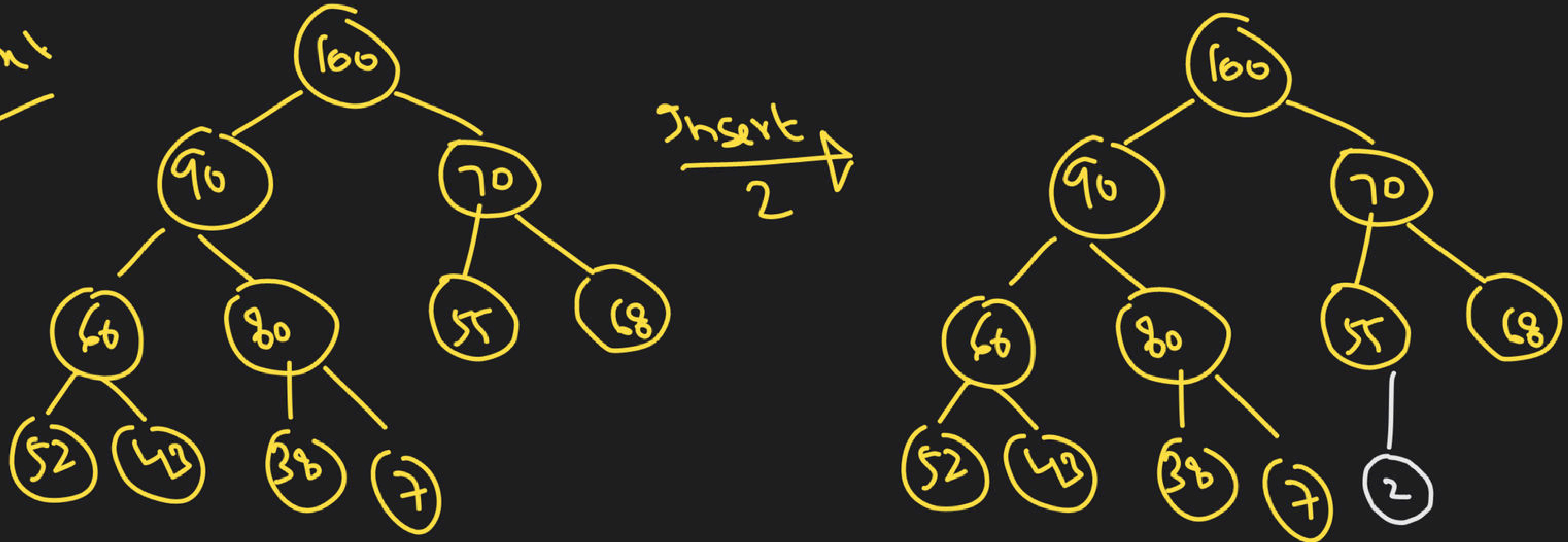
Existing heap with n nodes

key insert.

Ex:

Best case

1 comparison



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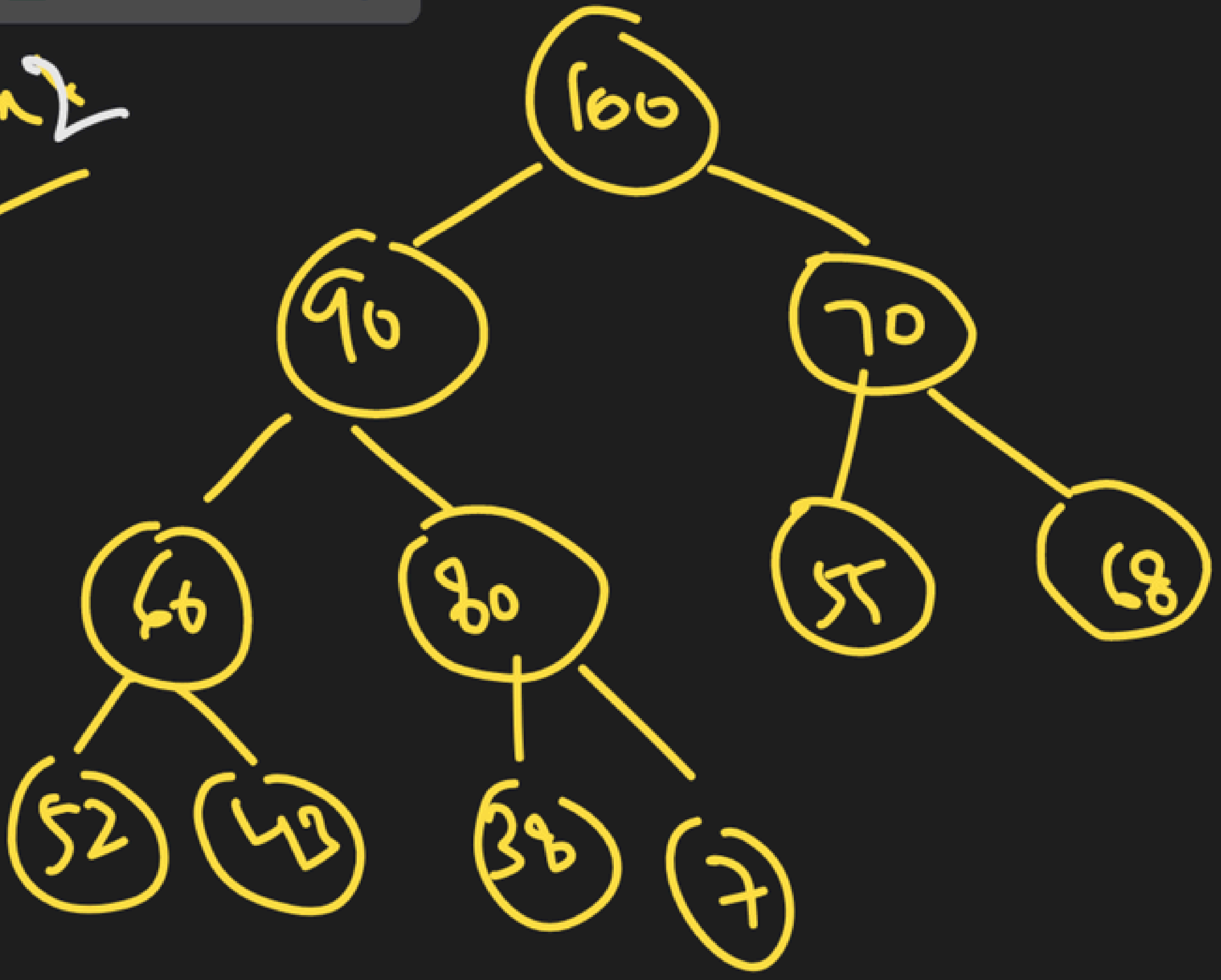
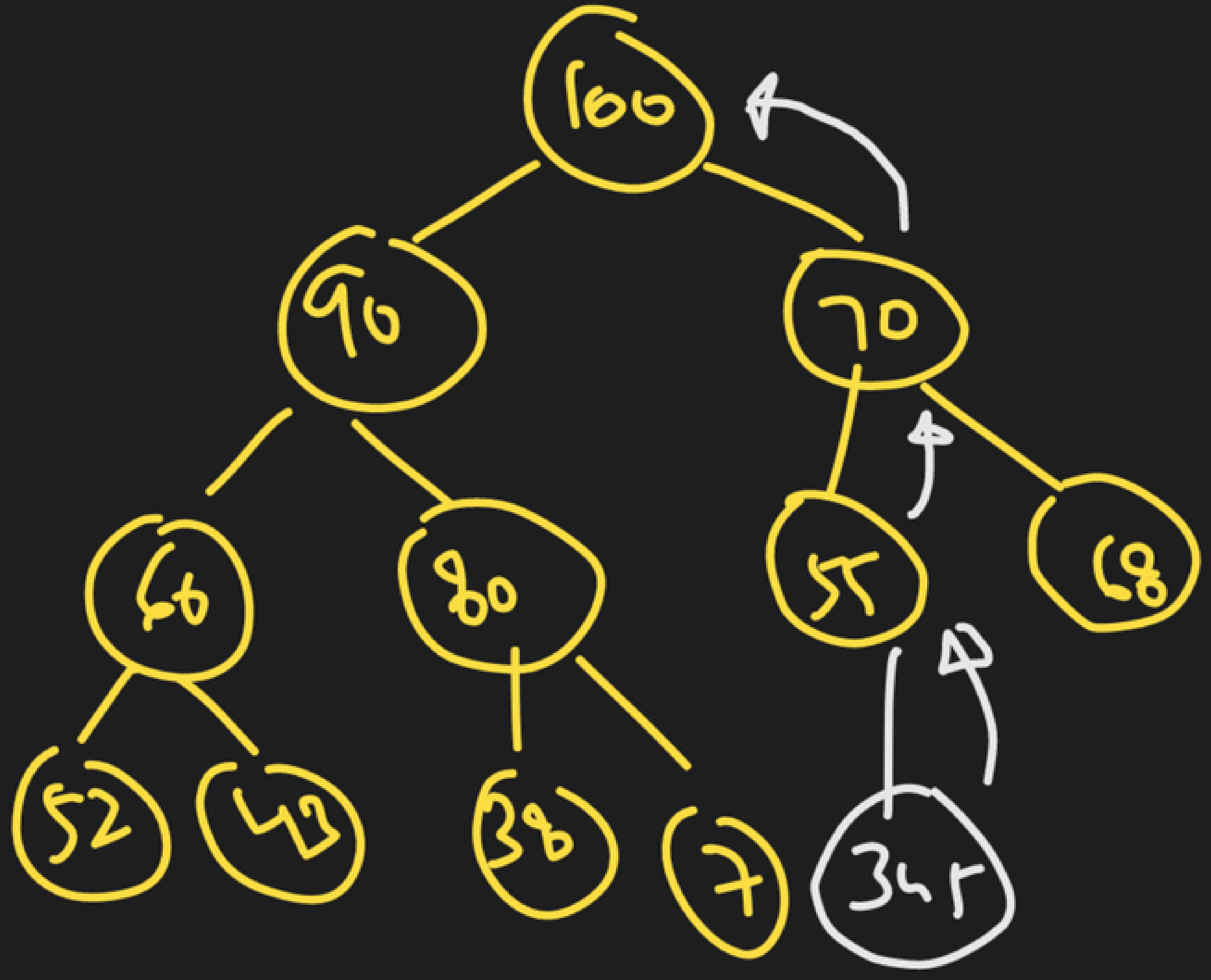
Ex 2



Insert

345



$\# \text{comp} \Rightarrow O(h) = O(\log_2 n)$

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Heap Const. by inserting keys one after another

$\Rightarrow O(n \log n)$

* Const. of heap using Build-Heap method

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THANK YOU!

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