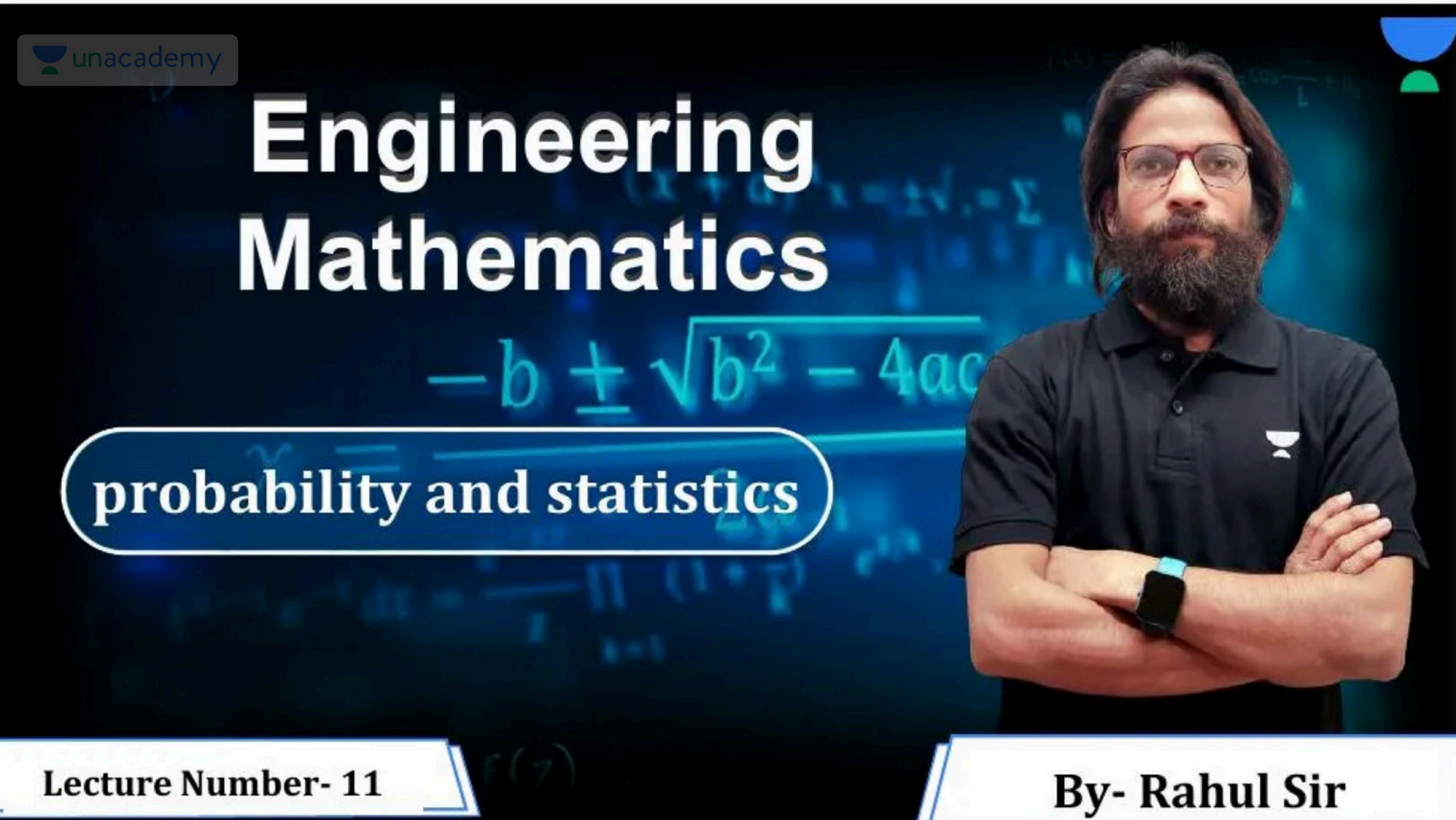
Probability Theory - Part VI

Course on Engineering Mathematics for GATE - CSE

Rahul • Lesson 12 • June 25, 2024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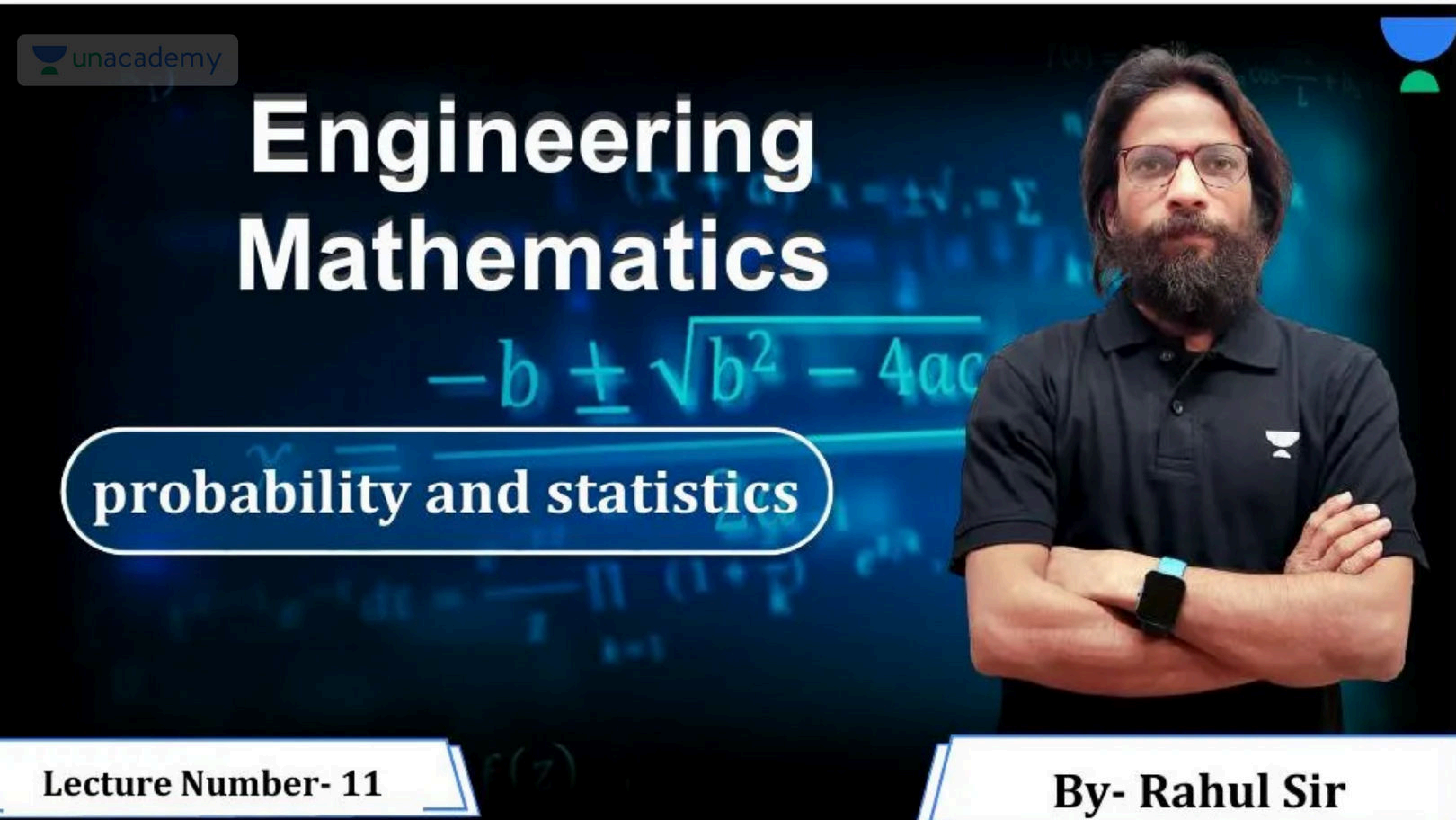
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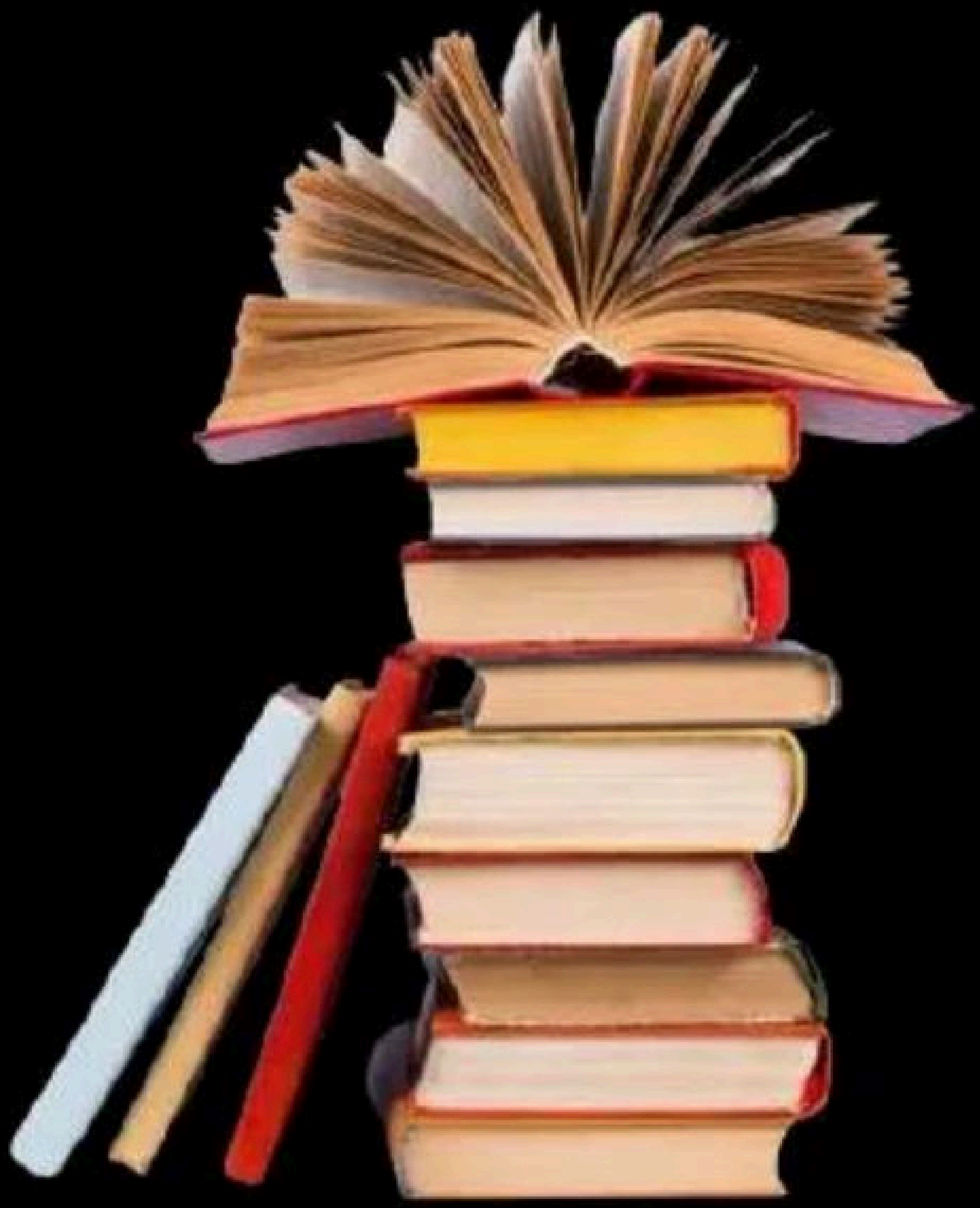
Topics

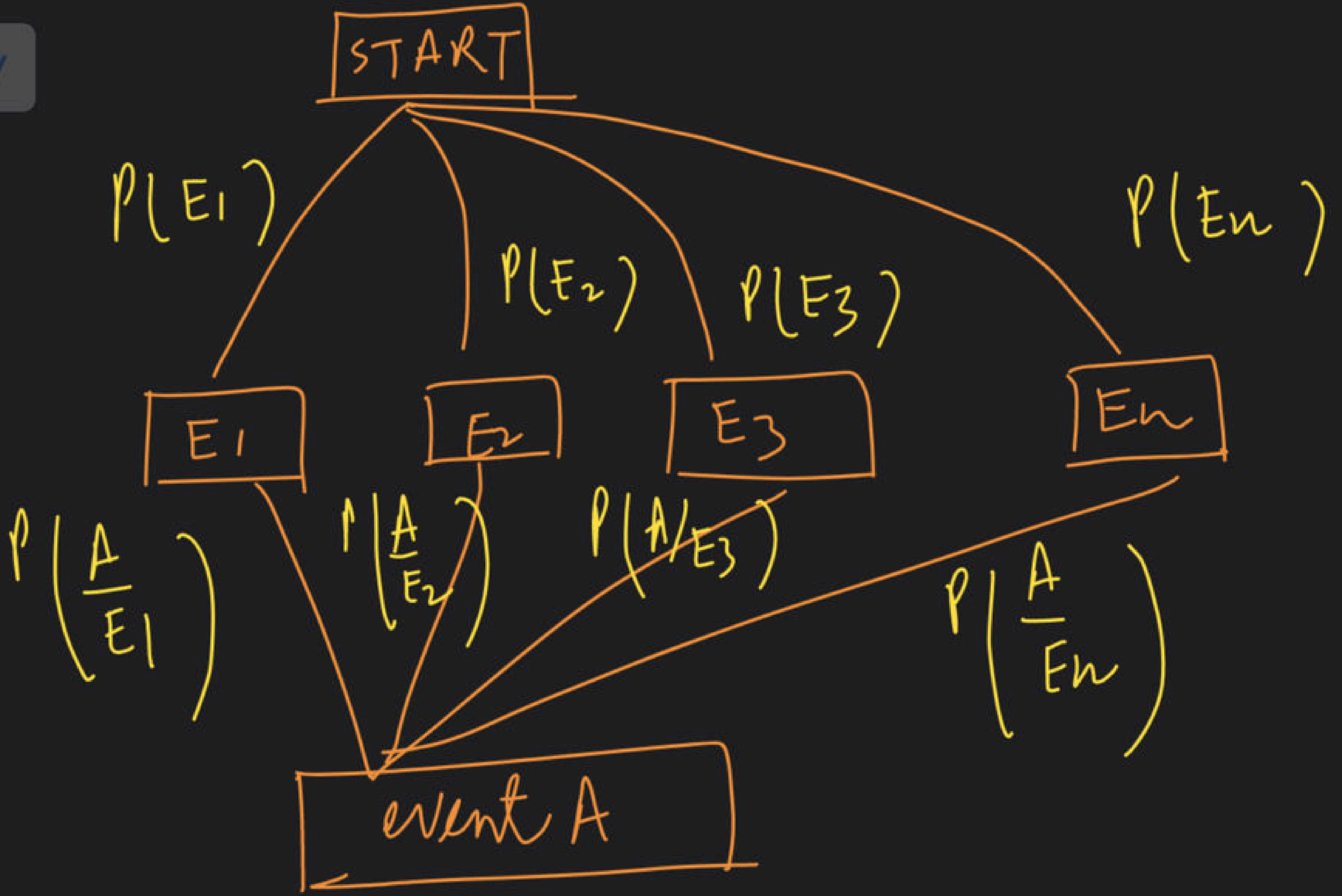
to be covered



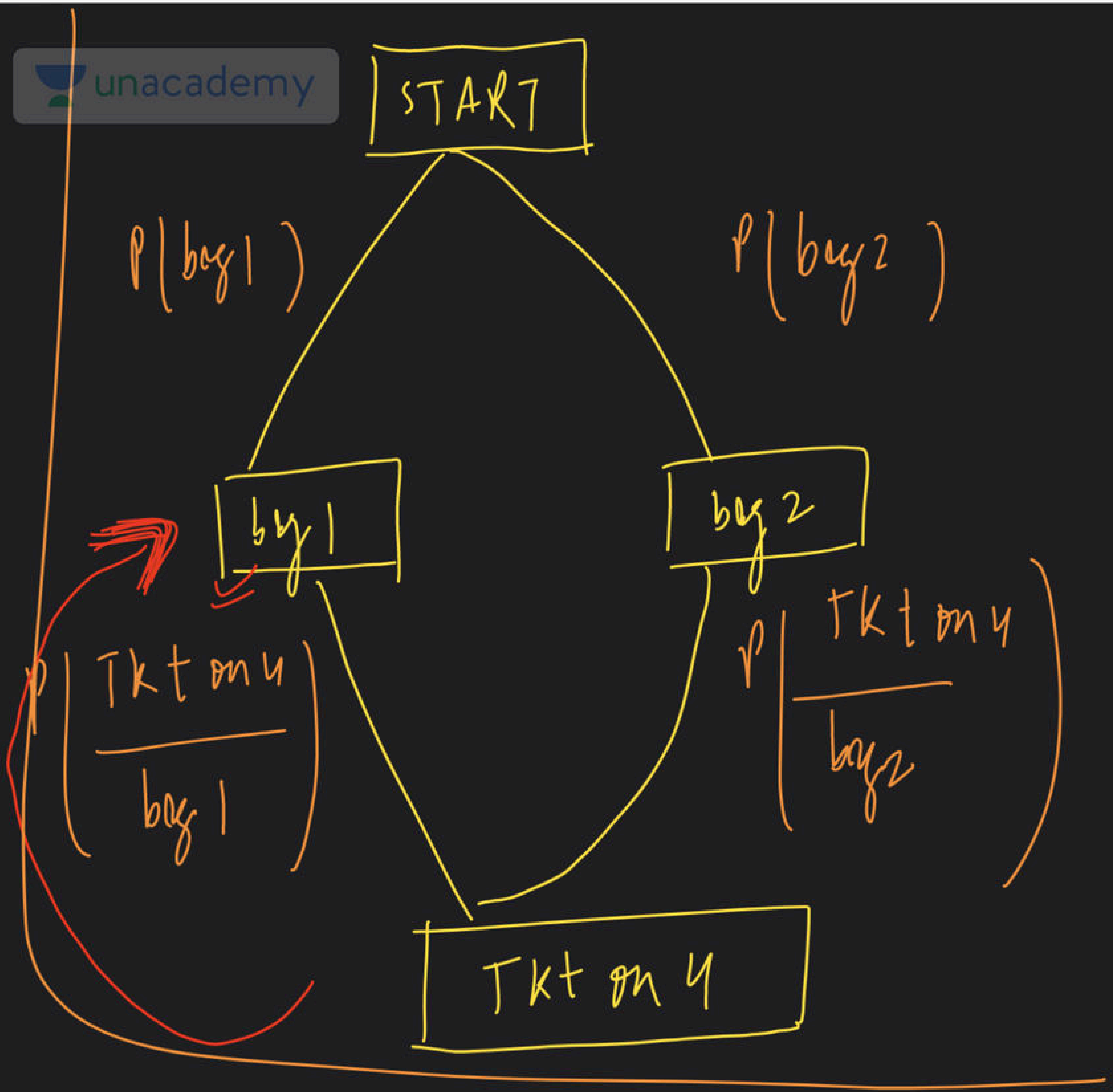
1. Problem solving class_I

DPP03

telegram group.



$$\boxed{P(A) = \text{Total prob} = P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right) + \cdots + \cdots + P(E_3)\,P\left(\frac{A}{E_n}\right)}$$



"What is the Prob.

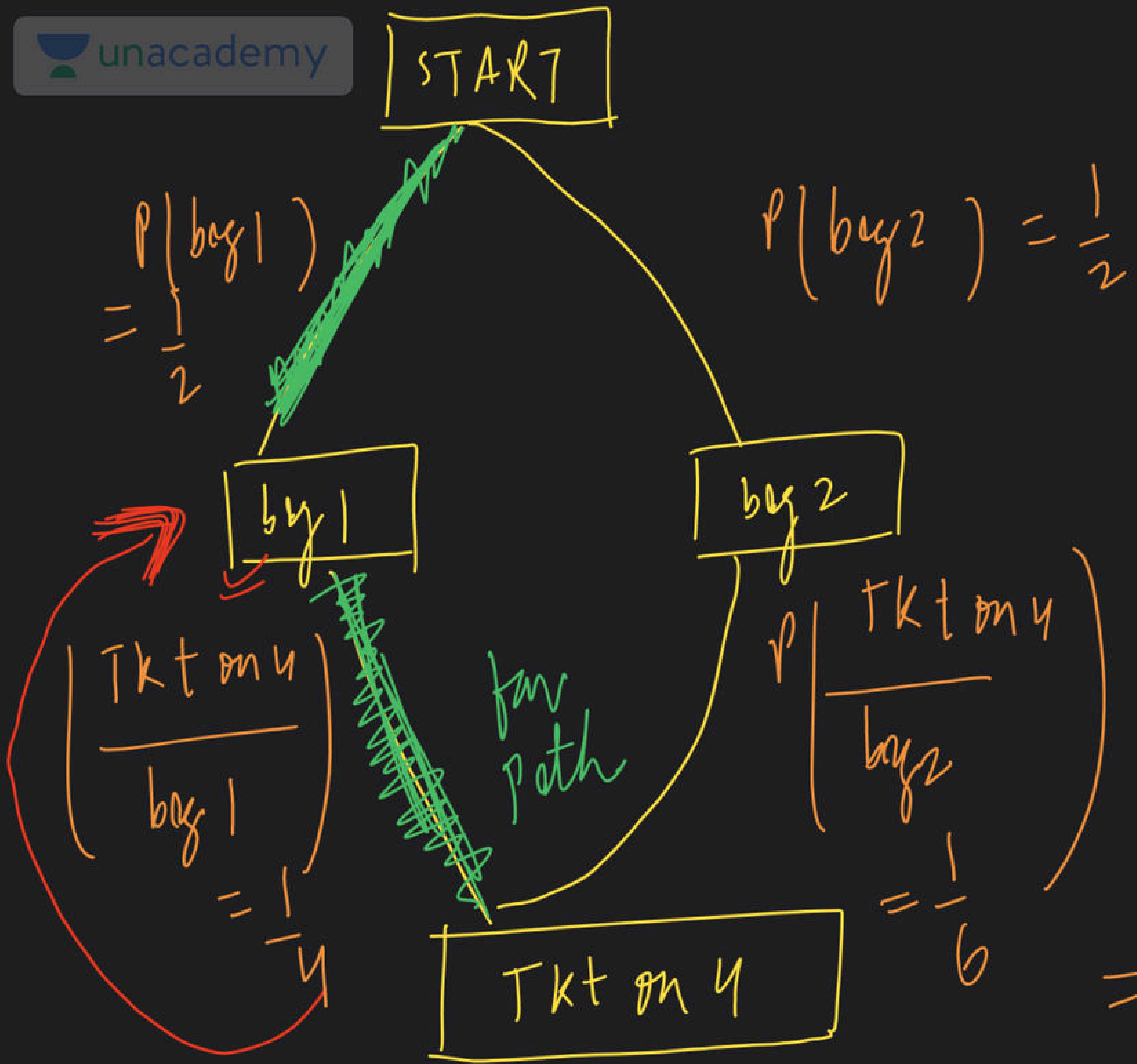
$$P\left(\frac{\text{bag 1}}{\text{Tkt on 4}}\right)$$

Tkt 4 is already Happened and

$\frac{\text{bag 1}}{\text{Tkt on 4}}$ Happening

→ past

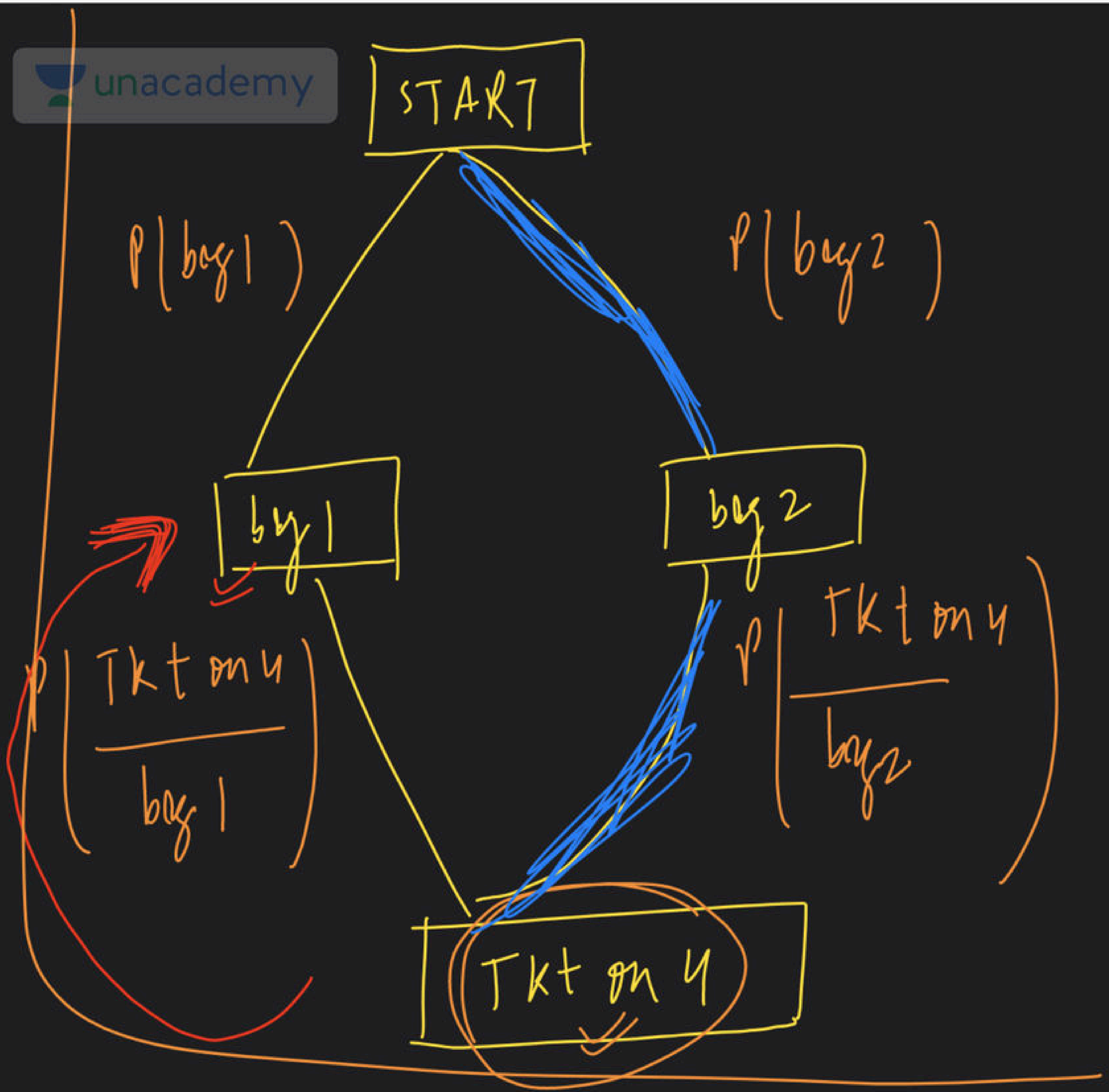
(Recoil)



$$P\left(\frac{\text{bag 1}}{\text{Tkt on 4}}\right) = \frac{\text{fav Path}}{\text{Total path}}$$

$$= \frac{P(\text{bag1})\,P\left(\frac{\text{Tkt on 4}}{\text{bag 1}}\right)}{P(\text{bag1})\,P\left(\frac{\text{Tkt on 4}}{\text{bag 1}}\right) + P(\text{bag2})\,P\left(\frac{\text{Tk 4}}{\text{bag2}}\right)}$$

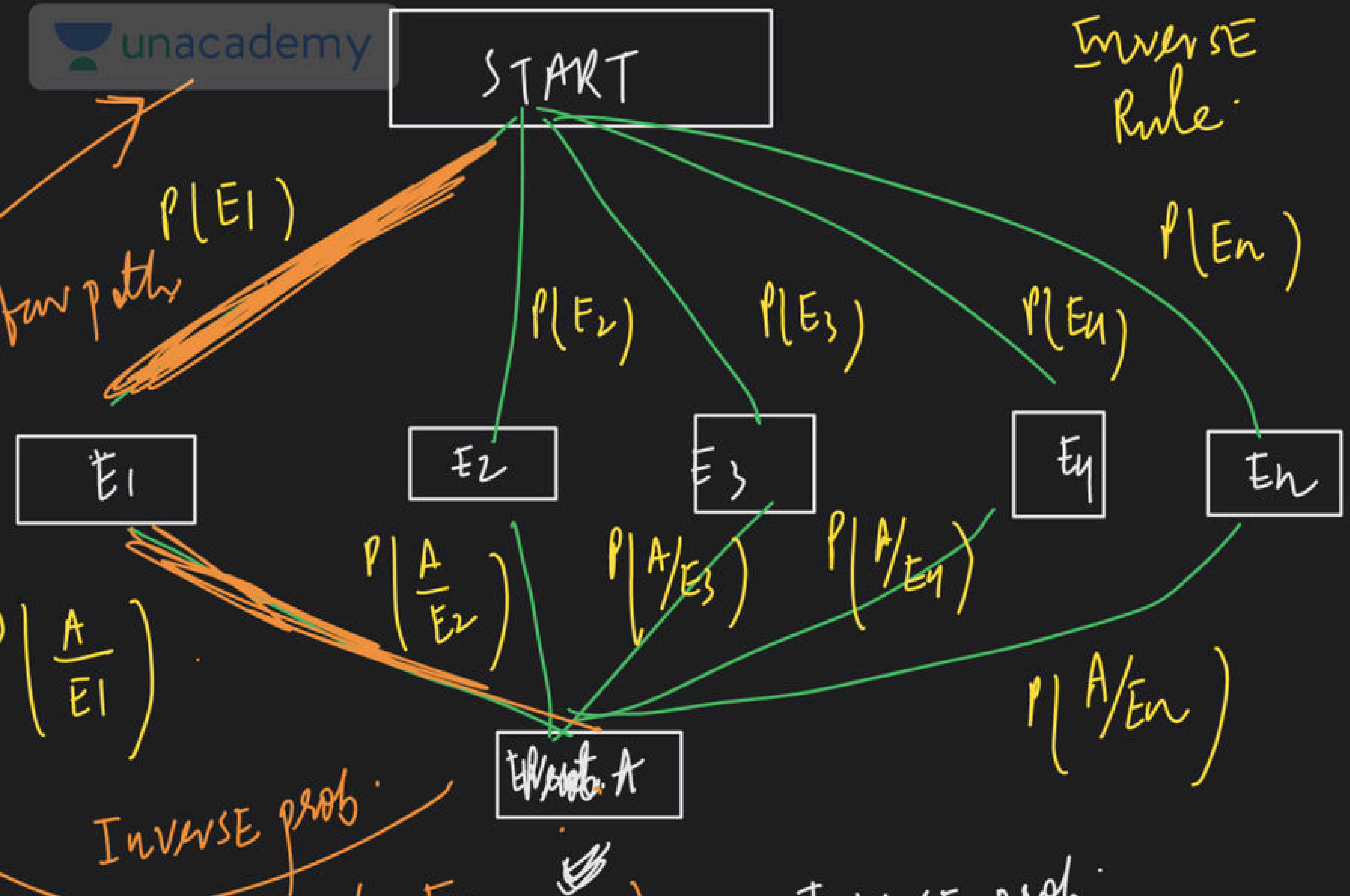
$$= \frac{\frac{1}{2} \times \frac{1}{4}}{5/24} = \frac{3}{5}$$



$$P\left(\frac{\text{bag 2}}{\text{Tkt 4}}\right)$$

$$= \frac{P(\text{bag}_2)\,P\left(\frac{\text{Tkt on 4}}{\text{bag}_2}\right)}{\text{Total}}$$

$$= \frac{\frac{1}{2} \times \frac{1}{6}}{\frac{5}{24}} = \frac{2}{5}$$



$$P\left(\frac{E_1}{A}\right) = \text{Inverse prob.}$$

Inverse Rule:

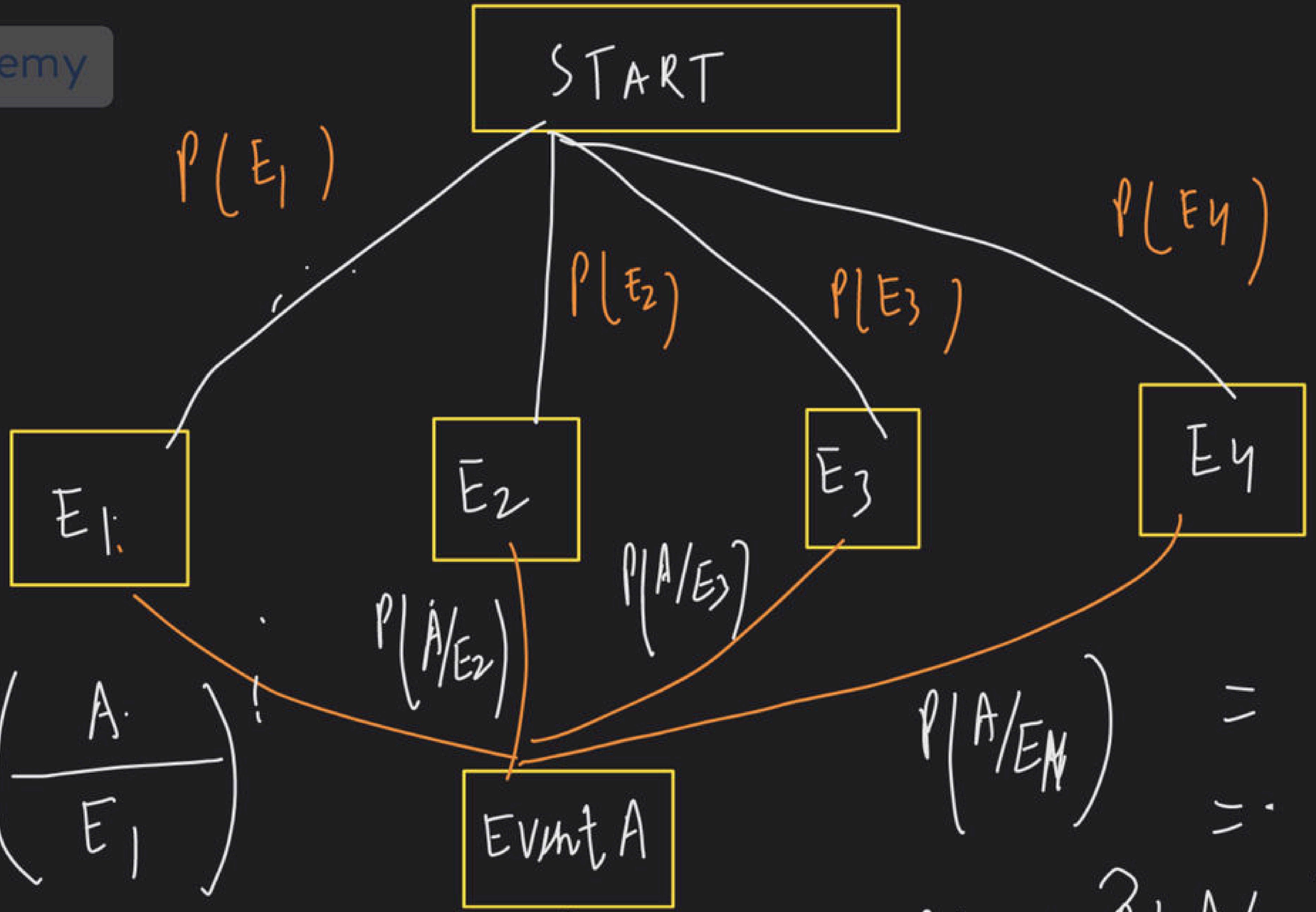
$$\boxed{P\left(\frac{E_1}{A}\right)} = \frac{P(E_1)\, P\left(\frac{A}{E_1}\right)}{P(A)}$$

$$= \frac{P(E_1)\, P\left(\frac{A}{E_1}\right) \text{ fav}}{\sum_{i=1}^{n} P(E_i)\, P\left(\frac{A}{E_i}\right) \text{ Total}}$$

Bayes THEOREM

for finding inverse prob. (past recall)

STARTED



$P\left(\frac{E_1}{A}\right)$

= inverse prob.

$P\left(\frac{A}{E_1}\right)$!

$P(A/E_1) = P(E_1 \cap A)$

$= P(E_1)\,P(A/E_1)$ for path

$\frac{A}{E_1}$ = occurring

$E_1 \rightarrow$ हो चुका है

$$P\left(\frac{A}{E_1}\right) = \frac{P(E_1)\,P(A/E_1)}{P(A)}$$

Depen

using condition

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\,P\left(\frac{A}{E_1}\right)}{P(A)} \quad P(E_1 \cap A)$$

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\,P\left(\frac{A}{E_1}\right)}{\sum_{i=1}^{n} P(E_i)\,P\left(\frac{A}{E_i}\right)}$$

= far path

= Total prob.

= prior

= posterior

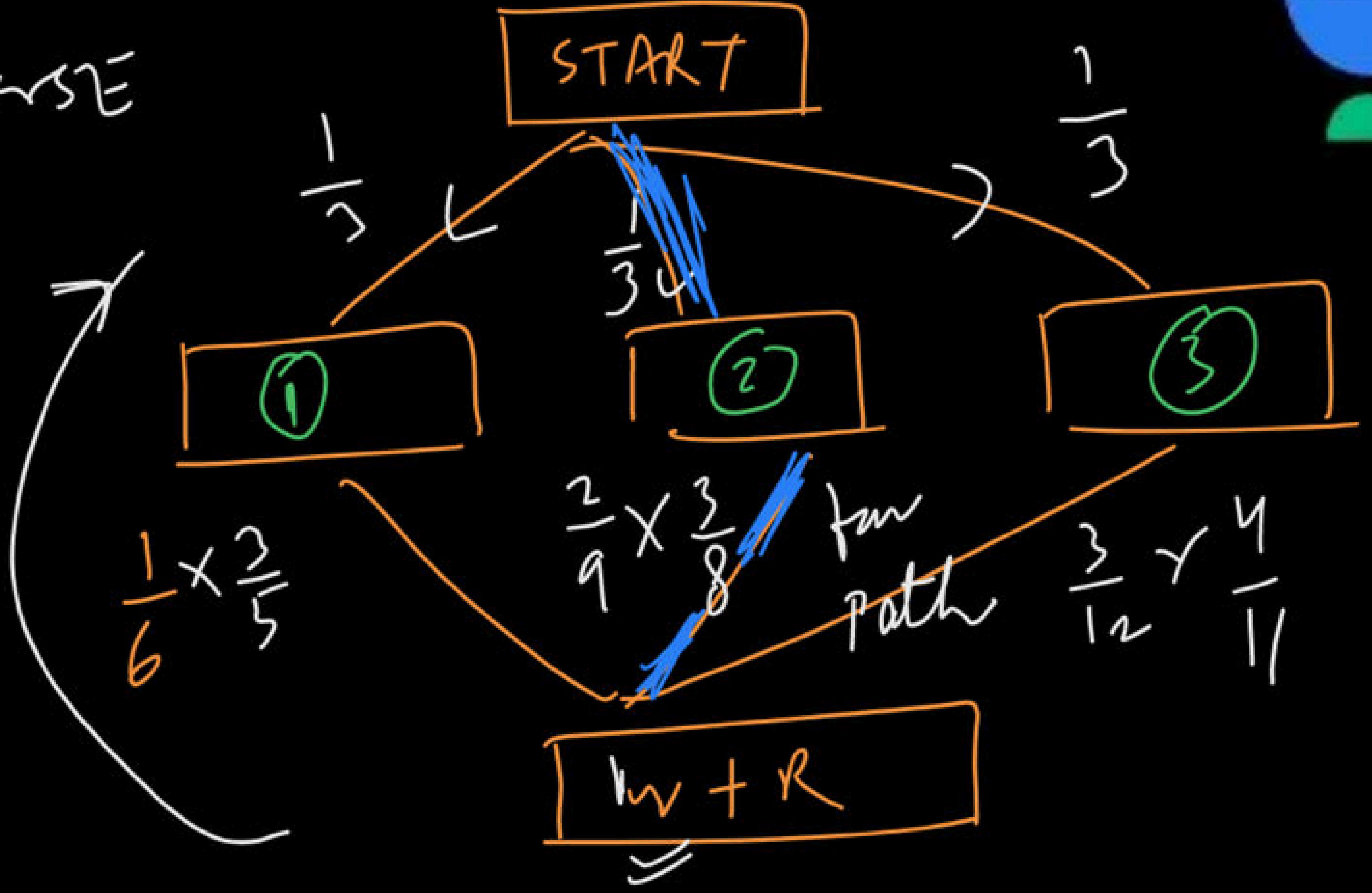
$\underbrace{\qquad}$ BAYES THEOREM (finding Inverse prob)

QUESTION

Q. # Three Boxes B_1 B_2 B_3 contains balls

$$\begin{Bmatrix} B_1 \rightarrow 1W, 2B, 3R \\ B_2 \rightarrow 2W, 4B, 3R \\ B_3 \rightarrow 3W, 5B, 4R \end{Bmatrix}$$

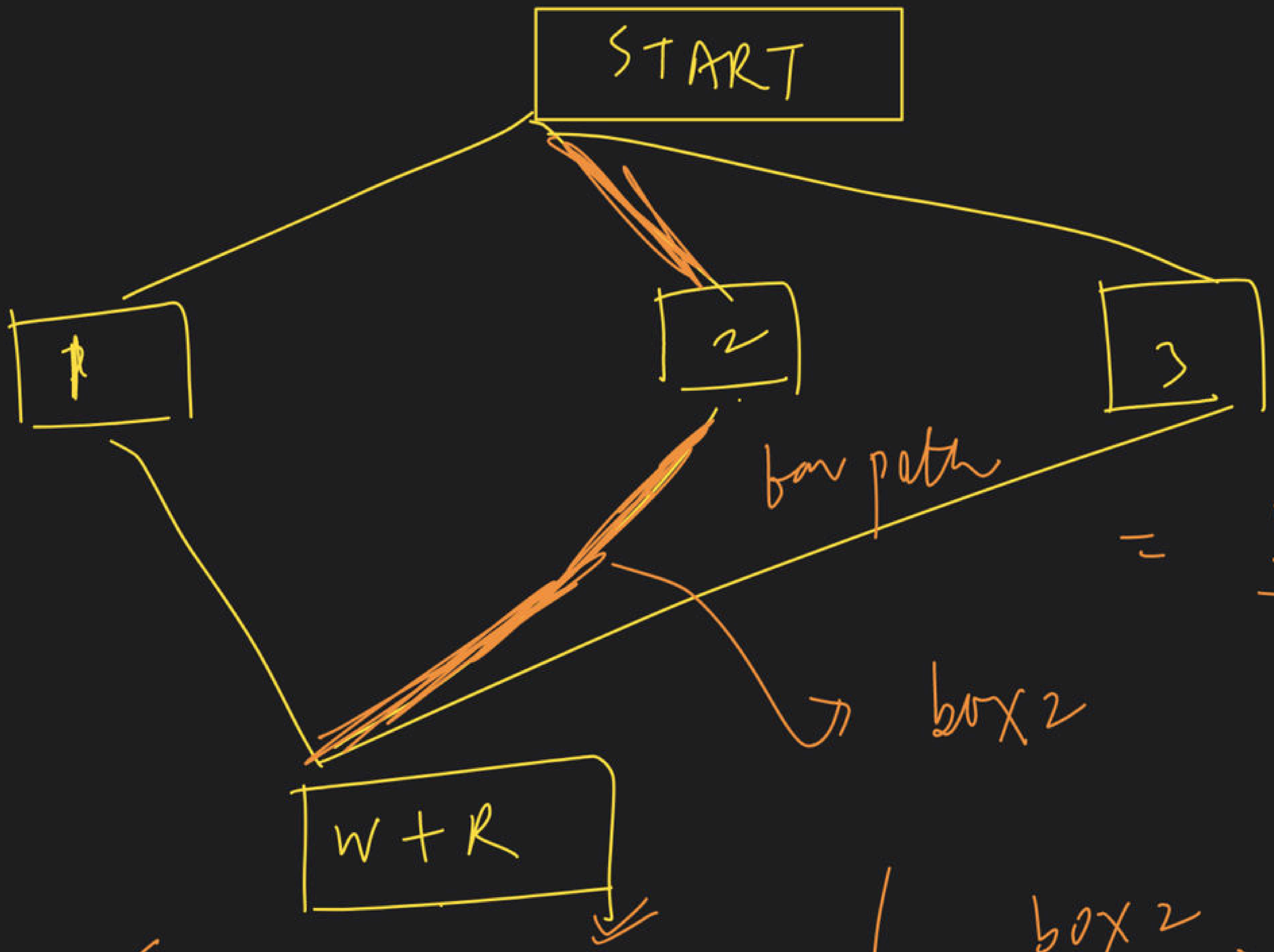
INVERSE



Without replacement, if $\boxed{\text{2 balls}}$ are drawn from randomly selected box. Find the probability one of the ball drawn is white and other ball is red from box 2 order is specified!

$$P\left(\frac{\text{box 2}}{\text{W+R}}\right) = \text{INVERSE prob} = \frac{\frac{1}{3} \times \frac{2}{9} \times \frac{3}{8}}{\frac{1}{3} \times \frac{2}{9} \times \frac{3}{8} + \frac{1}{3} \times \frac{1}{6} \times \frac{3}{5} + \frac{1}{3} \times \frac{3}{12} \times \frac{4}{11} \times \frac{1}{3}} = \frac{55}{181}$$

bayes THEOREM



$$= \frac{\text{fav path}}{\text{Total path}}$$

= bayes THEREM

$$P\left(\frac{\text{box 2}}{W+R}\right)$$

QUESTION

(Bayes THEOREM)

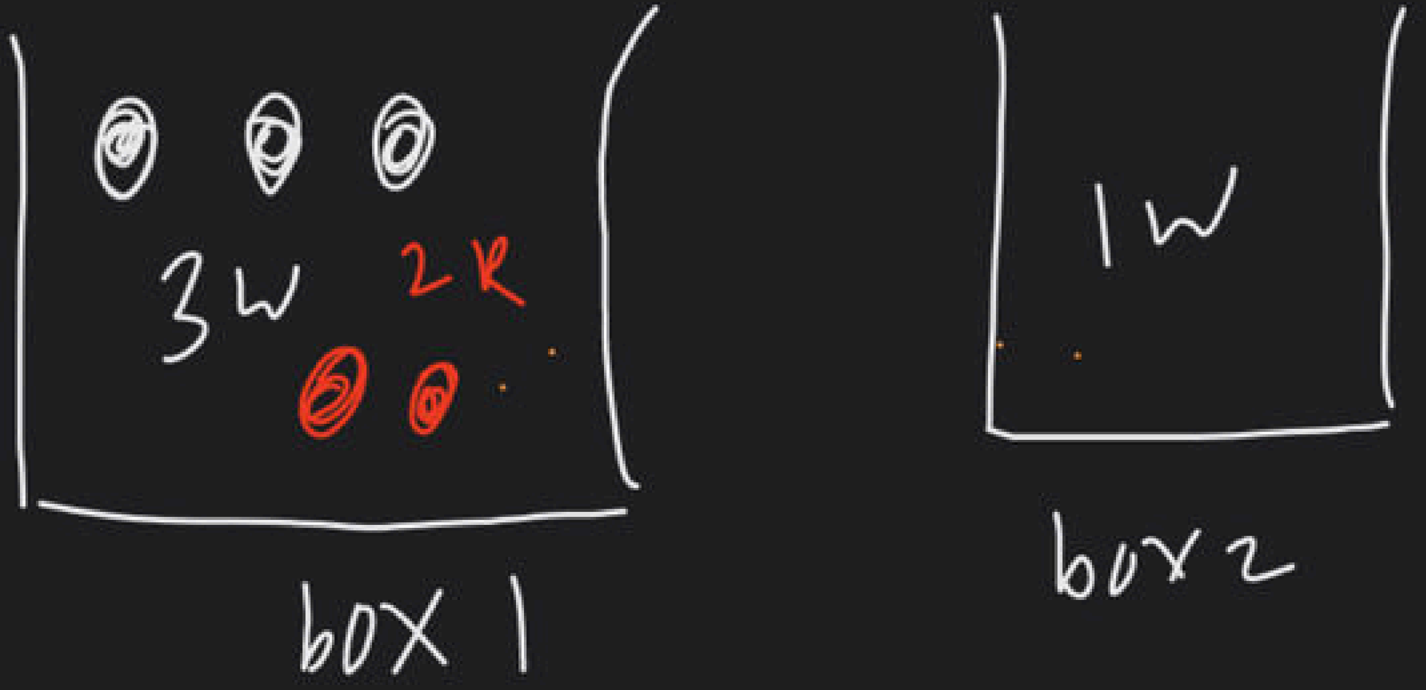
Q. Let V_1 and V_2 be two urns box such that V_1 contains 3 white and 2 Red balls and V_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from V_1 and put into v_2. However, if tail appears then 2 balls are drawn at random from v_1 and put into v_2. Now one ball is drawn at random from v_2 given that the drawn ball v_2 is white then the probability that head appeared on the coin.

✓ A) $8/9$

✓ B) $\frac{12}{23}$

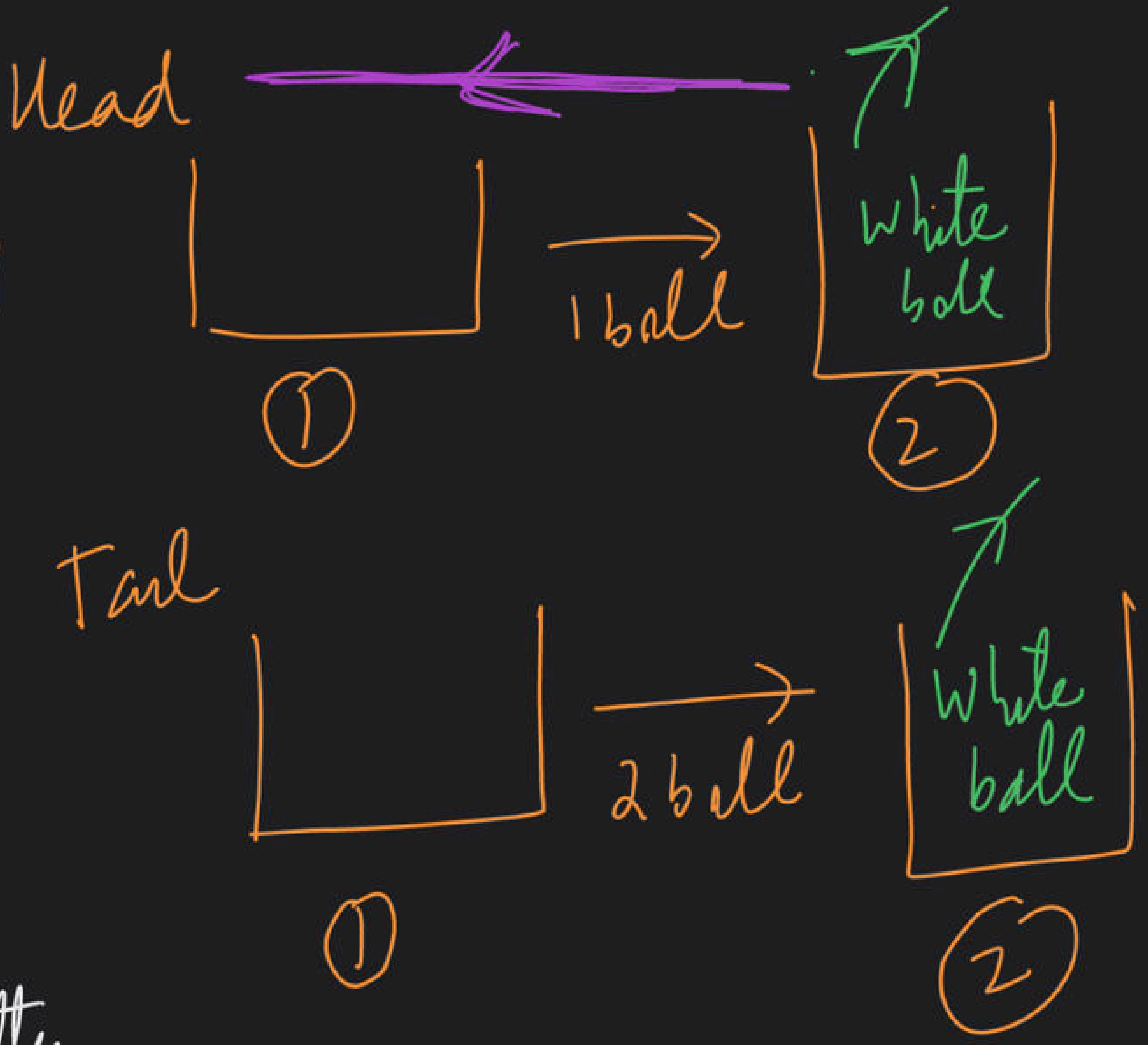
✓ C) $\frac{16}{23}$

✓ D) $\frac{16}{21}$



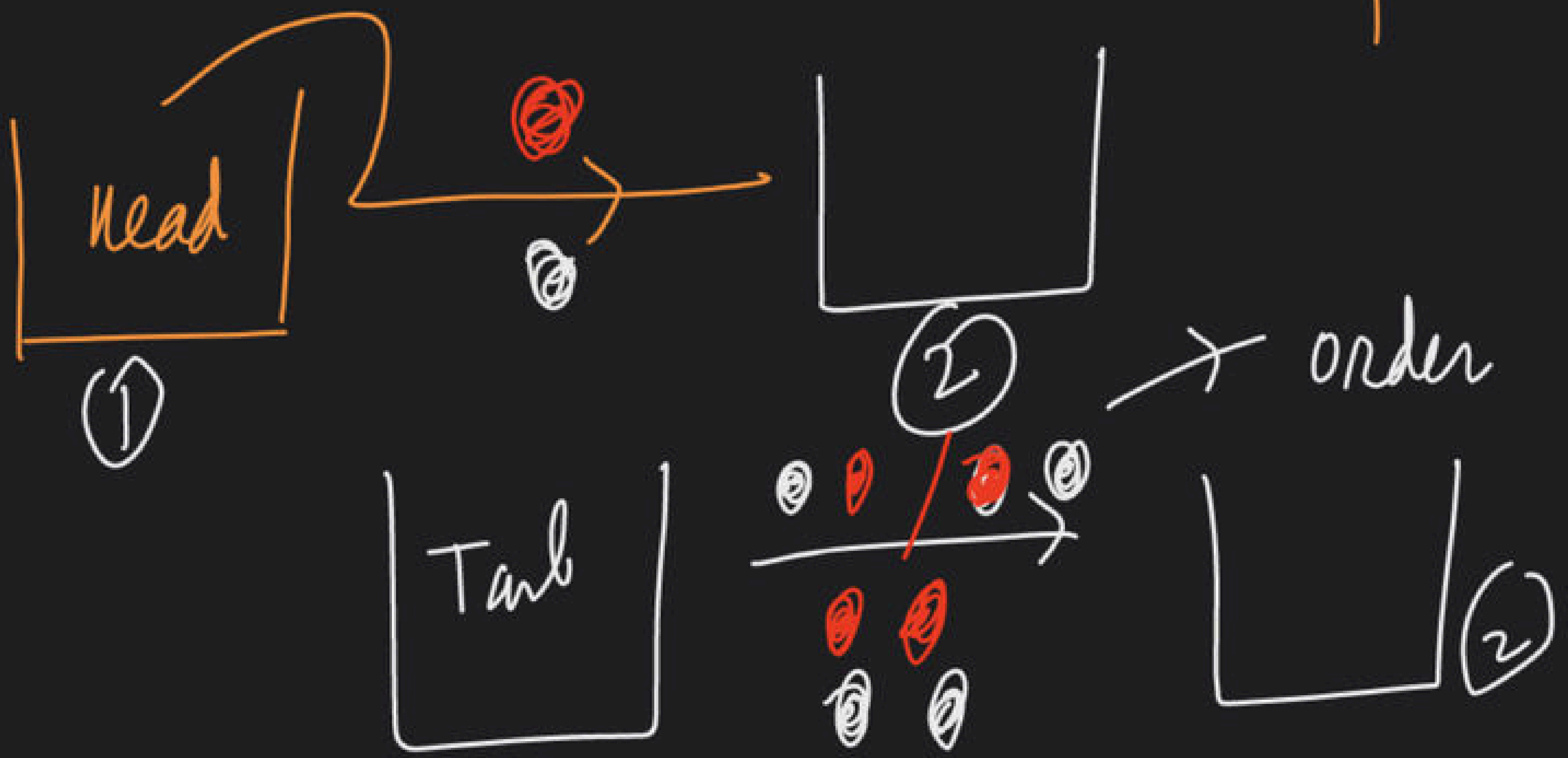
— coin Tossed

Shivansh

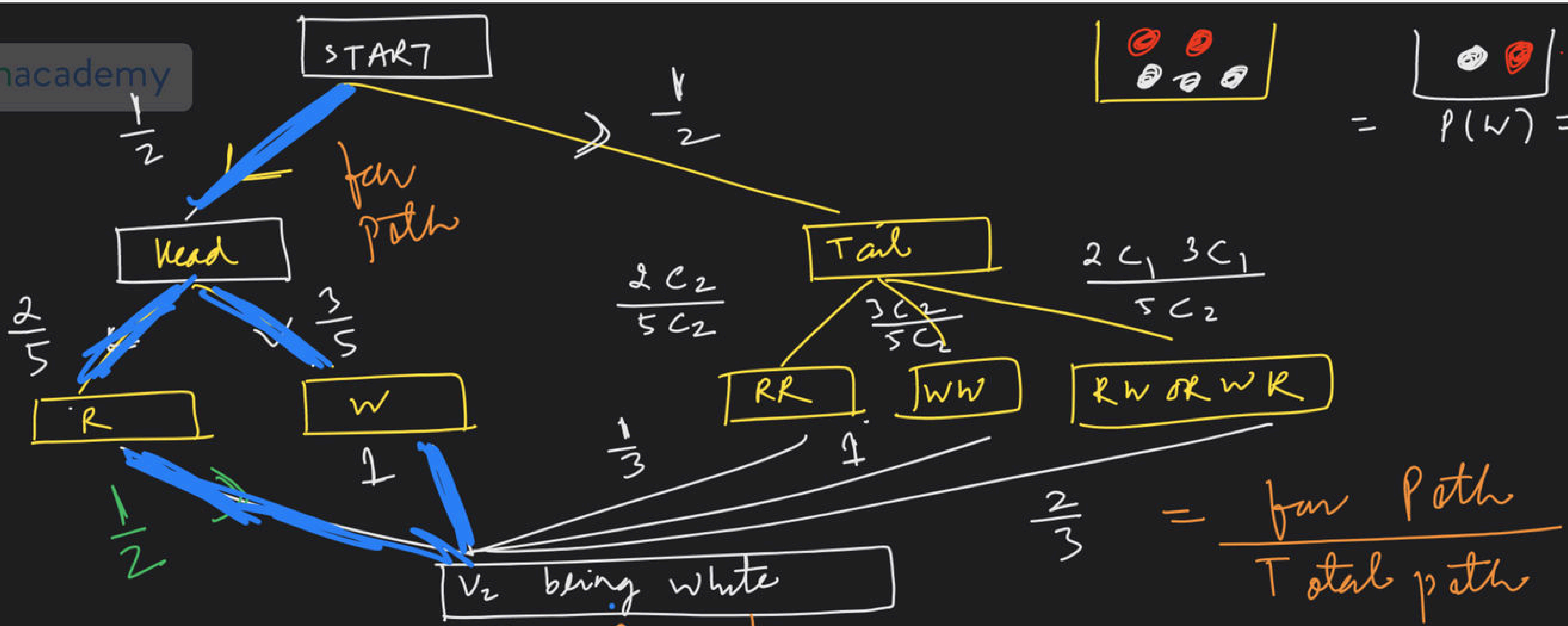


$$P\left(\frac{\text{Head appeared coin}}{\text{white ball}}\right) = \text{bayes THEOREM}$$

$$= \text{Inverse prob.}$$

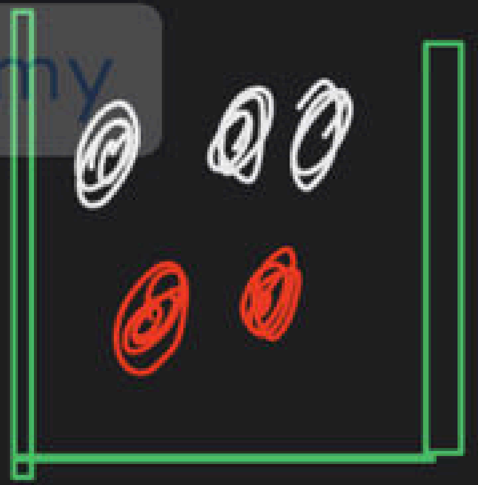


→ order does not matter

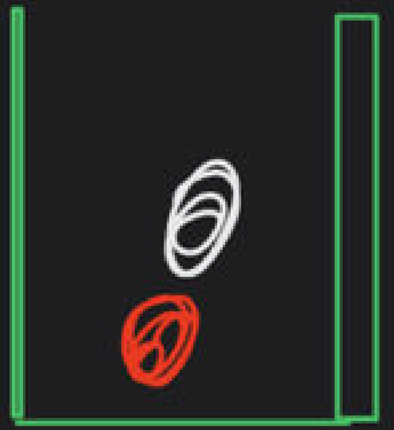


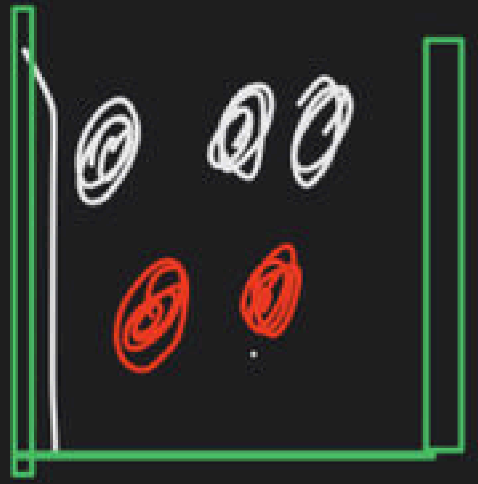
$$\Rightarrow \frac{\frac{1}{2}\times\frac{2}{5}\times\frac{1}{2}+\frac{1}{2}\times\frac{3}{5}\times 1}{\frac{1}{2}\times\frac{2}{5}\times\frac{1}{2}+\frac{1}{2}\times\frac{3}{5}\times 1+\frac{1}{2}\times\frac{^2C_2}{^5C_2}\times\frac{1}{3}+\frac{1}{2}\times\frac{^3C_2}{^5C_2}\times 1+\frac{1}{2}\times\frac{^2C_1\,{}^3C_1}{^5C_2}\times\frac{2}{3}}$$

$$= \frac{20}{23}$$ $\boxed{\text{answer}}$ ✓

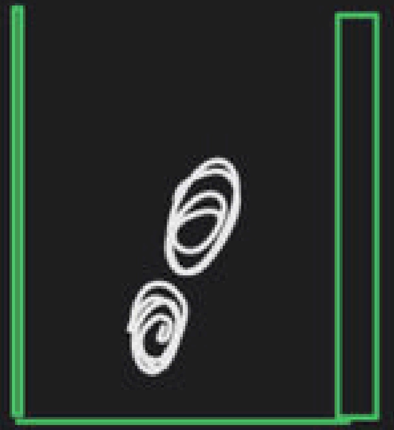


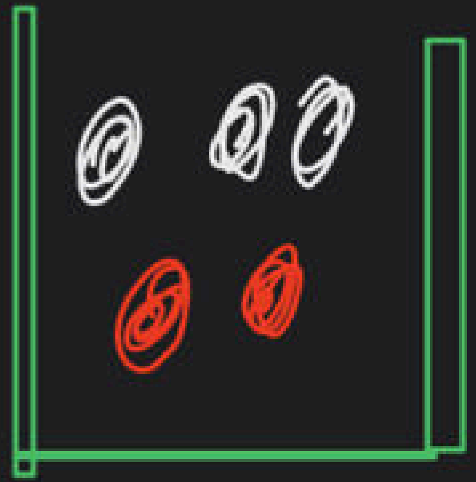
$P(W) \Rightarrow \frac{1}{2}$



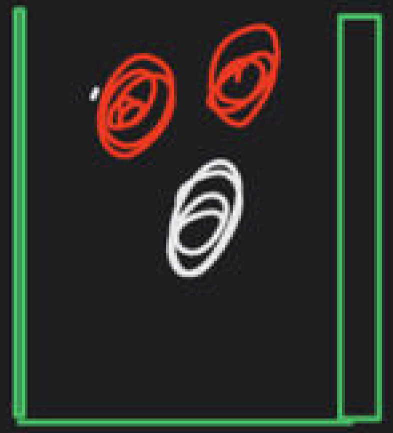


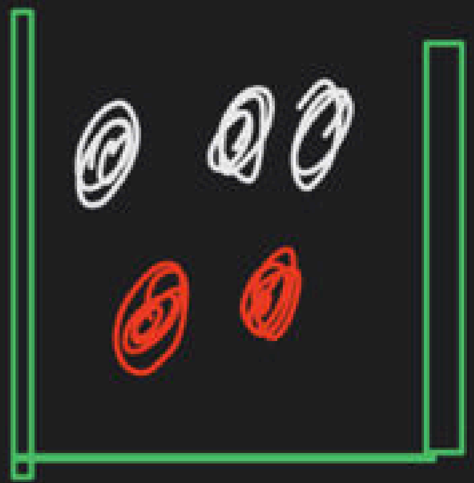
$\Rightarrow 1$





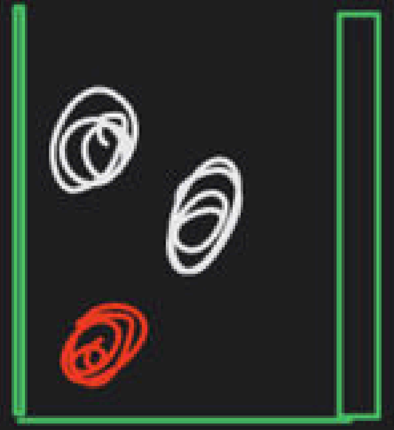
$\Rightarrow \frac{1}{3}$





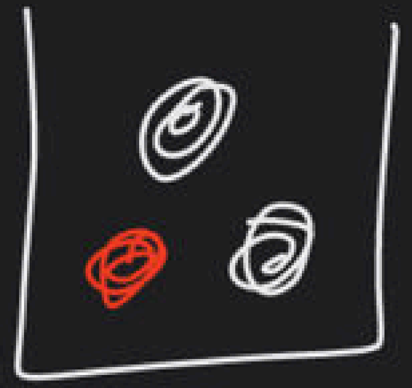
$\Rightarrow 1$

$\frac{2}{3}$



RW

$\frac{2}{3}$



WR

(all combinations)

QUESTION

Q. In a housing society, half of the families have a single child per family, while the remaining half have two children per family. The probability that a child picked at random, has a sibling is _________.

QUESTION

Q. If $P(X) = 1/4$, $P(Y) = 1/3$, and $P(X \cap Y) = 1/12$, then value of $P(Y/X)$ is

(A) $1/4$

(B) $4/25$

(C) $1/3$

(D) $29/50$

H.W

QUESTION

A signal which can be green or red with probability 4/5 and 1/5 respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is 3/4. If the signal received at station B is [illegible], then the probability that the original signal was [illegible]

GREEN

green =

$P(\text{GREEN}) = \frac{4}{5}$

$P(\text{Red}) = \frac{1}{5}$

$P(\text{correct}) = \frac{3}{4}$

(A) 3/5

(B) 6/7

(C) 20/23

(D) 9/20

original $\longrightarrow$ A $\longrightarrow$ B

original $\longleftarrow$ green fix

bayes THEOREM

$P(\text{GREEN}) = \frac{4}{5}$

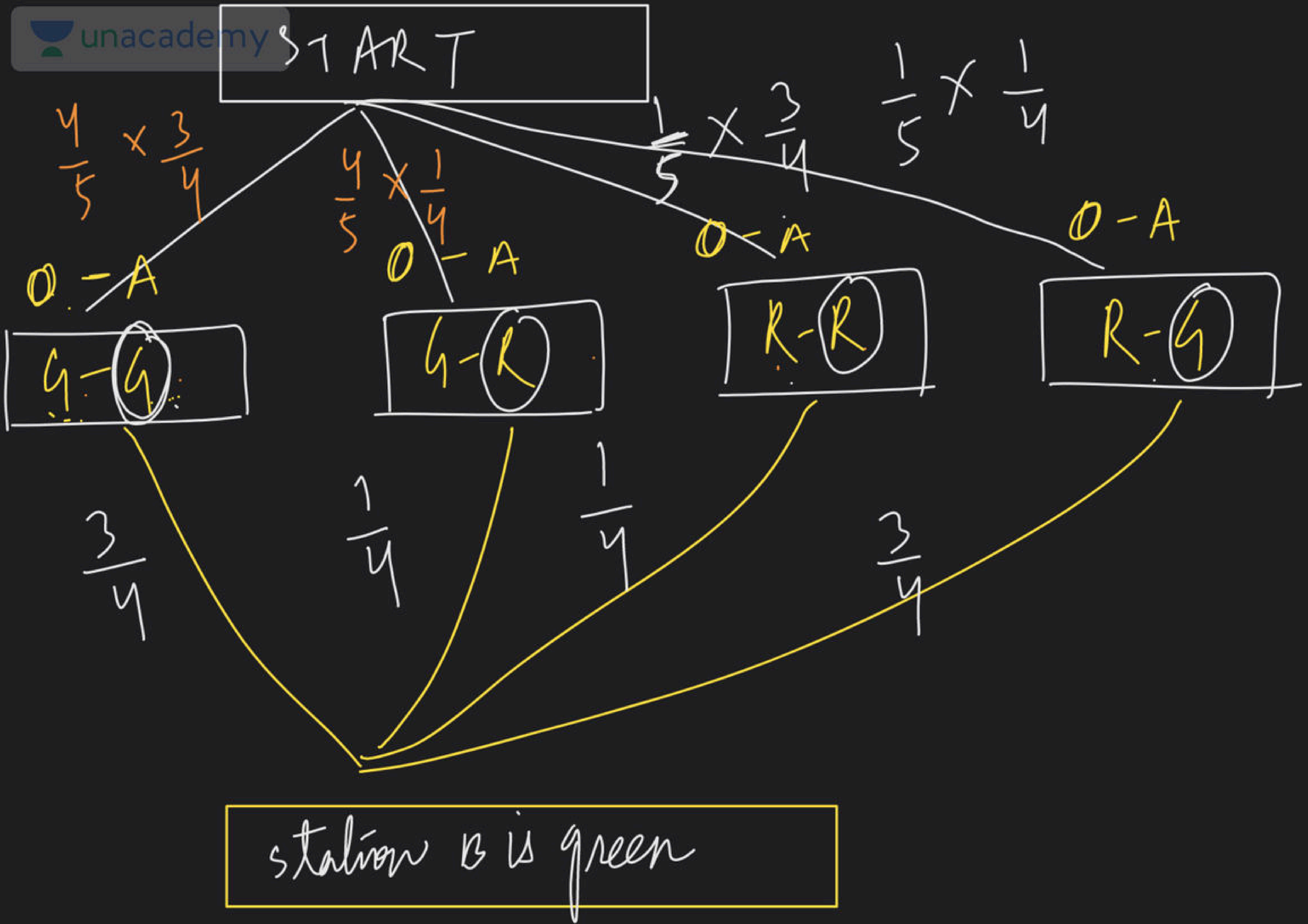
$P(\text{Red}) = \frac{1}{5}$

original	station A	station B
G ✓ →	G ✓	G
G ✓ →	R ✓	G
R ✓ →	R ✓	G
R ✓ →	G ✓	G

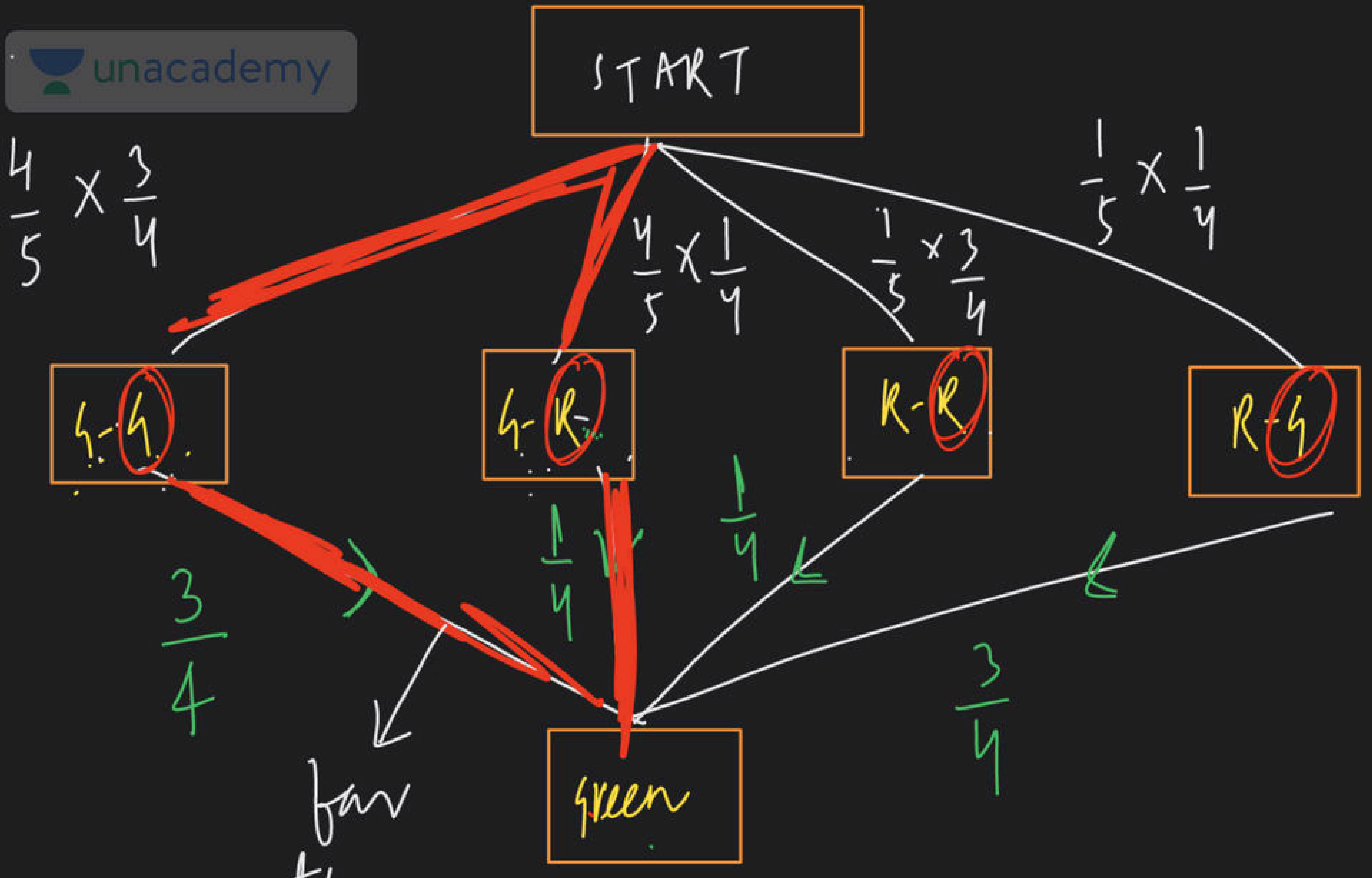
(bayes THEORM)

$= P\left(\frac{\text{original was green}}{\text{station B is green}}\right)$

original → station A → $\underbrace{\text{station B}}_{\text{green}}$



START
4/5 × 3/4
4/5 × 1/4
1/5 × 3/4
1/5 × 1/4
O - A
O - A
O - A
O - A
G - G
G - R
R - R
R - G
3/4
1/4
1/4
3/4
station B is green

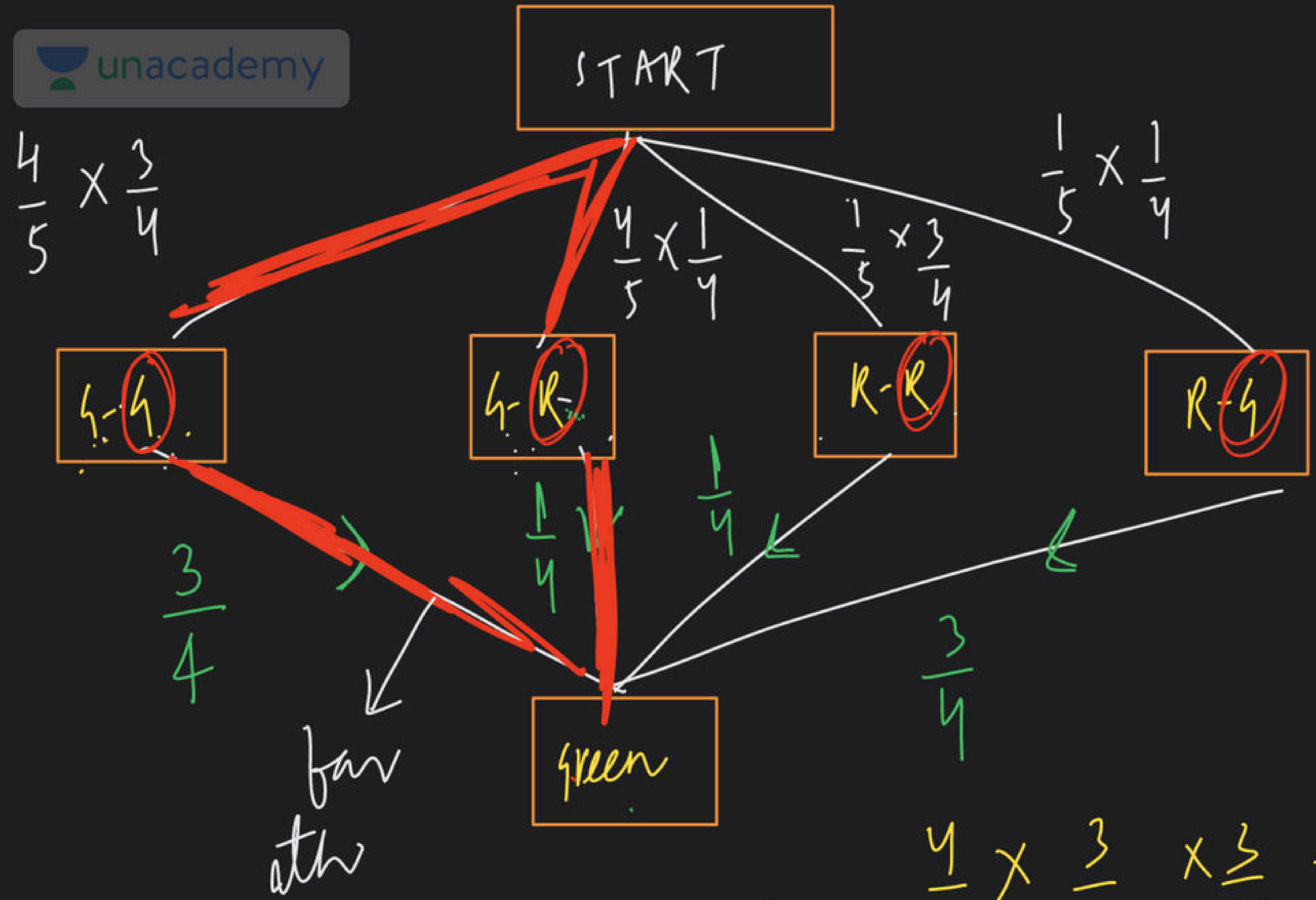


$$P\left(\frac{\text{original was green}}{\text{station B is green}}\right)$$

$$\Rightarrow \frac{\frac{4}{5} \times \frac{3}{4} \times \frac{3}{4} + \frac{4}{5} \times \frac{1}{4} \times \frac{1}{4}}{\text{Total}}$$

$$\Rightarrow \frac{20}{23} \text{ Ans}$$

$$\Rightarrow \frac{P(\text{original was green} \wedge \text{station B is green})}{P(\text{station B is green})}$$



$$= \frac{\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}+\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}}{\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}+\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{3}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4}\times\frac{3}{4}}$$

$$= \frac{\frac{\cancel{4}}{5} \times \frac{3}{\cancel{4}} \times \frac{3}{4} + \frac{\cancel{4}}{5} \times \frac{1}{\cancel{4}} \times \frac{1}{4}}{\frac{4}{5} \times \frac{3}{4} \times \frac{3}{4} + \frac{4}{5} \times \frac{1}{4} \times \frac{1}{4} + \frac{1}{5} \times \frac{3}{4} \times \frac{1}{4} + \frac{1}{5} \times \frac{1}{4} \times \frac{3}{4}}$$

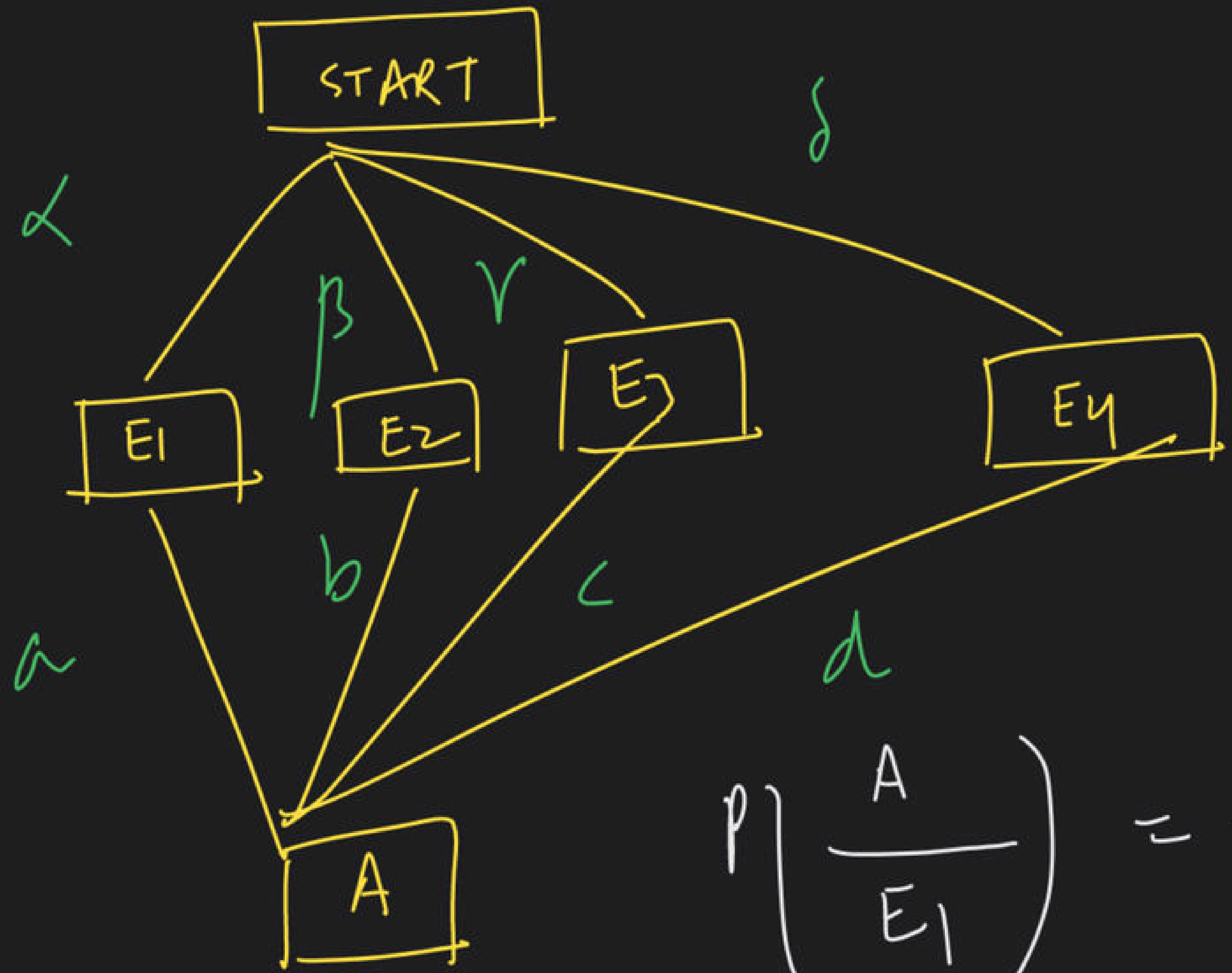
$$= \frac{\frac{9}{20} + \frac{1}{20}}{\frac{9}{20} + \frac{1}{20} + \frac{3}{80} + \frac{3}{80}}$$

$$= \frac{10}{20} \Big/ \frac{46}{80} = \frac{10 \times \cancel{80}^{\,4}}{\cancel{20} \times 46} = \frac{40}{46} = \frac{20}{23}$$

Ans.

saket - NCERT 12th

prob. (bayes THEOREM)



$$P\left(\frac{A}{E_1}\right) = \frac{a\alpha}{a\alpha + b\beta + c\gamma + d\delta}$$

(2) all PYQ solved

till bayes THEOREM

(4) DPP Pandc

27

DONE

✓ DPP – download = solve Pmd c

✓ Bayes THEOREM + NCERT ✓ (Revise)

QUESTION

(MSQ)

A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities 1/2, 1/4, 1/4. For the ship to be operational, $\boxed{\text{at least two of its engines must function.}}$ Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is/are true?

X = ship is operational

(A) $P\left(\frac{X_1^C}{X}\right) = \frac{3}{16}$

(B) P (exactly two engines of the ship functioning/X) = 7/8)

(C) $P\left(\frac{X}{X_2}\right) = \frac{5}{16}$

(D) $P\left(\frac{X}{X_1}\right) = \frac{7}{16}$

$E_1 \quad X_1 = \frac{1}{2}$

$E_2 \quad X_2 = \frac{1}{4}$

$E_3 \quad X_3 = \frac{1}{4}$

$\boxed{X = \text{ship is operational}}$

$X = \text{ship is working}$

E_1, E_2, E_3 Are Indep.

$P(E_1 \cap E_2 \cap E_3)$

$= P(E_1)\,P(E_2)\,P(E_3)$

Replace

$E_1 - X_1$

$E_2 - X_2$

$E_3 = X_3$

X_1

X_2

X_3

$X_1 \to X_2 \to X_3 \quad = X_1 \cap X_2 \cap X_3$ ✓

$X_1 \to X_2 \to \overline{X_3} \quad = X_1 \cap X_2 \cap \overline{X_3}$ ✓

$X_1 \to \overline{X_2} \to X_3 \quad = X_1 \cap \overline{X_2} \cap X_3$ ✓

$X_1 \to \overline{X_2} \to \overline{X_3} \quad = X_1 \cap \overline{X_2} \cap \overline{X_3}$ ✗

$\overline{X_1} \to X_2 \to X_3 \quad = \overline{X_1} \cap X_2 \cap X_3$ ✓

$\overline{X_1} \to X_2 \to \overline{X_3} \quad = \overline{X_1} \cap X_2 \cap \overline{X_3}$ ✗

$\overline{X_1} \to \overline{X_2} \to X_3 \quad = \overline{X_1} \cap \overline{X_2} \cap X_3$ ✗

$\overline{X_1} \to \overline{X_2} \to \overline{X_3} \quad = \overline{X_1} \cap \overline{X_2} \cap \overline{X_3}$ ✗

$X_1 \cap X_2 \cap X_3$

$\overline{X_1} \cap X_2 \cap X_3$

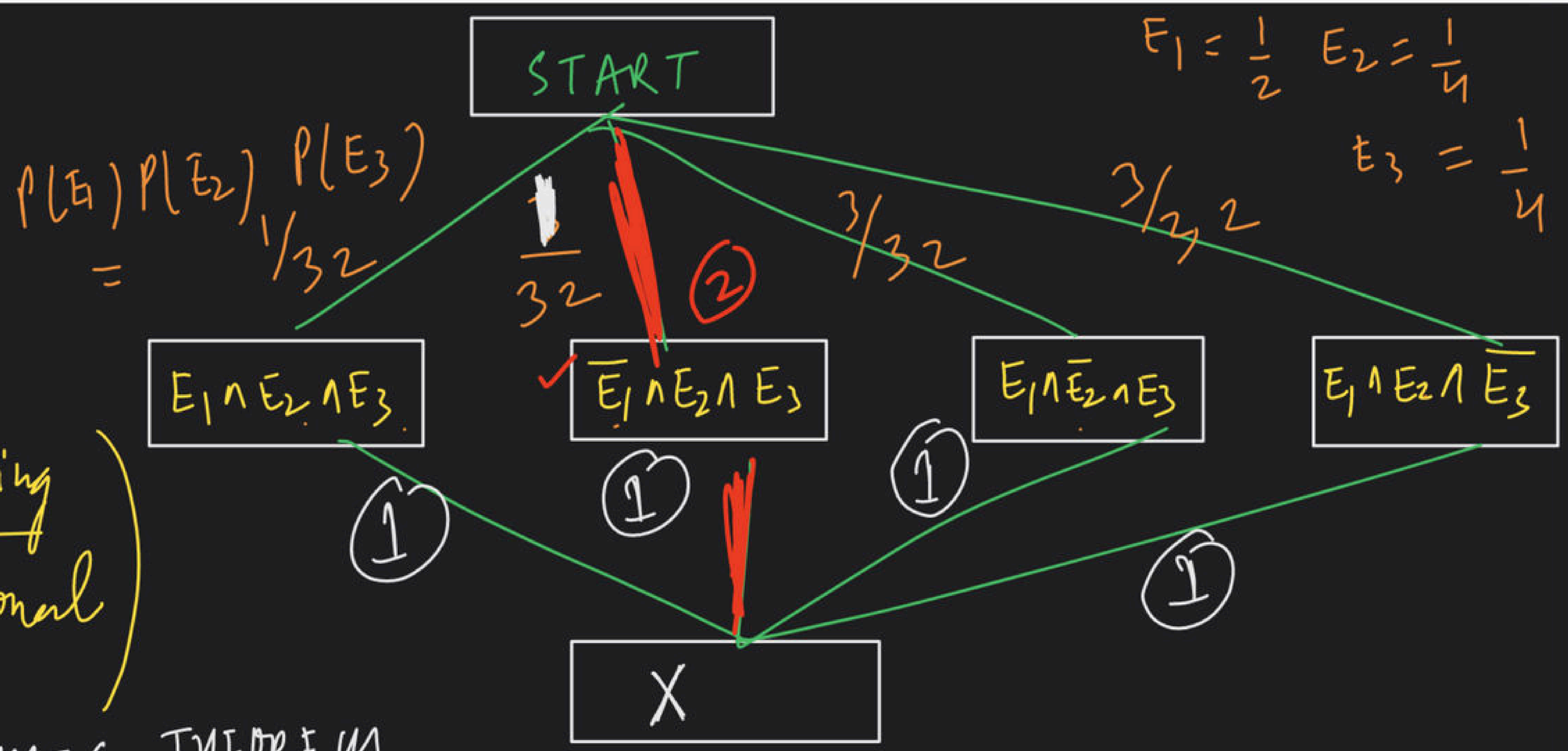
$X_1 \cap \overline{X_2} \cap X_3$

$X_1 \cap X_2 \cap \overline{X_3}$

$$P\left(\frac{x_1^c}{X}\right)$$

$E_1 = \frac{1}{2}$ $E_2 = \frac{1}{4}$

$E_3 = \frac{1}{4}$



ship is operational

$$= P\left(\frac{x_1 \text{ is not working}}{\text{ship is operational}}\right)$$

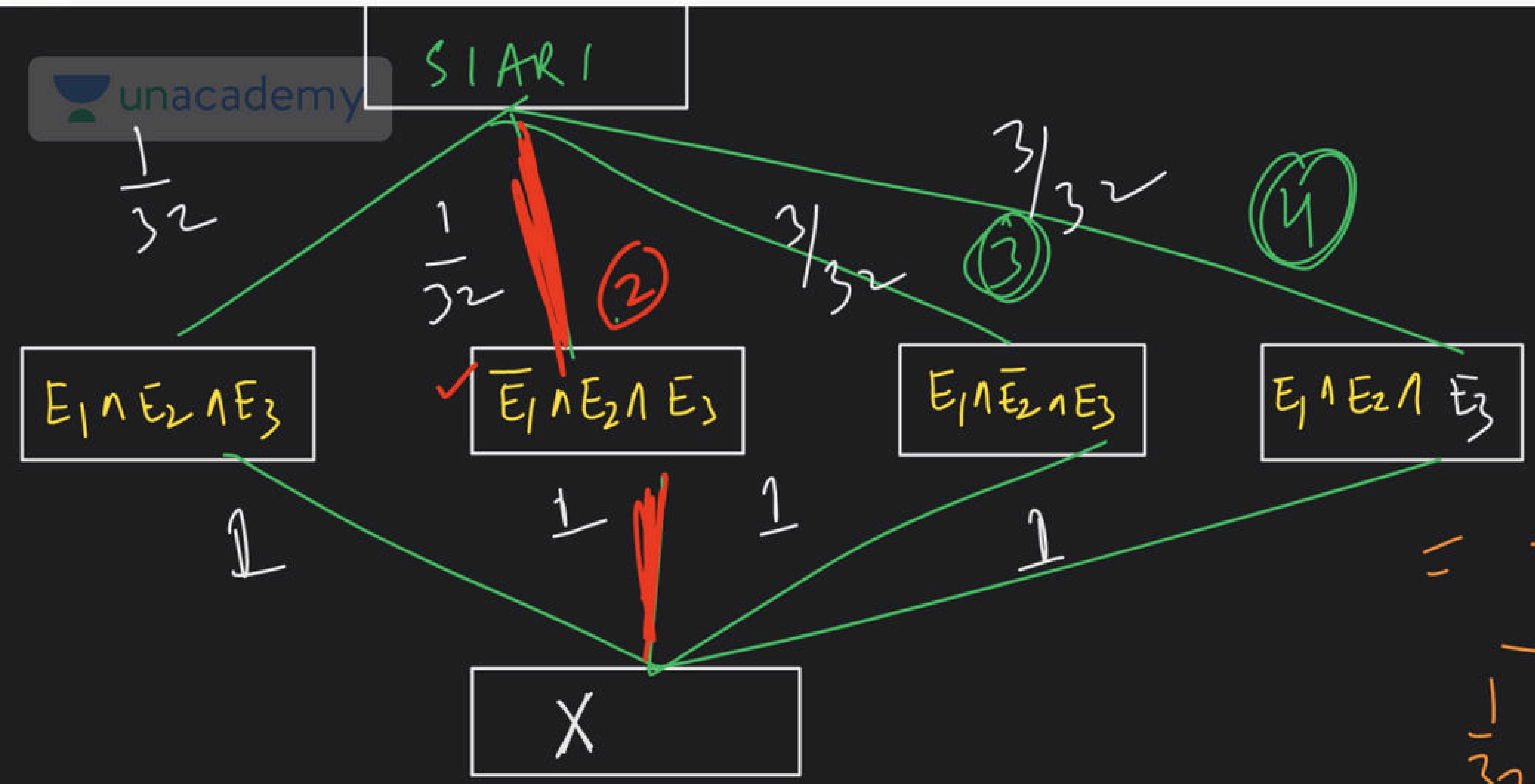
= bayes THEOREM

$$P\left(\frac{x_1^c}{X}\right) = \frac{P(x_1^c \cap X)}{P(X)} = \frac{\text{2nd path}}{\text{Total path}} \Rightarrow \frac{\frac{1}{32}}{\frac{1}{32} + \frac{1}{32} + \frac{3}{32} + \frac{3}{32}}$$

$$\Rightarrow \frac{1}{8}$$

X wrong

$$P\left(\frac{\text{exactly Two engines}}{\text{ship is operational}}\right)$$



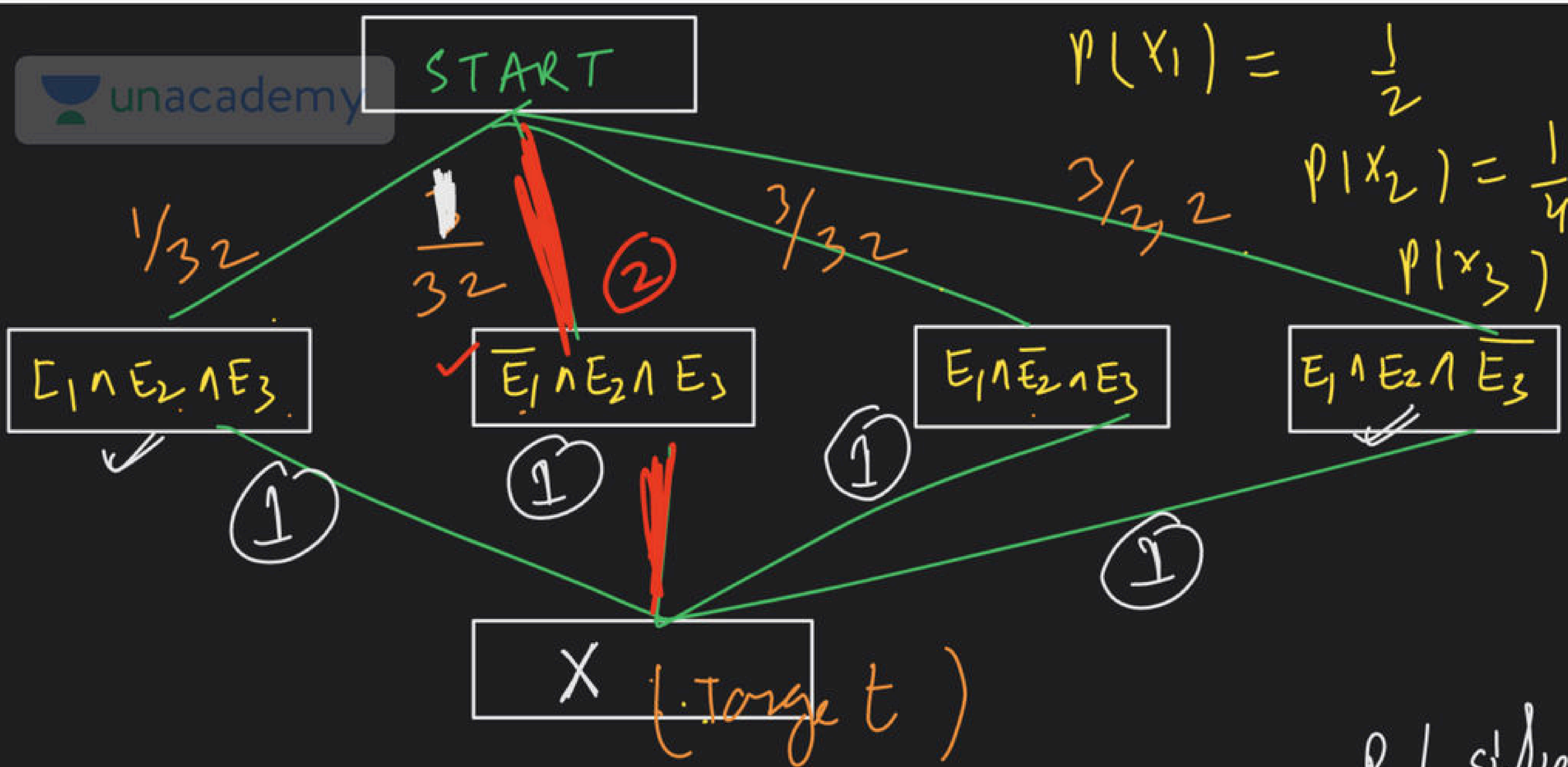
$$= \frac{(2) + (3) + (4) \text{ path for}}{\text{Total } (1) + (2) + (3) + (4)}$$

$$= \frac{\frac{1}{32} + \frac{3}{32} + \frac{3}{32}}{\frac{1}{32} + \frac{3}{32} + \frac{3}{32} + \frac{1}{32}}$$

$$P\left(\frac{\text{Exactly}}{X}\right) = \frac{P(\text{exactly Two engines working} \wedge \text{ship})}{P(\text{ship is operational})}$$

$$= \frac{7}{8} \checkmark$$

True (B) is correct



$P(X_1) = \frac{1}{2}$

$P(X_2) = \frac{1}{4}$

$P(X_3) = \frac{1}{4}$

$P\left(\frac{X}{X_2}\right)$ = conditional prob

$= P\left(\frac{\text{shlp is operation}}{E_2 \text{ is working}}\right)$

$$P\left(\frac{X}{X_2}\right) = \frac{P(X \cap X_2)}{P(X_2)} = \frac{P(\text{shlp is oper} \cap E_2 \text{ is working})}{\frac{1}{4}}$$

$$\Rightarrow \frac{① + ② + ④}{P(X_2)}$$

$$= \frac{\frac{1}{32} + \frac{1}{32} + \frac{3}{32}}{\frac{1}{4}} = \frac{5}{8}$$

C option wrong.

4)

$$P\left(\frac{X}{X_1}\right) = \frac{P(X \cap X_1)}{P(X_1)}$$

$$= \frac{1 + 3 + 4}{\frac{1}{2}}$$

$$= \frac{\frac{1}{32} + \frac{3}{32} + \frac{3}{32}}{\frac{1}{2}}$$

$$= \frac{7}{16}$$

D is correct

done

B

D

QUESTION

Parcels from sender S to receiver R pass sequentially through two post - offices.

Each post - office has a probability 1/5 of losing an incoming parcel, independently of all other parcels. Given that a parcel is lost, the probability that it was lost by the second post - office is ________.

X

QUESTION

An anti-aircraft gun can take a maximum of four shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first, second, third and fourth shot are 0.4, 0.3, 0.2, and 0.1, respectively. What is the probability that the gun hits the plane?

M.W

QUESTION

A box contains 2 black, 4 white and 3 red balls. One ball is drawn at random from the box and kept aside. From the remaining balls in the box, another ball is drawn at random and kept beside the first. This process is repeated till all the balls are drawn from the box. Find the probability that the balls drawn are in

X

QUESTION

Of the three independent events E_1, E_2 and E_3, the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is α. Let the probability P that none of events E_1, E_2 or E_3 occurs satisfy the equations $(\alpha - \beta).\,P = \alpha\beta$ and $(\beta - 3\alpha).\,P = 2\beta\alpha$. All the given probability are assumed to lie in the interval (0, 1).

Then, $\dfrac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$ is equal to:

X

QUESTION

Let $H_1, H_2, \dots H_n$ be mutually exclusive with $P(H_i) > 0$, $i = 1, 2, \dots, n$. Let E be any other event with $0 < P(E) < 1$.

Statement – 1: $P(H_i / E) > P(E / H_i).\,P(H_i)$ for $i = 1, 2, \dots, n$

Statement- 2: $\sum_{i=1}^{n} P(H_i) = 1$

A. Statement-1 is true, Statement-2 is true: Statement-2 is a correct explanation for Statement-1.

B. Statement-1 is true, Statement-2 is true: Statement-2 is not a correct explanation for Statement-1.

C. Statement-1 is true, Statement-2 is false.

D. statement -1 is false, statement-2 is true.

QUESTION

A person goes to office either by car, scoter, bus or train probability of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}$ and $\frac{1}{7}$, respectively. Probability that he reaches offices late, if he takes car, scooter, bus or train is $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}$ and $\frac{1}{9}$, respectively.

Given that he reached office in time, then what is the probability that he travelled by a car?

Office late $c - \frac{2}{9}$ $T \to \frac{1}{9}$

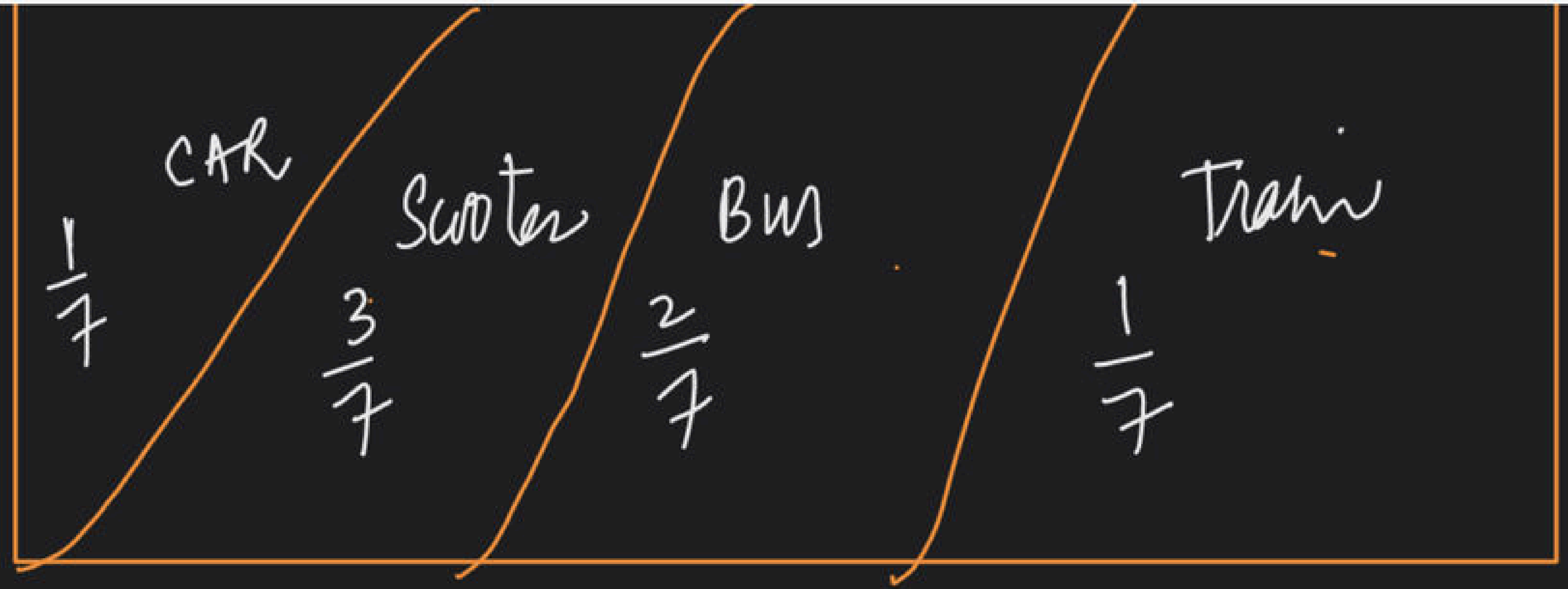
$s - \frac{1}{9}$

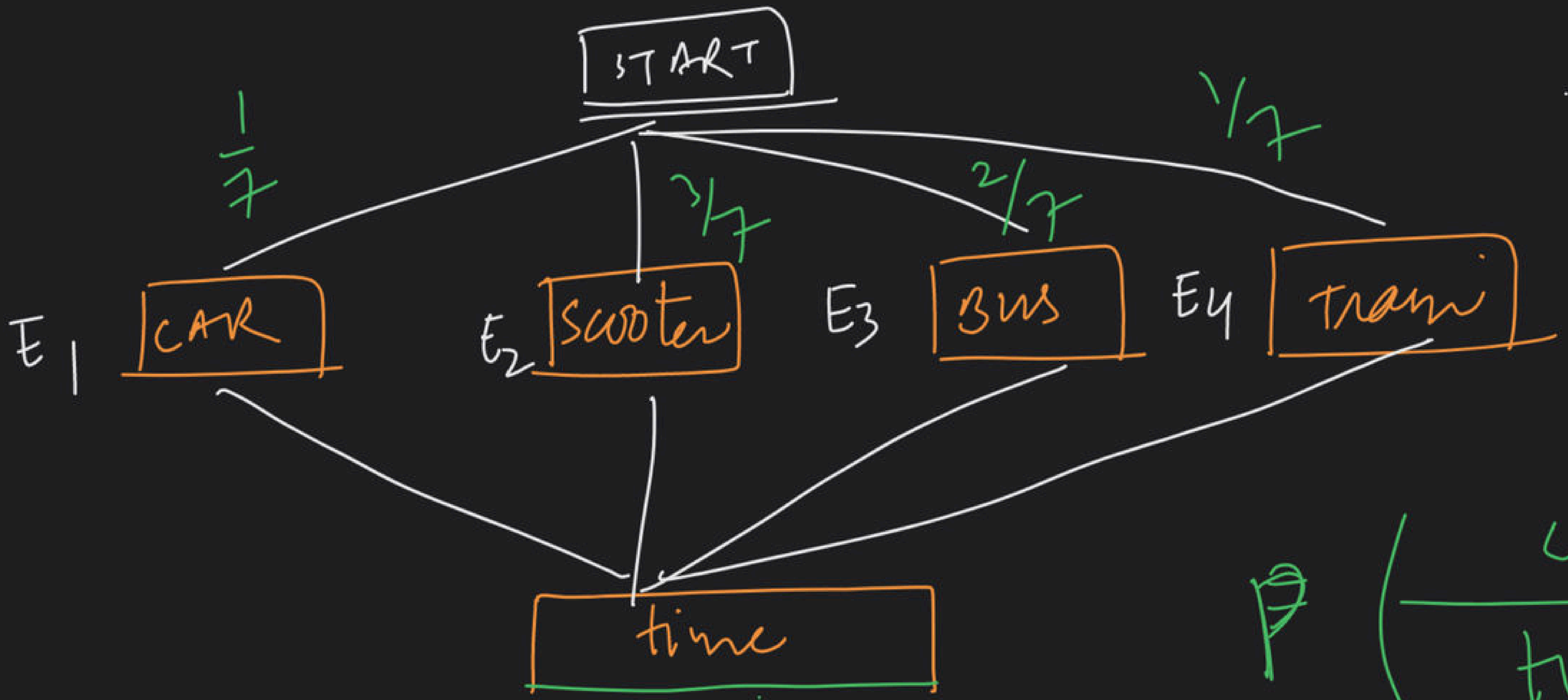
$b = \frac{4}{9}$

office late $c - \frac{2}{9}$ $T \to \frac{1}{9}$

$s - \frac{1}{9}$

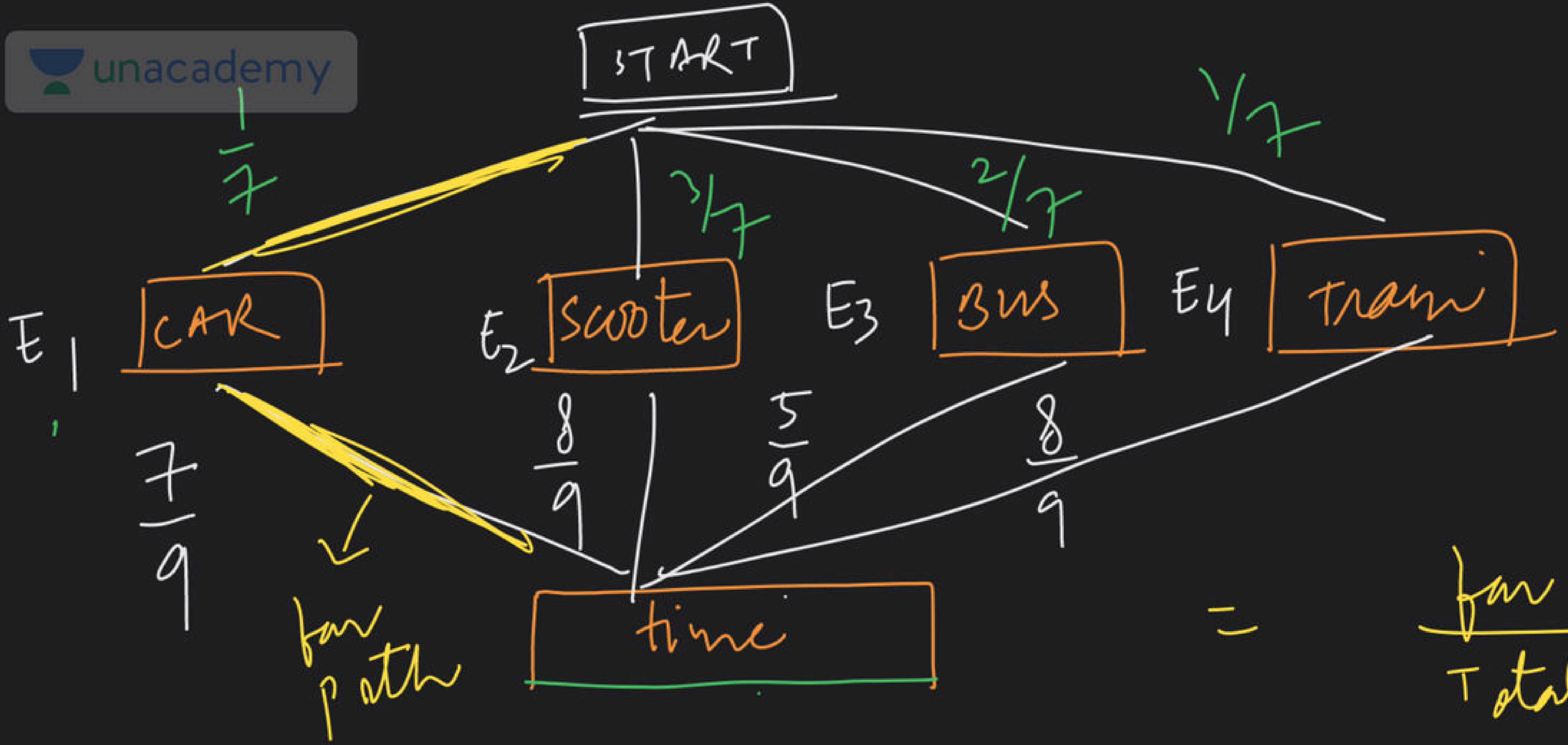
$b = \frac{4}{9}$





$$P\left(\frac{CAR}{time}\right) =$$

bayes THEOREM



$$P\left(\frac{CAR}{TIME}\right)$$

$$= \frac{\text{fav path}}{\text{Total path}}$$

$$P\left(\frac{CAR}{TIME}\right) \Rightarrow \frac{\frac{1}{7} \times \frac{7}{9}}{\frac{1}{7} \times \frac{7}{9} + \frac{3}{7} \times \frac{8}{9} + \frac{2}{7} \times \frac{5}{9} + \frac{1}{7} \times \frac{8}{9}} = \frac{1}{7}$$

(Inverse prob)

(bayes THEOREM)

QUESTION

bayes THEOREM.

In a test an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he make a guess is $\frac{1}{2}$ and the probability that he copies the answer $\frac{1}{6}$.

The probability that his answer is correct given that he copied it, is $\frac{1}{8}$.

Find the probability that he knew the answer to the question given that he correctly answered it.

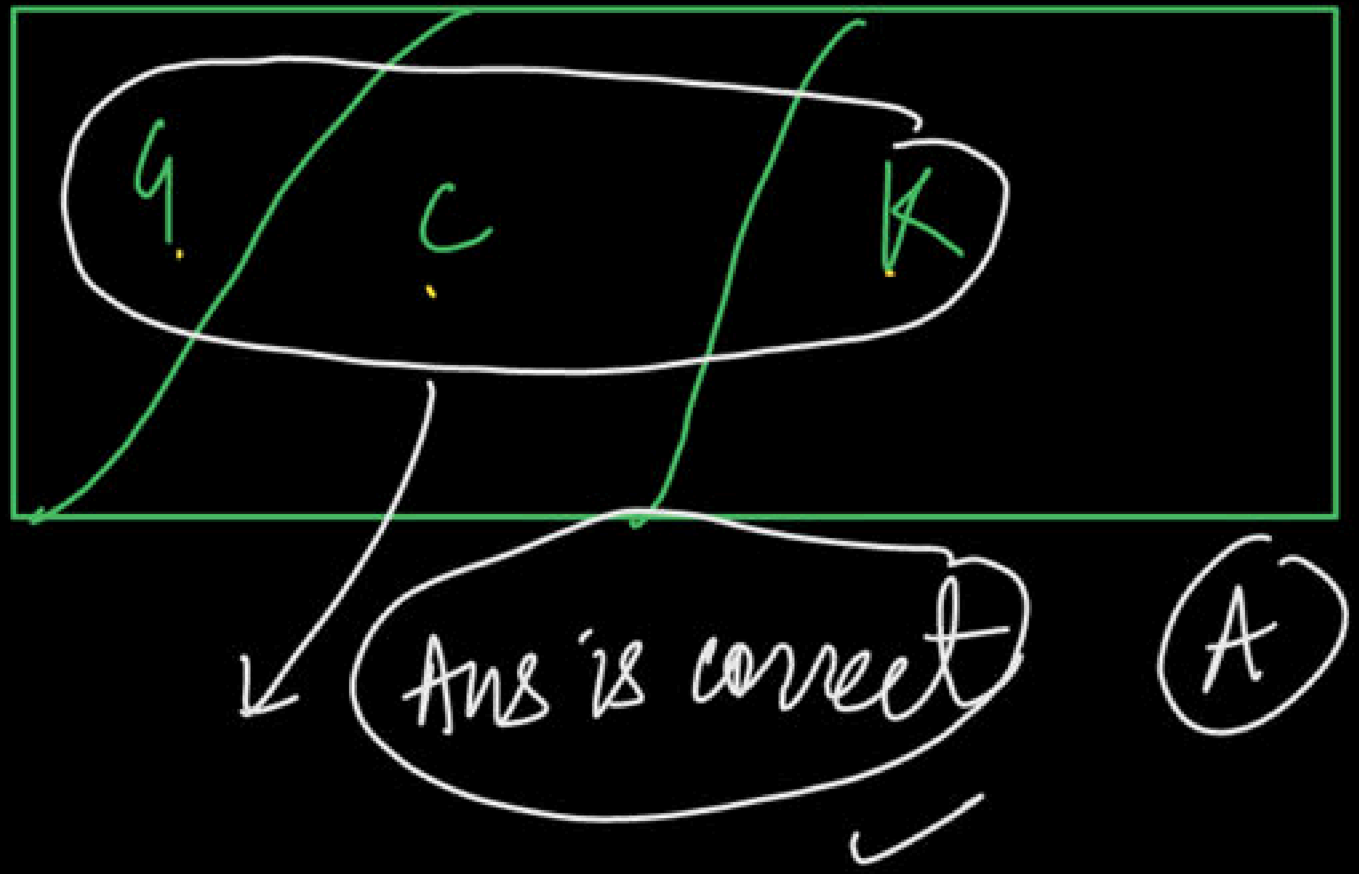
TEST

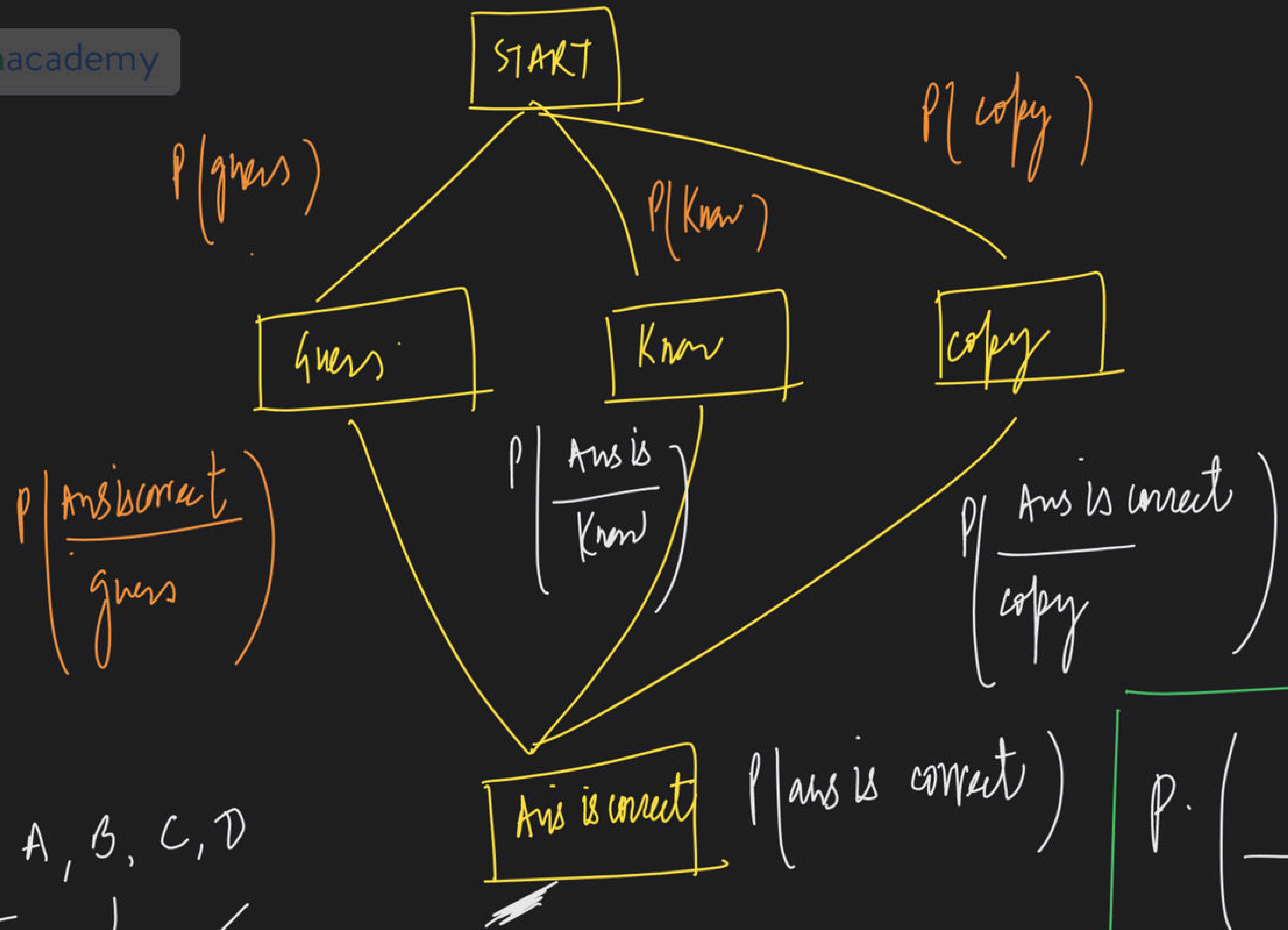
4 possible answer

GUESSES

copies

Knows.





Inverse prob.

⇑

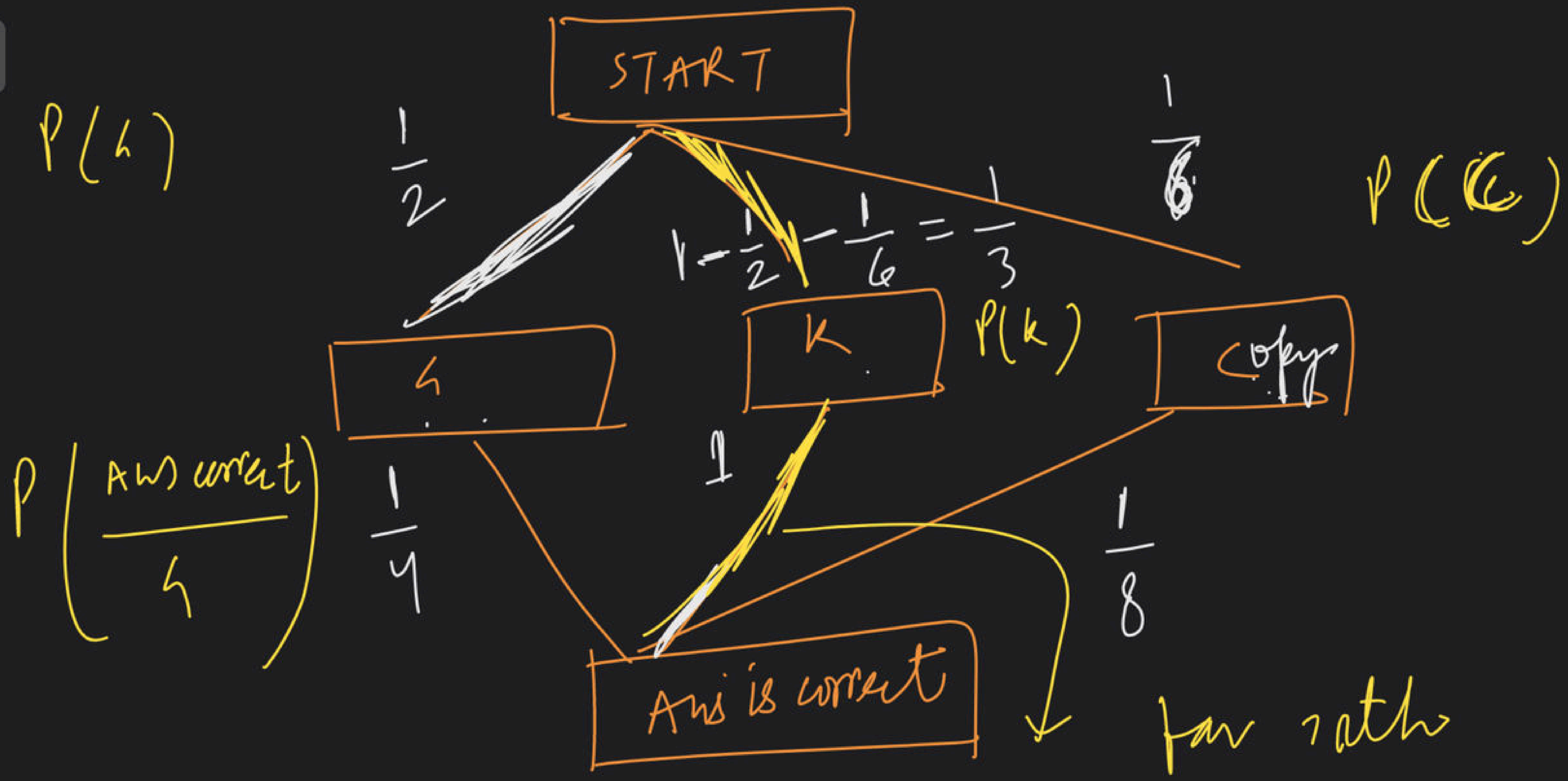
Bayes

↑ THEOREM

$$P\left(\frac{\text{Know}}{\text{Ans is correct}}\right)$$

A, B, C, D

$= \frac{1}{4}$ ✓



$$= \frac{\text{fav}}{\text{Total}} = P\left(\frac{\text{Ans correct}}{\text{Know}}\right) = \frac{1 \times \frac{1}{3}}{\frac{1}{2} \times \frac{1}{4} + 1 \times \frac{1}{3} + \frac{1}{6} \times \frac{1}{8}} = \frac{16}{23}$$

THANK YOU!

Here's to a cracking journey ahead!

✓ NCERT

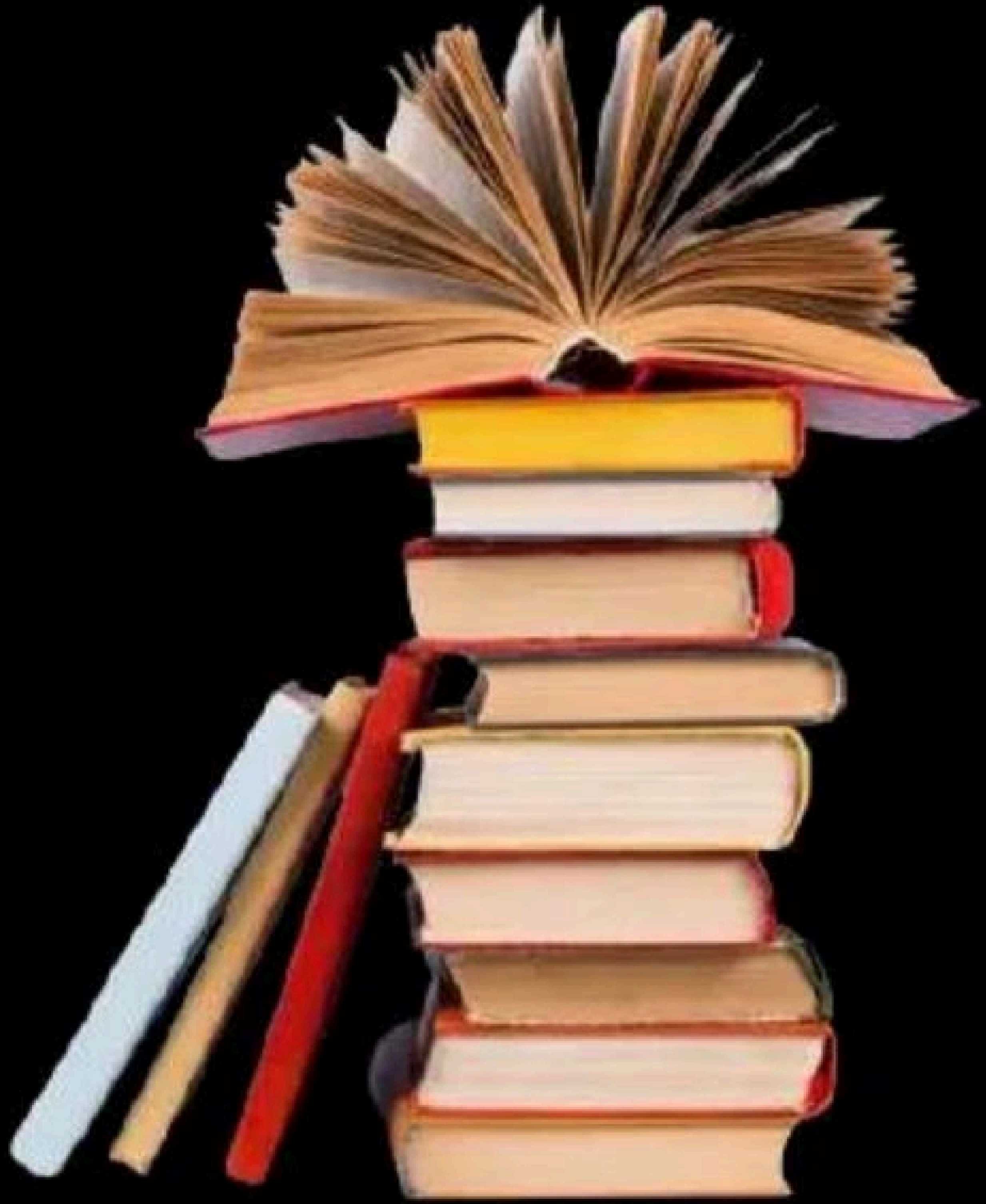
✓ Last lecture

✓ Random variable

NEXT topic

Topics

to be covered



1. Problem solving class_I

QUESTION

Q. Three Boxes B_1 B_2 B_3 contains balls

$$\begin{Bmatrix} B_1 \to 1W, 2B, 3R \\ B_2 \to 2W, 4B, 3R \\ B_3 \to 3W, 5B, 4R \end{Bmatrix}$$

Without replacement, if 2 balls are drawn from randomly selected box. Find the probability one of the ball drawn is white and other ball is red from box 2 order is specified.

QUESTION

Q. Let V_1 and V_2 be two urns box such that V_1 contains 3 white and 2 Red balls and V_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from V_1 and put into v_2. However, if tail appears then 2 balls are drawn at random from v_1 and put into v_2. Now one ball is drawn at random from v_2 given that the drawn ball v_2 is white then the probability that head appeared on the coin.

QUESTION

Q. In a housing society, half of the families have a single child per family, while the remaining half have two children per family. The probability that a child picked at random, has a sibling is ________.

QUESTION

Q. If $P(X) = 1/4$, $P(Y) = 1/3$, and $P(X \cap Y) = 1/12$, then value of $P(Y/X)$ is

(A) 1/4

(B) 4/25

(C) 1/3

(D) 29/50

THANK YOU!

Here's to a cracking journey ahead!