Ax

linear combination of columns of A

$Ax = 0$

$$\underline{c_1 a_1 + c_2 a_2 + c_3 a_3 = 0}$$

all $c_i = 0$ is called trivial sol^n

$Ax = 0$ has trivial sol^n then columns of A are

A) LD

B) LI

c) we can not say

$Ax = 0$ has trivial sol^n then columns of A are

A) LD

B) LI

~~c)~~ we can not say

$$\underset{\substack{\downarrow\\4}}{m} > \underset{\substack{\downarrow\\3}}{n}$$

$$A_{\underset{\substack{\downarrow\\4}}{m} \times \underset{\substack{\downarrow\\3}}{n}}$$

$$A_{4\times3}\, x = b$$

$$\begin{bmatrix} \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \ \\ \ \end{bmatrix}$$

ER9

T/F

$A_{4\times 3}\, x = b$ will always have sol^n for some A.

$\begin{bmatrix} \; & \; & \; \end{bmatrix}$

$\in \mathbb{R}^4$

You can never fill the space

you can have soln even if you do not fill the space

if

A

b

LI vectors

$$\begin{bmatrix} \vdots & \vdots \end{bmatrix} \qquad \overset{b}{\begin{bmatrix} \ \end{bmatrix}}$$

LI vectors

$$\begin{bmatrix} 0 & 0 \end{bmatrix}_{3\times 2} \begin{bmatrix} \quad \end{bmatrix}_{2\times 3} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}_{3\times 3}$$

LI

2 LI

$|A| = 0$

$$\begin{bmatrix} 2 & 2 & 4 \\ 3 & 3 & 6 \\ 4 & 5 & 10 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 4 \\ 3 & 3 & 6 \\ 4 & 5 & 10 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 8 \\ 3 & 3 & 11 \\ 4 & 5 & 19 \end{bmatrix}$$

No. of LI Columns = ?

= 2

$$\begin{bmatrix} 2 & 2 & 4 \\ 3 & 3 & 6 \\ 4 & 5 & 10 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 2 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{bmatrix}$$

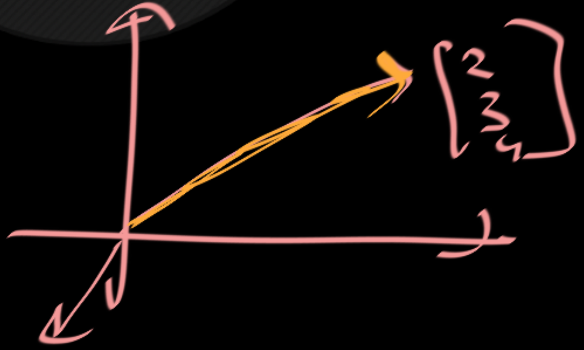
No. of LI Columns = 1

$$\begin{bmatrix} 2 & 2 & 4 \\ 3 & 3 & 6 \\ 4 & 5 & 10 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 3 & 3 \\ 4 & 5 \end{bmatrix}$$

R6

$$\begin{bmatrix} \\ \\ \\ \end{bmatrix} = c_1 \begin{bmatrix} \uparrow \\ v_1 \\ \downarrow \end{bmatrix} + c_2 \begin{bmatrix} \uparrow \\ v_2 \\ \downarrow \end{bmatrix}$$

$$\begin{bmatrix}2\\3\\4\end{bmatrix} = \overset{-5}{2}\begin{bmatrix}1\\0\\0\end{bmatrix} + \overset{3}{3}\begin{bmatrix}0\\1\\0\end{bmatrix} + \overset{9}{4}\begin{bmatrix}0\\0\\1\end{bmatrix}$$



$$\square + \square + 5\,\square + \square \cdots + \square = b$$

$$\square + \square + 3\,\square + \square \cdots + \square = b$$

$2V$

$\square + \square \ast + \square - - - + \square = b$

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Let $c_1 \ldots\ldots c_n$ be scalars, not all zero, such that $\sum_{i=1}^{n} c_i a_i = 0$ where a_i are column vectors in R^n.

a_i's are LD

Consider the set of linear equations

$Ax = b$

where $A = [a_1 \ldots\ldots a_n]$ and $b = \sum_{i=1}^{n} a_i$. The set of equations has

$a_1 + a_2 +$

A. a unique solution at $x = J_n$ where J_n denotes a n-dimensional vector of all 1.
B. no solution
C. infinitely many solutions
D. finitely many solutions

$\underset{A}{[\quad]}\underset{x}{[\quad]} = \underset{b}{[\quad]}$

Twinkle Chatterjee to Everyone 7:49 PM
But this question was so simple after your explanation

Harsh Koushle to Everyone 7:49 PM
no sir

Saurabh Gangwar to Everyone 7:49 PM
framing of this question was very good

Question

MSQ

3. Which of the following statement(s) is(are) true:

   a. Given a coefficient matrix $A_{m\times n}$, $m < n$, the number of solutions for $Ax = \vec{0}$ could either be zero or one.

   b. There exists a coefficient matrix $A_{m\times n}$, $\overset{4}{m} > \overset{3}{n}$, which doesn't give solution for any b .

      Some

   c. There exists a coefficient matrix $A_{m\times n}$, $m = n$, which doesn't give solution for some b.

   d. If a coefficient matrix $A_{m\times n}$ gives solution for every b, then $m = n$.

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}_{3\times 4} \underset{x}{\begin{bmatrix} \ \end{bmatrix}} = \underset{0}{\begin{bmatrix} \ \end{bmatrix}}$$

No such x exist ✗

only unique sol^n

$av_1 + bv_2 + cv_3 + v_4$

$$c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 v_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

there must be some Non trivial sol^n
(bcoz these are LD)

$\Rightarrow$ there are always many sol^{n}s

$$c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 v_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

there must be some Non trivial sol^n
(bcoz these are LD)

$\Rightarrow$ there are always many sol^{n}s

$$x = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}, \quad \begin{bmatrix} 2c_1 \\ 2c_2 \\ 2c_3 \\ 2c_4 \end{bmatrix}, \quad \begin{bmatrix} 3c_1 \\ 3c_2 \\ 3c_3 \\ 3c_4 \end{bmatrix}$$

b. There exists a coefficient matrix $A_{m\times n}$, $\overset{4}{m} > \overset{3}{n}$, which doesn't give solution for any b.

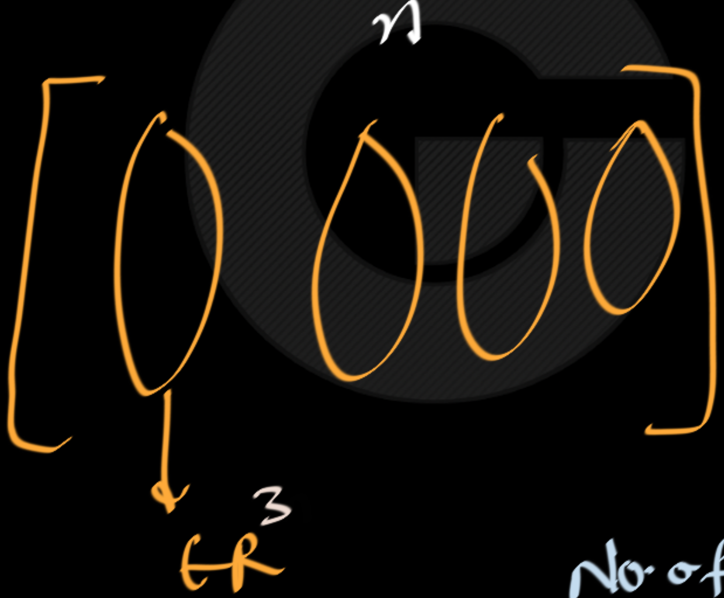
4×3

R^3

c. There exists a coefficient matrix $A_{m \times n}$, $m = n$, which doesn't give solution for some b.

d. If a coefficient matrix $A_{m \times n}$ gives solution for every b, then $m = n$.

$n \geq m$



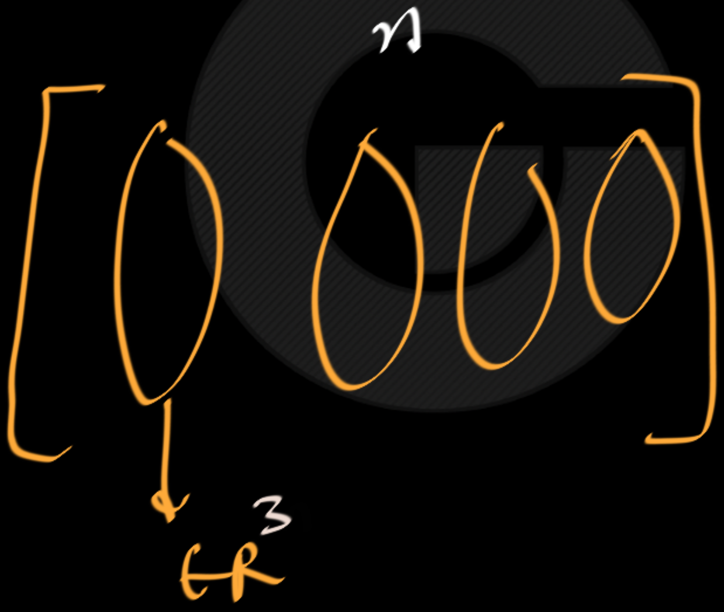
m Linearly indep. col^n.

$3 \times n$

No. of columns $\geq$ 3

$\Rightarrow n \geq 3$

d. If a coefficient matrix $A_{m \times n}$ gives solution for every b, then $m = n$.



$3 \times n$

$n \geq 3$

$n = 3, 4, 5$

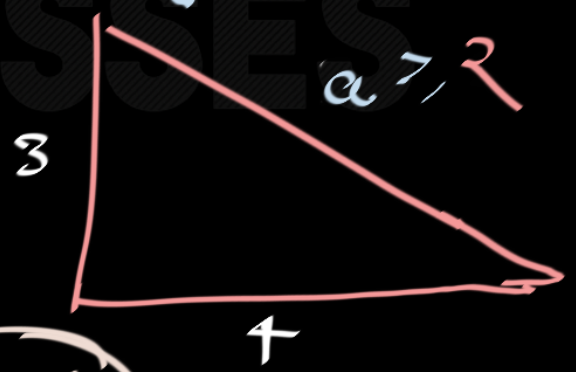
Twinkle Chatterjee to Everyone 8:04 PM

Why not exactly 3 columns?

$a \geq 3 \Rightarrow a = 3, 4, 5, 6$

side

$a = 3 \Rightarrow a \geq 3$



$a \geq 5$

$a \geq 6$

$a \geq 2$ ← True

$a = 2$ or $a=3$ or $a=4$ or $a=5$
T

or $a=6$ or $a=7$

True

Answer:

C

A is false, since the number of solutions can not be zero for the system, $Ax = \vec{0}$.

B is false. There always exists the zero column vector for which the given A would have solution.

C is true, since $m = n$ doesn't mean that there are n linearly independent vectors in $\mathbb{R}^n$.

D is false. If A gives solution for every b, then $m \leq n$. (We'll come back to this point while discussing Rank again.)

Question

4. Given the coefficient matrix $A_{m \times n}$, what can be said about the number of solutions if (zero/unique/infinite):

   a. $m = n$

   b. $m < n$

   c. $m > n$

Answer:

There isn't much we can say about the number of solutions just by looking at the dimensions of A. There can be zero or unique or infinite in all three above cases.

Question

Hit and trial

5. [Introduction to LA, Gilbert Strang, 5th ed., 2.1.7]

For the given matrix A:

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 5 \\ 2 & 3 & 8 \end{bmatrix}$$

a. Find the linear combination of first and second column which obtains the third column vector.

b. Check if there is a solution for the following b:

i. $b = \begin{bmatrix} 2 & 3 & 5 \end{bmatrix}^T$

ii. $b = \begin{bmatrix} 4 & 6 & 11 \end{bmatrix}^T$

$\begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$ or $\begin{bmatrix} 2 & 3 & 5 \end{bmatrix}^T$

$$\begin{bmatrix}1 & 1\\1 & 2\\2 & 3\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{matrix}3\\5\\8\end{matrix} \quad \leftarrow Ax = b$$

$\downarrow$

Sol^n for x

Answer:

a.

$$1 \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + 2 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 8 \end{bmatrix}$$

b.

i. Yes. $X = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$

ii. No.

Question

6. The given matrix has solution for:

$$\begin{array}{c} \begin{array}{ccc} & 2 & \end{array} \\ \begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 5 \\ 2 & 3 & 8 \end{bmatrix} \end{array}$$

a. All vectors b in $\mathbb{R}^3$.

b. No vector b in $\mathbb{R}^3$.

c. Some vectors b in $\mathbb{R}^3$.

d. Some vectors b in $\mathbb{R}^2$.

Answer:

C. Only for some vectors in $\mathbb{R}^3$.

Question

7. Check the number of solution (zero/unique/infinite) for the following system of linear equations

$$A = \begin{bmatrix} 1 & 4 & 3 \\ 3 & 8 & 9 \\ 0 & 0 & 0 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

since there are

$$\begin{bmatrix} 1 & 1 & 3 \\ 3 & 2 & 9 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Since these are LD hence there must be some non trivial soln.

$$A = \begin{bmatrix} 1 & 4 & 3 \\ 3 & 8 & 9 \\ 0 & 0 & 0 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

inf.

Answer:

There can be more than one solutions for the above system. Solving the above system is equivalent to checking if the columns of A are linearly independent/dependent. The columns are linearly dependent. The third column is a multiple of the first column.

Question

8. The following matrix A, doesn't give solution for any b, since its columns are linearly dependent. True/False?

$$\begin{bmatrix} 1 & 3 & 0 \\ 4 & 8 & 0 \\ 3 & 9 & 0 \end{bmatrix}$$

false

Twinkle Chatterjee to Everyone 8:25 PM

T

Sir if in question asked -> there is a sol or not, do we require the information that there is any LC or not?

Answer:

False. For example, $b = \begin{bmatrix} 1 & 4 & 3 \end{bmatrix}^T$ and get the solution as $X = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$.

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Question

9. Find a b such that the system $Ax = b$ has no solution.

$$\begin{bmatrix} 1 & 3 & 0 \\ 4 & 8 & 0 \\ 3 & 9 & 0 \end{bmatrix} \qquad \begin{bmatrix} 4 \\ 12 \\ 13 \end{bmatrix}$$

Answer:

$$b = \begin{bmatrix} 4 & 12 & 13 \end{bmatrix}.$$

Question

10. [https://www.uvm.edu/~mrombach/124_rombach_midterm1_sols.pdf Q5]

$$2x + 2z = 4$$
$$y = 3$$
$$2x + y + 3z = 8$$

a. Represent the system of Linear Equations in $Ax = b$ form.

b. Check if the columns of A are linearly independent.

c. Try to find the solution to the system.

[https://www.uvm.edu/~mrombach/124_rombach_midterm1_sols.pdf Q5]

Question

11. [https://math.berkeley.edu/~nikhil/courses/54.f18/midterm1sol.pdf Q2]

Give an example of each of the following, explaining why it has the required property, or explain why no such example exists.

a. A linear system with 3 equations in 2 variables which has solution. for some b.

b. A linear system with 2 equations in 3 variables which has no solution for any b

c. A linear system with 2 equations in 3 variables which has solution for no b ($b \neq \vec{0}$).

there exist

c. A linear system with 2 equations in 3 variables which ~~has solution for no~~ b ($b \neq \vec{0}$).

$$\left[\begin{array}{ccc|c} \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \\ & & & b \end{array}\right]$$

$\in R^2$

F a) does not have sol^n for any b.

T b) has sol^n for all the b's

A linear system with 3 equations in 2 variables which has solution.

$$\begin{bmatrix} 1 & \\ 2 & \\ 3 & \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$