System of linear equations

- Why solve System of linear equations ?
- Geometric Interpretation
- Understanding Ax = b intuitively
- A step by step method to find Solution for Ax=b (Gaussian Elimination)
    - Rank
    - Parametric form of solution

Question

Suppose a matrix $A_{20\times 50}$ has 10 pivot column, then –

- Number of Linearly independent solutions in $Ax = 0$?
- Number of Linearly independent columns in A ?

Question

Suppose a matrix $A_{20\times 50}$ has 10 pivot column, then –

- Number of Linearly independent solutions in $Ax = 0$?
- Number of Linearly independent columns in A ? $= 10$

free variable $= 50 - 10 = \underline{\underline{40}}$

GATE 2021

Suppose that P is a 4×5 matrix such that every solution of the equation $\mathrm{Px}=0$ is a scalar multiple of $\begin{bmatrix} 2 & 5 & 4 & 3 & 1 \end{bmatrix}^T$. The rank of P is ________

GATE 2021

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Sol^n of $Px = 0$

$K \begin{bmatrix} 2 \\ 5 \\ 4 \\ 3 \\ 1 \end{bmatrix}$

No. of free variables $= 1$

No. of pivots $= 4$

Question:

Consider a matrix A_{8x9}.

Suppose a system AX = 0 has some nontrivial solution such that solutions are linear combination of 3 linearly independent vectors.

What can you say about :

- Number of linearly independent columns in A $\Rightarrow 6$

free variables in $A = 3$

pivot variables $= 9 - 3 = 6$

Non Homogeneous equations solutions' Parametric form

We have already solved following homogeneous equation

Example Let

$$A = \begin{array}{cccc} x & y & z & w \\ \end{array}$$
$$A = \left[\begin{array}{cccc} 2 & 1 & -1 & 2 \\ 1 & 1 & 0 & 3 \end{array}\right]$$

Solve $A\mathbf{x} = 0$

Question

Now, Lets Solve Non homogeneous equation

Example Let

$$A = \begin{bmatrix} 2 & 1 & -1 & 2 \\ 1 & 1 & 0 & 3 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

Solve $A\mathbf{x} = \mathbf{b}$.

$$A = \begin{bmatrix} 2 & 1 & -1 & 2 \\ 1 & 1 & 0 & 3 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 2 & 1 & -1 & 2 & 0 \\ 1 & 1 & 0 & 3 & -2 \end{array}\right]$$

$R_2 \to 2R_2 - R_1$

$$\left[\begin{array}{cccc|c} 2 & 1 & -1 & 2 & 0 \\ 0 & 1 & 1 & 4 & -4 \end{array}\right]$$

$$\begin{array}{ccccc} x & y & z & w & \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 2 & 1 & -1 & 2 & 6 \\ 0 & 1 & 1 & 4 & -4 \end{array}\right]$$

$$2x \underline{-4-s} -4t -\underline{s} + 2t = 0$$

$$2x = 2s + 4 + 2t$$

free : z, w

$z = s$

$w = t$

$$y + s + 4t = -4$$

$$y = -4 - s - 4t$$

$$x = \begin{bmatrix} 2+s+t \\ -4-s-4t \\ s \\ t \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} s+t \\ -s-4t \\ s \\ t \end{bmatrix}$$

p

$s=0$

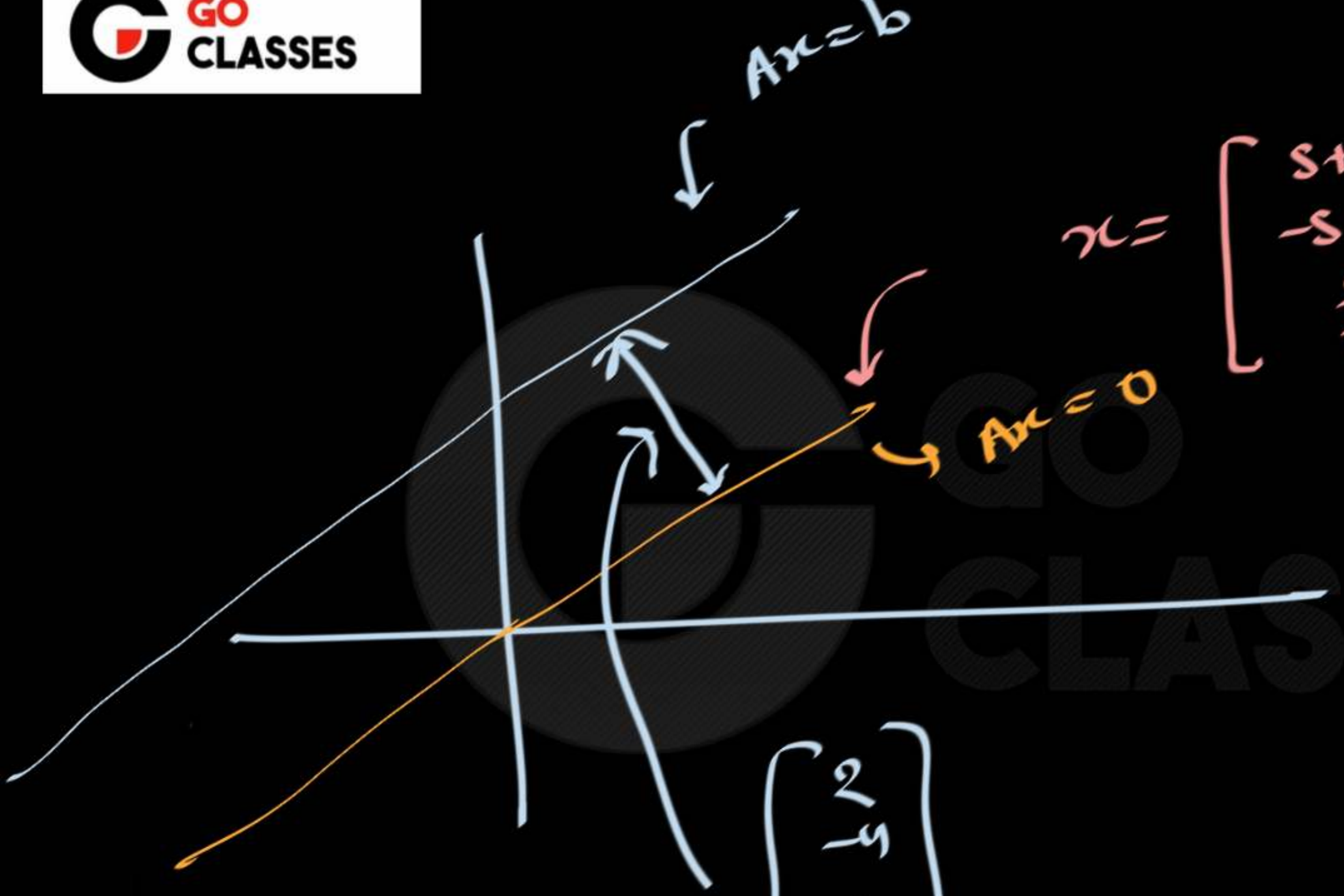
$t=0$

$$\mathbf{x} = \begin{bmatrix} s+t \\ -s-4t \\ s \\ t \end{bmatrix}$$

$$x = \begin{bmatrix} 2+s+t \\ -4-s-4t \\ s \\ t \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} s+t \\ -s-4t \\ s \\ t \end{bmatrix}$$

$s=0$
$t=0$

Answer for $Ax=b$



$$x = \begin{bmatrix} s+t \\ -s-4t \\ s \\ t \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ -4 \\ 0 \\ 0 \end{bmatrix}$$

$$x = \begin{bmatrix} 2+s+t \\ -4-s-4t \\ s \\ t \end{bmatrix} = \underset{\substack{s=0 \\ t=0}}{\begin{bmatrix} 2 \\ -4 \\ 0 \\ 0 \end{bmatrix}} + \begin{bmatrix} s+t \\ -s-4t \\ s \\ t \end{bmatrix} = \underset{\substack{s=0 \\ t=0}}{\begin{bmatrix} 2 \\ -4 \\ 0 \\ 0 \end{bmatrix}} + s\underset{\substack{s=1 \\ t=0}}{\begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}} + t\underset{\substack{t=1 \\ s=0}}{\begin{bmatrix} 1 \\ -4 \\ 0 \\ 1 \end{bmatrix}}$$

Common mistake

$$x = \begin{bmatrix} 2+s+t \\ -4-s-4t \\ s \\ t \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} 3 \\ -5 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 3 \\ -8 \\ 0 \\ 1 \end{bmatrix}$$

$s=0$, $t=0$

$s=1$, $t=0$

$t=1$, $s=0$

$$\mathbf{x} = \begin{bmatrix} 2+s+t \\ -4-s-4t \\ s \\ t \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} 1 \\ -4 \\ 0 \\ 1 \end{bmatrix} t$$

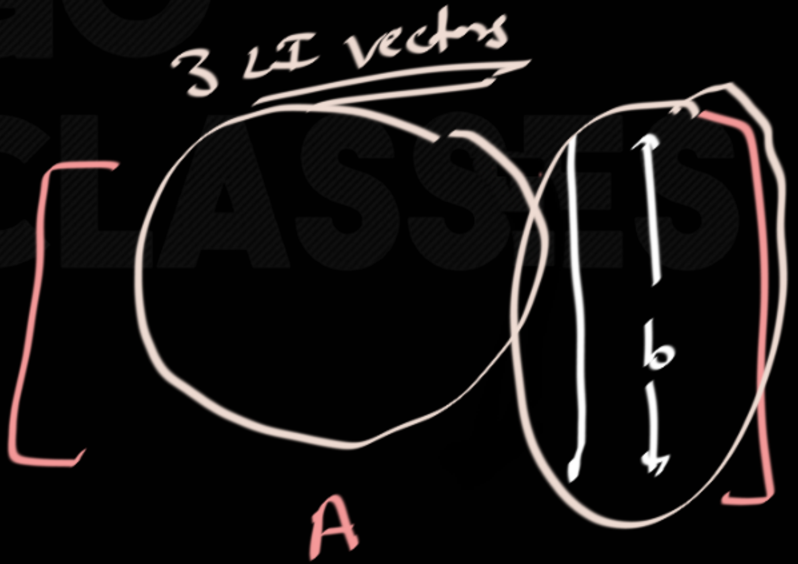
Now we know how to solve system of linear equations.

Lets now solve few more conceptual questions specially with rank.

Question

$Rank(A) \neq Rank(A\,|b)$ then system has

- No solution
- Unique Solution
- Infinite Solution



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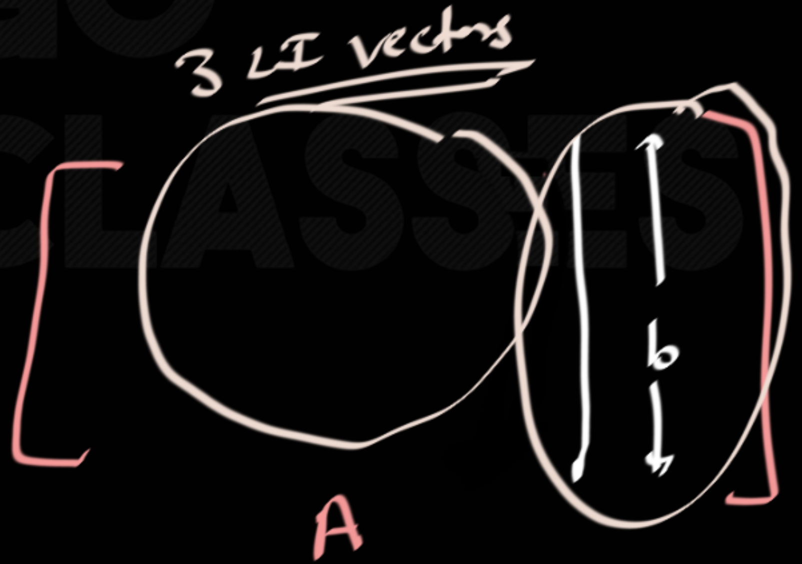

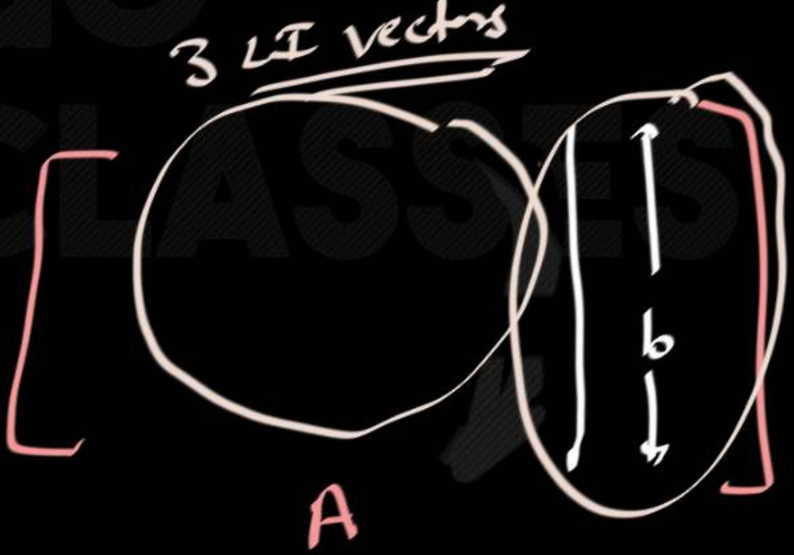

$\text{Rank}(A) \neq \text{Rank}(A|b)$

$\Updownarrow$

b is Not a LC of Columns of A

$\text{Rank}(A) = \text{Rank}(A|b) \iff$ b is a L.C. of vectors in A

$$\left[\begin{array}{c|c} A & b \end{array}\right]$$

$$\text{Rank}(A|b) = \text{Rank}(A)$$

or

$$\text{Rank}(A|b) = 1 + \text{Rank}(A)$$

$$\left[\begin{array}{c|c} A & b \end{array}\right]$$

$\text{Rank}(A) = \text{Rank}(A|b) \iff$ b is a LC of columns of A

sol^n exist

$\Updownarrow$

sol^n does not exist

$\Updownarrow$

$\text{Rank}(A) \neq \text{Rank}(A|b) \iff$ b is not a LC of columns of A

$$\left[\begin{array}{c|c} A & \uparrow \\ & b \\ & \downarrow \\ 0\ 0\ 0\ 0\ 0 & \text{Nonzero} \end{array}\right]$$

$[0\,0\,0\ \ldots \mid \text{Nonzero}]$ does not exist?

$\text{Rank}(A) = \text{Rank}(A|b)$

$$\left[\begin{array}{cccc|c} \square & * & * & * & * \\ 0 & 0 & \square & * & * \\ 0 & 0 & 0 & 0 & \square \end{array}\right]$$

zero

$\text{Rank}(A) = \text{Rank}(A|b)$

non zero

$\text{Rank}(A|b) = 1 + \text{Rank}(A)$

$\text{Rank}(A|b) \neq \text{Rank}(A)$

$$\left[\begin{array}{ccccc|c} 0 & 0 & 0 & 0 & 0 & \boxed{\cdot} \end{array}\right]$$

$$\text{Rank}(A) = \text{Rank}(A|b)$$

$$\left[\begin{array}{cccccc|c} 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\text{Rank}(A) = \text{Rank}(A|b)$$

$$\Updownarrow$$

b is a LC of columns of A

$$\Updownarrow$$

$[0\ 0\ 0\ \ldots\ldots\ |\text{Nonzero}]$ does not exist

$\text{Rank}(A) \neq \text{Rank}(A|b)$

$\Downarrow$

b is NOT a LC of columns of A

$\Downarrow$

$[0 \; 0 \; 0 \; \ldots\ldots \; | \text{Nonzero}]$ exists

Question

$Rank(A) = Rank(A\,|b)$ then system has

- No solution
- Unique Solution
- Infinite Solution

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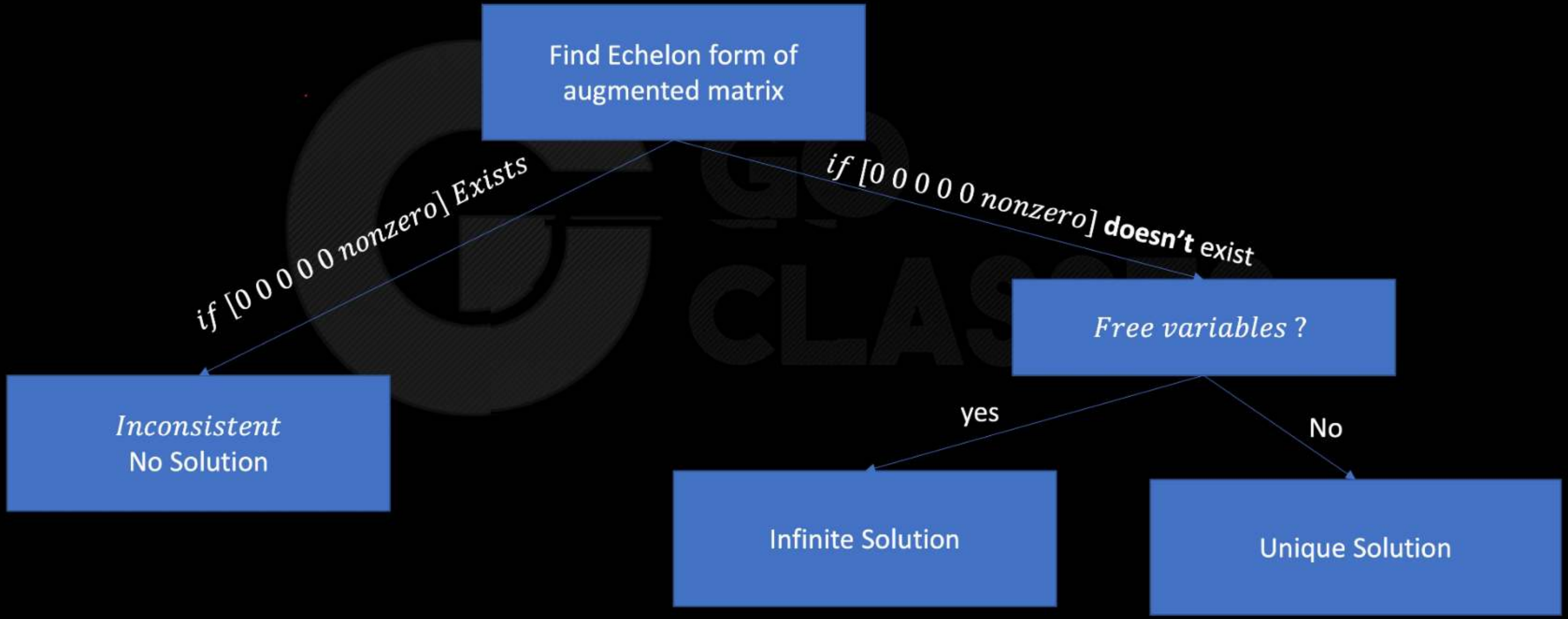
System of Linear Eqns flowchart (Ax = b)



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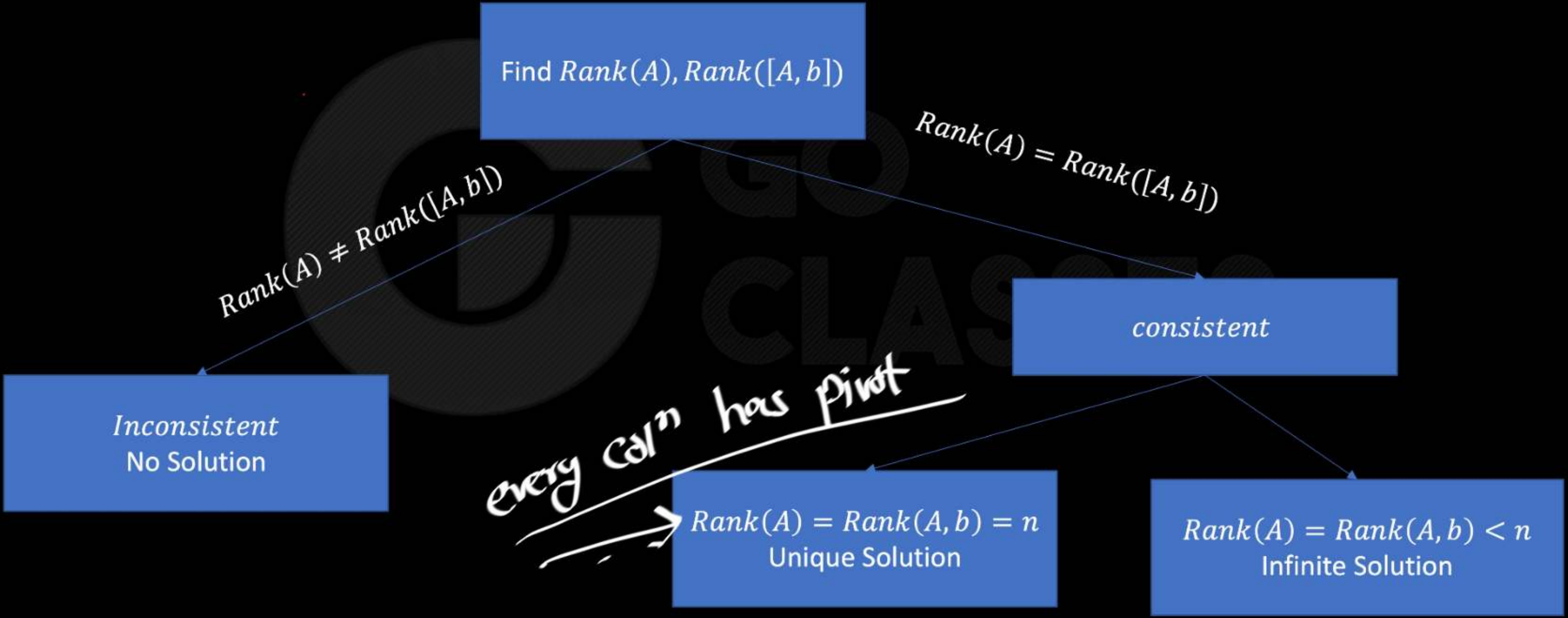

Question

Consider a matrix A of $m \times n$ size.

Let $rank(A) = m = n$.

Which of the following is true about solutions of $Ax = b$?

a) There are infinity solutions
b) There is exactly one solution
c) There is 0 or 1 solution
d) There is 0 or ∞ solutions

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Question

Consider a matrix A of $m \times n$ size.

Let $rank(A) < m$ and $rank(A) < n$.

Which of the following is true about solutions of $Ax = b$?

a) There are infinity solutions

b) There is exactly one solution

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Question

True/False

Let A be the $m \times (n+1)$ augmented matrix corresponding to a consistent system of equations in n variables, and suppose A has rank r. Then

1. the system has a unique solution if $r = n$
2. the system has infinitely many solutions if $r < n$

GATE CSE 1996 | Question: 1.7

Let $Ax = b$ be a system of linear equations where A is an $m \times n$ matrix and b is a $m \times 1$ column vector and X is an $n \times 1$ column vector of unknowns. Which of the following is false?

A. The system has a solution if and only if, both A and the augmented matrix $[Ab]$ have the same rank.

B. If $m < n$ and b is the zero vector, then the system has infinitely many solutions.

C. If $m = n$ and b is a non-zero vector, then the system has a unique solution.

D. The system will have only a trivial solution when $m = n$, b is the zero vector and $\text{rank}(A) = n$.

GATE CSE 1996 | Question: 1.7

asked in **Linear Algebra** Oct 9, 2014

17,188 views

37

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Answer C

GATE CSE 2016 Set 2 | Question: 04

asked in **Linear Algebra** Feb 12, 2016 11,590 views

- edited Jun 19, 2021 by **Lakshman Patel RJIT**

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Consider the systems, each consisting of m linear equations in n variables.

I. If $m < n$, then all such systems have a solution.
II. If $m > n$, then none of these systems has a solution.
III. If $m = n$, then there exists a system which has a solution.

Which one of the following is **CORRECT**?

A. I, II and III are true.
B. Only II and III are true.
C. Only III is true.
D. None of them is true.

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D. None of them is true.

Answer C

Consider a matrix A of $m \times n$ size.

Let $rank(A) = m = n$.

$$\begin{bmatrix} D & & \\ & D & \\ & & D \end{bmatrix}_n$$

Which of the following is true about solutions of $Ax = b$?

a) There are infinity solutions
b) There is exactly one solution
c) There is 0 or 1 solution
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Consider a matrix A of $m \times n$ size.

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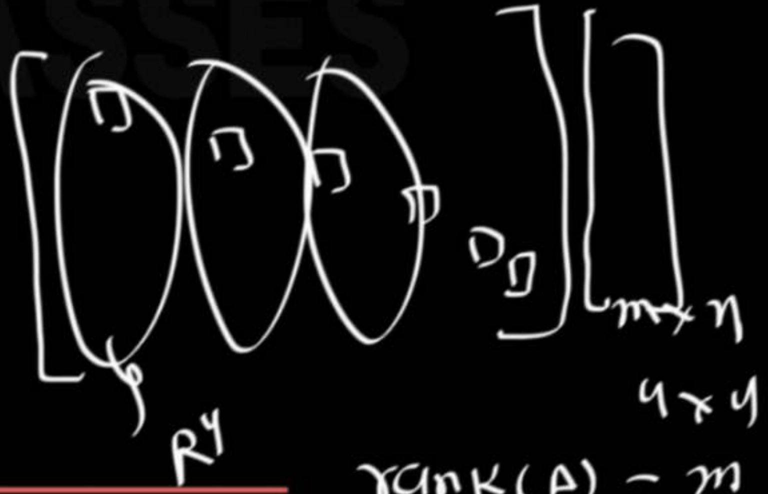

rank(A) = m

Consider a matrix A of $m \times n$ size.

Let $rank(A) = n$.

No Soln

Which of the following is true about solutions of $Ax = b$?

a) There are infinity solutions
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c) There is 0 or 1 solution
d) There is 0 or ∞ solutions

Consider a matrix A of $m\times n$ size.

Let $rank(A) = n$. $m \neq n$ $\boxed{m < n}$ 2×3

Which of the following is true about solutions of $Ax = b$?

a) There are infinity solutions

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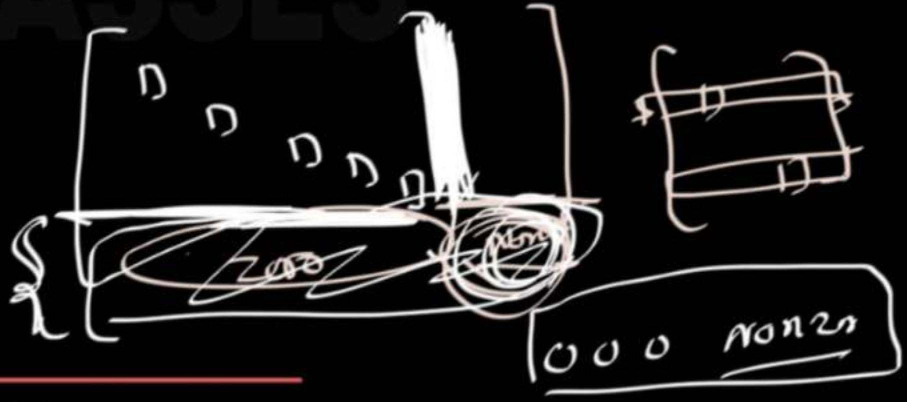

Consider a matrix A of $m \times n$ size.

Let $rank(A) = m$.

~~$m \neq n$~~

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Which of the following is true about solutions of $Ax = b$?

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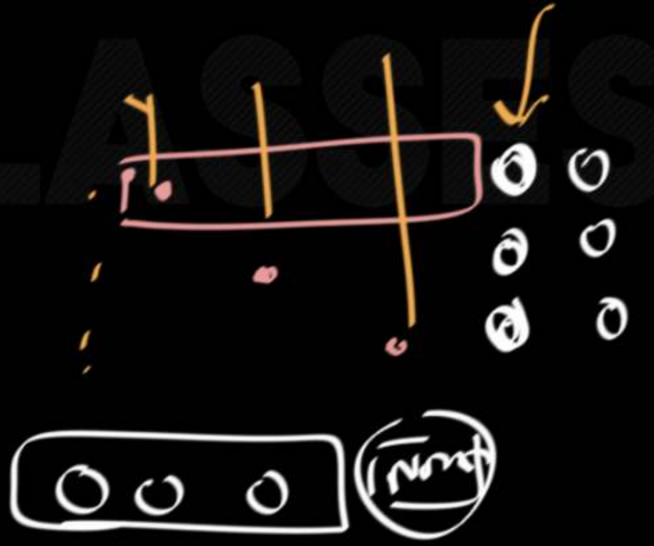

Consider a matrix A of $m \times n$ size.

No Soln

Let $rank(A) < m$ and $rank(A) < n$.

Which of the following is true about solutions of $Ax = b$?

a) There are infinity solutions
b) There is exactly one solution
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d) There is 0 or ∞ solutions

Wait a minute…

We are doing row operations and telling independent columns ?

LI Rows = 3

LI Columns = 3

$\boxed{\text{LI Rows}}$ = # LI Columns = 3

$C_3 = C_1 + C_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & 7 \end{bmatrix}$$

$R_1 \to R_1 + R_2$

$$\begin{bmatrix} 3 & 5 & 8 \\ 2 & 3 & 5 \\ 3 & 4 & 7 \end{bmatrix}$$

$R_2 \to R_3 - R_2$

$$\begin{bmatrix} 3 & 5 & 8 \\ 1 & 1 & 2 \\ 3 & 4 & 7 \end{bmatrix}$$

$c_3 = 2c_1 + c_2$

$$\begin{bmatrix} 1 & 4 & 6 \\ 2 & 5 & 9 \\ 3 & 6 & 12 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 2 & 4 & a \\ 3 & 6 & b \\ 5 & 7 & c \end{bmatrix}$$

unknown elementary row operations

$$\boxed{a = 8, \quad b = 12, \quad c = 17}$$

$a = ?$ $b = ?$ $c = ?$

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$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 8 & 11 \\ 4 & 12 & 16 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 6 & 6 \end{bmatrix} \longrightarrow \begin{bmatrix} \boxed{1} & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to 2R_1 - R_2$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - 4R_1$

$R_3 \to R_3 - 2R_2$

$R_4 \to R_4 - 6R_2$

First

free

$$\begin{bmatrix} a_1 & ka_1 \\ a_2 & ka_2 \\ a_3 & ka_3 \end{bmatrix} \longrightarrow \begin{bmatrix} a_1 & ka_1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$R_2 \rightarrow \frac{a_2}{a_1} R_1 - R_2$$

$$R_3 \rightarrow \frac{a_3}{a_1} R_1 - R_3$$

$$\begin{bmatrix} 1 & 5 \\ 2 & 10 \\ 3 & 15 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 5 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

free

Row echelon

LD

$c_4 = c_1 + c_2 + c_3$

Row Reduced Echelon form

or

Reduced Echelon form

$\boxed{\text{Row echelon form}}$

or

echelon form

Reduced echelon form

Every linear system is equivalent to a system whose augmented matrix is in reduced echelon form, e.g.:

$$\begin{bmatrix} \blacksquare & * & * & * & * & * & * \\ 0 & \blacksquare & * & * & * & * & * \\ 0 & 0 & 0 & \blacksquare & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \rightsquigarrow \quad \begin{bmatrix} 1 & 0 & * & 0 & * & * & * \\ 0 & 1 & * & 0 & * & * & * \\ 0 & 0 & 0 & 1 & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Reduced echelon form has the additional properties:

1. Each leading entry of a row is 1.
2. There are zeros below and above each leading 1.

$$
\begin{bmatrix} \ \ \ & \ \ \ \\ \ & \ \end{bmatrix}
\rightsquigarrow
\begin{bmatrix}
\blacksquare & * & * & * & * & * & * \\
0 & \blacksquare & * & * & * & * & * \\
0 & 0 & 0 & \blacksquare & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\rightsquigarrow
\begin{bmatrix}
1 & 0 & * & 0 & * & * & * \\
0 & 1 & * & 0 & * & * & * \\
0 & 0 & 0 & 1 & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

A

Row echelon form

Row reduced echelon form

Row reduced Echelon form

=

All pivots are 1.

all elements below + above pivots are zero

$$\begin{bmatrix} 1 & 3 & 1 & 9 \\ 1 & 1 & -1 & 1 \\ 3 & 11 & 5 & 35 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 1 & 9 \\ 0 & -2 & -2 & -8 \\ 0 & 2 & 2 & 8 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 1 & 9 \\ 0 & -2 & -2 & -8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\downarrow R_2 \rightarrow -\tfrac{1}{2}R_2$$

$$\begin{array}{cccc} \text{pivot} & \text{pivot} & \text{free} & \text{free} \\ \end{array}$$

$$\begin{bmatrix} \boxed{1} & 0 & -2 & -3 \\ 0 & \boxed{1} & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_1 \rightarrow R_1 - 3R_2} \begin{bmatrix} \boxed{1} & 3 & 1 & 9 \\ 0 & \boxed{1} & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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$R_2 \rightarrow -\frac{1}{2}R_2$

$$\begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix} = -2\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} + 1\begin{bmatrix} 3 \\ 1 \\ 11 \end{bmatrix}$$

$$\begin{array}{cccc} \text{pivot} & \text{pivot} & \text{free} & \text{free} \end{array}$$

$$\begin{bmatrix} \boxed{1} & 0 & -2 & -3 \\ 0 & \boxed{1} & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_1 \rightarrow R_1 - 3R_2} \begin{bmatrix} \boxed{1} & 3 & 1 & 9 \\ 0 & \boxed{1} & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} = -2\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 1\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix} = -3\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 4\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

GO CLASSES

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = -2\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 1\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \boxed{1} & 0 & -2 & -3 \\ 0 & \boxed{1} & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_1 \to R_1 - 3R_2} \begin{bmatrix} \boxed{1} & 3 & 1 & 9 \\ 0 & \boxed{1} & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 3 & 1 & 9 \\ 1 & 1 & -1 & 1 \\ 3 & 11 & 5 & 35 \end{bmatrix} \to \begin{bmatrix} 1 & 3 & 1 & 9 \\ 0 & -2 & -2 & -8 \\ 0 & 2 & 2 & 8 \end{bmatrix} \to \begin{bmatrix} 1 & 3 & 1 & 9 \\ 0 & -2 & -2 & -8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to -\frac{1}{2}R_2$

$$\begin{bmatrix} \boxed{1} & 3 & 1 & 9 \\ 0 & \boxed{1} & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \to R_1 - 3R_2$

$$\begin{array}{cccc} \text{pivot} & \text{pivot} & \text{free} & \text{free} \end{array}$$

$$\begin{bmatrix} \boxed{1} & 0 & -2 & -3 \\ 0 & \boxed{1} & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} = -2\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 1\begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 9 \\ 1 \\ 35 \end{bmatrix} = -3\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} + 4\begin{bmatrix} 3 \\ 1 \\ 11 \end{bmatrix}$$

Question

Let A be a 3×4 matrix which has 3 pivot columns.

Consider the following matrices.

I. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ II. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 2 & 0 & 1 \end{bmatrix}$ III. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Which of above matrices could possibly be the reduced row echelon form of A?

(a) I only
(b) II only
(c) III only
(d) I and II only
(e) II and III only

$$\begin{bmatrix} \underline{1} & 5 & 0 \\ 0 & 0 & \underline{1} \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & \underline{1} & 3 & 2 \\ 0 & 0 & \underline{1} & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

✓ ✗ ✓

Homework

Which of the following matrices are in row echelon form? Which are in reduced row echelon form?

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 3 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 2 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Which of the following matrices are in row echelon form? Which are in reduced row echelon form?

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 3 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 2 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Solution: The 2nd, 3rd, and 5th are in row echelon form. The 2nd is the only one in reduced row echelon form.

Bring all four matrices to reduced echelon form.

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 2 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \text{ and } D = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 \to R_1 - R_2} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_1 \to R_1 - R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Consider the matrix $A = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 2 & 4 & -2 \end{bmatrix}$ whose reduced echelon form is $\begin{bmatrix} 1 & 0 & a & b \\ 0 & 1 & c & d \end{bmatrix}$.

Find $a + b + c + d$:

(a) -7

(b) -4

(c) -3

(d) -1

(e) 0

$$\begin{bmatrix} 1 \\ 4 \end{bmatrix} = a\begin{bmatrix} 1 \\ 0 \end{bmatrix} + c\begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$2c = 4$

$c = 2$

$6 + a = 1$

$a = -5$

$$\begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 2 & 4 & -2 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 0 & a & b \\ 0 & 1 & c & d \end{bmatrix}.$$

Unknown elements

row operations

$$\begin{bmatrix} a \\ c \end{bmatrix} = a\begin{bmatrix} 1 \\ 0 \end{bmatrix} + c\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Consider the matrix $A = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 2 & 4 & -2 \end{bmatrix}$ whose reduced echelon form is $\begin{bmatrix} 1 & 0 & a & b \\ 0 & 1 & c & d \end{bmatrix}$.

Find $a+b+c+d$:

(a) -7

(b) -4

(c) -3

(d) -1

(e) 0

$$\begin{bmatrix} 0 \\ -2 \end{bmatrix} = b\begin{bmatrix} 1 \\ 0 \end{bmatrix} + d\begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$d = -1$

$b - 3 = 0$

$b = 3$

$$\begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 2 & 4 & -2 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 0 & a & b \\ 0 & 1 & c & d \end{bmatrix}.$$

Unknown elements

row operations

$$\begin{bmatrix} a \\ c \end{bmatrix} = a\begin{bmatrix} 1 \\ 0 \end{bmatrix} + c\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Gauss after
Echelon form

I quit

Gauss

You can do it

Jordan

No, I can't

Ok, Let me
help you

Row Reduced
Echelon form

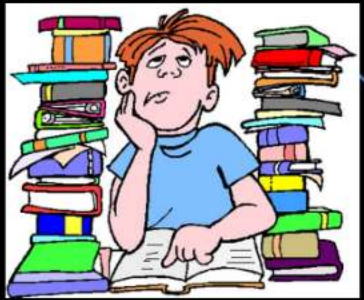
Time for shameless self promotion

Students Before
GO Classes

I quit

GO
CLASSES

You can do it



No, I can't

GO
CLASSES

Ok, Let me
help you

Admission in IIT/IISc