Multiplying 2 matrices

$$\begin{bmatrix}1 & 0 & 1\\ 0 & 0 & 1\\ 0 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 1 & 2 & 1\\ 1 & 1 & 2\end{bmatrix} = \begin{bmatrix}2 & 3 & \cdot\\ \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot\end{bmatrix}$$

What we just did

$$(AB)_{12} = (1^{st} \text{ row of } A) \cdot (2^{nd} \text{ column of } B)$$

dot products

Matrix × Vector = ?

extend this idea for 2 matrices?

Matrix × Vector = ?

$$\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ \underline{3} & \underline{5} \end{bmatrix} \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$1 \begin{bmatrix} 2 \\ 3 \end{bmatrix} + 0 \begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Can we multiply 2 matrices in similar way ?

Can we multiply 2 matrices in similar way ?

$$\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 7 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} \ \end{bmatrix}$$

Can we multiply 2 matrices in similar way ?

$$\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 7 \\ 1 & 2 \end{bmatrix}$$

$$1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 0 \begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Can we multiply 2 matrices in similar way ?

$$\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 7 \\ 1 & 2 \end{bmatrix}$$

$$2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$$

$$\overset{A}{\begin{bmatrix} & & \\ & & \end{bmatrix}}\begin{bmatrix} & \\ & \end{bmatrix} = \begin{bmatrix} & & \\ & & \end{bmatrix}$$

linear combination of columns of A.

$$\begin{bmatrix} a+2c & b+2d \\ 3a+4c & 3b+4d \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a\begin{bmatrix} 1 \\ 3 \end{bmatrix} + c\begin{bmatrix} 2 \\ 4 \end{bmatrix} & b\begin{bmatrix} 1 \\ 3 \end{bmatrix} + d\begin{bmatrix} 2 \\ 4 \end{bmatrix} \end{bmatrix}$$

Another interpretation

$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \left[\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right]$$

We will use this interpretation here and there

T/F

If A is an $m \times n$ matrix and B is an $n \times p$ matrix whose first column contains only zeros, then the first column of AB also contains only zeros.

$$\begin{bmatrix} & A & \end{bmatrix}_{m\times n} \begin{bmatrix} 0 & & \\ 0 & B & \\ 0 & & \\ 0 & & \end{bmatrix}_{n\times p} = \begin{bmatrix} 0 & & \end{bmatrix}_{m\times p}$$

T/F

If A is an $m \times n$ matrix and B is an $n \times p$ matrix whose first column contains only zeros, then the first column of AB also contains only zeros.

True. By the columnwise definition of matrix vector multiplication, the first column of B is b_1 then the first column of AB is Ab_1.

2 L.I. vectors in a set of 5 vectors

$$\begin{bmatrix}1\\2\end{bmatrix} \quad \begin{bmatrix}2\\3\end{bmatrix} \quad \begin{bmatrix}2\\4\end{bmatrix} \quad \begin{bmatrix}4\\6\end{bmatrix} \quad \begin{bmatrix}3\\5\end{bmatrix}$$

$\Rightarrow$ How many linearly Indep. vectors we can generate using linear combination

$\mathbb{R}^n$

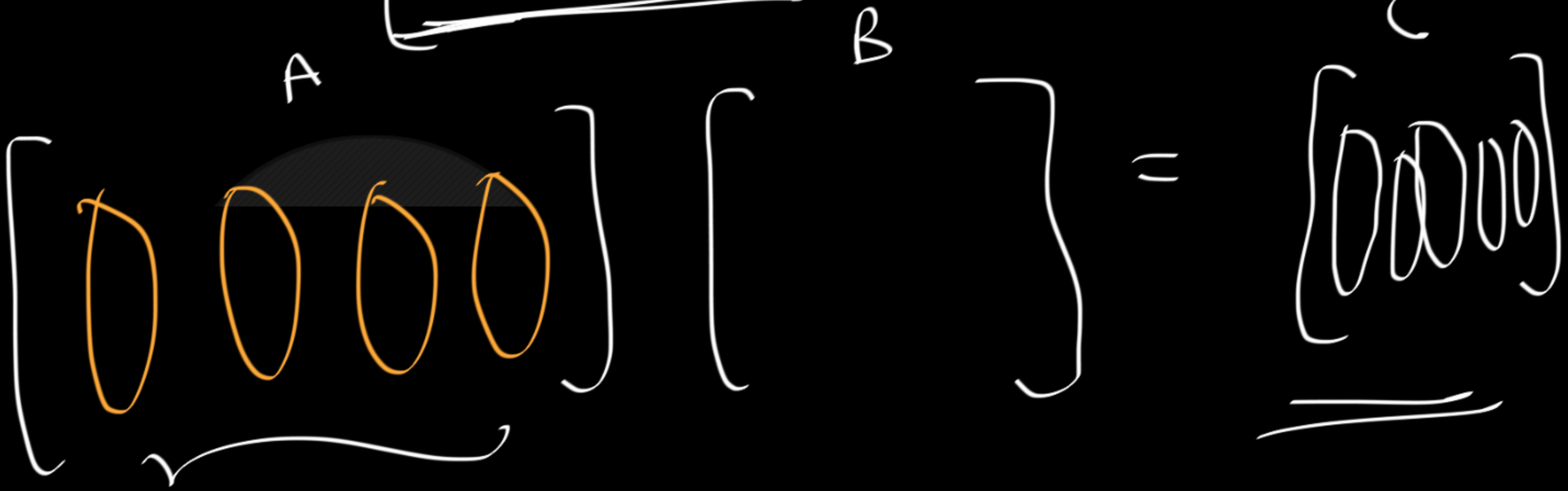
$u \quad v \quad w \quad x \quad y$

2

$1u + 2v + 0w + 3x + 7y$

$0u + (-1)v + 3w + 4x + 10 \cdot y$

walmart interview



given 2 LI vectors

No. of LI vectors in C? atmost 2

True/False

A. Every system of 3 equations in 8 unknowns has a solution.

B. Every set of five vectors in $\mathbb{R}^4$ is linearly dependent.

C. Every set of four vectors in $\mathbb{R}^4$ is linearly dependent.

D. If A and B are $m \times n$ matrices such that B can be obtained from A by column operations, then A can also be obtained from B by column operations .

https://sites.math.washington.edu/~smith/Teaching/308/2012-Summer-Midterm.pdf

GATE CSE 2016

Consider the systems, each consisting of m linear equations in n variables.

I. If $m < n$, then all such systems have a solution.
II. If $m > n$, then none of these systems has a solution.
III. If $m = n$, then there exists a system which has a solution.

Which one of the following is **CORRECT**?

A. I, II and III are true.
B. Only II and III are true.
C. Only III is true.
D. None of them is true.

GATE CSE 2016

Consider the systems, each consisting of m linear equations in n variables.

I. If $\overset{2}{m} < \overset{3}{n}$, then all such systems have a solution. — F

II. If $m > n$, then none of these systems has a solution. F

III. If $\underset{3}{m} = \underset{2}{n}$, then there exists a system which has a solution. T

all → F

Which one of the following is **CORRECT**?

$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$ $\in R^3$

$\begin{bmatrix} 1 & 1 & \cancel{1} \\ 2 & 2 & \cancel{2} \end{bmatrix}$ $\in R^2$

$\begin{bmatrix} \ \ & \ \ \end{bmatrix}$ $\in R^3$ $\in R^3$

A. I, II and III are true.

B. Only II and III are true.

C. Only III is true.

D. None of them is true.

$$\begin{bmatrix} \underbrace{\bigcirc}_{\in \mathbb{R}^3} & \underbrace{\bigcirc}_{\in \mathbb{R}^3} \end{bmatrix} \underline{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} = \begin{bmatrix} \; \end{bmatrix}$$

True/False

F Every system of 3 equations in 8 unknowns has a solution. → false

$\to A, b$

$\to$ 8 vectrs in $\mathbb{R}^3$

Every set of five vectors in $\mathbb{R}^4$ is linearly dependent. → T

F Every set of four vectors in $\mathbb{R}^4$ is linearly dependent. → F

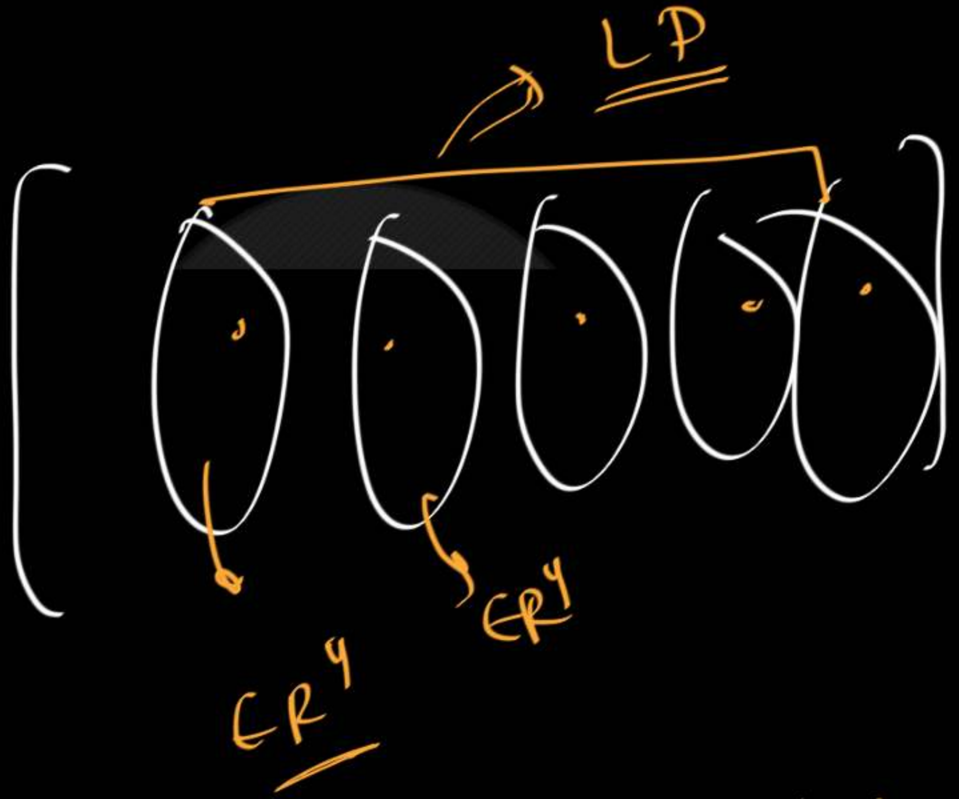
$$\begin{bmatrix}1\\0\\0\\0\end{bmatrix}\begin{bmatrix}0\\1\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\1\\0\end{bmatrix}\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

$\downarrow$

LI

If A and B are $m \times n$ matrices such that B can be obtained from A by column operations, then A can also be obtained from B by column operations. F

https://sites.math.washington.edu/~smith/Teaching/308/2012-Summer-Midterm.pdf



in R^n $\exists$ you can have (at most) n LI vectors

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LI

$$\left[\begin{array}{cccc} u & v & w & x \\ 0 & 0 & 0 & 0 \end{array}\right]$$

6:30 am

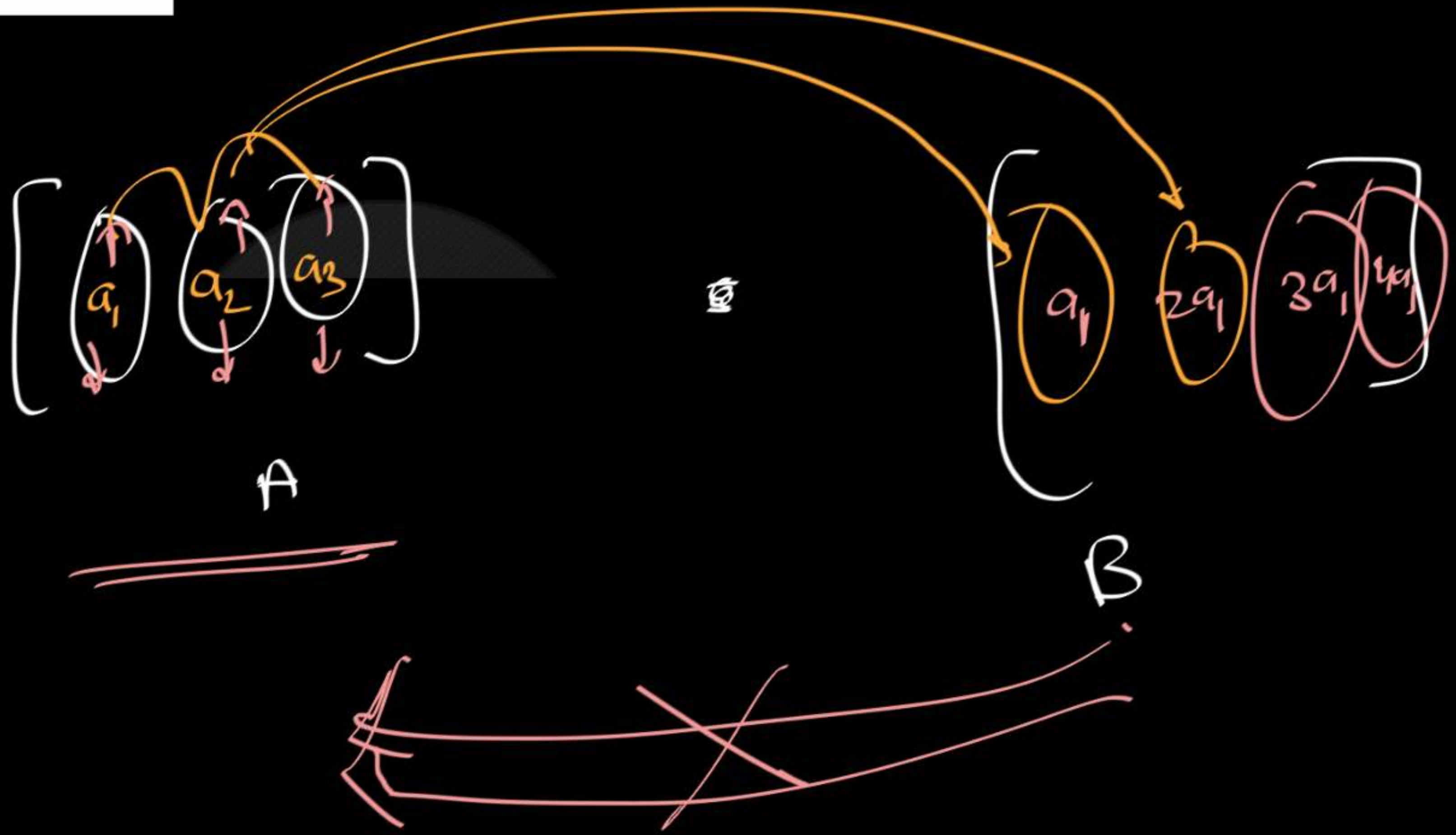
can you generate more than 4 LI?

NO

$u+v+w+x$

$2u+3v+4v+0$

$c_1u+c_2v+c_3w+c_4x$



a1
a2
a3
A
a1
2a1
3a1
4a1
B

a_1

$$\underbrace{\begin{bmatrix} \uparrow & \uparrow & \uparrow \\ a_1 & a_2 & a_3 \\ \downarrow & \downarrow & \downarrow \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{C} = \underbrace{\begin{bmatrix} a_1 & 2a_1 & 3a_1 \end{bmatrix}}_{B}$$

$$\underbrace{\begin{bmatrix} \; \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} \; \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} \; \end{bmatrix}}_{B}$$

$$2x + 3y + \tfrac{1}{2}z + 4w + 9p + 15q + 20r + 10s = 0$$

$$\begin{array}{c} x \quad y \quad \cdots \\ \begin{bmatrix} \quad \end{bmatrix} \end{array} \begin{bmatrix} \quad \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

$\in R^3$ R^3

multiply 2 matrices

$$\begin{bmatrix} 2 & 5 & 2 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} \; & \; & \; \\ & & \\ & & \end{bmatrix}$$

$$2\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + 2\begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix} + 0\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

multiply 2 matrices

$$\begin{bmatrix} 2 & 5 & 2 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix}$$

$$1 \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + 2 \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix} + 0 \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

multiply 2 matrices —

$$\begin{bmatrix} 2 & 5 \\ 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

$$2 \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + 2 \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$$

multiply 2 matrices

$$\begin{bmatrix}2 & 5\\1 & 0\\3 & 1\end{bmatrix}\begin{bmatrix}2 & 1 & 0\\2 & 2 & 1\end{bmatrix} = \begin{bmatrix} & \end{bmatrix}$$

$$1\begin{bmatrix}2\\1\\3\end{bmatrix} + 2\begin{bmatrix}5\\0\\1\end{bmatrix}$$

multiply 2 matrices

$$\begin{bmatrix} 2 & 5 \\ 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

$$0 \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + 1 \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$$

multiply 2 matrices

$$\underset{3\times1}{\begin{bmatrix}2\\1\\3\end{bmatrix}}\underset{1\times3}{\begin{bmatrix}2 & 1 & 0\end{bmatrix}} = \underset{3\times3}{\begin{bmatrix}0 & 0 & 0\end{bmatrix}}$$

$$2\begin{bmatrix}2\\1\\3\end{bmatrix}$$

multiply 2 matrices —

$$\begin{bmatrix}2\\1\\3\end{bmatrix}\begin{bmatrix}2 & 1 & 0\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\end{bmatrix}$$

3x1

1x3

3x3

$$1\begin{bmatrix}2\\1\\3\end{bmatrix}$$

multiply 2 matrices —

$$\underset{3\times 1}{\begin{bmatrix}2\\1\\3\end{bmatrix}}\underset{1\times 3}{\begin{bmatrix}2 & 1 & 0\end{bmatrix}} = \underset{3\times 3}{\begin{bmatrix}0 & 0 & 0\end{bmatrix}}$$

$$0\begin{bmatrix}2\\1\\3\end{bmatrix}$$

multiply 2 matrices

final answer

$$\underset{3\times 1}{\begin{bmatrix}2\\1\\3\end{bmatrix}}\underset{1\times 3}{\begin{bmatrix}2 & 1 & 0\end{bmatrix}} = \begin{bmatrix}4 & 2 & 0\\2 & 1 & 0\\6 & 3 & 0\end{bmatrix}$$

GATE CSE 2014

How many independent columns are present in A? (rank)

$$A = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix} \begin{bmatrix} 1 & 9 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 18 & 10 \\ -4 & -36 & -20 \\ 7 & 7\times 9 & 35 \end{bmatrix}$$

Question

If b is linear combination of columns of A then-

A. There is always a solution of $Ax = b$.
B. There is never a solution of $Ax = b$.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 7 \\ 3 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \\ 14 \end{bmatrix}$$

Q: $\underbrace{\begin{bmatrix} 1 & 3 & 2 \\ 2 & 6 & 4 \\ 3 & 9 & 6 \end{bmatrix}}_{A} \overset{x}{\begin{bmatrix} \\ \\ \\ \end{bmatrix}} = \underbrace{\begin{bmatrix} 4 \\ 7 \\ 12 \end{bmatrix}}_{b}$

given that b is NOT a linear combination of columns of A.

do you have a sol^n. $\Rightarrow$

Q: $\underbrace{\begin{bmatrix} 1 & 3 & 2 \\ 2 & 6 & 4 \\ 3 & 9 & 6 \end{bmatrix}}_{A} \overset{x}{\begin{bmatrix} \; \\ \; \\ \; \end{bmatrix}} = \underbrace{\begin{bmatrix} 4 \\ 7 \\ 12 \end{bmatrix}}_{b}$

given that b is NOT a linear combination of columns of A.

do you have a soln. $\Rightarrow$ NO

* if b is NOT a linear combination of columns of A then it does not matter how hard you try, You can never get the solution.

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* if b is a linear combination of columns of A then you will ALWAYS get the sol^n.

Question

If b is NOT linear combination of columns of A then-

A. There is always a solution of $Ax = b$.
B. There is never a solution of $Ax = b$.

Question

$A_{3\times 4}$

$m\times n$

$A_{m\times n}$ has m Linearly Independent Columns then –

A. There is always a solution of $Ax = b$.

B. There is never a solution of $Ax = b$ for any b.

C. There may or may not solution of $Ax = b$.

Question

$A_{m \times n}$ has m Linearly Independent Columns then –

A. There is always a solution of $Ax = b$.
B. There is never a solution of $Ax = b$ for any b.
C. There may or may not solution of $Ax = b$.

$\in \mathbb{R}^3$ $\quad A \quad x \quad b$

$$\underbrace{\begin{bmatrix} \ & \ & \ & \ \end{bmatrix}}_{3 \times 4} \begin{bmatrix} \ \end{bmatrix} = \begin{bmatrix} \ \end{bmatrix}$$

Question

$A_{m\times n}$ has m Linearly Independent Columns then –

A. There is always a solution of $Ax = b$.
B. There is never a solution of $Ax = b$ for any b.
C. There may or may not solution of $Ax = b$.

$\in R^3$

A

x

b

$=$

3×4

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -9 \end{bmatrix} \begin{bmatrix} 37 \\ -52 \\ 790 \\ 0 \end{bmatrix} = \begin{bmatrix} 37 \\ -52 \\ 790 \end{bmatrix}$$

Question

$A_{m\times n}$ does not have m Linearly Independent Columns then –

A. There is always a solution of $Ax = b$.
B. There is never a solution of $Ax = b$ for any b.
C. There may or may not solution of $Ax = b$.

$A_{3\times 4}$

b

Question

$A_{m\times n}$ does not have m Linearly Independent Columns then –

A. There is always a solution of $Ax = b$.
B. There is never a solution of $Ax = b$ for any b.
C. There may or may not solution of $Ax = b$.

$$\underbrace{\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 4 & 6 & 4 \\ 3 & 5 & 8 & 5 \end{bmatrix}}_{3\times 4} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\mathbb{R}^3$

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 4 & 6 & 4 \\ 3 & 5 & 8 & 5 \end{bmatrix}_{3\times 4} \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 10 \end{bmatrix}$$

$\downarrow$

R^3

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 4 & 6 & 4 \\ 3 & 5 & 8 & 5 \end{bmatrix}_{3\times 4} \begin{bmatrix} \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 7 \end{bmatrix}$$

$\downarrow$

$\mathbb{R}^3$

is not a linear comb. of

columns of A.

$\Rightarrow$ No soln

Solutions of $Ax = b$

$A_{m\times n}$ has m Linearly Independent Columns ?

(Total columns of A are n)

Yes → There is always a solution

No → May or may not be solutions

Solutions of $Ax = b$

b is linear combination of columns of A ?

Yes

There is always a solution
(Unique or infinite)

No

There is never a solution
(No Solution)

Solutions of $Ax = b$

b is linear combination of columns of A ?

Yes

There is always a solution
(Unique or infinite)

(Columns are LI)

Yes

Unique Soln

No

Infinite Soln

No

There is never a solution
(No Solution)

if a vector can be represented as linear combination of few vectors and further if those few vectors are LI then there is just one way to do it otherwise there are many ways.

if a vector can be represented as linear combination of few vectors and further if those few vectors are L.D then there are many way to do it otherwise there is a unique way.

Proof when "rank" comes

$$\boxed{b = c_1 v_1 + c_2 v_2 + \cdots c_n v_n}$$

given

$\{v_1 \cdots v_n\}$

LD

$b = d_1 v_1 + d_2 v_2 + \cdots$

$\{v_1 \cdots v_n\}$

LI

$b = c_1 v_1 + c_2 v_2 + \cdots$

intutively

(idea)

given $\rightarrow$ $b = c_1 v_1 + c_2 v_2 + \cdots c_n v_n$

case 1

$\{v_1 \cdots v_n\}$ are LD

$b = c_1 v_1 + c_2 v_2 + \cdots c_n v_n + 0$

$(k_1 v_1 + k_2 v_2 + \cdots k_n v_n)$

$= (k_1 + c_1) v_1 + (k_2 + c_2) v_2 + \cdots$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 4 \\ 8 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$\underbrace{[\;] + [\;] + [\;]}_{1d}$$

$$\begin{bmatrix} 0 \\ 0 \\ 4/3 \end{bmatrix}$$

$$\square + \square + 5\underline{\square} + \square \cdots + \square = b$$

$$\square + \square + 3\underline{\square} + \square \cdots + \square = b$$

intutively (idea)

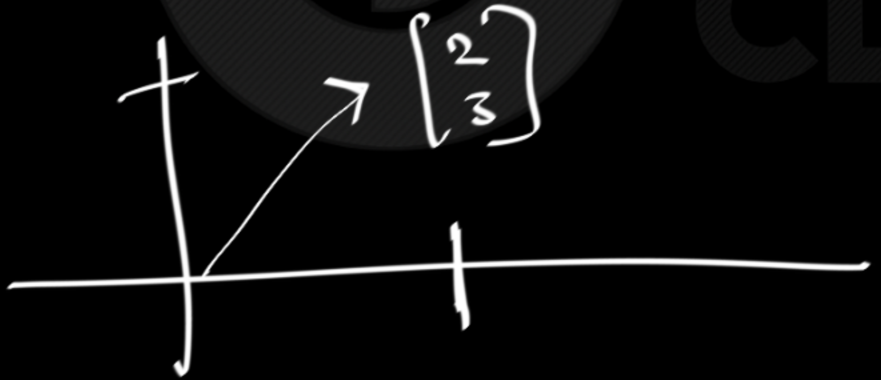
given $\rightarrow$ $b = c_1 v_1 + c_2 v_2 + \cdots c_n v_n$

$\{v_1 \cdots v_n\}$ are LI

case 2

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$2\begin{bmatrix}1\\0\end{bmatrix} + 3\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}2\\3\end{bmatrix}$$



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Proof -

Given

$b = c_1 v_1 + c_2 v_2 + \cdots c_n v_n$ & $\{v_1 \cdots v_n\}$

are L.I.

Sol

Suppose

$b = c_1 v_1 + c_2 v_2 + \cdots c_n v_n$ — ①

$b = d_1 v_1 + d_2 v_2 + \cdots d_n v_n$ — ②

$0 \quad (c_1 - d_1) v_1 + (c_2 - d_2) v_2 + \cdots (c_n - d_n) v_n$

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$$0 = (c_1 - d_1)v_1 + (c_2 - d_2)v_2 + \cdots (c_n - d_n)v_n$$

$$\underbrace{k_1 v_1 + k_2 v_2 + \cdots k_n v_n = 0}$$

$$\Downarrow$$

all $k_i = 0$

$k_1 = 0 \Rightarrow c_1 - d_1 = 0 \Rightarrow c_1 = d_1$

$c_2 = d_2$ $\quad$ $c_3 = d_3$ $\quad$ $c_4 = d_4$

$b = \boxed{\qquad\qquad\qquad}$

LI

unique

LD

inf.

Solutions of $Ax = b$

b is linear combination of columns of A ?

$b = 1$

Yes

There is always a solution
(Unique or infinite)

(Columns are LI)

Yes

Unique Soln

No

Infinite Soln

No

There is never a solution
(No Solution)

Question

If the linear equation $A\mathbf{x} = \mathbf{b}$ has a unique solution, then the columns of A are linearly independent.

(9) If the linear equation $A\mathbf{x} = \mathbf{b}$ has a unique solution, then the columns of A are linearly independent.

True

Question

Problem 2. How many solutions will the linear system $Ax = b$ have

(a) if b is in the column space of A and the columns of A are linearly independent?

is a Linear comb of columns of A

(b) if b is not in the column space of A

Question

Problem 2. How many solutions will the linear system $Ax = b$ have

(a) if b is in the column space of A and the columns of A are linearly independent?

is a Linear comb of columns of A

unique

(b) if b is not in the column space of A

a LC of columns

Sol^n can not exist

Problem 2. How many solutions will the linear system $Ax = b$ have
(a) if b is in the column space of A and the columns of A are linearly independent?
(b) if b is not in the column space of A
Explain your answer.
Solution.
(a) Since b is in the column space of A, it is a linear combination of columns of A, hence there is a solution of the system $Ax = b$. Since the columns of A are linearly independent in the reduced row echelon form of A every column will have a pivot. Therefore the system $Ax = b$ does not have free unknowns, hence it has exactly one solution.
(b) Since b is not in the column space of A, it is not a linear combination of columns of A, hence $Ax = b$ has no solutions.

GATE 2017

Let $c_1 \ldots \ldots c_n$ be scalars, not all zero, such that $\sum_{i=1}^{n} c_i a_i = 0$ where a_i are column vectors in R^n.

a_i's are LD

Consider the set of linear equations

$$Ax = b$$

where $A = [a_1 \ldots \ldots a_n]$ and $b = \sum_{i=1}^{n} a_i$. The set of equations has

$a_1 + a_2 +$

A. a unique solution at $x = J_n$ where J_n denotes a n-dimensional vector of all 1.
B. no solution
C. infinitely many solutions
D. finitely many solutions

$$\underset{A}{\begin{bmatrix} \quad \end{bmatrix}} \underset{x}{\begin{bmatrix} \vdots \end{bmatrix}} = \underset{b}{\begin{bmatrix} \quad \end{bmatrix}}$$

GATE 2017

51

Let $c_1 \ldots\ldots c_n$ be scalars, not all zero, such that $\sum_{i=1}^{n} c_i a_i = 0$ where a_i are column vectors in R^n.

L.D. $\{a_1 \ldots a_n\}$

Consider the set of linear equations

$Ax = b$

$x = \begin{bmatrix} \vdots \end{bmatrix}$

where $A = [a_1 \ldots\ldots a_n]$ and $b = \sum_{i=1}^{n} a_i$. The set of equations has

A. ~~a unique solution at~~ $x = J_n$ where J_n denotes a n-dimensional vector of all 1.

B. ~~no solution~~

C. infinitely many solutions

D. finitely many solutions

$\begin{bmatrix} \vdots \end{bmatrix}$

$b = a_1 + a_2 + \cdots a_n$

$b = a_1 + a_2 + a_3 + \cdots a_n$

(one of the way)

$a_1, \cdots a_n$ are LP

$b = 2a_1 + 3a_1 - 5a_3 + \cdots$

$$\begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ a_1 & a_2 & \cdots & a_n \\ \downarrow & \downarrow & & \downarrow \end{bmatrix} \begin{bmatrix} \\ \\ \\ \end{bmatrix} = \begin{bmatrix} \uparrow \\ b \\ \downarrow \end{bmatrix}$$

System of linear equations

- Why solve System of linear equations ?
- Geometric Interpretation
- Understanding Ax = b intuitively
- A step by step method to find Solution for Ax=b (Gaussian Elimination)
    - Rank
    - Parametric form of solution