unacademy

Counting Techniques - Part II

Course on Engineering Mathematics for GATE - CSE

Rahul • Lesson 2 • June 17, 2024

unacademy

Engineering Mathematics

Probability & Statistics

counting principles

Lecture Number- 01

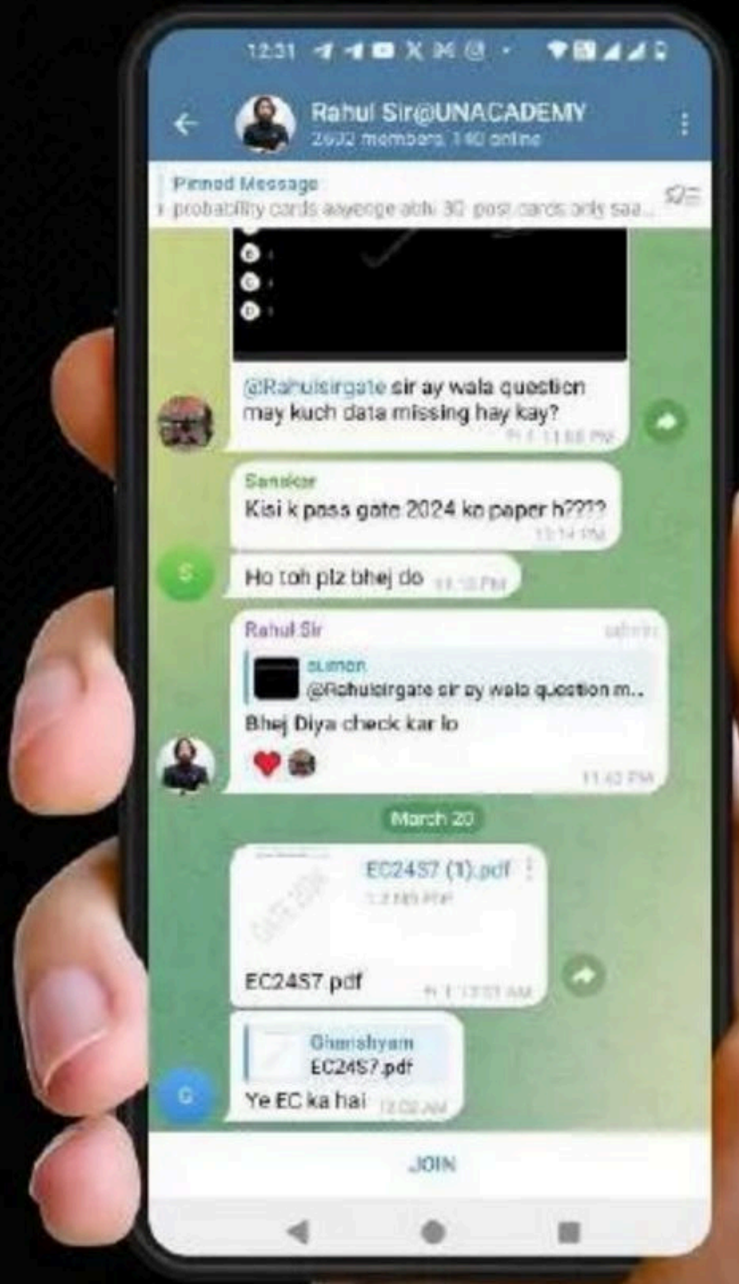
By- Rahul Sir

unacademy

JOIN MY TELEGRAM GROUP FOR

- Daily Quiz
- Weekly Test
- Best Quality Content
- Doubt Discussion
- Personal Guidance

Scan the QR code to join our Telegram Group or Search @RahulsirUA



unacademy

Topics

to be covered

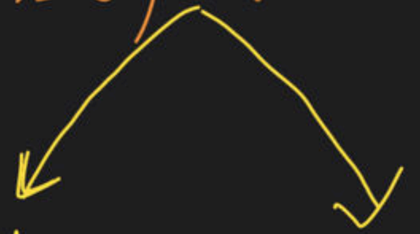
1. counting principles

✓ Counting Principles - 01

✓ DPP

Fundamental Principle of counting :-

Task (work) W



W_1 ways. W_2 ways.

If W_1 and W_2 Perform Together (Working Together)

both are simultaneous work.

Total No. of ways $= W_1 \times W_2$ = Multiplication Rule.

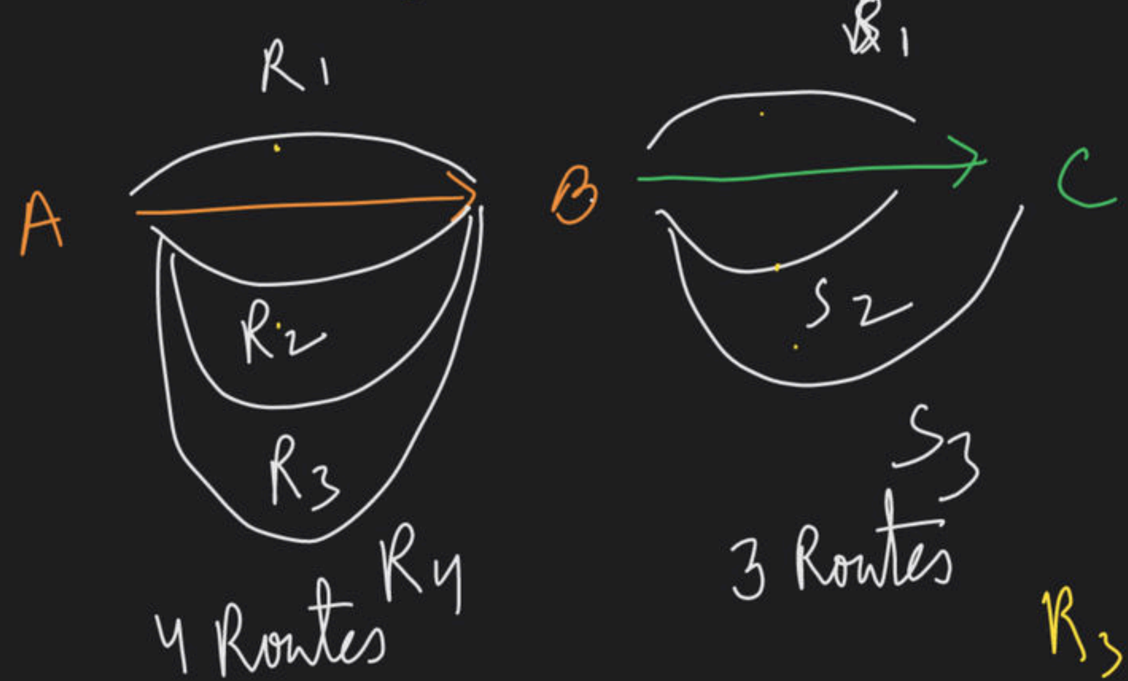
$= W_1 W_2$ ways.

A ⟶ B ⟶ C

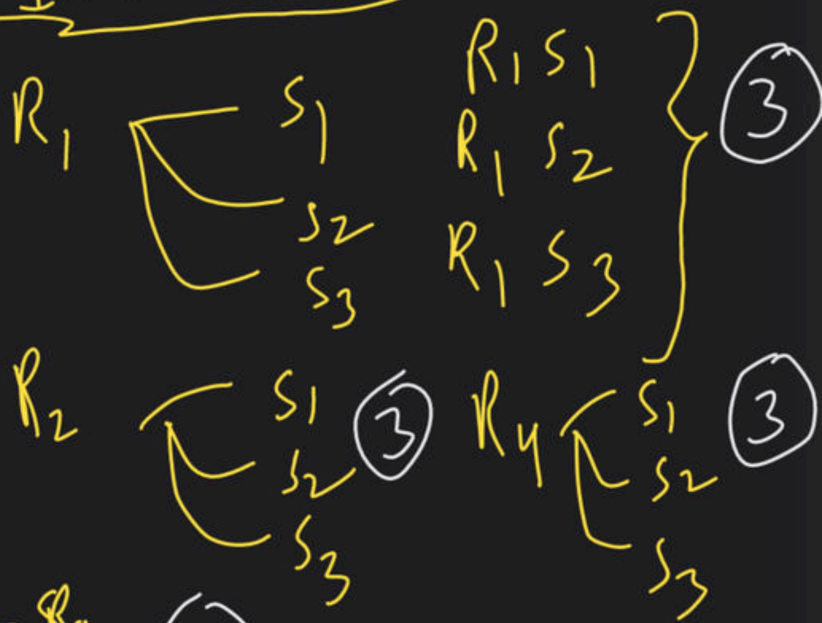
Lucknow Delhi Shimla

4 ways 3 ways

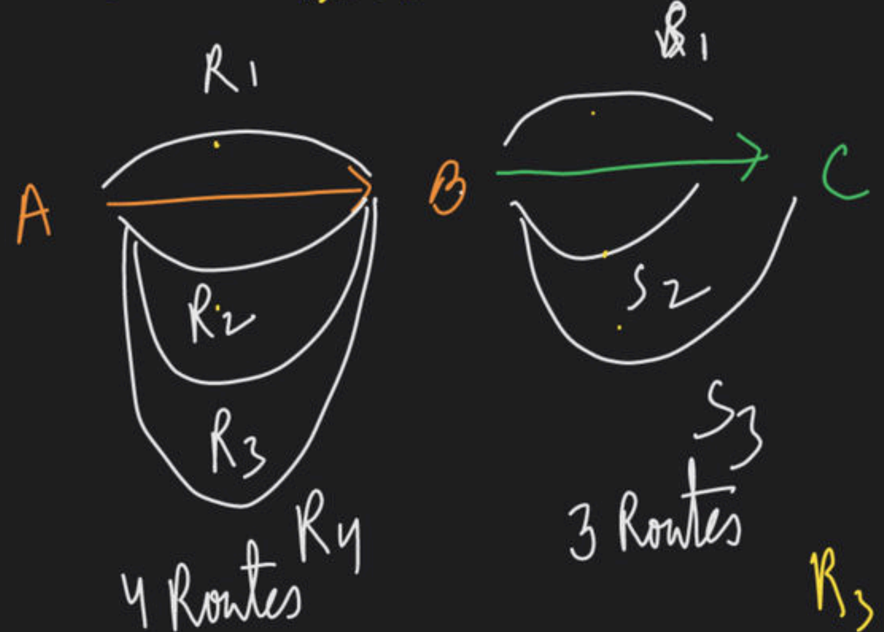
" How Many No. of ways Lucknow to Shimla via Delhi.



Ist view



11 How Many No. of ways known to shinder via Delhi.



1st view

R_1 → S_1, S_2, S_3 → R_1S_1, R_1S_2, R_1S_3 } 3 (A)

R_2 → S_1, S_2, S_3 (3) (3) (3) ; R_4 → S_1, S_2, S_3 (3) (C)

R_3 → R_4, S_2, S_3 (3) (D)

Total No. of ways = (A) + (B) + (C) + (D) = 3 added 4 tim

" A, B, C, D all are independent "

(A) OR (B) OR (C) OR (D) = 3+3+3+3

= 12

unacademy

$= \left(\text{No. of ways to Perform } W_2\right) \times \left(\text{No. of ways to Perform } W_1\right)$

$=$ 3 added 4 times.

$= 3 \times 4 = \underline{12 \text{ ways}}$.

If w jobs. Perform Simultaneously (Working Together) $= W_1 \times W_2 \times W_3 \times - - W_n$ ways.

(II)

Dal (3)
- (A)
- (3)
- (C)

} 3 ways.

Rice (2)
- ✓ simple Rice R_1
- ✓ Pea R_2

} 2 ways.

How Many No. of ways to eat the meal (Dal + chawal)

Dal A Rice R_1 | B R_1 | C R_1

Dal A Rice R_2 | B R_2 | C R_2

2 ways. | 2 ways. | 2 ways.

Independent

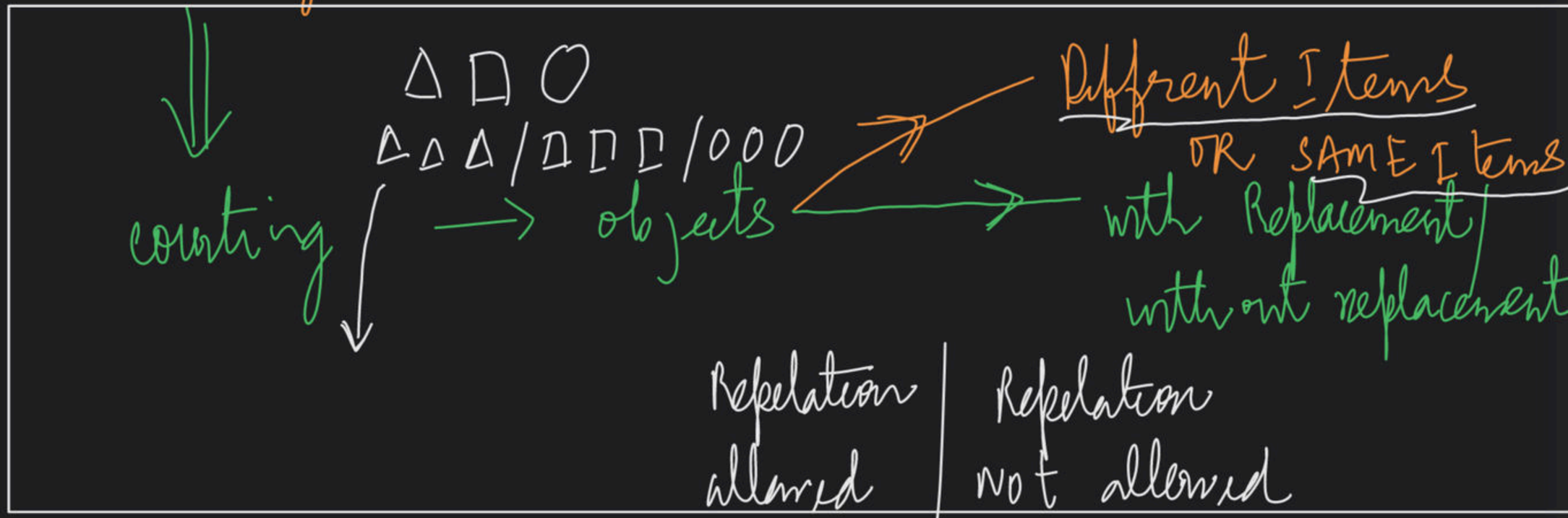
$2 + 2 + 2 = 2 \times 3 = 6$ ways.

Total No. of ways = 2 added 3 times.

unacademy

count # Addition → OR (W_1 perform OR W_2 perform)
(working Individual)

Multiplication → AND (W_1 perform and W_2 perform)
(working Together)



n Different Items Taken all at a time

(Repetitions allowed)
(with Replacement)

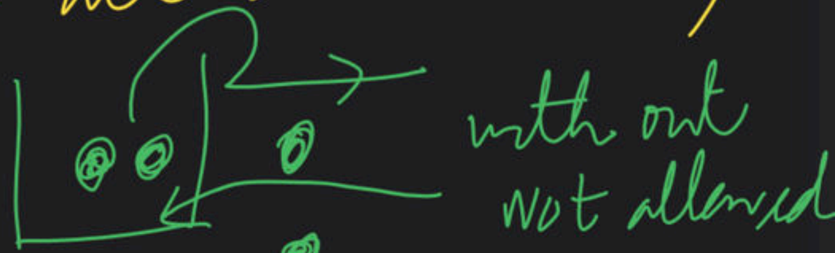
Throwing A 3 coins: Re 1, Re 2, Re 3

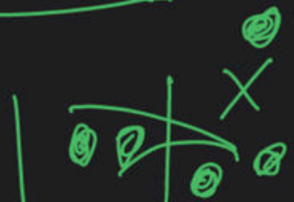
find The Total No. of ways.

Re 1, Re 2, Re 3 — n Diff. Items
(Taken all at a time)

Re 1 < H / T Re 3 < H / T

Re 2 < H / T





(N,T) W_1 Perform 2 ways.

(N,T) W_2 Perform 2 ways.

(N,T) W_3 Perform 2 ways.

W (Repetation allowed) (Simultaneous work) (Working Together)

Using FPC $= W_1 \times W_2 \times W_3$

$= 2 \times 2 \times 2 = (2^3)$ ✓ $\neq 3^2$

$= 8$ ways.

Die A  Die B 

W_1 W_2

W_1 perform = 6 Total No. of

W_2 perform = 6 ways

$= (6^2)$ $\neq 2^6$

✓ $W_1 = (1,2,3,4,5,6)$ $W_2 = (1,2,3,4,5,6)$

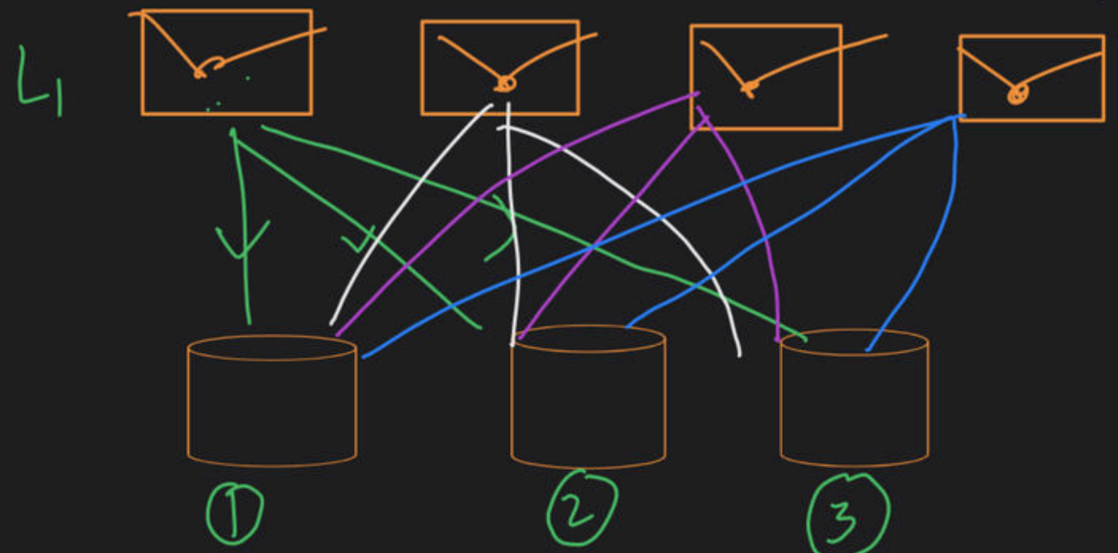
unacademy

#

Letter

P.O (simultaneous)

How many No. of ways SEND letter to post office.



Letter → ≠ 43

X Letter

Total ways

$= 3 \times 3 \times 3 \times 3 = 3^4$ times

unacademy

If n Different Items Taken all at a time
= Repetation allowed

$W_1 \rightarrow m$ ways. $W_2 = m$ ways. $W_3 = m$ ways.

Total no. of ways $= m \times m \times m \times \cdots n$ times

Total no. of ways $= (m)^n$

CASE 02

(n) Different Items Taken all at a time

(Repetation Not allowed)

(without Replacement)

A, B, C, D, E, F (6 Items)

Using Box method

6 blocks. Place (ONE Item)

W_1 W_2 W_3 W_4 W_5 W_6

A, B, C, D, E, F (6 Items)

Using BOX method

6 blocks. Place (ONE Item)

W_1 W_2 W_3 W_4 W_5 W_6

6 ways. Perform.

$W_2 \rightarrow$ 5 ways.

$W_3 \rightarrow$ 4 ways.

$W_4 \rightarrow$ 3 ways.

$W_5 \rightarrow$ 2 ways.

$W_6 \rightarrow$ 1 ways.

No. of ways = Dependent"

W_1 effect W_2

W_2 effect W_3

W_3 — W_4

W_4 — W_5

$W_5 \rightarrow W_6$

with out replacement

unacademy

Total No. of ways If Perform Together.

→ W = $\frac{B\ D\ E\ A\ C\ F}{}$

n Diff. + Taken all at a time
+ without Replacement

W ≠ B B D E A C

(Not allowed)

Total No. of ways = $W_1 \times W_2 \times W_3 \ldots W_6$

$= \underbrace{6 \times 5 \times 4 \times 3 \times 2 \times 1}_{\text{factorial}}$

$= 6!$

unacademy

7 objects

$w_1 \times w_2 \times w_3 \times w_4 \times w_5 \times w_6 \times w_7$

$= 7!$

n Diff. Items Taken all at a time

(Repetation allowed) / dependent / without Replacement

NOT

n	$(n-1)$	$(n-2)$	$(n-3)$	$(n-4)$		1
w_1	w_2	w_3	w_4			w_n

Total No. of ways $= n(n-1)(n-2)(n-3) \text{ ---- } 1$

$= n! = (n \text{ factorial})$

A) Choice - Reduced - DECREASE

B) Dependent

C) Next choice effected on previous ONE

$= 6 \times 5 \times 4 \times 3 \times 2 \times 1$

QUESTION

Q. How many n-digit numbers can be formed using 1, 2, 3, 7, 9 without any repetition of digits when:

(i) n = 5

n Different Items Taken all at a time (Repetation Not allowed

A. 120

B. 15

C. 6^5

D. 5^6

$= 5 \times 4 \times 3 \times 2 \times 1$

$= 5! = 120$

QUESTION

Q. How many n-digit numbers can be formed using 1, 2, 3, 7, 9 without any repetition of digits when:

(i) n = 3

5 Items

3 right

3 block

A. 12

B. 5^3

C. 60

D. 3^5

n Different Items Taken r at a time:

'5' Items Taken '3' at a a time

$= 5 \times 4 \times 3 = 60$ ways

unacademy

Total No. of ways = 5 X 4 X 3

= 60

5 Items

(without Replacement)

Default

n Different Items Taken r at a time

A B C D E

2 Items (Taken)

How Many No. of ways to arrange It.

n Diff.

A B C D E

→ 2 students at a time

EA AB BC CD DE

AE BA CB DC ED

Different

AB is Different → BA (order Matters)

AB | BA

both are Diff. + arrangement

n	(n-1)	(n-2)	....	n-(r-1)
w_1	w_2	w_3		
(n-0)	(n-1)	(n-2)	.....	n-(r-1)
↓0	↓1	↓2		↓(r-1)

If all are working Together $= n(n-1)(n-2)\cdots(n-(r-1))$

unacademy

Total No. of ways / arrangement

$$= n(n-1)(n-2)\cdots(n-(r-1)) \frac{(n-r)!}{(n-r)!}$$

No. of ways $= \dfrac{n!}{(n-r)!} = {}^{n}P_{r}$

Permutate = arrang. = order matter AB, BA (Diff)

n Diff. Taken r at a time (Repetition Not allowed)

(Arrangement Number) $= {}^{n}P_{r}$

QUESTION

Q. How many 3-letter words can be formed using a, b, c, d, e if:

(i) Repetition is not allowed

A. 60

B. 5^3

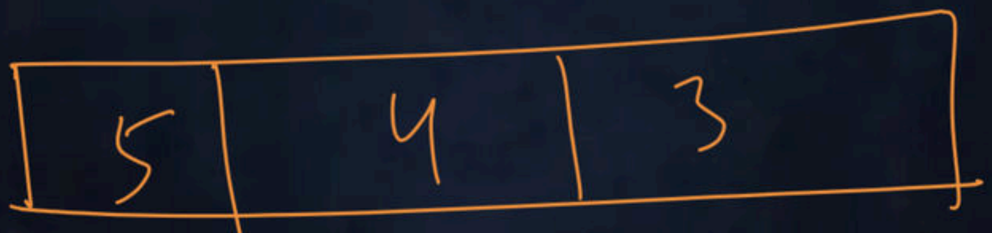
C. 3^5

D. 12

3 letters a, b, c, d, e

"n Diff. Items Taken r at a time"

$= {}^nP_r$



$w_1 \times w_2 \times w_3$

$= 60$

$= {}^5P_3 = \frac{L5}{L5-3}$

$= \frac{L5}{L2} = \underline{\underline{60}}$

QUESTION

Q. In how many ways can six persons be arranged in a row?

A. 6!

B. 6^6

C. 6^5

D. 5^6

n Diff. Items Taken all at a time $= {}^nP_n = n!$

$= 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6!$

n Diff. Items Taken r at a time $= {}^nP_r$

6 Diff. Items Taken '6' at a time

$= {}^6P_6 = 6! =$ SAME

QUESTION

Q. How many 5-digit odd numbers can be formed using digits 0, 1, 2, 3, 4, 5 without repeating digits?

5 Digit odd Numbers (without replacement)

(0, 1, 2, 3, 4, 5)

A. $4 \times 4!$

B. 288

C. 5!

D. 300

"Counting with Restrictions"

⇓ "odd Numbers"

0, 1, 2, 3, 4, 5 ⟶ Last digit 1, 3, 5 odd (Last digit)

W_5 Restrict job:

$W_5 \rightarrow$ 3 ways (1, 3, 5)

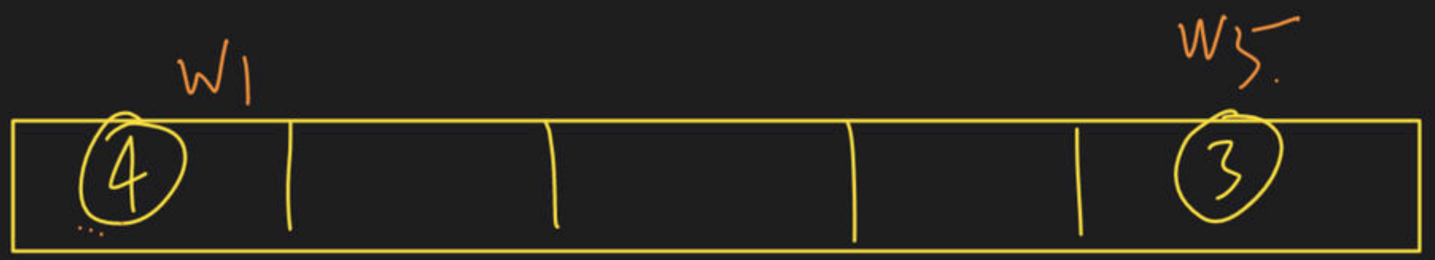
0, 1, 2, 3, 4, 5

0 1 2 3 4 5 ✗

0 - W_1 #

1, 5, 2, 3

0 ✗



Total No. of ways = 4 × 4 × 3 × 2 × 3

= 288 ways

QUESTION

Q. How many 5-digit numbers divisible by 2 can be formed using digits 0, 1, 2, 3, 4, 5 without repetition of digits.

A 120

B 192

C 312

D 208

5 digit Numbers divisible by 2

0, 1, 2, 3, 4, 5

counting with Restrictions

Total No. of ways = $5 \times 5!$ — odd

$= 5 \times 5! - 288 = 600 - 288$

$= 312$

0, 1, 2, 3, 4, 5

w_5 options (Last)

(0, 2, 4)

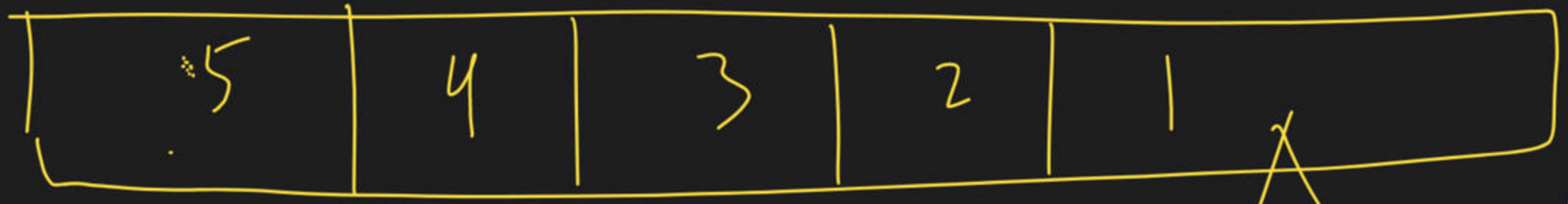
Even No.

0, 2, 4

$= 4 \times 4 \times 3 \times 2 \times 2 = 192$

192

+



$= 5 \times 4 \times 3 \times 2 \times 1$

$= 120$

120

included

$= 312$ Ans

QUESTION

B

Q. How many six-digit numbers divisible by 25 can be formed using 0, 1, 2, 3, 4, 5? (Repetition not allowed)

Homework

Task

A 24

B 42

C 256

D 100

= 24 + 18 = 42

W1	W2	W3	W4	W5	W6

25
50

divisible by 25

24
18
= 42

H.W

QUESTION

No. of arrangement

Q. Find number of different words which can be formed using all the letters of the word 'HISTORY'?

n Diff. Items Taken all at a time
(Repetation not allowed)
$= n!$

A 720

B 5040

C 2520

D 360

$= 7! = 5040$

7	6	5	4	3	2	1

$= 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

QUESTION

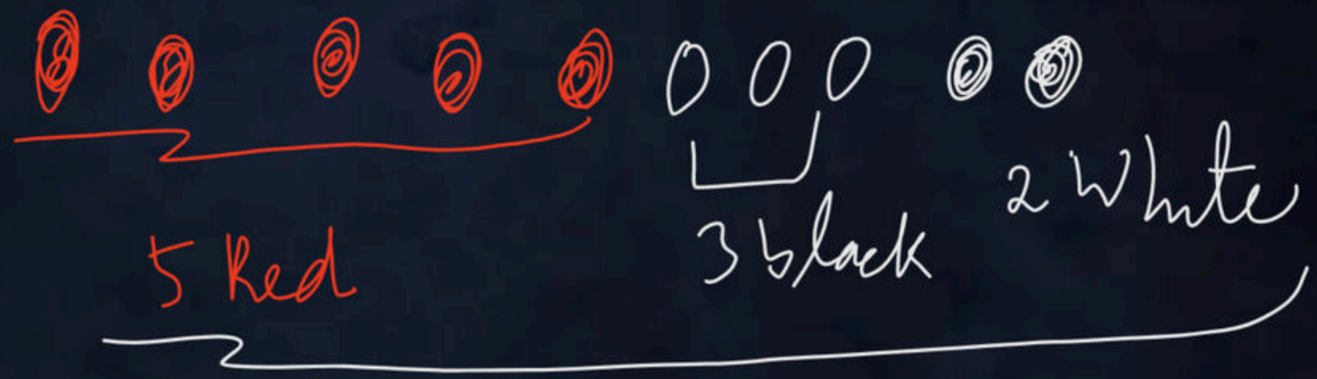
Q. In how many ways 5 different red balls, 3 different black balls and 2 different white balls can be arranged along a row?

A. $10!$

B. 10^{10}

C. $10^{10} - 10!$

D. None of these



Taken all at a time
(Repetation NOT allowed)

$= n! = 10! = \checkmark$

QUESTION

✓ Read the Problem.

Q. In how many ways can be letters of the word 'DELHI' be arranged so that the vowels occupy only even places?

A 6

B 12

C 24

D 48

DELHI

counting with Restrictions

odd	Even	odd	Even	odd
③	②	②	①	1
W_1	W_2	W_3	W_4	W_5

W_2 W_4 fixed

E I, 2!

D, L, H 3

$\Rightarrow 3 \times 2 \times 2 \times 1 \times 1 = 12$ ways.

Natural No.

0 Whole No

1 2 3 4 Natural No.

II nd method :. DELHI

W_1 | DLH | E, I W_2

Jobs. n Diff. Taken all at a time E | n Diff. Taken all at a time

$= 3!$ | $= 2!$

$W = W_1 \times W_2 = 3! \times 2!$

$= 12$ ways.

QUESTION

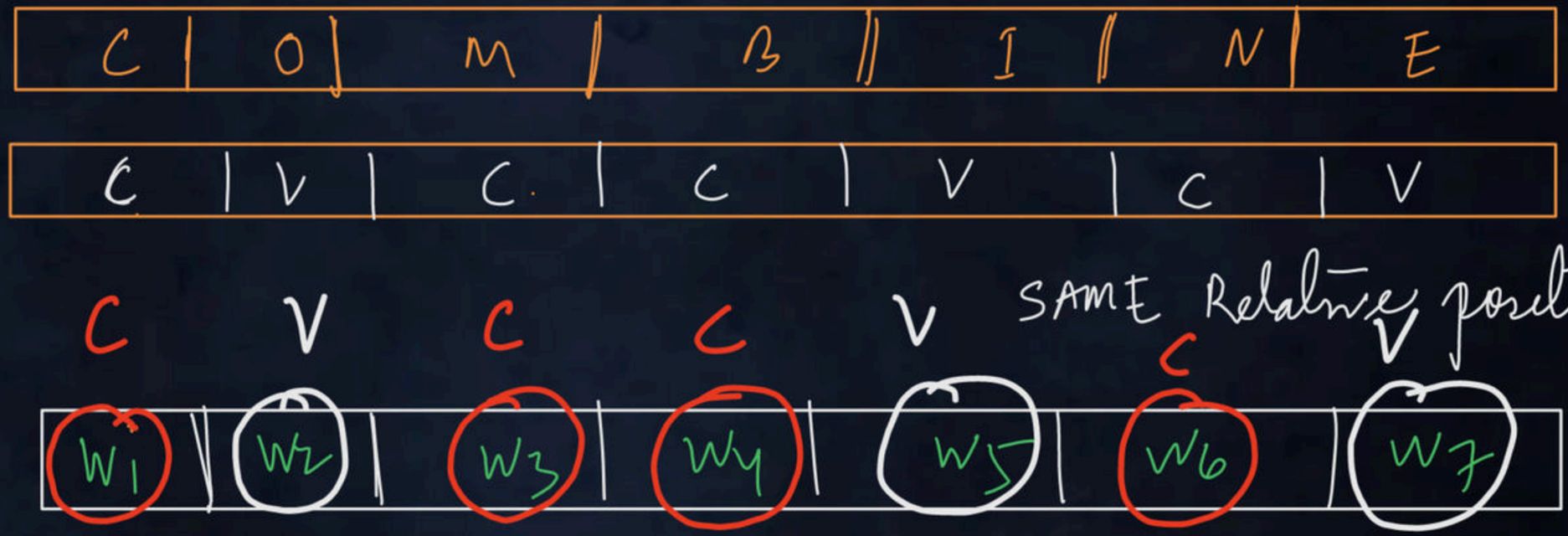
Q. In how many of these words do the vowels and the consonants occupy the same relative positions as in 'COMBINE'?

A 144

B 720

C 5040

D 360



C	O	M	B	I	N	E

C	V	C	C	V	C	V

4	3	3	2	2	1	1

$4C + 3V$

$= 4 \times 3 \times 3 \times 2 \times 2 \times 1 \times 1$

$= \underline{144}$ $\underline{\text{ways}}$

QUESTION

n Diff. Items

Q. How many words can be formed using letters of the word 'EQUATION' taken all the a time?

Taken all at a time

(A) $8!$

(B) $8 \times 7!$

(C) $7!$

(D) $4 \times 7!$

$= 8!$ Ans.

$8! = 8 \times 7!$ ✓

DONE

QUESTION

counting with Restrications

Q. How many words can be formed using letters of the word 'EQUATION' that begin with E and end with N?

1 X — n Diff. Token all at a time — X 1

= 1 X 6! X 1

= 6! answer

A. $2 \times 6!$

B. $7!$

C. $2 \times 7!$

D. $6!$

QUESTION

Q. How many words can be formed using letters of the word 'EQUATION' that begin and end with a consonant?

A 4320

B 720

C 1440

D 2880

Answer = 4320

QUESTION

Q. How many words can be formed using letters of the word 'EQUATION' such that vowels occupy the first, third, fourth, sixth & seventh positions?

A 360

B 720

C 120

D None of these

Homework

H.W

QUESTION

Q. In how many ways 5 different red balls, 3 different black balls, and 2 different white balls can be placed in 3 different boxes such that each box contains only 1 ball.

A 360

B 720

C 1440

D 1080

QUESTION

Q. In how many ways can 5 letters be posted in 4 letter boxes?

A 4^5

B 5^5

C $5!$

D $4!$

QUESTION

Q. Five persons entered the lift cabin on the ground floor of an 8-floor house. Suppose each of them can leave the cabin independently at any floor beginning with the first. Find the total numbers of ways in which each of the five persons can leave the cabin:

(i) at any one of the 7 floors

A 5^7

B 7^5

C $5!$

D $7!$

QUESTION

Q. There are 6 single choice questions in an examination. How many sequence of answers are possible, if the first three questions have 4 choices each and the next three have 5 each?

A 15625

B 8000

C 4000

D 4096

QUESTION

Q. How many triangles can be formed by joining the vertices of a hexagon?

A 10

B 20

C 30

D 60

QUESTION

Q. How many diagonals are these in a polygon with n sides?

A $\frac{n(n-1)}{2}$

B $\frac{n(n+1)}{2}$

C $\frac{n(n-3)}{2}$

D $\frac{n(n+3)}{2}$

QUESTION

Q. In how many ways can a cricket team be selected from a group of 25 players containing 10 batsmen, 8 bowlers, 5 all-rounders and 2 wicketkeepers? Assume that the team of 11 players requires 5 batsmen, 3 all-rounders, 2-bowlers and 1 wicketkeeper.

A $\frac{10!}{5!}$

B $\frac{14\times10!}{3\times5!}$

C $\frac{14\times10!}{5!}$

D $\frac{10!}{3\times5!}$

QUESTION

Q. A man has 7 relatives, 4 of them are ladies and 3 gentlemen, his wife has 7 relatives, 3 of them are ladies and 4 gentlemen. In how many ways can he invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of man's relatives and 3 of wife's relatives?

A 144

B 720

C 485

D 340

QUESTION

Q. A tea party is arranged for 16 people along two sides of a long table with 8 chairs on each side. Four men wish to sit on one particular side and two on the other side. In how many ways can they be seated?

A. $\frac{8!10!}{4!6!}$

B. $\frac{8!8!10!}{4!6!}$

C. $\frac{8!8!}{4!6!}$

D. $\frac{8!8!}{6!}$

unacademy

unacademy

orientation
Unacademy
Aditi

THANK YOU!

Here's to a cracking journey ahead!

LIVE
Recording

group - google drive
link

Whats app

resolution