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Engineering Mathematics

Probability & Statistics

counting principles

Lecture Number- 02

By- Rahul Sir

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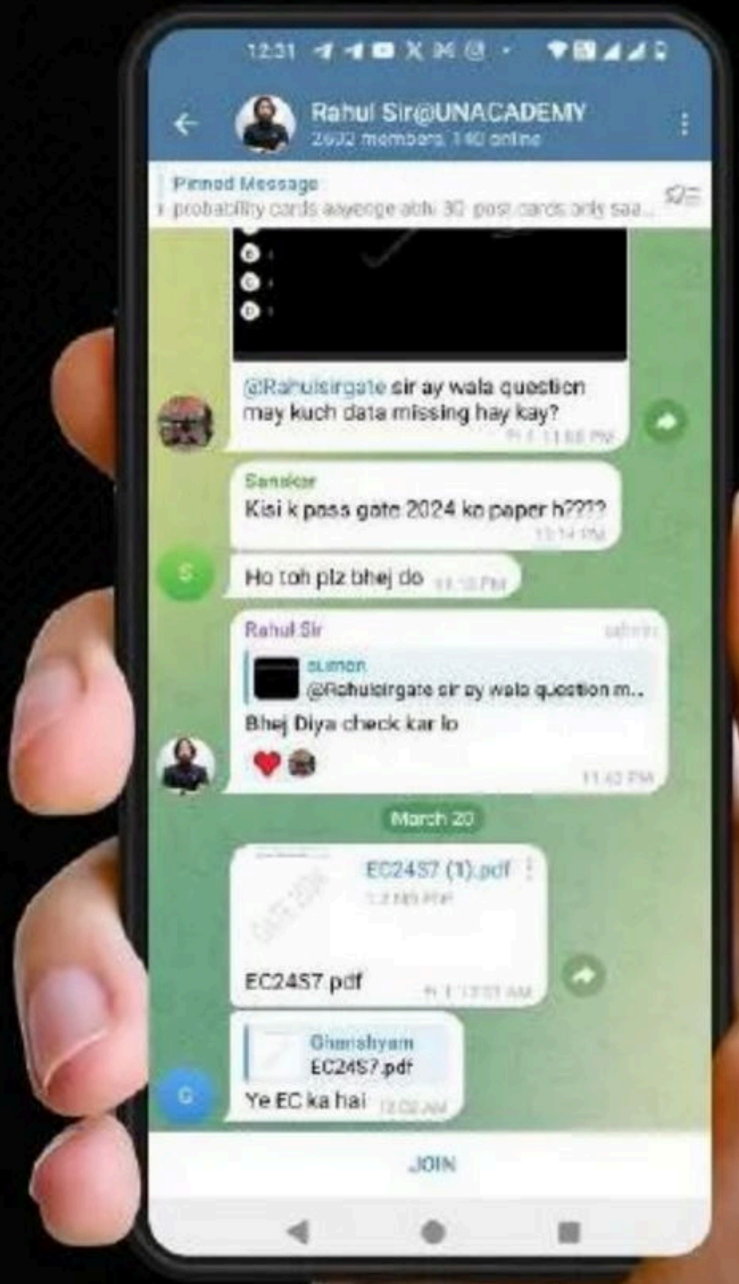

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Topics

to be covered

1. counting principles_II

QUESTION

(SELF assessment problem)

Illustration:

Q. How many 3 digit numbers can be formed by the digit 1, 2, 3, 4, 5 without repetition

$= 60$

QUESTION

Illustration:

Q. In an examination of 10 T/F question, How many sequence of answers are possible.

$= 2^{10}$

QUESTION

Illustration:

Q. 10 students complete in a swimming race. In how many ways can they occupy the first 3 positions.

$= 720$

QUESTION

Illustration:

Q. There are 7 flags of diffrent colour. Find the number of diffrent signals that can be transmitted by the use of 2 flags one above the other.

$= 42$

QUESTION

Illustration:

Q. A letter lock consists of 3 rings each marked with 10 different letters. In how many ways, its is possible to make an unsuccessful attempt to open the lock?

$= 999$

QUESTION

Illustration:

Q. How many 6 digits odd number greater than 6,00,000 can be formed from the digits 5,6,7, 8, 9, 0 if repetition of digit is allowed?

15552

QUESTION

Illustration:

Q. How many integers greater than 5000 can be formed with the digit 7, 6, 5, 4 & 3 using each digit at most once.

192

QUESTION

Illustration:

Q. How many natural number less than 30000 can be made from the digits 0, 1, 2, 3, 4, 5, 6.

$= 3 \times 7^4 - 1$

QUESTION

Illustration:

Q. Consider the word DAUGHTER. How many 4 letter word can be formed from the letter of above word so that each word contain letter G

$= 840$

QUESTION

Illustration:

Q. How many different words can be formed using all the letters in the word "MIRACLE".

(a) If vowels may accupy the even position.

(b) If vowels may occupy odd position.

A) 144

B) 576

QUESTION

Illustration:

Q. There are m men and n monkey. Number of ways in which every monkey has a master, if a man can have any number of monkey.

$= (m)^n$

QUESTION

Illustration:

Q. Number of ways in which m different toys can be distributed in "n" children if every child may receive any number of toys

$= n^m$

QUESTION

Illustration:

Q. Find the number of ways in which we can post 5 letters in 10 letter boxes.

$= 10^5$

QUESTION

Illustration:

Q. In a car plate number containing only 3 or 4 digits not containing the digit 0. What is the maximum numbers of cars that can be numbered?

$= 7290$

05

n Different Items Taken all at a time

(Repetation allowed) = $(n)^n$ — (A)

n Diff. Items Taken all at a time

(Repetation Not allowed) = $n!$ — (B)

(arrangement)

n Diff. Items Taken 'r' at a time

Repetation Not allowed = ^nP_r — (C)

$$^nP_r = \frac{\lfloor n}{\lfloor n-r}$$

$$\lfloor n = n(n-1)(n-2)\cdots 1$$

QUESTION

GEORGE Polya ⇒ Read The Problem: 99%
understand The Problem

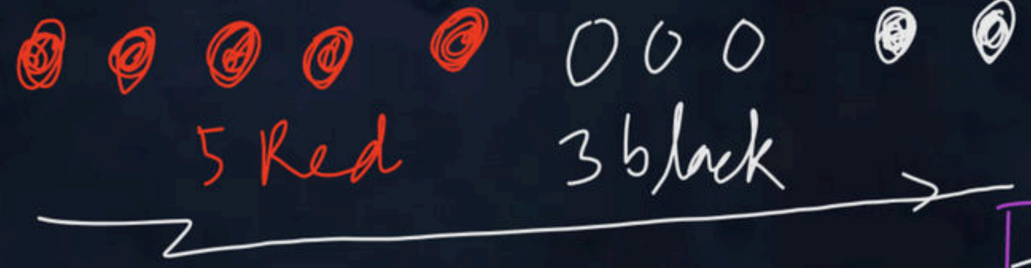
Q. In how many ways 5 different red balls, 3 different black balls, and 2 different white balls can be placed in 3 different boxes such that each box contains only 1 ball.

A 360

B 720

C 1440

D 1080



10 Diff. ball Taken 1 at a time

$= 10 \times 9 \times 8 = 720$

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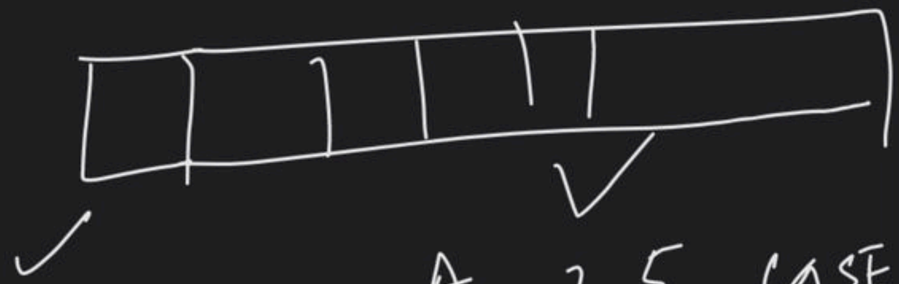
0, 1, 2, 3, 4, 5

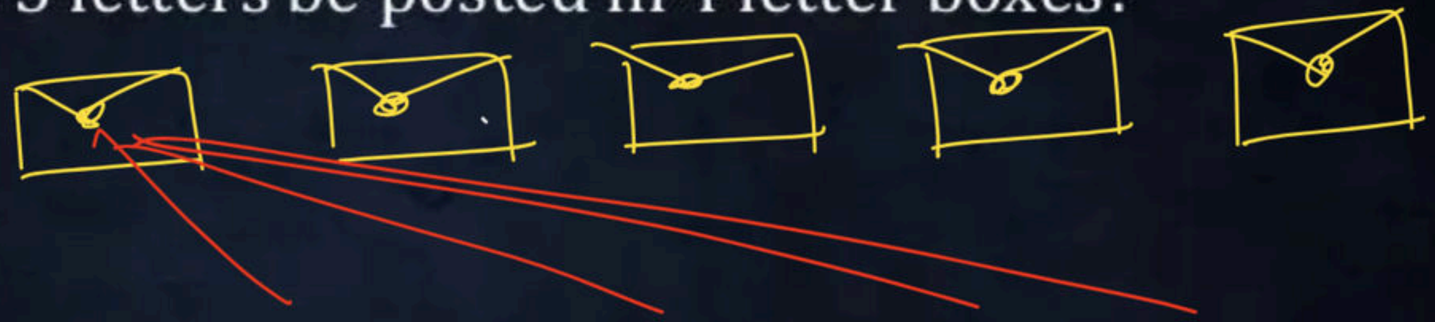


A 25 CASE

(B) 50

QUESTION

Q. In how many ways can 5 letters be posted in 4 letter boxes?



A 4^5

B 5^5

C 5!

D 4!

$= 4 \times 4 \times 4 \times 4 \times 4$

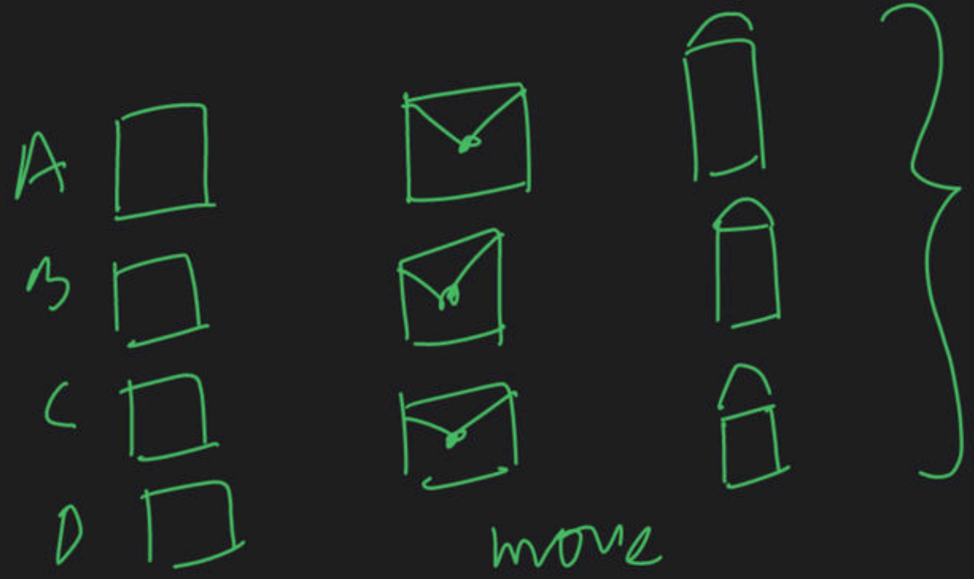
$= 4^5$ Total No. of ways.

Items

Replacement

Not replacement

(Default)



move

counting

Run

(No. of ways)

QUESTION

Q. Five persons entered the lift cabin on the ground floor of an 8-floor house. Suppose each of them can leave the cabin independently at any floor beginning with the first. Find the total numbers of ways in which each of the five persons can leave the cabin:

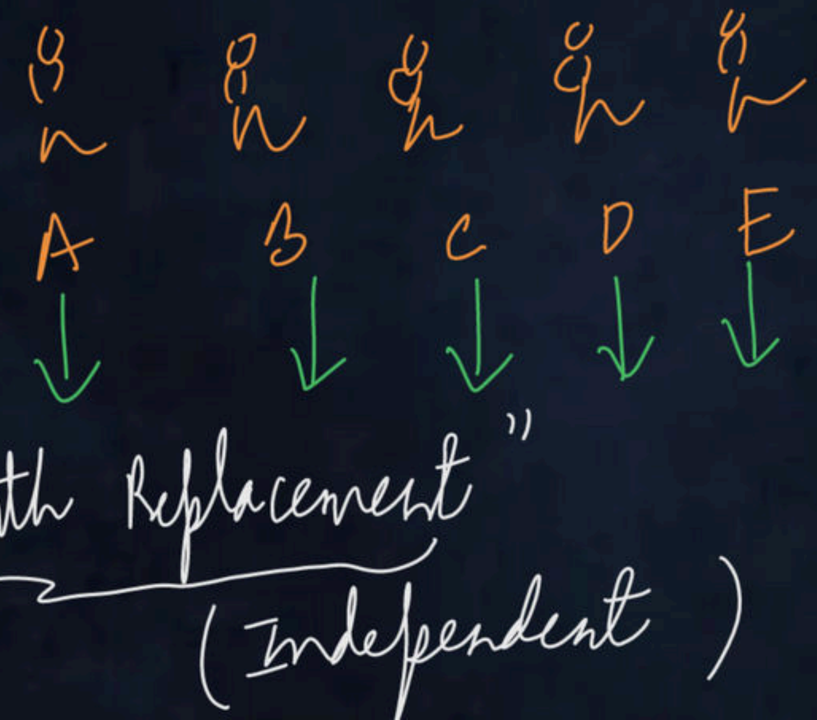
(i) at any one of the 7 floors

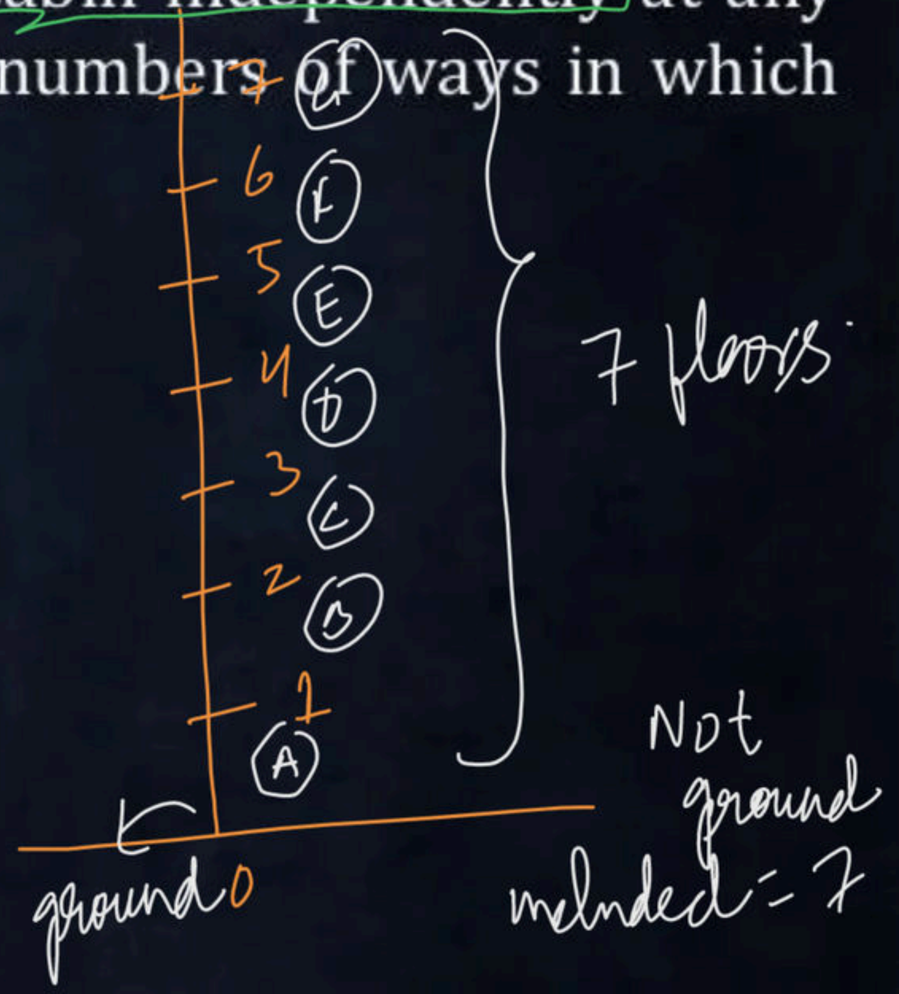
A 5^7

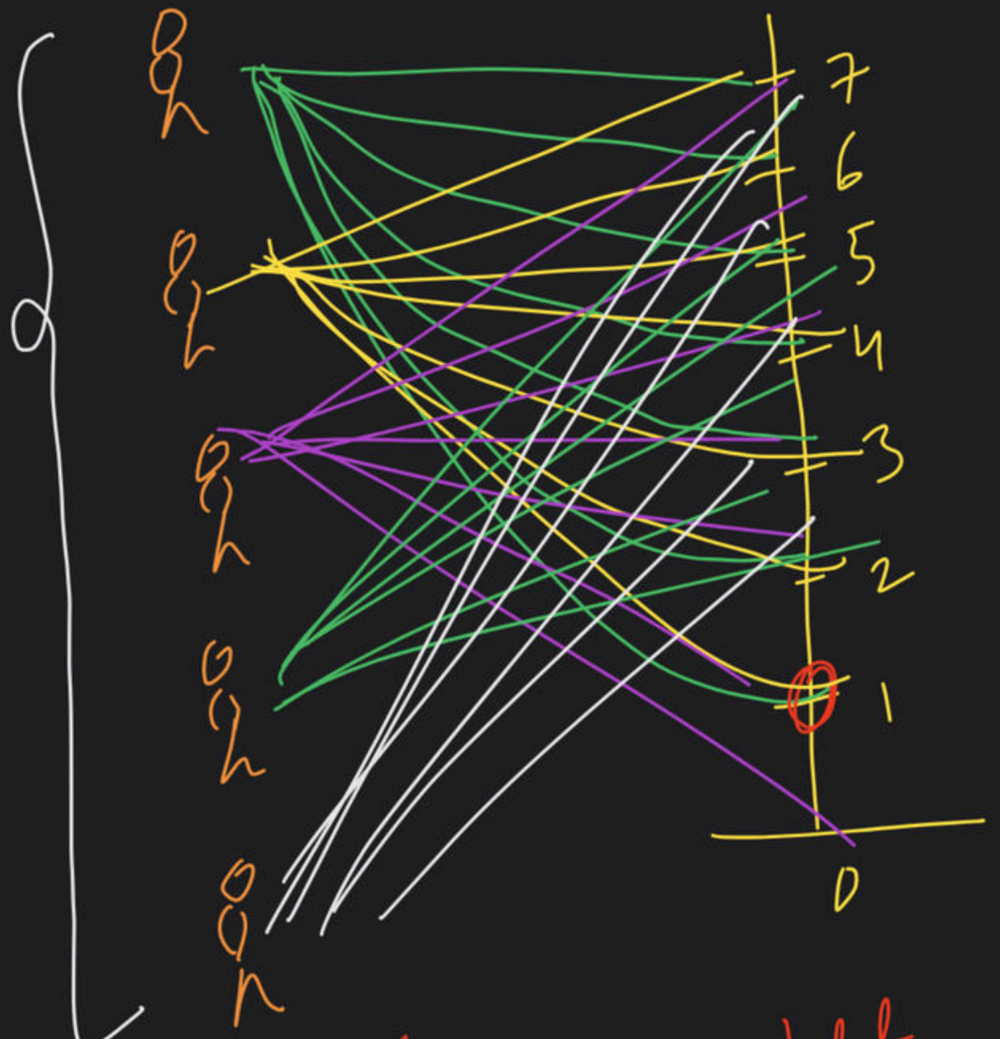
B 7^5

C $5!$

D $7!$







$= 7 \times 7 \times 7 \times 7 \times 7$

$= 7^5$ (answer)

person ← left

(Bijective function) = (bijective + surjective)

= No. of mappings

$= {}^{n}P_r = {}^{n}C_r \times r!$

{ONE ONE
+ONTO

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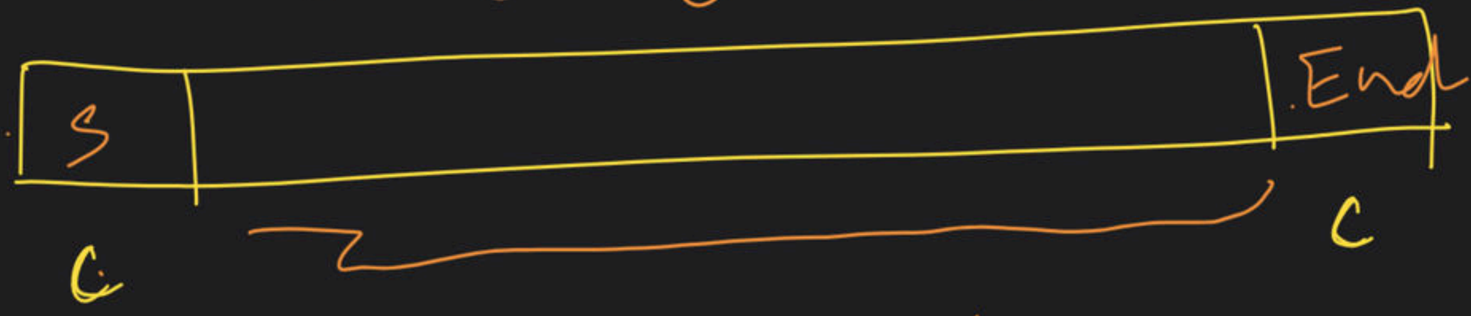
EQUATION

How many No. of ways # begin And End with The consonant



Q, T N

3 X 6.1 X 2

= answer = 720 X 6 = 4320 answer

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ARRANGEMENT

05

(A) # n Different Items Taken all at a time
(Repetation allowed) $= (n)^n$

(B) # n Diff. Items Taken all at a time
(Repetation Not allowed) $= n!$
(arrangement)

(C) # n Diff. Items Taken 'r' at a time
Repetation Not allowed $= {}^nP_r$

$${}^nP_r = \frac{\lfloor n}{\lfloor n-r}$$

$\lfloor n = n(n-1)(n-2)\cdots 1$

1) # n Diff. Items Selection all at a time

Selection → group / committee / choices / selection

objects

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c_1, c_2, c_3, c_4, c_5 — 5 objects (different)

Selection all at a time

Selection ⇒ (A) ORDER DOES NOT matter.

✓ (Rahul, akash) = {akash, Rahul}

group. group.

Permutation # (A, R)

(R, D)

(AB) (BA)

(ABC) (CBA)

✓ SAME group.

SAME group. = order does matter

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C_1, C_2, C_3, C_4, C_5

group: A $\{C_1\ C_2\ C_3\ C_4\ C_5\}$

group: B $\{C_2\ C_1\ C_3\ C_4\ C_5\}$

group: C $\{C_5\ C_1\ C_2\ C_3\ C_4\}$

" all are SAME group/ SAME committee "

No. of Ways to Select n Different Items Selection all at a time = 1

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n Different Items Selection 'r' at a time

⇓ group / selection / committee

1) A, B, C — 3 Diff. Items

" 3 Diff. Items **selection** 2 at a time "

AB BA

BC CB

AC CA

No. of selection = 3

No. of selection × ordering = Permutation

AB

BC

AC

3 ways selection

⇒ $^3P_{(2)}$ = Ist work Permutation

No. of selection = $\frac{^3P_2}{2!} = \frac{3}{}$ ✓

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A B C D

ABC
BCD
CDA
BAD
} CDA | ACB | equal (order does Not matter)

ACD
CDA

4

SAME

5 Items selection 2 at a time

A, B, C, D, E

$= \frac{5!}{3!\,2!} = \frac{120}{6} = 10$

$\frac{{}^5P_2}{L^2} = \frac{5!}{3!\,2!}$

4 Diff. Items selection

3 at a time

$= \frac{{}^4P_3}{L3} = \frac{4!}{3!} = 4$

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n Diff. Items Selection 'r' at a time

No. of Selection × ordering = No. of arrang.

No. of Selection × ordering = n Diff. Items Taken 'r' at a time

$$x \times r! = {}^{n}P_r$$

$$x = \frac{{}^{n}P_r}{r!}$$

No. of Selection Taken 'r' at a time

$$x = {}^{n}C_r$$

Dependent

n Diff. Items Taken all at a time $= (n)^n$
(Repetation allowed)

Independent

n Diff. Items Taken all at a time $= n!$
(Repetation not allowed)

n Diff. ——— Taken r at a time $= {}^nP_r$

n Diff. Items Selection all at a time $= 2^n$

Selection 'r' at a time $= {}^nC_r$

$${}^nC_r = \frac{\lfloor n}{\lfloor n-r \;\; \lfloor r}$$

$${}^nP_r = \frac{\lfloor n}{(\lfloor n-r)}$$

QUESTION

Q. There are 6 single choice questions in an examination. How many sequence of answers are possible, if the first three questions have 4 choices each and the next three have 5 each?

A 15625

B 8000

C 4000

D 4096

QUESTION

Q. How many triangles can be formed by joining the vertices of a hexagon?

A 10

B 20

C 30

D 60

QUESTION

Q. How many diagonals are these in a polygon with n sides?

A $\frac{n(n-1)}{2}$

B $\frac{n(n+1)}{2}$

C $\frac{n(n-3)}{2}$

D $\frac{n(n+3)}{2}$

QUESTION

Q. In how many ways can a cricket team be selected from a group of 25 players containing 10 batsmen, 8 bowlers, 5 all-rounders and 2 wicketkeepers? Assume that the team of 11 players requires 5 batsmen, 3 all-rounders, 2-bowlers and 1 wicketkeeper.

A. $\frac{10!}{5!}$

B. $\frac{14\times10!}{3\times5!}$

C. $\frac{14\times10!}{5!}$

D. $\frac{10!}{3\times5!}$

25 PERSONS
10 Batsman 5 all
8 bowlers 2 WK

11 Players.

(Team committee group selection)

5 batsman
3 all rounders
2 bowlers
1 WK

25 PERSONS
10 Batsman 5 all
8 bowlers 2 WK

→ 11 Players.
(Team committee group selection)

5 batsman
3 all rounders
2 bowlers
1 WK

W_2 W_3 W_4 — FPC W_1

TEAM = together = multiply

Total No. of Selection (CHOICES)

Batsman: 10 → 5 = $^{10}C_5$ W_1

all rounders: 5 → 3 = $^{5}C_3$ W_2

2 bowlers: 8 → 2 = $^{8}C_2$ W_3

2 WK: 2 → 1 = $^{2}C_1$ W_4

TEAM 11

Total No. of Selection $= {}^{10}C_5 \times {}^{5}C_3 \times {}^{8}C_2 \times {}^{2}C_1 =$ (13)

QUESTION

Q. A man has 7 relatives, 4 of them are ladies and 3 gentlemen, his wife has 7 relatives, 3 of them are ladies and 4 gentlemen. In how many ways can he invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of man's relatives and 3 of wife's relatives?

→ Selection / groups

(A) 144

(B) 720

(C) 485

(D) 340

Man → 7 Relatives 4 ladies / 3 gentlemen

wife → 7 Relatives 3 ladies / 4 gentleman

(3 ladies / 3 gentleman)

3 Man / 3 wife

M & W

Counting / combination with Restriction

wife

No. of selection

Ladies	gentleman	Ladies	gentl.	
4	3	3	4	$W_1 \times W_2 \times W_3 \times W_4$ = and
3	0	0	3	${}^4C_3 \times {}^3C_0 \times {}^3C_0 \times {}^4C_3$ = ✓ 16
0	3	3	0	${}^4C_0 \times {}^3C_3 \times {}^3C_3 \times {}^4C_0$ = ✓ 324.
2	1	1	2	${}^4C_2 \, {}^3C_1 \, {}^3C_1 \, {}^4C_2$ = ✓ 144
1	2	2	1	${}^4C_1 \times {}^3C_2 \, {}^3C_2 \, {}^4C_1$ = ✓ 1

OR OR OR OR

Ans = 485 ✓

QUESTION

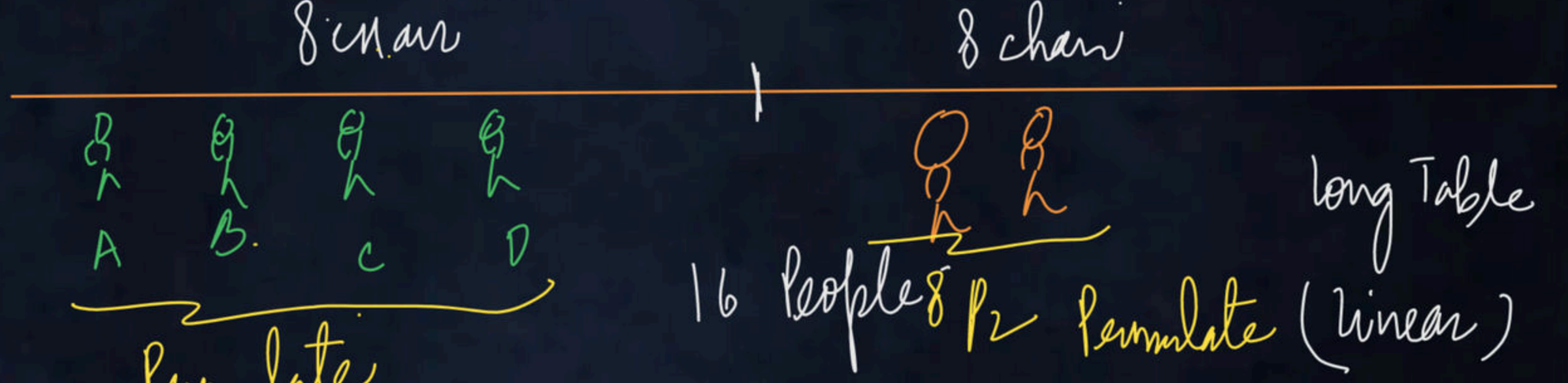
Q. A tea party is arranged for 16 people along two sides of a long table with 8 chairs on each side. Four men wish to sit on one particular side and two on the other side. In how many ways can they be seated?

(A) $\frac{8!10!}{4!6!}$

(B) $\frac{8!8!10!}{4!6!}$

(C) $\frac{8!8!}{4!6!}$

(D) $\frac{8!8!}{6!}$



$^8P_4 \times {}^8P_2 \times 10!$ answer ①

OR $^8C_4\ 4! \times {}^8C_2\ 2! \times 10!$ answer ②

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8 chars | 8 chars

4 Person | 2 Person

(arrange) | (arrange)

$= {}^{8}P_4 \; {}^{8}P_2 \times 10!$ answer

OR $= {}^{8}C_4 \; 4! \times {}^{8}C_2 \times 2! \times 10!$ answer

QUESTION

Q. In how many ways we can arrange letter A, A, B, B, B, C taken 3 at a time.

QUESTION

Q. How many four-letter words can be formed using the letters of the word 'INEFFECTIVE'?

A 840

B 1380

C 1422

D None of these

QUESTION

Q. In how many ways 3 letters can be selected from letter A, A, B, B, B, C?

QUESTION

Q. In how many ways 4 letters can be selected from letter of the word 'INEFFECTIVE'?

A 80

B 89

C 51

D None of these

QUESTION

Q. A man has 5 friends. In how many ways can be he invite one or more of them to a party?

A 32

B 31

C 30

D 16

QUESTION

Q. The question paper in the examination contain three sections – A, B, C. Therefore are 6, 4, 3 questions in sections A, B, C respectively. A student has the freedom to answer any number of question attempting at least one from each section. In how many ways can the paper be attempted by a student?

A 8192

B 6615

C 7168

D None of these

QUESTION

Q. In a box there are 10 balls, 4 red, 3 black, 2 white and 1 yellow. In how many ways can a child select 4 balls out of these 10 balls? (Assume that the balls of the same color are identical)

A 20

B 18

C 19

D None of these

QUESTION

Q. In how many ways a team of 11 players be selected from a list of 16 players where two particular players should always be included in the team.

QUESTION

Q. In how many ways a team of 11 players can be selected from a list of 16 players such that 2 particular player should be included in the selection.

QUESTION

Q. How many three-letter words can be made using the letters of the words 'SOCIETY', so that.

(i) 'S' is included in each word?

(ii) 'S' is not included in any word?

(iii) To include S in every word, we have two cases.

QUESTION

Q. How many words can be formed using the letters of the word "TRIANGLE" so that

(i) 'A' and 'N' are always together?

(ii) 'T', 'R', 'I' are always together?

QUESTION

Q. How many five-letter words containing 3 vowels and 2 consonants can be formed using the letters of the word 'EQUATION' so that the two consonants occur together in every word?

A 240

B 1440

C 720

D 480

QUESTION

ONE of MOST Important concepts

Q. There are 9 candidates for an examination out of which 3 are appearing in Mathematics and remaining 6 are appearing in different subjects. In how many ways can they be seated in a row so that no two mathematics candidates are together?

9 candidates C_1 C_2 C_3 C_4 C_5 C_6 C_7 C_8 C_9

M_1 M_2 M_3 "3" "6 are Different"

" NO TWO mathematics candidates are Together

" NO TWO boys are Together / NO TWO girls are Together

(alternative)

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$M_1\ M_2\ M_3\ O_1\ O_2\ O_3\ O_4\ O_5\ O_6$ 9 cards

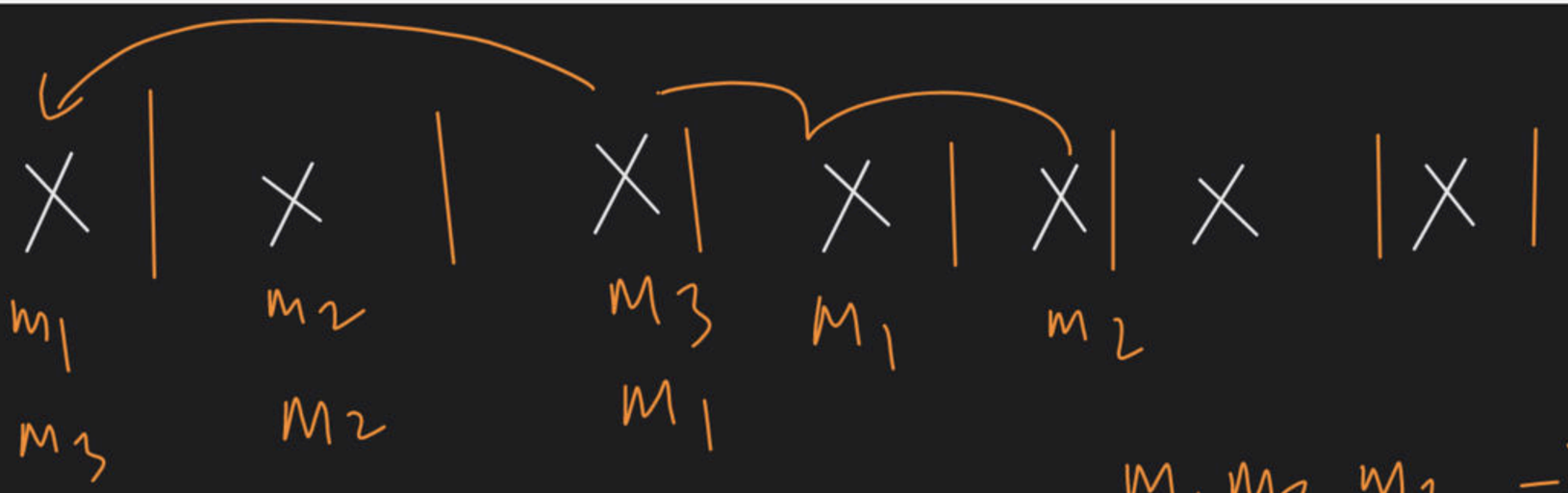
Using CROSS / BAR method (gap method)

O_1 O_2 O_3 O_4 O_5 O_6 other

(bar)

No. of Planes = 7 (n Different Position)
Select 3 at a time

No. of choices = $^7C_3$ No. of ways = $^7C_{3^0}$ (n)



m_1, m_2, m_3 → ordering

No. of ways $= {}^{7}C_3 \times 3!$ ✓

Total No. of ways = (Ways) × (Bar)

$= {}^{7}C_3 \times 3! \times \frac{6!}{2!}$

OR

$= {}^{7}P_3 \times 6!$

$=$ answer

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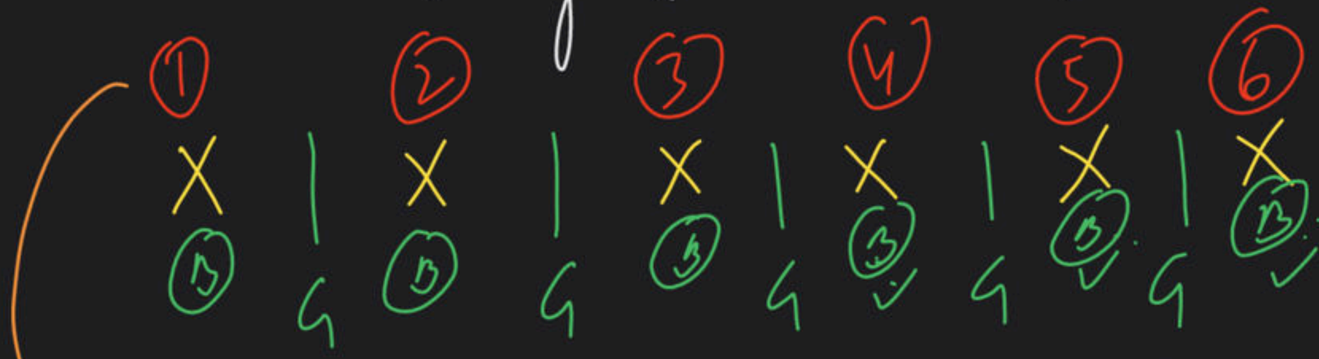
5 girls 3 boys

Arranging (bar + cross)

(No 2 boys are sit Together)



Girl - bar

Cross - boy.

6 Cross. → 3 boys. Select X order.

OR

3 boys. Permutate (arrang)

ABC
BAC
ACB
BCA

$= {}^6C_3 \cdot 3! = (\text{Cross})$

Total No. of ways = Cross X bar $= {}^6C_3\, 3! \times 5!$

QUESTION

Q. In how many ways 7 plus (+) signs and 5 minus (–) signs be arranged in a row so that no two minus (–) signs are together?

A. ${}^8C_5$

B. ${}^8C_5 \times 7! \times 5!$

C. ${}^8C_5 \times 5!$

D. ${}^8C_5 \times 7!$

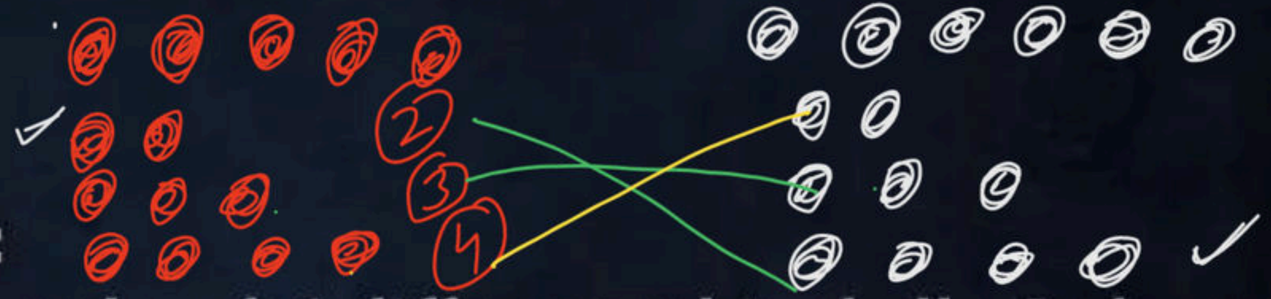
7 + 5 – Two Minus Signs are Together

X + X + X + X + X + X + X + X

= ${}^8C_5 \times 1 \times 1$ answer

No Job = Cross × Bar all are similar

QUESTION



Q. **Illustrating the concepts:**

A box contains 5 different red and 6 different white balls. In how many ways can 6 balls be selected so that are at least two balls of each colour?

Red (5)	White (6)		No. of selection
● ●	● ● ● ●	(6)	$5C_2 \times 6C_4 = 150$ ✓
● ● ●	● ● ●	(6)	$5C_3 \times 6C_3 = 200$ OR
● ● ● ●	● ●	(6)	$5C_4 \times 6C_2 = 75$ OR

✓ Total No. of selection / CHOICES

$150 + 200 + 75 = 425$

QUESTION

Q. In how many ways a team of 5 members can be selected form 4 ladies and 8 gentlemen such that selection includes at least 2 ladies ?

5 Members (at 2 ladies)

4 ladies	8 gentleman	No. of selection
(2)	(3)	$^4C_2 \times {}^8C_3 =$ ✓ ✓
(3)	(2)	$^4C_3 \times {}^8C_2 =$ ✓ ✓
(4)	(1)	$^4C_4 \times {}^8C_1 = 1 \times 8 = 8$
	Total = 8 + 112 + 336	= (456)

(A) 336

(B) 448

(C) 449

(D) 456

0, 1, 2, 3, 4, 5

2 5 .

5 0

25

1 case

× 1

3 × 3 × 2 × 1

= 18 ways.

18 + 24

= 42

4 × 3 × 2 × 1

50. = 24

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THANK YOU!

Here's to a cracking journey ahead!

Important lecture + PY 18 IIT advanced

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