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Counting Techniques - Part V

Course on Engineering Mathematics for GATE - CSE

Rahul • Lesson 5 • June 20, 2024

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Engineering Mathematics

$$-b \pm \sqrt{b^2 - 4ac}$$

Probability & Statistics

Counting principles

Lecture Number- 04

By- Rahul Sir

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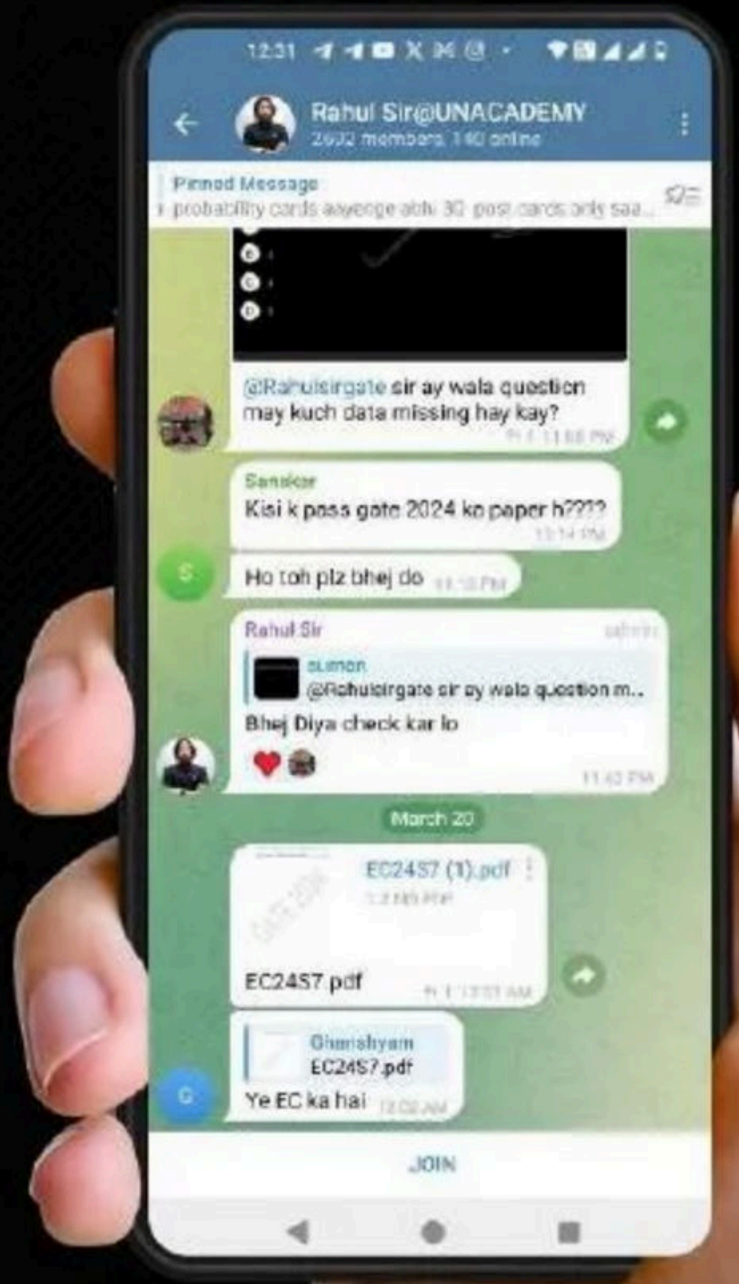

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Topics

to be covered

1. Problem solving class

n Different Items Taking all at a time $= (n_r{}^n)$
(Repetation allowed)

n Diff. Items Taken all at a time $\Rightarrow n!$
Repetation ✗ allowed

Not

n Diff. Items Taken r at a time $= {}^nP_r$
$=$ Permutation

n Diff. Items Selection / group $= {}^nC_r$.

$\leq n$ Diff. Items p alike, q alike, r alike
Taken all at a time $= n! / p!\,q!\,r!$

QUESTION

Q. The number of possible outcomes in a throw of n ordinary dice in which at least one of the dice shows an odd number are:

(A) $6^n - 1$

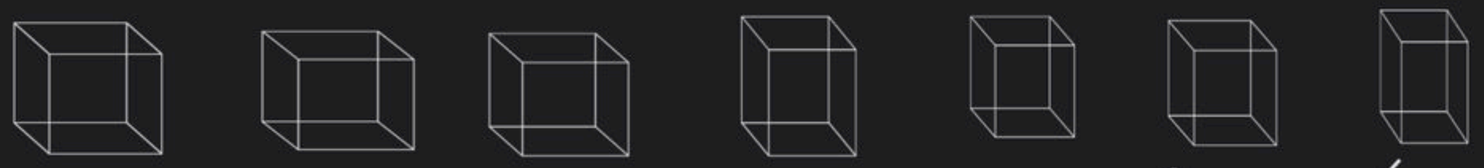
(B) $3^n - 1$

(C) $6^n - 3^n$

(D) $6^n - 2^n$

No. of possible outcomes = $6^n - 3^n$

at least ONE of the dice shows an odd number



Die A B C D E F G ---- n Die

$= 6 \quad 6 \quad 6 \quad 6 \quad 6 \quad 6 \quad 6 \quad \text{--- n times}$

$= 6^n =$ Total possible outcomes.

= atleast odd Number = Total possible outcomes − even Number.

Total Number

$= 6^n - 3^n$ Ans.

3 3 3 3 3 3 -- n times

-- n times $= 3^n$ times

QUESTION

Q. The number of five-digit telephone numbers having at least one of their digits repeated is:

A 90000

B 100000

C 30240

D 69760

five Digits Telephone Number.

(at least one of their digit is Repeated)

Total No. of Digits = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

without Repetation =

$= 10 \times 9 \times 8 \times 7 \times 6 = 30240$

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No. of atleast ONE of THEIR digit is repeated

= Total − without repeat

$= 10^5 - 10 \times 9 \times 8 \times 7 \times 6$

$= 69760$ Ans

answer

12345 = minus.

Total =

Total No. of ways.

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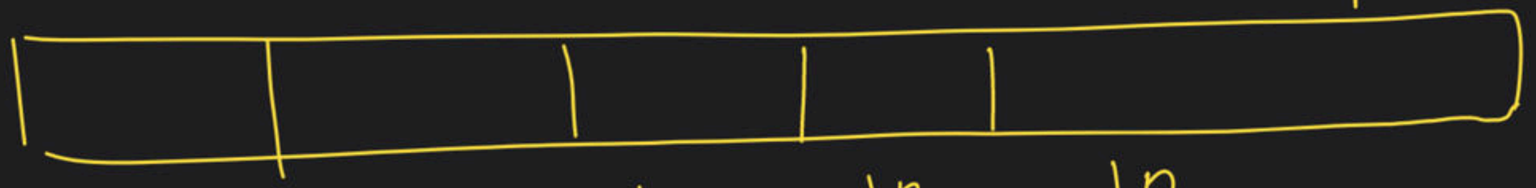
d_1	d_2	d_3	d_4	d_5

with repelation

$= 10 \times 9 \times 8 \times 7 \times 6$ = Total (without Repelation)



10 10 10 10 10

$= 10^5$

$Ans = 10^5 - 10 \times 9 \times 8 \times 7 \times 6$

0
1
2
3
4
5
6
7
8
9

n Different Taken all at a time (without Repelation) = Ans

QUESTION

Q. Let A = (x : x is a prime number and x < 30). The number of different rational numbers whose numerator and denominator belong to A is:

(A) 90

(B) 180

(C) 91

(D) 92

Let $A = \{x : x$ is a prime number$)$

$x < 30$

Number of Diff. Rational Number.

$\frac{\text{Num}}{\text{Den}}$ — belong to A

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$A = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$ (Prime Number)

" Rational Number $= \frac{p}{q}$ where $q \neq 0$ p, q both are Integer "

$\frac{p}{q}$ $p \neq q$ Num $\neq$ Den

$\frac{7}{5}$ Rational No = YES

$\Rightarrow \frac{2}{3}$ = Rational Number YES

$\Rightarrow \frac{5}{6}$ = Rational Number YES

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$\frac{p}{q}$

$\frac{NUM}{DEN}$

$\frac{NUM}{DEN}$

$NUM = DEN$ = Rational NO.

$p = q$ = 1

$NUM \neq DEN$ = Rational NO.

$p \neq q$

p have 10 options

	2	3	.5	.7	.	.
2.						
3.						
5.						
7.						
11						
13						
.						
.						
.						

Total NO = $10 \times 10 = 100$

Number of ways to belong A

$= 100 - 10 + 1$

$= 91$ cases

$\frac{p}{2}$ $p = 2$

$\frac{p}{q}$ $p \neq q$

$p = 2$

2 3 5 7 11 13 17 19 23 29

2
3
5
7
11
13
17
19
23
29

Total $= 100 - 10 + 1$

$= 91$

$= 100 - 10 + 1$

$= 91$

QUESTION

2W 3M

Q. Eight chairs are numbered 1 to 8. Two women and three men wish to occupy one chair each. First the women choose the chairs from amongst the chairs marked 1 to 4. and then the men select the chairs from amongst the remaining. The number of possible arrangements is:

(A) ${}^4C_3 \times {}^4C_2$

(B) ${}^4C_2 \times {}^4P_3$

(C) ${}^4P_2 \times {}^4P_3$

(D) ${}^4P_2 \times {}^6P_3$

1 2 3 4 5 6 7 8 ∴ 8 chairs.

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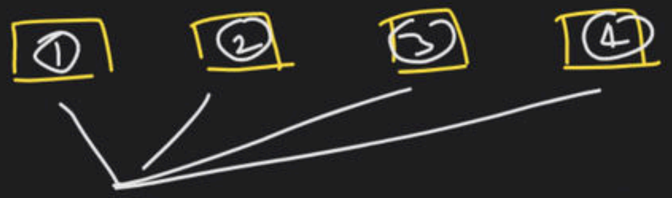
1	2	3	4	5	6	7	8
A	B	C	D	E	F	G	H

2 WOMEN (arrangement) $= {}^4C_2 \cdot 2! = {}^4P_2$ ✓

(Two chairs occupied)



3 MEN $\Rightarrow$ ${}^6C_3 \cdot 3! = {}^6P_3$ ✓

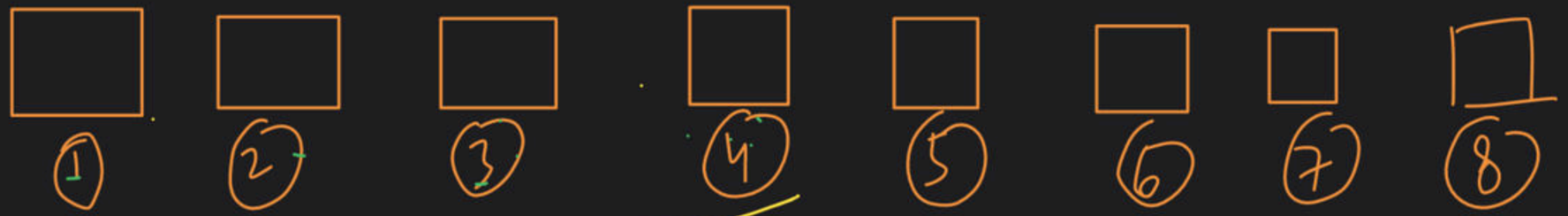
The No. of arrangements $= {}^4P_2 \times {}^6P_3 = w_1 w_2 = w_1 w_6$.

0 0 0 0 0 0 0 0 0

$\Downarrow$

${}^4C_2\, 2! \times {}^6C_3 \cdot 3!$

$=$



WOMEN = $^4C_2 \times 2!$ _ _ | _ _ _ _

Select × ordering 4 Remaining

MEN = $^6C_3 \times 3!$

Total = $^4C_2 \times 2! \times {}^6C_3 \times 3! = {}^4P_2 \times {}^6P_3$

= answer

QUESTION

Q. The number of arrangements of the letters of the word BANANA in which two Ns do not appear adjacently is:

A 40

B 60

C 80

D 100

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BANANA

2 N's do NOT appear adjacently

BANANA → NN, BAA, A

⇒ STAR/CROSS method

X | B X | A X | A X | A X |

Total 5 Places

$= {}^5C_2 \times \frac{L4}{L3} = \frac{5 \times 4^2}{2 \times 1} \times 4$

Selection × order

BAAA $= \frac{L4}{L3\,L1} = 4$

$= 40$ answer

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(Total - n's Together) = w's not Together.

(Homework)

(Read the problem)

5 girls 3 boys 2 boys sit Together

$\times G \times G \times G \times G \times G \times$

$= {}^{6}C_3 \times 3!$

Selection × ordering

QUESTION

Q. The number of ways in which two teams A and B of 11 players each can be made up from 22 players so that particular plavers are on the opposite sides is:

A 369512

B 184755

C 184756

D 369514

(A)

H.W

QUESTION

Q. We are required to form different words with the help of the letters of the word INTEGER. Let m_1 be the number of words in which I and N are never together and m_2 be the number of words which begin with I and end with R. then m_1/m_2 is given by:

A 42

B 30

C 6

D $\frac{1}{30}$

INTEGER

IN never Together ① m_1

I begin

R end m_2 ②

m_1 = number of ways.

m_2 =

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INTEGER IN | TEGER

TEGER — Using STAR / Cross method

X T X E X G X E X R X

① ② ③ ④ ⑤ ⑥

$m_1 = {}^6C_2 \times 2! \times \frac{5!}{2!} \Rightarrow 180$

TEGER

$= \frac{5!}{2!}$

I	N	T	E	G	E	R

$m_2 = 1 \times \frac{5!}{2!} \times 1 = 60$

$= \frac{180}{60} = 3$

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X T X E X 4 X A X R X

STAR / CROSS

$= {}^{6}C_{2} \times 2! \times 5! / 2!$ $= 1800$

IN

$\frac{m1}{m2} = \frac{180}{60} = 3$

QUESTION

Q. The number of permutations of k different things, in a row. taken not more than r at a time (each thing may be repeated any number of times) is equal to:

A $k^r - 1$

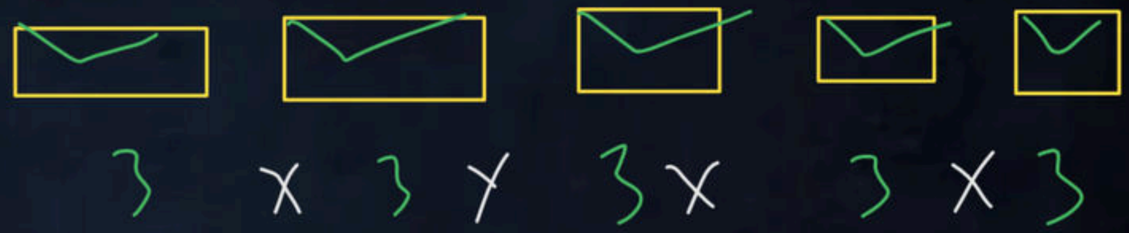
B k^r

C $k\left(\frac{k^r - 1}{k - 1}\right)$

D $\frac{\left(k^{r+1} - 1\right)}{(k - 1)}$

leave

QUESTION



Q. 5 letters can be posted into 3 letter boxes in:

$= 3^5$ Ans

(A) 3^5 ways

(B) 5^3 ways

(C) ${}^5C_3$ ways

(D) None of these

QUESTION

Q. A class has n students. We haw to form a team of the students including at least two students and also excluding at least two students. The number of ways of forming the team is:

(A) $2^n - 2n$

(B) $2^n - 2n - 2$

(C) $2^n - 2n - 4$

(D) $2^n - 2n - 1$

H.W

Ans = $2^n - 2n - 2$

QUESTION

Q. Find number of ways in which an arrangement of four letters can be made from the letters of the word ('PROPORTION')

A 754

B 758

C 752

D 750

n Diff. Items p alike, q alike r alike. (4 letter)

{ PROPORTION
PP, RR, OO, T, I, N

✓ CASE ① 4 alike → 0

② 3 alike 1 diff → 000 → P, R, T, I, N

$^5C_1 \times \frac{\lfloor 4}{3!} = 20$

selection

③ 2 alike, 2 alike → PP, RR, OO

$^3C_2 \times \frac{\lfloor 4}{\lfloor 2 \lfloor 2} = 0$ ✓ (18)

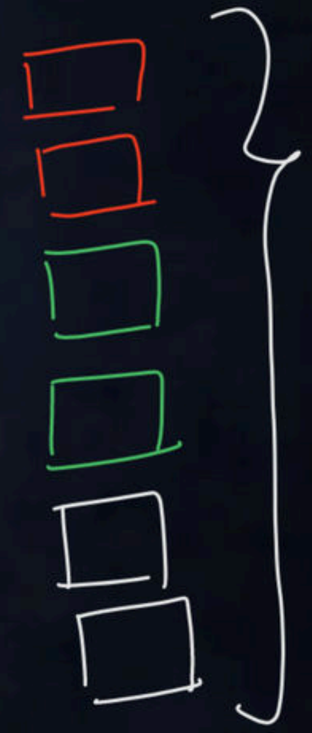
④ 2 alike, 2 diff → PP/RR/OO $= {}^3C_1 \times {}^5C_2 \times \frac{\lfloor 4}{\lfloor 2} = 360$

⑤ all Diff. → ${}^6C_4 \cdot \lfloor 4 = 360$

Total no. of cases = 758.

QUESTION

Q. The total number of flags with three horizontal strips, in order, that can be formed using 2 identical red. 2 identical green and 2 identical white strips, is equal to:



A. $4!$

B. $3.(4!)$

C. $2.(4!)$

D. $4.(4!)$

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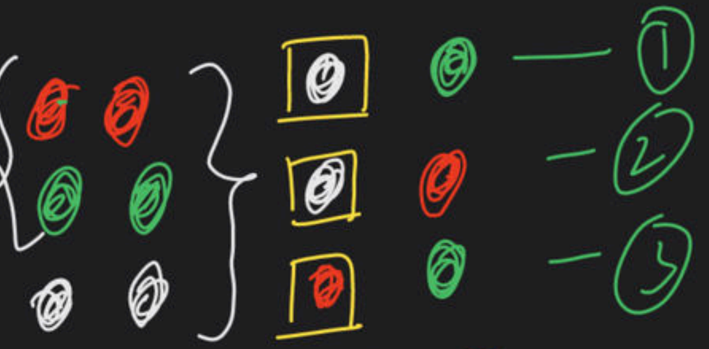
THREE Horizontal strips

n Diff. Items p alike, q alike, r alike Taken 'R' at a time

✓ CASE (A) all alike (THREE alike)

= 6

✓ CASE (B) 2 alike + 1 diff. = $^3C_1 \times {}^2C_1 \times \frac{L3}{L2}$



= $3 \times 2 \times 3 = 18$ ✓

4 lecture

CASE ③ all Diff. $= {}^{3}C_{3} \times 3! = 6$

Total No. of flags (THREE Horizontal strips)

$= 18 + 6$

$= 24$

DONE

QUESTION

Q. Find the number of permutations of the word ASSASSINATION taken 4 at a time.

A 900

B 917

C 719

D 791

(ASSASSINATION)

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n Diff. Items Taken 'R' at a time

(p alike, q alike, r alike)

ASSASSINATION

✓ SSSS	AAA	II NN	T, D

case (A) 4 alike = 1 — AAAA

case (B) 3 alike + 1 diff. = 40 — AAAB

case (C) 2 alike + 2 alike = 36 — AABB

case (D) 2 alike + 2 diff = 480 — AABC

case (E) all Diff. = 360 — ABCD (4 letter)

AAA, AAB, ABC — 3 letter

STrings.

Total No. of arrang = 1 + 40 + 36 + 480 + 360 = 917

2 alike + 2 diff ⇒

S,S	NN
A A	
I I	

$^4C_1$

A	T
I	
N	O

$^5C_2$

$= {}^4C_1 \times {}^5C_2 \times \frac{L4}{L2}$

$= 480$

QUESTION

Q. How many 4-letter words can be formed using letters of the word 'VIDYAMANDIR' such that the word always includes a letter 'I'.

(A) 1320

(B) 1546

(C) 1248

(D) 1456

Ans = 1248

H.W

DONE

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Week

Today work. = ① Revision (NOTES)
② NEW STUDENTS
4 Letters + backlog
✓ ③ H.W question + Division
Distribution
of objects
Tommorrow work

✓ Daily Practice question (25 question)

✓ Telegram — Rahul sir Unacademy | CS DPP
⇒ Whatsapp | NEW

✓ SAT - 2 SUN - 2

DPP

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THANK YOU!

Here's to a cracking journey ahead!