

Trees - Part X

Course on Data Structure



CS & IT Engineering

Data Structure
Tree





Topics

to be covered

1

Tree-IX



Const. of heap using build heap method

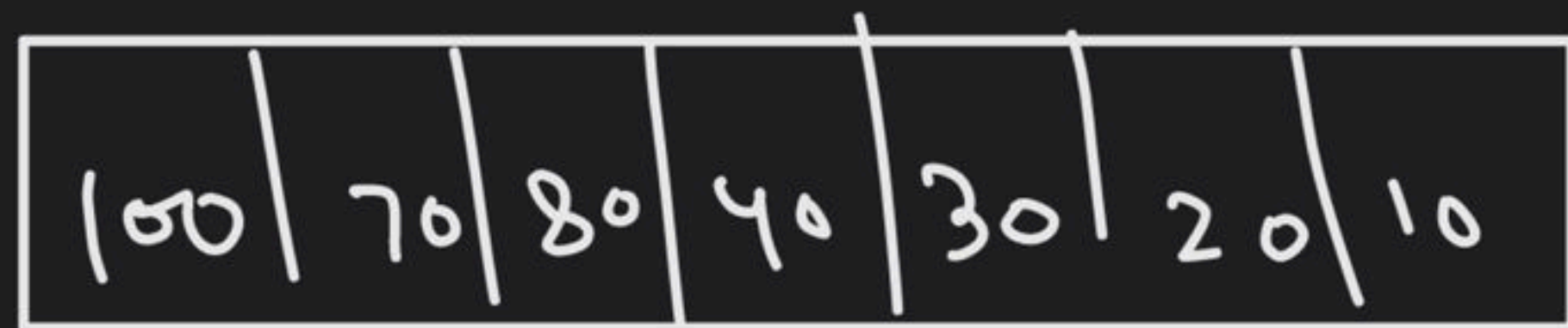
Given an array representation of a CBT,

convert it into a max-heap.

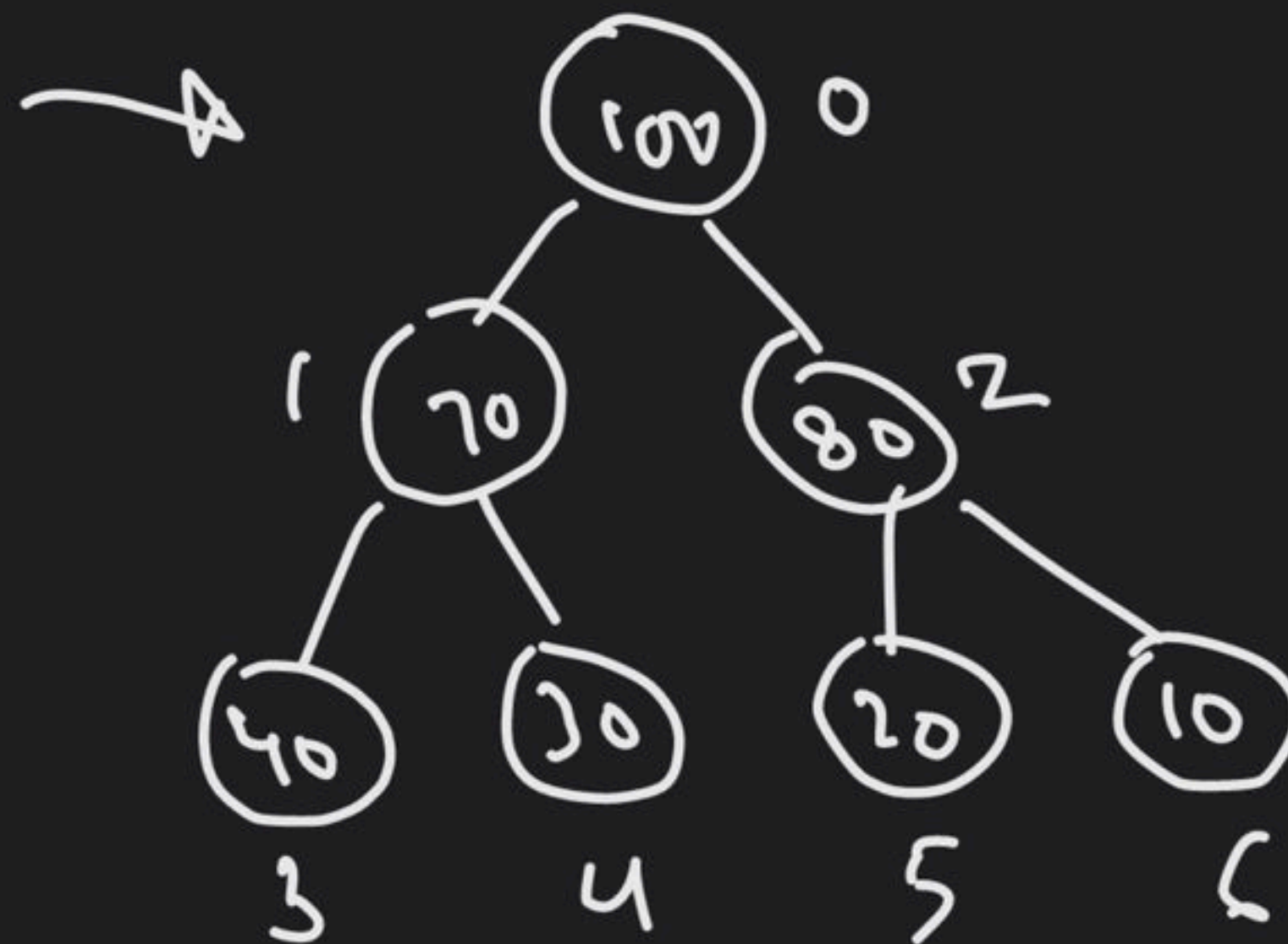
Heapify method / Algo.

Heap \rightarrow CBT

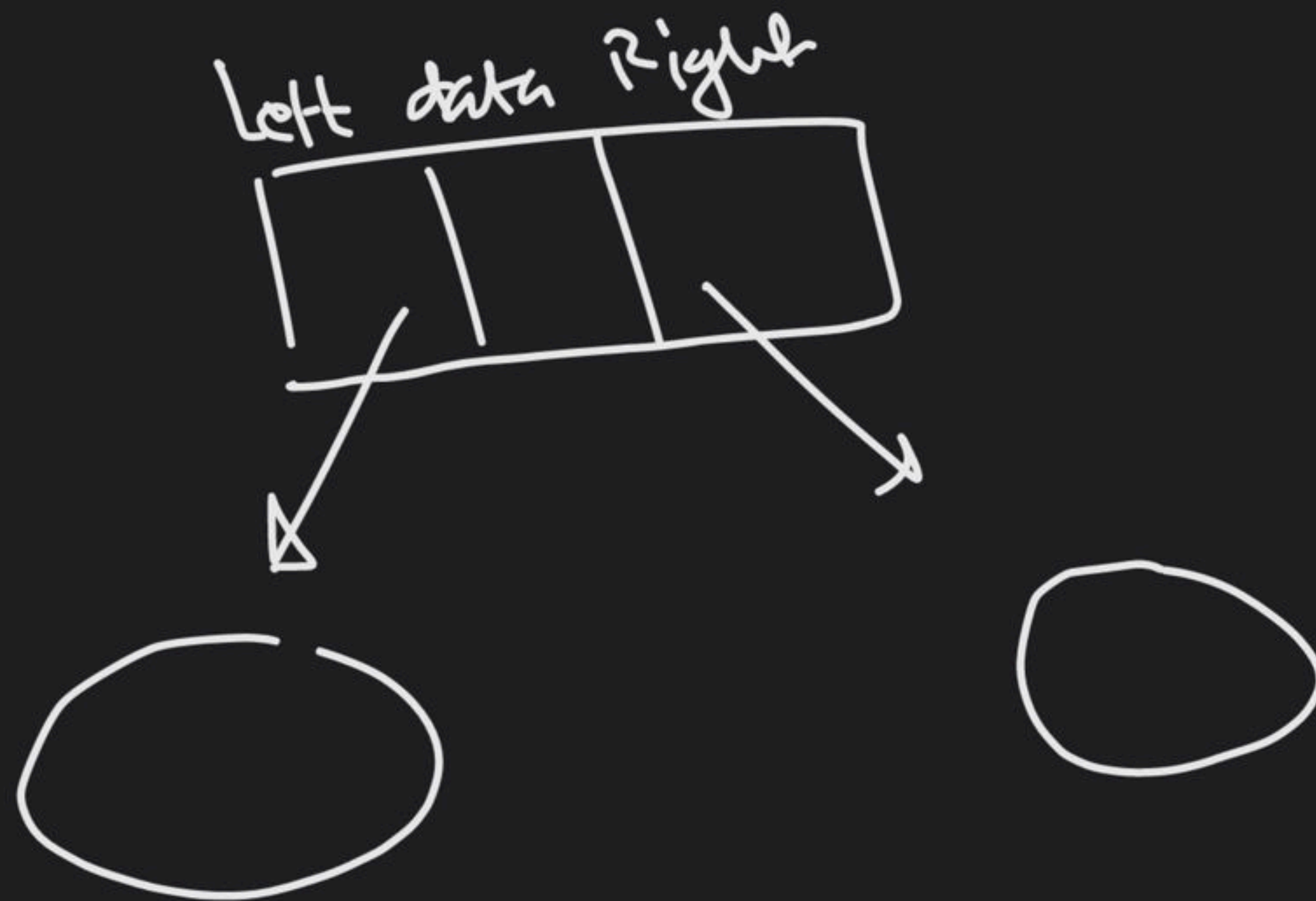
array representation



0 1 2 3 4 5 6

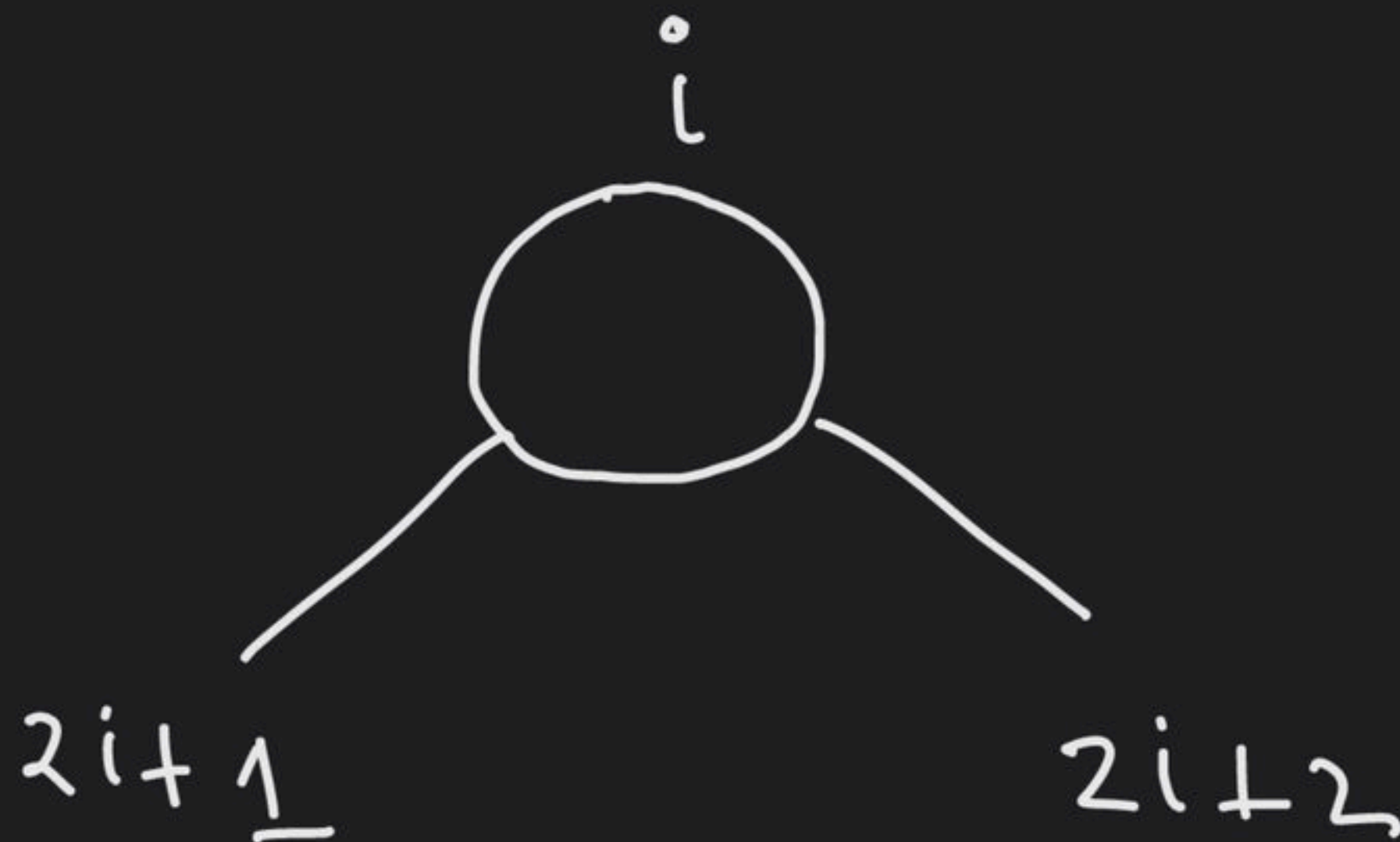


L.L. representation

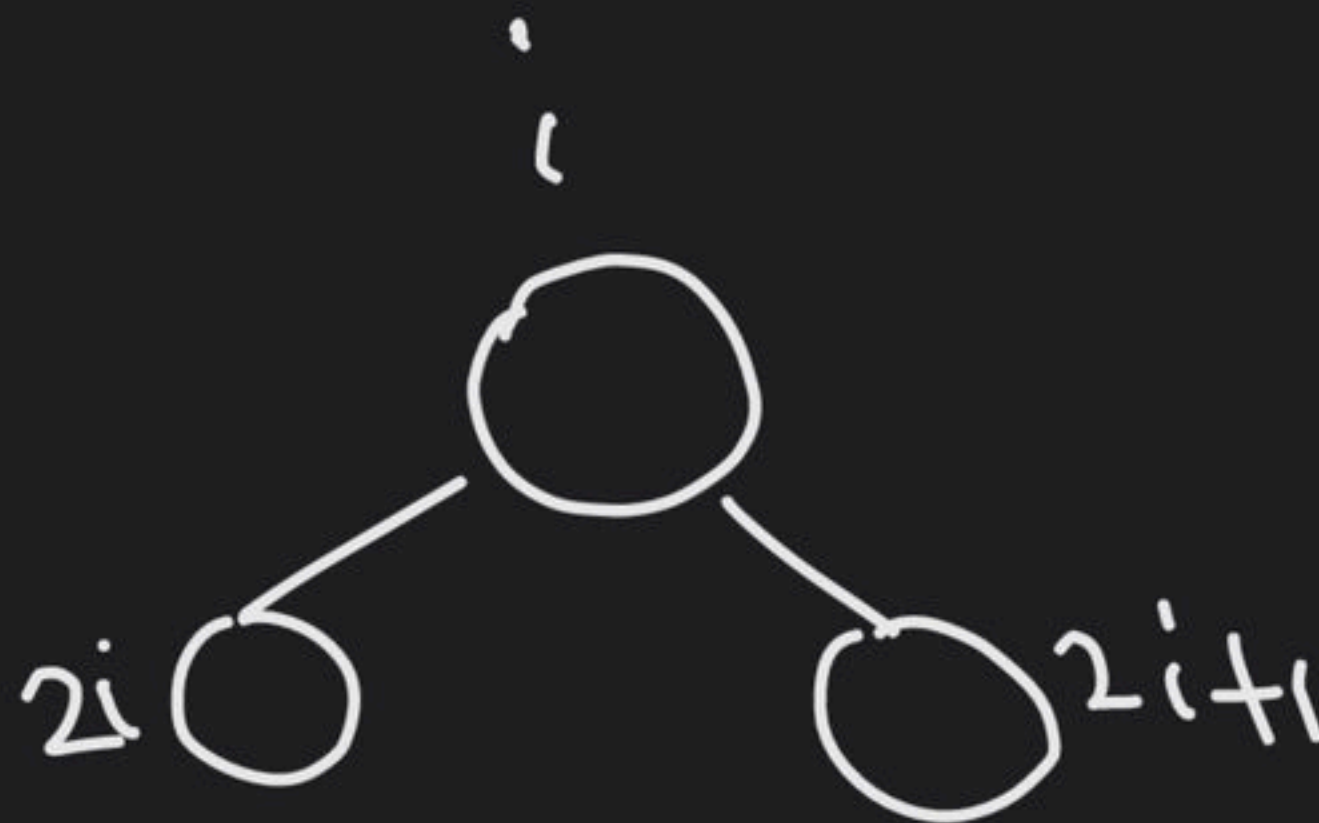


index

Cor²

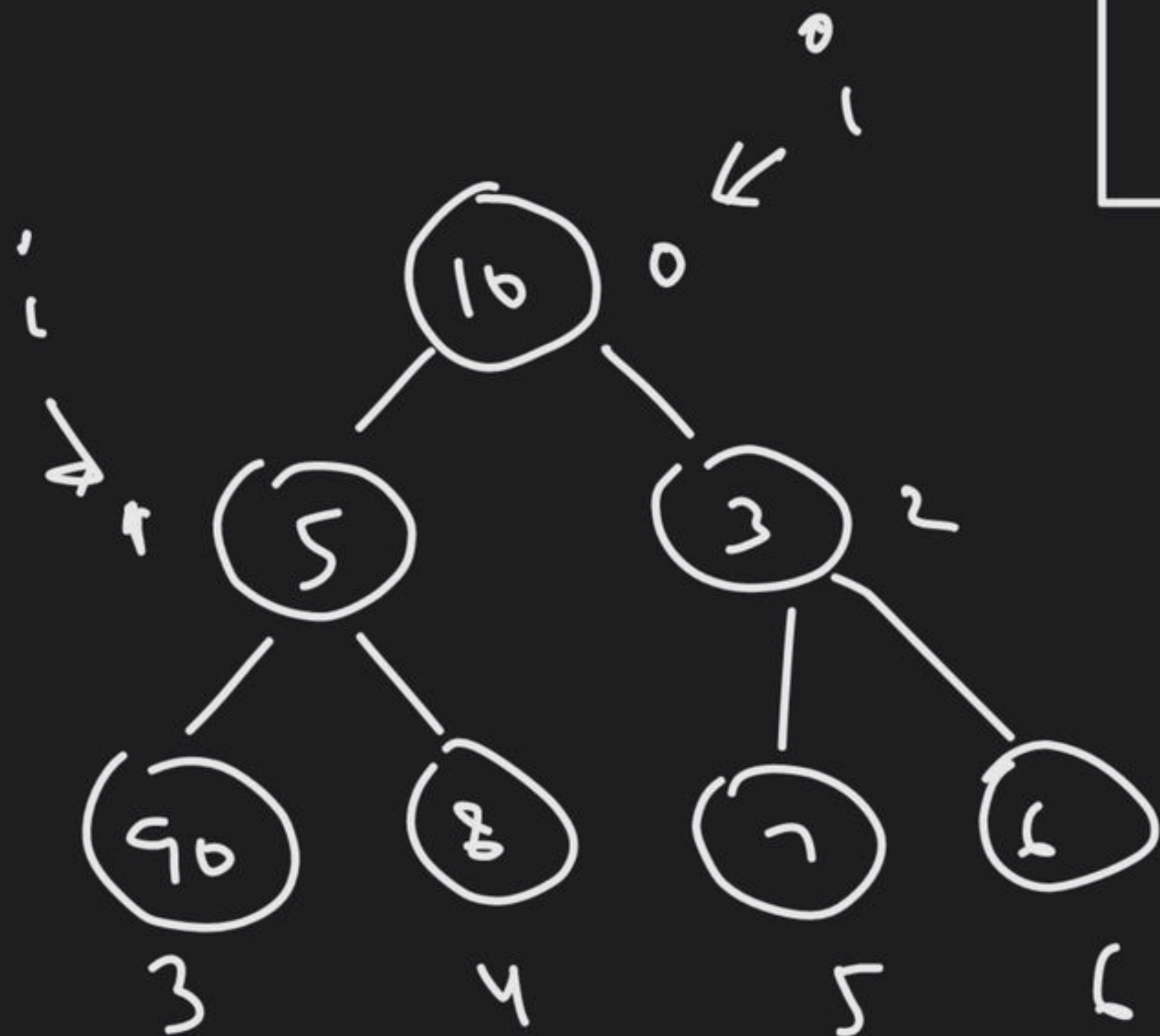


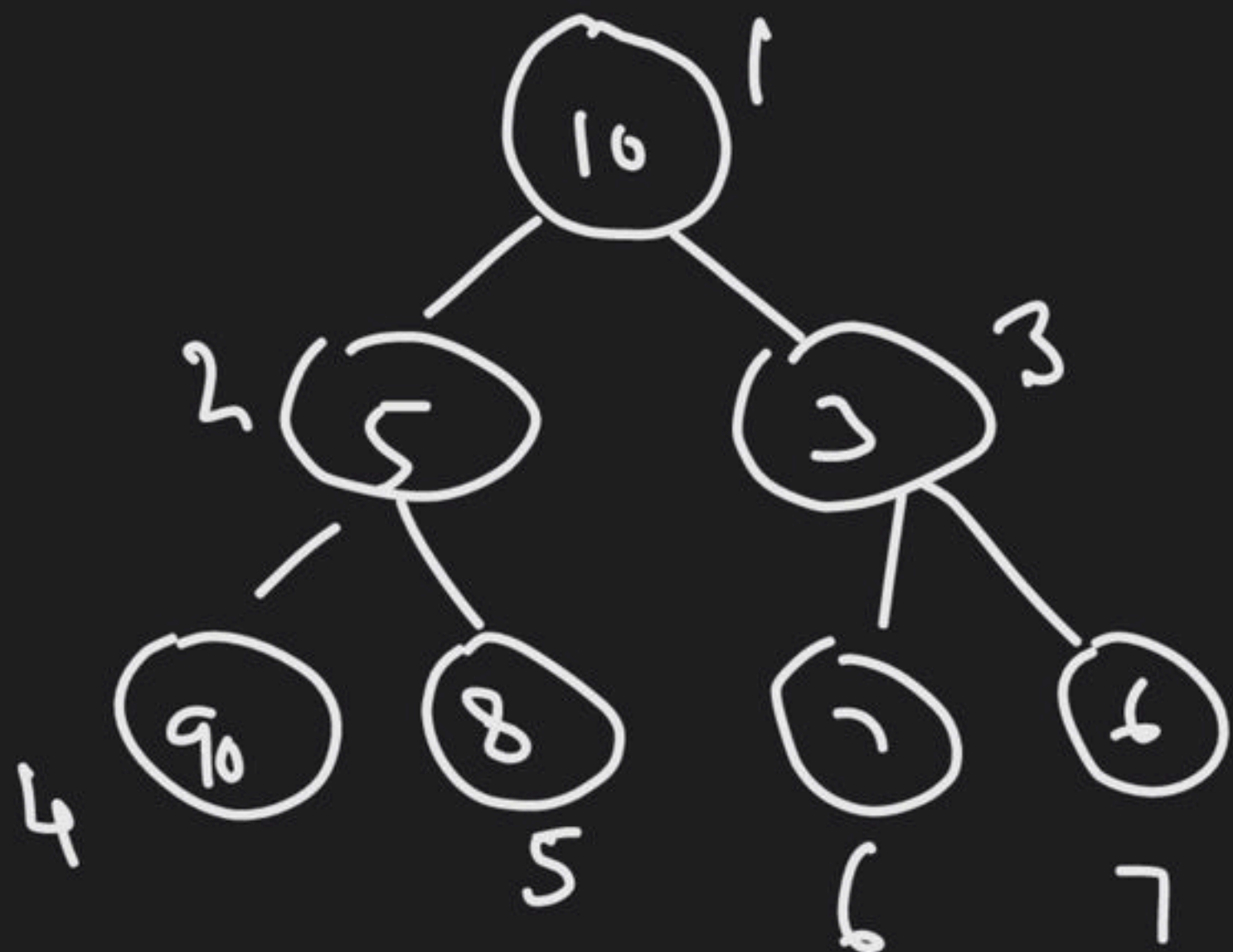
Theory



10	5	3	90	8	7	6
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0 1 2 3 4 5 6

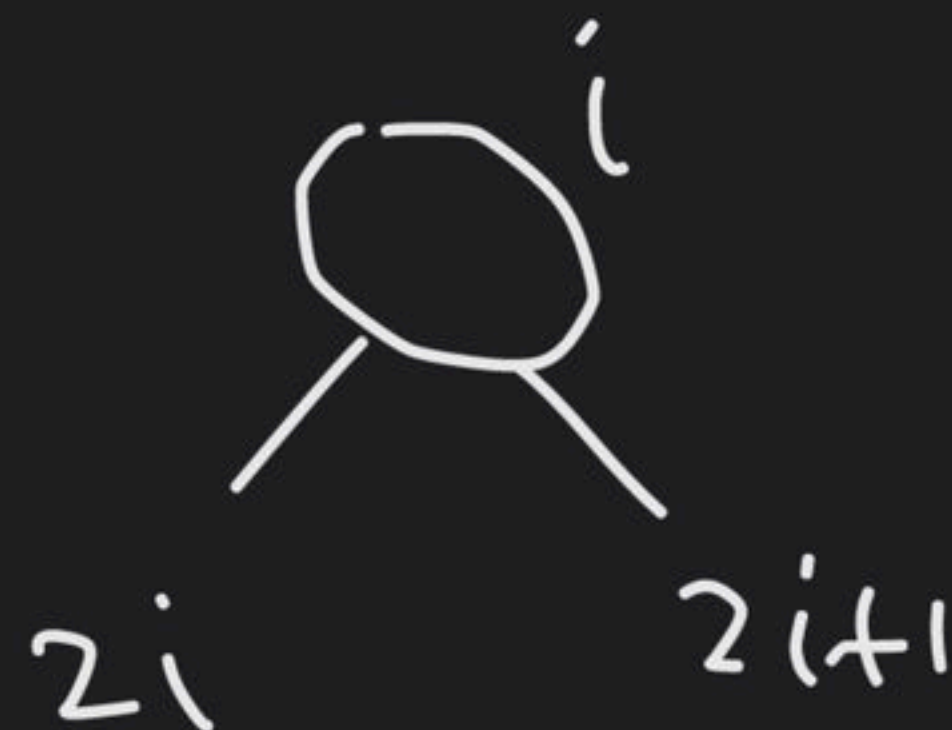
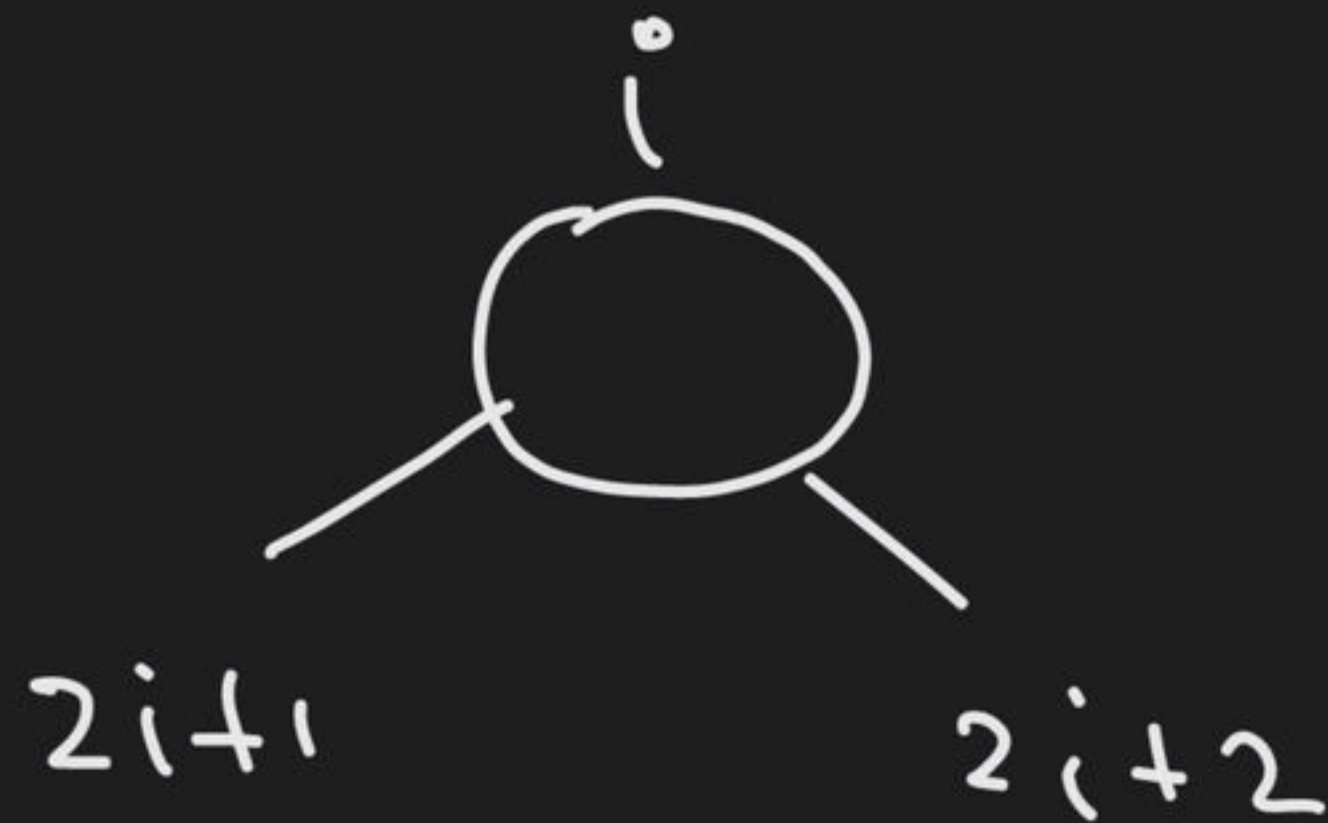




10	5	3	90	8	7	6
1	2	3	4	5	6	7

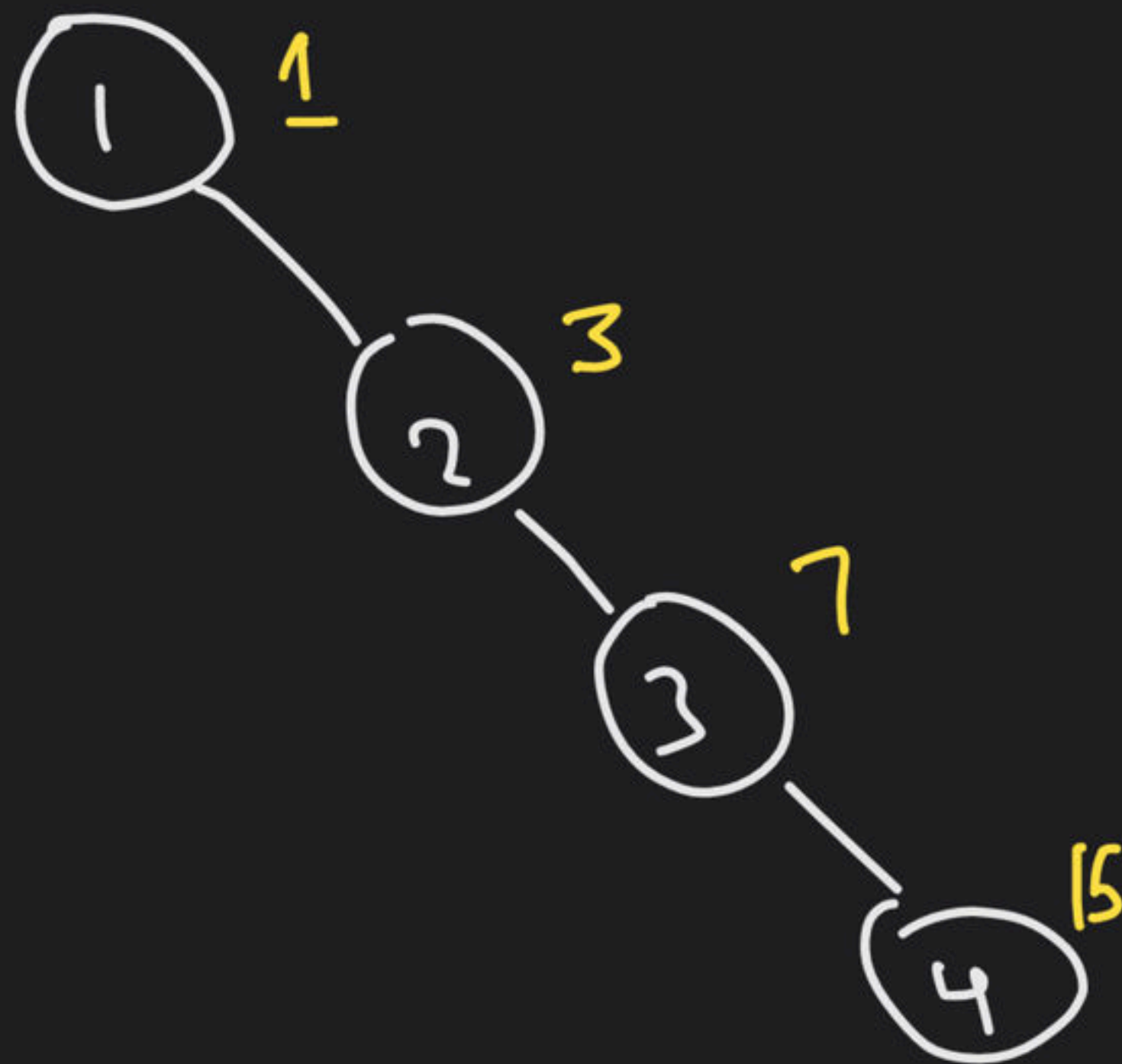
Why not for

BT?



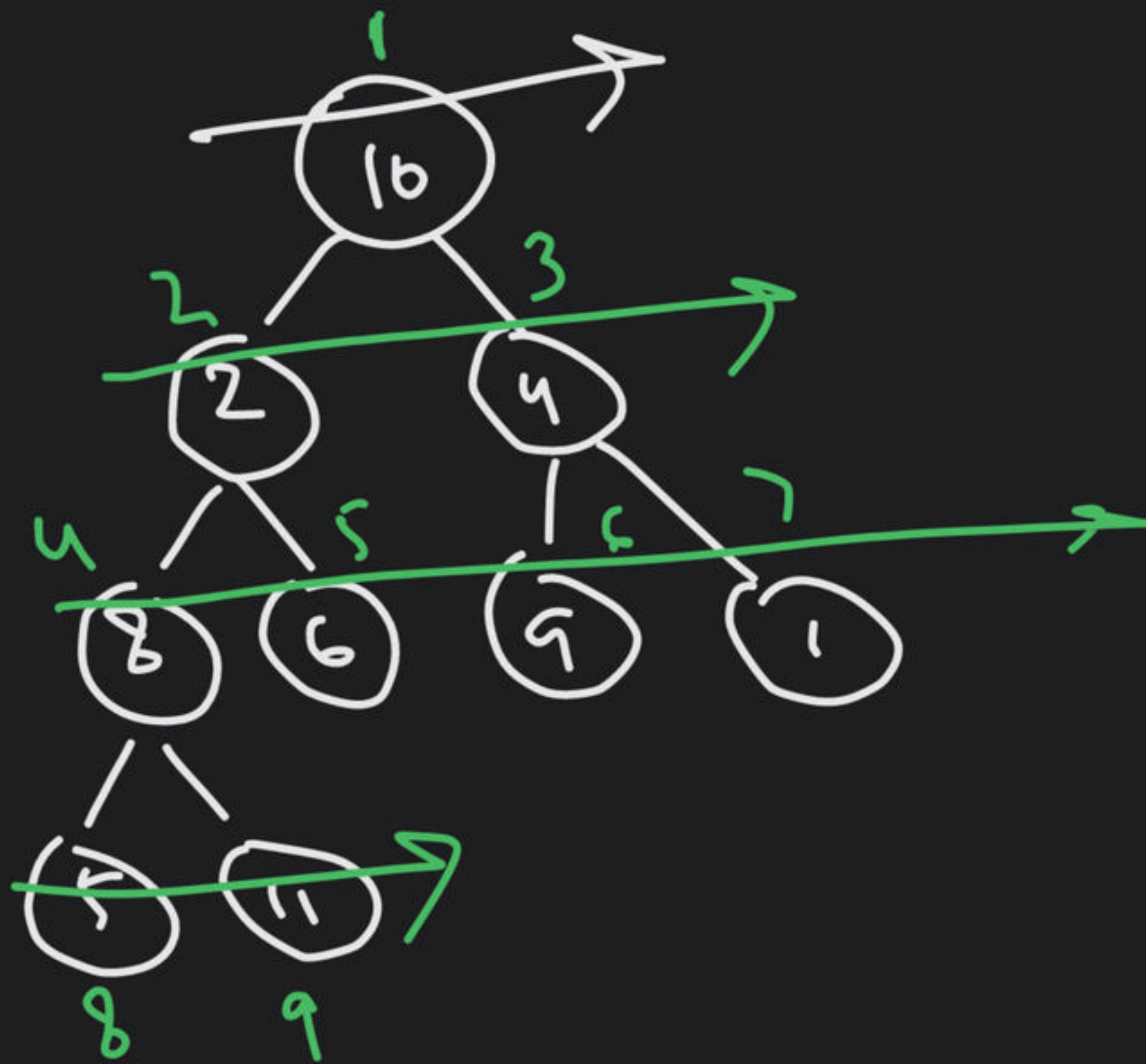
$4 \rightarrow \approx 16 \text{ size}$

$n \rightarrow O(2^n)$



10	2	4	8	6	9	1	5	11
1	2	3	4	5	6	7	8	9

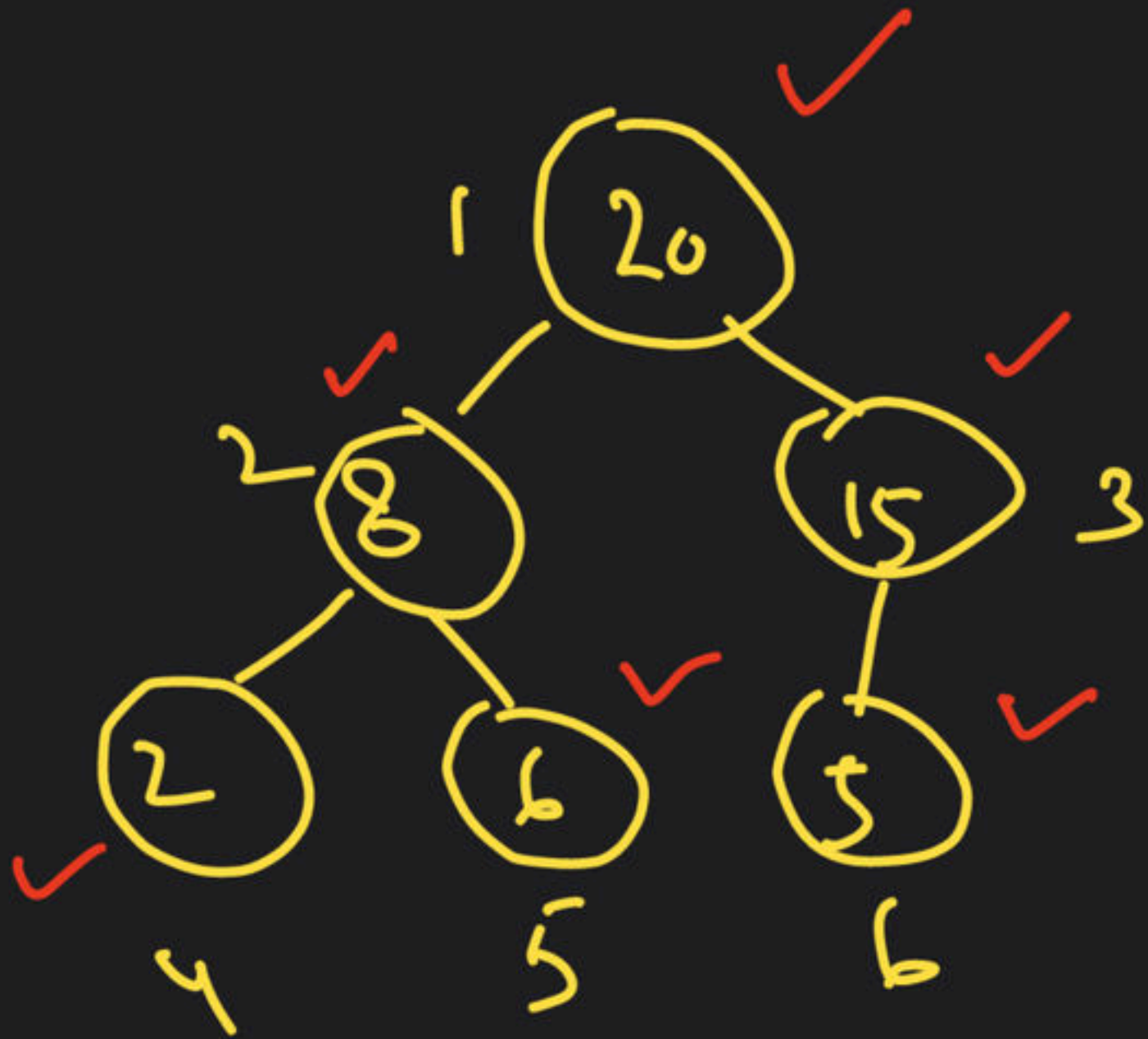
Array
represent.
of a
CBT



Max-heap?
NO

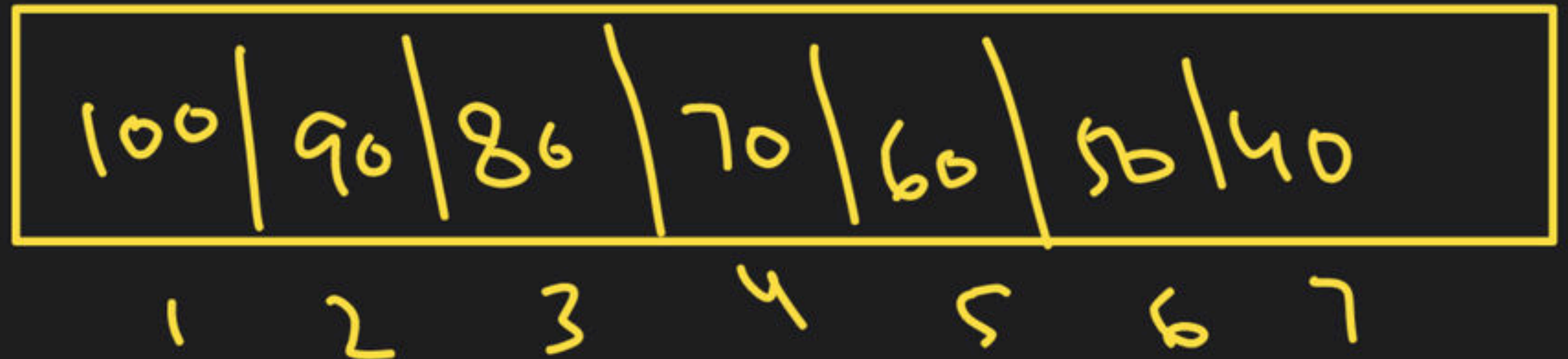
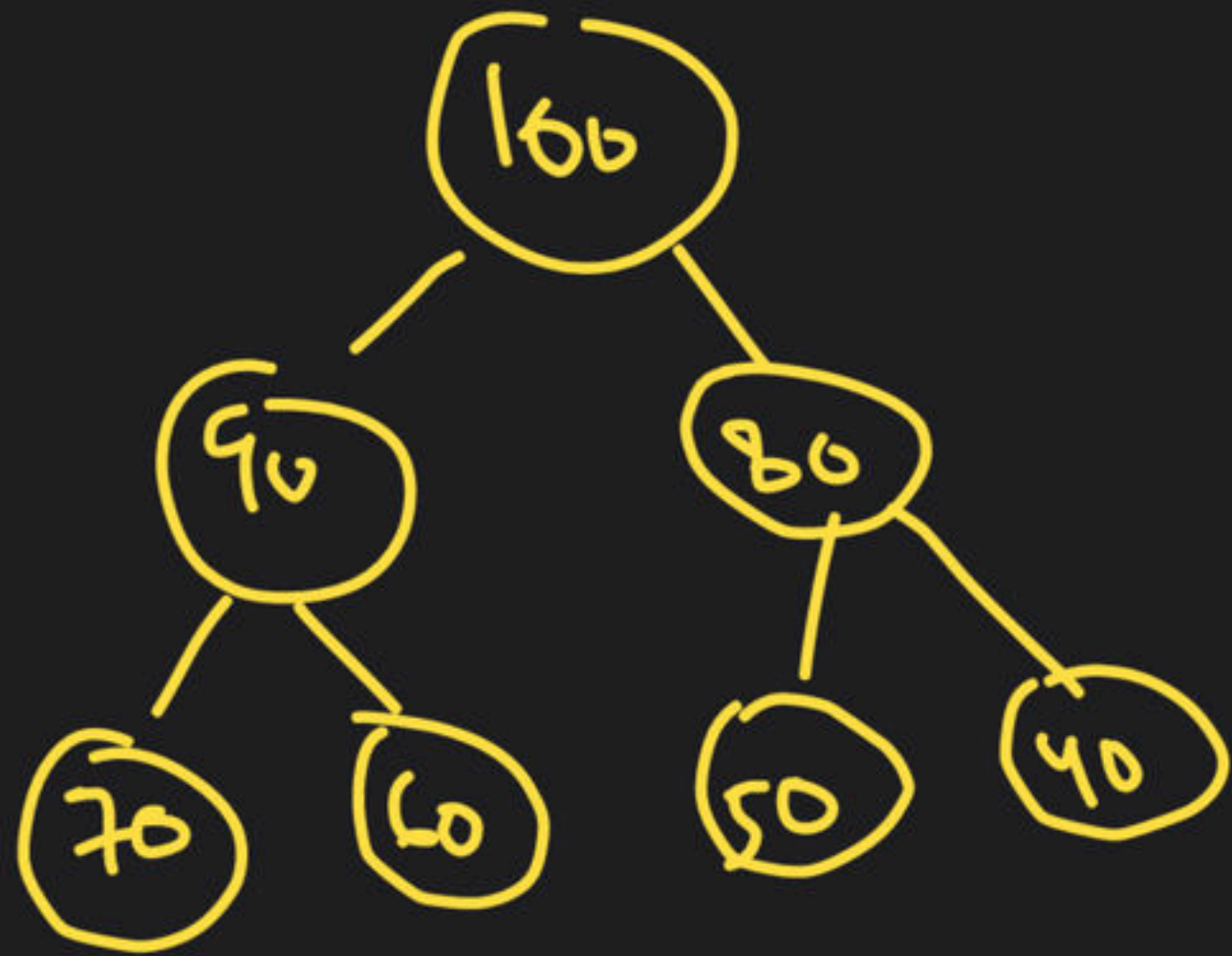
Given an array rep. a CBT 20, 8, 15, 2, 6, 5
Is it rep. a max heap?

20	8	15	2	6	5
1	2	3	4	5	6



Given an array rep. a CBT 100, 90, 80, 70, 60, 50, 40

Is it rep. a max-heap?



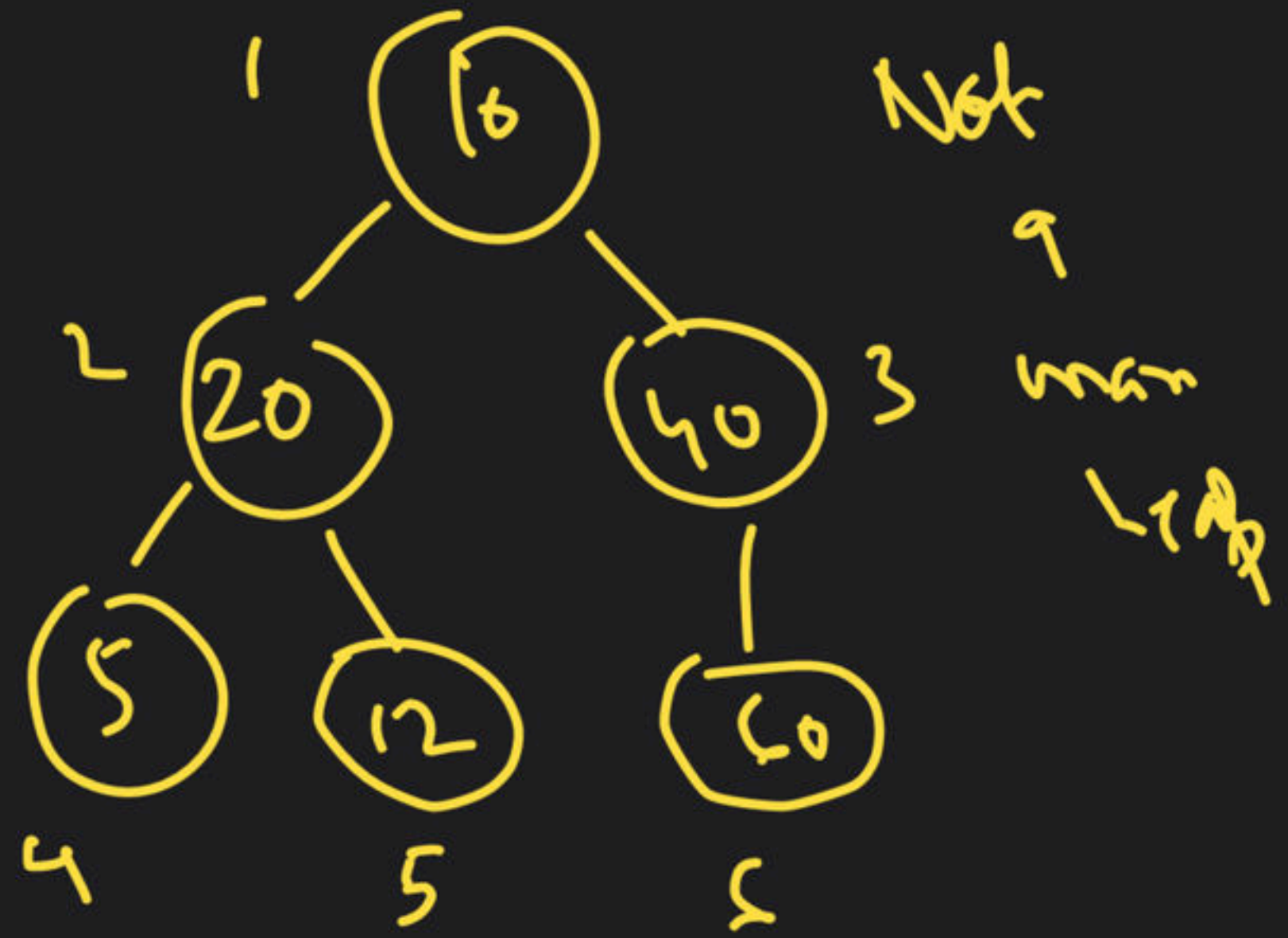
array dec. sorted \Rightarrow always rep a
max-heap

array inc. sorted \Rightarrow always rep a
min-heap

Given an array representing a CBT:

10, 20, 40, 5, 12, 60

Is it rep. a max-heap?



Not
a
max
Heap

10	20	40	5	12	60
1	2	3	4	5	6

Given an array representing a CBT : 16, 20, 30, 40, 50, 60, 70
Convert it into a max-heap.

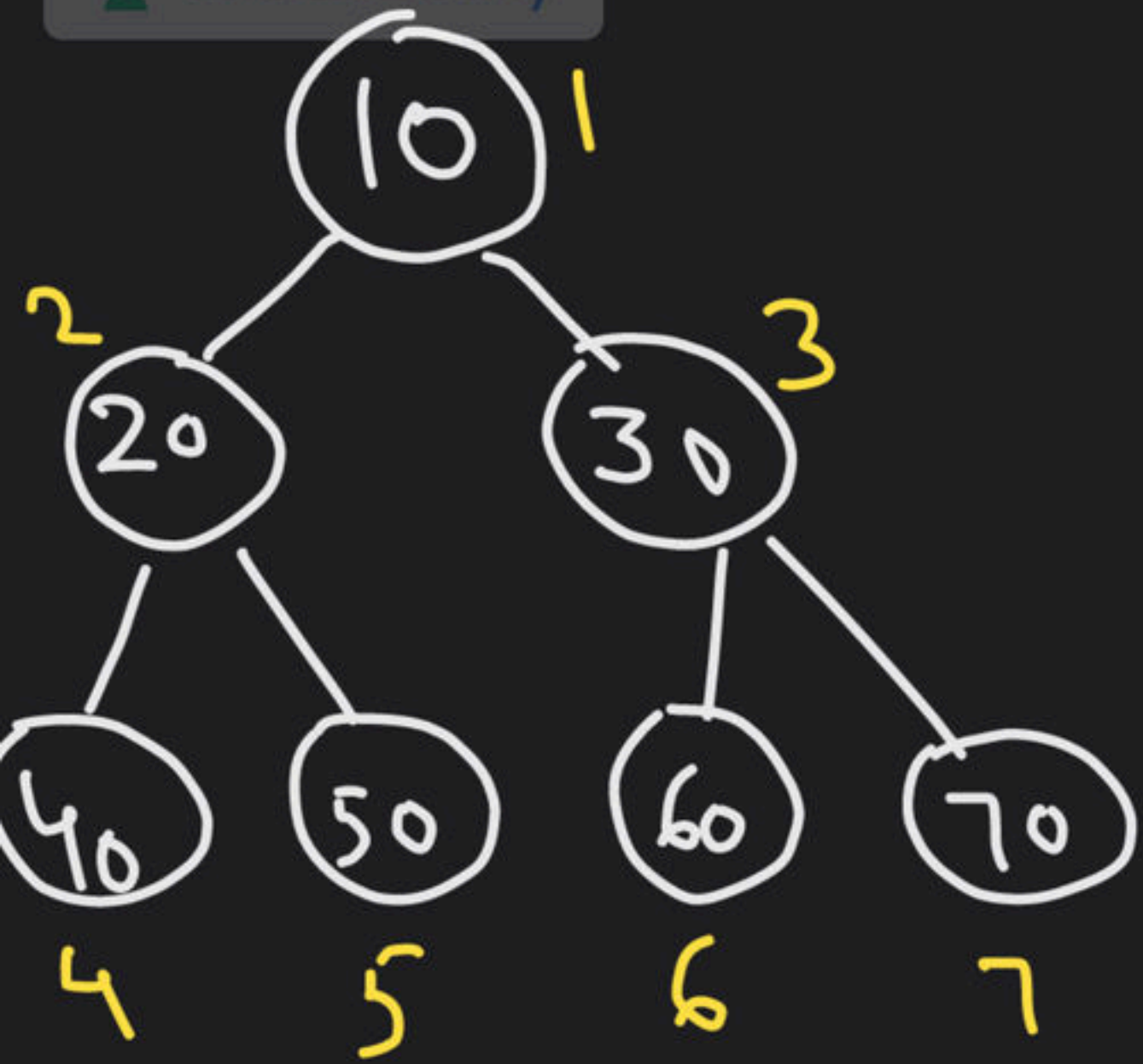
16	20	30	40	50	60	70
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Sort
 \rightarrow
 $O(n \log n)$

70, 60, 50, 40, 30, 20, 16

Better?

Build-heap method
 \rightarrow Heapify Algo

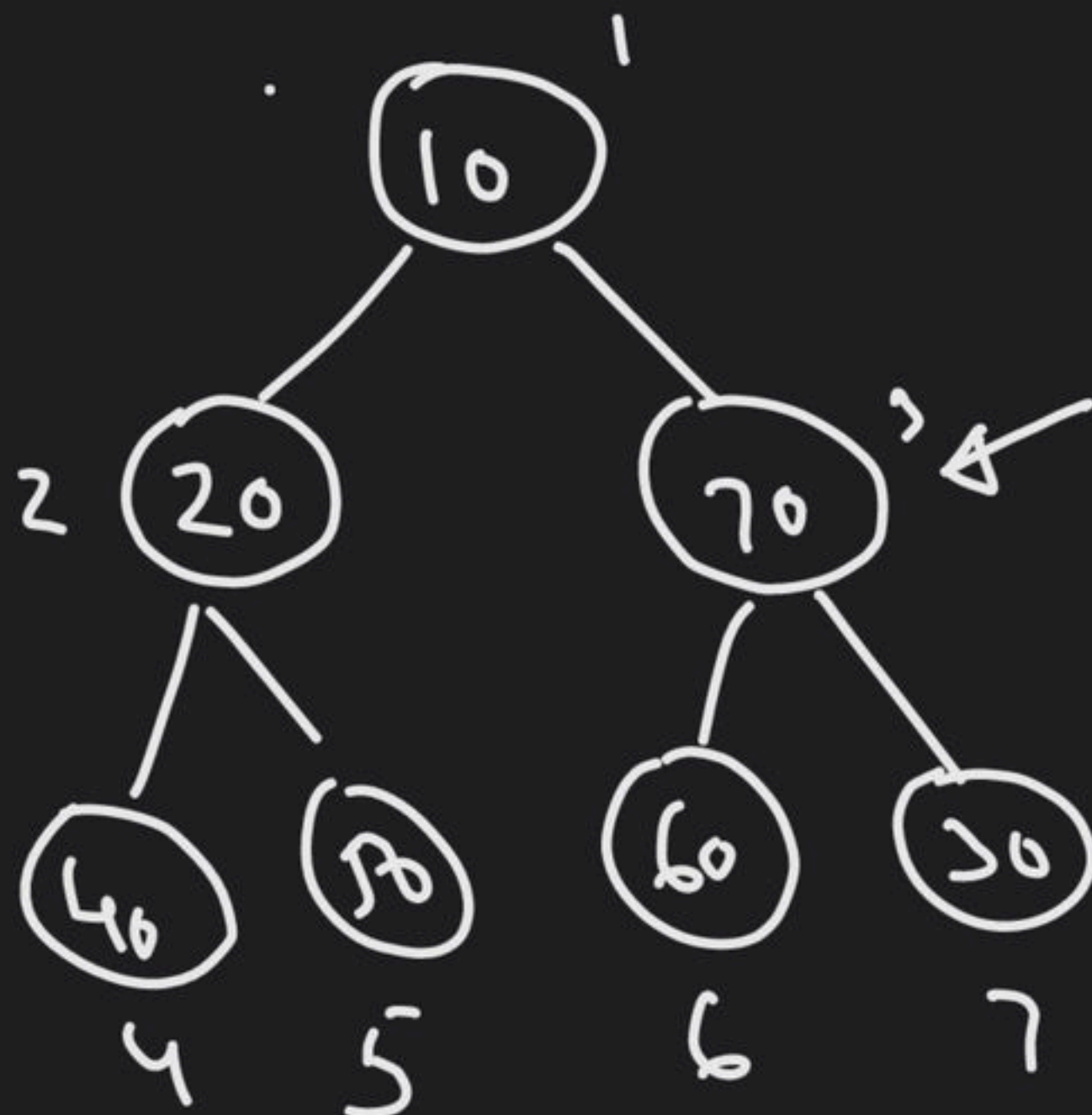
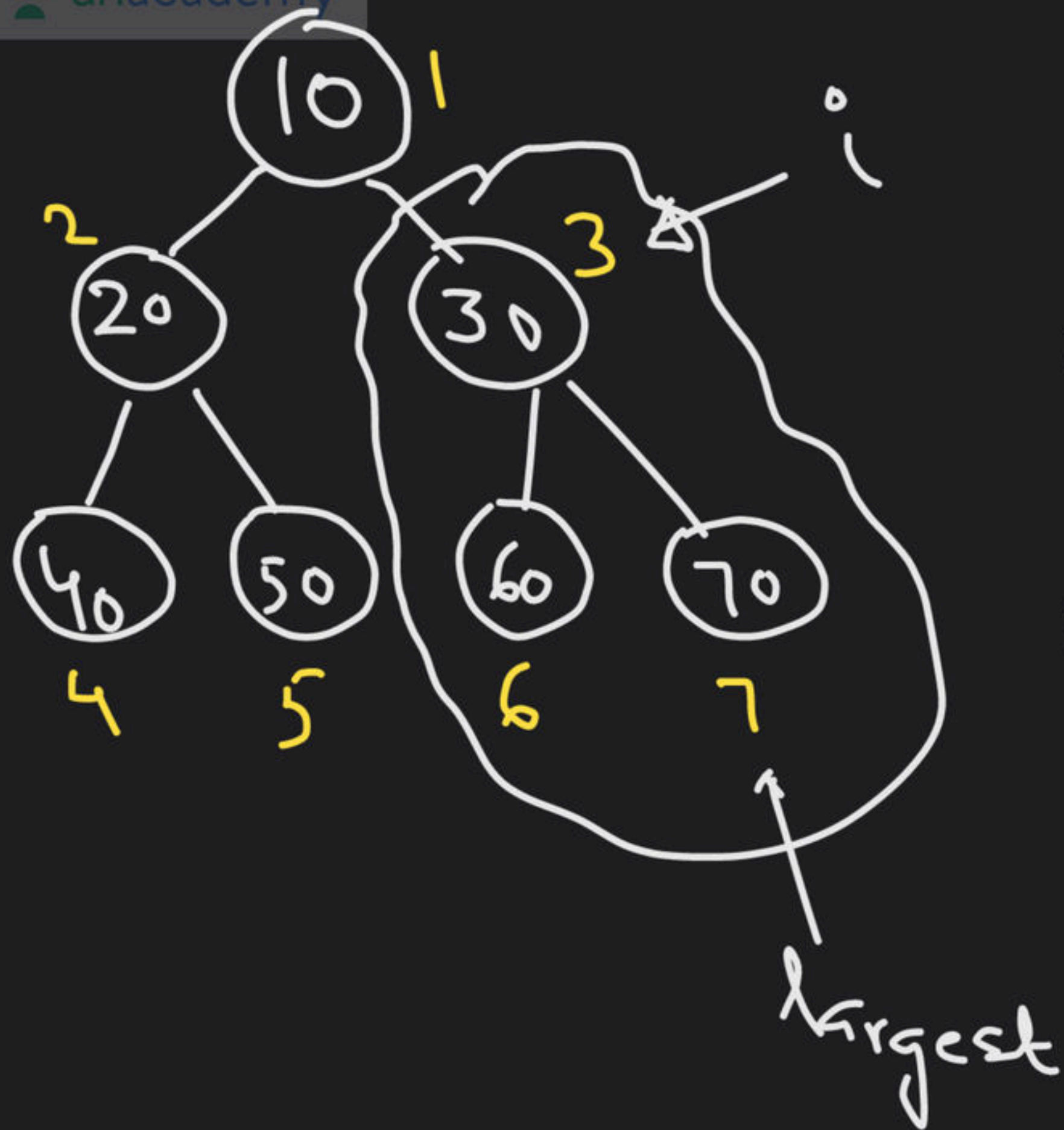


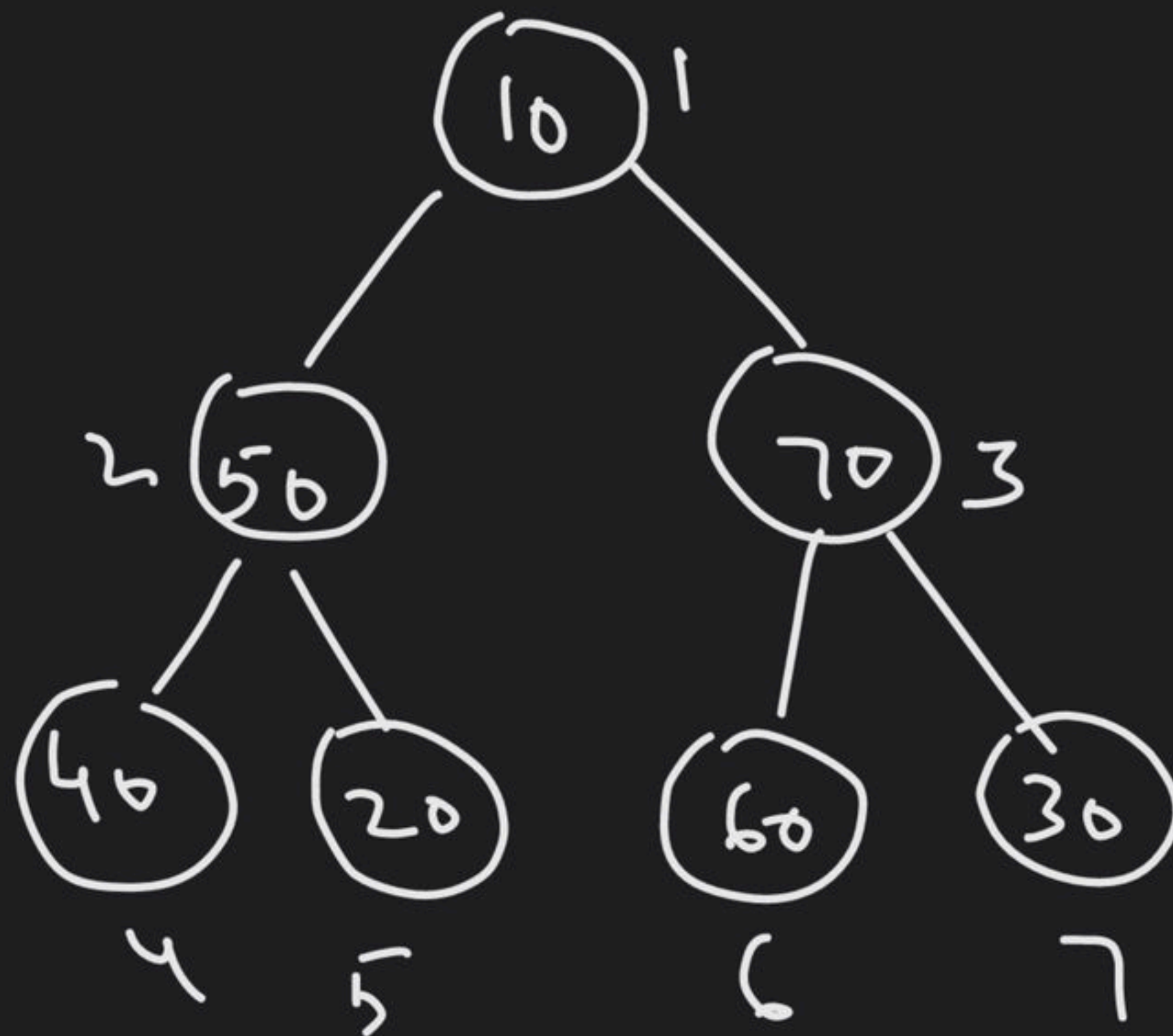
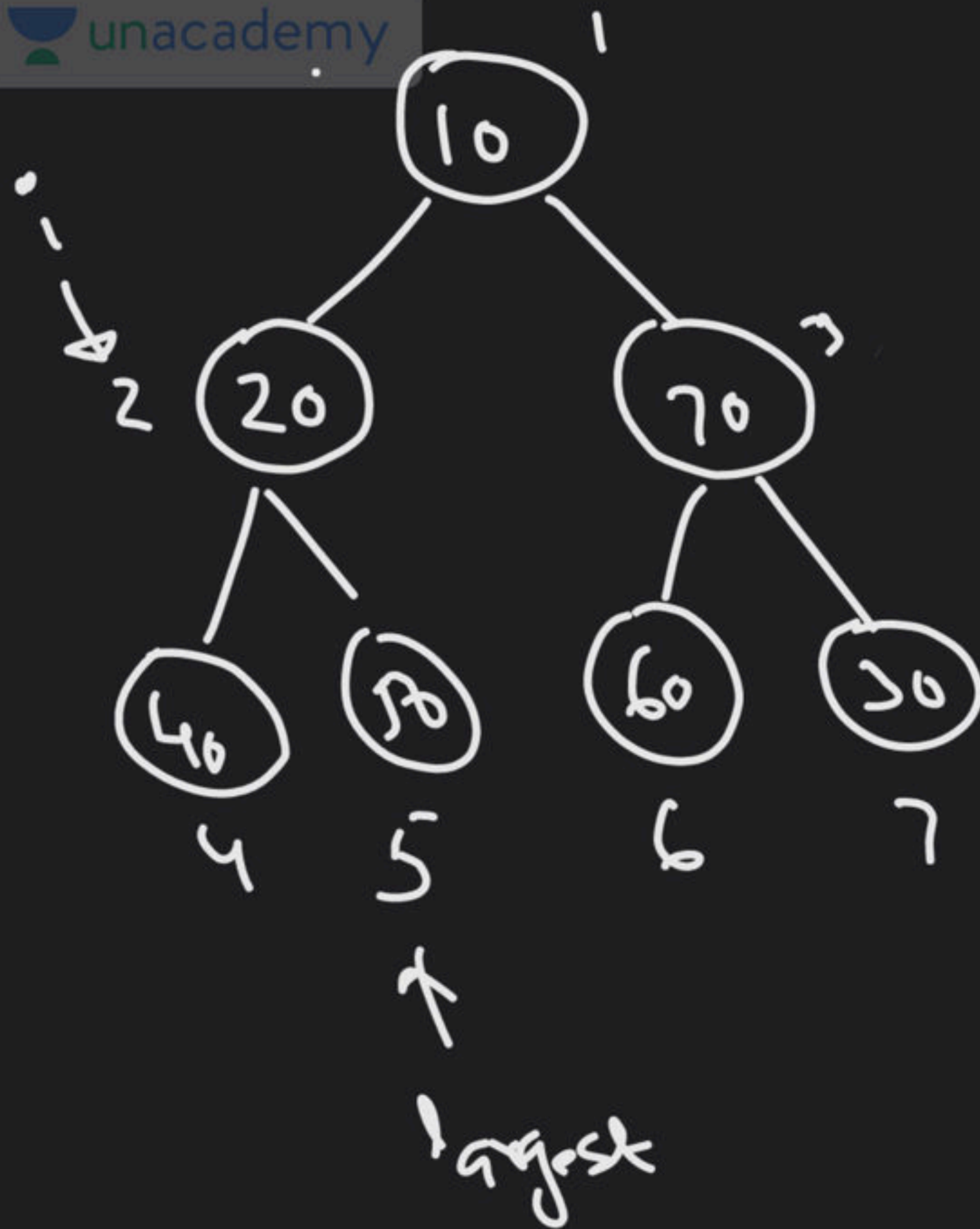
index of internal nodes

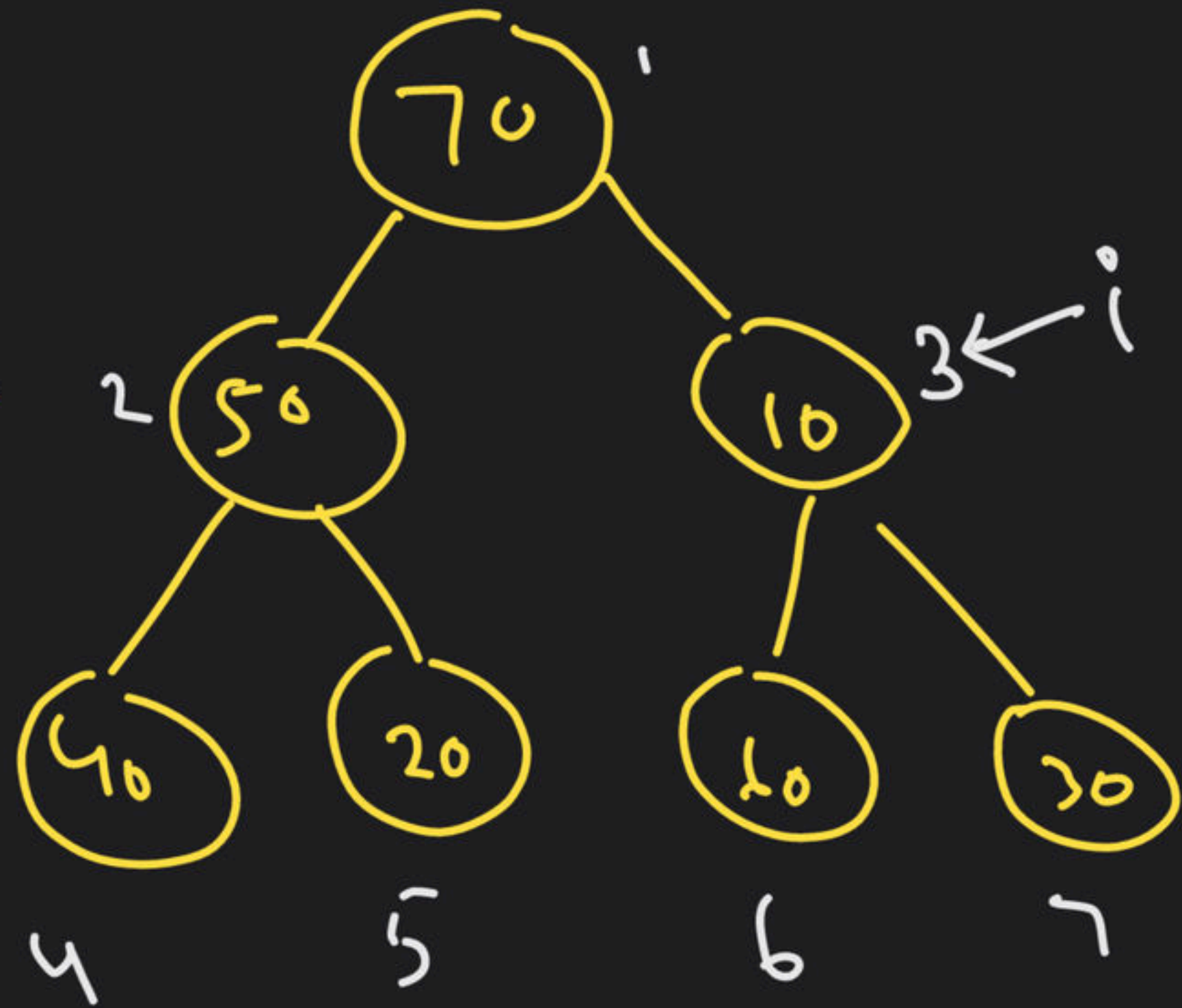
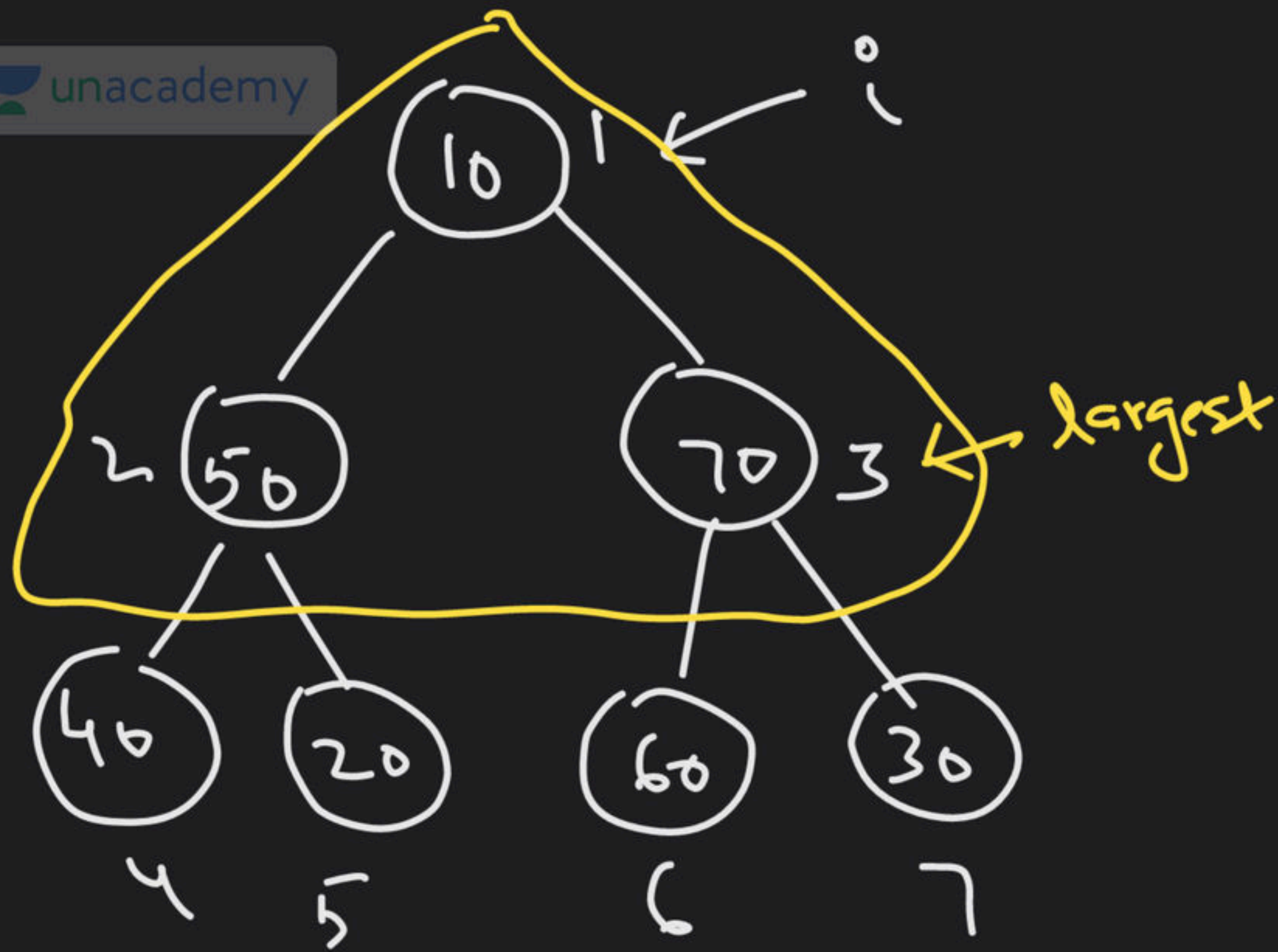
$$= 1 \text{ to } \lfloor \frac{n}{2} \rfloor \Rightarrow 1, 2, 3$$

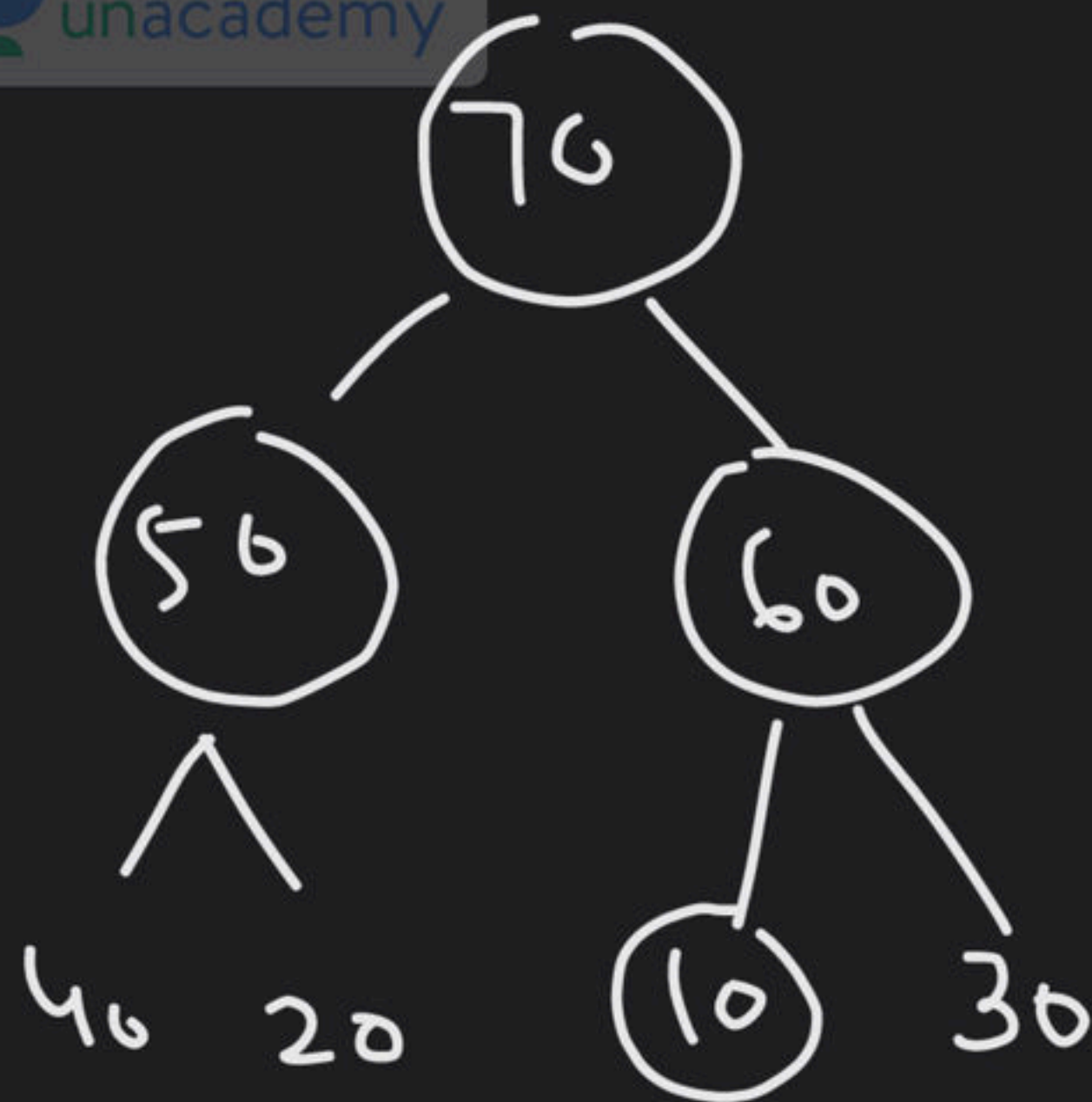
Every leaf node always satisfy
max-heap property.

index $\rightarrow 4, 5, 6, 7$

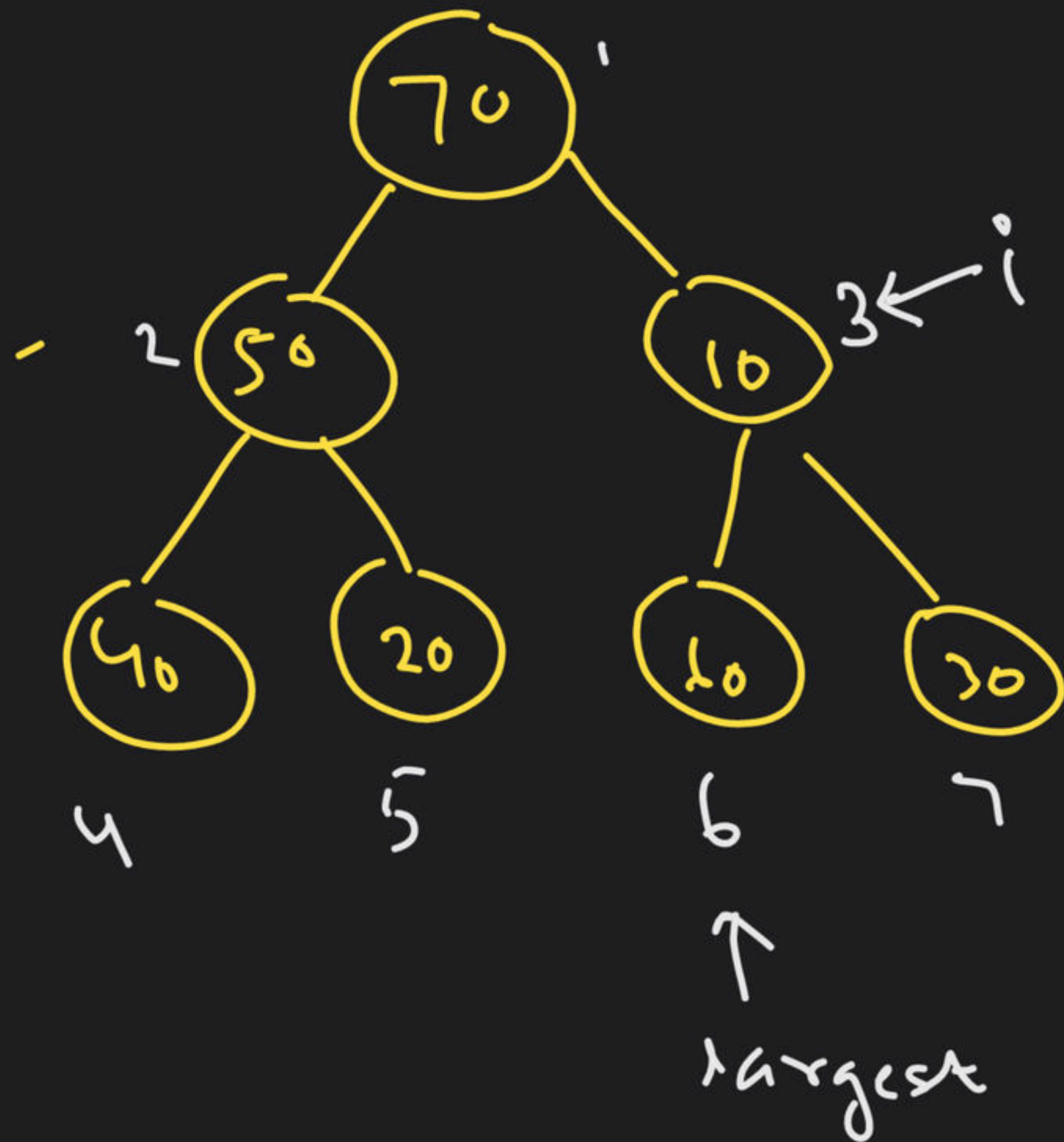


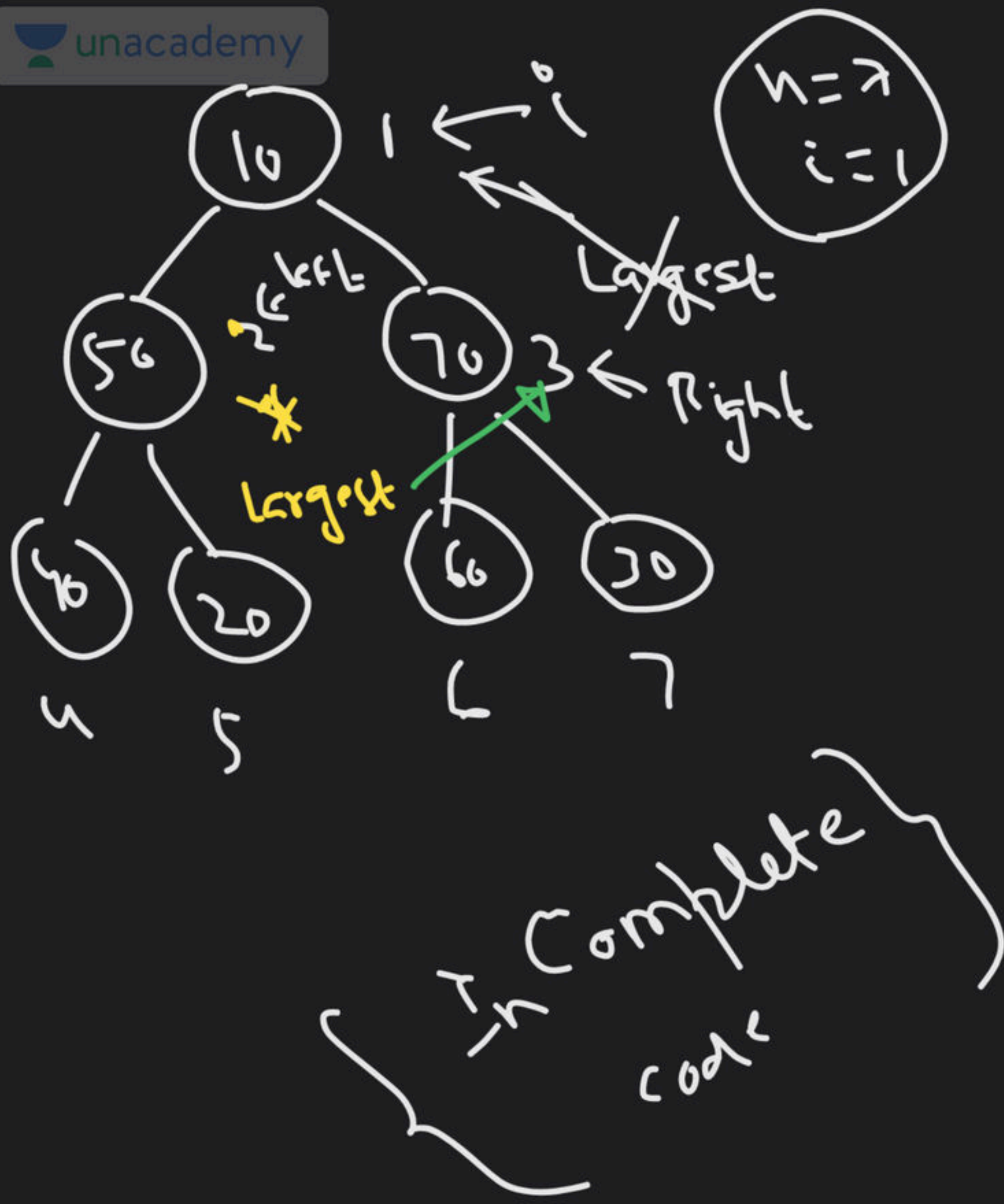






Max-heap ✓

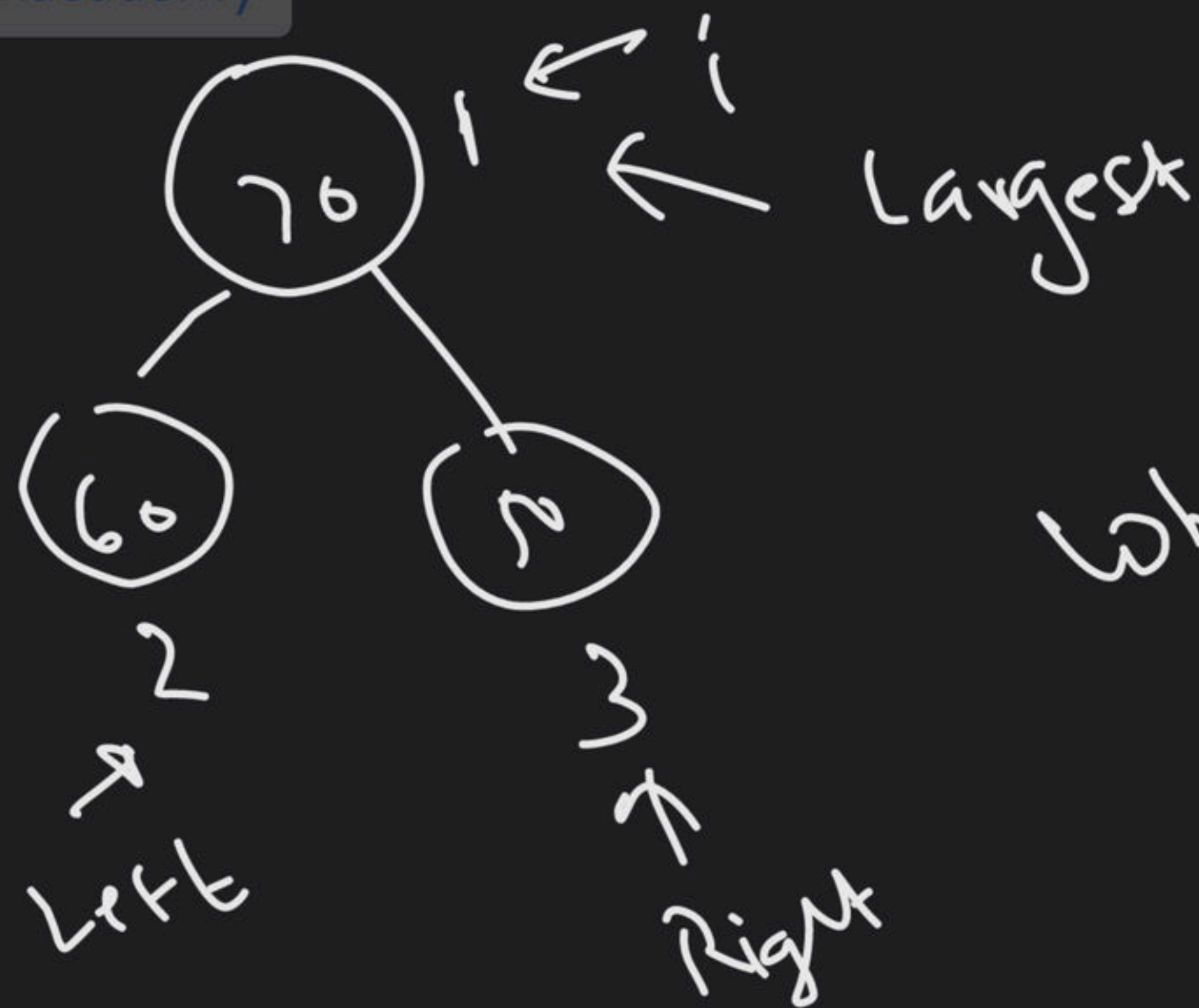




Heapify (A, i, n)

- 1.) $Left = 2 \cdot i$; $Right = 2 \cdot i + 1$;
 $Largest = i$;
- 2.) $If (A[Left] > A[Largest])$
 $Largest = Left$;
- 3.) $If (A[Right] > A[Largest])$
 $Largest = Right$;

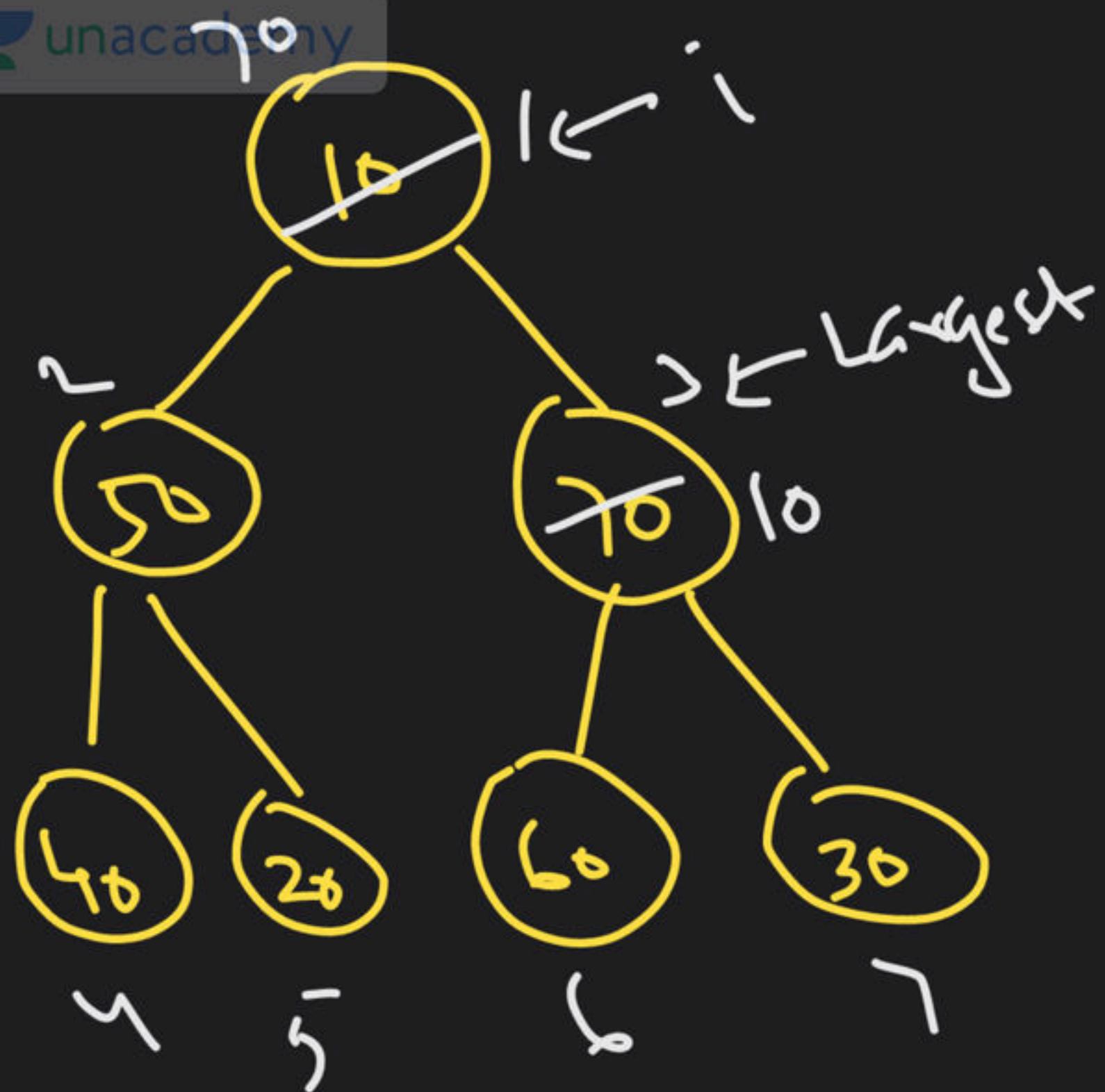
In this example \rightarrow swap.



When to swap

if ($i \neq \text{Largest}$)

swap ($A[i], A[\text{Largest}]$)



Apply same algo

at index
Largest

This is not
the exact
algo.

Heapify(A, i, n)

1.) Left = $2 \times i$; Right = $2 \times i + 1$;
Largest = i;

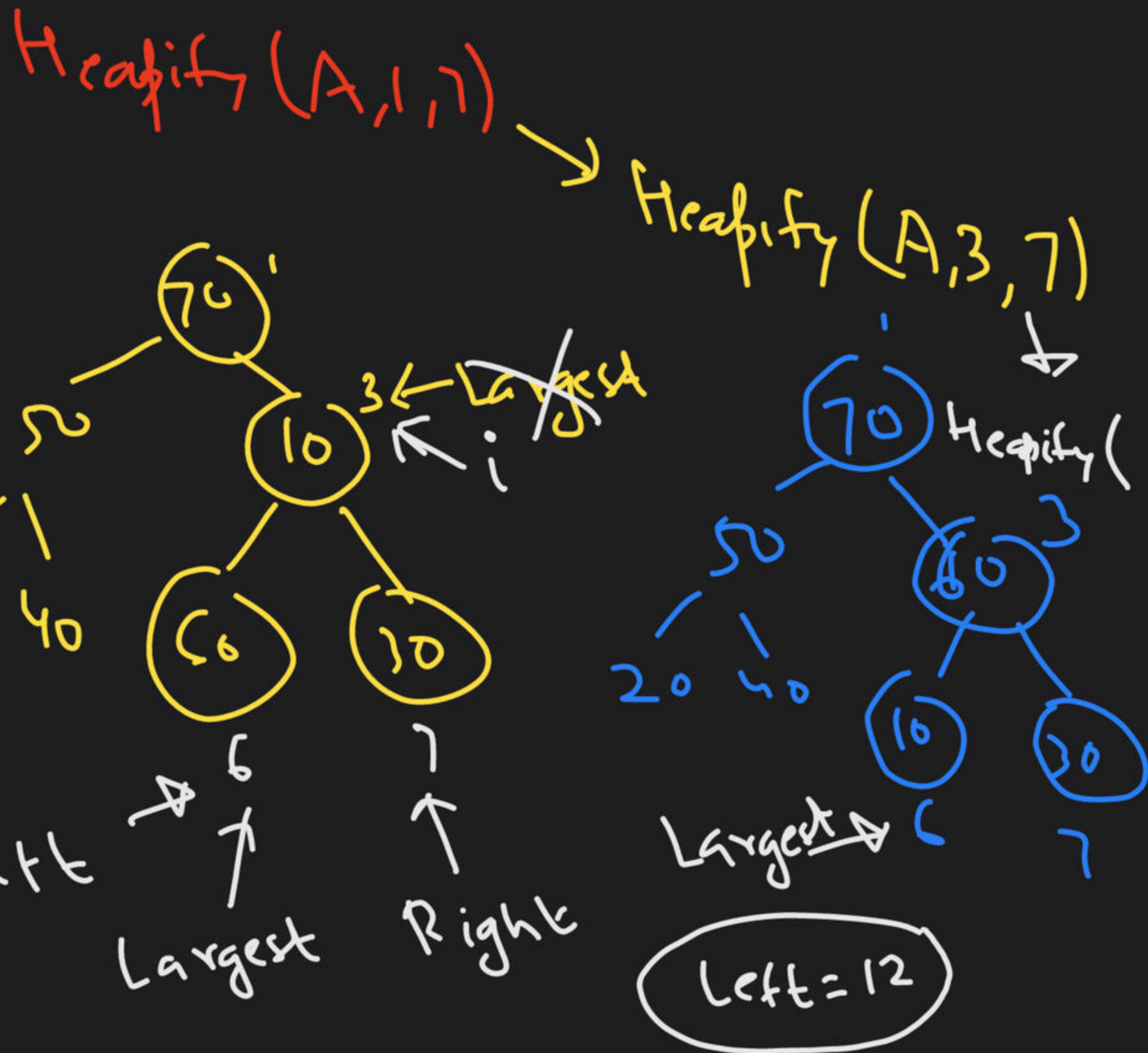
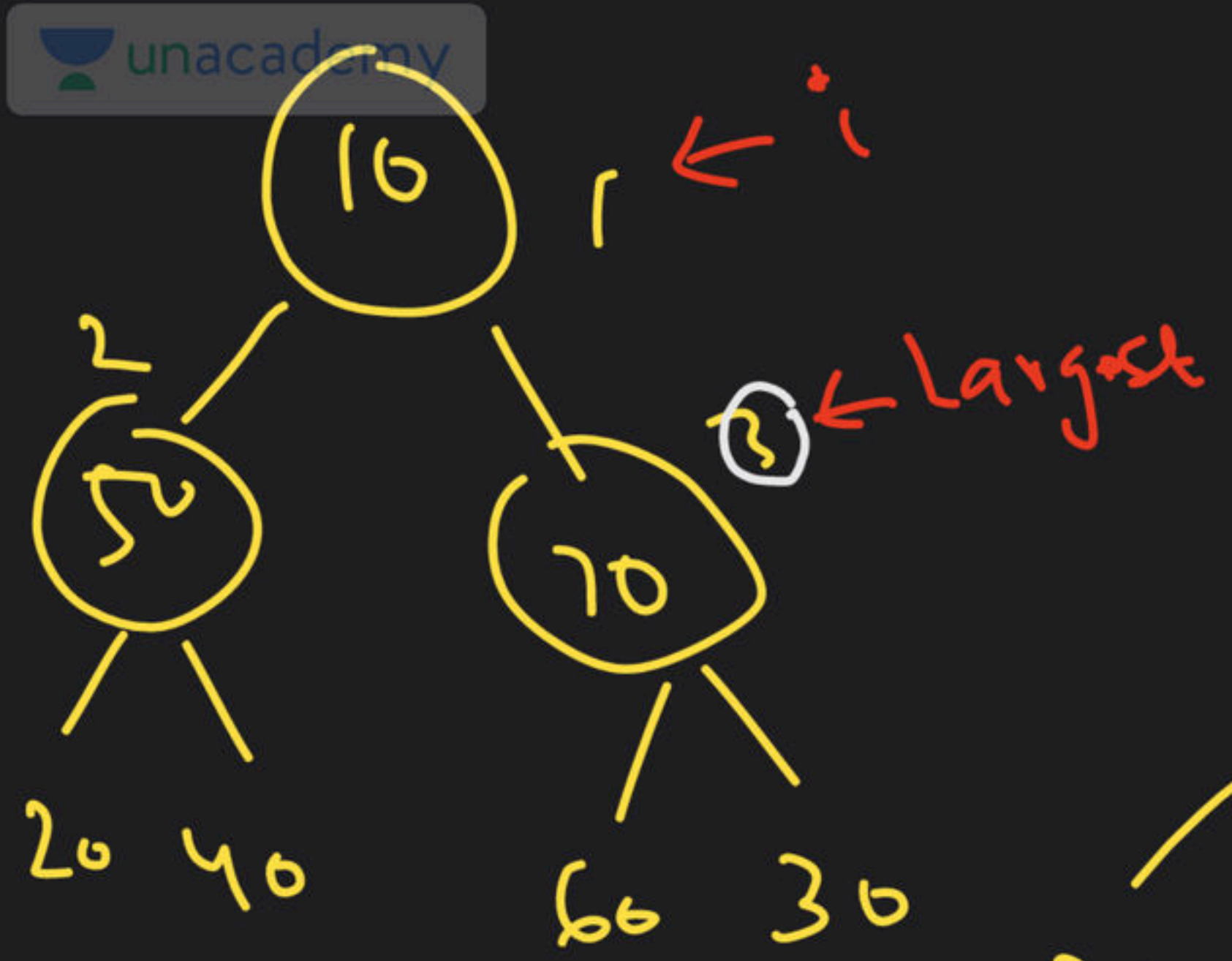
2) if ($A[\text{Left}] > A[\text{Largest}]$)

Largest = Left;

3) if ($A[\text{Right}] > A[\text{Largest}]$)

Largest = Right;

if ($i \neq \text{Largest}$) { swap($A[i], A_{\text{Largest}}$)
Heapify(A, Largest, n);



Heapify (A, i, n)

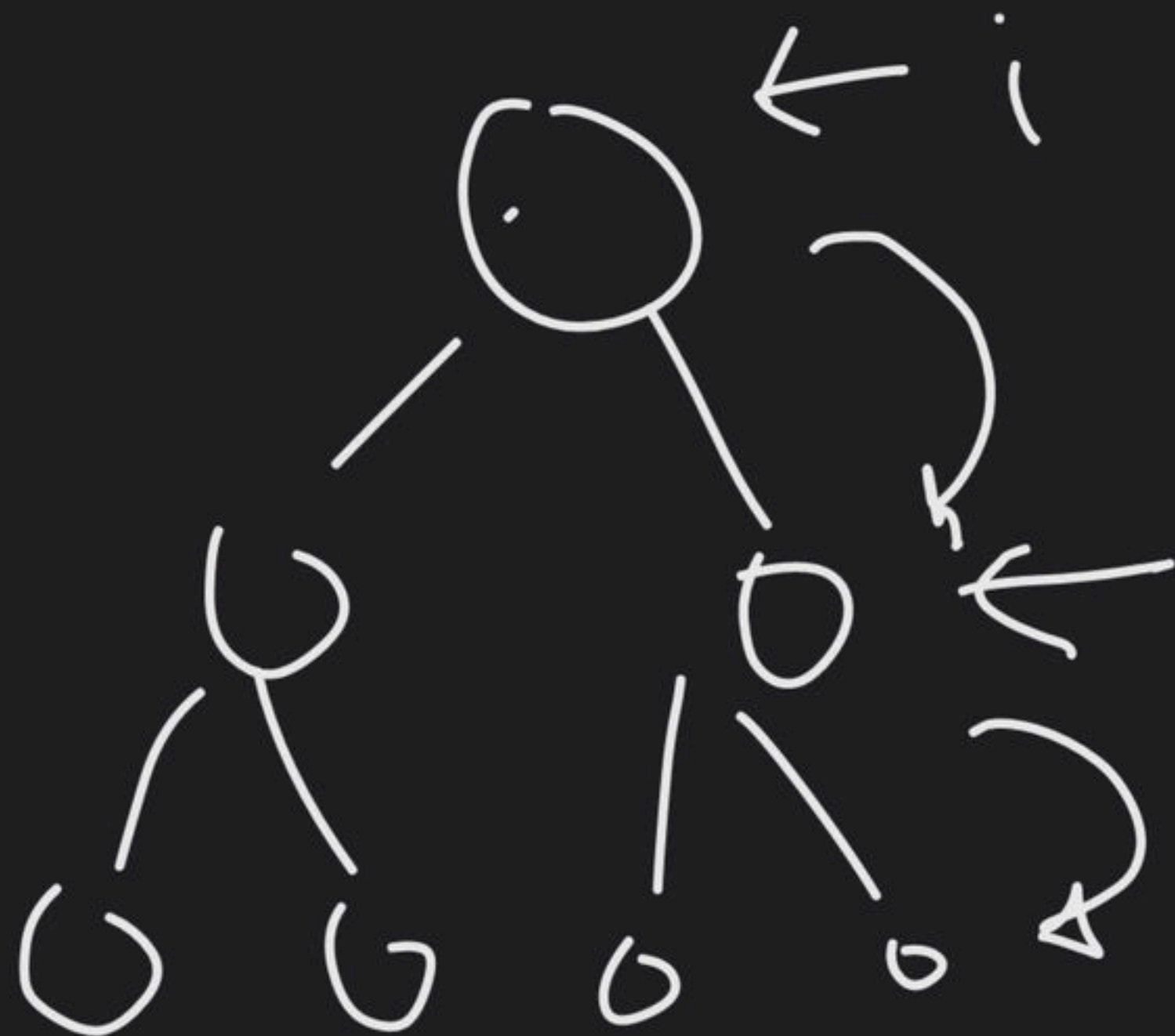
1) Left = $2 \times i$; Right = $2 \times i + 1$; Largest = i;

2) if (Left \leq n && A[Left] > A[Largest])
Largest = Left;

3) if (Right \leq n && A[Right] > A[Largest])
Largest = Right;

4) if (i \neq Largest) {
 swap(A[i], A[Largest]);
 Heapify(A, Largest, n);
}

Analysis



$$h \Rightarrow O(\log_2 n)$$

$$O(n)$$

internal nodes \Rightarrow \approx heapify

Max-heap

100	96	95	80	85	70	84
1	2	3	4	5	6	7

Build-heap $\Rightarrow O(n)$



① Find Max \Rightarrow return $A[1]$
 $O(1)$

② Find Min $\Rightarrow O(n)$

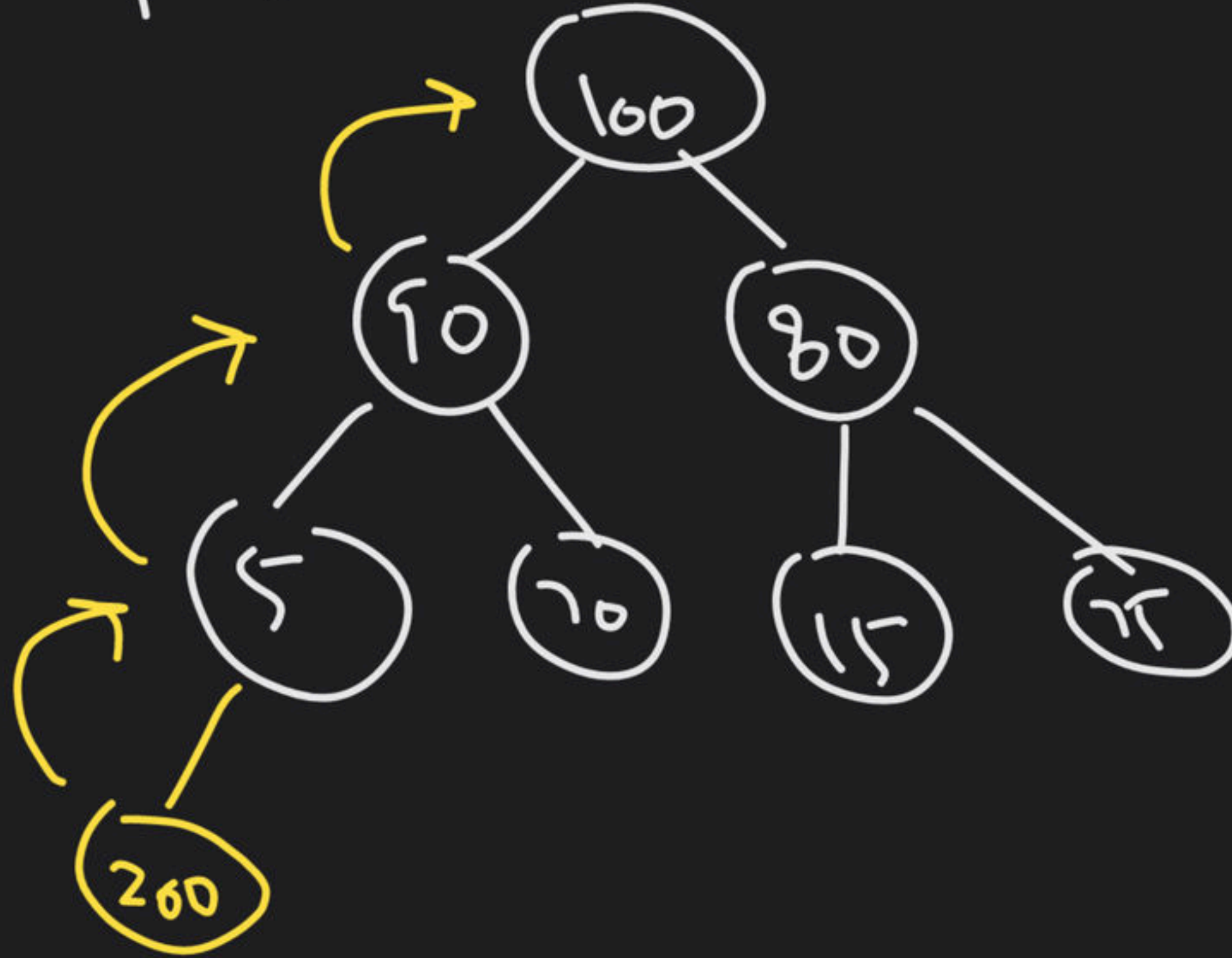
$$\# \text{ leafs} = \lceil \frac{n}{2} \rceil = \lceil \frac{7}{2} \rceil = \lceil 3.5 \rceil = 4$$

$$10 \Rightarrow 9$$

$$\left\lceil \frac{n}{2} \right\rceil \Rightarrow O(n)$$

$$\textcircled{3} \text{ Search } \Rightarrow O(n)$$

Insert a key



$O(\log_2 n)$

$\text{Par}(i)$

$$= \left\lfloor \frac{i}{2} \right\rfloor$$

Max-heap

- 1) Find Max - $O(1)$
- 2) Find Min - $O(n)$
- 3) Insert key - $O(\log_2 n)$
- 4) Search - $O(n)$
- 5) Extract Max : $O(\log_2 n)$

Min-heap

- 1) Find Min - $O(1)$
- 2) Find Max - $O(n)$
- 3) Insert $\rightarrow O(\log n)$
- 4) Search - $O(n)$
- 5) Extract-min $\rightarrow O(\log_2 n)$



THANK YOU!

Here's to a cracking journey ahead!