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Counting Techniques - Part VI

Course on Engineering Mathematics for GATE - CSE

Rahul • Lesson 6 • June 21, 2024

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Engineering Mathematics

Probability & Statistics

counting principles

Lecture Number- 05

By- Rahul Sir

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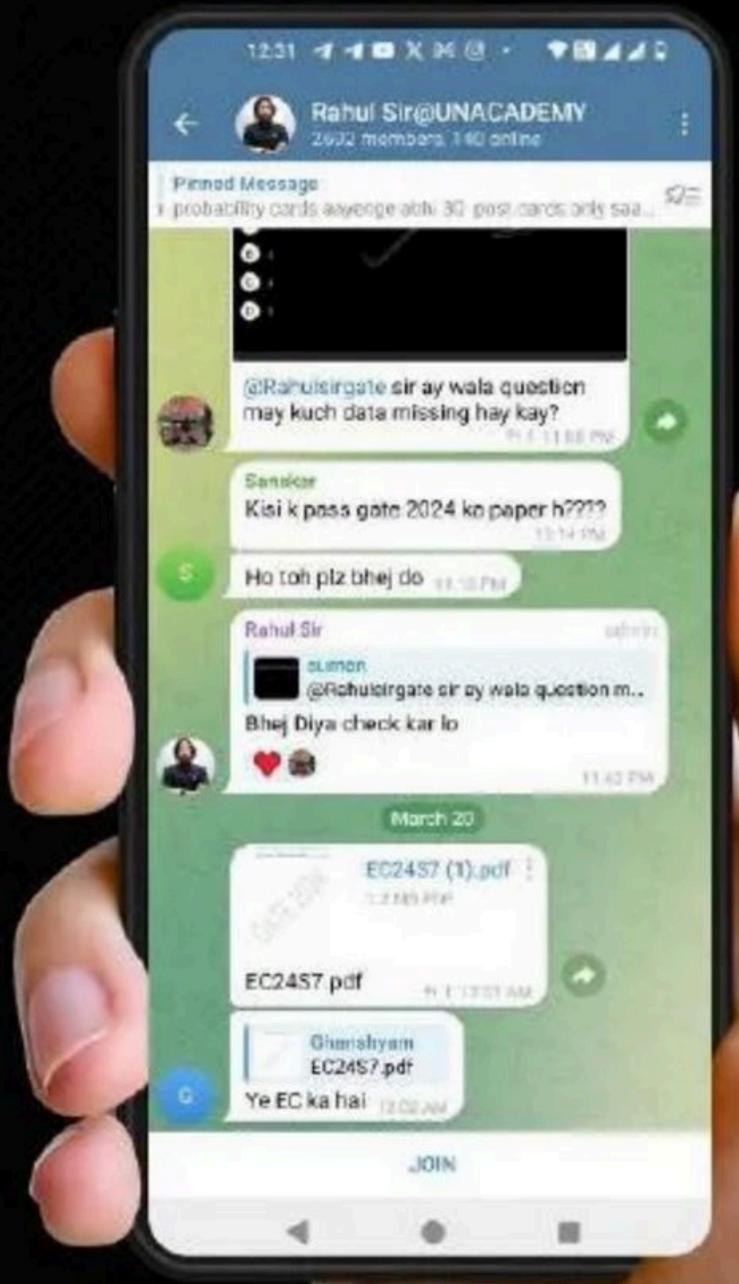

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Topics

to be covered

1. counting principles_III

✓ DPP01

Mrg upload
Telegram
whats app.

Distribution / division of Non Identical objects

⇓

basic flavour → combinatorics
→ Aptitude

Division | distribution of Non - Identical objects

DIVISION →

3 container

5 balls

How Many Number of ways to divide 5 balls in 3 container (0 or more balls are Recieve in every container)

0 or more balls

$= 3 \times 3 \times 3 \times 3 \times 3$

$= 3^5$ times

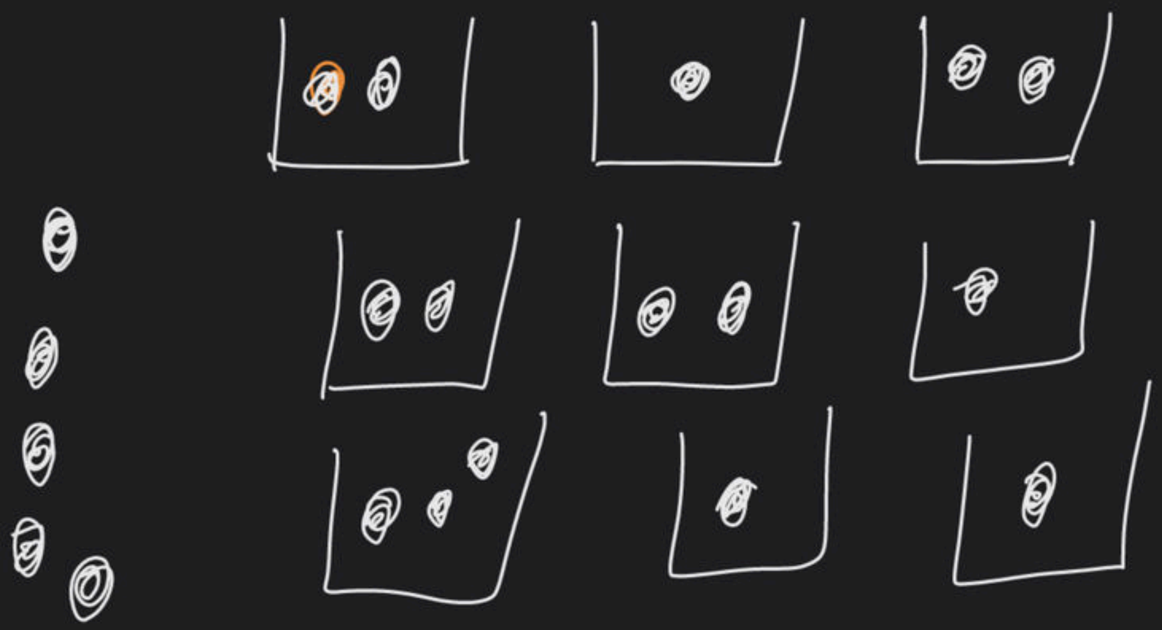
No. of divide ways.

1 OR more divide.

Receive

= (container) Receiver.

= No. of ways.



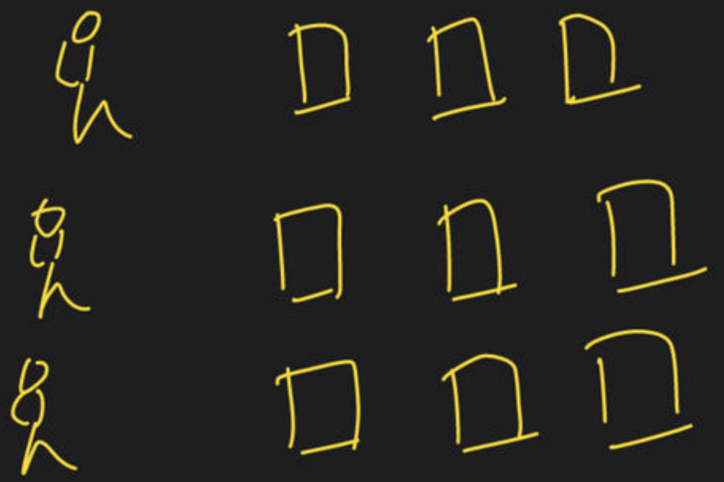
5 Tourists

3 Hotels

How Many No. of ways to divide

Each Hotel Reserve (Accomodate) 0 OR more Tourists.

$= 3^5$



3 ways.

Reserve (container)

n Items r bags (r containers)

Reverse:

0 OR more Items Reverse the bags = (container)

0 OR more Items Reverse The bags. = 2^n

CASE 02

" How Many No. of ways to divide. Every box contains atleast ONE ball "

CASE 02

"How Many No. of ways to divide Every box contains atleast ONE Ball"

X cancel This CASE

✓6 Toffee

Division

atleast ONE Toffee every person.

r box

n Items

0 0 0 0 0

Non

Identical

balls.

No. of ways to divide at least (1)

$$\Rightarrow r^n - {}^rC_1(r-1)^n + {}^rC_2(r-2)^n - {}^rC_3(r-3)^n$$

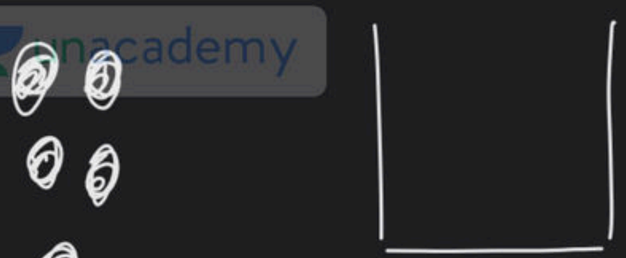
At least ONE

at least TWO

at least THREE

$$\Rightarrow 3^5 - {}^3C_1(3-1)^5 = 3^5 - 3 \times 2^5 = 3^5 - 32 \times 3$$

$$= 147$$ answer



n Items

r boxes

No. of ways to divide atleast ONE

$$= \underbrace{r^n - {}^rC_1(r-1)^n + {}^rC_2(r-2)^n + \cdots}_{\text{atleast ONE}}$$

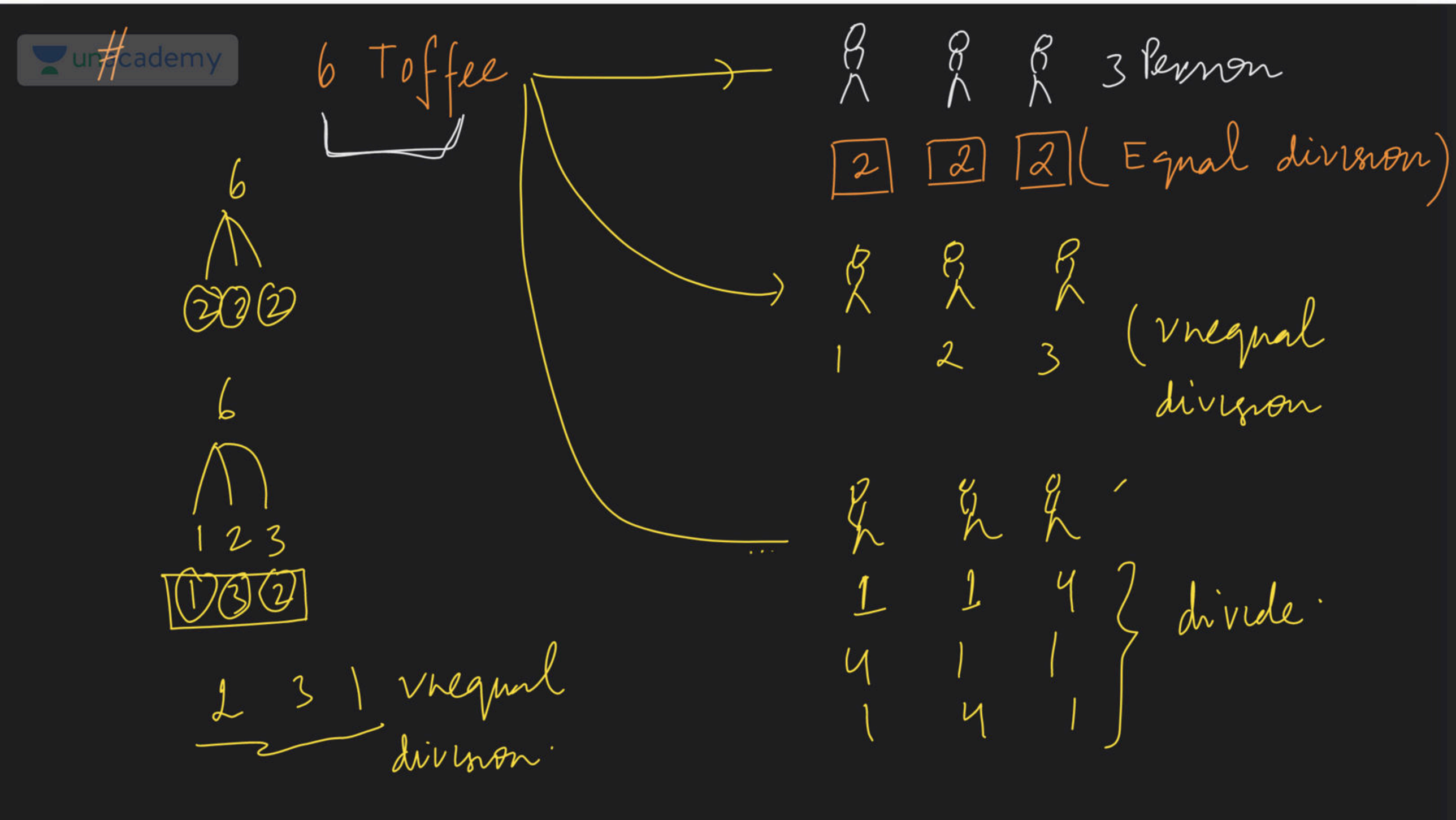
#

$$= 3^5 - {}^3C_1(3-1)^5 = 147$$

Division

n Non Identical Items → r boxes → Reserve 0 OR MORE ⇒ r^n | division

n Non Identical Items → r boxes Reserve atleast ONE $= r^n - {}^rC_1(r-1)^n + {}^rC_2(r-2)^n + {}^rC_3(r-3)^n + \cdots$


#cademy
6 Toffee
3 Person
2 2 2 (Equal division)
6
2 2 2
6
1 2 3
1 3 2
2 3 1 unequal division.
1 2 3 (unequal division
1 1 4
4 1 1
1 4 1
divide.

Unequal divide

Equal divide.

Equal as well Unequal.

✓ Division → Divide → boxes No change

✓ Division → 6 + distribution

1 2 3
3 2 1
2 3 1
} NEW way

2 2 2 NEW
2 2 2 way
2 2 2

Performing A jobs.

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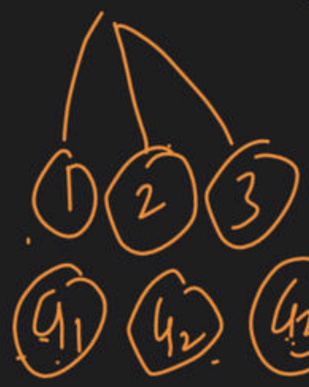
	division	distribution	Distribution among Equal groups
Unequal division	Basic Result	No. of ways to divide / No. of groups ×	×
Equal division	Basic Result / No. of groups	Basic Result	×
Equal as well unequal	basic Result / No. of Equal groups	No. of ways divide / No. of groups ×	Basic Result

	division	distribution	Distribution among Equal groups
Unequal division	Basic Result	No. of ways to divide / No. of groups	X
Equal division	Basic Result / No. of groups	Basic Result	X
Equal as well unequal	basic Result / No. of Equal groups	No. of ways divide / No. of groups	Basic Result

basic Result

6 Toffee



3 groups

No. of ways to divide

$= {}^{6}C_1 \; {}^{5}C_2 \; {}^{3}C_3$

$= \frac{\text{Total No. of objects}}{G_1(\text{No})! \; G_2(\text{No})! \; G_3! \; \ldots}$

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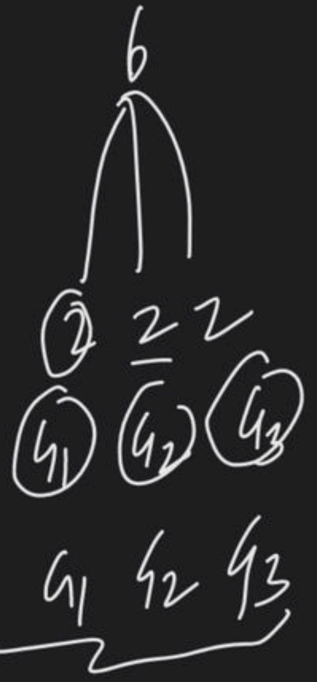
$= \text{No. of ways to divide It}$

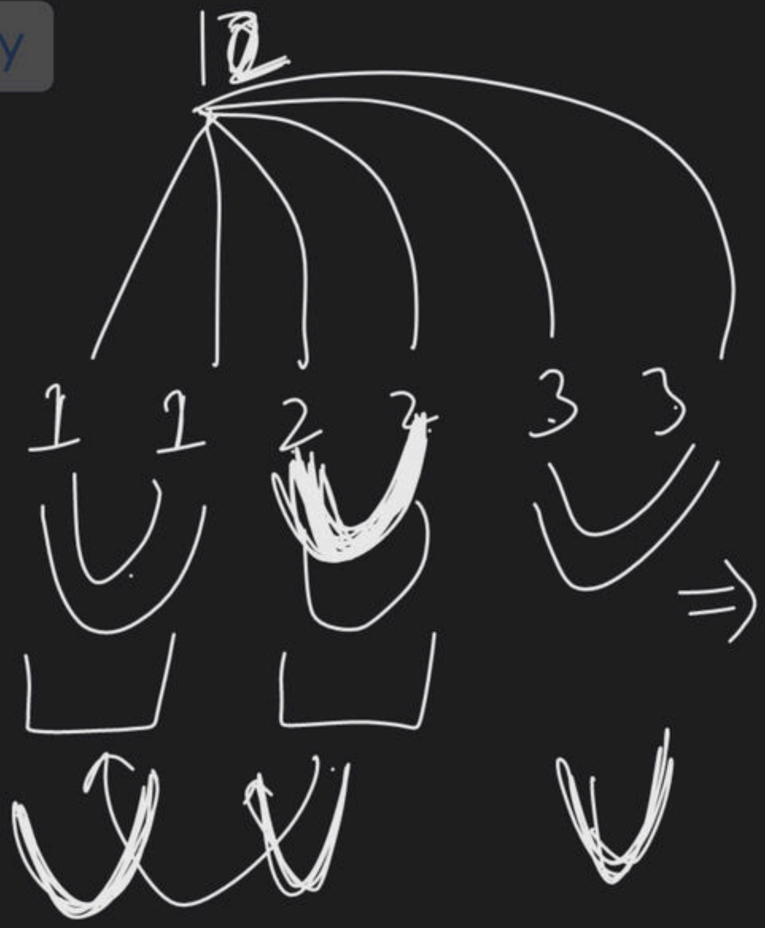
$= \frac{\lfloor 6}{\lfloor 1 \lfloor 2 \lfloor 3} = \frac{6!}{1!\,2!\,3!}$

basic Result



basic Result $= \frac{6!}{(2!)^3} \times \frac{1}{\lfloor 3}$

Key Point



$\Rightarrow$

$g_1 = g_2$

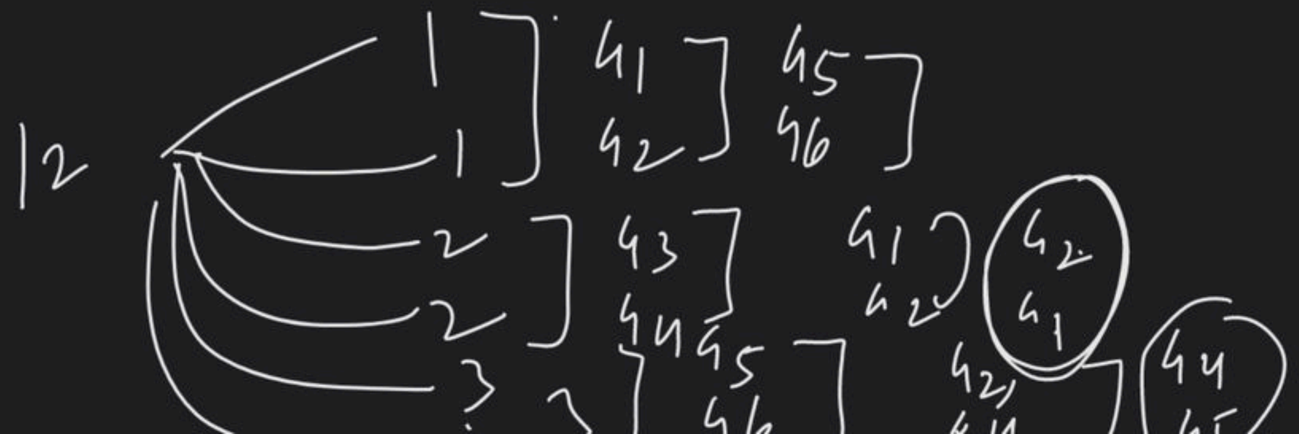
$g_3 = g_4$

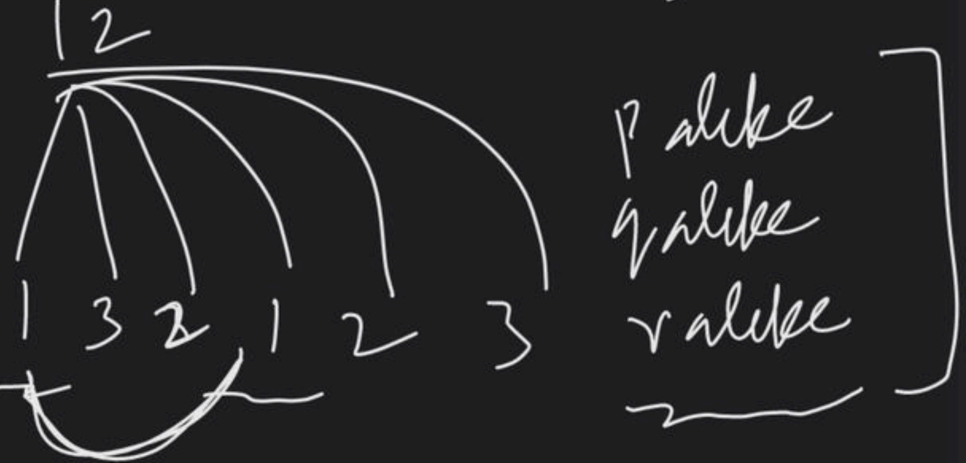
$g_5 = g_6$

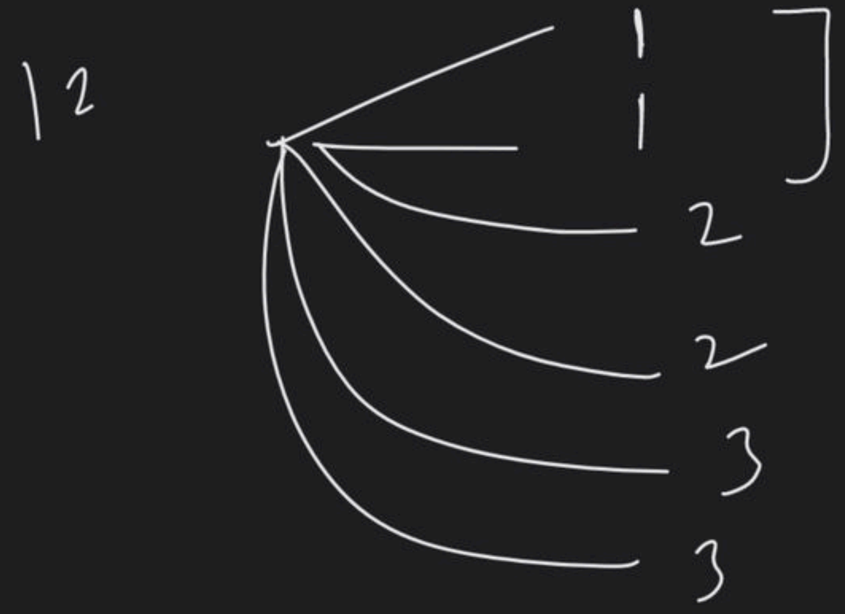
p alike, q alike, r alike

$\rightarrow$ counting over

$$\frac{12!}{(2!)^2\,(1!)^2\,(3!)^2} \times \frac{1}{(2!)^3}$$







$a_1 = a_2$ (a_2, a_1) (a_1, a_2)

$a_3 = a_4$ (a_4, a_3) (a_3, a_4)

$a_5 = a_6$

element

2

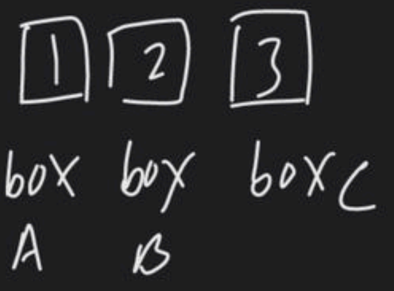
$= \frac{1}{21} \times \frac{1}{21} \times \frac{1}{21}$ ✓

2 counting correct

✓

	division	distribution	Distribution among Equal groups
Unequal division	Basic Result	No. of ways to divide / No. of groups	X
Equal division	Basic Result / No. of groups	Basic Result	X
Equal as well unequal	basic Result / No. of Equal groups	No. of ways divide / No. of groups	Basic Result

Unequal division



divide form

How Many No. of ways to divide

Division = basic Result

$$= \frac{\lfloor \text{Total No. of objects} \rfloor}{\lfloor \text{No. of objects } G_1 \rfloor \lfloor \text{No. of objects } G_2 \rfloor \lfloor \text{No. of objects } G_3 \rfloor}$$

Person = 1 = 1
= 2
= 3

✓ Division $= \frac{\lfloor 6}{\lfloor 1 \lfloor 2 \lfloor 3}$

A B C

1 2 3

2

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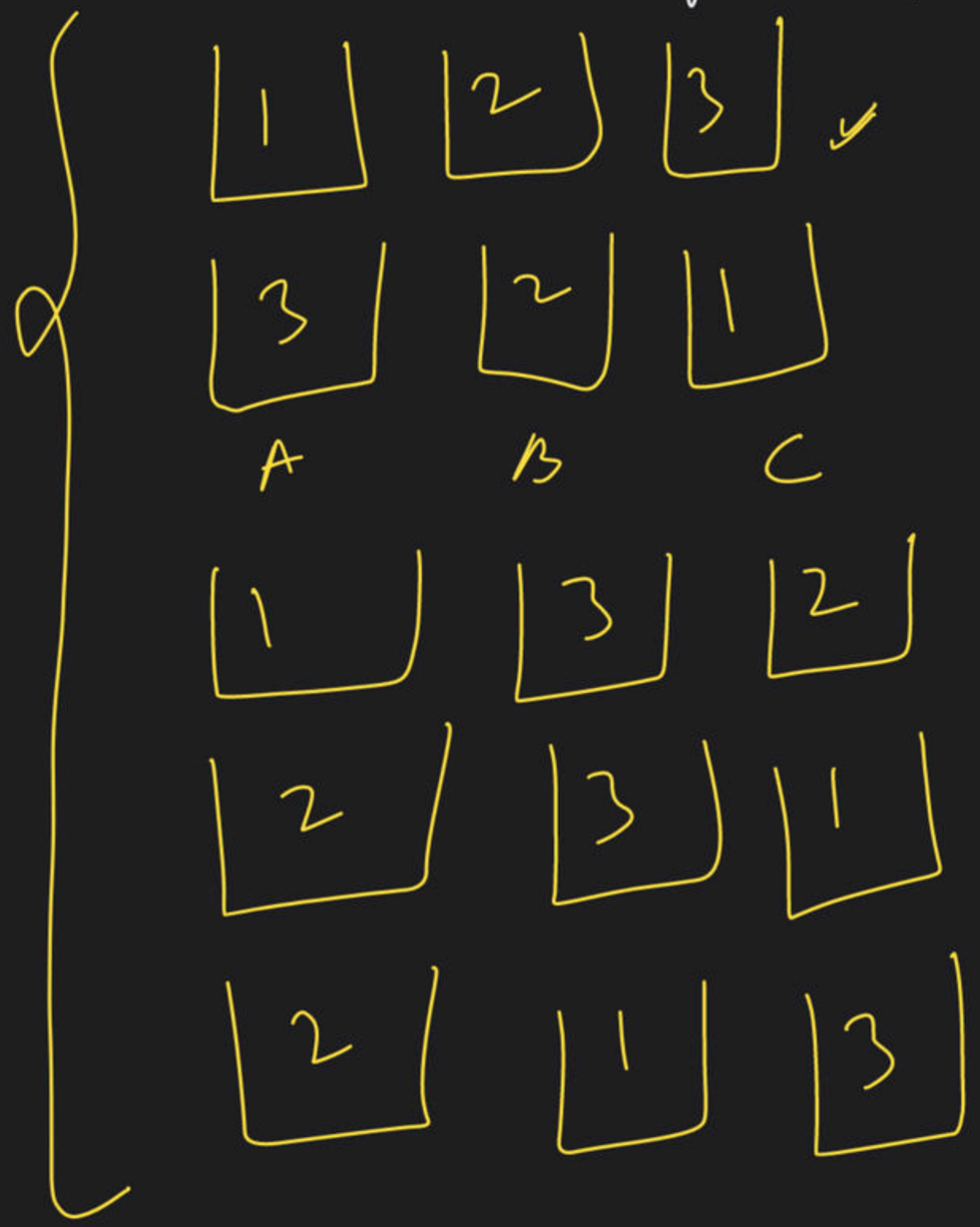
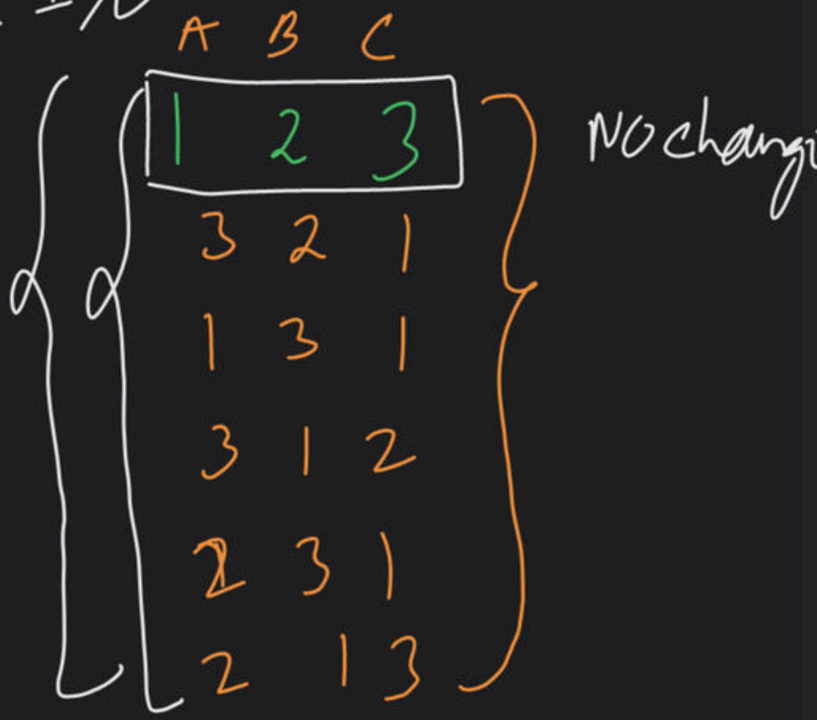
How Many ways to distribute It



Entries across the Row and column

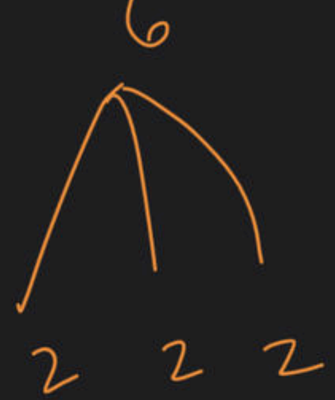


distribute

$= \text{basic Result} \times \frac{1}{\text{No. of groups}}$

$= \frac{\lfloor 6}{\lfloor 1 \lfloor 2 \lfloor 3} \times \lfloor 3 = \text{distribution}$

Equal division:



Division Rule $\Rightarrow$ $\frac{6!}{(\text{No. of objects } G_1)!\,(\text{No. of } G_2)!\,(\text{No. of objects } G_3)!} \times \frac{1}{3!}$

Ian Many No. of ways to divide It

basic Result $= \frac{6!}{2!\,2!\,2!} \times \frac{1}{3!}$

basic Result

Ans.

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How Many No. of ways to distribute It

$$\left\{\begin{array}{lll} \overset{2}{(G_1)} & \overset{2}{(G_2)} & \overset{2}{(G_3)} \\ (2) & (2) & (2) \\ (2) & (2) & (2) \end{array}\right. = \text{Basic Result} = \frac{\lfloor 6}{\lfloor 2 \; \lfloor 2 \; \lfloor 2}$$

Equal as well as Unequal divison.

20 Toffee

1 1 2 2 2 3 3 3 3

$a_1 = a_2$, $a_3 = a_4 = a_5$, $a_6 = a_7 = a_8 = a_9$

Equal as well unequal

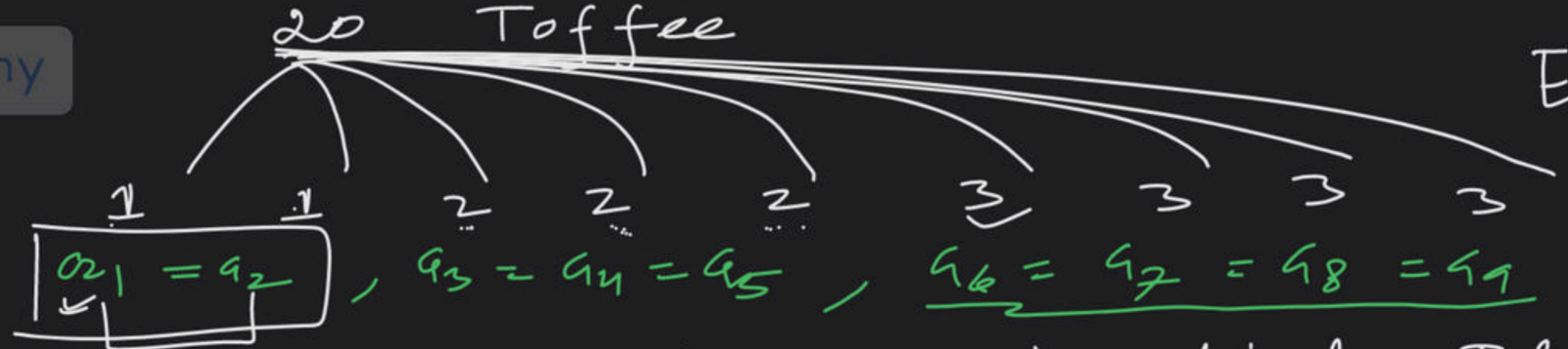
How Many No. of ways to divide It.

$$\Rightarrow \frac{20!}{(1!)^2(2!)^3(3!)^4} \times \frac{1}{2!^c\, 3!^b\, 4!^a}$$

No. of groups.

No. of division $= \left\{ \frac{20!}{(1!)^2(2!)^3(3!)^4} \times \frac{1}{\text{Divide of equal groups}} \right.$

$$= \left. \frac{20!}{(1!)^a(2!)^b(3!)^c} \times \frac{1}{2!^c\, 3!^b\, 4!^a} \right\} \text{DONE}$$



Equal as well unequal

How Many No. of ways to divide It.

$$\Rightarrow \frac{20!}{(2!)^2 (2!)^3 (3!)^4} \times \left[\frac{1}{2!}\ \frac{1}{3!}\ \frac{1}{4!}\right]$$

Equal No. of groups.

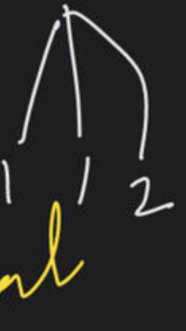
How many No. of distribute It

$$= \left\{ \frac{20!}{(2!)^2 (2!)^3 (3!)^4} \times \frac{1}{2!} \times \frac{1}{3!} \times \frac{1}{4!} \times \boxed{\text{No. of groups}} \right.$$

$$= \left\{ \frac{20!}{(2!)^2 (2!)^3 (3!)^4} \times \frac{1^6}{2!} \times \frac{1^6}{3!} \times \frac{1^6}{4!} \times \boxed{9} \right.$$

i: 1 to 9

ans.

	division	Distribution	Distri Among Equal groups
Unequal Division 	basic Result $\frac{\lfloor n}{\lfloor g_1 \lfloor g_2 \lfloor g_3}$	No. of ways to divide $\times$ $\frac{\text{No. of groups}}{\text{ordering}}$	X
Equal Division 	No. of ways to divide $\frac{}{\lfloor \text{No. of groups}}$	Basic Result $\frac{\lfloor n}{\lfloor g_1 \lfloor g_2 \lfloor g_3}$	X
Unequal Division as well equal 	No. of ways to divide $\frac{}{\lfloor \text{No. of Equal groups}}$	No of ways to divide $\times$ $\lfloor$ No. of groups	basic Result $\frac{\lfloor n}{\lfloor g_1 \lfloor g_2 \lfloor g_3}$

QUES01 In How Many ways can 7 department be divided among 3 ministers such That Every minister gets atleast ONE and atmost 4 departMENTS to control.

A) 630

B) 1050

C) 1680

D) NONE of THESE

7 DEPT → 3 Minister (Ruling Party)

✓ Every minister gets atleast ONE and atmost 4 Dep.

Atleast ONE ✓

M_1 M_2 M_3

4 dept max

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	Minister A	minister B	minister C	
✓	2	2	3	atleast one and atmost 4
✓	4	2	1	
✓	1	3	3	

(G_1, G_2, G_3)

$(2, 2, 3)$

A	B	C
$d_1 - d_2$	$d_3 - d_4$	$d_5 - d_6 - d_7$
$d_1 - d_4$	$d_3 - d_6$	d_7
d_7	$d_5 - d_6$	$d_1 - d_4$
$d_5 - d_6$	$d_4 - d_4$	d_7

atleast one and atmost

Distribution

A	B	C
2	2	3
4	2	1
1	3	3

No. of

Distribution

Equal division / Unequal

Unequal Division

(4, 2, 1)

$$\Rightarrow \frac{L7}{(L2)^2\, L3} \times \frac{1}{L2} \times L3 \quad (1)$$

$$\Rightarrow \frac{L7}{L4\, L2\, L1} \times L3 \quad (2)$$

UnEqual as well equal

$$= \frac{L7}{L3\, L3\, L1} \times \frac{1}{22} \times L3 \quad (3)$$

$$= 1680$$

answer

QUESTION

Illustration:

Q. How many 3 digit numbers can be formed by the digit 1, 2, 3, 4, 5 without repetition

QUESTION

Illustration:

Q. In an examination of 10 T/F question, How many sequence of answers are possible.

QUESTION

Illustration:

Q. 10 students complete in a swimming race. In how many ways can they occupy the first 3 positions.

QUESTION

Illustration:

Q. There are 7 flags of diffrent colour. Find the number of diffrent signals that can be transmitted by the use of 2 flags one above the other.

QUESTION

Illustration:

Q. A letter lock consists of 3 rings each marked with 10 different letters. In how many ways, its is possible to make an unsuccessful attempt to open the lock?

QUESTION

Illustration:

Q. How many 6 digits odd number greater than 6,00,000 can be formed from the digits 5,6,7, 8, 9, 0 if repetition of digit is allowed?

QUESTION

Illustration:

Q. How many integers greater than 5000 can be formed with the digit 7, 6, 5, 4 & 3 using each digit at most once.

QUESTION

Illustration:

Q. How many natural number less than 30000 can be made from the digits 0, 1, 2, 3, 4, 5, 6.

QUESTION

Illustration:

Q. Consider the word DAUGHTER. How many 4 letter word can be formed from the letter of above word so that each word contain letter G

QUESTION

Illustration:

Q. How many different words can be formed using all the letters in the word "MIRACLE".

(a) If vowels may accupy the even position.

(b) If vowels may occupy odd position.

QUESTION

Illustration:

Q. There are m men and n monkey. Number of ways in which every monkey has a master, if a man can have any number of monkey.

QUESTION

Illustration:

Q. Number of ways in which m different toys can be distributed in "n" children if every child may receive any number of toys

QUESTION

Illustration:

Q. Find the number of ways in which we can post 5 letters in 10 letter boxes.

QUESTION

Illustration:

Q. In a car plate number containing only 3 or 4 digits not containing the digit 0. What is the maximum numbers of cars that can be numbered?

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THANK YOU!

Here’s to a cracking journey ahead!