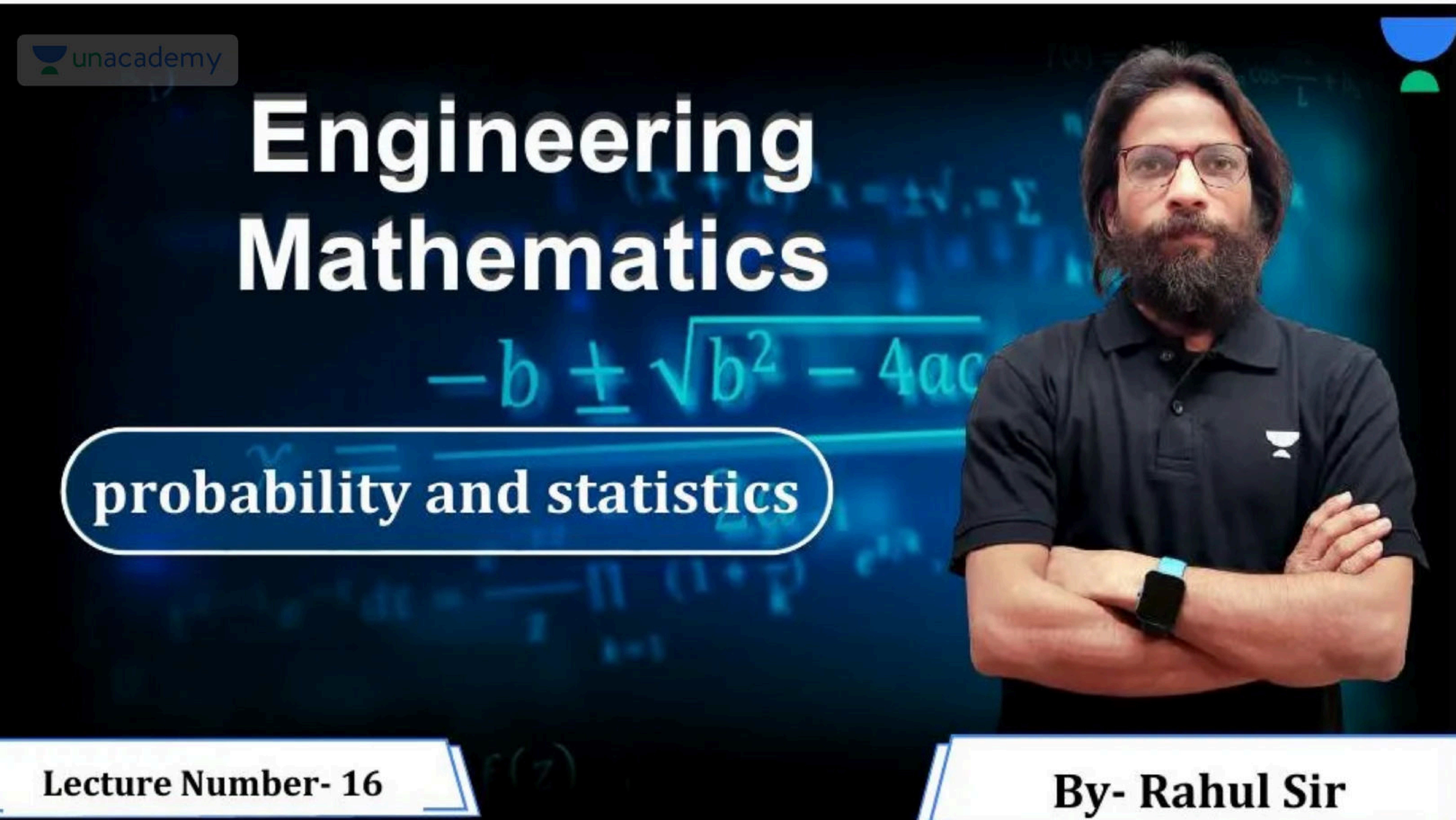
Random Variables Part-IV

Course on Engineering Mathematics for GATE - CSE

Rahul • Lesson 17 • June 30, 2024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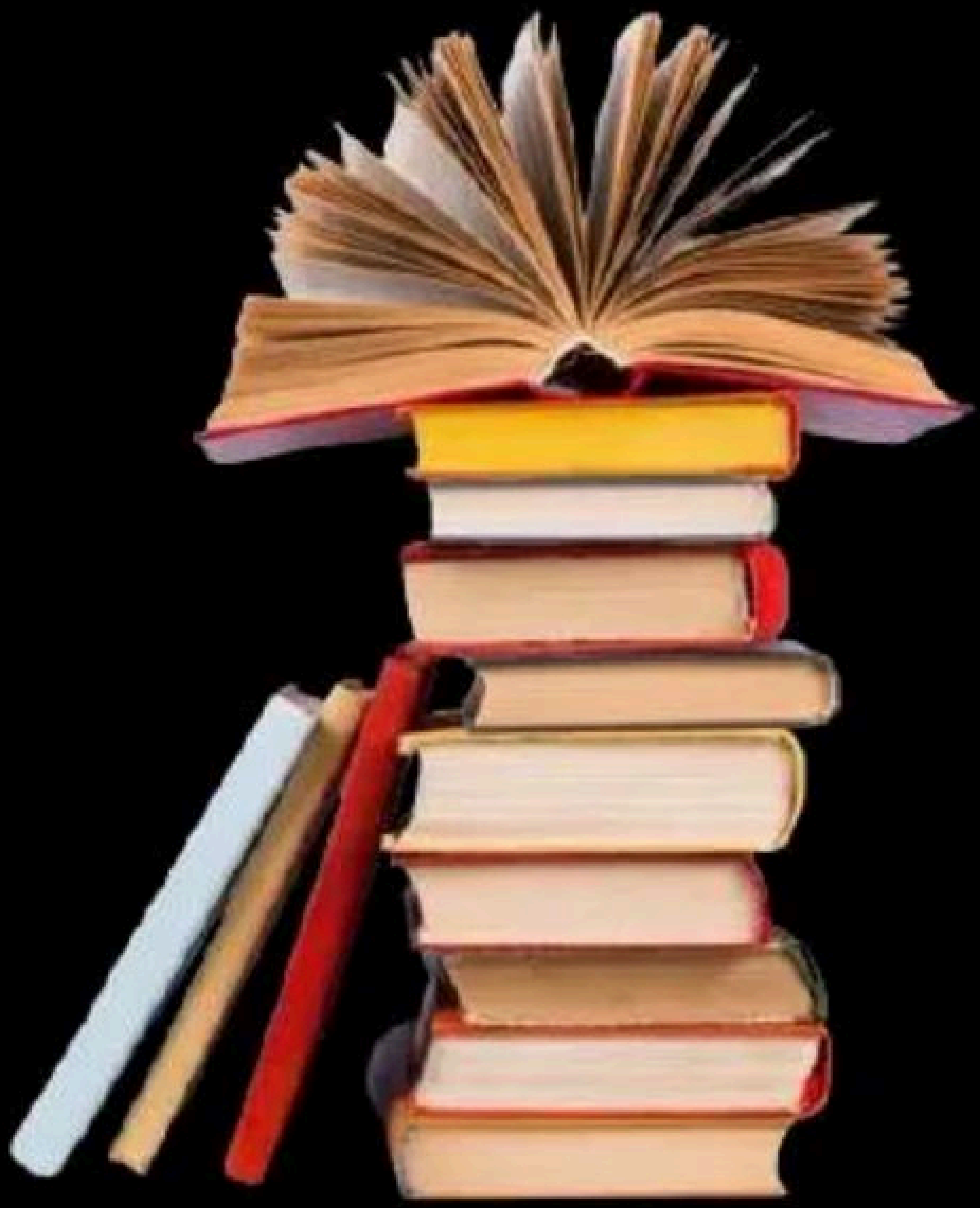
Engineering
Mathematics
probability and statistics
Lecture Number- 16
By- Rahul Sir

Topics

to be covered



1. Problem solving class

EXPECTED value $= \sum x_i p_i$

$\mu = E(X) =$ "X is DISCRETE Random variable"

EXPECTED value $= \int_a^b x \, f(x) \, dx$

$E(X) = \mu$

variance :- DISPERSION + Distance about MEAN

1
2
3
4
5
6

3.5

$= (1 - 3.5)^2$

$= (2 - 3.5)^2$

$= (3 - 3.5)^2$

$= (4 - 3.5)^2$

$= (5 - 3.5)^2$

$(6 - 3.5)^2$

$= \sum f_i x_i^2 - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2$

$$\boxed{\text{variance} = E(X^2) - [E(X)]^2}$$

x_0	x_1	x_2	x_3	$\dots$	x_n
p_0	p_1	p_2	p_3		p_n

$$E(X^2) = x_0^2 p_0 + x_1^2 p_1 + x_2^2 p_2 + \dots + x_n^2 p_n$$

$$\boxed{\begin{aligned} E(X^2) &= \sum x_i^2 p_i \\ \text{variance} &= \sum x_i^2 p_i - \left[\sum x_i p_i\right]^2 \end{aligned}}$$

standard deviation $= \sqrt{\text{variance}}$

variance can't be NegativE

In Continuous Random Variable:

$$Var(X) = E(X^2) - [E(X)]^2$$

$$\boxed{E(X^2) = \int^{b} x^2 f(x)\, dx}$$

$a \le X \le b$

$$E(X^3) = {}^{a}\int_{a}^{b} x^3 f(x)\, dx$$

$$\boxed{E(X^n) = \int_{a}^{b} x^n f(x)\, dx}$$

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$$\mathrm{var}(X) = \int_a^b x^2 f(x)\,dx - \left[\int_a^b x f(x)\right]^2$$

$$\text{standard deviation} = \sqrt{\text{variance}}.$$

Throwing A Die

X	$\boxed{1}$	2	3	4	5	6
P	1/6	1/6	1/6	1/6	1/6	1/6

$$E(X) = 3.5$$

$$E(X^2) = (1)^2 \times \frac{1}{6} + (2)^2 \frac{1}{6} + (3)^2 \frac{1}{6} + (4)^2 \frac{1}{6} + (5)^2 \frac{1}{6} + (6)^2 \frac{1}{6}$$

$$E(X^2) = \boxed{91/6.} = \underline{\underline{15.16}}$$

$$\text{var}(X) = E(X^2) - [E(X)]^2$$

$\text{variance} = 15.16 - (3.5)^2$

$$\sigma_X^2 = \text{variance} = 15.16 - 12.25 = 2.91$$

$$\sigma_X = \text{standard deviation} = \sqrt{2.91} = \underline{\underline{1.70}}$$

$\Downarrow$

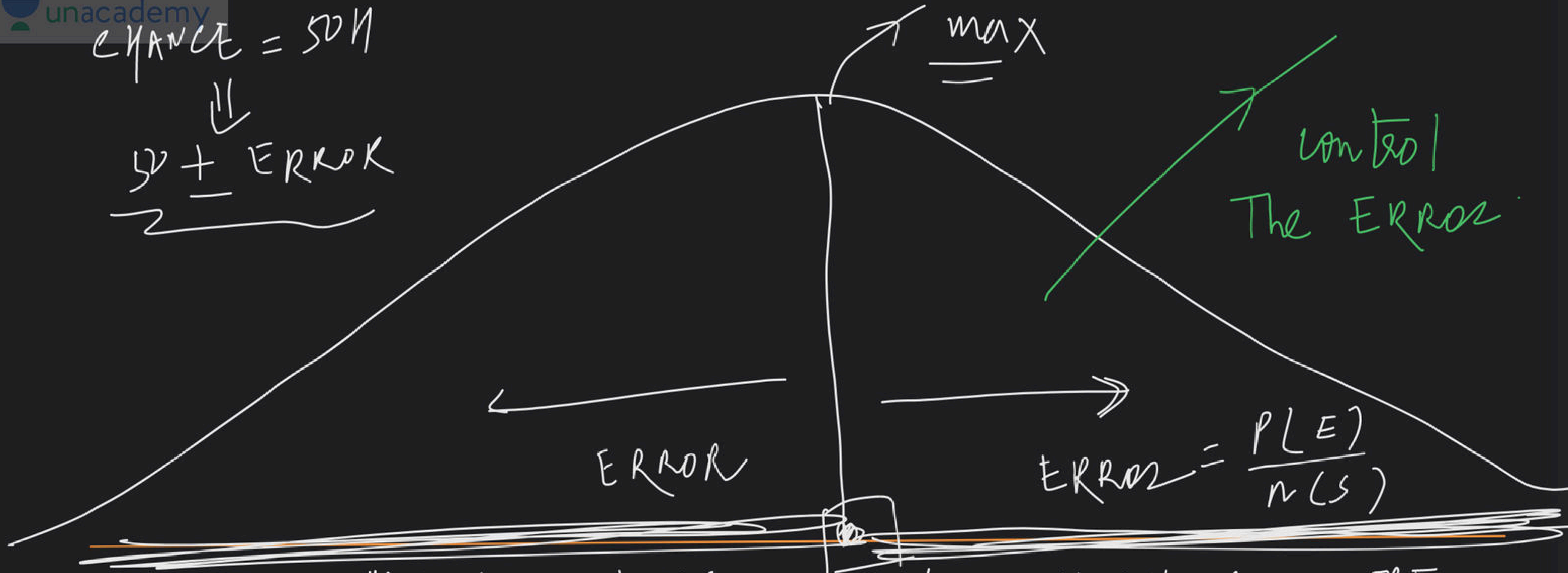
Standard Error large no. of Trials.

Error = 1.70 MEAN

CHANCE = 50%

⇓

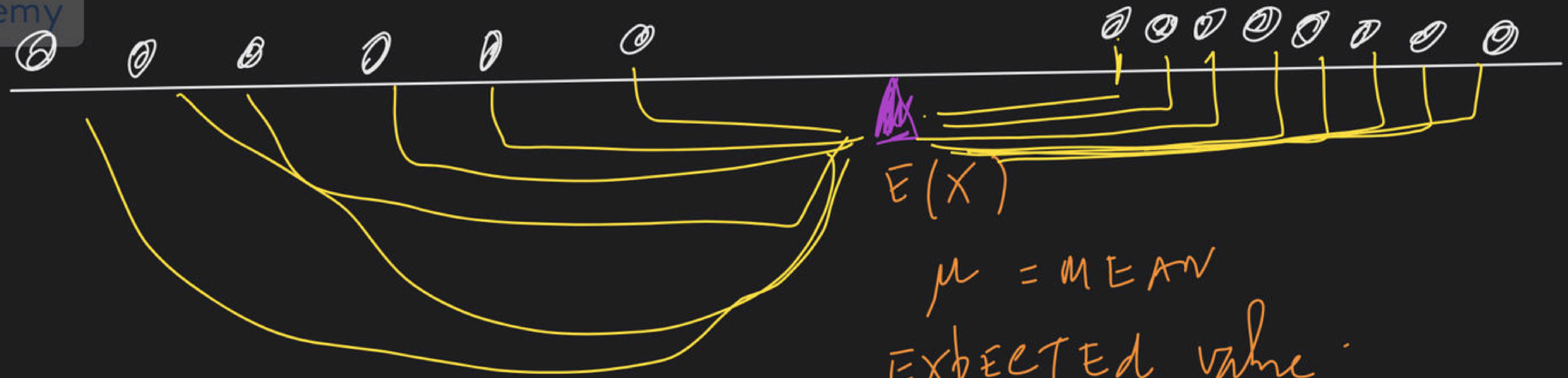
50 $\pm$ ERROR



MEAN - some LESS | MEAN | MEAN + SOME MORE

50 - S.D | 50 | 50 + S.D

$\mu - \sigma$ | | $\mu + \sigma$



μ = MEAN

EXPECTED value.

$\boxed{\text{MEAN = max likelihood number}}$

Tossing A coin 100 times

head	
47	→ 50
51	→ 50
43	→ 50
44	→ 50

$P(H) = \frac{1}{2}$ 50%

$P(T) = \frac{1}{2}$ 50%

Error ↑

↓ No Error

	Head	Diff.
50	50 $= (\text{Diff.})^2$	
50	41 $= (\text{Diff})^2$	
50	42	
50	43	
50	46	
50	54	
50	52	
50	49	
50	50	
50	52	
50	52	
50	51	

QUESTION

Q. Let $f(x)=\frac{k|x|}{(1+|x|)^4}, -\infty < x < \infty$

Then the value of k for which f(x) is a probability density function is

(A) 1/6

(B) 1/2

(C) 3

(D) 6

QUESTION

Q. A continuous random variable X has density function

$$f(x)=\begin{cases} 2x & 0<x<\dfrac{1}{2} \\ \dfrac{4-2x}{3} & \dfrac{1}{2}\leq x<2 \\ 0, & \text{elsewhere} \end{cases}$$

Find $P[0.25 < x \leq 1.25]$

QUESTION

Q. Let X be a continuous random variable with probability density function

$$f(x)=\frac{1}{2}e^{-|x-1|}, -\infty < x < \infty$$

Find the value of $P(1 < |X| < 2)$

QUESTION

X is DISCRETE Random variable

GATE

Q. A machine produces $\boxed{\text{0, 1 or 2 defective pieces in a day}}$ with associated probability of 1/6, 2/3 and 1/6, respectively. Then mean value and the variance of the number of defective pieces produced by

X	0	1	2
$P(X=x)$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$

$$MEAN = \sum x_i p_i$$

$$= 0 \times \frac{1}{6} + 1 \times \frac{2}{3} + 2 \times \frac{1}{6}$$

$$\boxed{MEAN = 1}$$

$$var = E(X^2) - [E(X)]^2$$

$$= (0)^2 \frac{1}{6} + (1)^2 \frac{2}{3} + (2)^2 \frac{1}{6} - (1)^2$$

$$\boxed{Variance = \frac{1}{3}}$$

$$S.D = \frac{1}{\sqrt{3}}$$

answer

A 1 and 1/3

B 1/3 and 1

C 1 and 4/3

D 1/3 and 4/3

QUESTION

GATE

Q. In the following table, x is a discrete random variable and P(x) is the probability density.
The standard deviation of x is:

standard deviation $= \sqrt{var(X)}$ $\quad var(X) > 0$

X	1	2	3
P(x)	0.3	0.6	0.1

$var(X) = E(X^2) - [E(X)]^2$

$V(X) = E(X^2) \times 0.3 + (2)^2\, 0.6 + (3)^2\, 0.1 - [1 \times 0.3 + 2 \times 0.6 + 3 \times 0.1]^2$

(A) 0.18

(B) 0.36

(C) 0.54

(D) 0.6

$= 0.3 + 2.4 + 0.9 - [0.3 + 1.2 + 0.3]^2$

$= 3.6 - [1.8]^2 = 3.6 - 3.24$

$= .36$

$S.D = \sqrt{.36} = .6$ ✓

QUESTION

Q. A random variable X has probability density function f(x) as given below:

$$f(x) = \begin{cases} a + bx & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

valid pdf $\int_0^1 (a+bx)\,dx = 1$

$=$ a b

If the expected value $\boxed{E|X| = 2/3}$ then Pr[X < 0.5] is_____.

a b

0.5

0 1

$\underbrace{a+b}x$

$E(X) = \frac{2}{3}$

$0 < x < 1$

$P(X < 0.5) = \int_0^{0.5} (a+bx)\,dx$

X is a continuous Random variable.

$=$ a or b term involve

If X is a valid prob. Density function

$$\int_0^1 (a+bx)\,dx = 1$$

$$\Rightarrow \quad a + \frac{b}{2} = 1 \quad — (1)$$

$E(X) = \frac{2}{3}$

X is a CRV

$$\int_a^b x f(x)\,dx = E(X)$$

$$\int_0^1 x(a+bx)\,dx = \frac{2}{3}$$

$$\int_0^1 ax + bx^2\,dx = \frac{2}{3}$$

$$\left[\frac{ax^2}{2} + \frac{bx^3}{3}\right]_0^1 = \frac{2}{3}$$

$$\frac{a}{2} + \frac{b}{3} = \frac{2}{3} \quad — (2)$$

Solve The Eqn (1) and (2)

$a = 0$

$b = 2$

$P(X < 0.5)$

$$= \int_0^{0.5} (a+bx)\,dx$$

$$= \int_0^{0.5} (0+2x)\,dx$$

$= (0.25)$ ans

QUESTION

6pm

Q. Consider the following probability mass function (p.m.f) of a random variable X.

$$p(x,q)=\begin{cases} q & \text{if } x=0 \\ 1-q & \text{if } x=1 \\ 0 & \text{otherwise} \end{cases}$$

if q = 0.4, the variance of X is ______.

DISCRETE Random variable

X	0	1
$P(X=x)$	q	$1-q$

$var(X) = (0)^2 \times q + (1)^2 \times (1-q) - [0 \times q + 1 \times (1-q)]^2$

$q = 0.4$

$= (1-q) - [1-q]^2$

$= [1-0.4] - [1-0.4]^2 = 0.24$

Variance $= 0.24$

QUESTION

GATE CSE

Q. Each of the nine words in the sentence "The Quick brown fox jumps over the lazy dog" is written on a separate price of paper. These nine pieces of paper ~~p~~are kept in a box. One of the pieces is drawn at random ~~for~~ the box. The expected length of the word drawn is____. (The answer should be rounded to one decimal place)

from

THE QUICK BROWN FOX JUMPS OVER THE LAZY DOG

Expected length of word

X = Random variable

X = No. of letters

$$\boxed{E(X) = 3.88}$$

THE QUICK BROWN FOX JUMPS OVER THE LAZY DOG
3 5 5 3 5 4 3 4 3

EXPECTED length of word

X = 3, 4, 5 (arrival / DRV / countable)

X = Random variable

X = No. of letters

X	3	4	5
$P(X=x)$	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{3}{9}$

#

$$E(X) = 3 \times \frac{4}{9} + 4 \times \frac{2}{9} + 5 \times \frac{3}{9} = \frac{35}{9}$$

$$\boxed{E(X) = 3.88}$$

answer

QUESTION

Q. The variance of the random variable X with probability density function $f(x)=\frac{1}{2}|x|e^{-|x|}$ is ____.

QUESTION

Q. A fair die with faces {1, 2, 3, 4, 5, 6} is thrown repeatedly till '3' is observed for the first time. Let X denote the number of times the dice is thrown. The expected value of X is______.

QUESTION

Q. A player tosses two unbiased coins. He wins Rs 5 if 2 heads appear, Rs 2 if one head appears and Rs 1 if no head appears. Find the expected value of the amount won by him.

QUESTION

Q. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the expected value for the number of aces.

QUESTION

Q. If it rains, a rain coat dealer can earn Rs 500 per day. If it is a dry day, he can lose Rs 100 per day. What is his expectation, if the probability of rain is 0.4?

QUESTION

Q. You toss a fair coin. If the outcome is head, you win Rs 100. if the outcome is tail, you win nothing. What is the expected amount won by you?

QUESTION

Q. A fair coin is tossed until a tail appears. What is the expectation of number of tosses?

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QUESTION

0 1 2

Q. The distribution of a continuous random variable X is defined by

$$f(x)\begin{cases} x^3, & 0<x\le 1 \\ (2-x)^3, & 1<x\le 2 \\ 0 & \text{elsewhere} \end{cases}$$

obtain the expected value of X.

$f(x) = \begin{cases} x^3 & 0 < x \le 1 \\ (2-x)^3 & 1 < x \le 2 \\ 0 & \text{elsewhere} \end{cases}$

$E(X)$ = Expected value of X

$E(X) = \int_0^1 x \cdot \underset{f(x)}{x^3}\, dx + \int_1^2 x \cdot (2-x)^3\, dx$

$(A-B)^3$

$2-x = t$

Expected value =

$$\int x^n \, dx = \frac{x^{n+1}}{n+1}$$

$$E(X) = \int_0^1 x \cdot x^3 \, dx + \int_1^2 x(2-x)^3 \, dx$$

$$= \underbrace{\int_0^1 x^4 \, dx}_{I_1} + \underbrace{\int_1^2 x(2-x)^3 \, dx}_{I_2}$$

$$I_1 = \frac{1}{5}$$

$$I_2 = \int_1^2 x(2-x)^3 \, dx = -\int_1^0 (2-t)\, t^3 \, dt$$

$2 - x = t \qquad 2 - t = x$

$-dx = dt$

$$= \int_0^1 t^3 (2-t) \, dt$$

$$\int_a^b = -\int_b^a$$

$$\boxed{I_2 = \frac{1}{10}}$$

$$E(X) = \frac{1}{5} + \frac{1}{10} = \frac{3}{10}$$

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QUESTION

2

Q. For a continuous distribution, whose probability density function is given by:

$$f(x)=\frac{3x}{4}(2-x), 0 \le x \le 2,$$

find the expected value of X.

$E(X) = \int_0^2 x \cdot \frac{3x}{4}(2-x)\,dx$

$f(x) = \frac{3x}{4}(2-x) \quad 0 \le x \le 2$

= Ans

H.W

QUESTION

2

Q. Given the following probability distribution

X	−2	−1	0	1	2
P(x)	0.15	0.30	0	0.30	0.25

Find (i) E(X)

(ii) E(2X + 3)

(iii) $E(X^2)$

(iv) E(4X – 5)

$E(X) = -2 \times 0.15 - 1 \times 0.30 + 0 + \cancel{0.30} + 2 \times 0.25$

$= -0.30 + 0.50$

$\boxed{E(X) = 0.20}$

$E(2X+3) = 2E(X) + 3$

$= 2 \times 0.20 + 3$

$= 0.4 + 3 = 3.4$

$E(X^2) = \checkmark$ H.W

$E(4X-5) = 4E(X) - 5 = 4 \times 0.2 - 5$

$= \checkmark$ H.W

QUESTION

Q. For each of the following, determine whether the given values can serve as the probability distribution of a random variable with the given range:

A $f(x) = \frac{x-2}{5}$ For $x = 1, 2, 3, 4, 5$;

B $f(x) = \frac{x^2}{30}$ For $x = 1, 2, 3, 4$;

C $f(x) = \frac{x}{5}$ For $x = 1, 2, 3, 4, 5$;

QUESTION

Q. Verify that $f(x)=\frac{2x}{k(k+1)}$ for x = 1, 2, 3,.........., k can serve as the probability distribution of a random variable with the given range.

QUESTION

Q. For each of the following, determine c so that the function can serve as the probability distribution of a random variable with the given range:

A $f(x) = cx$ for $x = 1, 2, 3, 4, 5$;

B $f(x) = c\binom{5}{x}$ for $x = 1, 2, 3, 4$;

C $f(x) = c\left(\frac{1}{4}\right)^x$ for $x = 1, 2, 3$____

D $f(x) = cx^2$ for $x = 1, 2, 3$_____k

QUESTION

Q. A random variable X has the following probability function:

x	0	1	2	3	4	5	6
f(x)	k	3k	5k	7k	9k	11k	13k

(i) Find k,

(ii) Find $P(X \geq 5)$, $P(3 < X \leq 6)$, $P(X < 4)$

11

5 + 6

$\frac{11}{49}$

33k

$\frac{33}{49}$

16

$\frac{16}{49}$

16k

H.W

QUESTION

Q. If $P(x) = \begin{cases} x/15 \,;\; x = 1,2,3,4,5 \\ 0 \quad ; \quad \text{otherwise} \end{cases}$

Find

(i) $P(X = 1 \text{ or } 2)$

(ii) $P\left(\frac{1}{2} < X < \frac{5}{2} / X > 1\right)$

X	1	2	3	4	5
$P(X=x)$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

$$P(X = 1 \text{ or } 2) = P(1) + P(2)$$

$$= \frac{1}{15} + \frac{2}{15}$$

$$= \frac{3}{15}$$

$$P\left(\frac{1}{2} < X < 5/2 \,/\, X > 1\right) = \frac{P\left(\frac{1}{2} < X < \frac{5}{2} \cap X > 1\right)}{P(X > 1)}$$

H.W

QUESTION

Q. For each of the following, determine whether the given values can serve as the values of a distribution function of a random variable with the range $x = 1, 2, 3$ and 4;

(A) $F(1) = 0.3$, $F(2) = 0.5$, $F(3) = 0.8$, and $F(4) = 1.2$;

(B) $F(1) = 0.5$, $F(2) = 0.4$, $F(3) = 0.7$, and $F(4) = 1.0$;

(C) $F(1) = 0.25$, $F(2) = 0.61$, $F(3) = 0.83$, and $F(4) = 1.0$;

QUESTION

Q. Given that the discrete random variable X has the distribution function

$$f(x)=\begin{cases} x/6 \; ; \; x=1,2,3 \\ 0 \quad \text{elsewhere} \end{cases} \quad \text{Find } F(x)$$

QUESTION

Q. If X has the distribution function

$$f(x)=\begin{cases} 0 & \text{for } x<1 \\ \frac{1}{3} & \text{for } 1\le x<4 \\ \frac{1}{2} & \text{for } 4\le x<6 \\ \frac{5}{6} & \text{for } 6\le x<10 \\ 1 & \text{for } x\ge 10 \end{cases}$$

Find

(a) $P(2 < X \le 6)$;

(b) $P(X = 4)$

(c) $P(X \ge 10)$

(d) $P(X < 4)$

(e) $P(X > 4)$

(f) $P(X \ge 4)$

QUESTION

Find

Q. If X has the distribution function

$$f(x)=\begin{cases} 0 & \text{for } x<-1 \\ \frac{1}{4} & \text{for } -1\leqq x<1 \\ \frac{1}{2} & \text{for } 1\leqq x<3 \\ \frac{3}{4} & \text{for } 3\leqq x<5 \\ 1 & \text{for } x\geqq 5 \end{cases}$$

(a) $P(X \le 3)$;

(b) $P(X = 3)$;

(c) $P(X < 3)$;

(d) $P(X \ge 5)$;

(e) $P(-0.4 < X < 4)$;

(f) $P(X = 5)$;

(g) $P(3 < X < 5)$;

(i) $P(3 \le X < 5)$;

(j) $P(3 \le X \le 5)$

QUESTION

Q. Find distribution fx^n of the random variable that has the prob. distribution

$$f(x) = \frac{x}{15}; x = 1, 2, 3, 4, 5$$

QUESTION

Q. Let $X_1, X_2 \ldots\ldots\ldots X_n$ be random sample from the following density function

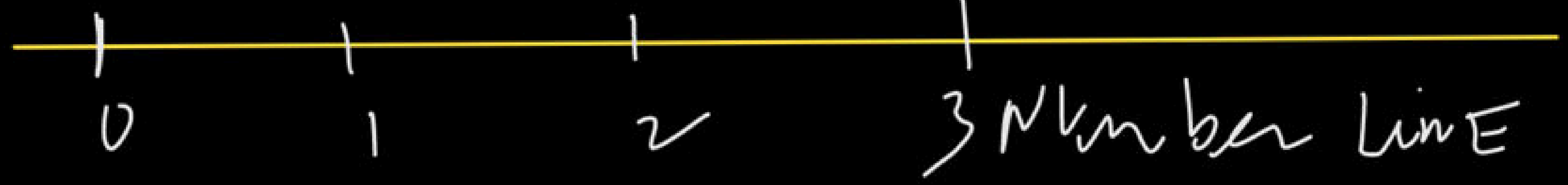
$$f(x;\theta) = \frac{kx}{\theta^2}; 0 < x < \theta,\ \theta > 0$$

Find k such that above is a valid density function.

QUESTION

Q. Let X be a continuous random variable with p.d.f:

$$f(x)=\begin{cases} ax; 0<x<1 \\ a; 1\le x\le 2 \\ -ax+3a; 2\le x\le 3 \\ 0;\ \text{elewhere} \end{cases}$$



(i) Determine constant a

pdf = 1

(ii) P (X ≤ 1.5)

Key Point

$$\int_0^{1.5} = \int_0^{1} + \int_1^{1.5}$$

H.W

QUESTION

H.W

Q. The probability density of the random variable Y is given by

$$f(y)=\begin{cases}\frac{1}{8}(y+1) & \text{for } 2<y<4\\ 0 & \text{elsewhere}\end{cases}$$

$P(Y<3.2) =$

$P(2.9<Y<3.2) =$

Find P (Y < 3.2) and P (2.9 < Y < 3.2).

2 3 4

H.W

QUESTION

Q. The p.d.f of the random variable X is given by

$$f(x)=\begin{cases}\dfrac{c}{\sqrt{x}} & \text{for } 0<y<4\\ 0 & \text{elsewhere}\end{cases}$$

H.W

Find (a) The value of c;

(b) $P\left(X<\dfrac{1}{4}\right)$ and $P(X>1)$

QUESTION

Q. The density function of the random variable X is given by

$$g(x)=\begin{cases}6x(1-x) & \text{for } 0<\cancel{x}<1\\ 0 & \text{elsewhere}\end{cases}$$

Find $P\left(X<\frac{1}{4}\right)$ and $P\left(X>\frac{1}{2}\right)$

NUMBER LINE

Draw

$$g(x)=\begin{cases}6x(1-x) & 0<x<1\\ 0 & \text{elsewhere}\end{cases}$$

$$P\left(X<\frac{1}{4}\right)=\int_0^{1/4}6x(1-x)\,dx$$

$$P\left(X>\frac{1}{2}\right)=\int_{\frac{1}{2}}^{1}6x(1-x)\,dx$$

QUESTION

done

Q. (a) Show that $f(x) = 3x^2$ for $0 < x < 1$ represents a density function.

(b) Calculate $P(0.1 > X < 0.5)$

H.W

QUESTION

Q. If X has the prob. density fx^n

$$f(x)\begin{cases} ke^{-3x} & ; \quad x>0 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

Find k and P $(0.5 \le X \le 1)$

H.W

$$f(x) = \begin{cases} Ke^{-3x} & x>0 \\ 0 & \text{elsewhere} \end{cases}$$

find K

valid prob. Density function

$P(0.5 \le X \le 1) = \int_{0.5}^{1} Ke^{-3x}\,dx$

= answer

QUESTION

Q. The probability density of the continuous random variable X is given by

$$f(x)\begin{cases}\frac{1}{5} & \text{for } 2<x<7\\ 0 & \text{elsewhere}\end{cases}$$

$$f(x) = \begin{cases}\frac{1}{5} & 2<x<7\\ 0 & \text{elsewhere}\end{cases}$$

Find P(3 < X< 5)

$P(3<x<5)$

H.W

$$P(3<x<5) = \int_3^5 \frac{1}{5}\,dx = \frac{2}{5}$$

Ans.

QUESTION

Q. Find the distribution function of the random variable X whose Probability density is given by

$$f(x)\begin{cases} \dfrac{1}{3} & \text{for } 0<x<1 \\ \dfrac{1}{3} & \text{for } 2<x<4 \\ 0 & \text{elsewhere} \end{cases}$$

QUESTION

Q. The distribution fx^n of the random Variable X is given by

$$F(x)=\begin{cases}1-(1+x)^{e^{-x}} & \text{for } x>0\\ 0 & \text{for } x\leq 0\end{cases}$$

Find

(i) $P(X \leq 2)$

(ii) $P(1 < X < 3)$

(iii) $P(X > 4)$

QUESTION

Q. Find the distribution function of the random variable X whose probability density is given by

$$f(x)=\begin{cases} x & \text{for } 0<x<1 \\ 2-x & \text{for } 1\le x\le 1 \\ 0 & \text{elsewhere} \end{cases}$$

QUESTION

Q. Find the distribution function of the random variable X whose probability density is given by

$$f(x)=\begin{cases}\dfrac{x}{2} & \text{for } 0<x\leqq 1\\[2ex] \dfrac{1}{2} & \text{for } 1<x\leqq 2\\[2ex] \dfrac{3-x}{2} & \text{for } 2<x<3\\[2ex] 0 & \text{elsewhere}\end{cases}$$

QUESTION

Q. Find a prob. density fx^n for the random variable whose distribution fx^n is given by

$$f(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ x & \text{for } 0 < x < 1 \\ 1 & \text{for } x > 1 \end{cases}$$

QUESTION

H.W

Q. The ~~distribution~~ density function of the random variable Y is given by

$$f(y) = \begin{cases} 1 - \dfrac{9}{y^2} & \text{for } y > 3 \\ 0 & \text{elsewhere} \end{cases}$$

Find $P(Y \le 5)$ and $P(Y > 8)$

$$f(y) = \begin{cases} 1 - \dfrac{9}{y^2} & y > 3 \\ 0 & \text{elsewhere} \end{cases}$$

3 4 5 8

$$P(Y \le 5) = \int_3^5 \left(1 - \frac{9}{y^2}\right) dy$$

$$P(Y > 8) = \int_8^\infty 1 - \frac{9}{y^2} \, dy$$

QUESTION

Q. A random variable X which can be used in certain circumstances as a model for claim sizes has cumulative distribution function

$$f(x)=\begin{cases}0 & ,x<0\\ 1-\left(\dfrac{2}{2+x}\right)^{3} & ,x>0\end{cases}$$

Calculate the value of the conditional probability P (X > 3/X >1)

QUESTION

Q. The probability density of the random variable Z is given by

$$f(z) = \begin{cases} kze^{-z^2} & \text{for } z > 0 \\ 0 & \text{for } z \leqq 0 \end{cases}$$

QUESTION

Q. A random variable X has the following probability distribution

X	–2	–1	0	1	2
P(X)	1/6	p	1/4	p	1/6

(i) Find the value of p.

(ii) Calculate E (X + 2), E (2X² + 3X + 5)

$$\frac{1}{6}+p+\frac{1}{4}+p+\frac{1}{6}=1$$

pmf = 1

$$E(X+2) = E(X)+2$$

$$E[(2X^2)+3X+5]$$

$$= 2E(X^2)+3E(X)+5$$

=

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$\int_0^\infty \frac{e^{-\frac{x}{2}}}{k} dx = 1$

$$\frac{1}{k}\int_0^\infty e^{-\frac{x}{2}} dx = 1$$

$$\frac{x}{2} = t$$

$$\frac{1}{k}\int_0^\infty e^{-t} \cdot 2\,dt = 1$$

$$\frac{2}{k}\int_0^\infty e^{-t} dt = 1$$

$$\frac{2}{k}\left[-e^{-t}\right]_0^\infty = 1$$

$$\frac{2}{k}\left[0+1\right] = 1$$

$$\frac{2}{k} = 1 \quad \boxed{k=2}$$

Shivansh problem

QUESTION

Q. If X is the number of points rolled with a balanced die, find the expected value of
$g(X) = 2X^2 + 1$.

QUESTION

Q. Let X be a random variable with the following probability fxn

$x:$	-3	6	9
$P(X = x)$	$1/6$	$1/2$	$1/3$

Find E (X) and $E(X^2)$ and evaluate $E(2X + 1)^2$

QUESTION

Q. Find the expected value of the random variable Y whose probability density is given by

$$f(y)=\begin{cases}\frac{1}{8}(y+1) & \text{for}\, -1<y<1\\ 0 & \text{elsewhere}\end{cases}$$

Expected value

$$\boxed{E(Y)=\int_{-1}^{1} y\cdot\frac{1}{8}(y+1)\,dy}$$

QUESTION

Q. If X has the probability density

$$f(x)=\begin{cases} e^{-x} & \text{for } x>0 \\ 0 & \text{elsewhere} \end{cases}$$

Find the expected value of $g(X) = e^{3X/4}$.

QUESTION

Q. If the probability density of X is given by

$$f(x) = \begin{cases} 2x^{-3} & \text{for } x > 1 \\ 0 & \text{elsewhere} \end{cases}$$

check whether its mean and its variance exist.

QUESTION

Q. If the probability density of X is given by

$$f(x)=\begin{cases}2(1-x) & \text{for } 0<x<1\\ 0 & \text{elsewhere}\end{cases}$$

(a) Show that $E(X^r)=\dfrac{2}{(r+1)\,(r+2)}$

(b) and use this result to evaluate $E\,[(2X+1)^2]$

QUESTION

Q. A continuous random variable X has the p.d.f,

$$f(x) = \begin{cases} a(1-x^2) & 2 \le x \le 5 \\ 0 & \text{elsewhere} \end{cases}$$

(i) find a

(ii) Find E(X)

$$f(x) = \begin{cases} a(1-x^2) & 2 \le x \le 5 \\ 0 & \text{elsewhere} \end{cases}$$

$$\int_2^5 a(1-x^2)\,dx = 1 \quad \boxed{a = ?}$$

$$E(X) = \int_2^5 x \cdot f(x)\,dx = \underline{\underline{\text{answer}}}$$

QUESTION

Q. Certain coded measurements of the pitch diameter of threads of a fitting have the probability density

$$f(x)=\begin{cases}\dfrac{4}{\pi(1+x^2)} & \text{for } 0<x<1\\ 0 & \text{elsewhere}\end{cases}$$

Find the expected value of this random variable.

$f(x) = \dfrac{4}{\pi(1+x^2)}$

$0 < x < 1$

expected value $= \int_0^1 x \cdot f(x)\, dx$

done

THANK YOU!

Here's to a cracking journey ahead!