

# Multiplying 2 matrices

$$\begin{bmatrix} \textcircled{1} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 3 & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

# What we just did

$$(A \cdot B)_{12} = (1^{\text{st}} \text{ row of } A) \cdot (2^{\text{nd}} \text{ column of } B)$$

dot products

Matrix  $\times$  Vector = ?



extend  
→

this idea for 2 matrices,

GO CLASSES

Matrix  $\times$  Vector = ?

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$



$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$
$$1 \begin{bmatrix} 2 \\ 3 \end{bmatrix} + 0 \begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Can we multiply 2 matrices in similar way ?





Can we multiply 2 matrices in similar way ?

---

$$\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 7 \\ 1 & 2 \end{bmatrix}$$
$$= \boxed{\quad}$$



Can we multiply 2 matrices in similar way ?

---

$$\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 7 \\ 1 & 2 \end{bmatrix}$$

$$1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 0 \begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$



Can we multiply 2 matrices in similar way ?

---

$$\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \begin{bmatrix} 7 \\ 2 \end{bmatrix}$$

$$2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$$

$$A \begin{bmatrix} & \\ & \end{bmatrix} \begin{bmatrix} & \\ & \end{bmatrix} = \begin{bmatrix} w \\ w \end{bmatrix}$$

linear  
combination  
of columns at  
A.



$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \left[ \begin{array}{c|c} a \begin{bmatrix} 1 \\ 3 \end{bmatrix} + c \begin{bmatrix} 2 \\ 4 \end{bmatrix} & b \begin{bmatrix} 1 \\ 3 \end{bmatrix} + d \begin{bmatrix} 2 \\ 4 \end{bmatrix} \\ \hline \end{array} \right]$$

Handwritten annotations:

- A curly bracket above the first column of the second matrix groups  $a$  and  $c$ . An arrow points from this bracket to the term  $a + 2c$  in the result.
- A curly bracket above the second column of the second matrix groups  $b$  and  $d$ . An arrow points from this bracket to the term  $b + 4d$  in the result.
- A curly bracket below the first row of the result matrix groups the terms  $a + 2c$  and  $b + 4d$ .
- A curly bracket below the second row of the result matrix groups the terms  $3a + 4c$  and  $3b + 4d$ .

# Another interpretation

$$\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \xrightarrow{\quad} \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} \xrightarrow{\quad} \begin{bmatrix} 1 & 6 \\ 2 & 6 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \xrightarrow{\quad} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

We will use this interpretation here and there

T/f

If  $A$  is an  $m \times n$  matrix and  $B$  is an  $n \times p$  matrix whose first column contains only zeros, then the first column of  $AB$  also contains only zeros.

$$\left[ \begin{array}{c} A \\ \vdots \end{array} \right]_{m \times n} \cdot \left[ \begin{array}{c} \begin{matrix} 0 \\ 0 \\ \vdots \\ 0 \end{matrix} \\ B \end{array} \right]_{n \times p} = \left[ \begin{array}{c} 0 \\ \vdots \end{array} \right]_{m \times p}$$



T/f

If  $A$  is an  $m \times n$  matrix and  $B$  is an  $n \times p$  matrix whose first column contains only zeros, then the first column of  $AB$  also contains only zeros.

True. By the columnwise definition of matrix vector multiplication, the first column of  $B$  is  $b_1$  then the first column of  $AB$  is  $Ab_1$ .



2 L.I. vectors in a set of s vectors

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$
$$\begin{bmatrix} 2 \\ 4 \end{bmatrix} \quad \begin{bmatrix} 4 \\ 6 \end{bmatrix} \quad \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

How many linearly Indep. vectors  
we can generate using  
linear combination



A diagram of a cylinder with a curved front edge. At the front edge, there are three vectors labeled  $u$ ,  $v$ , and  $w$ . Above the cylinder, there are three force vectors labeled  $F^n$ ,  $F^x$ , and  $F^y$ . A small shaded area at the bottom front of the cylinder is indicated by a hatched pattern.

$$\begin{cases} u + 2v + 0w + 3x + 7y \\ 0u + (-1)v + 3w + 4x + 10y \end{cases}$$

walmart interview

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} = C$$

given  
=

2 LI vectors

No. of LI vectors  
in C ? at most 2

# True/False

- A. Every system of 3 equations in 8 unknowns has a solution.
- B. Every set of five vectors in  $\mathbb{R}^4$  is linearly dependent.
- C. Every set of four vectors in  $\mathbb{R}^4$  is linearly dependent.
- D. If  $A$  and  $B$  are  $m \times n$  matrices such that  $B$  can be obtained from  $A$  by column operations, then  $A$  can also be obtained from  $B$  by column operations .

## GATE CSE 2016

Consider the systems, each consisting of  $m$  linear equations in  $n$  variables.

- I. If  $m < n$ , then all such systems have a solution.
- II. If  $m > n$ , then none of these systems has a solution.
- III. If  $m = n$ , then there exists a system which has a solution.

Which one of the following is **CORRECT**?

- A. I, II and III are true.
- B. Only II and III are true.
- C. Only III is true.
- D. None of them is true.

GATE CSE 2016

Consider the systems, each consisting of  $m$  linear equations in  $n$  variables.

- I. If  $m < n$ , then all such systems have a solution. — F
- II. If  $m > n$ , then none of these systems has a solution. F
- III. If  $m = n$ , then there exists a system which has a solution. T

Which one of the following is **CORRECT?**

- A. I, II and III are true.
- B. Only II and III are true.
- C. Only III is true.
- D. None of them is true.

$$\begin{array}{c} \text{all } \rightarrow F \quad [Q60] \\ \cancel{\text{ER}^3} \quad \cancel{\text{ER}^2} \\ \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 1 \end{array} \right] \end{array}$$

$$\begin{bmatrix} Q & Q \end{bmatrix}_{\in \mathbb{R}^3} \begin{bmatrix} I \\ 0 \end{bmatrix} = P$$



## True/False

F  $\xrightarrow{\text{P} \perp b}$

Every system of 3 equations in 8 unknowns has a solution.

false

8 vectors in  $\mathbb{R}^3$

Every set of five vectors in  $\mathbb{R}^4$  is linearly dependent.  $\rightarrow T$

F Every set of four vectors in  $\mathbb{R}^4$  is linearly dependent.  $\rightarrow F$

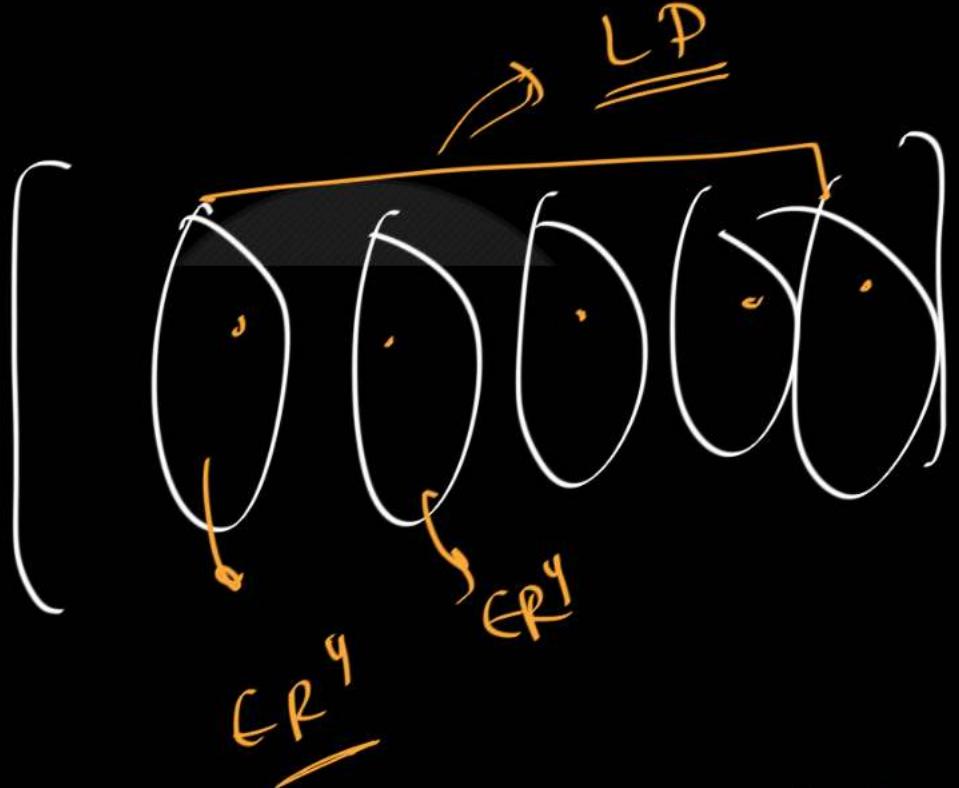
If  $A$  and  $B$  are  $m \times n$  matrices such that  $B$  can be obtained from  $A$  by column operations, then  $A$  can also be obtained from  $B$  by column operations.

$$\left[ \begin{array}{c|c|c|c|c} 1 & 6 & 6 & 6 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{\text{J}} \left[ \begin{array}{c|c|c|c|c} 1 & 6 & 6 & 6 & 6 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

T

<https://sites.math.washington.edu/~smith/Teaching/308/2012-Summer-Midterm.pdf>



in  $\mathbb{R}^n \Rightarrow$  you can have (at most)  $n$  LI vectors

$$\begin{bmatrix} u & v & w & x \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\xrightarrow{\text{LT}}$

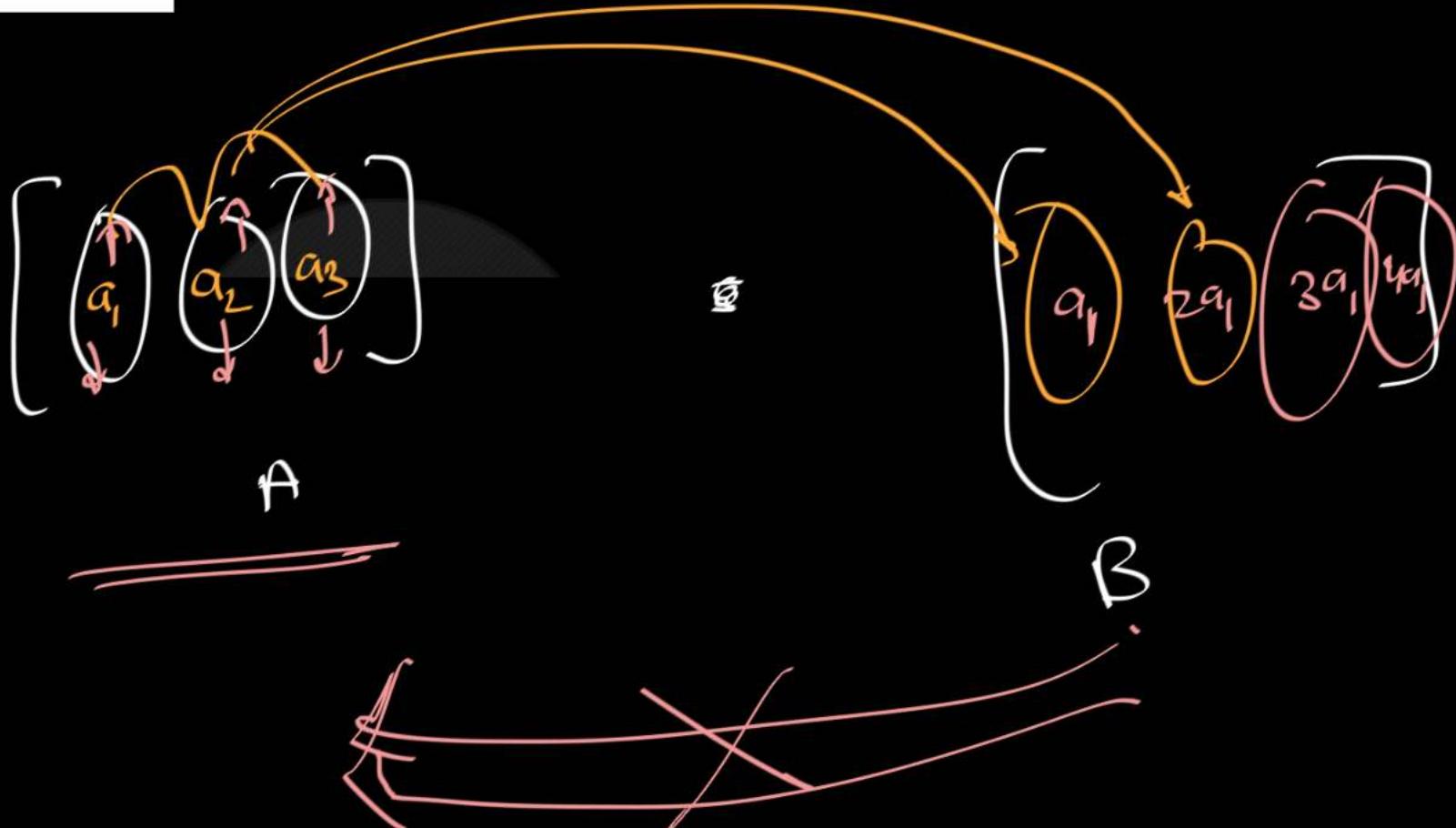
$$\xrightarrow{6:30 \text{ am}}$$

can you generate more  
than  $+ LT$ ?

if  $\sqrt{u+v+w+x}$

no  $\sqrt{\sum u + 3v + uv + 0}$

$c_1 u + c_2 v + c_3 w + c_4 x$



$$\begin{matrix}
 & & & a_1 \\
 & & & \downarrow \\
 \left[ \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ a_1 & a_2 & a_3 \\ \downarrow & \downarrow & \downarrow \end{array} \right] & \times & \left[ \begin{array}{ccc} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right] & = & \left[ \begin{array}{ccc} a_1 & 2a_1 & 3a_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right] \\
 A & & C & & B
 \end{matrix}$$

$$\left[ \begin{array}{c} \text{ } \\ \text{ } \\ \text{ } \end{array} \right] \quad \left[ \begin{array}{c} \text{ } \\ \text{ } \\ \text{ } \end{array} \right] \quad = \quad \left[ \begin{array}{c} \text{ } \\ \text{ } \\ \text{ } \end{array} \right]$$

$C$        $A$        $B$

$$-2x + 3y + \frac{1}{2}z + 4w + 9p + 15q + 20r + 10s = 0$$

$$\begin{bmatrix} x & y \\ \vdots & \vdots \end{bmatrix} = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$$

multiply 2 matrices —

$$\begin{bmatrix} 2 & 5 & 2 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}$$

$$2 \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + 2 \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix} + 0 \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

multiply 2 matrices -

$$\begin{bmatrix} 2 & 5 & 2 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$1 \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + 2 \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix} + 0 \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

multiply 2 matrices -

$$\begin{bmatrix} 2 & 5 \\ 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \downarrow \end{bmatrix}$$

$$2 \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + 2 \begin{bmatrix} s \\ o \\ 1 \end{bmatrix}$$

multiply 2 matrices -

$$\begin{bmatrix} 2 & 5 \\ 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

$$1 \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + 2 \begin{bmatrix} s \\ o \end{bmatrix}$$

multiply 2 matrices -

$$\begin{bmatrix} 2 & 5 \\ 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$0 \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + 1 \begin{bmatrix} s \\ o \\ 1 \end{bmatrix}$$

multiply 2 matrices -

$$\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}_{3 \times 1} \begin{bmatrix} 2 & 1 & 0 \end{bmatrix}_{1 \times 3} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}_{3 \times 3}$$

2  $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

multiply 2 matrices -

$$\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$\downarrow$   
 $3 \times 1$

$\downarrow$   
 $1 \times 3$

$3 \times 3$

$$1 \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

multiply 2 matrices -

$$\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}_{3 \times 1} \begin{bmatrix} 2 & 1 & 0 \end{bmatrix}_{1 \times 3} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}_{3 \times 3}$$

0  $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

multiply 2 matrices -

final answer

$$\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 0 \\ 2 & 1 & 0 \\ 6 & 3 & 0 \end{bmatrix}$$



**↓**  
**1x3**

**3x1**

## GATE CSE 2014

How many independent columns are present in A? (rank)

$$A = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix} [1 \quad 9 \quad 5] = \begin{bmatrix} 2 & 18 & 10 \\ -4 & -36 & -20 \\ 7 & 719 & 35 \end{bmatrix}$$

## Question

If  $b$  is linear combination of columns of  $A$  then-

- A. There is always a solution of  $Ax = b$ .
- B. There is never a solution of  $Ax = b$ .

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \\ 14 \end{bmatrix}$$

$$Q: \begin{bmatrix} 1 & 3 & 2 \\ 2 & 6 & 4 \\ 3 & 9 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 12 \end{bmatrix}$$

Given that  $b$  is not a linear combination  
of columns of  $A$ .

do you have a sol'n.  $\Rightarrow$

$$Q: \begin{bmatrix} 1 & 3 & 2 \\ 2 & 6 & 4 \\ 3 & 9 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 12 \end{bmatrix}$$

Given that  $b$  is not a linear combination  
of columns of  $A$ .

do you have a sol'n.  $\Rightarrow$   $\text{No}$

\* if  $b$  is NOT a linear combination  
of columns of  $A$  then it does not  
matter how hard you try, you can  
never get the solution.



\* if  $b$  is a linear combination  
of columns of  $A$  then you will **ALWAYS**  
get the sol<sup>n</sup>.

## Question

If  $b$  is ~~not~~ linear combination of columns of  $A$  then-

- A. There is always a solution of  $Ax = b$ .
- ~~B.~~ There is never a solution of  $Ax = b$ .

## Question

$$A_{3 \times 4}$$
  
 $m \times n$

$A_{m \times n}$  has  $m$  Linearly Independent Columns then –

- A. There is always a solution of  $Ax = b$ .
- B. There is never a solution of  $Ax = b$  for any  $b$ .
- C. There may or may not solution of  $Ax = b$ .

$$\left[ Q \right]_{m \times n} \in \mathbb{R}^m$$

$$\left[ 000 \right]_{\mathbb{R}^3}^{3 \times 4}$$

## Question

$A_{m \times n}$  has  $m$  Linearly Independent Columns then –

- A. There is always a solution of  $Ax = b$ .
- B. There is never a solution of  $Ax = b$  for any  $b$ .
- C. There may or may not solution of  $Ax = b$ .

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3 \times 4} \in \mathbb{R}^3 \quad A \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad b$$

## Question

$A_{m \times n}$  has  $m$  Linearly Independent Columns then –

- A. There is always a solution of  $Ax = b$ .
- B. There is never a solution of  $Ax = b$  for any  $b$ .
- C. There may or may not solution of  $Ax = b$ .

The diagram illustrates a linear system  $Ax = b$ . On the left, there is a matrix  $A$  with dimensions  $3 \times 4$ , represented by a bracket under four columns. Above the matrix, the text  $\in \mathbb{R}^3$  indicates the dimension of the vector  $x$ . To the right of the matrix, there is a bracket under two entries, labeled  $x$  above the first entry. An arrow points from the label  $x$  to the first entry of the bracket. To the right of the bracket, there is an equals sign followed by another bracket under two entries, labeled  $b$  above the second entry. An arrow points from the label  $b$  to the second entry of the bracket.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ -9 \end{bmatrix} = \begin{bmatrix} 37 \\ -52 \\ 790 \end{bmatrix}$$

The matrix multiplication is shown as follows:

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ -9 \end{bmatrix} = \begin{bmatrix} 37 \\ -52 \\ 790 \end{bmatrix}$

Annotations:

- A large orange bracket on the left side of the first matrix indicates its columns.
- An orange bracket above the second matrix indicates its components.
- An orange bracket on the right side of the result matrix indicates its components.
- An orange arrow points from the second matrix to the result matrix.
- An orange arrow points from the result matrix to the right.

## Question

$A_{m \times n}$  does not have  $m$  Linearly Independent Columns then –

- A. There is always a solution of  $Ax = b$ .
- B. There is never a solution of  $Ax = b$  for any  $b$ .
- C. There may or may not solution of  $Ax = b$ .

$A_{3 \times 4}$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

## Question

$A_{m \times n}$  does not have  $m$  Linearly Independent Columns then –

- A. There is always a solution of  $Ax = b$ .
- B. There is never a solution of  $Ax = b$  for any  $b$ .
- C. There may or may not be a solution of  $Ax = b$ .

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 4 & 6 & 4 \\ 3 & 5 & 8 & 5 \end{bmatrix}_{3 \times 4} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \text{in } \mathbb{R}^3$$

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 4 & 6 & 4 \\ 3 & 5 & 8 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 10 \end{bmatrix}$$

$\downarrow R^3$

$3 \times 4$

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 4 & 6 & 4 \\ 3 & 5 & 8 & 5 \end{bmatrix} \begin{bmatrix} ? \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 7 \end{bmatrix}$$

*3x4*

$\downarrow R^3$

is not a linear comb. of

columns of A.

= No sol'n

Solutions of  $Ax = b$

$A_{m \times n}$  has  $m$  Linearly Independent Columns ?



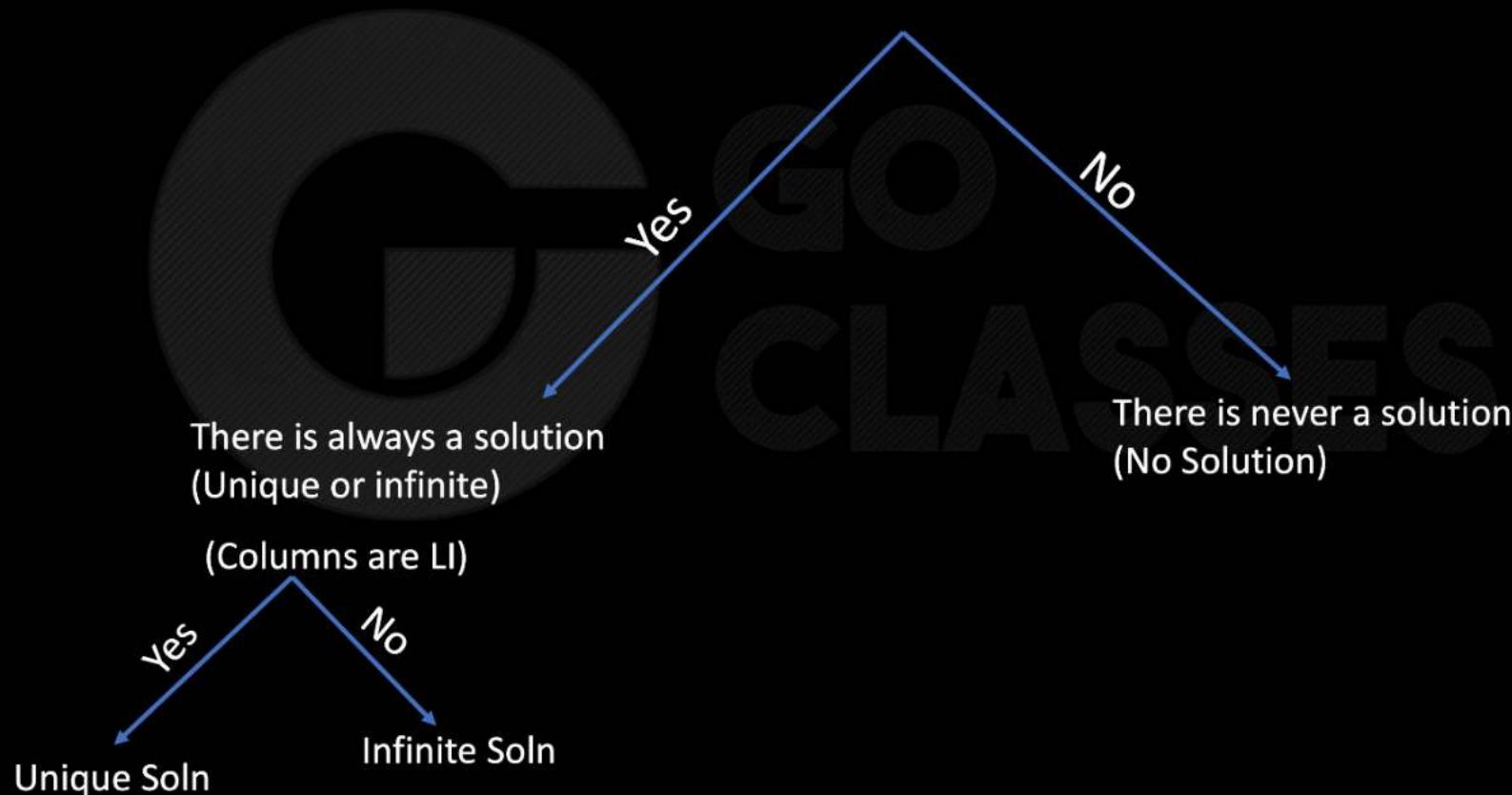
Solutions of  $Ax = b$

$b$  is linear combination of columns of A ?



## Solutions of $Ax = b$

$b$  is linear combination of columns of A ?



if a vector can be represented as  
linear combination of few vectors and  
further if those few vectors are LI  
then there is just one way to do it  
otherwise there are many ways.

if a vector can be represented as

linear combination of few vectors and

if those few vectors are L.P.

then there are many way to do it

otherwise there is a unique way.

↳ Proof when "rank" comes

$$b = c_1 v_1 + c_2 v_2 + \dots + c_n v_n$$

given

$$\{v_1, \dots, v_n\}$$

$$b = d_1 v_1 + d_2 v_2 + \dots$$

L.B

$$b = c_1 v_1 + c_2 v_2 + \dots$$

intuitively

(idea)

given

$\{v_1, \dots, v_n\}$

$$b = c_1 v_1 + c_2 v_2 + \dots + c_n v_n$$

case 1

are L.D

$$b = c_1 v_1 + c_2 v_2 + \dots + c_n v_n + 0$$

$$= (k_1 + c_1) v_1 + (k_2 + c_2) v_2 + \dots + (k_n + c_n) v_n$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ 3 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix}$$

$\boxed{\quad} + \boxed{\quad} + \boxed{\quad}$

$\curvearrowright b$

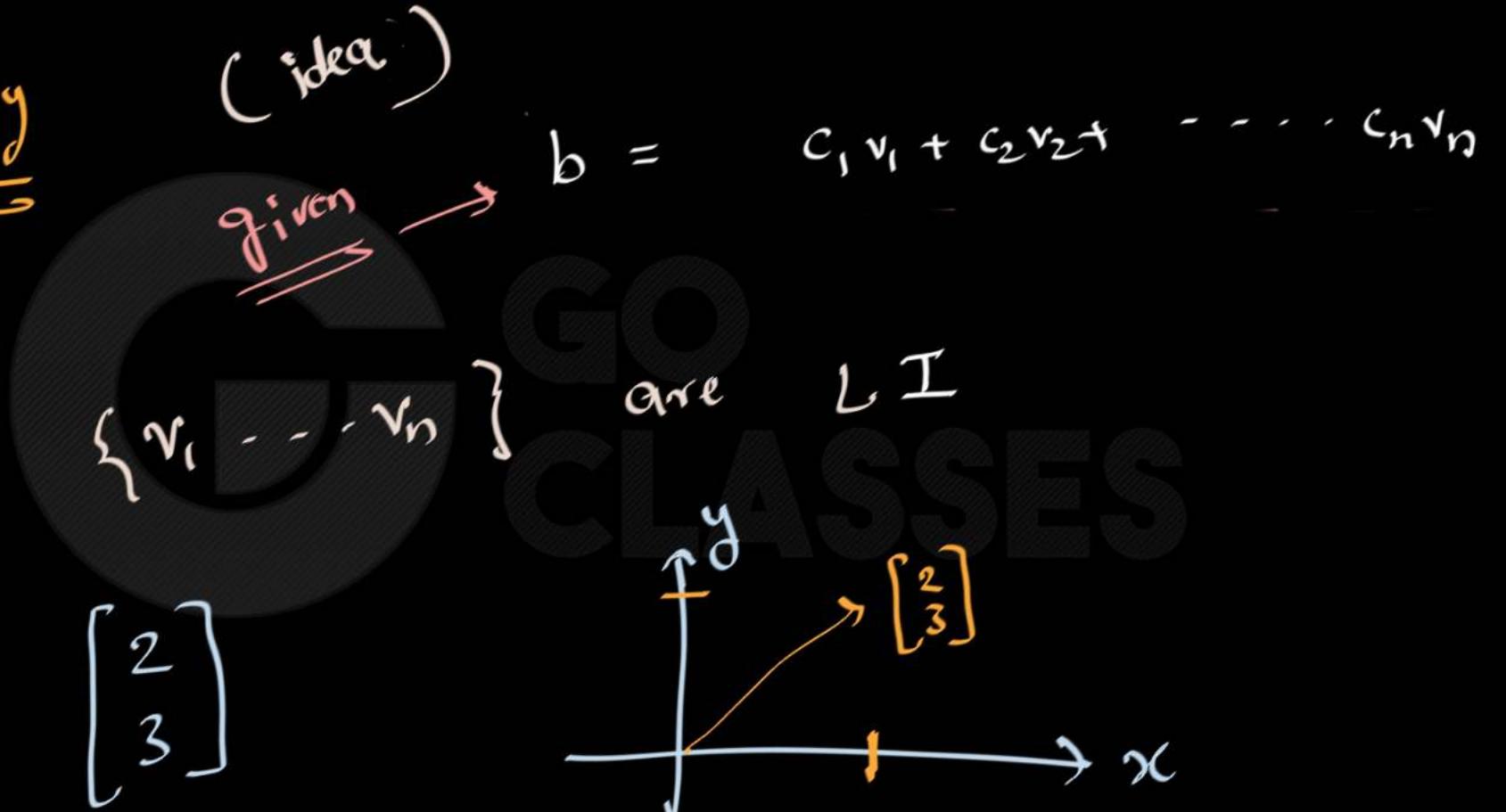
$$\boxed{\phantom{0}} + \boxed{\phantom{0}} + 5\boxed{\phantom{0}} + \boxed{\phantom{0}} \cdots + \boxed{\phantom{0}} = b$$



$$\boxed{\phantom{0}} + \boxed{\phantom{0}} + 3\boxed{\phantom{0}} \equiv \boxed{\phantom{0}} \cdots + \boxed{\phantom{0}} = b$$

intuitively

case 2



$$2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 3 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$


A diagram illustrating vector addition in a 2D Cartesian coordinate system. Two vectors are shown originating from the same point on the origin. One vector is orange and has components (1, 0), pointing along the positive x-axis. The other vector is also orange and has components (0, 1), pointing along the positive y-axis. A large black circle labeled 'G' contains a diagonal line with arrows at both ends, representing the resultant vector. This resultant vector is white and has components (2, 3), pointing into the first quadrant. A white arrow points from the origin to the tip of this vector.

GO  
CLASSES

Proof

Given

$$b = c_1 v_1 + c_2 v_2 + \dots + c_n v_n \quad \& \quad \{v_1, \dots, v_n\}$$

are

L.I.

Suppose

$$b = c_1 v_1 + c_2 v_2 + \dots + c_n v_n \quad \text{--- } ①$$

$$\frac{b = d_1 v_1 + d_2 v_2 + \dots + d_n v_n}{0 = (c_1 - d_1) v_1 + (c_2 - d_2) v_2 + \dots + (c_n - d_n) v_n} \quad \text{--- } ②$$

$$0 = (c_1 - d_1)v_1 + (c_2 - d_2)v_2 + \dots + (c_n - d_n)v_n$$

$$\underbrace{k_1 v_1 + k_2 v_2 + \dots + k_n v_n = 0}_{\Downarrow}$$

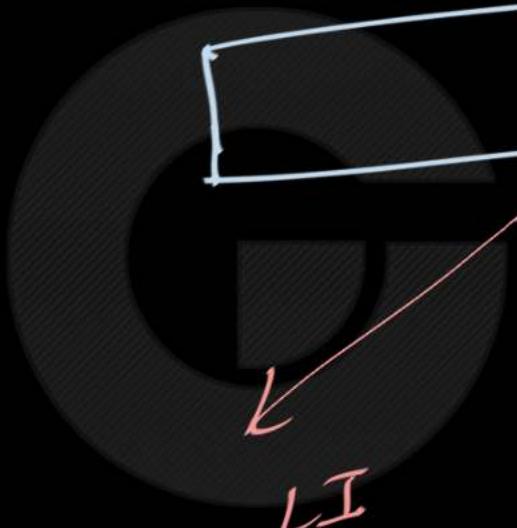
all  $k_i = 0$

$$k_1 = 0 \Rightarrow c_1 - d_1 = 0 \Rightarrow c_1 = d_1$$

$$c_2 = d_2 \quad c_3 = d_3$$

$$c_4 = d_4$$

b =



unique



GO  
CLASSES

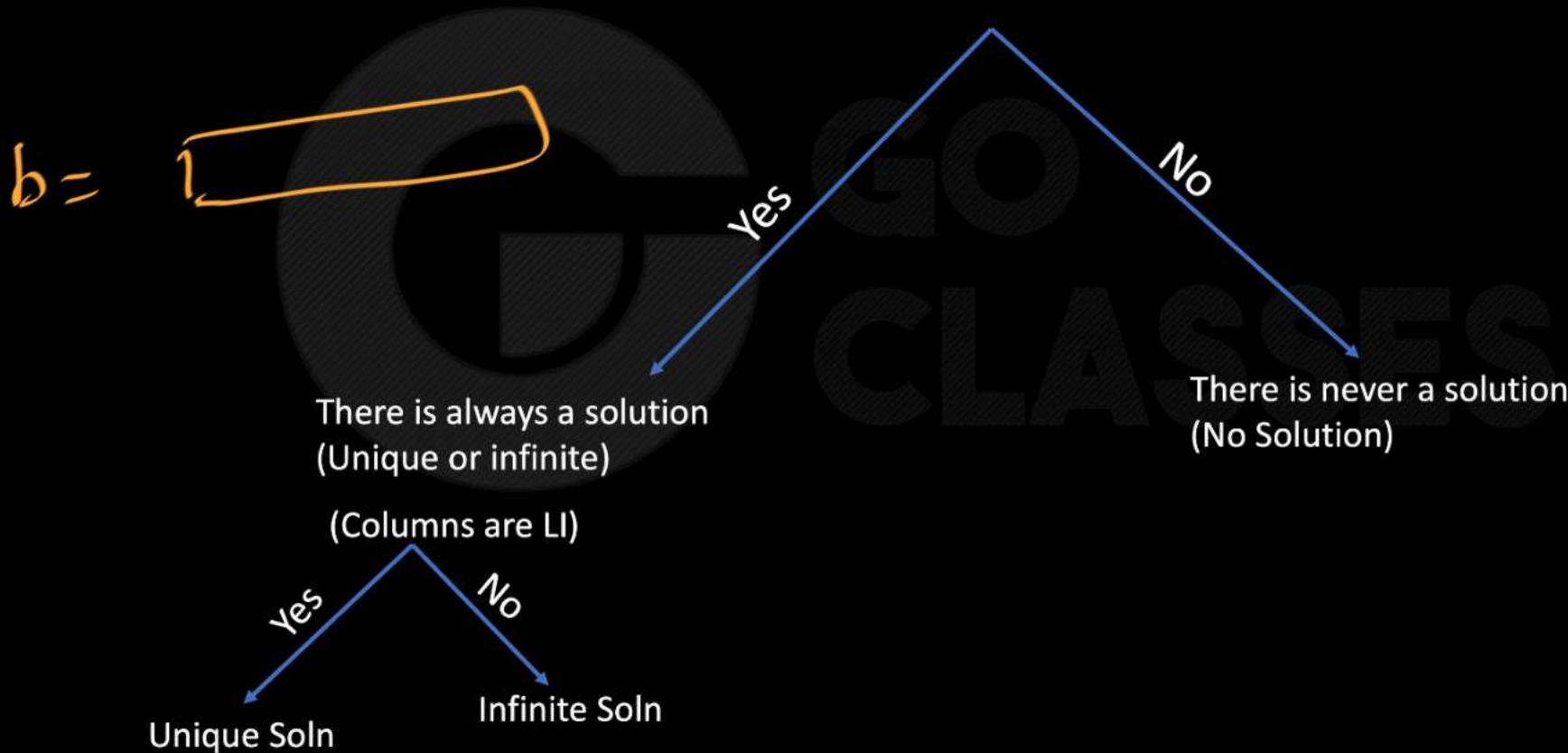
L<sup>D</sup>

inf.



Solutions of  $Ax = b$

$b$  is linear combination of columns of A ?



## Question

If the linear equation  $A\mathbf{x} = \mathbf{b}$  has a unique solution, then the columns of  $A$  are linearly independent.



(9) If the linear equation  $A\mathbf{x} = \mathbf{b}$  has a unique solution, then the columns of  $A$  are linearly independent.

**True**



## Question

**Problem 2.** How many solutions will the linear system  $Ax = b$  have

- (a) if  $b$  is in the column space of  $A$  and the columns of  $A$  are linearly independent?

is a linear comb of columns of A

- (b) if  $b$  is not in the column space of  $A$

## Question

**Problem 2.** How many solutions will the linear system  $Ax = b$  have

- (a) if  $b$  is in the column space of  $A$  and the columns of  $A$  are linearly independent?

is a linear comb of columns of  $A$  Unique

- (b) if  $b$  is not in the column space of  $A$

a LC of columns

Sol<sup>n</sup> can not exist

**Problem 2.** How many solutions will the linear system  $Ax = b$  have

- (a) if  $b$  is in the column space of  $A$  and the columns of  $A$  are linearly independent?
- (b) if  $b$  is not in the column space of  $A$

Explain your answer.

**Solution.**

(a) Since  $b$  is in the column space of  $A$ , it is a linear combination of columns of  $A$ , hence there is a solution of the system  $Ax = b$ . Since the columns of  $A$  are linearly independent in the reduced row echelon form of  $A$  every column will have a pivot. Therefore the system  $Ax = b$  does not have free unknowns, hence it has exactly one solution.

(b) Since  $b$  is not in the column space of  $A$ , it is not a linear combination of columns of  $A$ , hence  $Ax = b$  has no solutions.

# GATE 2017

13,798 views

51

Let  $c_1, \dots, c_n$  be scalars, not all zero, such that  $\sum_{i=1}^n c_i a_i = 0$  where  $a_i$  are column vectors in  $\mathbb{R}^n$ .

51

Consider the set of linear equations

$$Ax = b$$

where  $A = [a_1, \dots, a_n]$  and  $b = \sum_{i=1}^n c_i a_i$ . The set of equations has

- A. a unique solution at  $x = J_n$  where  $J_n$  denotes a  $n$ -dimensional vector of all 1.
- B. no solution
- C. infinitely many solutions
- D. finitely many solutions

$$\left[ \begin{array}{c|c} A & b \\ \hline x & \end{array} \right] = \left[ \begin{array}{c} \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \end{array} \right]$$

$a_i$ 's are L.D

$a_1, a_2, \dots$



## GATE 2017

13,798 views



51



Let  $c_1, \dots, c_n$  be scalars, not all zero, such that  $\sum_{i=1}^n c_i a_i = 0$  where  $a_i$  are column vectors in  $\mathbb{R}^n$ .

Consider the set of linear equations

$$\underline{Ax = b}$$

where  $A = [a_1, \dots, a_n]$  and  $b = \sum_{i=1}^n c_i a_i$ . The set of equations has

$$x = \begin{bmatrix} | \\ | \\ | \end{bmatrix} \quad L \cdot D \rightsquigarrow \left\{ a_1, \dots, a_n \right\}$$

- A. a unique solution at  $x = J_n$  where  $J_n$  denotes a  $n$ -dimensional vector of all 1.
- B. no solution
- C. infinitely many solutions
- D. finitely many solutions

$$\begin{bmatrix} | \\ | \\ | \end{bmatrix}$$

$$b = a_1 + a_2 + \dots + a_n$$

$$b = a_1 + a_2 + a_3 + \dots + a_n$$

(one of the ways)

$a_1, \dots, a_n$

are LP

$$b = 2a_1 + 3a_1 - 5a_3 + \dots$$

$$\left[ \begin{matrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{matrix} \right] = \left[ \begin{matrix} b \\ b \\ \vdots \\ b \end{matrix} \right]$$

# System of linear equations

- Why solve System of linear equations ?
- Geometric Interpretation
- Understanding  $Ax = b$  intuitively
- A step by step method to find Solution for  $Ax=b$  (Gaussian Elimination)
  - Rank
  - Parametric form of solution