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Probability Theory - Part I

Course on Engineering Mathematics for GATE - CSE

Rahul • Lesson 7 • June 22, 2024

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Engineering Mathematics

Probability & Statistics

counting principles

Lecture Number- 06

By- Rahul Sir

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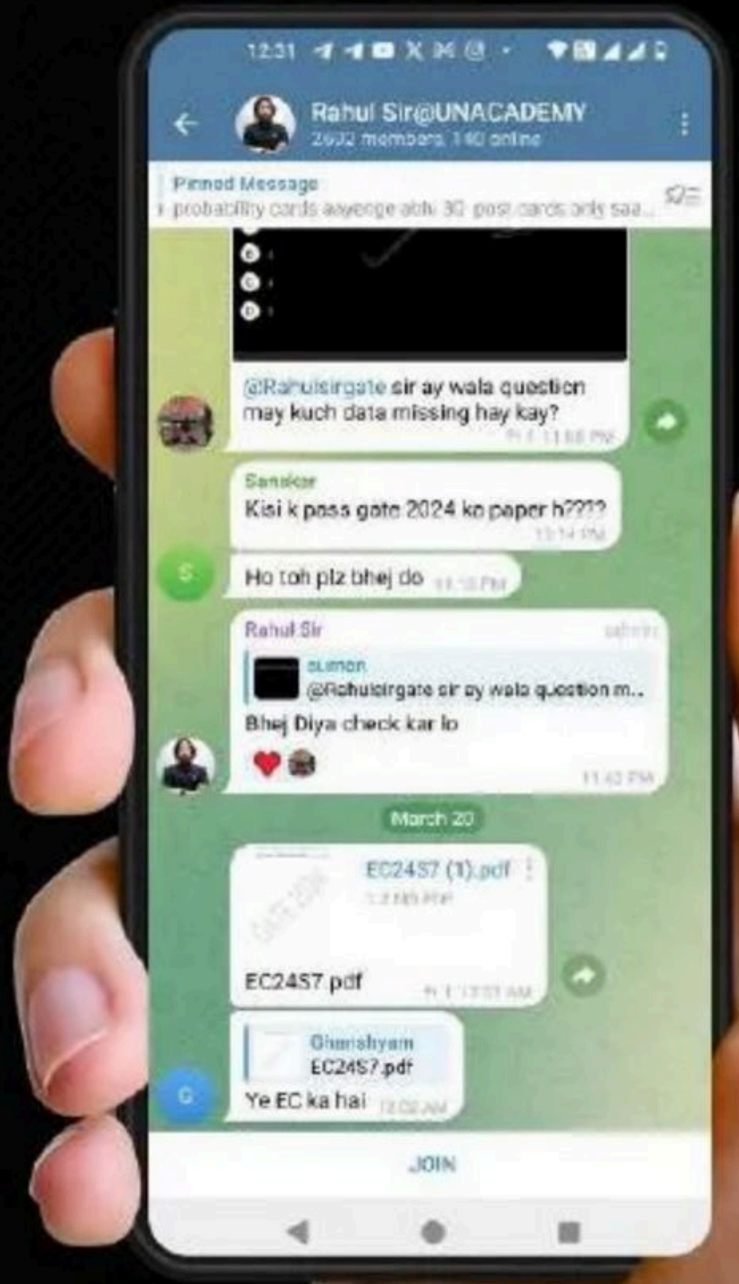

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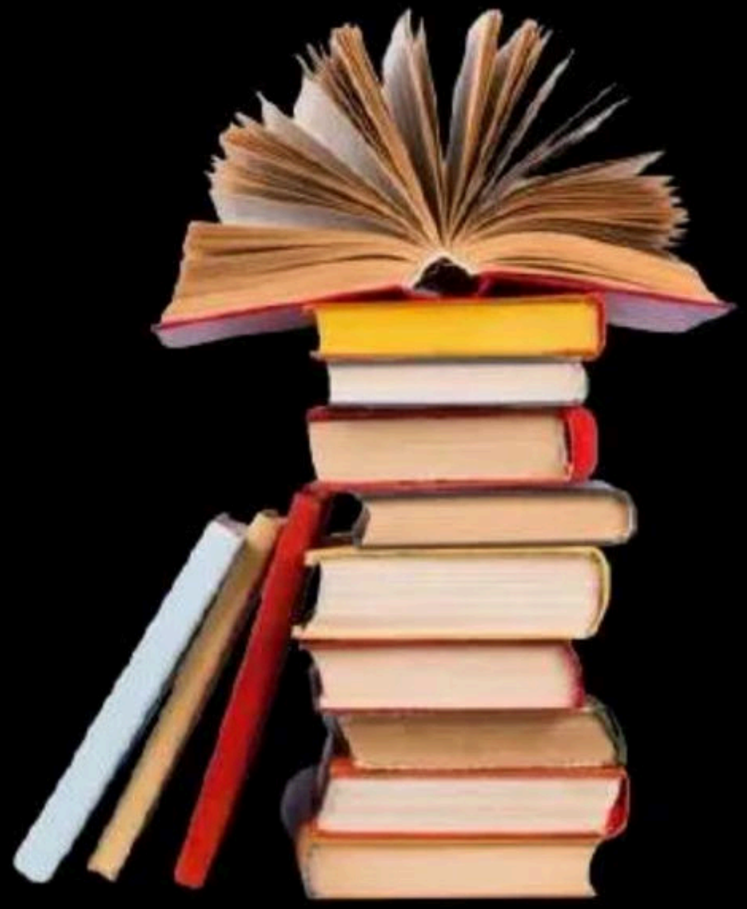
Topics

to be covered



1. Problem solving class part_II

Probability

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Probability :-

DETERMINISTIC EVENT

If experiments REPEAT n times Then results also SAME (every Trial)

experiment - REPEAT - Deterministic event

✓ # 1 month - 4 SUNday | SUN-RISE east

✓ 1 month - 5 SUNday → event = Random event.

(deterministic)

What is Probability

Random events (Random Phenomenon)

If REPEATS the experiments (Trial)

. every Trial HAVE a NEW way (NEW result)

⇒ Random exp.

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· Random exp :- Tossing A coin / Throwing A Die / pick A card / lottery / compan /

(Random experiment)

Probability :- STUDY of chance / STUDY of uncertainty

Study of Entropy (Disorder) / STUDY of Randomness /

STUDY of belief

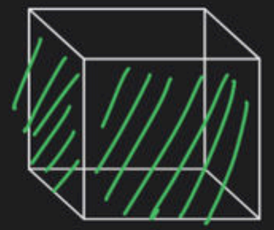
(Prob. THEORY)

Probability :- A) Random EXPERIMENT (1)

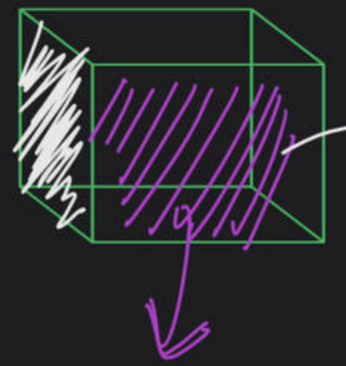
(1) Fair COIN → mass distribution both surface (Equal)
(unbiased coin)
biased coin → unEqual ⇒ deterministic Event

Fair DIE → all surface Mass distribution
Unbiased die SAME.

#



6 surface – mass distri – SAME
(unbiased)
(blindness / fairness)



→ weight
Due to gravity
= biased DIE

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(B) Tossing A coin (fair coin) Performed Throwing

(2)

HEAD Tail

$S = \{HEAD, Tail\}$

outcomes

Sample space $S = \{H, T\} = SET$

H, T

(3)

C) EVENTS Tossing A coin { HEAD occur, Tail occur } (SUBSET)

Sample Point

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Probability

- Random experiment (1)
- Sample space (2)
- Event (3)

Definition (Prob) School days - Defination

$$P(E) = \frac{\text{Number of favourable events}}{\text{Total No. of Possible events}} = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{\text{No. of desired outcomes}}{\text{Total No. of possible outcomes}} = \frac{n(E)}{n(S)}$$ Ratio

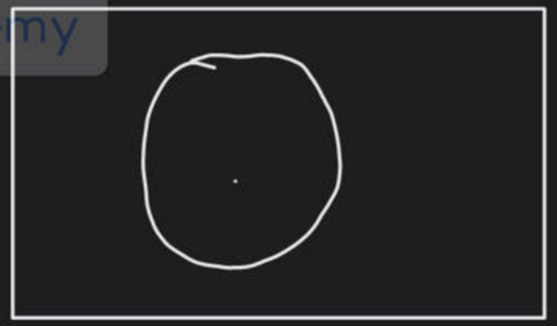
P(E) = Frequency (Relative)

100 times - experiment

Mukul	Krishan
55	45

$P(\text{Mukul}) = \frac{55}{100} = \checkmark$

$P(\text{Krishan}) = \frac{45}{100} = \checkmark$



$$P(E) = P(\text{circle event})$$

$$= \frac{\text{No. of Fav region}}{\text{Total region}}$$

$$P(\text{circle}) = \frac{\bigcirc}{\square + \bigcirc} = \frac{\text{No. of fav region (Area)}}{\text{Total region}}$$

Tossing A coin $n(S) = 2$

A = Head appears.

B = Tail appears.

$$P(\text{Head}) = \frac{n(A)}{n(S)} = \frac{1}{2} = 50\%$$

$$P(\text{Tail}) = \frac{n(B)}{n(S)} = \frac{1}{2} = 50\%$$

#

Throwing A Die

$S = \{1, 2, 3, 4, 5, 6\}$

EVENT $\begin{cases} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{cases}$

$P(1) = \frac{1}{6}$ $P(3) = \frac{1}{6}$

$P(2) = \frac{1}{6}$ $P(4) = \frac{1}{6}$

$P(5) = \frac{1}{6}$ $P(6) = \frac{1}{6}$

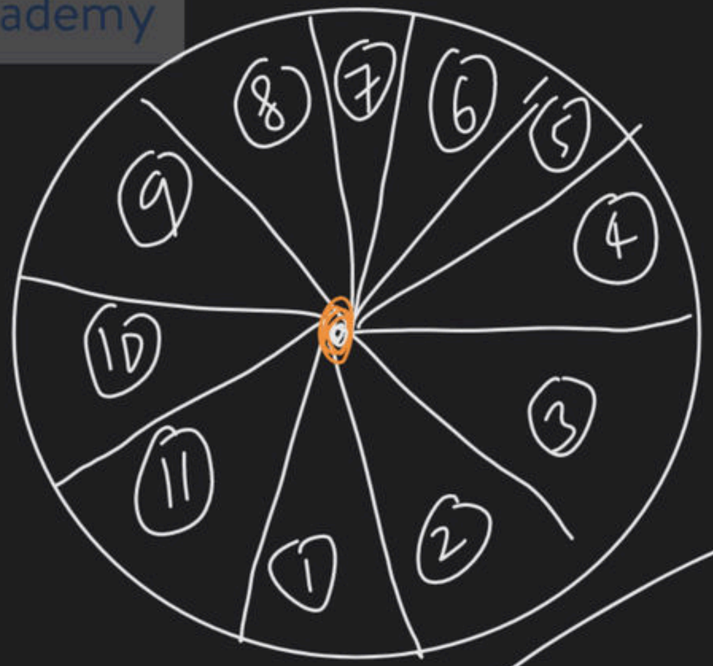
Roulette (gambling)

$P(\text{orange}) = 1/3$

$P(\text{Purple}) = 1/3$

$P(\text{white}) = 1/3$

$S = \{O, W, P\}$



$P(\text{any Number}) = \frac{1}{11}$ ✓

May I ask you

Die Throw 5 4 6 4 3 2

I II III IV V VI

$P(1) = \frac{1}{6}$ $P(2) = \frac{1}{6}$ $P(3) = \frac{1}{6}$ $P(5) = \frac{1}{6}$

$P(4) = \frac{1}{6}$ $P(6) = \frac{1}{6}$

$P(1) = 0$ ✓ (What ?)

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TOSSING A COIN EXPERIMENT 10 times

H H H H H H H H T T

$P(H) = \frac{8}{10}$

$P(T) = \frac{2}{10}$

H H H H H H H H H H

(NO Tail appears)

(LARGE NUMber of Tosses)

(what's going on)

Tossing A coin

SINGLE coin $n = 1$

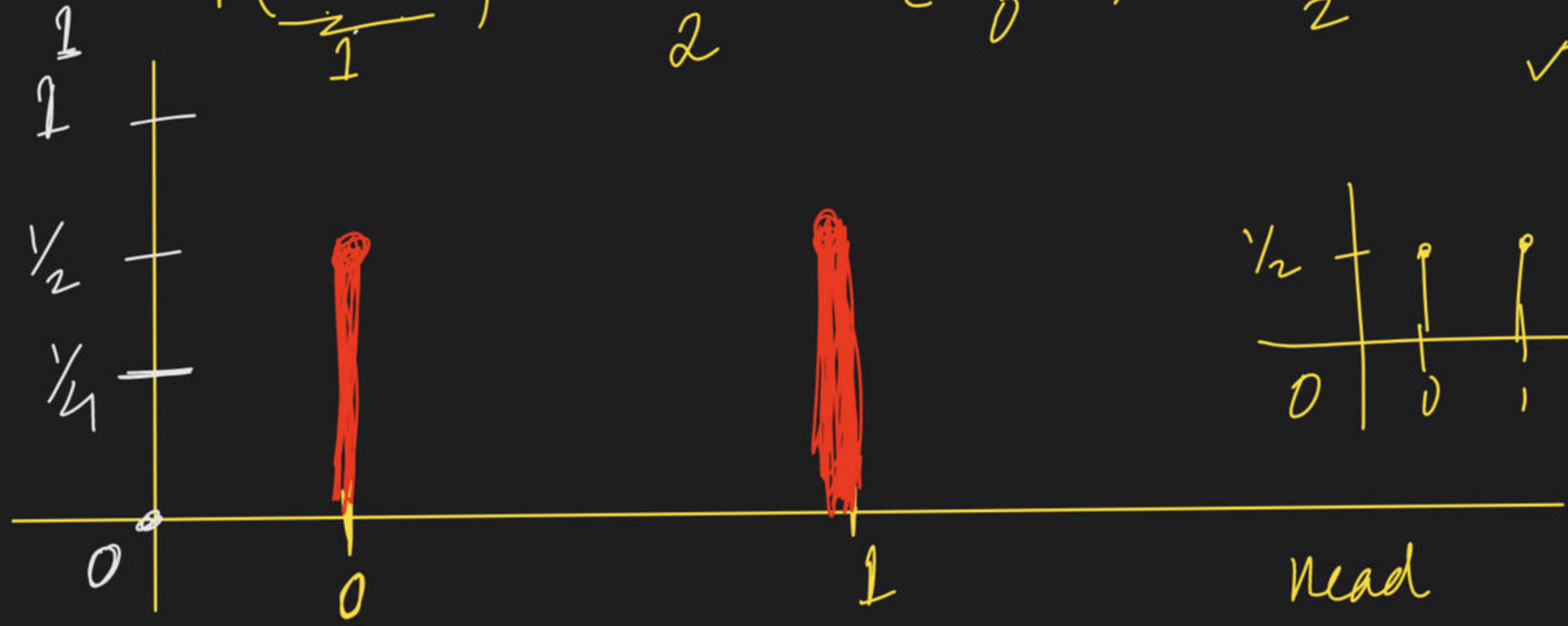
$S = \{H, T\}$ (Head, Tail)

$P(\text{Head}) = \frac{1}{2}$ $P(\text{Tail}) = \frac{1}{2}$

Head = 1

Tail = 0

ABSENT



X = sample Point

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Tossing A Two coin

Re 1, Re 2

(Simultaneously Throw)

Re 1 < H, T

Re 2 < H, T

Total No. of outcomes $= 2 \times 2 = 2^2$

A B

H T

T H

H H

T T

H < H = HH, T = HT

T < H = TH, T = TT

Tree diagram

(working Together)

(using FPC)

(using n Diff. Items Taken all at a time (Repetation allowed)

$= n^n$

$= 2^2$

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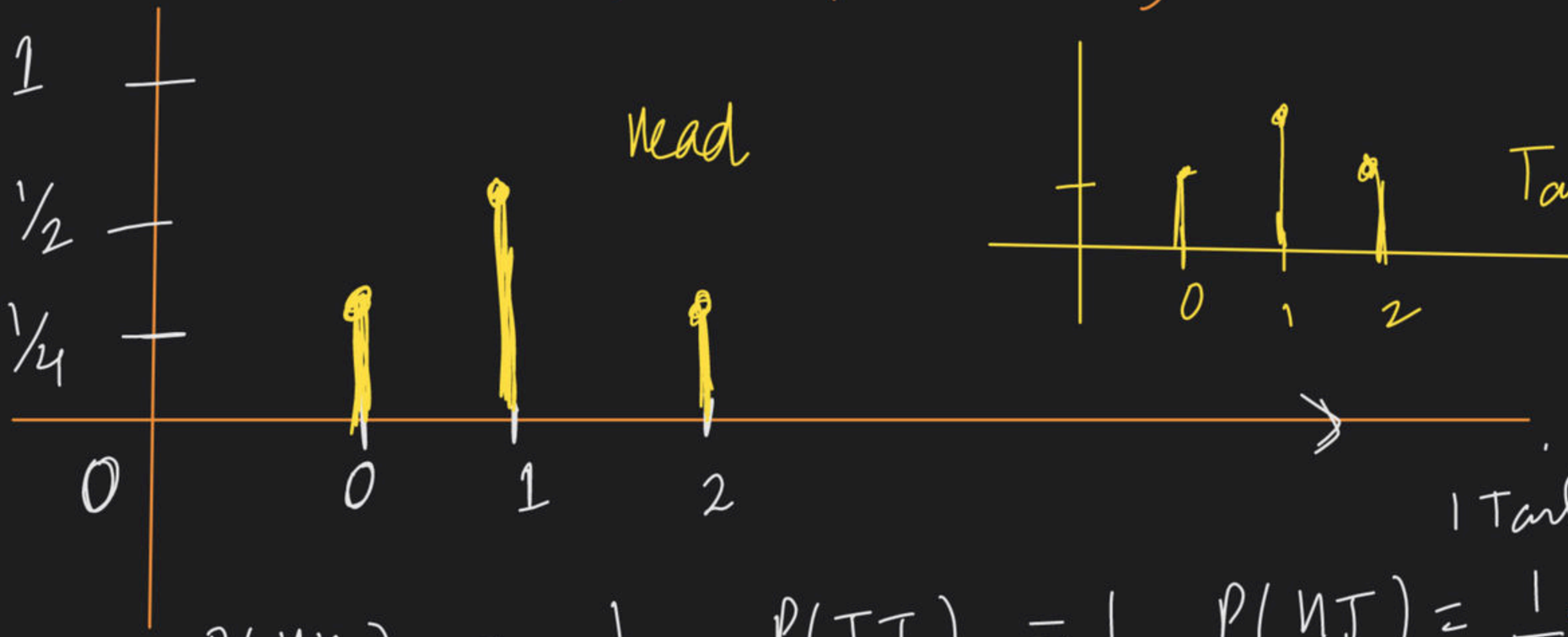
$S = \{HH, HT, TH, TT\}$

X-axis < Head / Tail



$P(HH) = \frac{1}{4}$ — 2 Head, 0 Tail — Srishti

$P(TT) = \frac{1}{4}$ — 0 Head, 2 Tail — Ravi

$P(HT) = \frac{1}{4}$ — 1 Head

$P(TH) = \frac{1}{4}$ — 1 Head, 1 Tail

No. of Head / No. of Tail — Add

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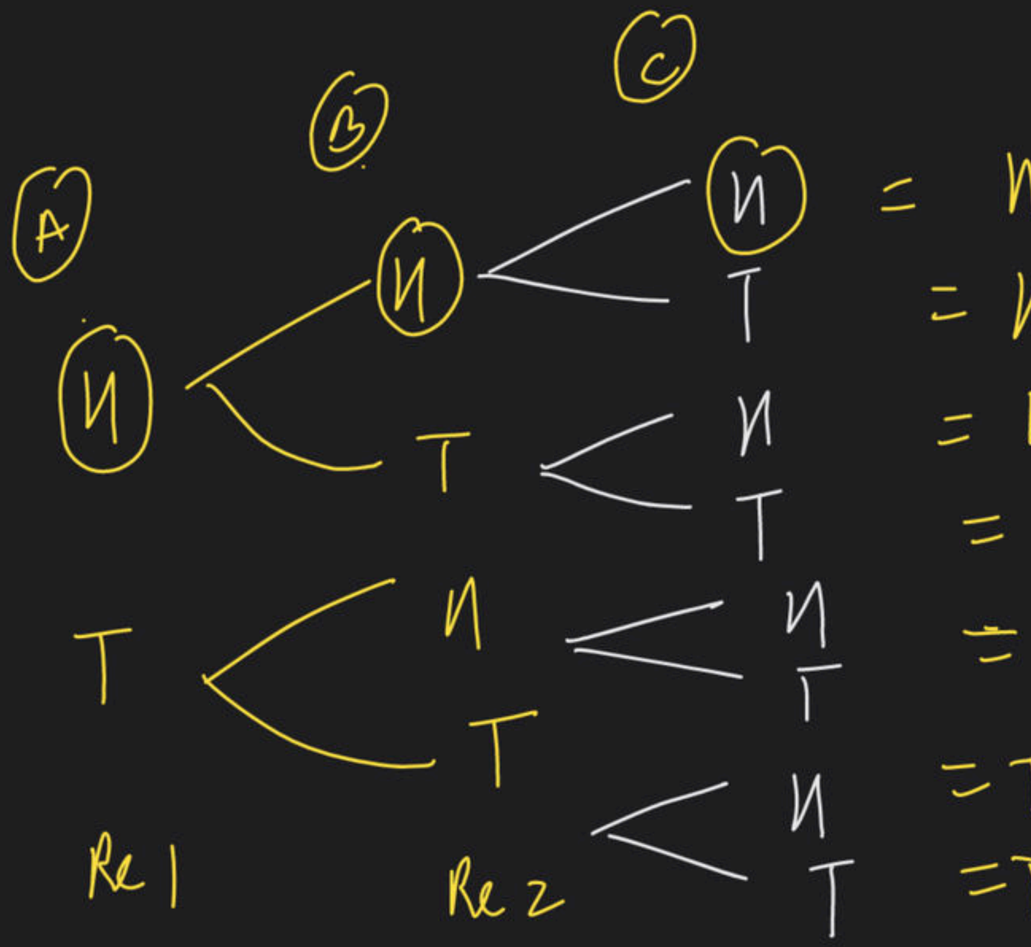
Tossing A THREE coin



Re 1, Re 2, Re 3 simult.

Total No. of ways $= 2^3$
$= 8$

$= HHH = \frac{1}{8}$	$=$	3H / 0T
$= HHT = \frac{1}{8}$	$=$	2H / 1T
$= HTH = \frac{1}{8}$	$=$	2H / 1T
$= HTT = \frac{1}{8}$	$=$	1H / 2T
$= THH = \frac{1}{8}$	$=$	1T / 2H
$= THT = \frac{1}{8}$	$=$	1H / 2T
$= TTH = \frac{1}{8}$	$=$	2T / 1H
$= TTT = \frac{1}{8}$	$=$	3T / 0H

X-axis
= No. of Heads
= 0, 1, 2, 3

OR

X-axis
No. of Tails
= 0, 1, 2, 3

Tossing A THREE coin

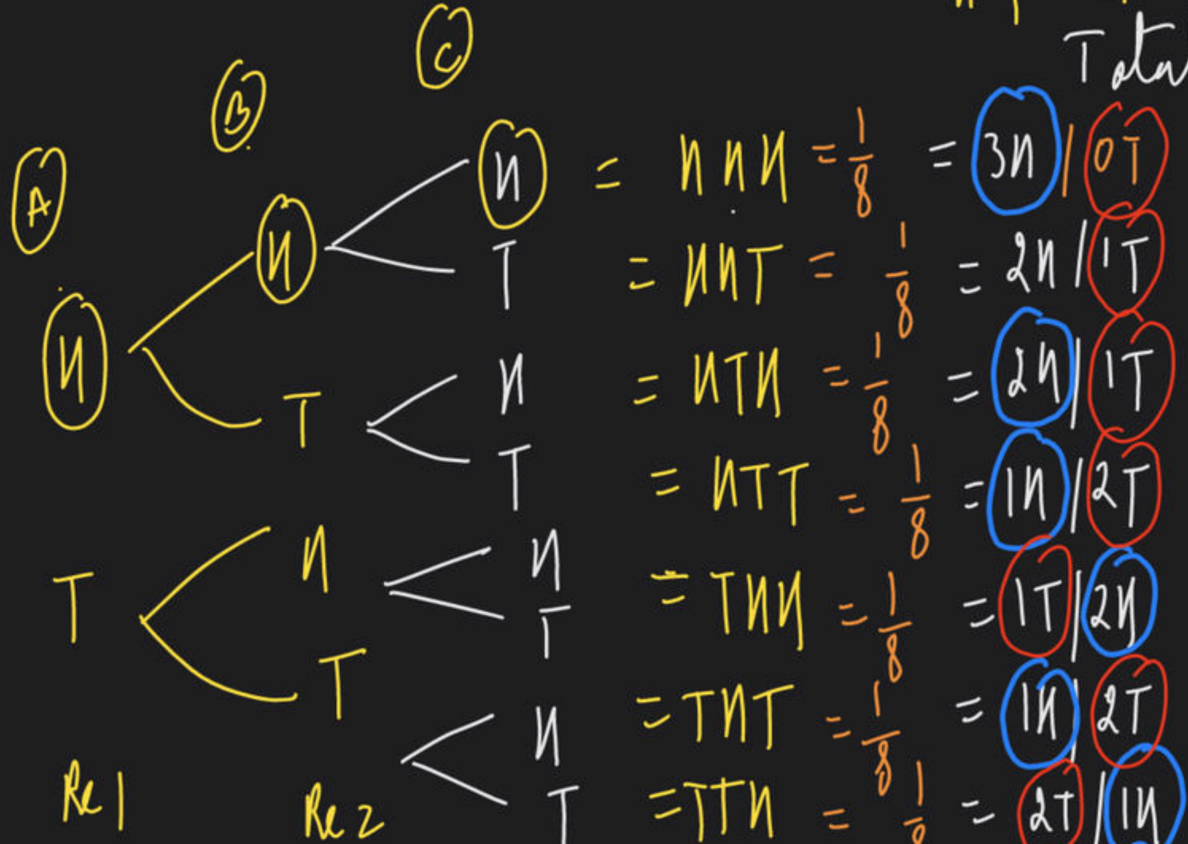
Re1 Re2, Re3

(2) ∧ (2) ∧ (2) ∧ simult.

H T H T H T

Total No. of ways $= 2^3$

$= 8$ ✓



$= HHH = \frac{1}{8} = (3H \mid 0T)$

$= HHT = \frac{1}{8} = (2H \mid 1T)$

$= HTH = \frac{1}{8} = (2H \mid 1T)$

$= HTT = \frac{1}{8} = (1H \mid 2T)$

$= THH = \frac{1}{8} = (1T \mid 2H)$

$= THT = \frac{1}{8} = (1H \mid 2T)$

$= TTH = \frac{1}{8} = (2T \mid 1H)$

$= TTT = \frac{1}{8} = (3T \mid 0H)$

X-axis
= No. of Heads
= 0, 1, 2, 3

OR

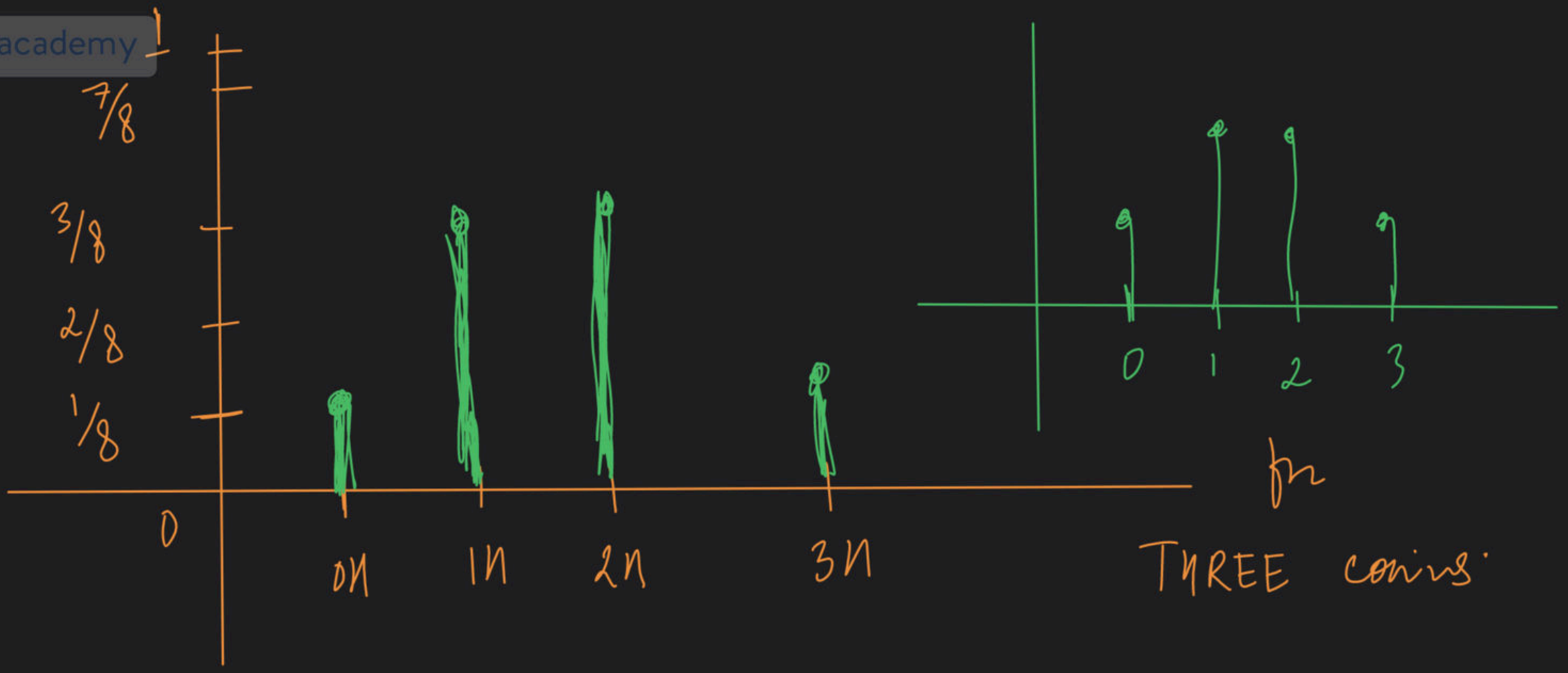
X-axis
No. of Tails
= 0, 1, 2, 3

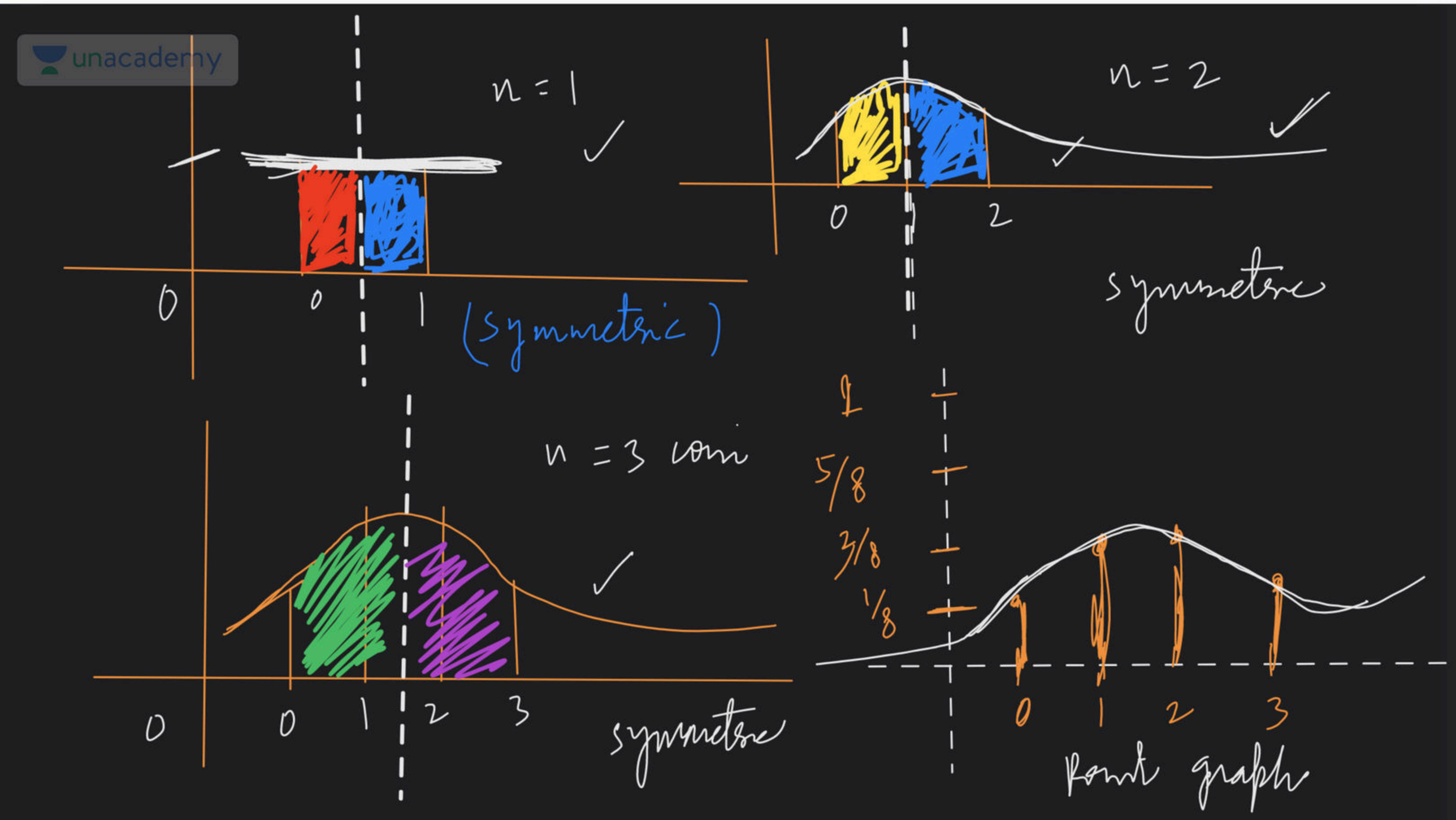
$P(0\text{ head}) = 1/8$

$P(1\text{ Head}) = 3/8$

$P(2\text{ head}) = 3/8$

$P(3\text{ head}) = 1/8.$


1
7/8
3/8
2/8
1/8
0
0H
1H
2H
3H
0
1
2
3
fx
THREE coins.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n = 1
0
0
1
(symmetric)
n = 2
0
1
2
symmetric
n = 3 coin
0
0
1
2
3
symmetric
1
5/8
3/8
1/8
0
1
2
3
Bar graph

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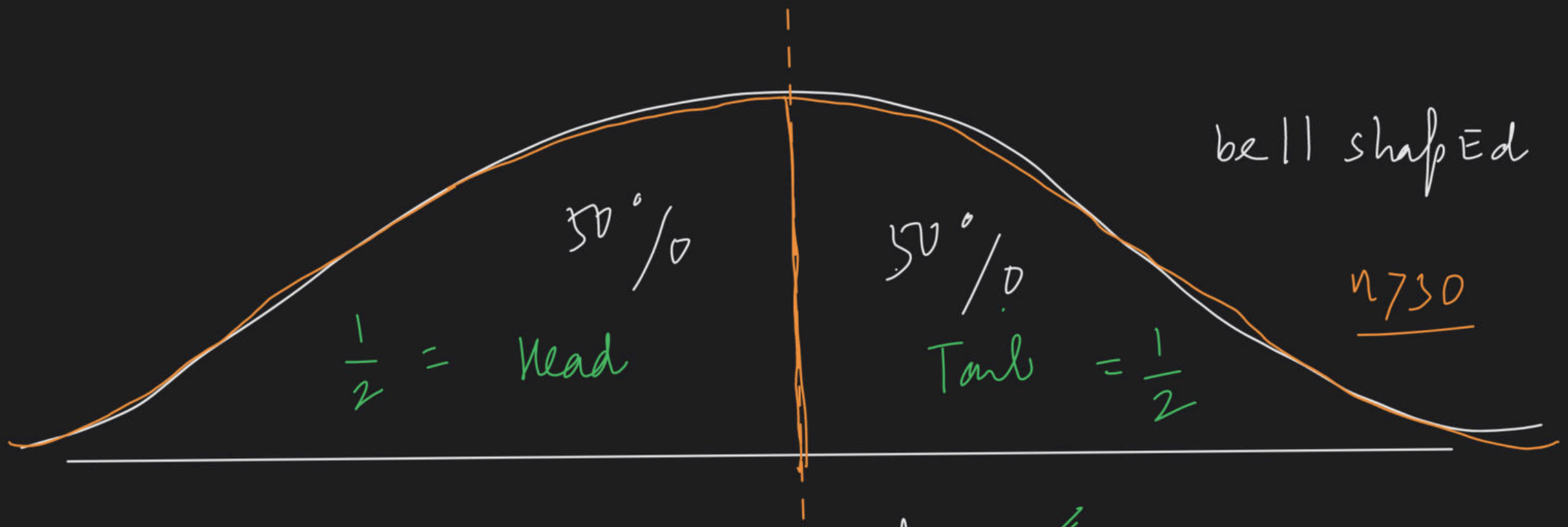
for $n \geq 30$ minimum 30 Trials.



large no. of Trials.

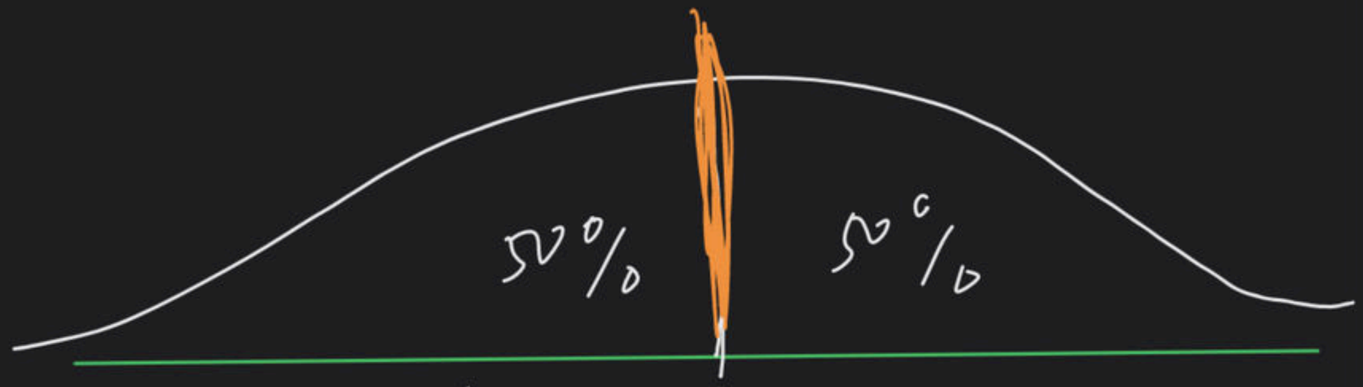
symmetric ✓

∴

If Experiment repeats $n(A)$ times then prob. of event

$$P(E) = \lim_{n \to \infty} \frac{n(A)}{n(S)}$$

bell shaped



" LARGE Number of Trials."

n is suffiient Number of large Trials.

$$P(E) = \lim_{n \to \infty} \frac{n(A)}{n(S)} = \text{constant} = \text{Ratio} = \text{prob. of an Event}$$

$$= 0 < P(A) \leq 1$$

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Throwing A Die :- (balanced Die)

$S = (1, 2, 3, 4, 5, 6)$

(Arrival)

$$\begin{cases} P(1) = \frac{1}{6} \quad P(2) = \frac{1}{6} \\ P(3,4,5,6) = \frac{1}{6} \end{cases}$$

X axis = No. of Dots

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Throwing A Two Die (Die A or Die B)

Two Die

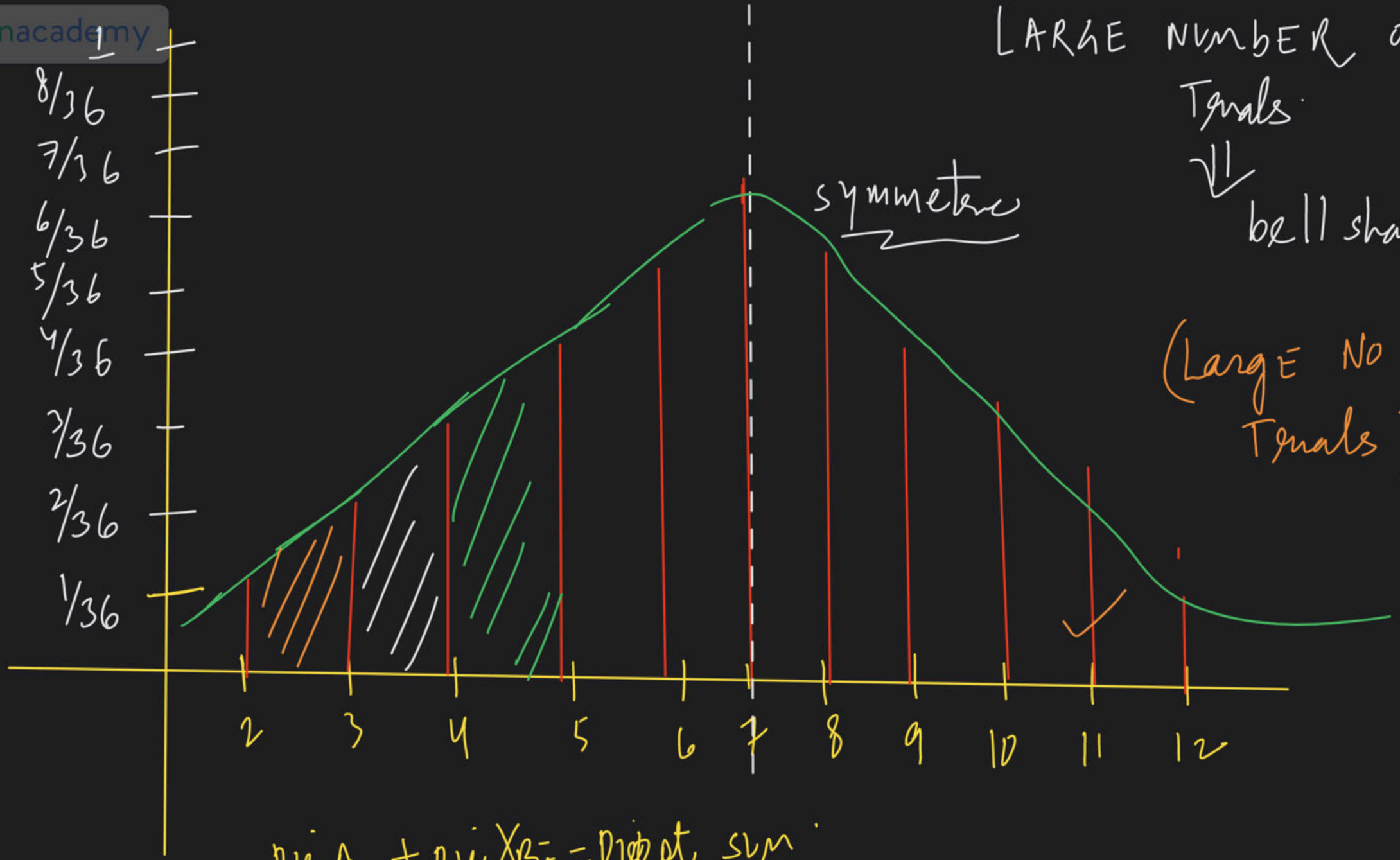
(Die A + Die B) = Sum

2	1/36
3	2/36
4	3/36
5	4/36
6	5/36
7	6/36
8	5/36
9	4/36
10	3/36
11	2/36
12	1/36

+	1	2	3	4	5	6
1	(1,1) 2	(1,2) 3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Total outcomes = 6×6

$= 6^2$



LARGE NUMBER of Trials

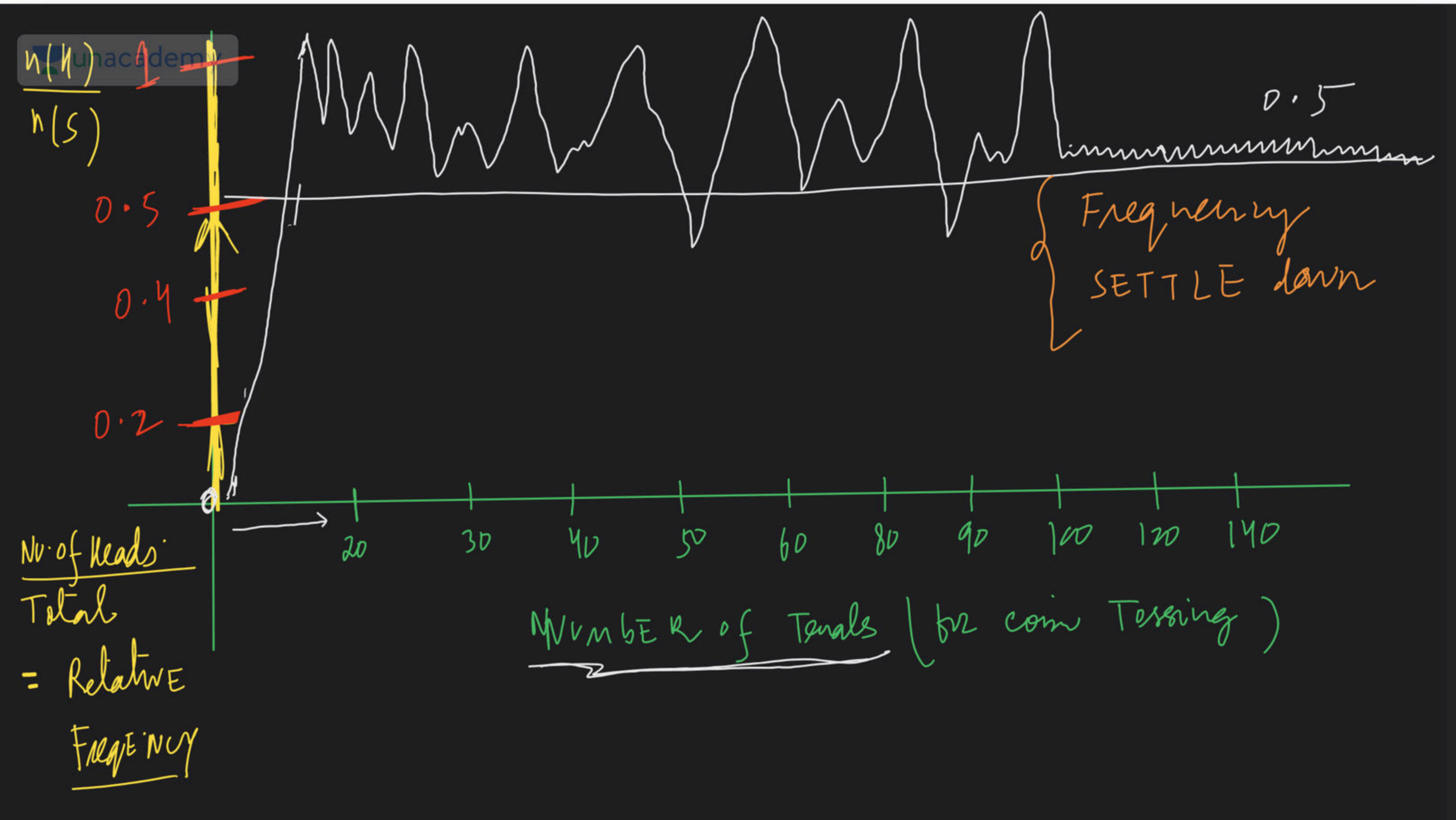
⇓

bell shaped.

symmetric

(Large No. of Trials)

Die A + Die B = Dist of sum


n(H)
n(S)
1
0·5
0·4
0·2
0
0·5
Frequency
SETTLE down
20
30
40
50
60
80
90
100
120
140
Nv. of Heads
Total
= Relative
Frequency
NUMBER of Tosses (for coin Tossing)

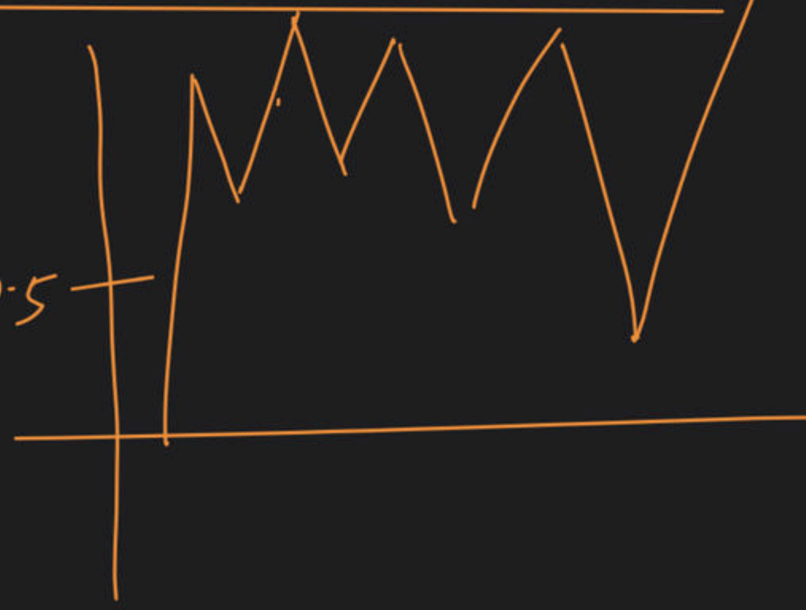
$$\frac{\text{Proportion of Head}}{\text{Total outcomes}} = \text{Prob. of Event}$$

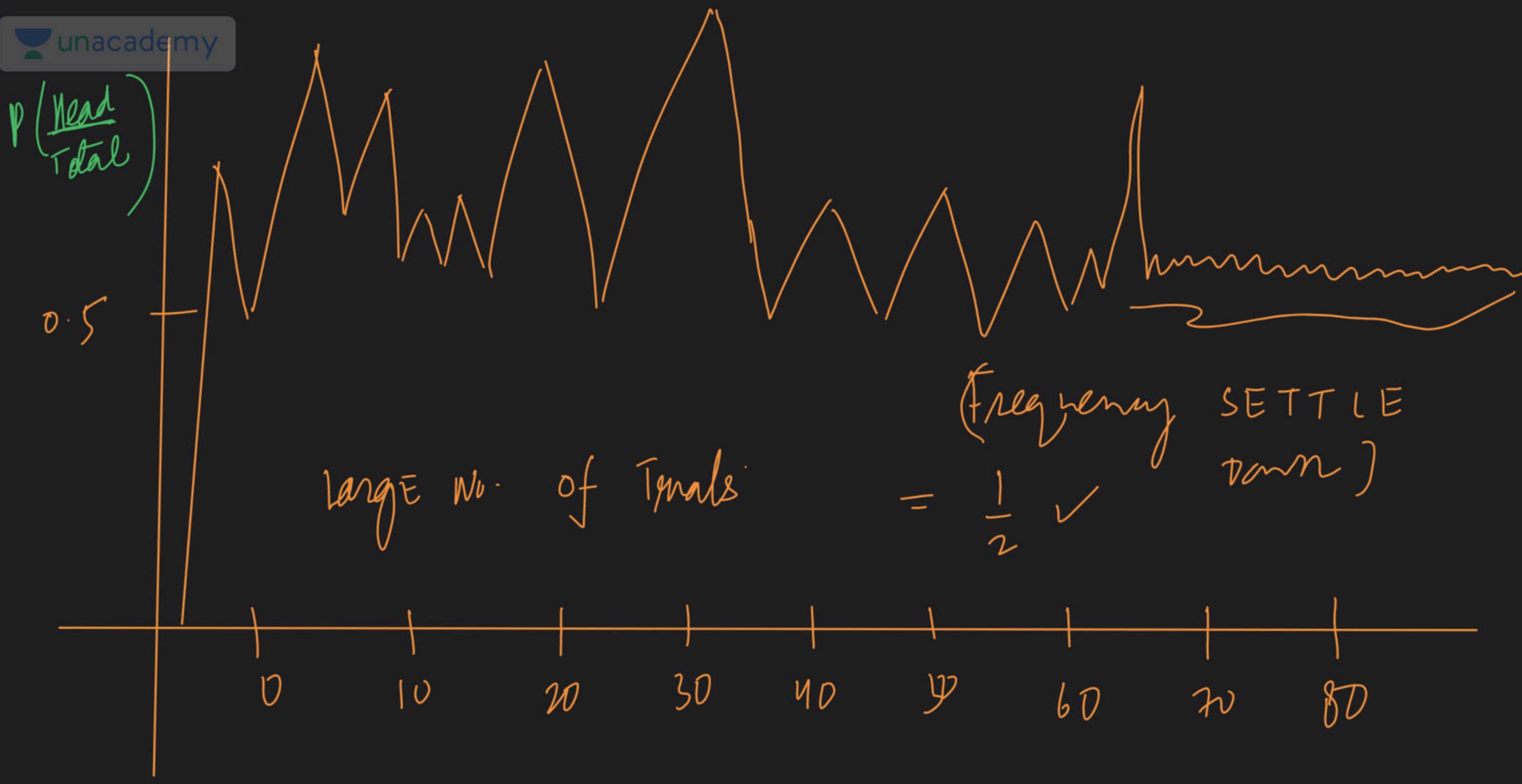
100 Tosses:

H H T T H H H
H H H T H T H
H H T T T H T
H H H T H T H
H T H T H H H

$= \frac{22}{35}$

$=$

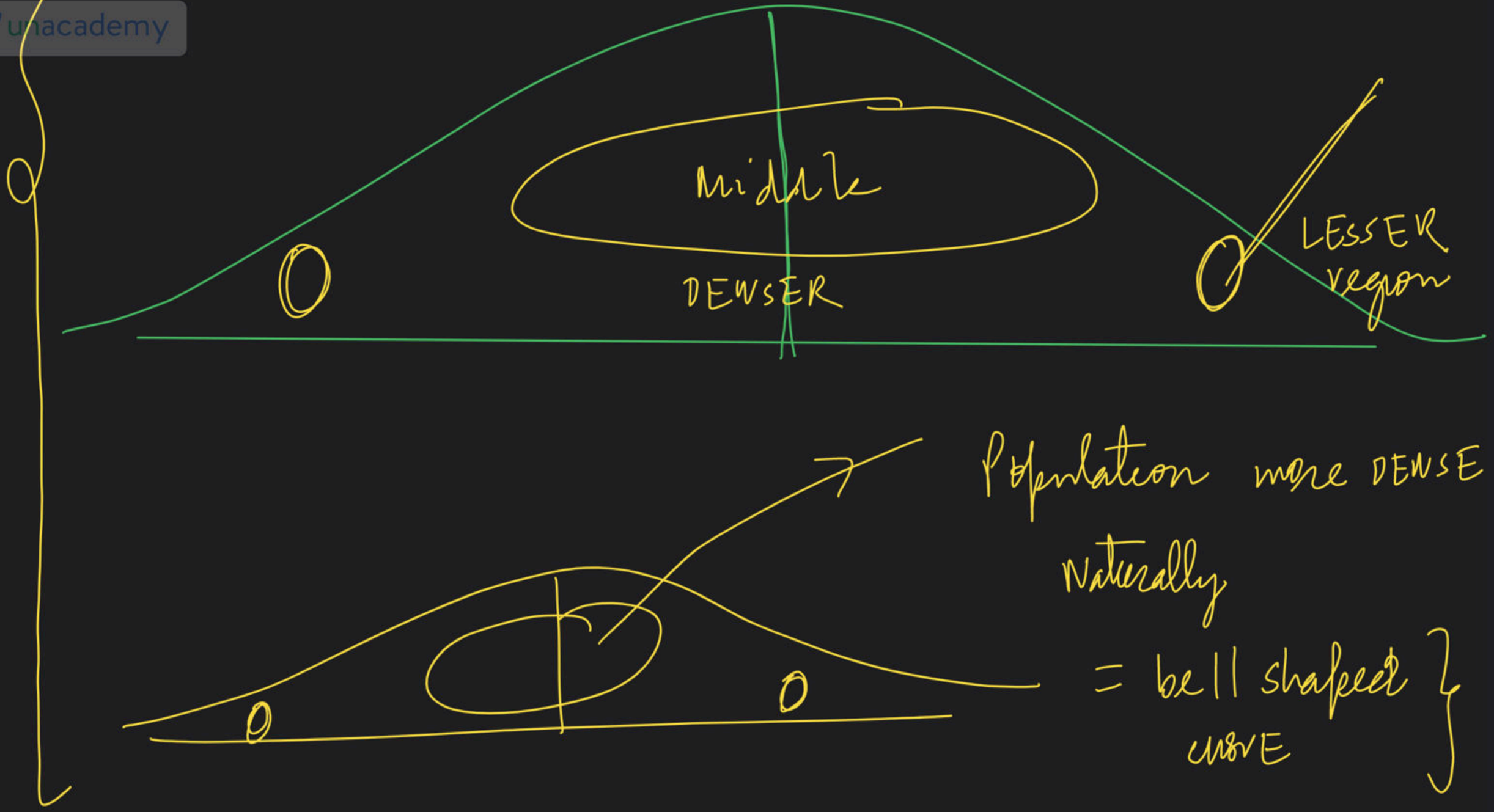




Large No. of Trials

= $\frac{1}{2}$ ✓

(Frequency SETTLE down)


Middle
DENSER
LESSER
region
Population more DENSE
Naturally
= bell shaped
curve

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Playing CARDS

52

26 cards

(13) Diamond

(13) Heart

cardeod

26 cards

(13) club

(13) spade

black

13 cards

Face card: J, Q, K

Non face card: A, 2, 3, ..., 10

Total = 13

face card = 12

Non face card = 40

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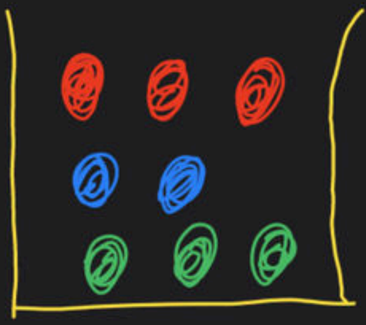
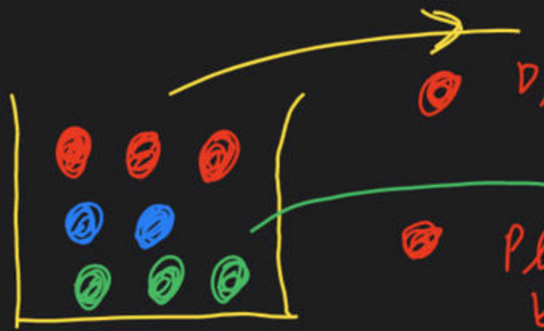
Counting strategies :- (without Replacement)

=> (Repetation Not allowed)

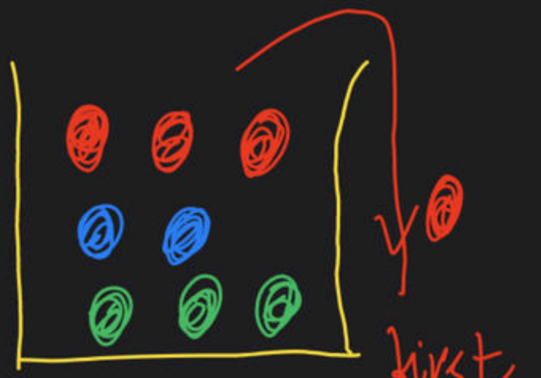
What is The Prob. 3 red ball are drawn Taken ONE at a time)

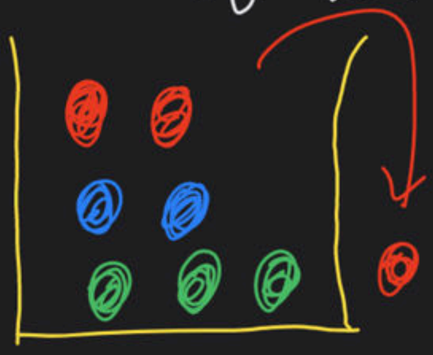


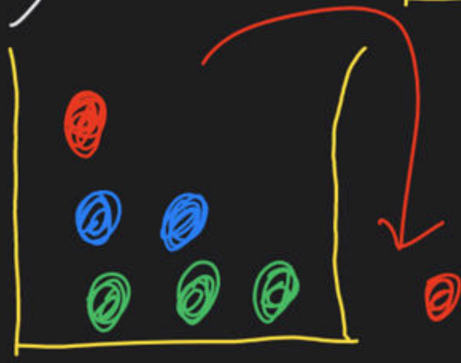


(Not allowed)

$P(E) = \frac{n(E)}{n(S)}$







$P(\text{first red}) = \frac{3}{8}$

$P\left(\frac{\text{SECOND red}}{\text{first red}}\right) = \frac{2}{7}$

$P\left(\frac{\text{NUM}}{\text{DEN}}\right)$

$P\left(\frac{\text{Third red}}{\text{first red, SECOND red}}\right) = \frac{1}{6}$

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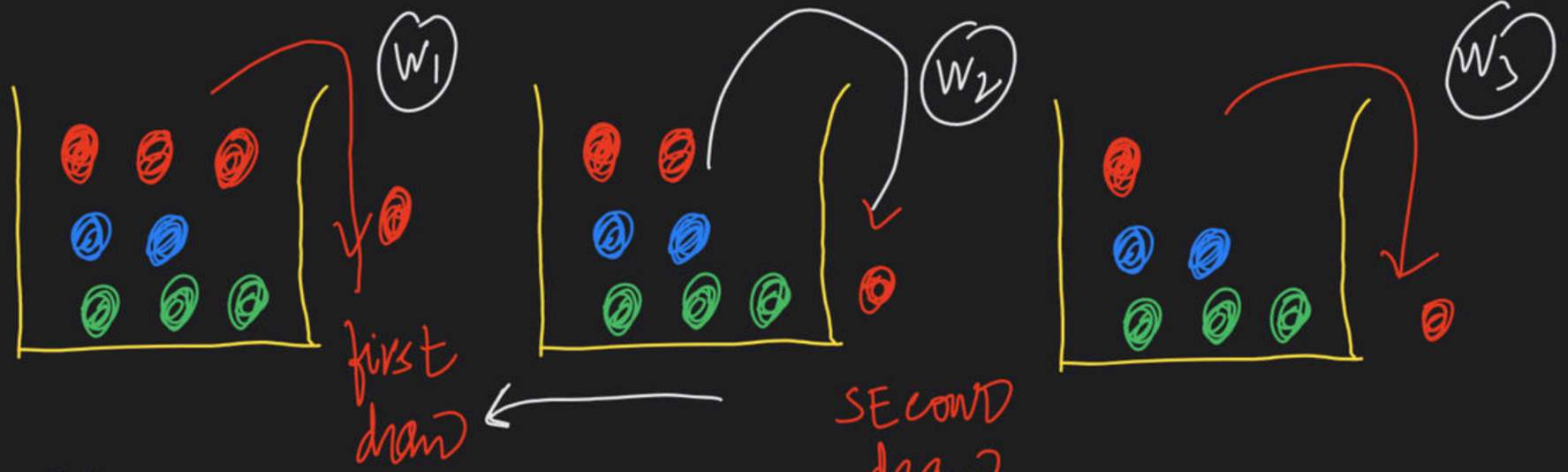
$$P\left(\frac{NUM}{DEN}\right) = P\left(\frac{Future}{past}\right) = P\left(\frac{HAPPENING}{Happened}\right)$$

$$= P(Happ, Happened)$$



$$P(\text{first red}) = \frac{3}{8}$$

$$P\left(\frac{\text{SECOND red}}{\text{first red}}\right) = \frac{2}{7}$$

$$P\left(\frac{\text{Third red}}{\text{first red, SECOND red}}\right) = \frac{1}{6}$$

$P(\text{3 red balls are drawn}) = P(\text{red 1})\, P\left(\frac{\text{red 2}}{\text{red 1}}\right) P\left(\frac{\text{red 3}}{\text{red 1, red 2}}\right)$

$= \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6}$ (Working Together)

(A) sample space / chance Are change

(B) $P(E) = \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6}$

NEXT prob. effected on previous prob/event.

c) DEPENDENT Events

#

10 I
10 T

What is the probability If 3 coupons are drawn making word $P(\text{I I T}) =$

10 I
10 T
$\xrightarrow{I_1}$ $P(I_1) = \frac{10}{20}$ (W1)

9 I
10 T
$\xrightarrow{I_2}$ $P\left(\frac{I_2}{I_1}\right) = \frac{9}{19}$ (W2)

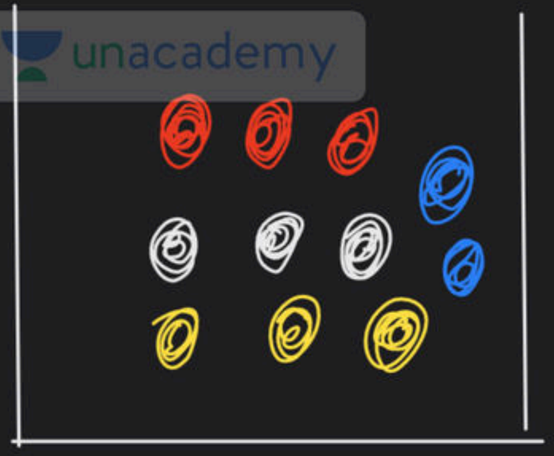
8 I
10 T
$\xrightarrow{T}$ $P\left(\frac{T}{I_1 ; I_2}\right) = \frac{10}{18}$ (W3)

better approach.

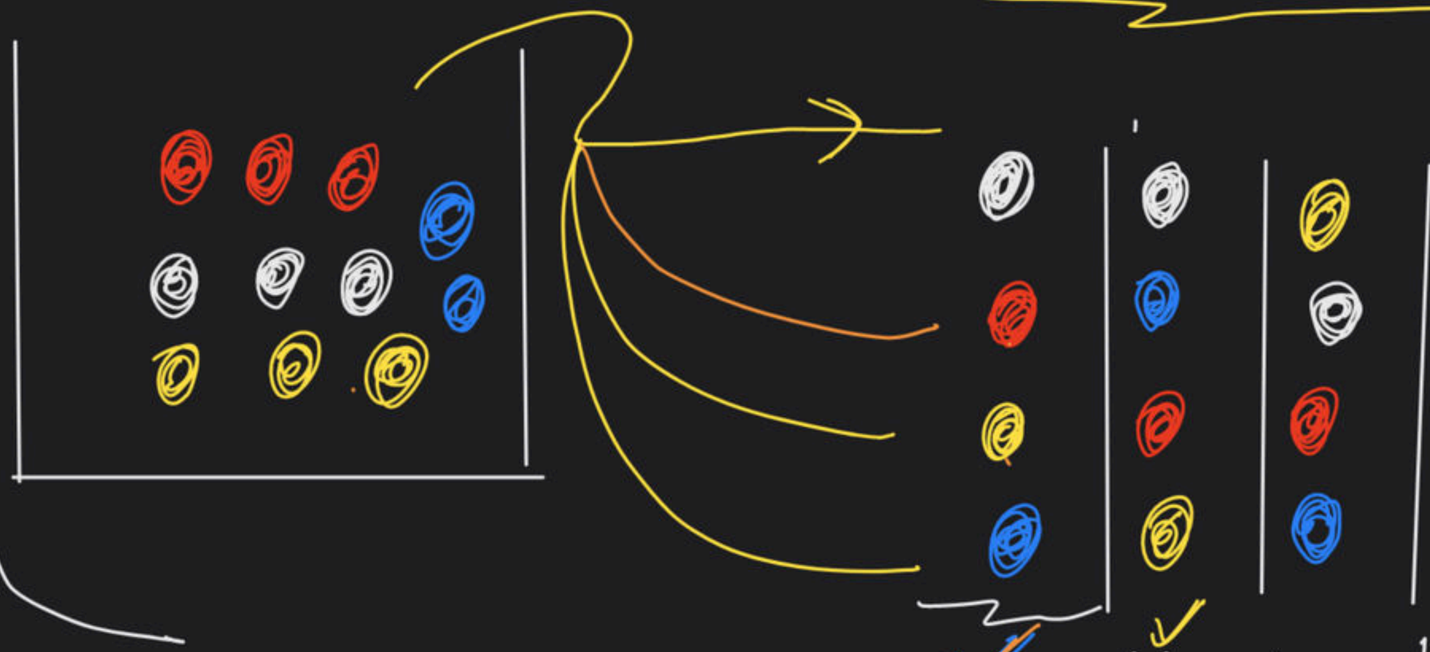
$= \frac{10}{20} \times \frac{9}{19} \times \frac{10}{18}$

Ans.

$= \frac{{}^{10}C_2 \; {}^{10}C_1}{{}^{20}C_3}$



If 4 balls ARE drawn at Random ONE at a time What is prob. " ONE of THOSE balls ie Red, White, blue, Yellow (Order DEcide)



$$\Rightarrow \frac{3}{11} \times \frac{3}{10} \times \frac{3}{9} \times \frac{2}{8} \times 4!$$

= answer

n Diff. Items Taken all at a time (Repetation Not allowed)

4J 4K, 4Q
52 cards.

If 3 cards are drawn at random.
What is prob. ONE of THOSE cards.
J, Q, K. (order Not specified)

$= \frac{4}{52} \times \frac{4}{51} \times \frac{4}{50}$ (J, Q, K)

(A) 52 4J 4Q 4K

(B) 4J 4K 4Q

(C) 4J 4K 4Q

(D) 4J 4K 4Q

(E) 4J 4K 4Q

(F) 4J 4K 4Q

JKQ OR JQK OR KJQ OR KQJ OR QJK OR QKJ

$= P(JKQ) + P(JQK) + P(KJQ) + P(KQJ) + P(QJK) + P(QKJ)$ ✓

= = P(one way) × n Diff. Items Taken all at a time
(repetation not allowed)

$= P(JKQ) \times 3!$

$= \frac{4C_1 \; 4C_1 \; 4C_1}{52C_3} \times 3!$

$= \frac{4}{52} \times \frac{4}{51} \times \frac{4}{50} \times 3!$

QUESTION

Q. 4 squares are chosen at random on a chessboard. Find the probability that they lie along the same diagonal line adjacent to each other.

QUESTION

Q. 2 squares are chosen on a chessboard. Find the probability that they have a side in common.

QUESTION

Q. A fair coin is tossed n times, the probability that the difference between the number of heads and tail is (n-3)/is_____.

QUESTION

Q. Three boys and two girls stand in a queue. The probability that the number of boys ahead of every girl is atleast one more than the number of girls ahead of her is

A 1/2

B 1/3

C 2/3

D 3/4

QUESTION

Q. Four fair dice D1, D2, D3, D4, each having six faces numbered 1, 2, 3, 4, 5, 6 are rolled simultaneously. The probability that D4 shows a number appearing on one of D1, D2 and D3 is_____.

A 91/216

B 108/216

C 125/216

D 127/216

QUESTION

Q. Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently_____.

A 1/2

B 7/15

C 2/15

D 2/3

QUESTION

Q. Three of the six vertices of a regular hexagon are chosen at random. The probability that a triangle with three vertices is equilateral.

QUESTION

Q. If P and Q are chosen randomly from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} with replacement) Determine the probability that the roots of the equation $x^2 + px + q = 0$ are real.

QUESTION

Q. The probability that two friends share the same birth month is_____.

QUESTION

Q. Seven car accidents occurred in a week. What is the probability that they all occurred on the same day?

QUESTION

Q. box contains 10 screw, 3 of which are defective. Two screws are drawn at random with replacement. The probability that none of the two screws is defective will be

QUESTION

Q. A bag contains 10 blue marbles, 20 black marbles and 30 red marbles. A marble is drawn from the bag, its colour is recorded and it is put back in the bag. This process is repeated 3 times. The probability that no two of the marbles drawn have the same colour.

QUESTION

Q. A box contains 4 white balls and 3 red balls. In succession, two balls are randomly selected and removed from the box, given that the first removed ball is white, the probability that the second removed ball is red is_____.

QUESTION

Q. There are two containers, with one containing 4 Red and 3 Green balls and the other containing 3 Blue balls and 4 Green balls. One ball is drawn at random from each container. The probability that one of the balls is Red and the other is Blue will be_____.

QUESTION

Q. An urn contains 5 red and 7 green balls. A ball is drawn at random and its colour is noted. The ball is placed back into the urn along with another ball of the same colour. The probability of getting a red ball in the next draw is_____.

QUESTION

Q. If from each of the three boxes containing 3 white and 1 black balls, 2 white and 2 black balls, 1 white and 3 black balls, one ball is drawn at random, then the probability that 2 white and 1 black ball will be drawn is_____.

QUESTION

Q. Let w be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2, r_3 are the numbers obtained on the die, then the probability that $\omega^{r1} + \omega^{r2} + \omega^{r3} = 0$ is_____.

A 1/18

B 1/9

C 2/9

D 1/36

QUESTION

Three boys and two girls stand in a queue. The probability that the number of boys ahead of every girl is at least one more that the number of girls ahead of her, is:

A $\frac{1}{2}$

B $\frac{1}{3}$

C $\frac{2}{3}$

D $\frac{3}{4}$

QUESTION

Four fair dice D_1, D_2, D_3 and D_4 each having six faces numbered 1, 2, 3, 4, 5 and 6 are rolled simultaneously. The probability that D4 shows a number appearing on one of D_1, D_2 and D_3 is:

A $\frac{91}{216}$

B $\frac{108}{216}$

C $\frac{125}{216}$

D $\frac{127}{216}$

QUESTION

Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$, is:

A 1/18

B 1/9

C 2/9

D 1/36

QUESTION

If three distinct numbers are chosen randomly from the first 100 natural numbers, then the probability that all three of them are divisible by both 2 and 3, is :

A $\frac{4}{55}$

B $\frac{4}{35}$

C $\frac{4}{33}$

D $\frac{4}{1155}$

QUESTION

Two numbers are selected randomly from the set $S = \{1, 2, 3, 4, 5, 6\}$ without replacement one by one. The
probability that minimum of the two numbers is less than 4, is:

A $\frac{1}{15}$

B $\frac{14}{15}$

C $\frac{1}{5}$

D $\frac{4}{5}$

QUESTION

If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5, equals:

A 1/4

B 1/7

C 1/8

D 1/49

QUESTION

Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently, equals:

A 1/2

B 7/15

C 2/15

D 1/3

QUESTION

Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with three vertices is equilateral, equals:

A 1/2

B 1/5

C 1/10

D 1/20

QUESTION

Three identical dice are rolled. The probability that the same number will appear on each of them, is:

A 1/6

B 1/36

C 1/18

D 3/28

QUESTION

Fifteen coupons are numbered 1, 2,.....,15, respectively. Seven coupons are selected at random one at a time with replacement. The probability that the largest number appearing on a selected coupon is 9, is:

A $\left(\frac{9}{16}\right)^6$

B $\left(\frac{8}{15}\right)^7$

C $\left(\frac{3}{5}\right)^7$

D None of these

QUESTION

Consider the system of equations $ax + by = 0$, $cx + dy = 0$ where $a, b, c, d \in \{0, 1\}$.

Statement-I: The probability that the system of equations has a unique solution, is $3/8$.

Statement-II: The probability that the system of equations has a solution, is 1.

A. Statement-I is true, Statement-II is also true; Statement-II is the correct explanation of Statement-I

B. Statement-I is true, Statement-II is also true; Statement-II is not the correct explanation of Statement-I.

C. Statement-I is true; Statement-II is false.

D. Statement-I is false; Statement-II is true.

QUESTION

Paragraph for Questions

Box I contains three cards bearing numbers 1, 2, 3 ; box II contains five cards bearing numbers 1, 2, 3, 4, 5; box III contains seven cards bearing numbers 1, 2, 3, 4, 5, 6, 7. A card is drawn from each of the boxes. Let x_i be the number on the card drawn from the ith box i = 1,2,3.

Q. The probability that $x_1 + x_2 + x_3$ is odd, is:

A $\frac{29}{105}$

B $\frac{53}{105}$

C $\frac{57}{105}$

D $\frac{1}{2}$

QUESTION

Paragraph for Questions

Box I contains three cards bearing numbers 1, 2, 3 ; box II contains five cards bearing numbers 1, 2, 3, 4, 5; box III contains seven cards bearing numbers 1, 2, 3, 4, 5, 6, 7. A card is drawn from each of the boxes. Let x_i be the number on the card drawn from the ith box i = 1,2,3.

Q. The probability that x_1, x_2 and x_3 are in an arithmetic progression, is:

A $\frac{9}{105}$

B $\frac{10}{105}$

C $\frac{11}{105}$

D $\frac{7}{105}$

QUESTION

Three faces of a fair die are yellow, two faces red and one face blue. The die is tossed three times. The probability that the colours, yellow, red and blue, appear in the first, second and the third tosses respectively, is_____.

QUESTION

$\frac{1+3p}{3}, \frac{1-p}{4}$ and $\frac{1-2p}{2}$ are the probabilities of three mutually exclusive events, then the set of all values of p is_____.

QUESTION

A box contain 100 tickets numbered 1, 2,....., 100. Two tickets are chosen at random. It is given that the minimum number on the two chosen tickets is not more than 10. The maximum number on them is 5 with probability_____.

QUESTION

A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that the value of the determinant chosen is positive, is_____.

QUESTION

If the letters of the word 'ASSASSIN' are written down at random is a row, the probability that no two S's occur together is 1/35.

QUESTION

An unbiased die, with faces numbered 1, 2, 3, 4, 5 and 6 is thrown n times and the list of n numbers showing up is noted. What is the probability that among the numbers 1, 2, 3, 4, 5 and 6 only three numbers appear in this list ?

QUESTION

If p and q are chosen randomly from the set {1, 2, 3, 4, 5, 6, 7, 8, 9 and 10} with replacement, determine the probability that the roots of the equation $x^2 + px + q = 0$ are real.

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THANK YOU!

Here's to a cracking journey ahead!

CSE full marks
full
Knowledge