Question

Determine if the following vectors are linearly dependent or linearly independent.

(a) $\left\{\begin{pmatrix} -4 \\ 0 \\ 1 \\ 5 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 4 \\ 3 \\ 6 \end{pmatrix}\right\}$,

(b) $\left\{\begin{pmatrix} 4 \\ 4 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \begin{pmatrix} 8 \\ 1 \end{pmatrix}\right\}$,

(c) $\left\{\begin{pmatrix} -8 \\ 12 \\ -4 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}\right\}$.

http://math.bu.edu/people/popovic/MA242/midterm1_sol.pdf

Solution.

(a) The set is linearly dependent, since it contains the zero vector $\mathbf{0}$. (Any set containing the zero vector is linearly dependent.)

(b) The set is linearly dependent, since it consists of four vectors in $\mathbb{R}^2$. (Any set containing more vectors than each vector has entries is linearly dependent.)

(c) The set is linearly independent, since it contains two vectors neither of which is the zero vector, and since these vectors are not multiples of each other.

Question

Practice problems Determine whether each set is linearly dependent or independent. If the set is dependent, come up with a set of coefficients that illustrates their dependence.

a) $\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2.000001 \end{bmatrix} \right\}$

Practice problems Determine whether each set is linearly dependent or independent. If the set is dependent, come up with a set of coefficients that illustrates their dependence.

a) $\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2.000001 \end{bmatrix} \right\}$

Answers

a) Dependent (1,0,0)

b) Independent

Question

Label the following sets as independent or dependent. For dependent sets, determine whether it is possible to modify only one element of one vector to change it to an independent set.

a) $\left\{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}2\\1\end{bmatrix}\right\}$

b) $\left\{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}1\\-2\end{bmatrix}\right\}$

c) $\left\{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}-1\\-2\end{bmatrix}\right\}$

d) $\left\{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}1\\-2\end{bmatrix}, \begin{bmatrix}-1\\-2\end{bmatrix}\right\}$

e) $\left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}3\\0\end{bmatrix}\right\}$

f) $\left\{\begin{bmatrix}5\\12\end{bmatrix}, \begin{bmatrix}-3\\4\end{bmatrix}, \begin{bmatrix}10\\23\end{bmatrix}\right\}$

g) $\left\{\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}1\\2\\4\end{bmatrix}\right\}$

h) $\left\{\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}8\\16\\24\end{bmatrix}\right\}$

a) Independent

b) Independent

c) Dependent, yes

d) Dependent, no

e) Dependent, yes

f) Dependent, no

g) Independent

h) Dependent, yes

Question

Determine the value of λ that would make the following sets of vectors dependent.

a) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4.5 \\ \lambda \\ 13.5 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ \lambda \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

a) $\lambda = 9$

b) Any λ

MSQ Question

Consider the following set of 2 vectors.

For what value of λ these 2 vectors are linearly dependent ?
(More than one could be correct options)

$$\left\{ \begin{bmatrix} 0 \\ 0 \\ \lambda \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} a \\ b \\ 5 \\ c \\ d \end{bmatrix} \right\}$$

a) For all values of λ vectors are linearly dependent

b) For $\lambda = 0$ vectors are linearly dependent

c) If a,b,c and d are zeros then vectors are linearly dependent only if λ is multiple of 5

d) If a,b,c and d are zeros then vectors are linearly dependent for all values of λ

Answer: B, D

Option A is trivial and has been discussed in lecture.

Option D: for a=b=c=d=0, set becomes -

$$\left\{ \begin{bmatrix} 0 \\ 0 \\ \lambda \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 5 \\ 0 \\ 0 \end{bmatrix} \right\}$$

For any λ first vector is multiple of other.
For example λ = 3 then we need to multiple 5/3 in first vector to get second vector.

Question

In R^6,

a) Any six vectors fill the space

b) Any fifty vectors fill the space

c) Any six linearly dependent vectors fill the space

d) Any three linearly independent vectors fill the space

e) Any ten vectors fill the space provided there are 6 linearly independent vectors out of ten

Answer: E