Practice Questions on Intuition of Ax=b

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Question:

True/False

fill

Any set of 5 vectors in $\mathbb{R}^4$ spans $\mathbb{R}^4$.

https://math.berkeley.edu/~awzorn/spring14/2-18%20Solutions.pdf

Question:

True/False

fill

Any set of 5 vectors in $\mathbb{R}^4$ spans $\mathbb{R}^4$.

to fill $R^4 \Rightarrow$ 4 linearly indep. vecter

false

https://math.berkeley.edu/~awzorn/spring14/2-18%20Solutions.pdf

Question:

Let A be an $m \times n$ matrix with columns $\underline{A}_1, \ldots, \underline{A}_n$. Express $A\underline{x}$ as a linear combination of the columns of A.

$$A = \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ A_1 & A_2 & \cdots & A_n \\ \downarrow & \downarrow & & \downarrow \end{bmatrix}$$

https://sites.math.washington.edu//~smith/Teaching/308/2012-Summer-Mterm-Answers.pdf

Let A be an $m \times n$ matrix with columns $\underline{A}_1, \ldots, \underline{A}_n$. Express $A\underline{x}$ as a linear combination of the columns of A.

Answer: $A\underline{x} = x_1\underline{A}_1 + \cdots + x_n\underline{A}_n$

$$Ax = x_1 A_1 + x_2 A_2 + \cdots x_n A_n$$

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

https://sites.math.washington.edu//~smith/Teaching/308/2012-Summer-Mterm-Answers.pdf

Question:

Write down a 4×3 matrix A such that $A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = z\begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} + y\begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} + x\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$.

https://sites.math.washington.edu//~smith/Teaching/308/2012-Summer-Mterm-Answers.pdf

Question:

Write down a 4×3 matrix A such that $A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = z \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} + y \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} + x \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}.$

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

easy

https://sites.math.washington.edu//~smith/Teaching/308/2012-Summer-Mterm-Answers.pdf

Write down a 4×3 matrix A such that $A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = z \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} + y \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} + x \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$.

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Question:

Write down a 4×3 matrix A such that $A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z \\ y+z \\ x+y+z \\ 0 \end{pmatrix}$.

https://sites.math.washington.edu//~smith/Teaching/308/2012-Summer-Mterm-Answers.pdf

$$A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z \\ y+z \\ x+y+z \\ 0 \end{pmatrix}.$$

decomposition

$$= x\underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}}_{x=1,\ y=0,\ z=0} + y\underbrace{\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}}_{y=1,\ x=0,\ z=0} + z\underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}}_{z=1,\ x=0,\ y=0}$$

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

H.W.

$$\left\{ \begin{pmatrix} z \\ y+z \\ x+y+z \\ 0 \end{pmatrix} \;\middle|\; x, y, z \text{ are real number} \right\}$$

Set notation

Q: in the above set, how many vectors are LI?

Write down a 4×3 matrix A such that $A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z \\ y+z \\ x+y+z \\ 0 \end{pmatrix}$.

This question is "identical" to the previous one so the answer is the same, namely

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

Comments: Understand why this question is the same as the previous one.

Question:

Let $\underline{A}_1, \underline{A}_2, \underline{A}_3, \underline{A}_4$ be the columns of a 4×4 matrix A and suppose that $2\underline{A}_1 + 2\underline{A}_4 = \underline{A}_3 - 3\underline{A}_2$. Write down a solution to the equation $A\underline{x} = \underline{0}$ of the form

$$\underline{x} = \begin{pmatrix} 6 \\ ? \\ ? \\ ? \end{pmatrix}$$

$$2A_1 + 2A_4 = A_3 - 3A_2$$

$$2A_1 + 3A_2 - A_3 - 2A_4 = 0$$

$$\Rightarrow \quad x = \begin{bmatrix} 2 \\ 3 \\ -1 \\ -2 \end{bmatrix}$$

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Question:

Let $\underline{A}_1, \underline{A}_2, \underline{A}_3, \underline{A}_4$ be the columns of a 4×4 matrix A and suppose that $2\underline{A}_1 + 2\underline{A}_4 = \underline{A}_3 - 3\underline{A}_2$. Write down a solution to the equation $A\underline{x} = \underline{0}$ of the form

$$\underline{x} = \begin{pmatrix} 6 \\ ? \\ ? \\ ? \end{pmatrix}$$

$$\begin{bmatrix} \uparrow & \uparrow & \uparrow & \uparrow \\ A_1 & A_2 & A_3 & A_4 \\ \downarrow & \downarrow & \downarrow & \downarrow \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2A_1 + 2A_4 = A_3 - 3A_2$$

$$2A_1 + 3A_2 - A_3 + 2A_4 = 0$$

$$\Rightarrow x = \begin{bmatrix} 2 \\ 3 \\ -1 \\ +2 \end{bmatrix}$$

$$\begin{bmatrix} 6 \\ 9 \\ -3 \\ 6 \end{bmatrix} \checkmark \text{ Answer}$$

nmer-Mterm-Answers.pdf

asses.in

$$\begin{bmatrix} \uparrow & \uparrow & \uparrow & \uparrow \\ A_1 & A_2 & A_3 & A_4 \\ \downarrow & \downarrow & \downarrow & \downarrow \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A\,x = 0$$

$$x = \begin{bmatrix} 2 \\ 3 \\ -1 \\ 2 \end{bmatrix}$$

if x is the sol^n of $Ax = 0$

kx is also Solution of

$$Ax = 0$$

$$A\,x = 0$$

$$A\,\underline{(kx)} = 0$$

Answer: $\underline{x} = \begin{pmatrix} 6 \\ 9 \\ -3 \\ 6 \end{pmatrix}$

Comments: You are told that $2\underline{A}_1 + 3\underline{A}_2 - \underline{A}_3 + 2\underline{A}_4 = \underline{0}$. Since $A\underline{x} = x_1\underline{A}_1 + \cdots + x_n\underline{A}_n$ the equation $2\underline{A}_1 + 3\underline{A}_2 - \underline{A}_3 + 2\underline{A}_4 = \underline{0}$ is telling you that

$$A\begin{pmatrix} 2 \\ 3 \\ -1 \\ 2 \end{pmatrix} = \underline{0}.$$

But you are asked for a solution to $A\underline{x}$ in which $x_1 = 6$. If $A\underline{x} = \underline{0}$, then $Ac\underline{x} = \underline{0}$ for all $c \in \mathbb{R}$. Hence

$$3\begin{pmatrix} 2 \\ 3 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 9 \\ -3 \\ 6 \end{pmatrix}$$

is a solution to $A\underline{x} = \underline{0}$.

Question:

Let A be the 2×2 matrix such that

$$A \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ 0 \end{pmatrix}.$$

Then which of the following is/are Non-Trivial solutions to $Ax = 0$?

(A) $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ (B) $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ (C) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (D) $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$

Q73 at https://sites.math.washington.edu//~smith/Teaching/308/2012-Summer-Mterm-Answers.pdf

Question:

Let A be the 2×2 matrix such that

$$A\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}x_2\\0\end{pmatrix}.$$

$A\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}2\\0\end{bmatrix}$

Then which of the following is/are Non-Trivial solutions to $Ax = 0$?

(A) $\begin{pmatrix}1\\1\end{pmatrix}$ ~~(B)~~ $\begin{pmatrix}1\\0\end{pmatrix}$ (C) $\begin{pmatrix}0\\1\end{pmatrix}$ ~~(D)~~ $\begin{pmatrix}-1\\0\end{pmatrix}$ E. $\begin{bmatrix}0\\0\end{bmatrix}$

Not answer because it is trivial

$A\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$

$A\begin{bmatrix}5\\9\end{bmatrix} = \begin{bmatrix}9\\0\end{bmatrix}$

$A\begin{bmatrix}x_1\\0\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

Q73 at https://sites.math.washington.edu//~smith/Teaching/308/2012-Summer-Mterm-Answers.pdf

Question:

Let A be the 2×2 matrix such that

$$A \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ 0 \end{pmatrix}.$$

Then which of the following is/are Non-Trivial solutions to $Ax = 0$?

(A) $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ (B) $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ (C) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (D) $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$x_2 = 0$

$\Rightarrow$

$A \begin{bmatrix} x_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$A \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Q73 at https://sites.math.washington.edu//~smith/Teaching/308/2012-Summer-Mterm-Answers.pdf

Question: T/F

The equation $A\underline{x} = \underline{b}$ has a solution if and only if $\underline{b}$ is linear combination of the rows of A.

Q66 at https://sites.math.washington.edu//~smith/Teaching/308/2012-Summer-Mterm-Answers.pdf

Question: T/F

The equation $A\underline{x} = \underline{b}$ has a solution if and only if $\underline{b}$ is linear combination of the rows of A.

non sense

false

Q66 at https://sites.math.washington.edu//~smith/Teaching/308/2012-Summer-Mterm-Answers.pdf

The equation $A\underline{x} = \underline{b}$ has a solution if and only if $\underline{b}$ is linear combination of the rows of A.

False.

Q66 at https://sites.math.washington.edu//~smith/Teaching/308/2012-Summer-Mterm-Answers.pdf

Question:

The equation $A\underline{x} = \underline{b}$ has a solution if and only if $\underline{b}$ is a linear combination of the columns of A.

Q18 at https://sites.math.washington.edu//~smith/Teaching/308/2012-Summer-Mterm-Answers.pdf

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Question:

The equation $A\underline{x} = \underline{b}$ has a solution if and only if $\underline{b}$ is a linear combination of the columns of A.

True

$$\begin{bmatrix} & \\ & \end{bmatrix}\begin{bmatrix} \\ \end{bmatrix} = \underset{b}{\begin{bmatrix} \\ \end{bmatrix}}$$

Q18 at https://sites.math.washington.edu//~smith/Teaching/308/2012-Summer-Mterm-Answers.pdf

The equation $A\underline{x} = \underline{b}$ has a solution if and only if $\underline{b}$ is a linear combination of the columns of A.

True.

Q18 at https://sites.math.washington.edu//~smith/Teaching/308/2012-Summer-Mterm-Answers.pdf

Solutions of $Ax = b$

b is linear combination of columns of A ?

Yes

There is always a solution
(Unique or infinite)

No

There is never a solution
(No Solution)

$$Ax = b$$

$$\underbrace{\begin{bmatrix} & & \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} \\ \\ \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} \\ \\ \end{bmatrix}}_{b}$$

Solutions of $Ax = b$

b is linear combination of columns of A ?

it will help to solve many many questions

Yes

There is always a solution
(Unique or infinite)

(Columns are LI)

Yes

Unique Soln

No

Infinite Soln

No

There is never a solution
(No Solution)

$$\underbrace{\begin{bmatrix} \uparrow & \uparrow & \uparrow \\ a_1 & a_2 & a_3 \\ \downarrow & \downarrow & \downarrow \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} \\ \\ \\ \end{bmatrix}}_{b}$$

$$c_1 a_1 + c_2 a_2 + c_3 a_3 = \boxed{b}$$

$$c_1 a_1 + c_2 a_2 + c_3 a_3 = \boxed{b}$$

$$\begin{bmatrix} \\ \\ \\ \end{bmatrix} \begin{bmatrix} \\ \\ \\ \end{bmatrix} \begin{bmatrix} \\ \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

LI $\Rightarrow$ only one combination

$$5\begin{bmatrix}1\\0\\0\end{bmatrix}+9\begin{bmatrix}0\\1\\0\end{bmatrix}+7\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}5\\9\\7\end{bmatrix}$$

only one combination is possible

T/F

Assume A is an $m \times n$ matrix. If $A\vec{x} = \vec{b}$ has a solution for every vector $\vec{b}$ then the columns of A fill $\mathbf{R^m}$.

T/F

Assume A is an $m \times n$ matrix. If $A\vec{x} = \vec{b}$ has a solution for every vector $\vec{b}$ then the columns of A fill $\mathbf{R^m}$.

R^m R^m

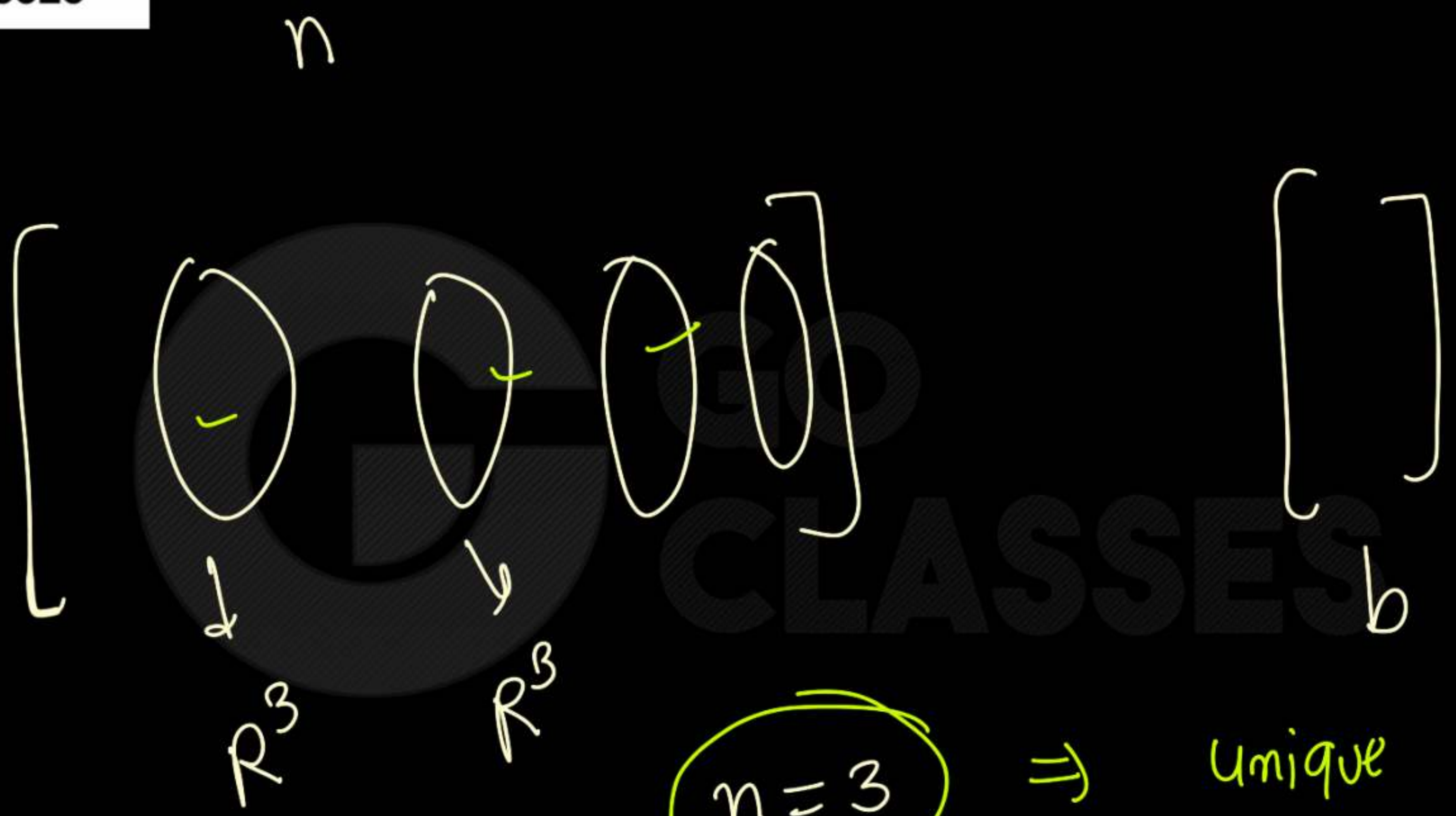
$m \times n$ x b

$n = m$

$$\overset{n}{\begin{bmatrix} \vdots & \vdots & \vdots \\ \end{bmatrix}} \qquad \begin{bmatrix} \ \\ \end{bmatrix}_{b}$$

$\mathbb{R}^3$ $\mathbb{R}^3$

$n = 3 \Rightarrow$ unique Sol^n



$n=3 \Rightarrow$ unique Soln

$n=4 \Rightarrow$ LD vectors $\Rightarrow$ inf Soln

$$\begin{array}{cc} \begin{bmatrix} \; \cdot \; & \; \cdot \; & \end{bmatrix} & \begin{bmatrix} \; \end{bmatrix} \\ \quad R^3 \quad R^3 & b \end{array}$$

we need 3 LI vectors

R^3 R^3

b

we need 3 LI vectors

$R^3 \quad R^3 \quad R^3 \quad R^3 \qquad\qquad b$

we have inf sol^n for every b.

T/F

Assume A is an $m \times n$ matrix. If $A\vec{x} = \vec{b}$ has ~~a solution~~ for every vector $\vec{b}$ then the columns of A fill $\mathbf{R^m}$.

TRUE

$$\begin{bmatrix} \vdots & \cdots \end{bmatrix}_{m \times n} \begin{bmatrix} \; \end{bmatrix}_{x} = \begin{bmatrix} \; \end{bmatrix}_{b}$$

R^m R^m

$n \geq m$

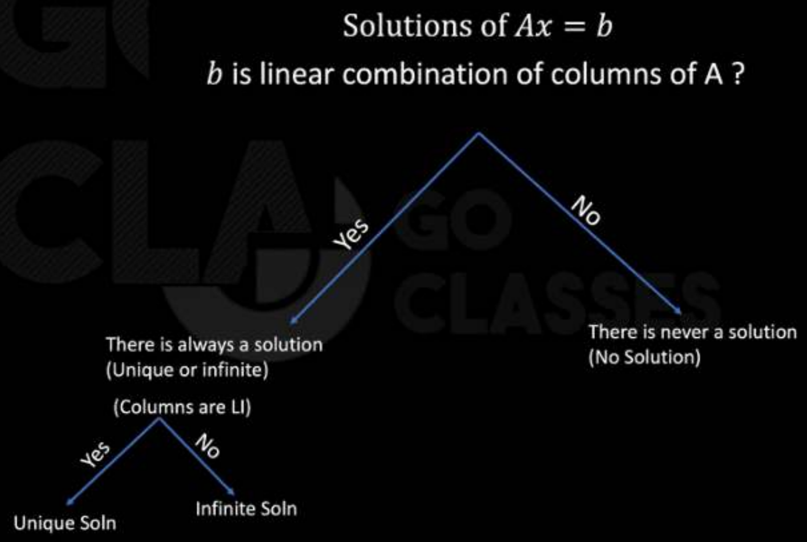
T/F

Assume A is a 4×4 matrix.

It is possible that $A\vec{x} = \vec{b}$ has infinite solutions for some $\vec{b}$ and unique solution for other $\vec{b}$.

$\boxed{\text{unique} \Rightarrow \text{LI}}$

inf $\Rightarrow$ LD



T/F

false

Assume A is a 4×4 matrix.

It is possible that $A\vec{x} = \vec{b}$ has infinite solutions for some $\vec{b}$ and unique solution for other $\vec{b}$.

$$\underbrace{\begin{bmatrix} \text{LD} & & \\ & & \\ & & \end{bmatrix}_{4\times 4}}_{A} \begin{bmatrix} \\ \\ \\ \end{bmatrix} = \underbrace{\begin{bmatrix} \\ \\ \\ \end{bmatrix}}_{b}$$

if inf sol^n are there then col^{n}s must be LD and if we have LD col^{n}s then unique sol^{n}s are not possible.

T/F

Assume A is a 4×4 matrix.

It is possible that $A\vec{x} = \vec{b}$ has infinite solutions for some $\vec{b}$ and no solution for other $\vec{b}$.

T/F

TRUE

Assume A is a 4×4 matrix.

It is possible that $A\vec{x} = \vec{b}$ has infinite solutions for some $\vec{b}$ and no solution for other $\vec{b}$.

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{bmatrix}}_{LP} \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 5 \end{bmatrix}$$

inf sol^n for this b

No sol^n for this b.

T/F

Assume A is a 4×4 matrix.

It is possible that $A\vec{x} = \vec{b}$ has unique solution for some $\vec{b}$ and no solution for other $\vec{b}$.

T/F

false

Assume A is a 4×4 matrix.

It is possible that $A\vec{x} = \vec{b}$ has unique solution for some $\vec{b}$ and no solution for other $\vec{b}$.

$$\underbrace{\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}_{4\times y}}_{LI} \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

unique sol^{n}s $\Rightarrow$ LI col^{n}s $\Rightarrow$ 4 col^{n}s in R^4 $\Rightarrow$ they fill space $\Rightarrow$ sol^n for all b.

T/F

Assume A is a 4×5 matrix.

It is possible that $A\vec{x} = \vec{b}$ has infinite solutions for some $\vec{b}$ and no solution for other $\vec{b}$.

T/F

true

Assume A is a 4×5 matrix.

It is possible that $A\vec{x} = \vec{b}$ has infinite solutions for some $\vec{b}$ and no solution for other $\vec{b}$.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 & 4 \end{bmatrix} \begin{bmatrix} \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 5 \end{bmatrix}$$

R^4 R^4 R^4 x b

inf sol^n

No sol^n

T/F

Assume A is a 4×5 matrix.

It is possible that $A\vec{x} = \vec{b}$ has infinite solutions for some $\vec{b}$ and unique solution for other $\vec{b}$.

T/F

false

Assume A is a 4×5 matrix.

It is possible that $A\vec{x} = \vec{b}$ has infinite solutions for some $\vec{b}$ and unique solution for other $\vec{b}$.

R^4

these vectors has to be LD hence unique sol^n is not possible

T/F

Assume A is a 4×5 matrix.

It is possible that $A\vec{x} = \vec{b}$ has unique solution for some $\vec{b}$ and no solution for other $\vec{b}$.

T/F

Assume A is a 4×5 matrix.

It is possible that $A\vec{x} = \vec{b}$ has unique solution for some $\vec{b}$ and no solution for other $\vec{b}$.

Not possible

false

T/F

Assume A is a 5×4 matrix.

It is possible that $A\vec{x} = \vec{b}$ has infinite solutions for some $\vec{b}$ and no solution for other $\vec{b}$.

T/F

TRUE

Assume A is a 5×4 matrix.

It is possible that $A\vec{x} = \vec{b}$ has infinite solutions for some $\vec{b}$ and no solution for other $\vec{b}$.

$\Rightarrow$ LD

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \\ 5 & 5 & 5 & 5 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 6 \end{bmatrix}$$

R^5

inf soln

No soln

True/ false

Assume A is a 5×4 matrix.

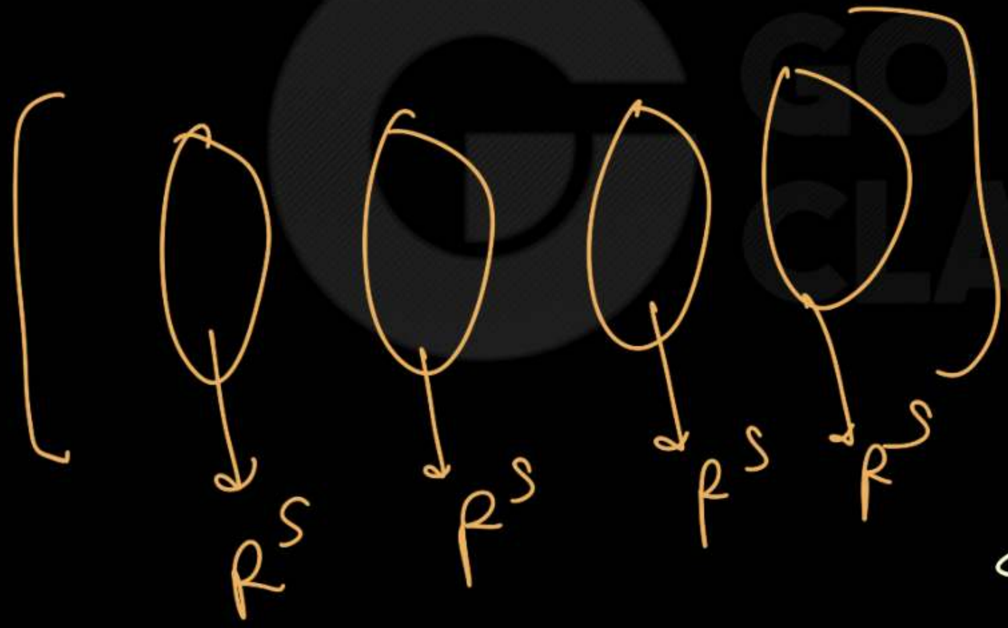
It is possible that $A\vec{x} = \vec{b}$ has unique solution for some $\vec{b}$ and no solution for other $\vec{b}$.

True/false

TRUE

Assume A is a 5×4 matrix.

It is possible that $A\vec{x} = \vec{b}$ has unique solution for some $\vec{b}$ and no solution for other $\vec{b}$.



$\Rightarrow$ LI (bcoz unique sol^n is given) but does not fill the space hence No sol^n for some b.

$$\underbrace{\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix} \begin{bmatrix}0\\1\\0\\0\\0\end{bmatrix} \begin{bmatrix}0\\0\\1\\0\\0\end{bmatrix} \begin{bmatrix}0\\0\\0\\1\\0\end{bmatrix}}_{4 \text{ LI vectors in } R^5} \qquad \begin{bmatrix}2\\3\\4\\5\\0\end{bmatrix} \qquad \begin{bmatrix}3\\2\\5\\7\\9\end{bmatrix}$$

unique sol^n

No sol^n

True/ false

Assume A is a 5×4 matrix.

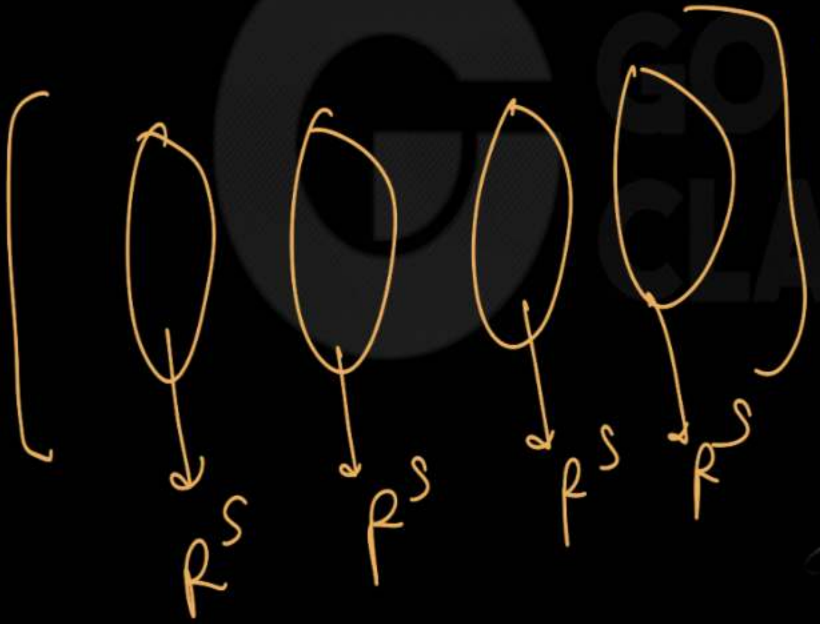
It is possible that $A\vec{x} = \vec{b}$ has infinite solutions for some $\vec{b}$ and unique solution for other $\vec{b}$.

True/ false

false

Assume A is a 5×4 matrix.

It is possible that $A\vec{x} = \vec{b}$ has infinite solutions for some $\vec{b}$ and unique solution for other $\vec{b}$.



LD

LI

can not happen at same time

Question: T/F

The equation $A\underline{x} = \underline{b}$ has a unique solution for all $\underline{b} \in \mathbb{R}^n$ if A is an $n \times n$ matrix with rank n.

Q67 at https://sites.math.washington.edu//~smith/Teaching/308/2012-Summer-Mterm-Answers.pdf

The equation $A\underline{x} = \underline{b}$ has a unique solution for all $\underline{b} \in \mathbb{R}^n$ if A is an $n \times n$ matrix with rank n.

True .

Q67 at https://sites.math.washington.edu//~smith/Teaching/308/2012-Summer-Mterm-Answers.pdf

Question:

Consider a matrix $A = \begin{bmatrix} \uparrow & \uparrow & \cdots & \uparrow \\ a_1 & a_2 & \cdots & a_n \\ \downarrow & \downarrow & \cdots & \downarrow \end{bmatrix}$, where a_i are column vectors in $\mathbb{R}^n$.

Let's assume that for EXACTLY one column vector, say $a_j = 2a_{j+1}$, while for the other columns, this condition does NOT hold.

Now, let's consider the set of linear equations $Ax = b$, where $b = \sum_{i=1}^{n} i^2 a_i$.

Which of the following statements is TRUE for such a system of equations?

A. No Solution

B. Unique Solution

C. Infinitely many solutions

D. None of these

Question:

Simillar to GATE 2017 question

Consider a matrix $A = \begin{bmatrix} \uparrow & \uparrow & \cdots & \uparrow \\ a_1 & a_2 & \cdots & a_n \\ \downarrow & \downarrow & \cdots & \downarrow \end{bmatrix}$, where a_i are column vectors in $\mathbb{R}^n$.

Let's assume that for EXACTLY one column vector, say $a_j = 2a_{j+1}$, while for the other columns, this condition does NOT hold.

col^n s are linearly depend.

Now, let's consider the set of linear equations $Ax = b$, where $b = \sum_{i=1}^{n} i^2 a_i$.

b is a linear combination of col^n of A.

Which of the following statements is TRUE for such a system of equations?

A. No Solution

B. Unique Solution

C. Infinitely many solutions ✓

D. None of these

$b = 1[\ ] + 4[\ ] + 9[\ ]$

Question: True/False

Given two sets of vectors, $\{\underline{u}_1, \ldots, \underline{u}_r\}$ and $\{\underline{v}_1, \ldots, \underline{v}_s\}$, if they fill the same space

(formally $\text{span}\{\underline{u}_1, \ldots, \underline{u}_r\} = \text{span}\{\underline{v}_1, \ldots, \underline{v}_s\}$),

then every vector $\underline{u}_i$ can be expressed as a linear combination of vectors from $\{\underline{v}_1, \ldots, \underline{v}_s\}$, and every vector $\underline{v}_i$ can be expressed as a linear combination of vectors from $\{\underline{u}_1, \ldots, \underline{u}_r\}$.

Q 43 https://sites.math.washington.edu//~smith/Teaching/308/2012-Summer-Mterm-Answers.pdf

Question: True/False

Given two sets of vectors, $\{\underline{u}_1, \ldots, \underline{u}_r\}$ and $\{\underline{v}_1, \ldots, \underline{v}_s\}$, if they fill the same space

(formally $\text{span}\{\underline{u}_1, \ldots, \underline{u}_r\} = \text{span}\{\underline{v}_1, \ldots, \underline{v}_s\}$),

then every vector $\underline{u}_i$ can be expressed as a linear combination of vectors from $\{\underline{v}_1, \ldots, \underline{v}_s\}$, and every vector $\underline{v}_i$ can be expressed as a linear combination of vectors from $\{\underline{u}_1, \ldots, \underline{u}_r\}$.

True

Question: True/False

Every subset of a linearly independent set is linearly independent.

H.W.

Q 38 https://sites.math.washington.edu//~smith/Teaching/308/2012-Summer-Mterm-Answers.pdf

Every subset of a linearly independent set is linearly independent.

True.

Q 38 https://sites.math.washington.edu//~smith/Teaching/308/2012-Summer-Mterm-Answers.pdf

Question: True/False

Every subset of a linearly dependent set is linearly dependent.

H.W.

Q 40 https://sites.math.washington.edu//~smith/Teaching/308/2012-Summer-Mterm-Answers.pdf

Every subset of a linearly dependent set is linearly dependent.

False.

Q 40 https://sites.math.washington.edu//~smith/Teaching/308/2012-Summer-Mterm-Answers.pdf

Question: True/False

Suppose v_1, v_2, and v_3 are linearly dependent vectors in $\mathbf{R}^4$. Then v_1 must be a linear combination of v_2 and v_3.

TRUE FALSE

H.W.

https://chrisj.math.gatech.edu/23s/1553/materials/e2-rev-again-solutions.pdf

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TRUE $\boxed{\text{FALSE}}$

False. Similar to many past examples including an example in the 2.5 slides. Out of pure coincidence, it is nearly identical to #5 in the 2.5-3.1 Supplement. If the set is linearly dependent, we only know that *at least one* of the vectors is a linear combination of the others, not that *every* vector is. For example, the vectors below form a linearly dependent set but v_1 is not a linear combination of v_2 and v_3.

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \end{pmatrix}.$$

Question: True/False

If A is an $n \times n$ matrix and the equation $Ax = b$ has at least one solution for each b in $\mathbf{R}^n$, then A must be invertible.

TRUE FALSE

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$\boxed{\text{TRUE}}$ FALSE

MSQ Question:

Suppose $\{v_1, v_2, v_3, v_4\}$ is a **linearly independent** set of vectors in $\mathbf{R}^4$. Which of the following statements are true? Clearly circle all that apply.

(i) For each b in $\mathbf{R}^4$, the vector equation

$$x_1v_1 + x_2v_2 + x_3v_3 + x_4v_4 = b$$

is consistent and has a unique solution.

(ii) It is possible that the set $\{v_1, v_2, v_3\}$ is linearly dependent.

(iii) $\text{Span}\{v_1, v_2, v_3, v_4\} = \mathbf{R}^4$.

b) (3 points) Suppose $\{v_1, v_2, v_3, v_4\}$ is a **linearly independent** set of vectors in $\mathbf{R}^4$. Which of the following statements are true? Clearly circle all that apply.

$\boxed{\text{(i)}}$ For each b in $\mathbf{R}^4$, the vector equation

$$x_1v_1 + x_2v_2 + x_3v_3 + x_4v_4 = b$$

is consistent and has a unique solution.

(ii) It is possible that the set $\{v_1, v_2, v_3\}$ is linearly dependent.

$\boxed{\text{(iii)}}$ $\text{Span}\{v_1, v_2, v_3, v_4\} = \mathbf{R}^4$.

Question:

possible	impossible	
○	○	Two nonzero vectors $\vec{v}_1, \vec{v}_2$ such that $\{\vec{v}_1, \vec{v}_2\}$ is linearly independent and $\{\vec{v}_1 - \vec{v}_2, \vec{v}_1 + \vec{v}_2\}$ is linearly dependent.
○	○	A matrix A of size 4×3 with linearly dependent columns.

https://sbarone7.math.gatech.edu/ma1554s23_exam1_key.pdf

possible	impossible	
○	●	Two nonzero vectors $\vec{v}_1, \vec{v}_2$ such that $\{\vec{v}_1, \vec{v}_2\}$ is linearly independent and $\{\vec{v}_1 - \vec{v}_2, \vec{v}_1 + \vec{v}_2\}$ is linearly dependent.
●	○	A matrix A of size 4×3 with linearly dependent columns.

MCQ Question:

(c) (2 points) If A is an $m \times 5$ matrix and $A\vec{x} = 0$ has a unique solution, then which of the following is true. *Select only one.*

- ○ $m \geq 5$
- ○ $m = 5$
- ○ $m \leq 5$
- ○ m can be any natural number

LI

SLI fix

$R^m \geq 5$

LT

https://sbarone7.math.gatech.edu/ma1554s23_exam1_key.pdf

(c) (2 points) If A is an $m \times 5$ matrix and $A\vec{x} = 0$ has a unique solution, then which of the following is true. *Select only one.*

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https://sbarone7.math.gatech.edu/ma1554s23_exam1_key.pdf

Question: True/False

True/False Questions

For each of the following statements, determine if they are true or false. You do not have to show your work or justify your answer. You just have to write "true" or "false."

A. (2 points) For any $m \times n$ matrix A, if $m > n$ then the columns of A must be linearly independent.

B. (2 points) Any set of distinct standard basis vectors in $\mathbb{R}^n$ is linearly independent.

C. (2 points) For any 20×24 matrix A, the equation $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution.

D. (2 points) For any vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ in $\mathbb{R}^n$, if $\{\mathbf{v}_1, \mathbf{v}_2\}$ is linearly dependent, then so is $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

https://nmmull.github.io/CS132-F23/Midterm/v1-Midterm/midterm-solution.pdf

A. False

B. True

C. True

D. True

Q.48	Which of the following statements is/are **TRUE**? **Note**: $\mathbb{R}$ denotes the set of real numbers.
(A)	There exist $\boldsymbol{M} \in \mathbb{R}^{3\times 3}, \boldsymbol{p} \in \mathbb{R}^3$, and $\boldsymbol{q} \in \mathbb{R}^3$ such that $\boldsymbol{M}\mathbf{x} = \boldsymbol{p}$ has a unique solution and $\boldsymbol{M}\mathbf{x} = \boldsymbol{q}$ has infinite solutions.
(B)	There exist $\boldsymbol{M} \in \mathbb{R}^{3\times 3}, \boldsymbol{p} \in \mathbb{R}^3$, and $\boldsymbol{q} \in \mathbb{R}^3$ such that $\boldsymbol{M}\mathbf{x} = \boldsymbol{p}$ has no solutions and $\boldsymbol{M}\mathbf{x} = \boldsymbol{q}$ has infinite solutions.
(C)	There exist $\boldsymbol{M} \in \mathbb{R}^{2\times 3}, \boldsymbol{p} \in \mathbb{R}^2$, and $\boldsymbol{q} \in \mathbb{R}^2$ such that $\boldsymbol{M}\mathbf{x} = \boldsymbol{p}$ has a unique solution and $\boldsymbol{M}\mathbf{x} = \boldsymbol{q}$ has infinite solutions.
(D)	There exist $\boldsymbol{M} \in \mathbb{R}^{3\times 2}, \boldsymbol{p} \in \mathbb{R}^3$, and $\boldsymbol{q} \in \mathbb{R}^3$ such that $\boldsymbol{M}\mathbf{x} = \boldsymbol{p}$ has a unique solution and $\boldsymbol{M}\mathbf{x} = \boldsymbol{q}$ has no solutions.

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$M_{3\times3}$

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(D)	There exist $\boldsymbol{M} \in \mathbb{R}^{3\times 2}, \boldsymbol{p} \in \mathbb{R}^3$, and $\boldsymbol{q} \in \mathbb{R}^3$ such that $\boldsymbol{M}\mathbf{x} = \boldsymbol{p}$ has a unique solution and $\boldsymbol{M}\mathbf{x} = \boldsymbol{q}$ has no solutions.

$M_{3\times 3}$

$[\,0\,0\,0\,][\;] = [\;]$

$[\,0, 0\,]$ R^3 R^3