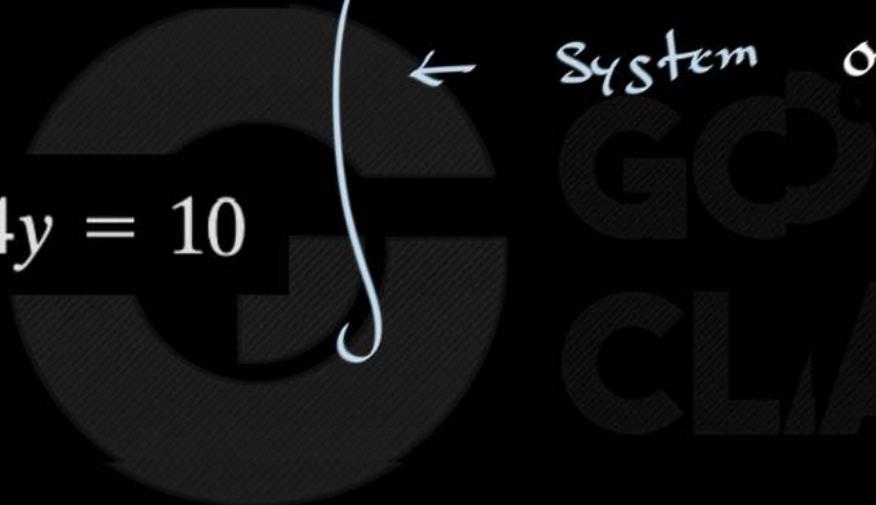


New Topic -

System of linear equations

$$\begin{array}{l} x - 3y = -5 \\ 2x + 4y = 10 \end{array} \left. \begin{array}{c} \\ \end{array} \right\} \leftarrow \text{System of linear equations}$$



$$x^2 + y = 5 \quad \times$$

$$x - xy = 4 \quad \times$$

$$x_1^2 + x_2^2 = 4 \quad \times$$

$$-\sqrt{x} + y = 2 \quad \times$$

← Non linear equations

CLASSES

variables

$$\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{array}$$

System of linear equations

coefficient

The diagram illustrates a system of linear equations. It shows four equations arranged vertically. The first three equations have three variables each, while the fourth equation has two variables. The variables are circled in red, and the coefficients are circled in orange. A bracket on the right side groups all four equations together, labeled 'System of linear equations'. A curved arrow points from the word 'variables' at the top left down towards the variables in the equations. Another curved arrow points from the word 'coefficient' at the bottom left up towards the circled coefficients in the equations.

CLASSES

$$x + y - z = 8$$

$$x - 2y + z = 3$$

$$x + 3y + 2z = 7$$

Example 1

$$x, y, z = ?$$

$$2x - y + z = -4$$

$$-x + y + 3z = -7$$

$$x + 3y - 4z = 22$$

Example 2

Why we want to solve

System of linear equations

?



Application 1: Price prediction

	x_1	x_2	x_3	x_4	x_5	x_6	
	Square feet	BHK	Number of bedrooms	Number of washrooms	Area of dinning room	Year built	Price
House1	1200	2	1	1	300	1990	35
House2	1500	3	2	3	900	2000	40
House3	1100	2	2	2	350	2020	45

House100



Application 1: Price prediction

	x_1	x_2	x_3	x_4	x_5	x_6
Square feet	BHK	Number of bedrooms	Number of washrooms	Area of dinning room	Year built	
House1	1200	2	1	1	300	1990
House2	1500	3	2	3	400	2006
House3	1100	2	2	2	350	2020

Price
35
40
45

House100

$$x_1, x_2, \dots, x_6 =$$

$$\left\{ \begin{array}{l} 1200 x_1 + 2x_2 + 1x_3 + 1x_4 + 300x_5 + 1990x_6 = 35 \\ 1500 x_1 + 3x_2 + 2x_3 + \dots = 40 \\ \dots \end{array} \right.$$

$$= 45$$

Application 1: Price prediction

	x_1	x_2	x_3	x_4	x_5	x_6
Square feet	BHK	Number of bedrooms	Number of washrooms	Area of dinning room	Year built	
House 1	1200	2	1	1	300	1990
House 2	1500	3	2	3	400	2006
House 3	1100	2	2	2	350	2020

Price
35
40
45



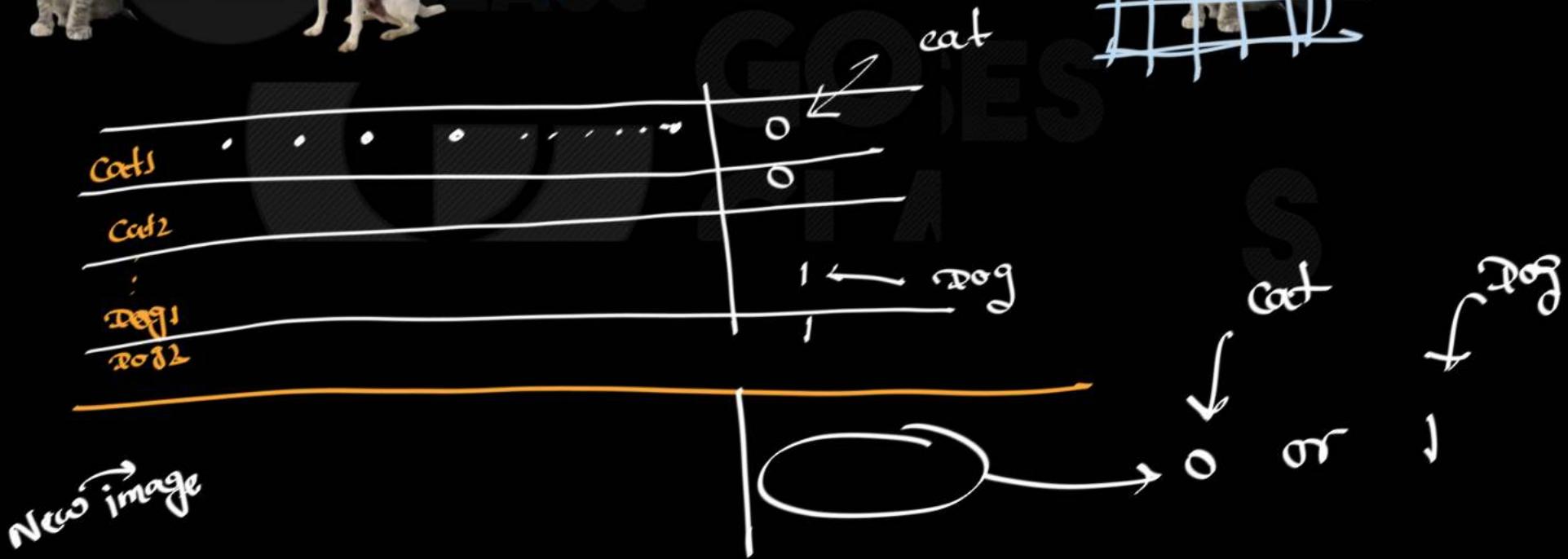
$$x_1, x_2, \dots, x_6 =$$

$$1550x_1 + 3x_2 + 2x_3 + 3x_4 + 500x_5 + 2022x_6 = ?$$

Application 2: Image Classification



Application 2: Image Classification

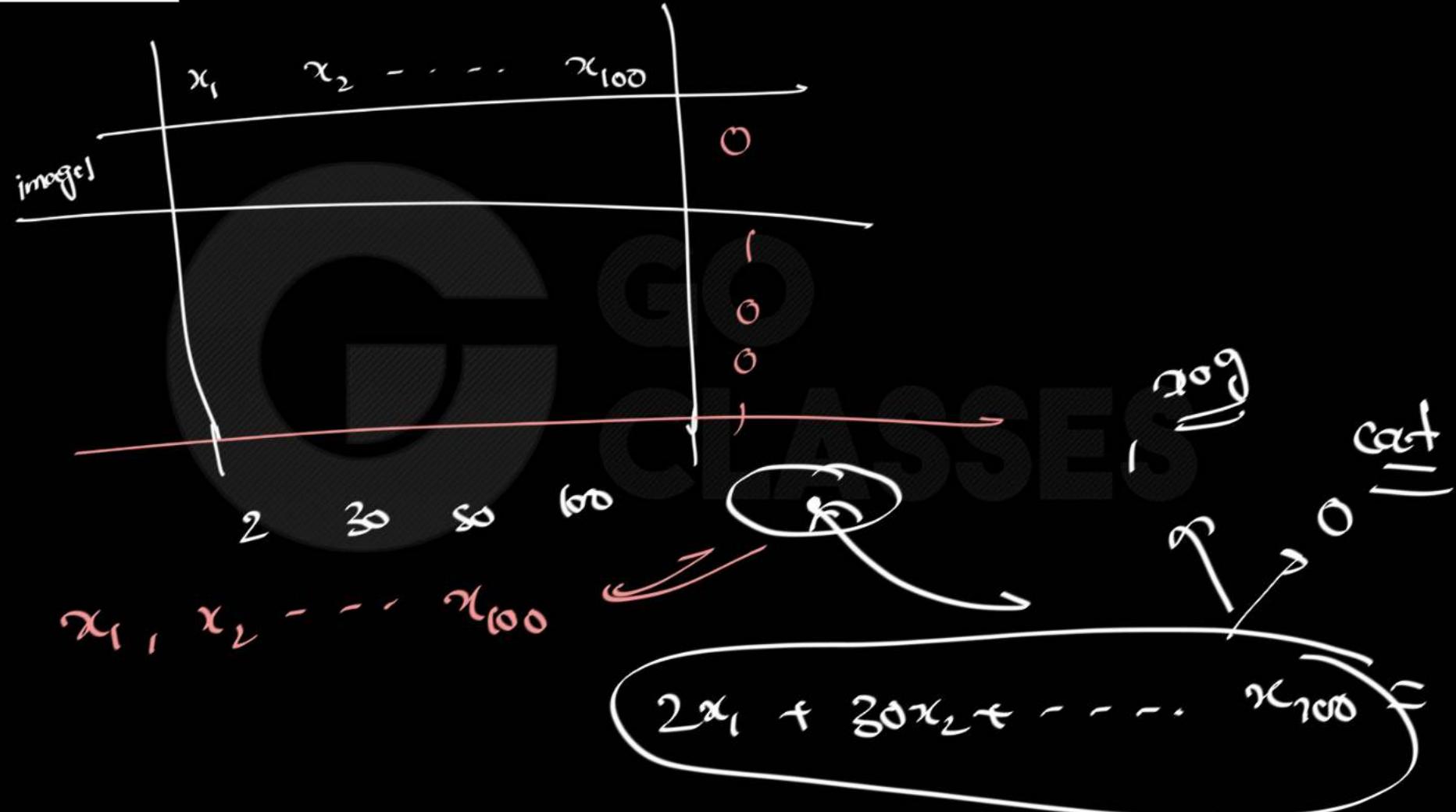




08	02	22	97	58	15	00	60	00	75	04	05	07	78	52	12	50	77	91	00
49	49	99	40	17	81	18	57	60	87	17	40	98	43	69	44	05	56	62	00
81	49	31	73	55	79	14	29	93	71	40	67	55	88	30	03	49	13	36	65
52	70	95	23	04	60	11	42	69	21	68	56	01	32	56	71	37	02	36	91
22	31	16	71	51	62	03	89	41	92	36	54	22	40	40	28	66	33	13	80
24	47	33	60	99	03	45	02	44	75	33	53	78	36	84	20	35	17	12	50
32	98	81	28	64	23	67	10	26	38	40	67	59	54	70	66	18	38	64	70
67	26	20	68	02	62	12	20	95	63	94	39	63	08	40	91	66	49	94	21
24	55	58	05	66	73	99	26	97	17	78	78	96	83	14	88	34	89	63	72
21	36	23	09	75	00	26	44	20	45	35	14	00	61	33	97	34	31	33	95
78	17	53	28	22	75	31	67	15	94	03	80	04	62	16	14	09	53	56	92
16	39	05	42	96	35	31	47	55	58	88	24	00	17	54	24	36	29	85	57
86	56	00	48	35	71	89	07	05	44	44	37	44	60	21	58	51	54	17	58
19	80	81	61	05	94	47	69	28	73	92	13	86	52	17	77	04	89	55	40
04	52	08	13	97	35	99	16	07	97	57	32	16	26	26	79	33	27	98	66
04	46	68	87	57	62	20	72	03	46	33	67	46	55	12	32	63	93	53	69
04	42	16	73	35	95	39	11	24	94	72	18	08	46	29	32	40	62	76	36
20	69	31	41	72	30	23	88	34	65	99	69	82	67	59	85	74	04	36	16
20	73	31	29	78	31	90	01	74	31	49	71	46	84	31	16	23	57	05	54
01	70	51	71	83	51	54	69	16	92	33	48	61	43	52	01	89	47	48	00

What the computer sees





System of linear equations

- Why solve System of linear equations ?
- Geometric Interpretation
- Understanding $Ax = b$ intuitively and finding the number of solution
- A step by step method to find Solution for $Ax=b$ (Gaussian Elimination)
 - Rank
 - Parametric form of solution

The Matrix form of equations

$$x + 2y + 3z = 6$$

$$2x + 5y + 2z = 4$$

$$6x - 3y + z = 2$$

$$Ax = b \quad \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 2 \\ 6 & -3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 2 \\ 6 & -3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = 2 \text{ times column 3} = \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}.$$



CLASSES

$$Ax = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}$$

 CLASSES

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$



Two equations
Two unknowns

$$\begin{cases} x - 2y = 1 \\ 3x + 2y = 11 \end{cases}$$

Matrix equation

$$\underline{Ax = b}$$

$$\begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}.$$

for our
understanding

$$x \begin{bmatrix} 1 \\ 3 \end{bmatrix} + y \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix} \text{ S}$$

$$x + 2y + 3z = 4$$

$$3x + 4y + z = 5$$

$$2x + y + 3z = 6$$

can be written as

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

This is called coefficient matrix



Geometric interpretation of $Ax = b$

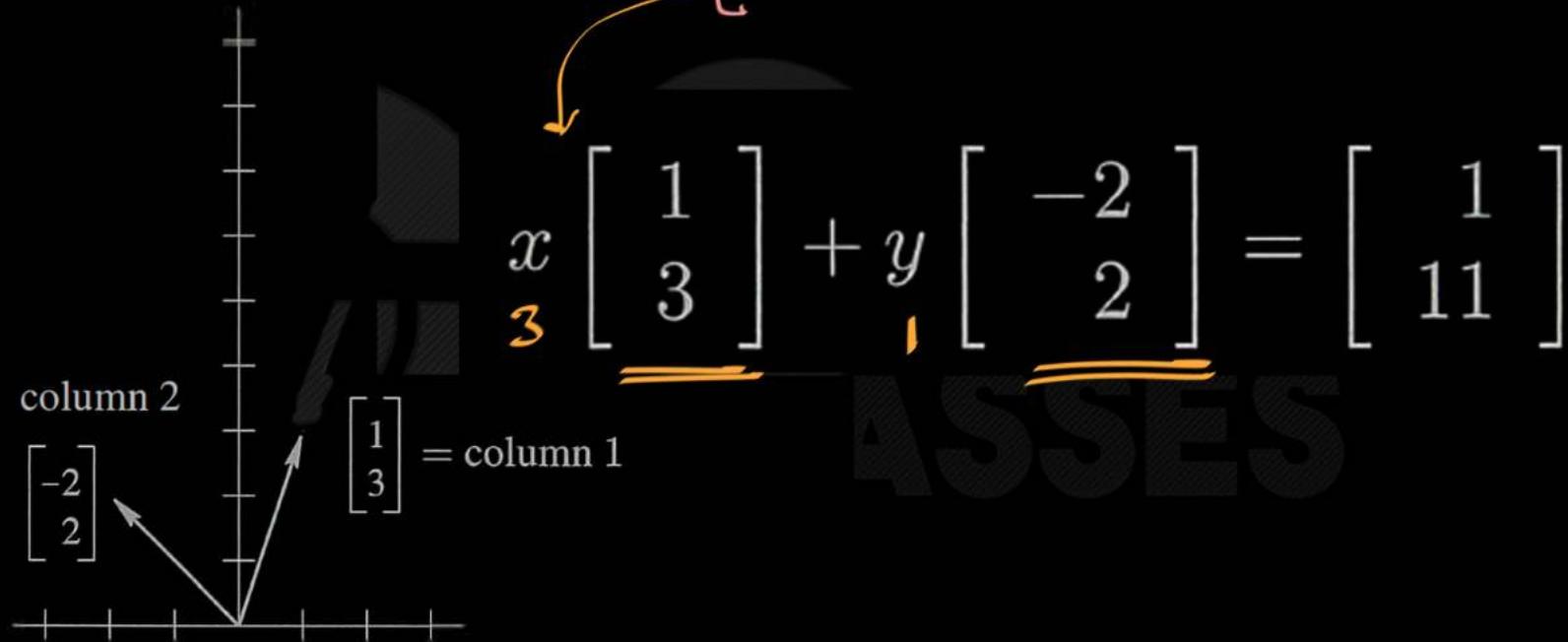
Geometric interpretation(1)

$$\begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$$

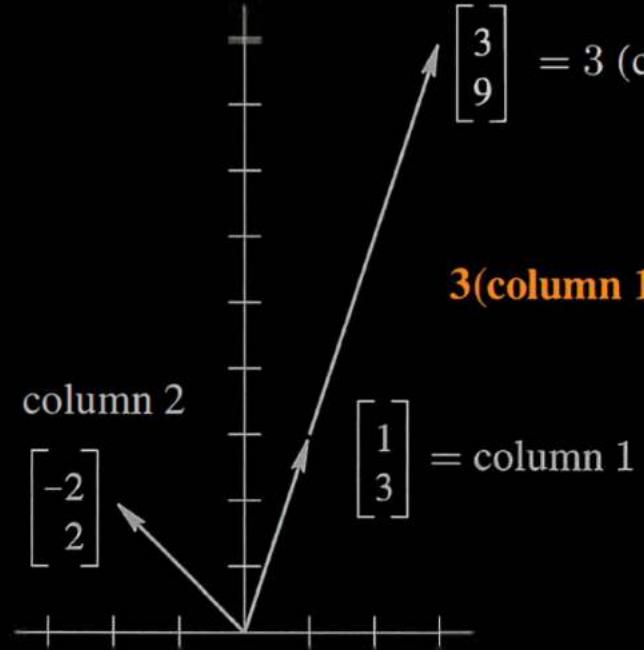
$A x = b$

$$x \begin{bmatrix} 1 \\ 3 \end{bmatrix} + y \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$$

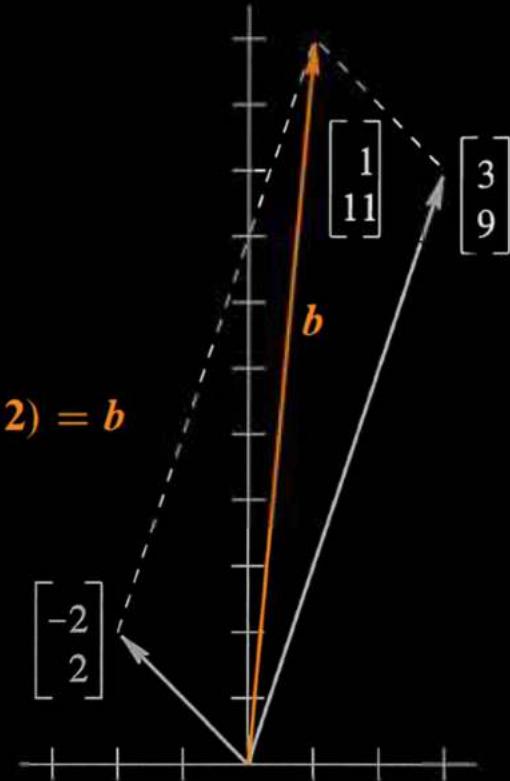


This interpretation is called **Column Picture**



$$\begin{bmatrix} 3 \\ 9 \end{bmatrix} = 3 \text{ (column 1)}$$

$$3(\text{column 1}) + 1(\text{column 2}) = b$$



This interpretation is called **Column Picture**

Geometric interpretation(2)

$$\begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$$

$\overbrace{\quad\quad\quad}^{\text{A } x = b}$

$$\left\{ \begin{array}{l} x - 2y = 1 \\ 3x + 2y = 11 \end{array} \right.$$



column picture

$$x \begin{bmatrix} 1 \\ 3 \end{bmatrix} + y \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$$

Geometric interpretation(2)

$$\begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\xrightarrow{\text{A } x = b}$

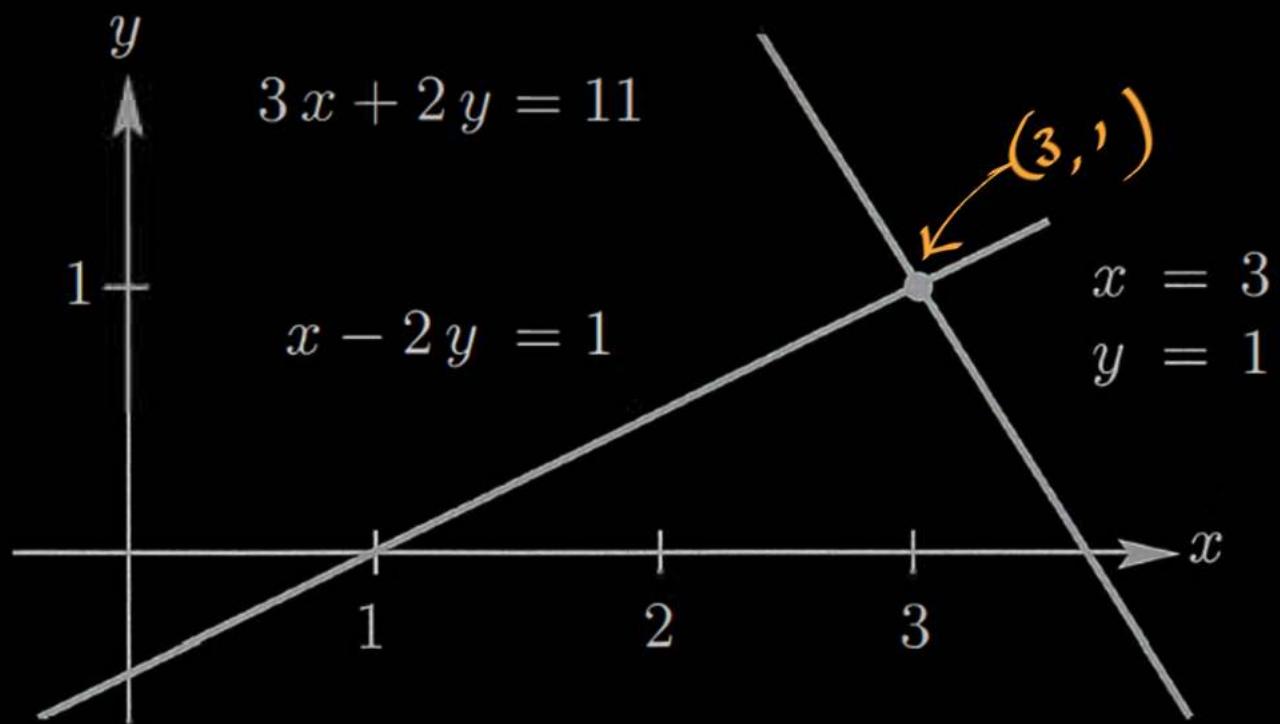
Row picture

$$\left\{ \begin{array}{l} x - 2y = 1 \\ 3x + 2y = 1 \end{array} \right.$$

**Two equations
Two unknowns**

$$\begin{aligned}x &- 2y = 1 \\3x &+ 2y = 11\end{aligned}$$





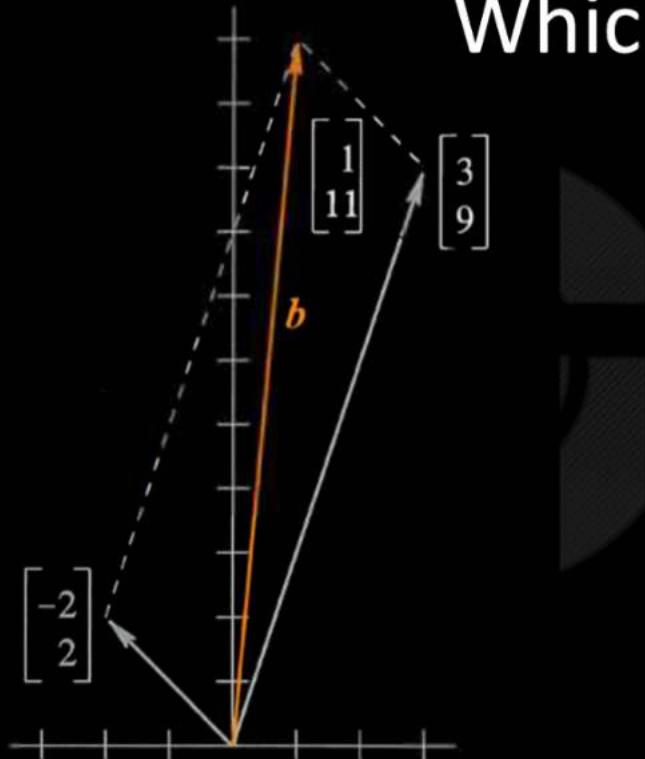
This interpretation is called **Row Picture**



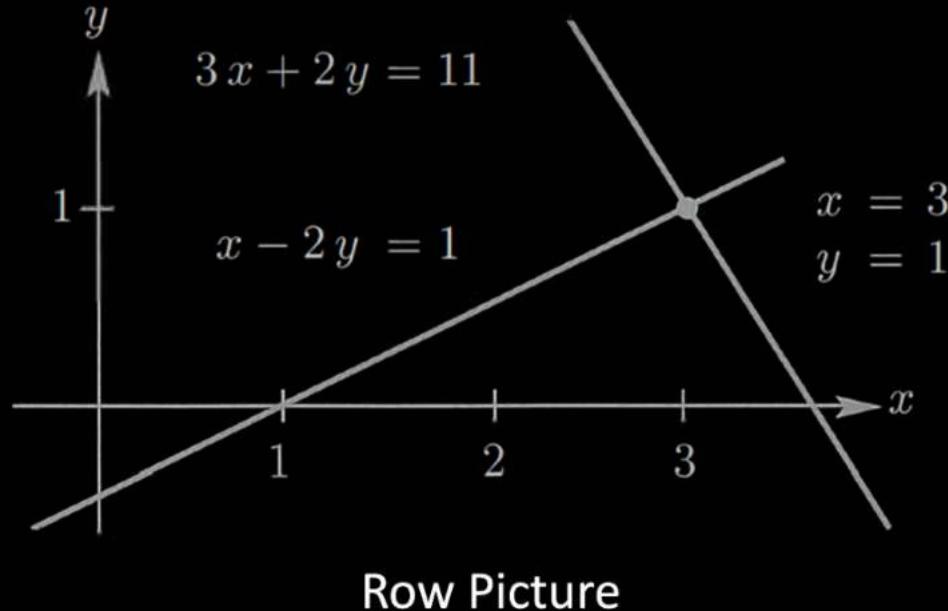
Which Interpretation you like ?



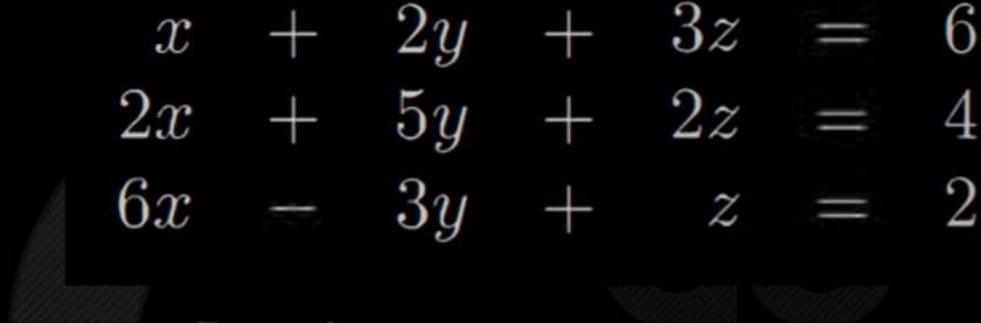
Which Interpretation you like ?



Column Picture

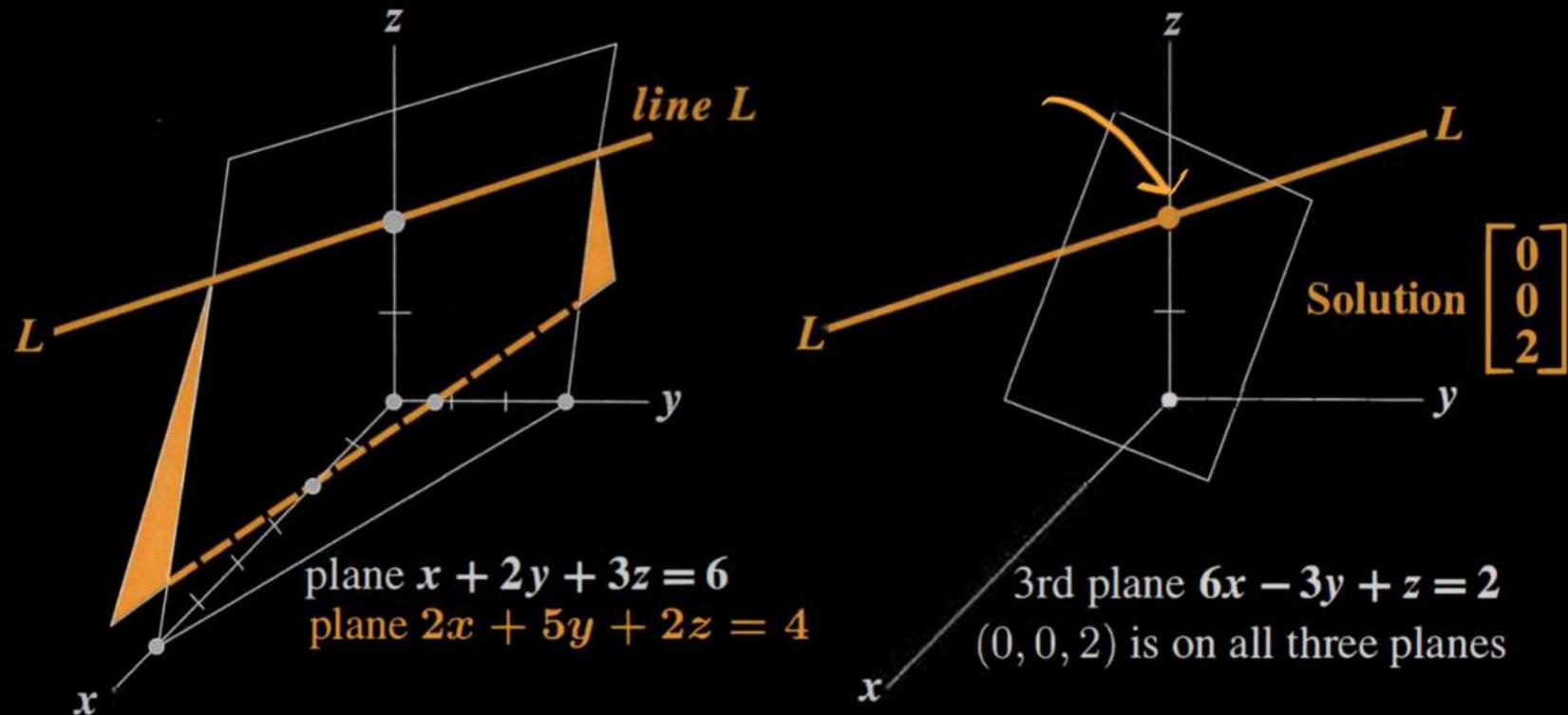


Row Picture

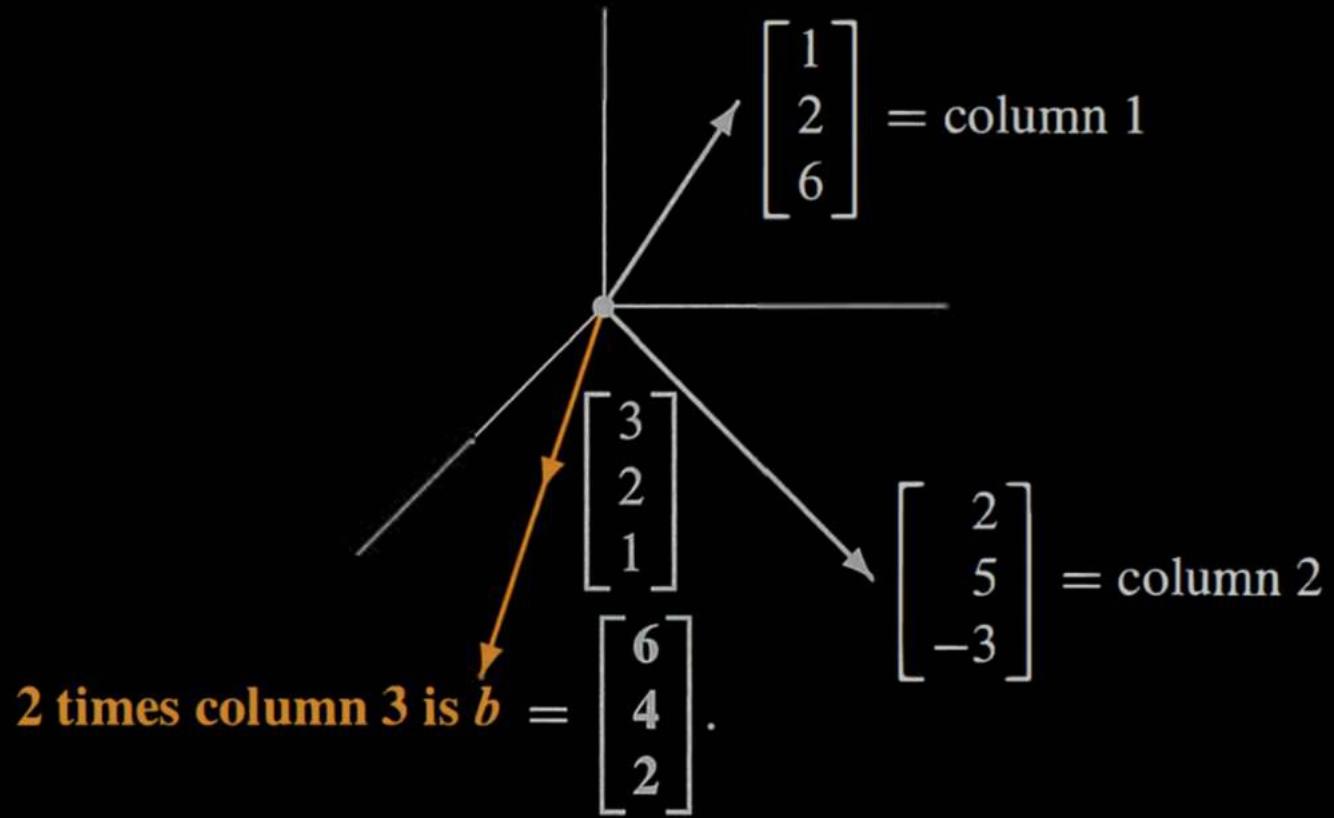

$$\begin{array}{rclcl} x & + & 2y & + & 3z = 6 \\ 2x & + & 5y & + & 2z = 4 \\ 6x & - & 3y & + & z = 2 \end{array}$$

ROW *The row picture shows three planes meeting at a single point.*

COLUMN *The column picture combines three columns to produce $b = (6, 4, 2)$.*



Row picture: Two planes meet at a line *L*. Three planes meet at a point.



Column picture: Combine the columns with weights $(x, y, z) = (0, 0, 2)$.

interesting

Do we always have solution of $Ax = b$?



Do we always have solution?

$$\begin{array}{rcl} x_1 - 2x_2 & = & -1 \\ -x_1 + 3x_2 & = & 3 \end{array}$$

Example1

$$\begin{array}{rcl} x_1 - 2x_2 & = & -1 \\ -x_1 + 2x_2 & = & 3 \end{array}$$

Example2

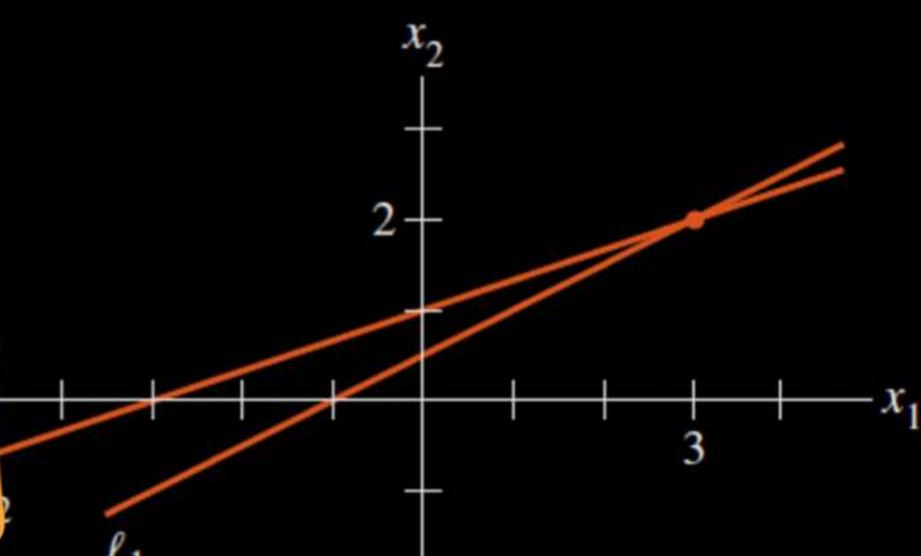
$$\begin{array}{rcl} x_1 - 2x_2 & = & -1 \\ -x_1 + 2x_2 & = & 1 \end{array}$$

Example3

Case 1: Unique solution

$$\begin{aligned}x_1 - 2x_2 &= -1 \\-x_1 + 3x_2 &= 3\end{aligned}$$

Example 1

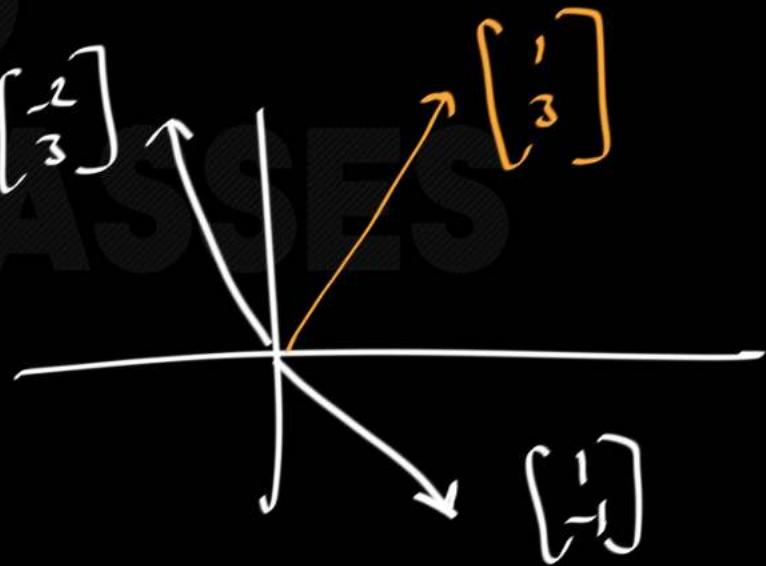
$$\begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad \ell_2$$


Case 1: Unique solution

$$\begin{aligned}x_1 - 2x_2 &= -1 \\-x_1 + 3x_2 &= 3\end{aligned}$$

Example 1

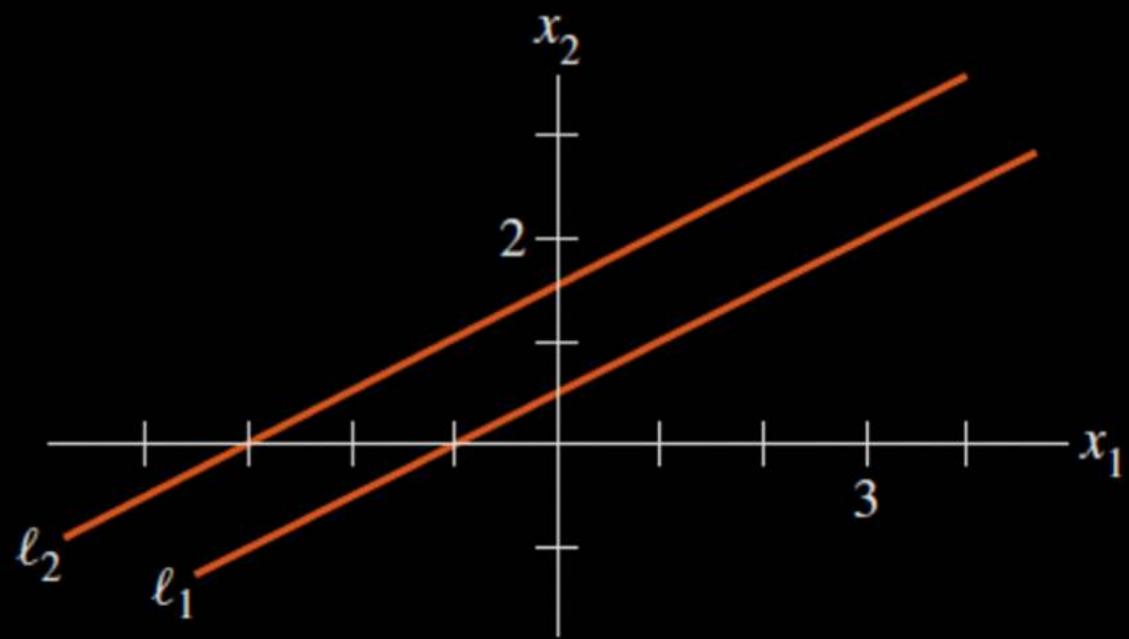
$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$



Case 2: No solution

$$x_1 - 2x_2 = -1$$

$$-x_1 + 2x_2 = 3$$



$$\left\{ \begin{array}{l} x+y = 3 \\ x+y = 5 \end{array} \right.$$

not agreeing
to each other

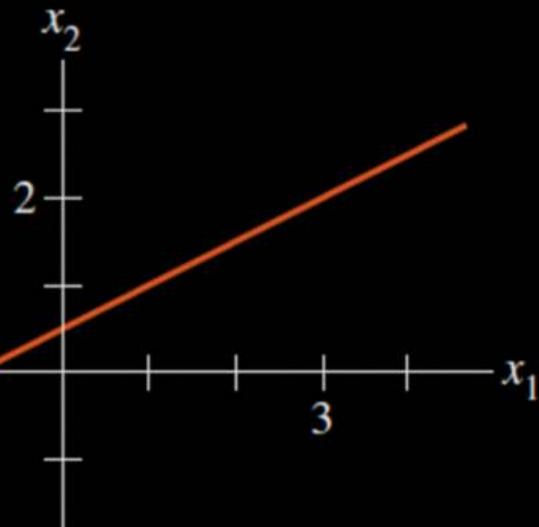
↳ No sol'



Case 3: Infinite Solution

$$\begin{cases} x_1 - 2x_2 = -1 \\ -x_1 + 2x_2 = 1 \end{cases}$$

$$-1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} + 0 \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$



$$x = k$$

$$y = 5 - k \quad (k, 5 - k)$$

$$\left[\begin{array}{l} x + y = 5 \\ 2x + 2y = 10 \end{array} \right]$$

$$x + y = 5$$

x	y	
1	4	$(1, 4)$
2	3	$(2, 3)$
-1	6	$(-1, 6)$

System of linear equations

- Why solve System of linear equations ?
- Geometric Interpretation
- Understanding $Ax = b$ intuitively
- A step by step method to find Solution for $Ax=b$ (Gaussian Elimination)
 - Rank
 - Parametric form of solution

A large, semi-transparent watermark logo for "GO CLASSES" is centered on the slide. The word "GO" is in a smaller, lighter font above the word "CLASSES", which is in a larger, bolder font.

Understanding $Ax = b$ intuitively

$$Ax = b$$

Matrix vector vector

$$\begin{bmatrix} & & \end{bmatrix} \begin{bmatrix} & \\ & \end{bmatrix} = \begin{bmatrix} & \end{bmatrix}$$

A x b

$$\begin{bmatrix}
 \begin{array}{ccc}
 \uparrow & \uparrow & \uparrow \\
 a_1 & a_2 & a_3 \\
 \downarrow & \downarrow & \downarrow
 \end{array}
 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix}] \\] \\] \end{bmatrix} b$$

A x b

$$\underbrace{\begin{bmatrix} \uparrow \\ a_1 \\ \downarrow \end{bmatrix} x_1 + \begin{bmatrix} \uparrow \\ a_2 \\ \downarrow \end{bmatrix} x_2 + \begin{bmatrix} \uparrow \\ a_3 \\ \downarrow \end{bmatrix} x_3}_{\text{CLASSE}}$$

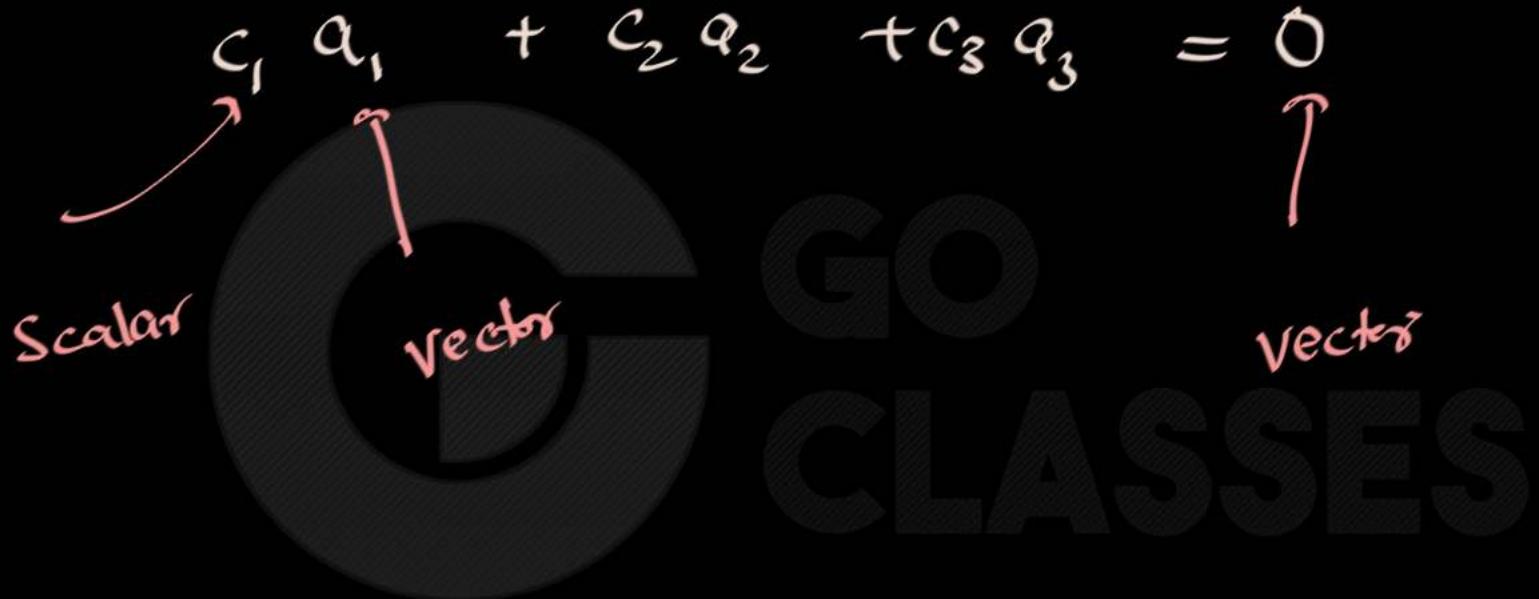
$$\begin{bmatrix} \overset{\uparrow}{a_1} & \overset{\uparrow}{a_2} & \overset{\uparrow}{a_3} \\ \downarrow & \downarrow & \downarrow \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \xrightarrow{\quad} \text{linear combination of columns of } A.$$

A x

$$\left[\begin{array}{c} \overset{\uparrow}{a_1} \\ \downarrow \end{array} \right] x_1 + \left[\begin{array}{c} \overset{\uparrow}{a_2} \\ \downarrow \end{array} \right] x_2 + \left[\begin{array}{c} \overset{\uparrow}{a_3} \\ \downarrow \end{array} \right] x_3$$

$$\underline{Ax = 0} \text{ has some soln.}$$
$$c_1 \begin{bmatrix} 1 \\ a_1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ a_2 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ a_3 \\ 1 \end{bmatrix} = 0$$

$$Ax = 0$$



$$Ax = 0$$

$$\underline{c_1} \alpha_1 + \underline{c_2} \alpha_2 + \underline{c_3} \alpha_3 = 0$$

$$x = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

is there any non zero c_i ?

yes no

L.D

L.I



If $Ax = 0$ has some solution then what can you say about linearly dependency of columns of A.



Nothing can be inferred.

GO
CLASSES

True/False

If $Ax = 0$ has some nontrivial solution then columns of A are linearly dependent.

$$c_1 a_1 + c_2 a_2 + c_3 a_3 = 0$$



True/False

If $Ax = 0$ has some nontrivial solution then columns of A are linearly dependent.

$$c_1 q_1 + c_2 q_2 + c_3 q_3 = 0$$



there is some nonzero c_i

let $c_2 \neq 0$ $q_2 = -\frac{c_1}{c_2} q_1 - \frac{c_3}{c_2} q_3$

True/False

If $Ax = 0$ has ONLY trivial solution then columns of A are linearly dependent.

$$x = \begin{bmatrix} : \\ : \\ : \end{bmatrix}$$

$$c_1 a_1 + c_2 a_2 + c_3 a_3 = 0$$

$$\downarrow$$

if a non zero $c_i = ?$

True/False

If $Ax = 0$ has ONLY trivial solution then columns of A are linearly dependent.

$$x = \begin{bmatrix} : \\ : \\ : \end{bmatrix}$$

$$c_1 a_1 + c_2 a_2 + c_3 a_3 = 0$$

$$\text{if } \exists \text{ a non zero } c_i = ?$$

No

linearly indep.

Question: Suppose a matrix $A_{3 \times 3}$ contains 3 linearly independent columns. What can you say about the solutions to $Ax = b$? (here $b \neq 0$)

YES, solⁿ always exists

$$\begin{bmatrix} - & 0 & 0 \\ - & 0 & 0 \\ - & 0 & 0 \end{bmatrix}_{3 \times 3} \in \mathbb{R}^3 \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \quad A \quad = \quad \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in \mathbb{R}^3$$

3 LI columns in \mathbb{R}^3
 \Rightarrow it will fill whole space
 (we can generate any b)

$$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \] \\ \] \end{bmatrix}_b$$

Diagram illustrating a linear transformation from \mathbb{R}^3 to \mathbb{R}^3 . A vector $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is multiplied by a matrix A (represented by three columns) to produce a vector in \mathbb{R}^3 . The result is labeled b .

$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \Rightarrow 3 \text{ LI vectors in } \mathbb{R}^3$

$$\begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix} = -5 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 7 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Question: Suppose a matrix $A_{3 \times 4}$ contains 3 linearly independent columns. What can you say about the solutions to $Ax = b$? (here $b \neq \underline{0}$)

Redundant

$$\left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \right] = \left[\begin{array}{c} b_1 \\ b_2 \\ b_3 \end{array} \right]$$

$\in \mathbb{R}^3$

always solⁿ

Question: Suppose a matrix $A_{5 \times 4}$ contains 3 linearly independent columns. What can you say about the solutions to $Ax = b$? (here $b \neq 0$)

$$\left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \left[\begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array} \right] = \left[\begin{array}{c} b \\ b \\ b \\ b \\ b \end{array} \right]$$

depends on b . if b is a linear combination of cols of A \Rightarrow yes

Redundant

$\epsilon \mathbb{R}^5$

$$\left[\begin{array}{ccc}
 1 & 2 & 3 \\
 2 & 4 & 6 \\
 3 & 6 & 15 \\
 4 & 9 & 12 \\
 5 & 10 & 15
 \end{array} \right] = \left[\begin{array}{c}
 5 \\
 10 \\
 15 \\
 20 \\
 25
 \end{array} \right]$$

L.I. \Rightarrow
 Colⁿs

Yes solⁿ exists
 $\left[\begin{array}{c} 1 \\ 0 \\ 6 \\ 0 \end{array} \right]$

$$\left[\begin{array}{ccc}
 1 & 2 & 3 \\
 2 & 4 & 6 \\
 3 & 6 & 15 \\
 4 & 9 & 12 \\
 5 & 10 & 15 \\
 \hline
 \end{array} \right] \left[\begin{array}{c}
 5 \\
 10 \\
 15 \\
 20 \\
 25 \\
 \hline
 \end{array} \right] = \left[\begin{array}{c}
 - \\
 - \\
 - \\
 - \\
 - \\
 \hline
 \end{array} \right]$$

L.I. Colⁿs

Not a linear
 combination
 of these 3



Question: Suppose a matrix $A_{5 \times 2}$ contains 2 linearly independent columns. What can you say about the solutions to $Ax = b$? (here $b \neq 0$)

$$\left[\begin{array}{cc} 0 & 0 \\ \vdots & \vdots \end{array} \right] \in \mathbb{R}^5 \quad \left[\begin{array}{c} 0 \\ \vdots \\ 0 \end{array} \right] = \left[\begin{array}{c} 0 \\ \vdots \\ 0 \end{array} \right]$$

may or may not

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \\ 4 & 8 \\ 5 & 20 \end{bmatrix}$$

$$\begin{bmatrix} \quad \end{bmatrix} =$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \checkmark$$

No soln

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \\ 4 & 8 \\ 5 & 20 \end{bmatrix}$$

$$\begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

=

$$\begin{bmatrix} 3 \\ 6 \\ 9 \\ 12 \\ 24 \end{bmatrix}$$

(assume) not a
linear comb
of these two
cols

① { if you can fill space }

Always solⁿ

② { if you can not fill space }
may ¹ or may not be

{ in fⁿ if we have n LI col's then only we can fill space }



Question: [Suppose a matrix $A_{2 \times 5}$ contains 3 linearly independent columns.] What can you say about the solutions to $Ax = b$? (here $b \neq 0$)

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\epsilon \mathbb{R}^2$

NOT even possible
bcuz in \mathbb{R}^2 , we can
not have 3 LI cols



Linear Algebra

$Ax = 0$ always have one trivial solution which is $x = 0$.





Consider a matrix A with dimension $m \times n$. For system $Ax = b$, what can you say about below statement ?

(Note that b may or may not be zero)

A & b

Statement: if $m < n$ then $Ax = b$ always have solution (True/False)

$$\left[\begin{array}{ccc} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right] \in \mathbb{R}^3 \quad \left[\begin{array}{c} \quad \\ \quad \\ \quad \end{array} \right] = \left[\begin{array}{c} \quad \\ \quad \\ \quad \end{array} \right]$$

3×4



Consider a matrix A with dimension $m \times n$. For system $Ax = b$, what can you say about below statement ?
(Note that b may or may not be zero)

Statement: if $m < n$ then there exist such system $Ax = b$ which has solution even if b is nonzero vector. (True/False)

$$\xrightarrow{(A,b)}$$

$$\begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

\checkmark \checkmark

$\in \mathbb{R}^2$



Consider a matrix A with dimension $m \times n$. For system $Ax = b$, what can you say about below statement ?
(Note that b may or may not be zero)

Statement: if $m > n$ then none of the system $Ax = b$ has solution
(True/False) 3 2

A hand-drawn diagram on a black background. At the top right, there is a yellow circle containing the text "A₁ b". An arrow points from this circle to a large, faint watermark-like text "CLASSES" in the center. Below this, a 3x2 matrix is shown in white: $\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$. To the right of the matrix is an equals sign (=). To the right of the equals sign is a vertical bracket [] with a horizontal underline underneath it. Below the matrix, the text "3x2" is written. At the bottom left, the text "E R³" is written above a horizontal line, with a bracket [] drawn around the first two columns of the matrix. A curved arrow originates from the "E R³" text and points towards the first two columns of the matrix.



Consider a matrix A with dimension $m \times n$. For system $Ax = b$, what can you say about below statement ?

(Note that b may or may not be zero)

Statement: for $m < n$ there is a matrix A which gives solution of $Ax = b$ for any given b (True/False)

2 LI columns to ALWAYS have soln

$$\begin{bmatrix} (1) & (0) \\ (0) & (1) \end{bmatrix}_{2 \times 3} \begin{bmatrix} \ } \end{bmatrix} = \begin{bmatrix} \ } \end{bmatrix}$$

$\in \mathbb{R}^2$



Consider a matrix A with dimension $m \times n$. For system $Ax = b$, what can you say about below statement ?

(Note that b may or may not be zero)

there is no matrix
which can fill whole
space.

Statement: for $m > n$ there is a matrix A which gives solution of $Ax = b$ for any given b (True/False)

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\in \mathbb{R}^3$



Consider a matrix A with dimension $m \times n$. For system $Ax = b$, what can you say about below statement ?

(Note that b may or may not be zero)

Statement: for $m = n$ there is a matrix A which gives solution of $Ax = b$ for any given b (True/False)

✓

$$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}_{3 \times 3} \in \mathbb{R}^3$$