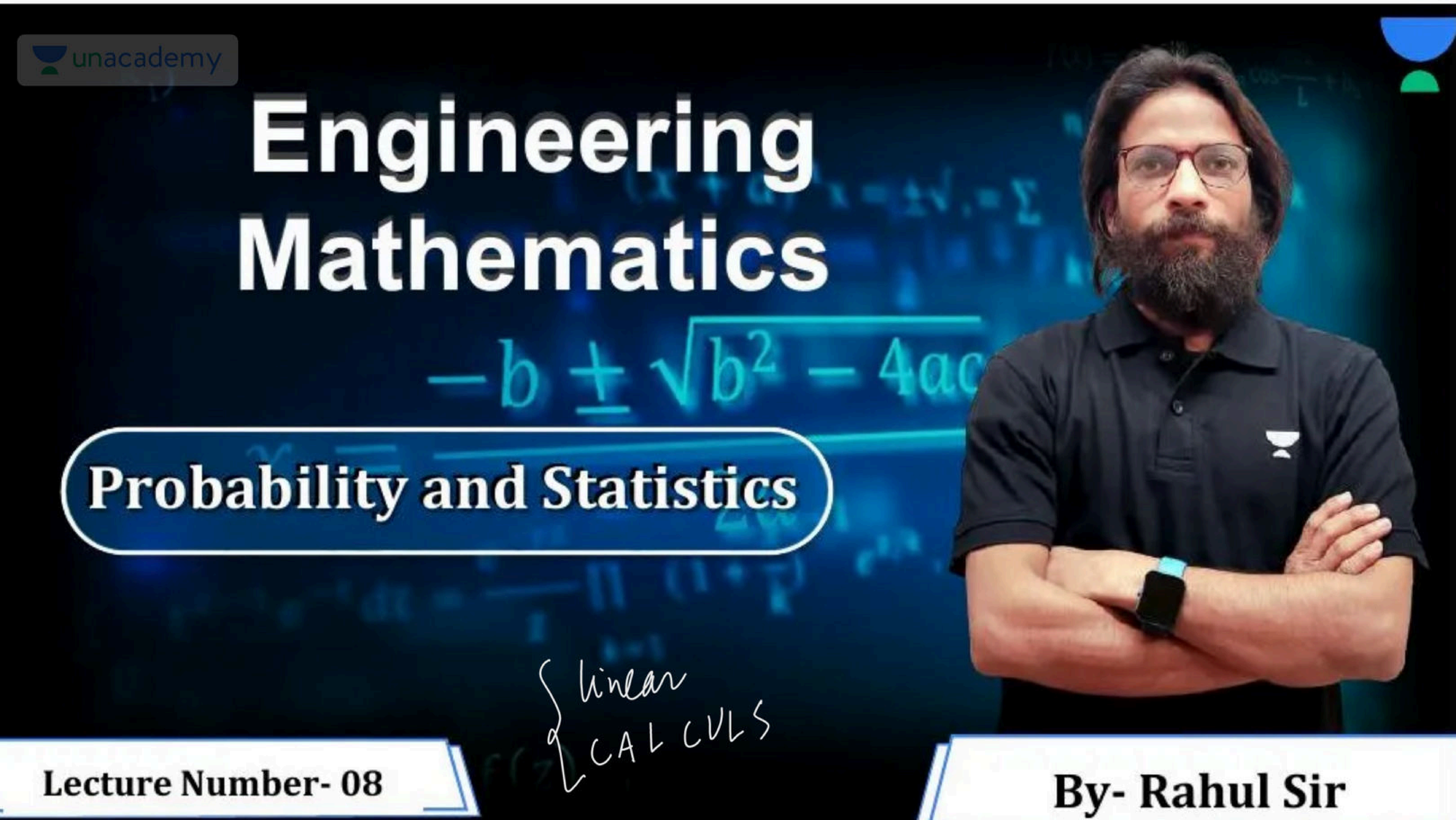
Probability Theory - Part III

Course on Engineering Mathematics for GATE - CSE

Rahul • Lesson 9 • June 23, 2024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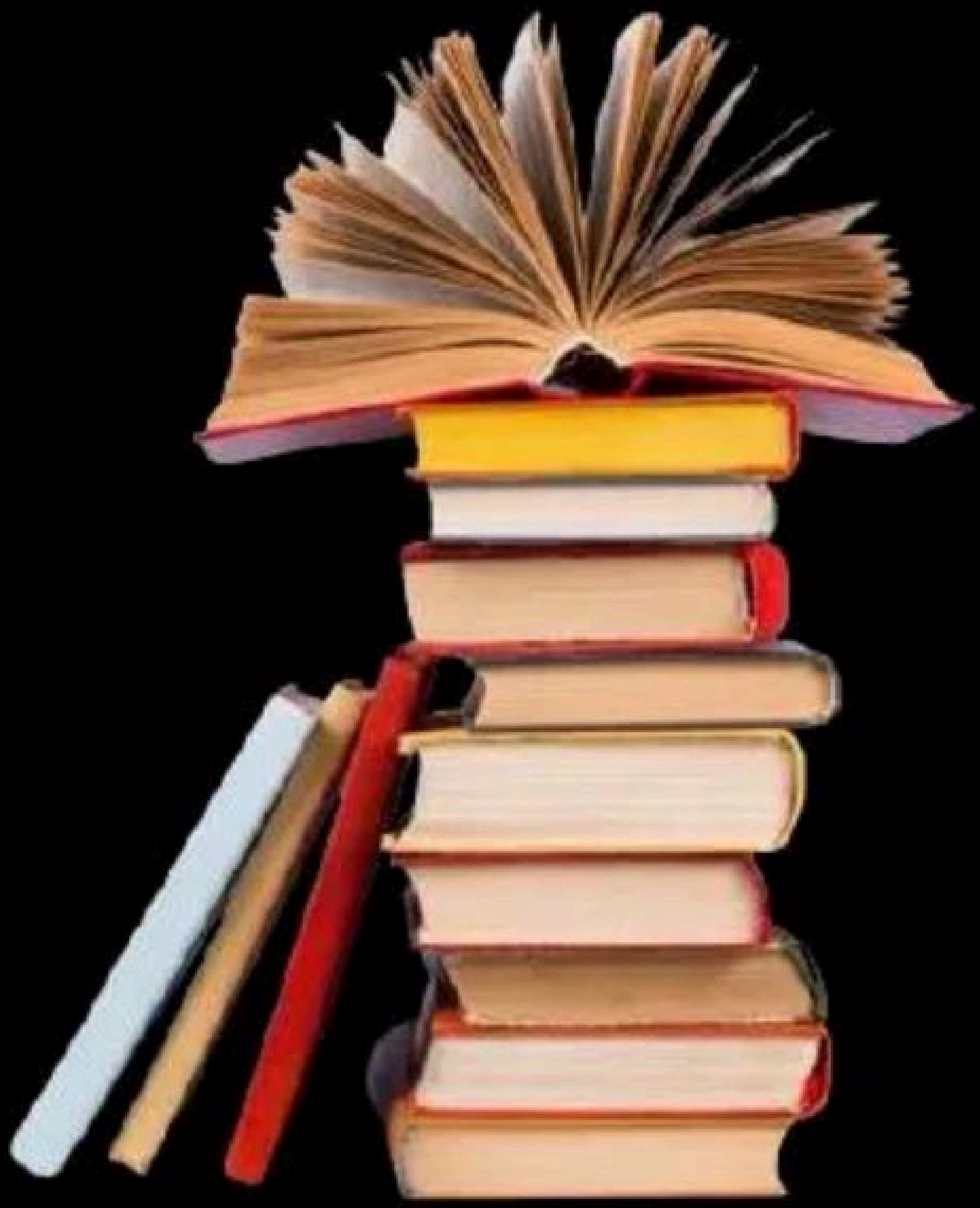
Lecture Number- 08
By- Rahul Sir

Topics *to be covered*



1. Problem solving class

QUESTION

Q. 4 squares are chosen at random on a chessboard. Find the probability that they lie along the same diagonal line adjacent to each other.

done

QUESTION

Q. 2 squares are chosen on a chessboard. Find the probability that they have a side in common.

done

QUESTION

Q. A fair coin is tossed n times, the probability that the difference between the number of heads and tail is (n-3)/is______.

✓ done

QUESTION

Q. Three boys and two girls stand in a queue. The probability that the number of boys ahead of every girl is atleast one more than the number of girls ahead of her is

(A) 1/2

(B) 1/3

(C) 2/3

(D) 3/4

QUESTION

Q. Let w be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2, r_3 are the numbers obtained on the die, then the probability that $\omega^{r1} + \omega^{r2} + \omega^{r3} = 0$ is______.

(A) 1/18

(B) 1/9

(C) 2/9

(D) 1/36

Let w be a complex cube Root of unity $w \neq 0$

In school days:

complex NO

$$\boxed{\begin{array}{c} 1 + w + w^2 = 0 \\ w^3 = 1 \end{array}}$$

Die A Die B Die C

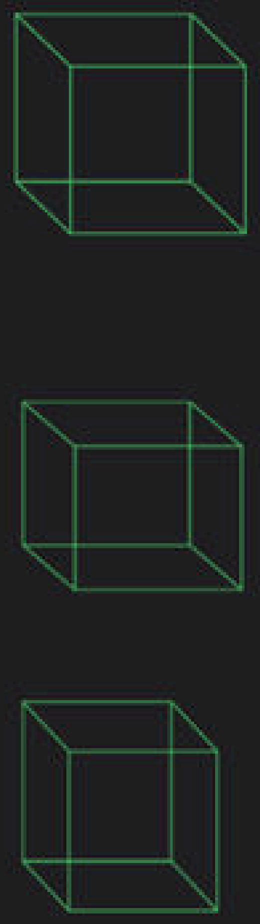
r_1 r_2 r_3

$r_1 \in (1,2,3,4,5,6)$

$r_2 \in (1,2,3,4,5,6)$

$r_3 \in (1,2,3,4,5,6)$

$w^{3k}+w^{3k+1}+w^{3k+2} = 0$

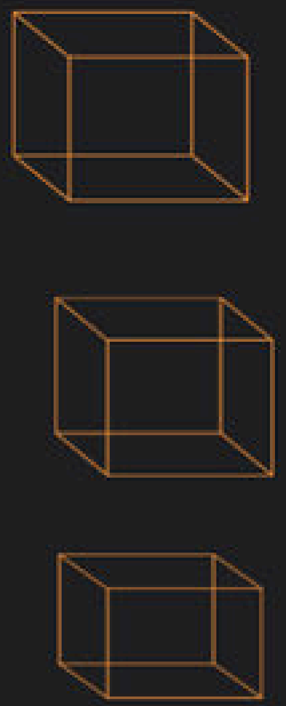


1 r_1

2 r_2

3 r_3

$$w^{r_1}+w^{r_2}+w^{r_3}$$
$$= w^1+w^2+w^3$$
$$= w+w^2+1$$
$$= 0 \checkmark$$



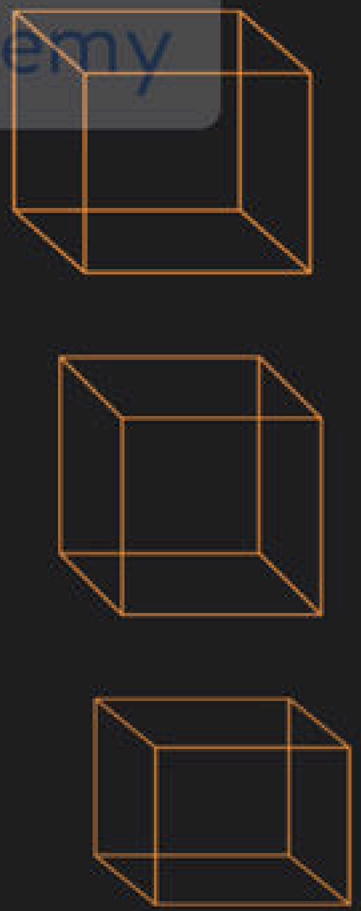
2

1

3

$w^2+w^1+w^3 = 0$

= fav No. of case + ORDER



2

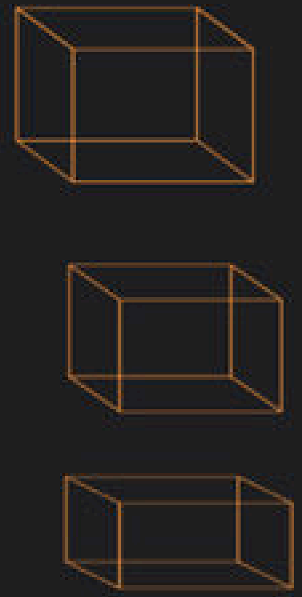
4

6

$$w^2 + w^4 + w^6 = 0$$

$$= w^2 + w^3 \cdot w + w^3 \cdot w^3$$

$$= w^2 + w + 1 = 0$$



4

6

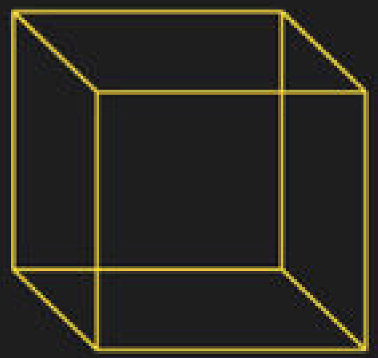
2

$$w^4 + w^6 + w^2 = 0$$

$r_1 = 3k$ $\qquad$ $r_3 = (3k+2)$

$r_2 = 3k+1$

r_1	r_2	r_3
$3k$	$3k+1$	$3k+2$
3, 6,	1, 4,	2, 5,



→ 1

2

3

4

5

6

r_1	r_2	r_3
$3k$	$3k+1$	$3k+2$
3, 6	1, 4,	2, 5

$r_1 = 2$ ways.

$r_2 = 2$ ways.

$r_3 = 2$ ways.

Favorable No. of outcomes $= 2 \times 2 \times 2 \times 3!$

$$P(E) = \frac{2 \times 2 \times 2 \times 3!}{6 \times 6 \times 6} = \frac{2}{9}$$

$r_1 \to 3, 6$

$r_2 \to 1, 4$

$r_3 \to 2, 5$

- 3
  - 1
    - 2 = 312
    - 5 = 315
  - 4
    - 2 = 342
    - 5 = 345
- 6
  - 1
    - 2 = 612
    - 5 = 615
  - 4
    - 2 = 642
    - 5 = 645

8 cases

order

$$= \frac{8 \times 6}{6 \times 6 \times 6}$$

$$= \frac{2}{9}$$ answer

$w^3 = 1 \qquad w^{3k} = 1$

$1 + w + w^2 = 0$

$w^{3k} + w^{3k} w + w^2 w^{3k} = 0$

$$\boxed{w^{3k} + w^{3k+1} + w^{3k+2} = 0}$$

(Multiply with w^{3k})

QUESTION

Three boys and two girls stand in a queue. The probability that the number of boys ahead of every girl is at least one more that the number of girls ahead of her, is:

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$ H.W

(C) $\frac{2}{3}$

(D) $\frac{3}{4}$

QUESTION

If three distinct numbers are chosen randomly from the first 100 natural numbers, then the probability that all three of them are divisible by both 2 and 3, is :

(A) $\frac{4}{55}$

(B) $\frac{4}{35}$

(C) $\frac{4}{33}$

(D) $\frac{4}{1155}$

H.W

QUESTION

H.W

Two numbers are selected randomly from the set S = {1, 2, 3, 4, 5, 6} without replacement one by one. The probability that minimum of the two numbers is less than 4, is:

(A) $\frac{1}{15}$

(B) $\frac{14}{15}$

(C) $\frac{1}{5}$

(D) $\frac{4}{5}$

S = {1, 2, 3, 4, 5, 6}
with out replacement
one by one
min of Two No.
less Than 4.

QUESTION

If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5, equals:

(A) 1/4

(B) 1/7

(C) 1/8

(D) 1/49

"m, n ARE CHOSEN at Random between 1 and 100.
$7^m + 7^n$ is divisible by 3 Equals"

m, n ARE CHOSEN at Random between 1 and 100

$7^m + 7^n$ is $\boxed{\text{divisible by 3 Equals}}$?

✓ m	7^m	$\frac{7^m}{5}$ = Remainder
1	7^1	$= 7^1/5 = 2$
2	7^2	$= 49/5 = 4$
3	7^3	$= 343/5 = 3$
4	7^4	$= 7^4/5 = 1$
5	7^5	$= 7^5/5 = 2$
6	7^6	$= 7^6/5 = 4$
7	7^7	$= 7^7/5 = 3$

m	7^m	$\frac{7^m}{5}$ = Remainder
1	7^1	$= 7^1/5 = 2$
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4	7^4	$= 7^4/5 = 1$
5	7^5	$= 7^5/5 = 2$
6	7^6	$= 7^6/5 = 4$
7	7^7	$= 7^7/5 = 3$

Remainder

2
4
3
1
2
4
3

$7 + 243$

$= 250$

together divisible by 5

m	7^m	$\frac{7^m}{5}$ = Remainder
1	7^1	$= 7^1/5 = 2$
2	7^2	$= 49/5 = 4$
3	7^3	$= 343/5 = 3$
4	7^4	$= 7^4/5 = 1$
5	7^5	$= 7^5/5 = 2$
6	7^6	$= 7^6/5 = 4$
7	7^7	$= 7^7/5 = 3$

cases: (m, n)

(1, 3)	(2, 4)
(5, 7)	(6, 8)
(9, 11)	(10, 12)
(13, 15)	(14, 16)
(17, 19)	(18, 20)
⋮	⋮

$(4k+1, 4k+3)$

$(4k+2, 4k+4)$

25

$$2\left({}^{25}C_1 \times {}^{25}C_1\right) + 2\left({}^{25}C_1 \times {}^{25}C_1\right) \Big/ \text{Total} = \text{Ans}$$

$$\underset{m}{100} \times \underset{n}{100}$$

$7^m + 7^n$ $\qquad$ $\underline{\underline{3, 5}}$

$7^3 + 7^5$

$=$ Remainder. 5 divisible.

$$(m, n) = (1, 3)$$

$$= 2\left({}^{25}C_1 \times {}^{25}C_1\right) + 2\left({}^{25}C_1 \times {}^{25}C_1\right)$$

$$= \underline{\text{Favourable outcomes}}$$

QUESTION

Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently, equals:

(A) 1/2

(B) 7/15

(C) 2/15

(D) 1/3

QUESTION

Fifteen coupons are numbered 1, 2,....,15, respectively. Seven coupons are selected at random one at a time with replacement. The probability that the largest number appearing on a selected coupon is 9, is:

(A) $\left(\frac{9}{16}\right)^6$

(B) $\left(\frac{8}{15}\right)^7$

H.W

(C) $\left(\frac{3}{5}\right)^7$

(D) None of these

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QUESTION

Consider the system of equations $ax + by = 0$, $cx + dy = 0$ where $a, b, c, d \in \{0, 1\}$.

Statement-I: The probability that the system of equations has a unique solution, is 3/8.

det A ≠ 0

Statement-II: The probability that the system of equations has a solution, is 1.

wrong

A. Statement-I is true, Statement-II is also true; Statement-II is the correct explanation of Statement-I

B. Statement-I is true, Statement-II is also true; Statement-II is not the correct explanation of Statement-I.

C. Statement-I is true; Statement-II is false.

D. Statement-I is false; Statement-II is true.

system of Eqn

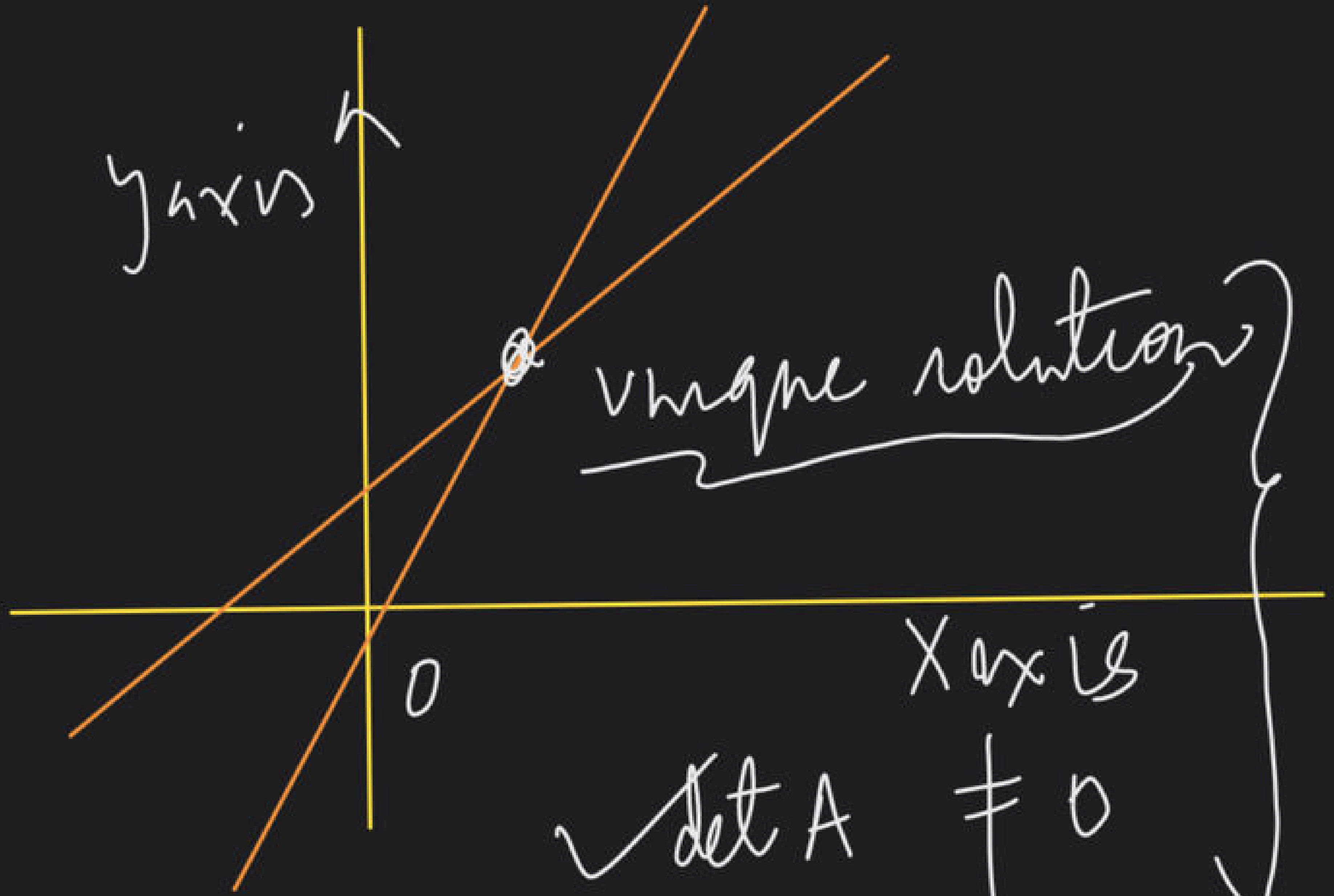
$ax + by = 0$ $AX = 0$

$cx + dy = 0$ always give a solution

$ax + by = 0$

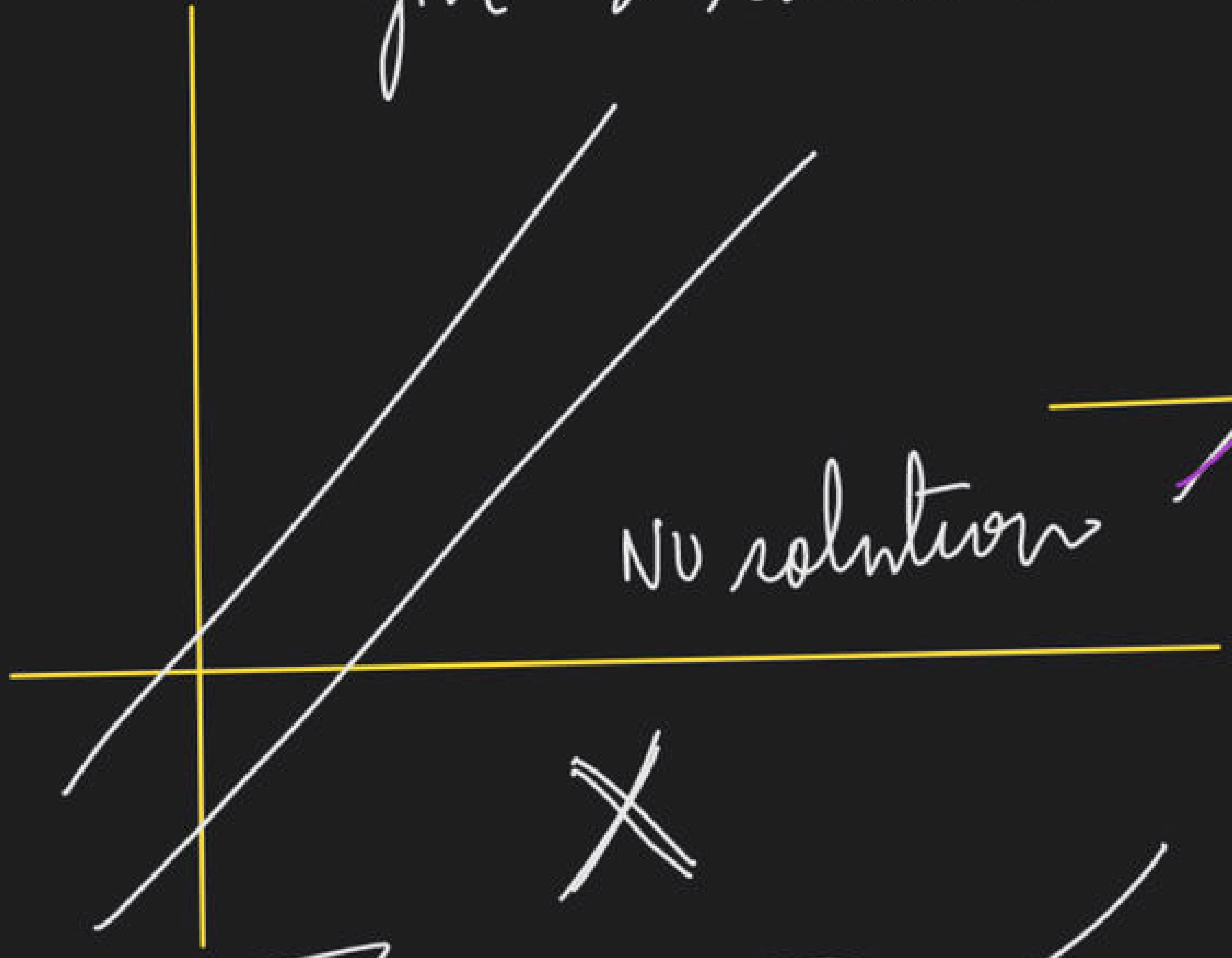
$cx + dy = 0$

$a, b, c, d \in \{0, 1\}$



unique solution

✓ $\det A \neq 0$



No solution

✗

Not allowed

Infinite solution

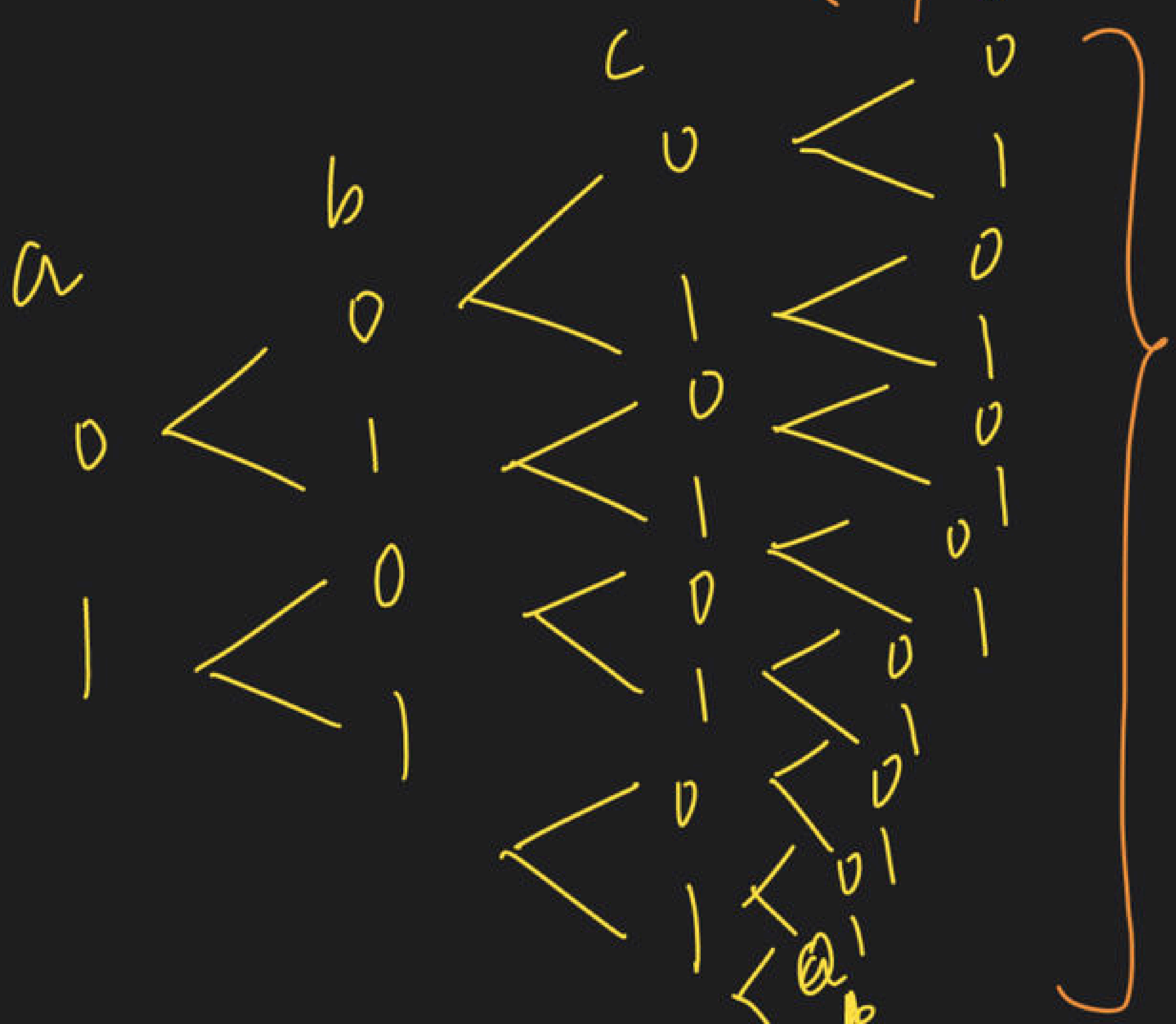
✓ $\boxed{\det A = 0}$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$ax + by = 0$$
$$cx + dy = 0$$

$a \begin{cases} 0 \\ 1 \end{cases}$ $b \begin{cases} 0 \\ 1 \end{cases}$ $c \begin{cases} 0 \\ 1 \end{cases}$ $a, b, c, d \in \{0, 1\}$

$D \begin{cases} 0 \\ 1 \end{cases}$



Total No. of ways: a, b, c, d

$= 2 \times 2 \times 2 \times 2 = 2^4$

$\Rightarrow 16$ ✓

$$\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

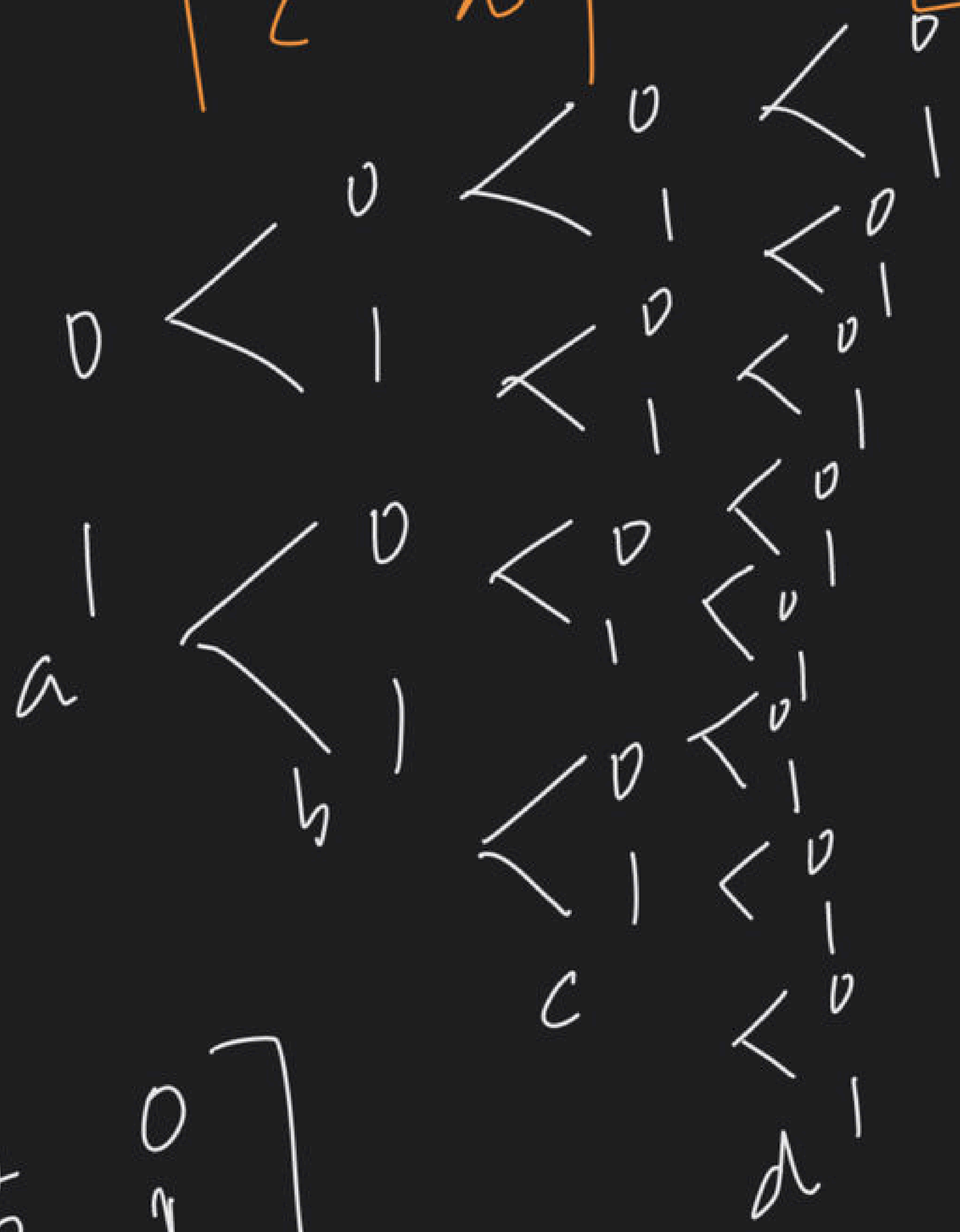
$\boxed{ad - bc \neq 0}$

$\det A \neq 0$

$$A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \underbrace{ad - bc \neq 0}$$

✓ $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

Det $A \neq 0$



Fav. no. of outcomes

$= \underline{\underline{6}}$

$$P(E) = \frac{6}{16} = \frac{3}{8}$$ unique solution

✓ $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$ad - bc \neq 0$ Unique solution $= \frac{6}{16}$

$ad - bc = 0$ Infinite solution $= \frac{10}{16}$ (Not 1)

Have a solution

Statement ① is correct but statement ② is Not correct explanation of statement ①

(B)

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ What is The Prob. p. (positive determinant)

$ad - bc > 0$

$a, b, c, d \in \{-1, 1\}$

#

QUESTION

Three faces of a fair die are yellow, two faces red and one face blue. The die is tossed three times. The probability that the colours, yellow, red and blue, appear in the first, second and the third tosses respectively, is______.

H.W

QUESTION

A box contain 100 tickets numbered 1, 2,....., 100. Two tickets are chosen at random. It is given that the minimum number on the two chosen tickets is not more than 10. The maximum number on them is 5 with probability______.

M.W

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QUESTION

A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that the value of the determinant chosen is positive, is______.

H.W

A deter $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

0, 1

$$P(ad-bc>0) = \frac{n(ad-bc>0)}{\text{Total}} = \frac{}{2^4}$$

QUESTION

If the letters of the word 'ASSASSIN' are written down at random is a row, the probability that no two S's occur together is 1/35.

QUESTION

An unbiased die, with faces numbered 1, 2, 3, 4, 5 and 6 is thrown n times and the list of n numbers showing up is noted. What is the probability that among the numbers 1, 2, 3, 4, 5 and 6 only three numbers appear in this list ?

H.W

QUESTION

Two distinct number x & y are chosen at random from the set $\{1,2,3..... 30\}$. The probability that $x^2 - y^2$ is divisible by 3 is

(A) 3/29

(B) 4/29

(C) 5/29

(D) None of these

Two distinct number x, y Are chosen at random.

$\{1, 2, 3, 4, 5, 6 - - 30\}$

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$(x^2 - y^2)$ is divisible by 3.

$\Downarrow$

$\underbrace{(x-y)}\ \underbrace{(x+y)}$

Diff. of number

sum of number.

divisible by 3

$3k$

$3k+1$

$3k+2$

divisible

$(1-30)$

$1, 2, 3, 4, 5, 6 - - .$

$\boxed{3, 6, 9, 12, - - .30}$ $\boxed{1, 4, 7, - - .}$ $\boxed{2, 5, 8 - - - .}$

10 Number 10 Number 10 Number.

$3k$ $3k+1$ $3k+2$

$k=1$

$k=1$

$k=0$

$3, \underline{\underline{4}}, 5$

$x \rightarrow$ box

$y \rightarrow$ box

(x^2-y^2) is div by 3

3, 6, 9, 12 - - - 30	4, 7, - - -	5, 8 - - - -

$3k$ divisible
$3k+1$ divisible
$3k+2$ divisible

CASE 01

(x^2-y^2) is divisible by 3

$\left.\begin{array}{l} x \\ y \end{array}\right]$ $\begin{array}{l} 3k_1 \\ 3k_2 \end{array}$

$$(x-y)(x+y) = (3k_1 - 3k_2)(3k_1 + 3k_2)$$

$$= \underbrace{3 \times 3 (k_1 - k_2)(k_1 + k_2)}_{\text{divisible}}$$

$\Rightarrow {}^{10}C_2$ ✓

$k = 0 \quad 3k+1 = 1$

$k = 1 \quad 3k+1 = 4$

$k = 2 \quad 3k+2 = 7$

$k = 3 \quad 3k+2 = 10$

$k = 4 \quad = 13$

$k = 5 \quad = 16$

$k = 6 \quad = 19$

$k = 7 \quad = 22$

$k = 8 \quad = 25$

$k = 9 \quad = 28$

$k = 0$

$k = 1$

$k = 2$

$k = 3$

$k = 4$

$k = 5$

$k = 6$

$k = 7$

$k = 8$

$k = 9$

$$(x^2-y^2) = (x-y)(x+y)$$

$x = 3k_1+1$

$y = 3k_2+1$

$$= (3k_1+\cancel{1}-3k_2-\cancel{1})(3k_1+1+3k_2+1)$$

$$= \boxed{3}(k_1-k_2)\,3((k_1+k_2)+2)$$

$=$ Multiple of 3. $= {}^{10}C_2$

$$(x^2-y^2) = (x-y)(x+y)$$

$$= (3k_1+\cancel{2}-(3k_2+\cancel{2}))(3k_1+2+3k_2+2)$$

$=$ multiple of 3 $= {}^{10}C_2$

$$= {}^{10}C_2 + {}^{10}C_2 + {}^{10}C_2$$

CASE = $\boxed{3k}$ $\boxed{3k+1}$ $\boxed{3k+2}$

NO

NO

$$\underbrace{{}^{10}C_1 \times {}^{10}C_1}_{\text{divisible by 3}}$$

favourable outcomes.

$$n(E) = {}^{10}C_2 + {}^{10}C_2 + {}^{10}C_2 + {}^{10}C_1\,{}^{10}C_1$$

$$\boxed{P(E) = \frac{{}^{10}C_2 + {}^{10}C_2 + {}^{10}C_2 + {}^{10}C_1\,{}^{10}C_1}{{}^{30}C_2}} = \underline{\underline{\text{Ans}}}$$

QUESTION

If $\overline{E}$ and $\overline{F}$ are the complementary events of events E and F respectively and if $0 < P(F) < 1$, then:

A $P(E/F) + P(\overline{E}/F) = 1$

B $P(E/F) + P(E/\overline{F}) = 1$

C $P(\overline{E}/F) + P(E/\overline{F}) = 1$

D $P(E/\overline{F}) + P(\overline{E}/\overline{F}) = 1$

QUESTION

A coin is tossed $(m + n)$ times, $(m > n)$. Find the probability of getting exactly m consecutive heads.

QUESTION

An unbiased die, with faces numbered 1, 2, 3, 4, 5, 6, is thrown n times and the list of n numbers showing up is noted. What is the probability that among the numbers 1, 2, 3, 4, 5, 6 only three numbers appear in this list.

QUESTION

There are n points in a plane of which no three are in a straight line except 'm' which are all in a straight line. Then the number of different quadrilaterals, that can be formed with the given points as vertices, is:

(A) ${}^{n}C_4 - {}^{m}C_3\left({}^{n-m+1}C_1\right) - {}^{m}C_4$

(B) ${}^{n}C_4 - {}^{m}C_3\left({}^{n-m}C_1\right) - {}^{m}C_4$

(C) ${}^{n}C_4 - {}^{m}C_3\left({}^{m-n}C_1\right) - {}^{m}C_4$

(D) ${}^{n}C_4 + {}^{n}C_3 . {}^{m}C_1$

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THANK YOU!

Here's to a cracking journey ahead!