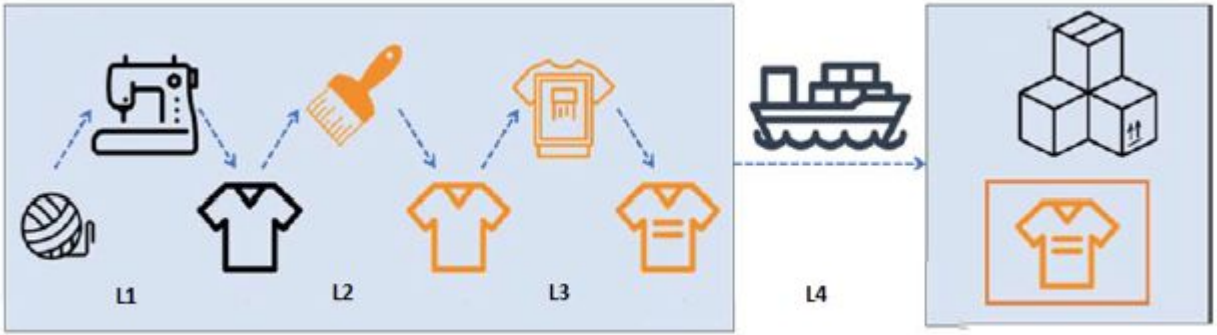
OR7240 Project

1. Robust Inventory Optimization
2. Charlie Card Kiosk Visits

Team Members: Soumyakant Padhee
Prerit Samria

Robust Inventory Optimization



Style: SSC, LSC, SSV

Color: Blue, Red, White, Green, Magenta

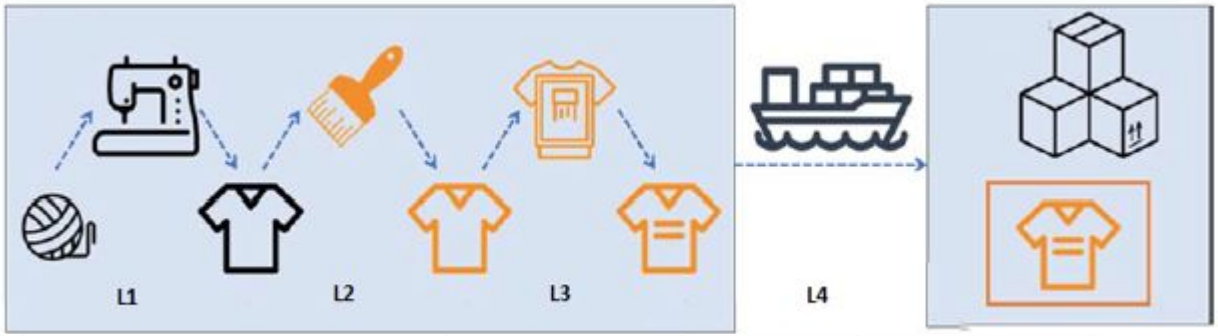
Caption: 5 types

Gender: Male, Female

Size: S, M, L, XL , XXL

750 variants

Robust Inventory Optimization



	Cut & Sew	Dye	Embellish
Cost (in $)	2	0.5	1.25
Lead time (in weeks)	2	1	1

Robust Inventory Optimization

$$Total\ cost\ at\ the\ end\ of\ a\ period\ with\ the\ order\ quantity\ of\ Q = T(Q)$$

$$T(Q) = c_0 \max\left((Q - D, 0) + c_u Max(0, D - Q)\right.$$

where $c_0 = cost\ of\ average, c_u = cost\ of\ underage, D = demand\ in\ the\ period.$

$$E_D[T(Q)] = \int_0^{\infty} T(Q)\, f(x) dx = c_0 \int_0^{Q} (Q - x) f(x)\, dx + c_u \int_Q^{\infty} (x - Q) f(x) dx$$

$$E_D[T(Q)] = c_0[z + L(z)]\sigma_D + c_u L(z)\sigma_D$$

where, $L(z)$ is the loss function to estimate standard deviation z.

Robust Inventory Optimization

$$\alpha = probability\ of\ no\ stock\ out = 1 - \int_{s}^{\infty} f(x)\,dx = F(s)$$

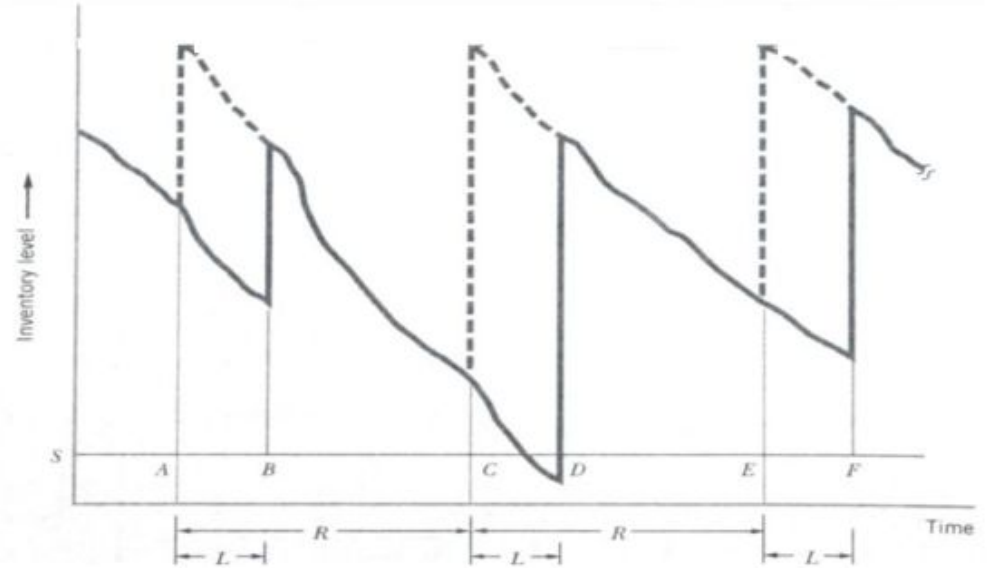
$$\beta = propertion\ of\ order\ filled\ from\ stock = 1 - \frac{L(z)\sigma_{DLTR}}{\mu_D T},$$

$L(z)=$ is the standardized loss function

$$\beta = 1 - \frac{L(z)\sigma_{DLTR}}{\mu_D T\ (\ average\ demand\ per\ cycle)}$$

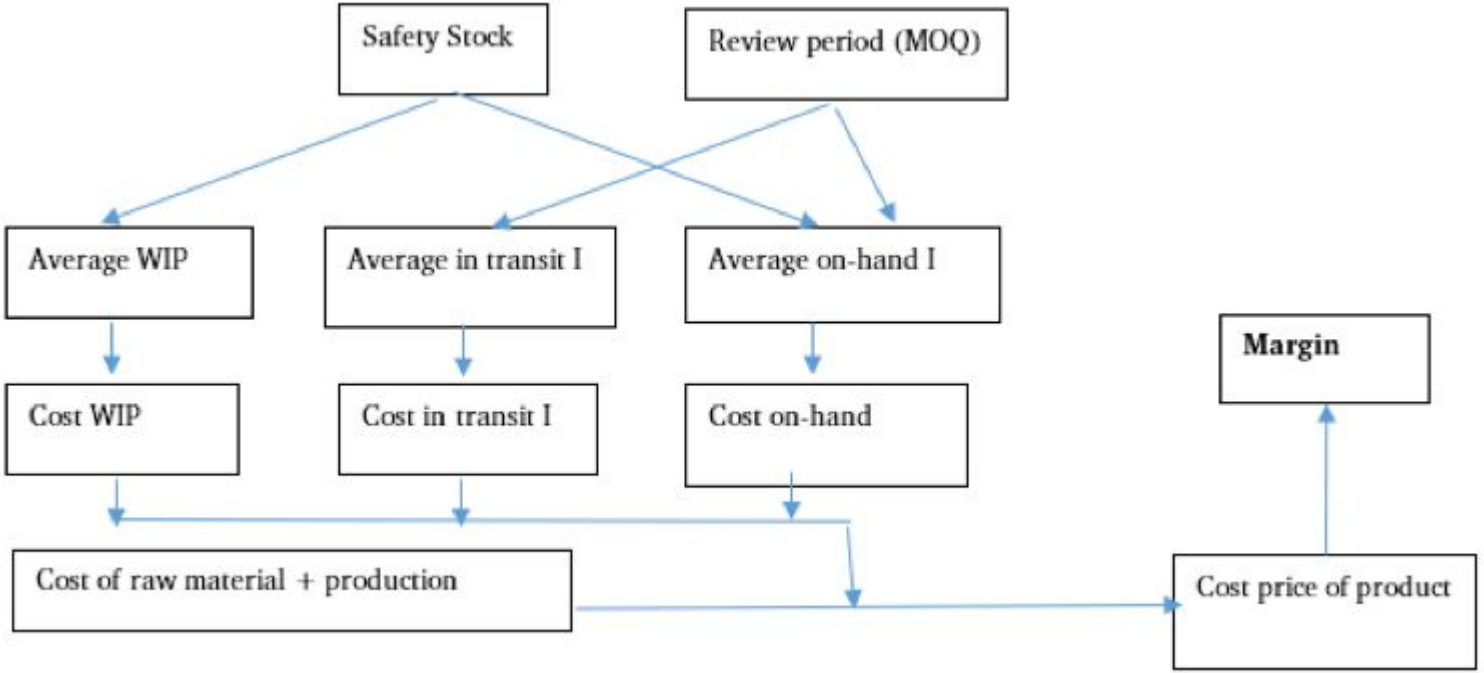
$$= 1 - \frac{\sigma_{DLTR}}{\mu_D T} L\left(\frac{s - \mu_{DLTR}}{\sigma_{DLTR}}\right)$$

$$s = \mu_{DLTR} + z * \sigma_{DLTR}, \text{ where } z = L^{-1}\left((1-\beta)\frac{\mu_D * T}{\sigma_D * \sqrt{(\mu_L + T)}}\right)$$



Typical inventory level and flow in S, T policy
(periodic review, safety stock)

Robust Inventory Optimization



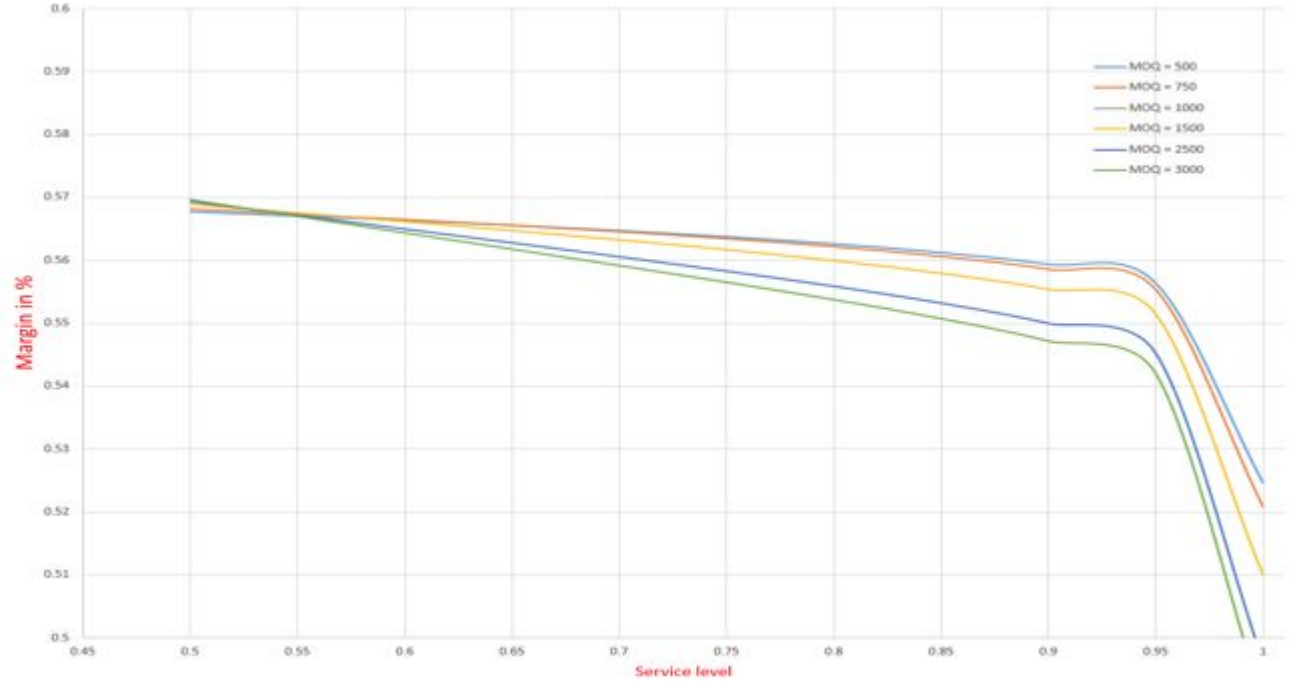
Robust Inventory Optimization

Objective function: $\max margin$

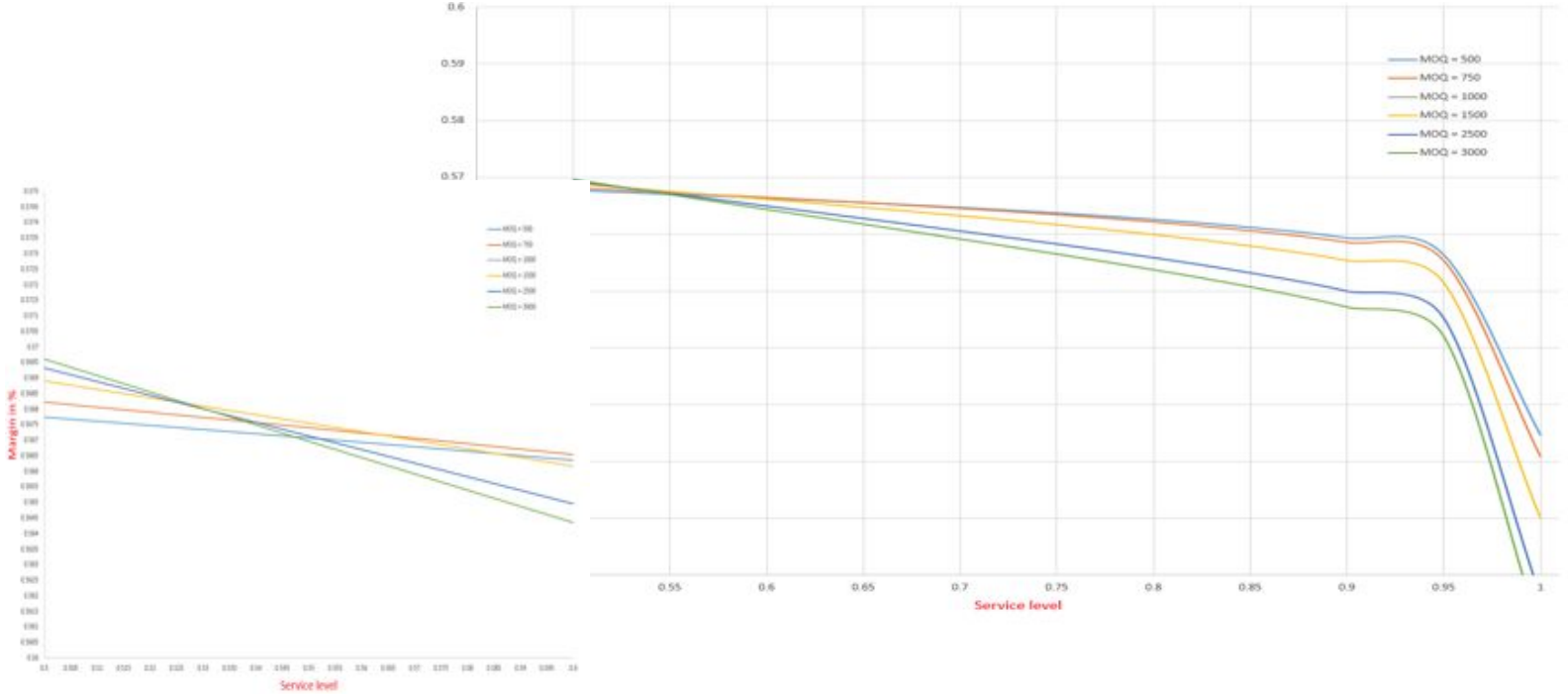
Constraints: *Replenishment lead times,*
Various inventory costs, transit cost
demand distribution.
$Shirts \in \boldsymbol{Z}^{+}$

Decision variables: $fill\ rate, Review\ period\ -T$

Robust Inventory Optimization



Robust Inventory Optimization



Robust Inventory Optimization

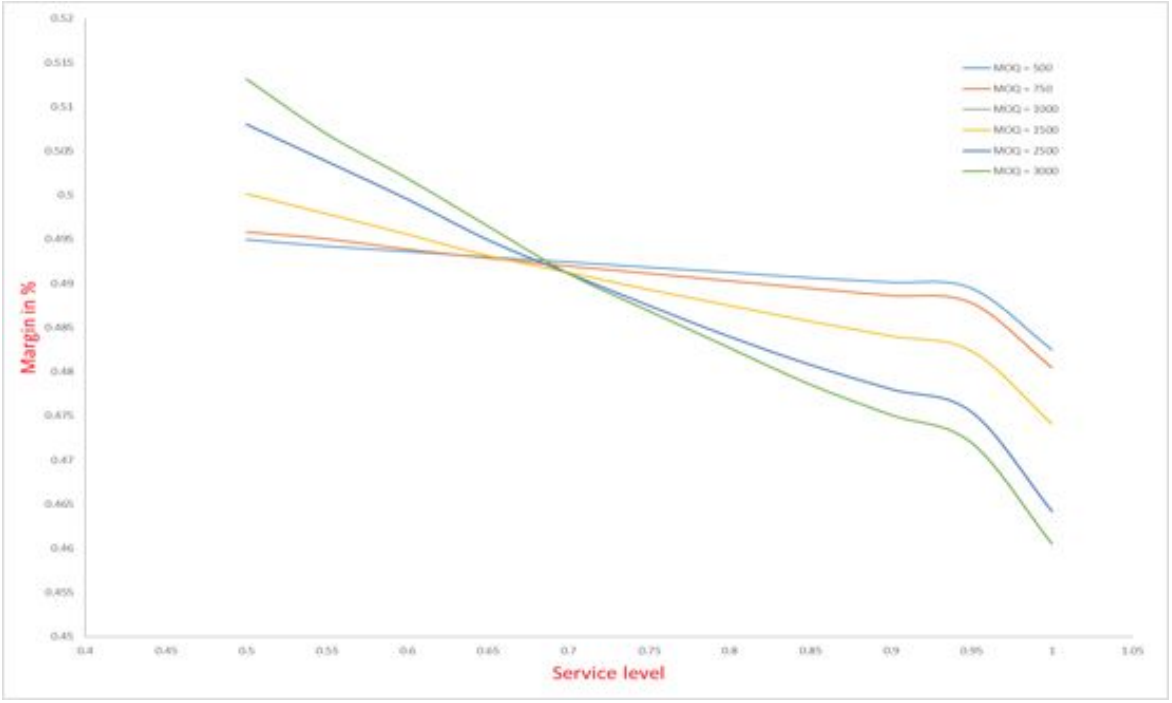
Sensitivity Analysis

- Demand distribution
- Change in overage and underage cost
- Stochastic Lead times and transit time

$$Std.dev\ demand\ -\ DLTR = \sqrt{\mu_L \sigma_D^2 + \mu_D \sigma_L^2 + \sigma_D^2 T}$$

Robust Inventory Optimization

Demand Distribution



Charlie Card Problem - Motivation

- To find the Optimal number of visits to vending machine Kiosks and reducing congestion during card loading.
- Goal is to minimize the number of visits to the Kiosk machine to load Charlie card for N trips.
- Given a menu of reload options M at the kiosk

Goal

- Given the menu of refill values available to subway riders, what is the minimum number of visits to vending machine Kiosks?
- Given the distribution of types of riders in terms of different values of N and R (robustness test), what menu of refill values will decrease the number of visits to the Kiosk and decrease congestion for travelers.

Optimization Problem

- Decision Variables: x_i denotes the number of times one visits a Kiosk machine to refill on i^{th} value from the menu M.
- x_i belongs to non-negative Integer
- Let R denote the residual amount of money in the card at the end of the month.
- Let cost of each subway ride be $2.40

Optimization Problem

- Integer Linear Programming Problem with linear constraints
- Different optimization problems formed for different values of N, R and menu M.
- Range of N = 1 to 120 with increments of 5
- Range of R = 0.25 to 100 with increments of 5

Optimization Problem

- The integer programming problem is solved using the *Intlinprog* in the optimization toolbox in MATLAB.
- The integrality gap between the integer programming problem and its linear program relaxation is 10^{-5}

Model

- Menu, M={1:$10, 2:$20 , 3:$40, 4:$50 }
- **Objective function**: $Min\ x_1 + x_2 + x_3 + x_4$

  Subject to

  1. $10x_1 + 20x_2 + 40x_3 + 50x_4 \geq 2.40N$
  2. $10x_1 + 20x_2 + 40x_3 + 50x_4 - 2.40N \geq R$

N/R	0.25	5.25	10.25	15.25	20.25	25.25	30.25	35.25	40.25	45.25	50.25	55.25	60.25	65.25	70.25	75.25	80.25	85.25	90.25	95.25
1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2
6	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	3	3
11	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	3	3	3	3	3
16	1	1	1	2	2	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3
21	2	2	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3
26	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	4	4
31	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	4	4	4	4
36	2	2	2	3	3	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4
41	2	3	3	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	4
46	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	4	4	5	5
51	3	3	3	3	3	3	4	4	4	4	4	4	4	4	4	4	5	5	5	5
56	3	3	3	3	4	4	4	4	4	4	4	4	4	4	5	5	5	5	5	5
61	3	4	4	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5	5
66	4	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5	5	5	6
71	4	4	4	4	4	4	5	5	5	5	5	5	5	5	5	5	6	6	6	6
76	4	4	4	4	5	5	5	5	5	5	5	5	5	5	6	6	6	6	6	6
81	4	4	5	5	5	5	5	5	5	5	5	5	6	6	6	6	6	6	6	6
86	5	5	5	5	5	5	5	5	5	6	6	6	6	6	6	6	6	6	6	7
91	5	5	5	5	5	5	5	6	6	6	6	6	6	6	6	6	7	7	7	7
96	5	5	5	5	6	6	6	6	6	6	6	6	6	6	7	7	7	7	7	7
101	5	5	6	6	6	6	6	6	6	6	6	6	7	7	7	7	7	7	7	7
106	6	6	6	6	6	6	6	6	6	6	7	7	7	7	7	7	7	7	7	7
111	6	6	6	6	6	6	6	7	7	7	7	7	7	7	7	7	7	8	8	8
116	6	6	6	6	6	7	7	7	7	7	7	7	7	7	7	8	8	8	8	8
121	6	6	7	7	7	7	7	7	7	7	7	7	8	8	8	8	8	8	8	8

Solution

- For N = 120 and R = $10, optimal solution is: $x_1 = 0,\ x_2 = 0,\ x_3 = 0,\ x_4 = 6$
- For N = 10 and R = $10, optimal solution is: $x_1 = 0,\ x_2 = 0,\ x_3 = 0,\ x_4 = 1$
- In general, the optimal solution tends to favor to refill higher values in menu M to minimize the number of visits to the kiosk.

Sensitivity Analysis

- Changing the refill values in the menu M
  - Menu, M={1:$10, 2:$20 , 3:$30, 4:$40}
  - Menu, M={1:$10, 2:$20 , 3:$25, 4:$30}
  - Menu, M={1:$5, 2:$10, 3:$20, 4:$25}

Menu, M={1:$10, 2:$20 , 3:$30, 4:$40}

N/R	0.25	5.25	10.25	15.25	20.25	25.25	30.25	35.25	40.25	45.25	50.25	55.25	60.25	65.25	70.25	75.25	80.25	85.25	90.25	95.25
1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	3	3	3	3
6	1	1	1	1	1	1	2	2	2	2	2	2	2	2	3	3	3	3	3	3
11	1	1	1	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	4
16	1	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	4	4	4
21	2	2	2	2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4	4
26	2	2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4
31	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	5	5
36	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	5	5	5	5	5
41	3	3	3	3	3	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5
46	3	3	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5	6	6
51	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5	6	6	6	6
56	4	4	4	4	4	4	5	5	5	5	5	5	5	5	6	6	6	6	6	6
61	4	4	4	5	5	5	5	5	5	5	5	6	6	6	6	6	6	6	6	7
66	4	5	5	5	5	5	5	5	5	6	6	6	6	6	6	6	6	7	7	7
71	5	5	5	5	5	5	6	6	6	6	6	6	6	6	7	7	7	7	7	7
76	5	5	5	5	6	6	6	6	6	6	6	6	7	7	7	7	7	7	7	7
81	5	5	6	6	6	6	6	6	6	6	7	7	7	7	7	7	7	7	8	8
86	6	6	6	6	6	6	6	7	7	7	7	7	7	7	7	8	8	8	8	8
91	6	6	6	6	6	7	7	7	7	7	7	7	7	8	8	8	8	8	8	8
96	6	6	7	7	7	7	7	7	7	7	8	8	8	8	8	8	8	8	9	9
101	7	7	7	7	7	7	7	7	8	8	8	8	8	8	8	8	9	9	9	9
106	7	7	7	7	7	7	8	8	8	8	8	8	8	8	9	9	9	9	9	9
111	7	7	7	8	8	8	8	8	8	8	8	9	9	9	9	9	9	9	9	10
116	7	8	8	8	8	8	8	8	8	9	9	9	9	9	9	9	9	10	10	10
121	8	8	8	8	8	8	9	9	9	9	9	9	9	9	10	10	10	10	10	10

Menu, M={1:$10, 2:$20 , 3:$25, 4:$30}

N/R	0.25	5.25	10.25	15.25	20.25	25.25	30.25	35.25	40.25	45.25	50.25	55.25	60.25	65.25	70.25	75.25	80.25	85.25	90.25	95.25
1	1	1	1	1	1	1	2	2	2	2	2	2	3	3	3	3	3	3	4	4
6	1	1	1	1	2	2	2	2	2	2	3	3	3	3	3	3	4	4	4	4
11	1	2	2	2	2	2	2	3	3	3	3	3	3	4	4	4	4	4	4	5
16	2	2	2	2	2	3	3	3	3	3	3	4	4	4	4	4	4	5	5	5
21	2	2	3	3	3	3	3	3	4	4	4	4	4	4	5	5	5	5	5	5
26	3	3	3	3	3	3	4	4	4	4	4	4	5	5	5	5	5	5	6	6
31	3	3	3	3	4	4	4	4	4	4	5	5	5	5	5	5	6	6	6	6
36	3	4	4	4	4	4	4	5	5	5	5	5	5	6	6	6	6	6	6	7
41	4	4	4	4	4	5	5	5	5	5	5	6	6	6	6	6	6	7	7	7
46	4	4	5	5	5	5	5	5	6	6	6	6	6	6	7	7	7	7	7	7
51	5	5	5	5	5	5	6	6	6	6	6	6	7	7	7	7	7	7	8	8
56	5	5	5	5	6	6	6	6	6	6	7	7	7	7	7	7	8	8	8	8
61	5	6	6	6	6	6	6	7	7	7	7	7	7	8	8	8	8	8	8	9
66	6	6	6	6	6	7	7	7	7	7	7	8	8	8	8	8	8	9	9	9
71	6	6	7	7	7	7	7	7	8	8	8	8	8	8	9	9	9	9	9	9
76	7	7	7	7	7	7	8	8	8	8	8	8	9	9	9	9	9	9	10	10
81	7	7	7	7	8	8	8	8	8	8	9	9	9	9	9	9	10	10	10	10
86	7	8	8	8	8	8	8	9	9	9	9	9	9	10	10	10	10	10	10	11
91	8	8	8	8	8	9	9	9	9	9	9	10	10	10	10	10	10	11	11	11
96	8	8	9	9	9	9	9	9	10	10	10	10	10	10	11	11	11	11	11	11
101	9	9	9	9	9	9	10	10	10	10	10	10	11	11	11	11	11	11	12	12
106	9	9	9	9	10	10	10	10	10	10	11	11	11	11	11	11	12	12	12	12
111	9	10	10	10	10	10	10	11	11	11	11	11	11	12	12	12	12	12	12	13
116	10	10	10	10	10	11	11	11	11	11	11	12	12	12	12	12	12	13	13	13
121	10	10	11	11	11	11	11	11	12	12	12	12	12	12	13	13	13	13	13	13

Menu, M={1:$5, 2:$10, 3:$20, 4:$25}

N/R	0.25	5.25	10.25	15.25	20.25	25.25	30.25	35.25	40.25	45.25	50.25	55.25	60.25	65.25	70.25	75.25	80.25	85.25	90.25	95.25
1	1	1	1	1	1	2	2	2	2	2	3	3	3	3	3	4	4	4	4	4
6	1	1	1	2	2	2	2	2	3	3	3	3	3	4	4	4	4	4	5	5
11	2	2	2	2	2	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5
16	2	2	2	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5	6	6
21	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5	6	6	6	6	6
26	3	3	3	4	4	4	4	4	5	5	5	5	5	6	6	6	6	6	7	7
31	3	4	4	4	4	4	5	5	5	5	5	6	6	6	6	6	7	7	7	7
36	4	4	4	5	5	5	5	5	6	6	6	6	6	7	7	7	7	7	8	8
41	4	5	5	5	5	5	6	6	6	6	6	7	7	7	7	7	8	8	8	8
46	5	5	5	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	9	9
51	5	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	9	9	9	9
56	6	6	6	6	7	7	7	7	7	8	8	8	8	8	9	9	9	9	9	10
61	6	7	7	7	7	7	8	8	8	8	8	9	9	9	9	9	10	10	10	10
66	7	7	7	7	8	8	8	8	8	9	9	9	9	9	10	10	10	10	10	11
71	7	8	8	8	8	8	9	9	9	9	9	10	10	10	10	10	11	11	11	11
76	8	8	8	8	9	9	9	9	9	10	10	10	10	10	11	11	11	11	11	12
81	8	8	9	9	9	9	9	10	10	10	10	10	11	11	11	11	11	12	12	12
86	9	9	9	9	10	10	10	10	10	11	11	11	11	11	12	12	12	12	12	13
91	9	9	10	10	10	10	10	11	11	11	11	11	12	12	12	12	12	13	13	13
96	10	10	10	10	11	11	11	11	11	12	12	12	12	12	13	13	13	13	13	14
101	10	10	11	11	11	11	11	12	12	12	12	12	13	13	13	13	13	14	14	14
106	11	11	11	11	11	12	12	12	12	12	13	13	13	13	13	14	14	14	14	14
111	11	11	12	12	12	12	12	13	13	13	13	13	14	14	14	14	14	15	15	15
116	12	12	12	12	12	13	13	13	13	13	14	14	14	14	14	15	15	15	15	15
121	12	12	13	13	13	13	13	14	14	14	14	14	15	15	15	15	15	16	16	16

Observations

- As N and R increases, the optimal number of visits to the kiosk also increases.
- The optimal solution for an individual with a high value of N and low value of R is to refill using the highest refill value.
- If you want to take more trips (high N) and have more residual money (high R), then you have to refill your card with more money.
- If this is the case, then it is optimal to refill using the higher values of refill amount on the menu.

Observations

Menu, M	Highest Number of Visits	x_1	x_2	x_3	x_4
{1:$10, 2:$20, 3:$40, 4:$50}	8	0	0	0	8
{1:$10, 2:$20, 3:$30, 4:$40}	10	0	0	0	10
{1:$10, 2:$20, 3:$25, 4:$30}	13	0	0	0	13
{1:$5, 2:$10, 3:$20, 4:$25}	16	0	0	0	16

Observations

- In general, the optimal solution tends to favor to refill higher values to minimize the number of visits to the kiosk.
- As menu M changes, the optimal number of visits change.
- The optimal number of visits have higher values for lower refill values in the menu M.
- However, in general subway users do not refill using higher refill amounts.
- Thus, we can put an upper bound for decision variables x_3 and x_4 to be 5.
- Now the optimal number of visits to the kiosk would change.

Change in Model

- Menu, M={1:$10, 2:$20 , 3:$40, 4:$50 }
- **Objective function**: $Min\ x_1 + x_2 + x_3 + x_4$

Subject to

1. $10x_1 + 20x_2 + 40x_3 + 50x_4 \geq 2.40N$
2. $10x_1 + 20x_2 + 40x_3 + 50x_4 - 2.40N \geq R$
3. x_i = non-negative Integer
4. x_3, x_4 <= 5

For Menu = {1:$10, 2:$20, 3:$40, 4:$50}

N/R	0.25	5.25	10.25	15.25	20.25	25.25	30.25	35.25	40.25	45.25	50.25	55.25	60.25	65.25	70.25	75.25	80.25	85.25	90.25	95.25
1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2
6	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	3	3
11	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	3	3	3	3	3
16	1	1	1	2	2	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3
21	2	2	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3
26	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	4	4
31	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	4	4	4	4
36	2	2	2	3	3	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4
41	2	3	3	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	4
46	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	4	4	5	5
51	3	3	3	3	3	3	4	4	4	4	4	4	4	4	4	4	5	5	5	5
56	3	3	3	3	4	4	4	4	4	4	4	4	4	4	5	5	5	5	5	5
61	3	4	4	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5	5
66	4	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5	5	5	6
71	4	4	4	4	4	4	5	5	5	5	5	5	5	5	5	5	6	6	6	6
76	4	4	4	4	5	5	5	5	5	5	5	5	5	5	6	6	6	6	6	6
81	4	4	5	5	5	5	5	5	5	5	5	5	6	6	6	6	6	6	6	6
86	5	5	5	5	5	5	5	5	5	6	6	6	6	6	6	6	6	7	7	7
91	5	5	5	5	5	5	5	6	6	6	6	6	6	6	6	7	7	7	7	7
96	5	5	5	5	6	6	6	6	6	6	6	6	7	7	7	7	7	7	7	7
101	5	5	6	6	6	6	6	6	6	6	7	7	7	7	7	7	7	7	8	8
106	6	6	6	6	6	6	6	6	7	7	7	7	7	7	7	8	8	8	8	8
111	6	6	6	6	6	7	7	7	7	7	7	7	7	8	8	8	8	8	8	8
116	6	6	6	7	7	7	7	7	7	7	7	8	8	8	8	8	8	8	8	9
121	7	7	7	7	7	7	7	7	8	8	8	8	8	8	8	8	9	9	9	9

For Menu = {1:$10, 2:$20, 3:$30, 4:$40}

N/R	0.25	5.25	10.25	15.25	20.25	25.25	30.25	35.25	40.25	45.25	50.25	55.25	60.25	65.25	70.25	75.25	80.25	85.25	90.25	95.25
1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	3	3	3	3
6	1	1	1	1	1	1	2	2	2	2	2	2	2	2	3	3	3	3	3	3
11	1	1	1	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	4
16	1	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	4	4	4
21	2	2	2	2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4	4
26	2	2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4
31	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	5	5
36	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	5	5	5	5	5
41	3	3	3	3	3	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5
46	3	3	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5	6	6
51	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5	6	6	6	6
56	4	4	4	4	4	4	5	5	5	5	5	5	5	5	6	6	6	6	6	6
61	4	4	4	5	5	5	5	5	5	5	5	6	6	6	6	6	6	7	7	7
66	4	5	5	5	5	5	5	5	5	6	6	6	6	6	6	7	7	7	7	7
71	5	5	5	5	5	5	6	6	6	6	6	6	7	7	7	7	7	7	8	8
76	5	5	5	5	6	6	6	6	6	6	7	7	7	7	7	7	8	8	8	8
81	5	5	6	6	6	6	6	6	7	7	7	7	7	7	8	8	8	8	8	8
86	6	6	6	6	6	7	7	7	7	7	7	8	8	8	8	8	8	9	9	9
91	6	6	6	7	7	7	7	7	7	8	8	8	8	8	8	9	9	9	9	9
96	7	7	7	7	7	7	8	8	8	8	8	8	9	9	9	9	9	9	10	10
101	7	7	7	7	8	8	8	8	8	8	9	9	9	9	9	9	10	10	10	10
106	7	7	8	8	8	8	8	8	9	9	9	9	9	9	10	10	10	10	10	10
111	8	8	8	8	8	9	9	9	9	9	9	10	10	10	10	10	10	11	11	11
116	8	8	8	9	9	9	9	9	9	10	10	10	10	10	10	11	11	11	11	12
121	9	9	9	9	9	9	10	10	10	10	10	10	11	11	11	11	12	12	12	12

For Menu = {1:$10, 2:$20, 3:$25, 4:$30}

N/R	0.25	5.25	10.25	15.25	20.25	25.25	30.25	35.25	40.25	45.25	50.25	55.25	60.25	65.25	70.25	75.25	80.25	85.25	90.25	95.25
1	1	1	1	1	1	1	2	2	2	2	2	2	3	3	3	3	3	3	4	4
6	1	1	1	1	2	2	2	2	2	2	3	3	3	3	3	3	4	4	4	4
11	1	2	2	2	2	2	2	3	3	3	3	3	3	4	4	4	4	4	4	5
16	2	2	2	2	2	3	3	3	3	3	3	4	4	4	4	4	4	5	5	5
21	2	2	3	3	3	3	3	3	4	4	4	4	4	4	5	5	5	5	5	5
26	3	3	3	3	3	3	4	4	4	4	4	4	5	5	5	5	5	5	6	6
31	3	3	3	3	4	4	4	4	4	4	5	5	5	5	5	5	6	6	6	6
36	3	4	4	4	4	4	4	5	5	5	5	5	5	6	6	6	6	6	7	7
41	4	4	4	4	4	5	5	5	5	5	5	6	6	6	6	6	7	7	7	7
46	4	4	5	5	5	5	5	5	6	6	6	6	6	7	7	7	7	7	8	8
51	5	5	5	5	5	5	6	6	6	6	6	7	7	7	7	7	8	8	8	8
56	5	5	5	5	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	9
61	5	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	9	9	9	9
66	6	6	6	6	7	7	7	7	7	8	8	8	8	8	9	9	9	9	9	10
71	6	7	7	7	7	7	8	8	8	8	8	9	9	9	9	9	10	10	10	10
76	7	7	7	7	8	8	8	8	8	9	9	9	9	9	10	10	10	10	10	11
81	7	7	8	8	8	8	8	9	9	9	9	9	10	10	10	10	10	11	11	11
86	8	8	8	8	9	9	9	9	9	10	10	10	10	10	11	11	11	11	12	12
91	8	8	9	9	9	9	9	10	10	10	10	10	11	11	11	11	12	12	12	12
96	9	9	9	9	10	10	10	10	10	11	11	11	11	12	12	12	12	13	13	13
101	9	9	10	10	10	10	10	11	11	11	11	12	12	12	12	13	13	13	13	14
106	10	10	10	10	10	11	11	11	11	12	12	12	12	13	13	13	13	14	14	14
111	10	10	11	11	11	11	12	12	12	12	13	13	13	13	14	14	14	14	15	15
116	11	11	11	11	12	12	12	12	13	13	13	13	14	14	14	14	15	15	15	15
121	11	12	12	12	12	13	13	13	13	14	14	14	14	15	15	15	15	16	16	16

For Menu = {1:$5, 2:$10, 3:$20, 4:$25}

N/R	0.25	5.25	10.25	15.25	20.25	25.25	30.25	35.25	40.25	45.25	50.25	55.25	60.25	65.25	70.25	75.25	80.25	85.25	90.25	95.25
1	1	1	1	1	1	2	2	2	2	2	3	3	3	3	3	4	4	4	4	4
6	1	1	1	2	2	2	2	2	3	3	3	3	3	4	4	4	4	4	5	5
11	2	2	2	2	2	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5
16	2	2	2	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5	6	6
21	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5	6	6	6	6	7
26	3	3	3	4	4	4	4	4	5	5	5	5	5	6	6	6	6	7	7	7
31	3	4	4	4	4	4	5	5	5	5	5	6	6	6	6	7	7	7	7	8
36	4	4	4	5	5	5	5	5	6	6	6	6	7	7	7	7	8	8	8	8
41	4	5	5	5	5	5	6	6	6	6	7	7	7	7	8	8	8	8	9	9
46	5	5	5	6	6	6	6	7	7	7	7	8	8	8	8	9	9	9	9	10
51	5	6	6	6	6	7	7	7	7	8	8	8	8	9	9	9	9	10	10	10
56	6	6	6	7	7	7	7	8	8	8	8	9	9	9	9	10	10	10	10	11
61	7	7	7	7	8	8	8	8	9	9	9	9	10	10	10	10	11	11	12	12
66	7	7	8	8	8	8	9	9	9	9	10	10	10	10	11	11	12	12	13	13
71	8	8	8	9	9	9	9	10	10	10	10	11	11	12	12	13	13	14	14	15
76	8	9	9	9	9	10	10	10	10	11	11	12	12	13	13	14	14	15	15	16
81	9	9	9	10	10	10	10	11	11	12	12	13	13	14	14	15	15	16	16	17
86	10	10	10	10	11	11	12	12	13	13	14	14	15	15	16	16	17	17	18	18
91	10	10	11	11	12	12	13	13	14	14	15	15	16	16	17	17	18	18	19	19
96	11	12	12	13	13	14	14	15	15	16	16	17	17	18	18	19	19	20	20	21
101	12	13	13	14	14	15	15	16	16	17	17	18	18	19	19	20	20	21	21	22
106	13	14	14	15	15	16	16	17	17	18	18	19	19	20	20	21	21	22	22	23
111	15	15	16	16	17	17	18	18	19	19	20	20	21	21	22	22	23	23	24	24
116	16	16	17	17	18	18	19	19	20	20	21	21	22	22	23	23	24	24	25	25
121	17	18	18	19	19	20	20	21	21	22	22	23	23	24	24	25	25	26	26	27

Observations

Menu, M	Highest Number of Visits	x_1	x_2	x_3	x_4
{1:$10, 2:$20, 3:$40, 4:$50}	9	0	0	4	5
{1:$10, 2:$20, 3:$30, 4:$40}	12	0	2	5	5
{1:$10, 2:$20, 3:$25, 4:$30}	16	0	6	5	5
{1:$5, 2:$10, 3:$20, 4:$25}	27	0	17	5	5

Further Work

- Personalized for each user
- An application can be designed for smartphones that can help users decide the number of visits to the kiosks required for their personalized menu M and a fixed number of trips N per month.
- This would help reduce the congestion on the kiosks which is the main objective of this study

Any Questions?

Thank You!