Sabanci University, FENS
CS419 Digital Image and Video Analysis, Fall 2023
Assignment 1

October 31, 2023

Question 1 (30 points)

The definition for the convolution of two continuous functions $f(x)$ and $g(x)$ is given by:

$$(f * g)(x) = \int_{-\infty}^{+\infty} f(y)\, g(x-y)\, \mathrm{d}y. \tag{1}$$

Prove that the convolution operation is commutative:

$$(f * g) = (g * f). \tag{2}$$

and then prove that the cross-correlation operation:

$$(f \star g)(x) = \int_{-\infty}^{+\infty} f(y)\, g(x+y)\, \mathrm{d}y. \tag{3}$$

is not commutative.
And finally, prove that the convolution operation is associative:

$$(f * g) * h = f * (g * h). \tag{4}$$

Question 2 (30 points)

The Laplacian operator as applied on a 2D image f is defined as:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \tag{5}$$

Your task is to prove that the Laplacian operator is rotation invariant, in other words that it is not affected by rotations. More explicitly, assuming that we

rotate the coordinates (x, y) of an image f through our familiar rotation matrix:

$$\begin{aligned} x' &= xcos(\theta) - ysin(\theta) \\ y' &= xsin(\theta) + ycos(\theta) \end{aligned} \tag{6}$$

prove that:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 f}{\partial x'^2} + \frac{\partial^2 f}{\partial y'^2} \tag{7}$$

Question 3 (20 points)

You've decided to develop a new filter h and after many hours of design you come up with the following function:

$$h(x, y) = 3f(x, y) + 2f(x-1, y) + 2f(x+1, y) - 17f(x, y-1) + 99f(x, y+1) \tag{8}$$

Is h a linear filter? Prove your answer. And then provide the convolution mask corresponding to h.

Question 4 (20 points)

Implement the morphological convex hull operator (as shown in class) and apply it to the attached binary image. You are not allowed to use any image processing libraries (OpenCV or otherwise) except for image loading and image displaying purposes. I expect you to code every aspect of the solution.

Instructions

1. **Integrity:** Plagiarism is strongly prohibited and may lead to failure of this course.
2. **Questions:** Contact the TA for any questions you might have.
3. **Write-up:** Please submit your answers as a zip file containing the documented python source code of your implementation and a single pdf file type-set with LateX (I recommend using overleaf) containing your proofs and answers. Please do not submit scans or photographs of handwritten documents, or pdfs prepared in word/libreffice, they will not be accepted for evaluation.
4. **Collaboration:** You can work in groups, however each student must submit their own work.