Meshes

Some Slides/Images adapted from Marschner and Shirley and David Levin

Announcements

Assignment 4 due Thursday (11/06)

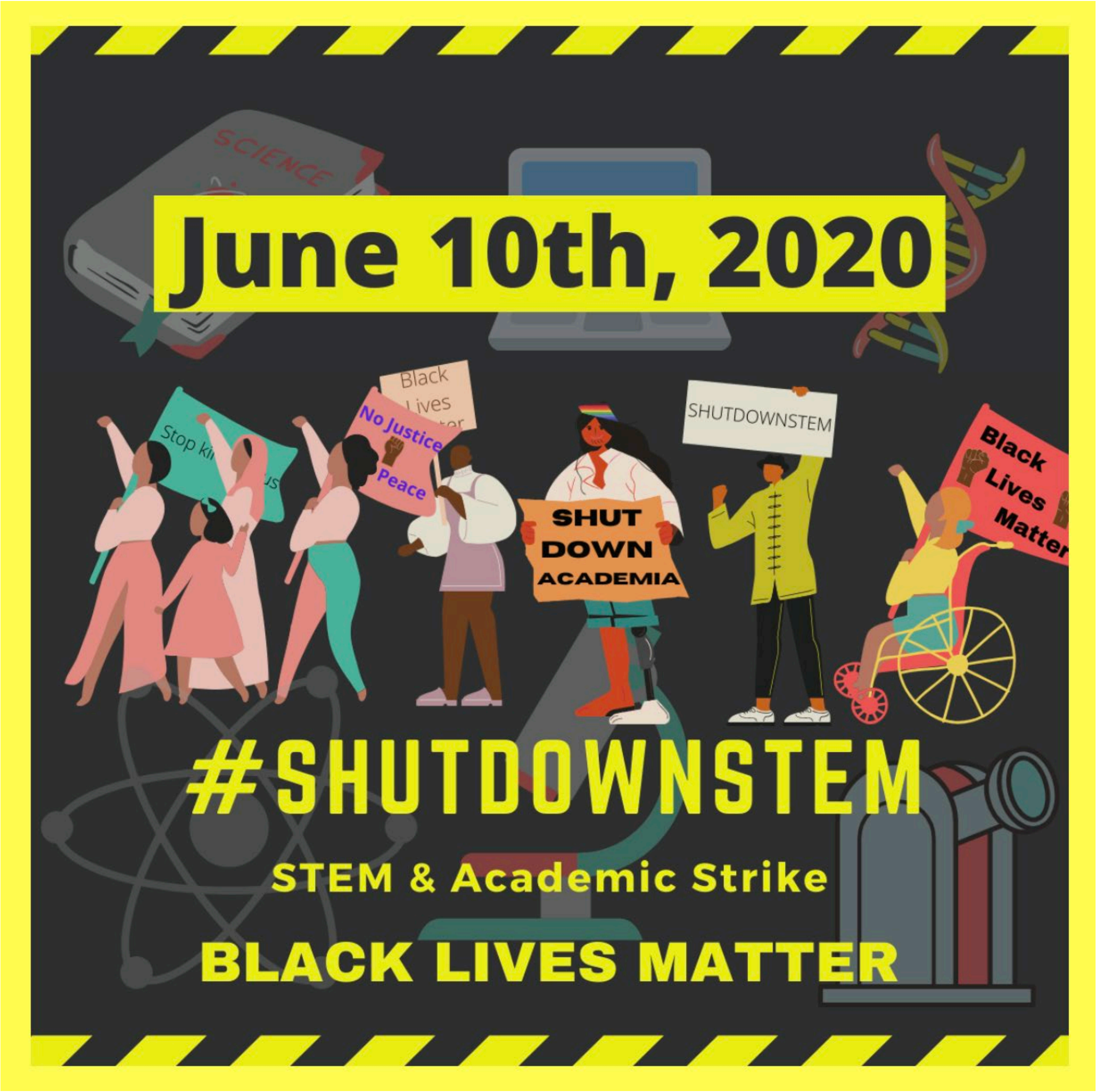
Mid-course evaluations

Announcements

No class Wednesday

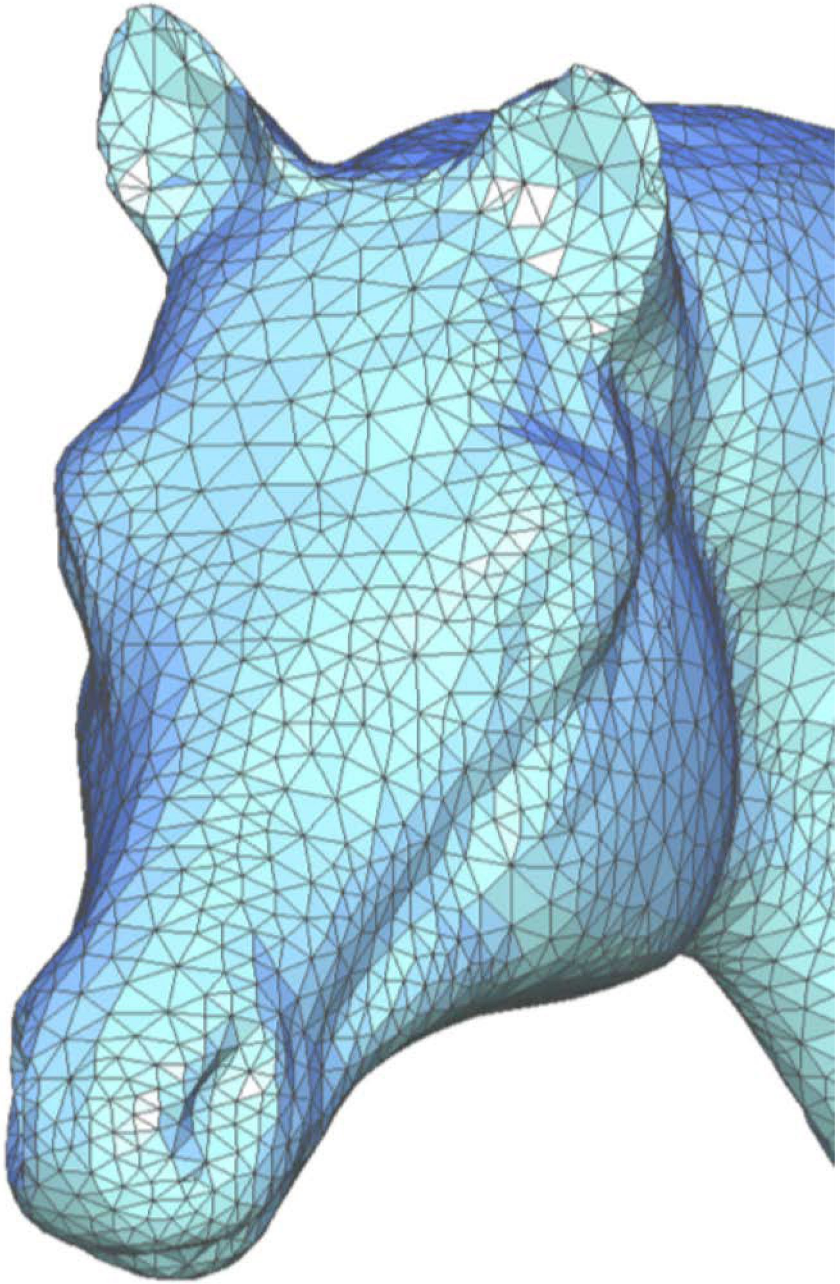
https://www.shutdownstem.com/action

https://blacklivesmatters.carrd.co/



Meshes

Ottawa Convention Center



Meshes

Today:

Types of Surfaces

Triangles

Data Structures for Triangle Meshes

Normals for Meshes

Next Monday:

Texture Mapping

Subdivision Surfaces

Any Questions?

Surface Representations in Graphics

What are the two main types of surface representations?

Surface Representations in Graphics

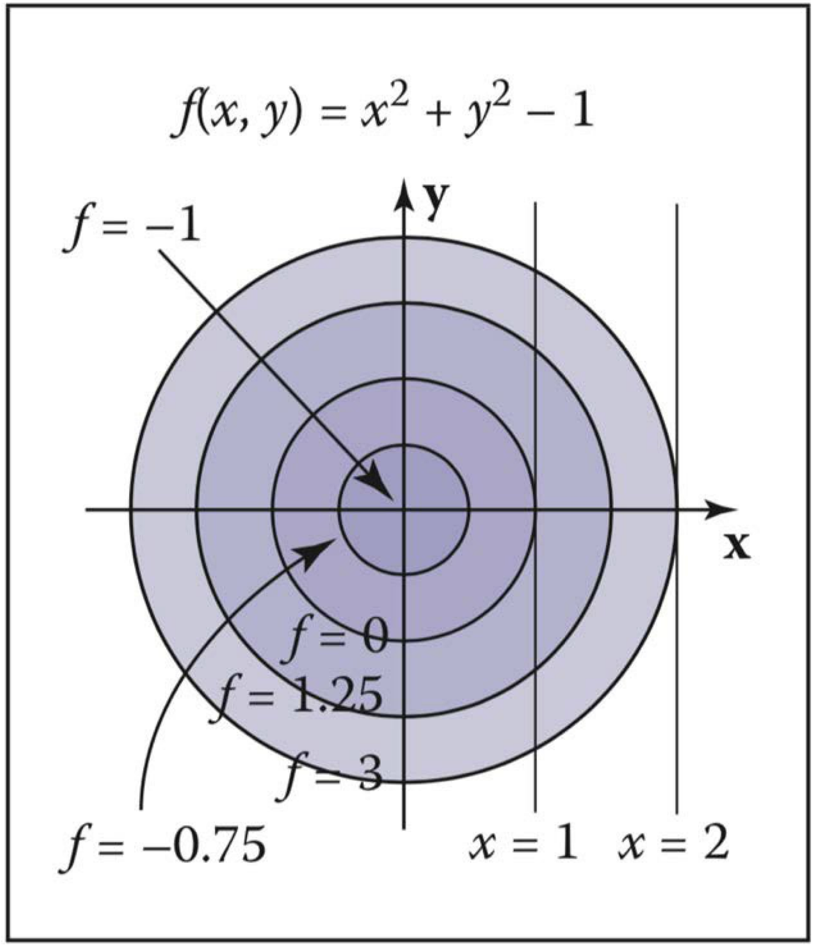
What are the two main types of surface representations ?

Implicit and Parametric

How do you define each type?

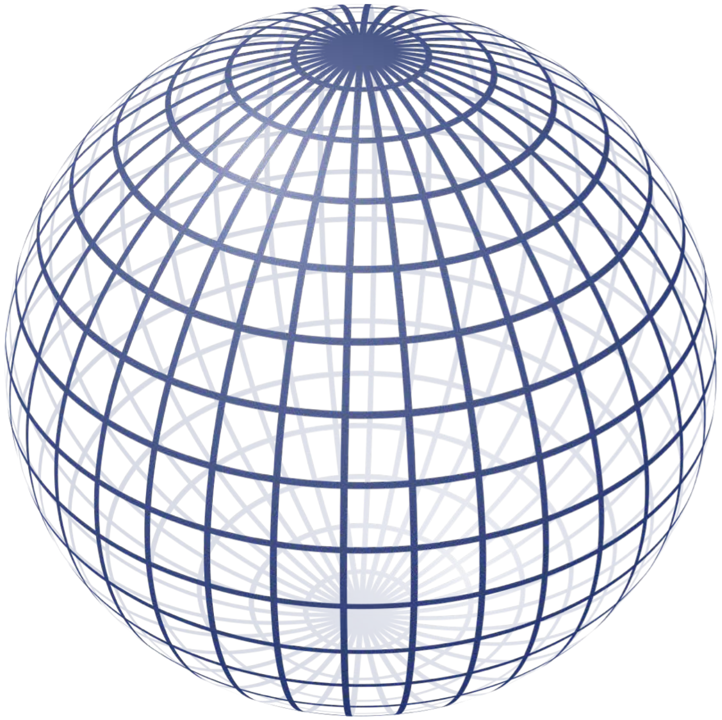
Surface Representations in Graphics

Implicit Surface

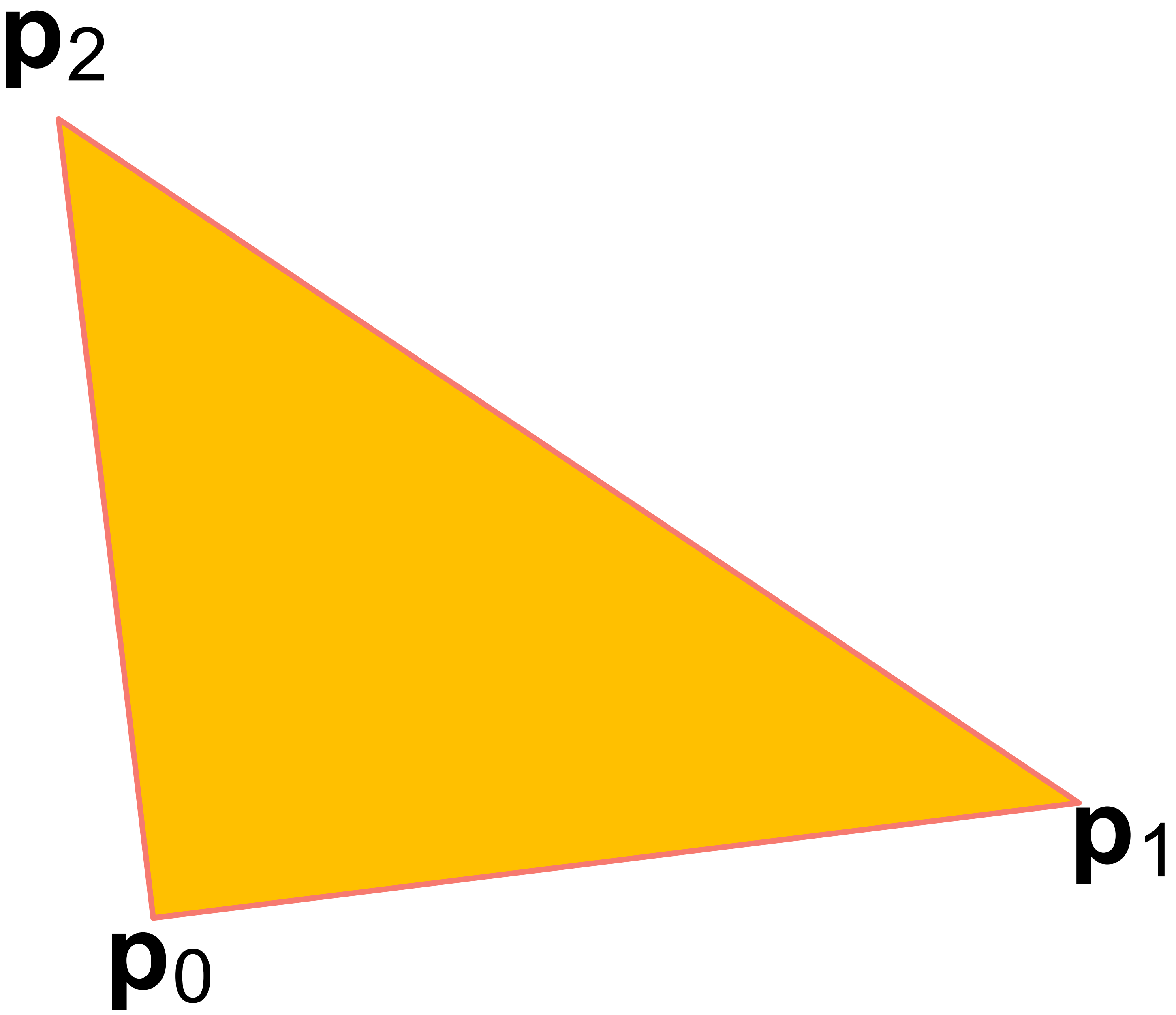


Parametric Surface

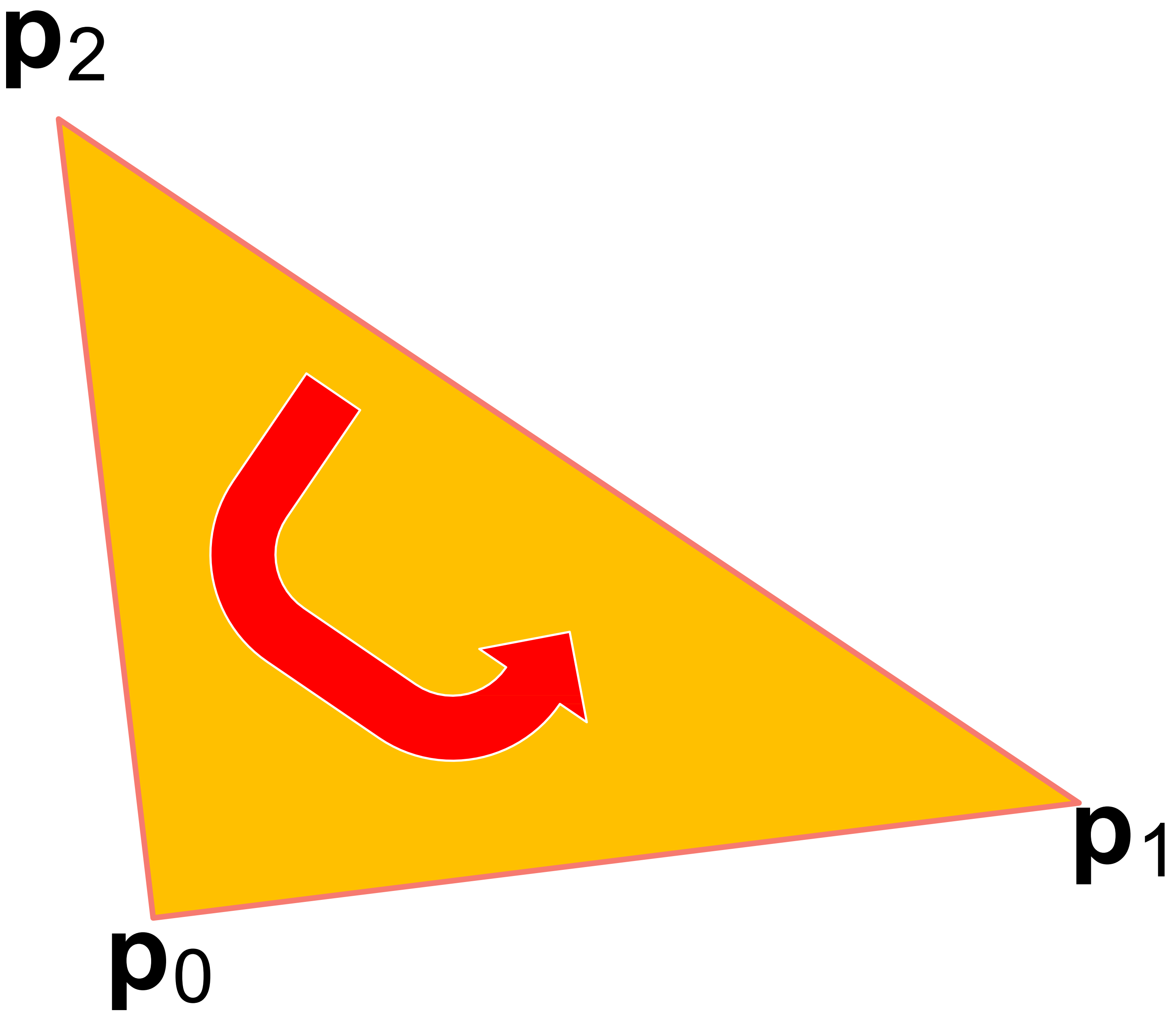
$$x = r\cos\phi\sin\theta,$$
$$y = r\sin\phi\sin\theta,$$
$$z = r\cos\theta.$$



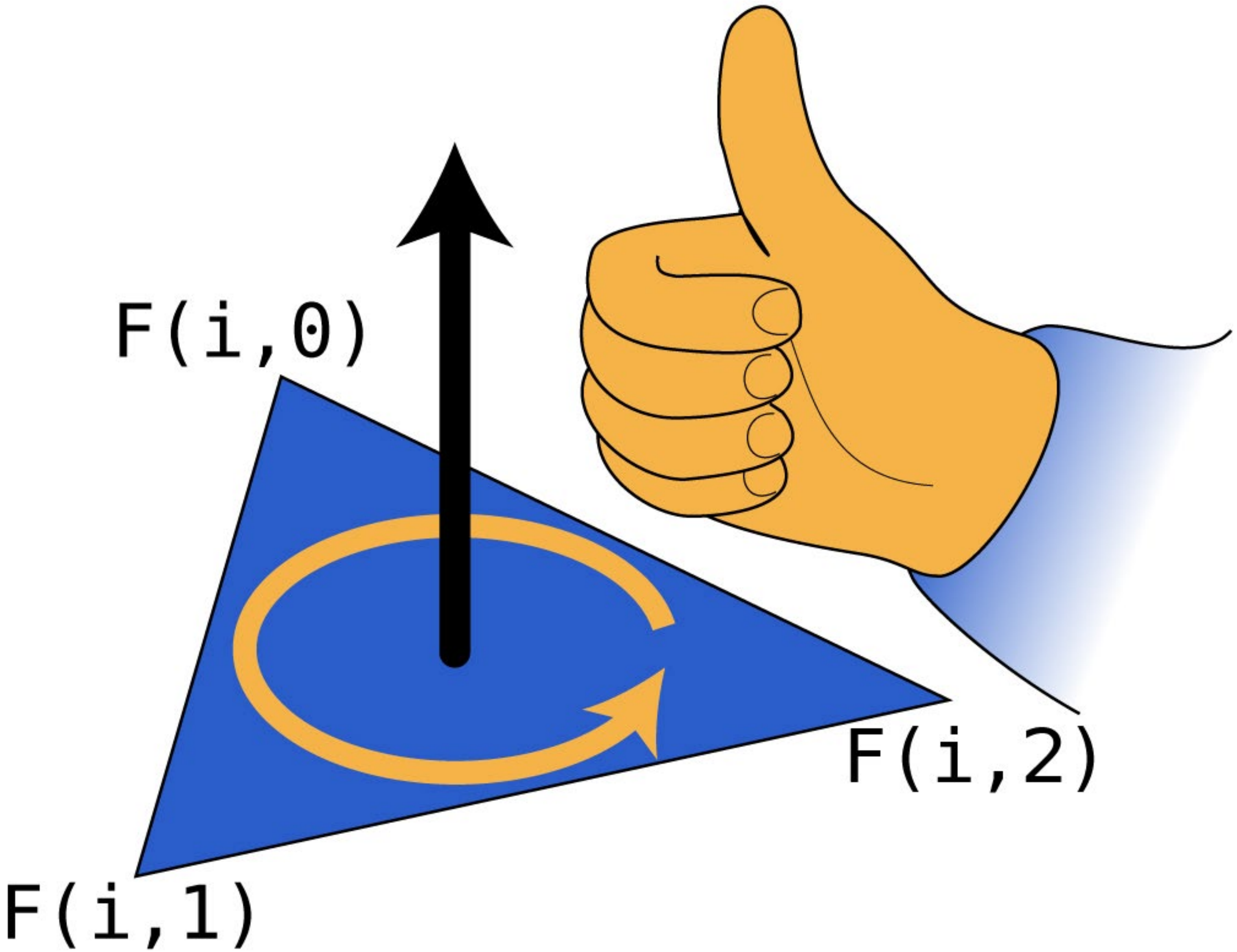
Triangles



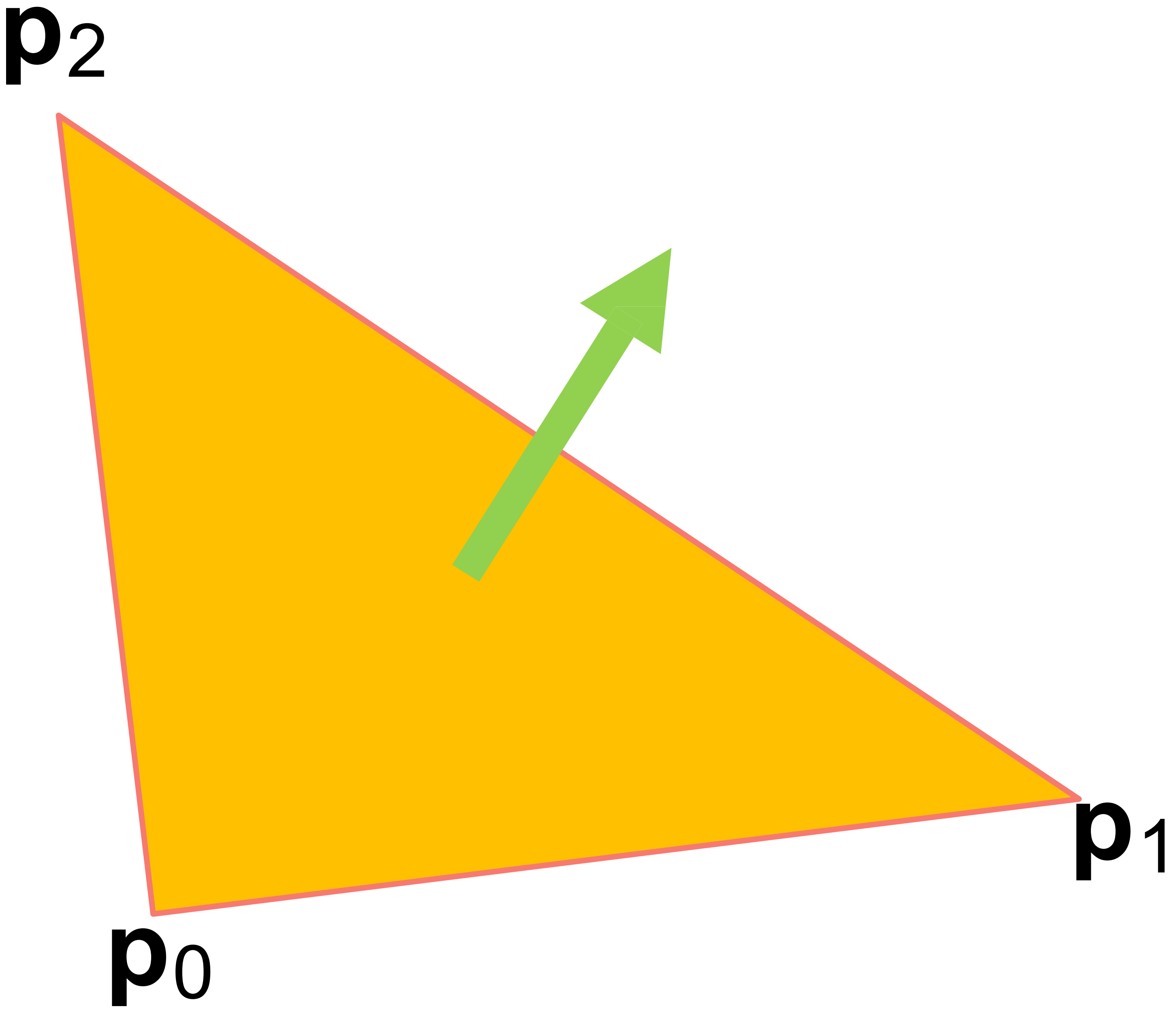
Triangles



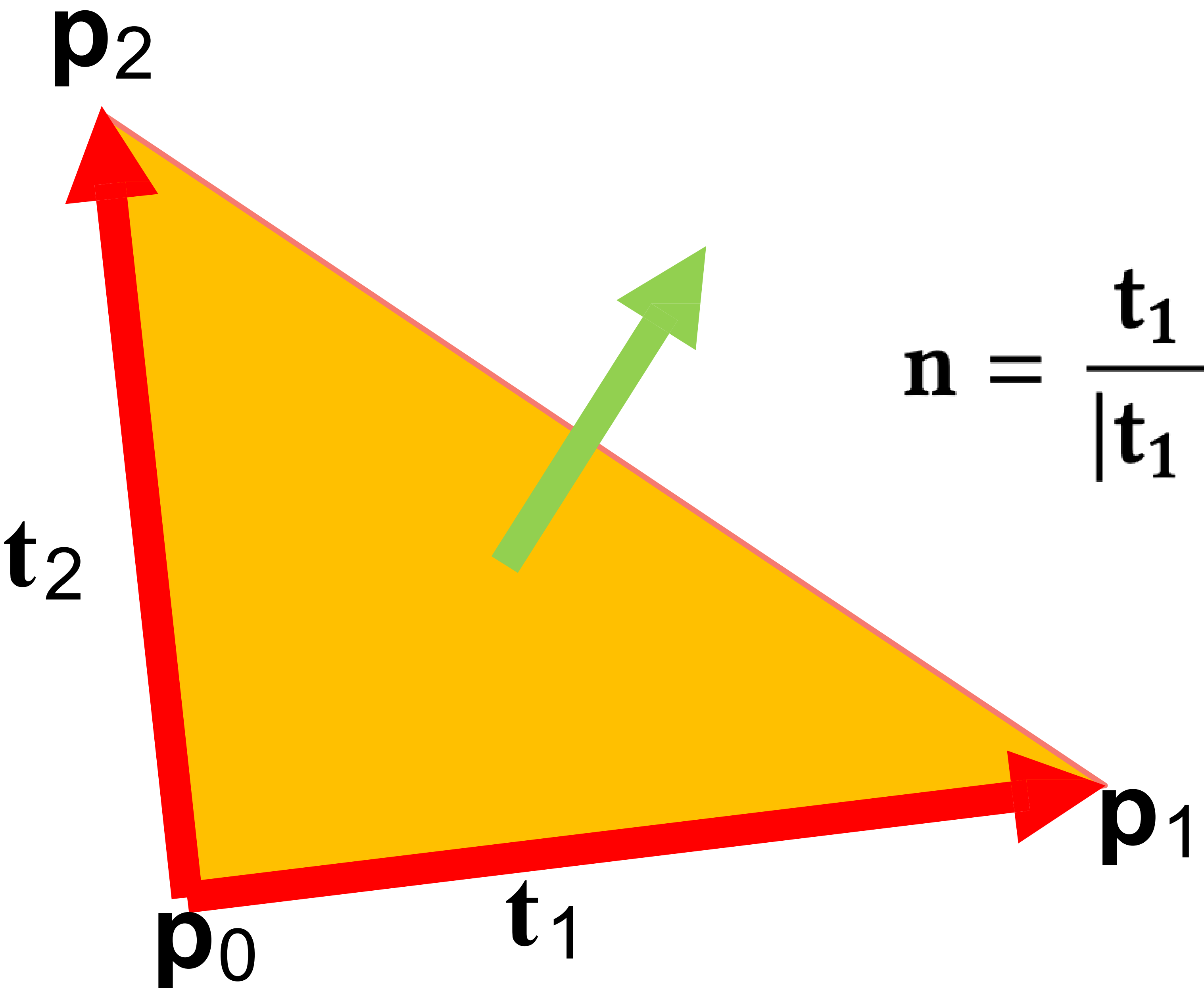
Triangles



Triangles



Triangles



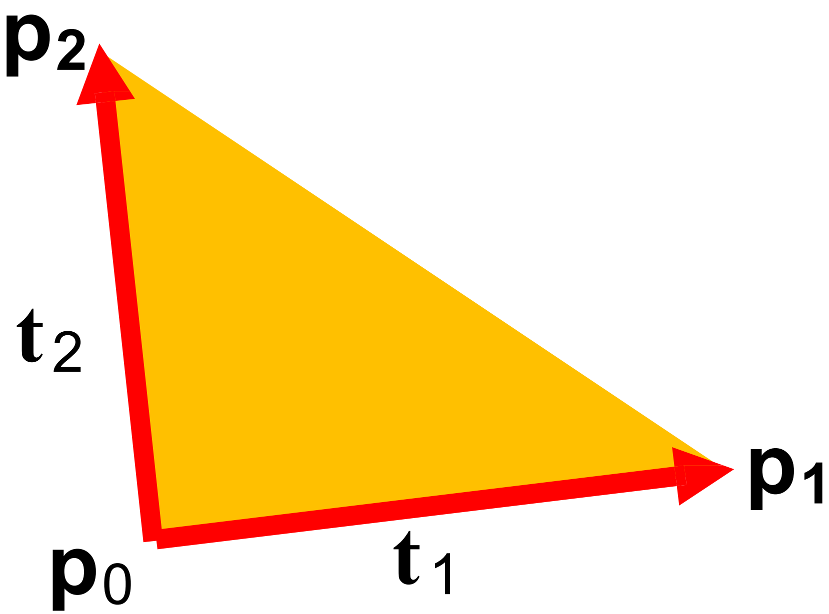
$$\mathbf{n} = \frac{\mathbf{t}_1 \times \mathbf{t}_2}{|\mathbf{t}_1 \times \mathbf{t}_2|}$$

Barycentric Coordinates

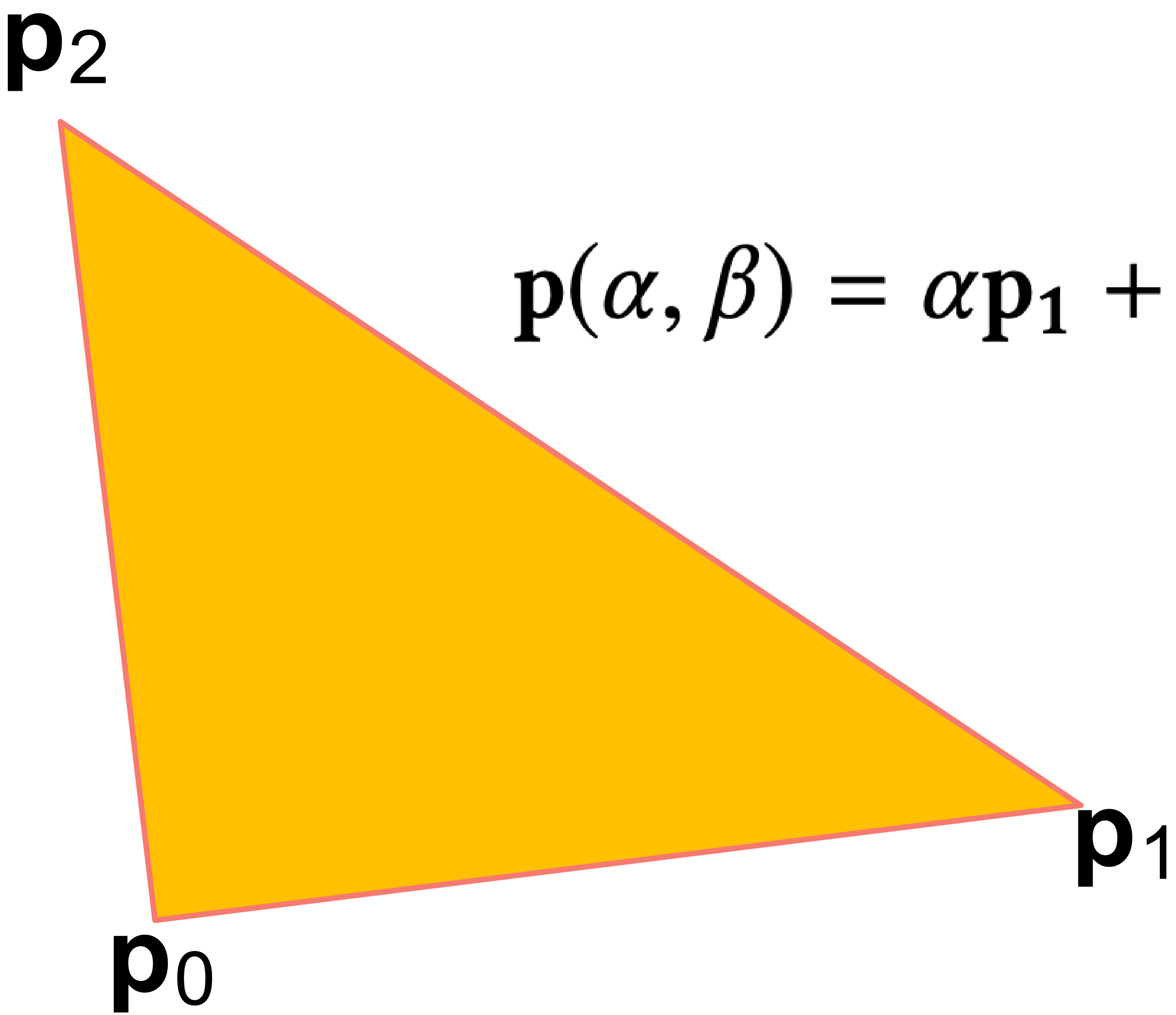
$$\mathbf{p}_1 = \mathbf{p}_0 + \alpha \mathbf{t}_1 + \beta \mathbf{t}_2$$

$$\mathbf{p}(\alpha, \beta) = \mathbf{p}_0 + \alpha(\mathbf{p}_1 - \mathbf{p}_0) + \beta(\mathbf{p}_2 - \mathbf{p}_0)$$

$$\mathbf{p}(\alpha, \beta) = \alpha \mathbf{p}_1 + \beta \mathbf{p}_2 + (1 - \alpha - \beta)\mathbf{p}_0$$

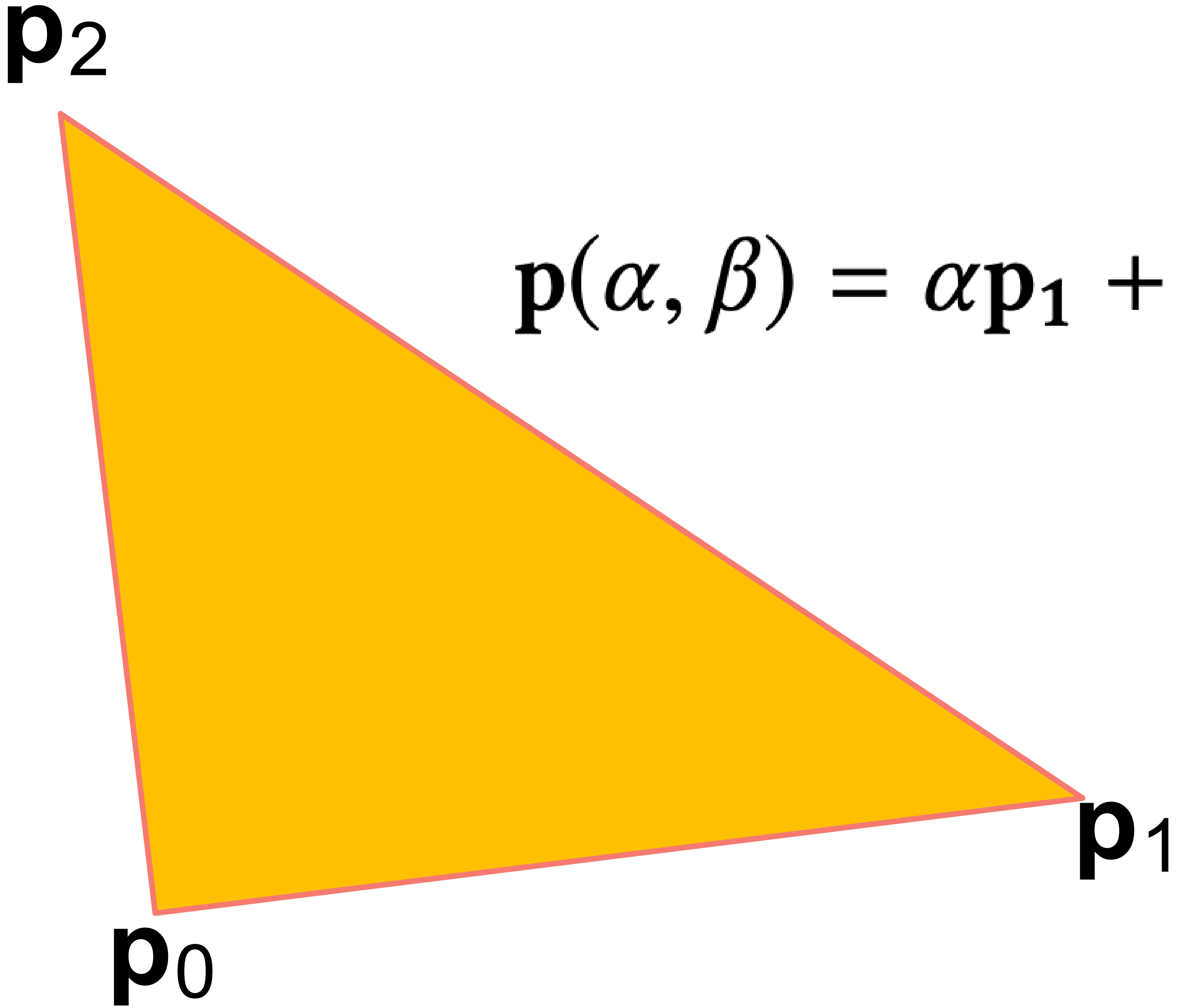


Barycentric Coordinates



$$\mathbf{p}(\alpha, \beta) = \alpha\mathbf{p_1} + \beta\mathbf{p_2} + (1 - \alpha - \beta)\mathbf{p_0}$$

Barycentric Coordinates



$$\mathbf{p}(\alpha, \beta) = \alpha\mathbf{p_1} + \beta\mathbf{p_2} + (1 - \alpha - \beta)\mathbf{p_0}$$

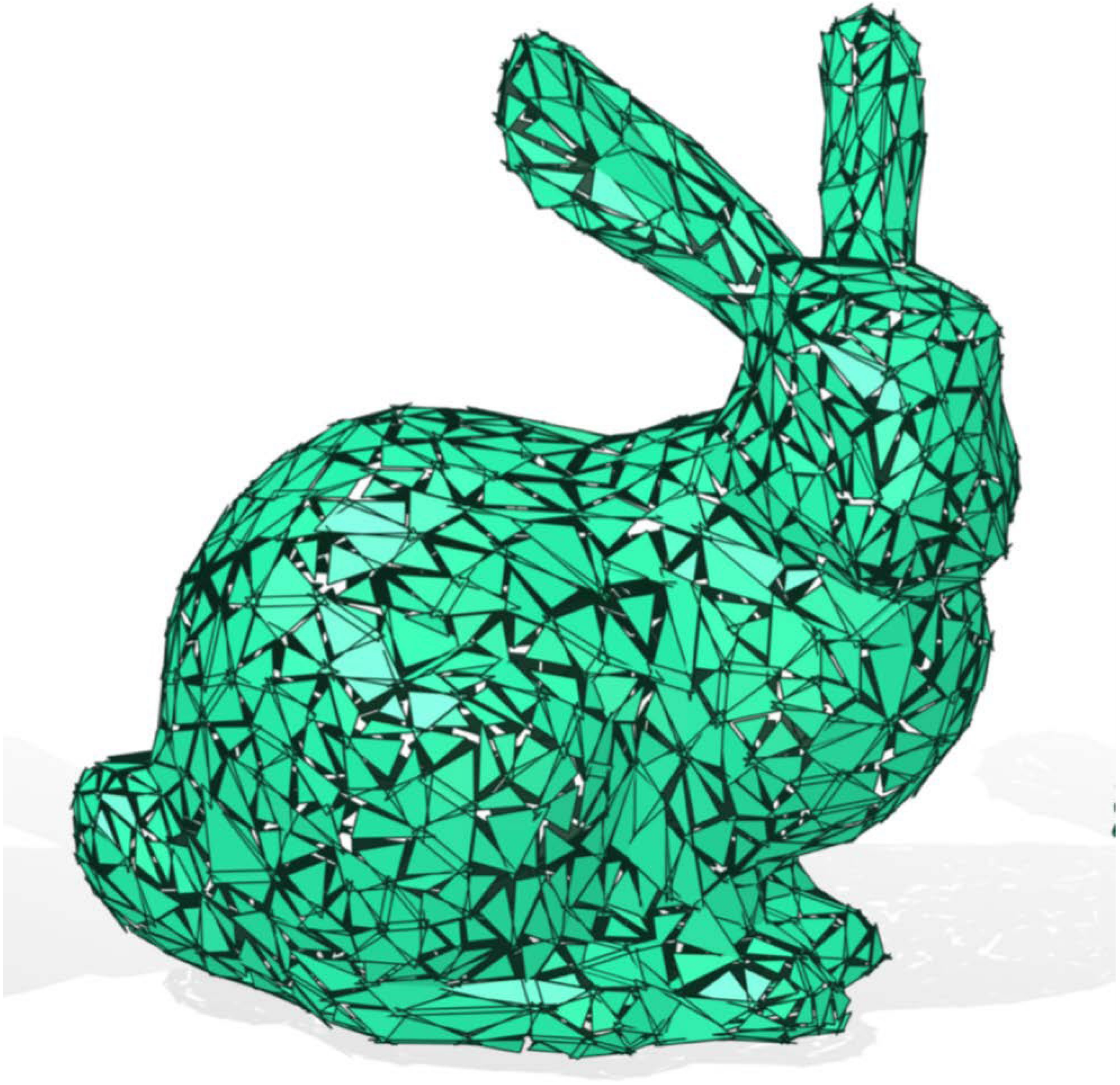
$$\alpha \geq 0$$

$$\beta \geq 0$$

$$\alpha + \beta \leq 1$$

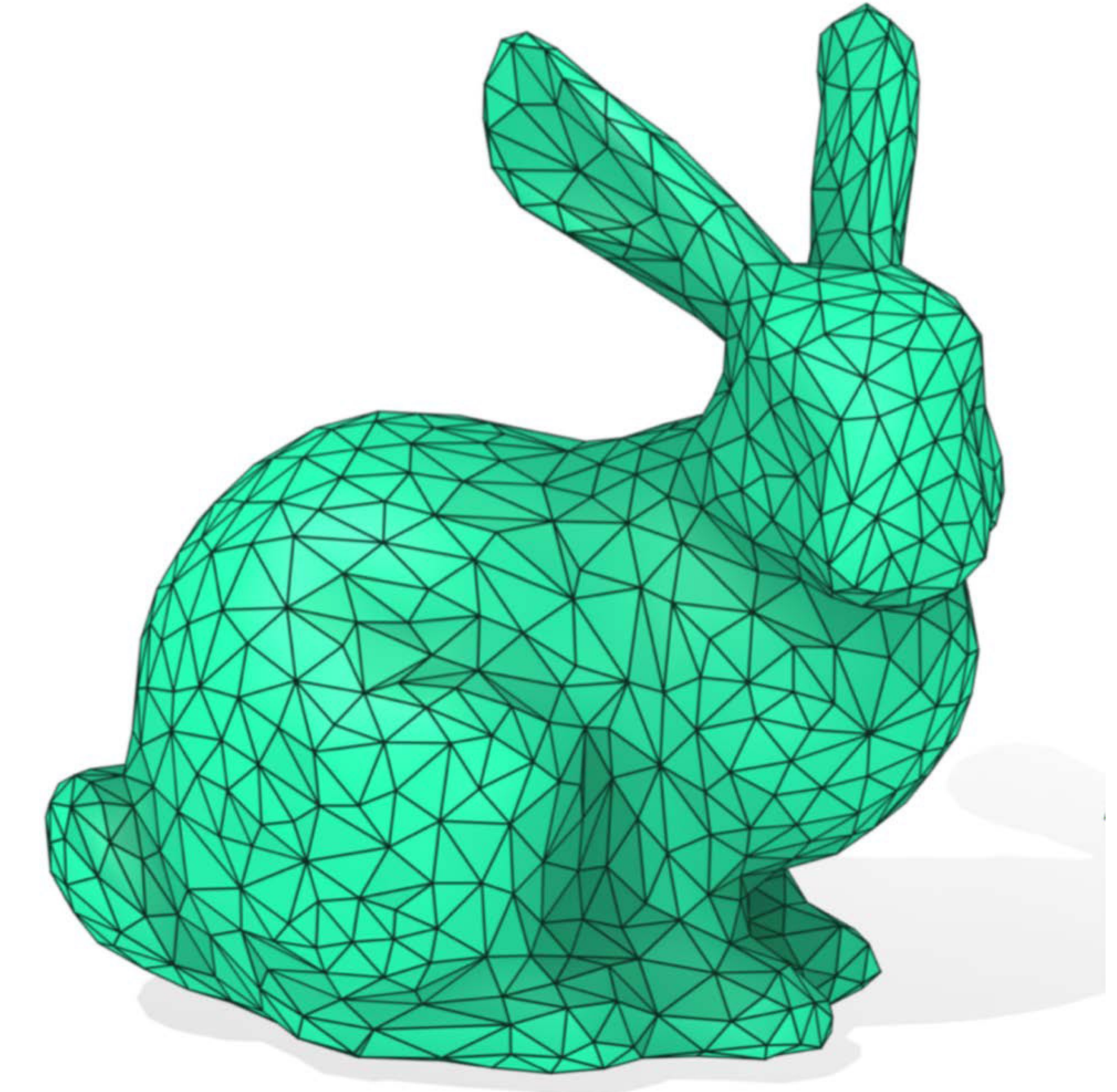
$$\gamma = 1 - \alpha - \beta$$

Triangle Soup



Triangle Mesh

Soup



Mesh

Topology

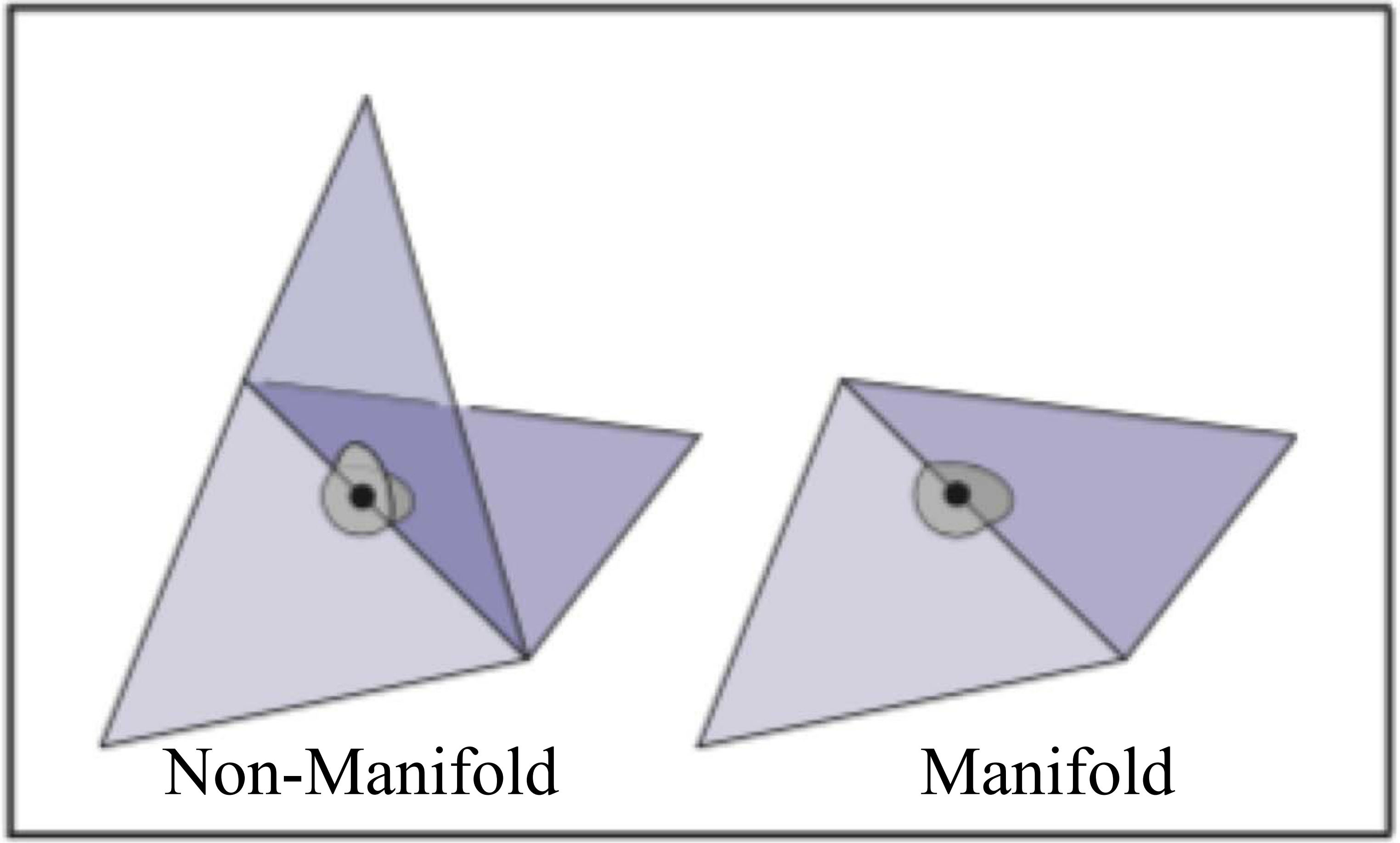
Topology is concerned with the connectivity of a mesh

Many algorithms are easier to implement or more efficient when connectivity data is available (we'll see an example of this later on).

We are going to assume that our meshes are *2-manifolds*

Manifold

A *2-manifold* is a surface for which the neighbourhood around any point can be flattened onto the plane



Manifold

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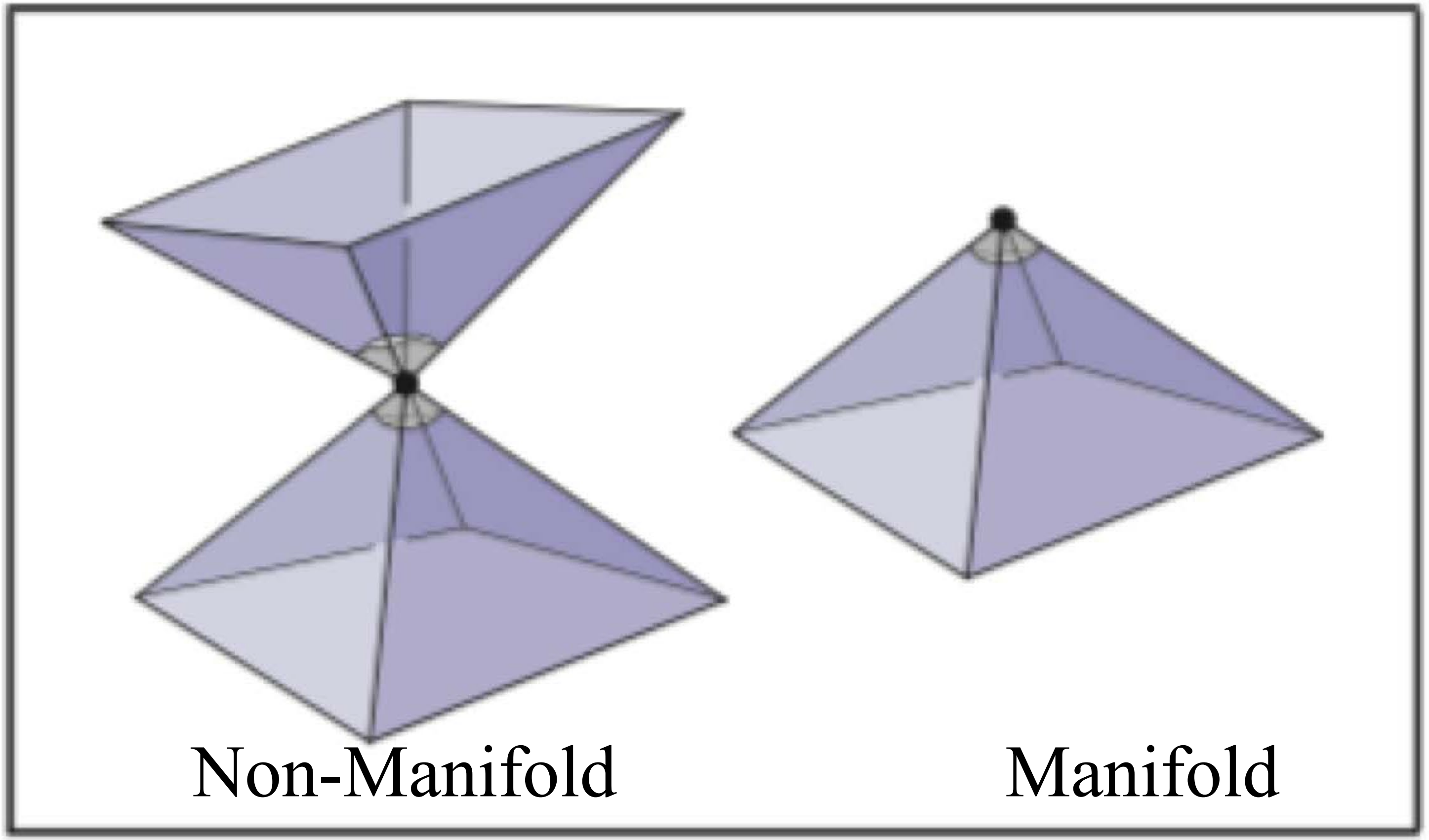

Manifold

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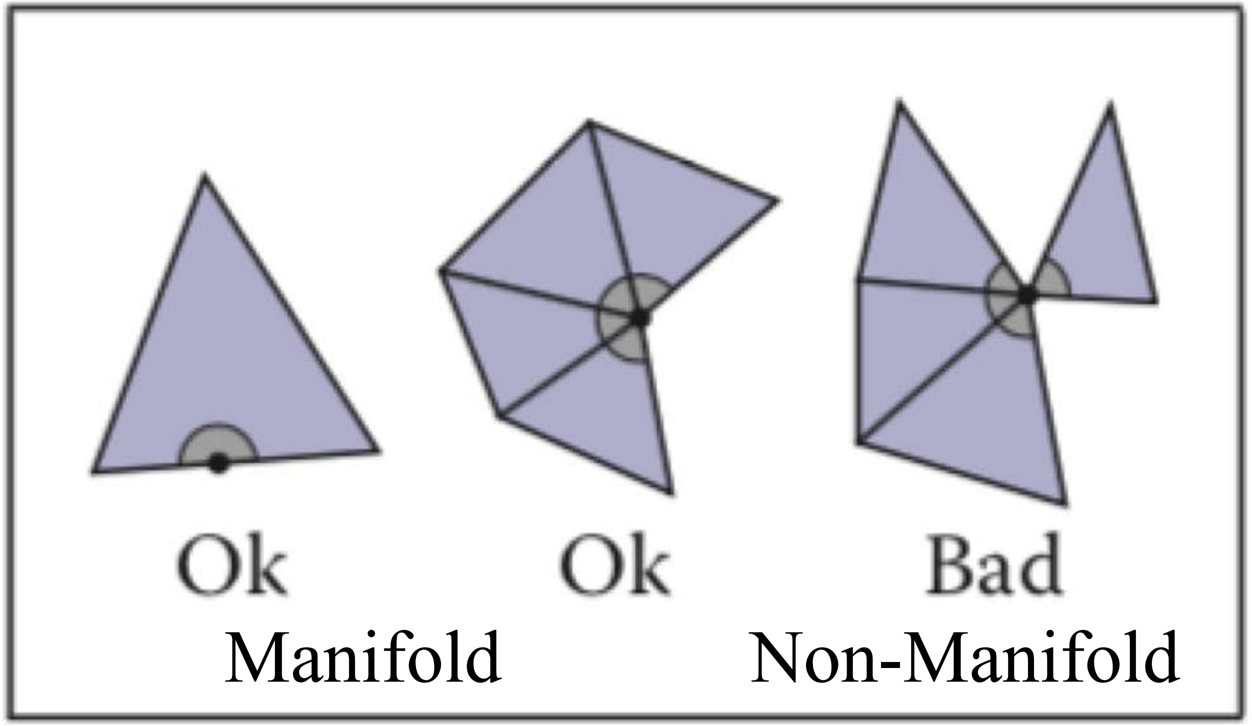


Manifold

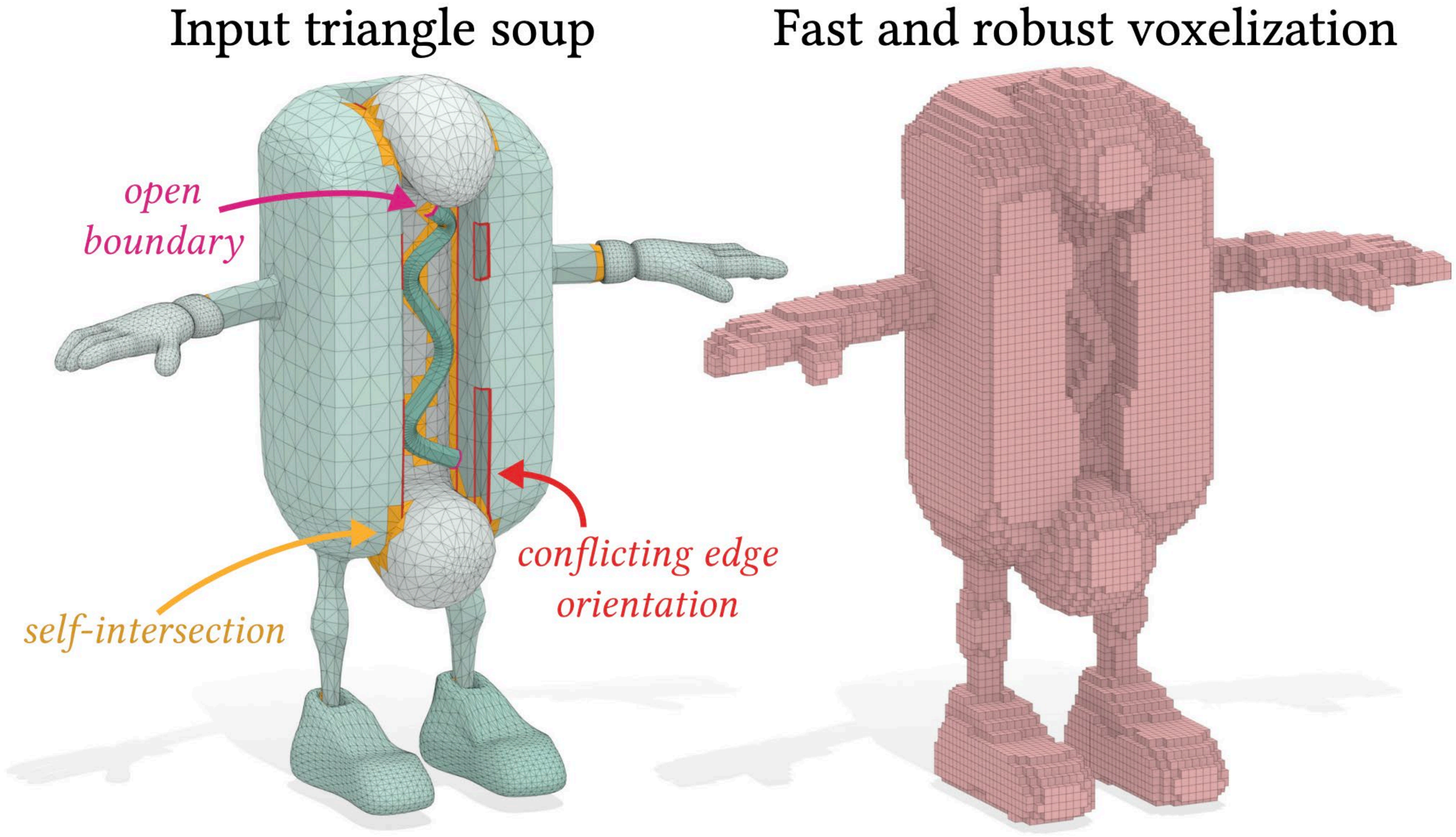
A *2-manifold* is a surface for which the neighbourhood around any point can be flattened onto the plane

Manifold with boundaries

https://www.dgp.toronto.edu/projects/cubic-stylization/

Watertight

Watertight meshes have no holes



https://www.dgp.toronto.edu/projects/fast-winding-numbers/

Geometry

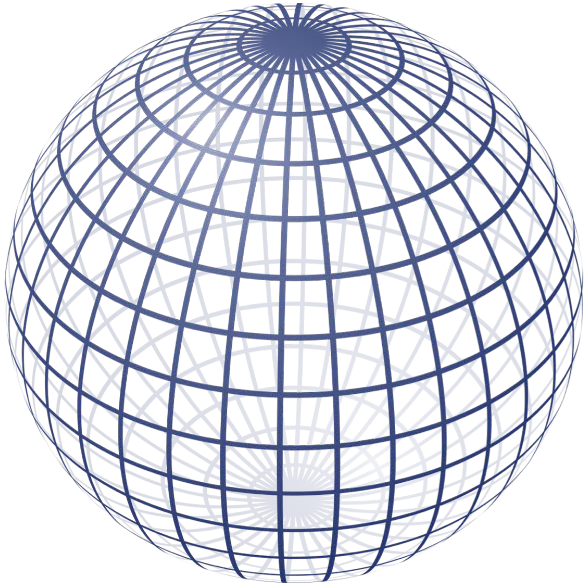
Geometrically, a mesh is a **piecewise planar** surface

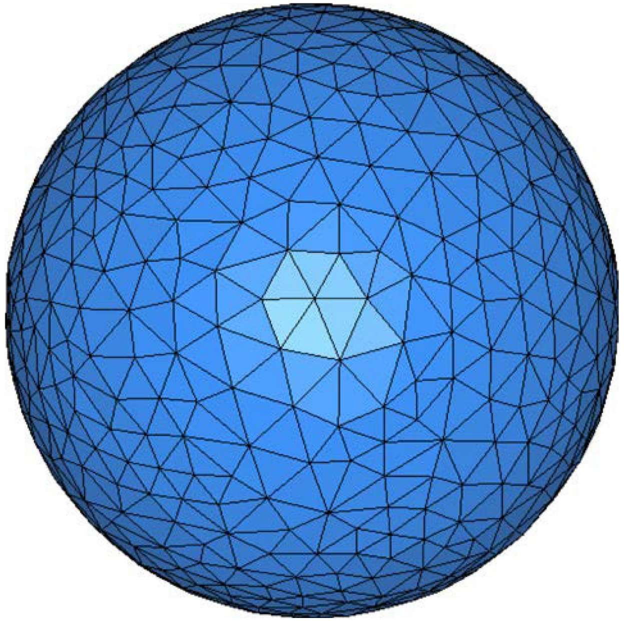
-almost everywhere, it is planar

-exceptions are at the edges where triangles join

Often, it's a piecewise planar approximation of a smooth surface

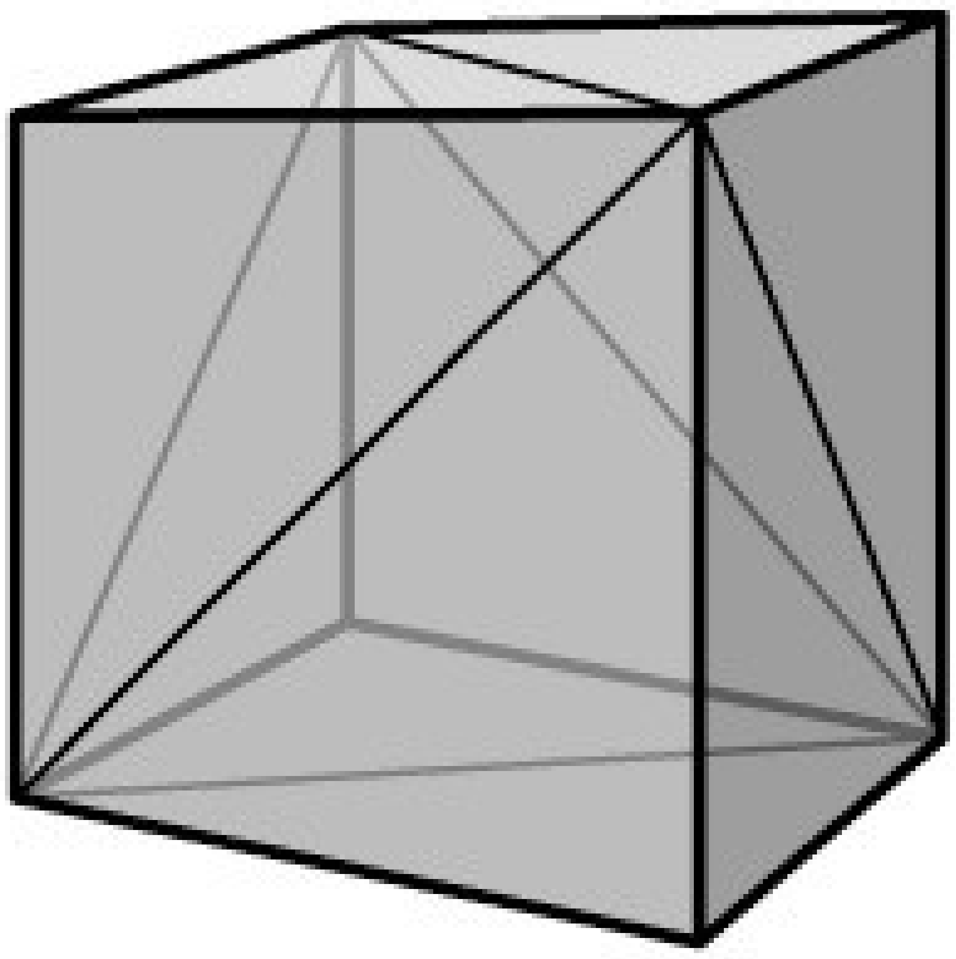
$$x = r\cos\phi\sin\theta,$$
$$y = r\sin\phi\sin\theta,$$
$$z = r\cos\theta.$$





Rineau & Yvinec CGAL manual

Examples of Meshes



12 triangles, 8 vertices

Examples of Meshes

10 million triangles from a high-resolution 3D scan

Traditional Thai sculpture—scan by XYZRGB, inc., image by MeshLab project

About a trillion triangles from automatically processed satellite and aerial photography.

Storing Triangle Meshes

What do we care about ?

Storing Triangle Meshes

What do we care about?

1. Compactness
2. Efficiency of queries
    - all vertices of a triangle
    - all triangles around a vertex
    - neighboring triangles of a triangles

Data Structures for Triangle Meshes

Separate Triangles (soup)

Indexed Triangle Set

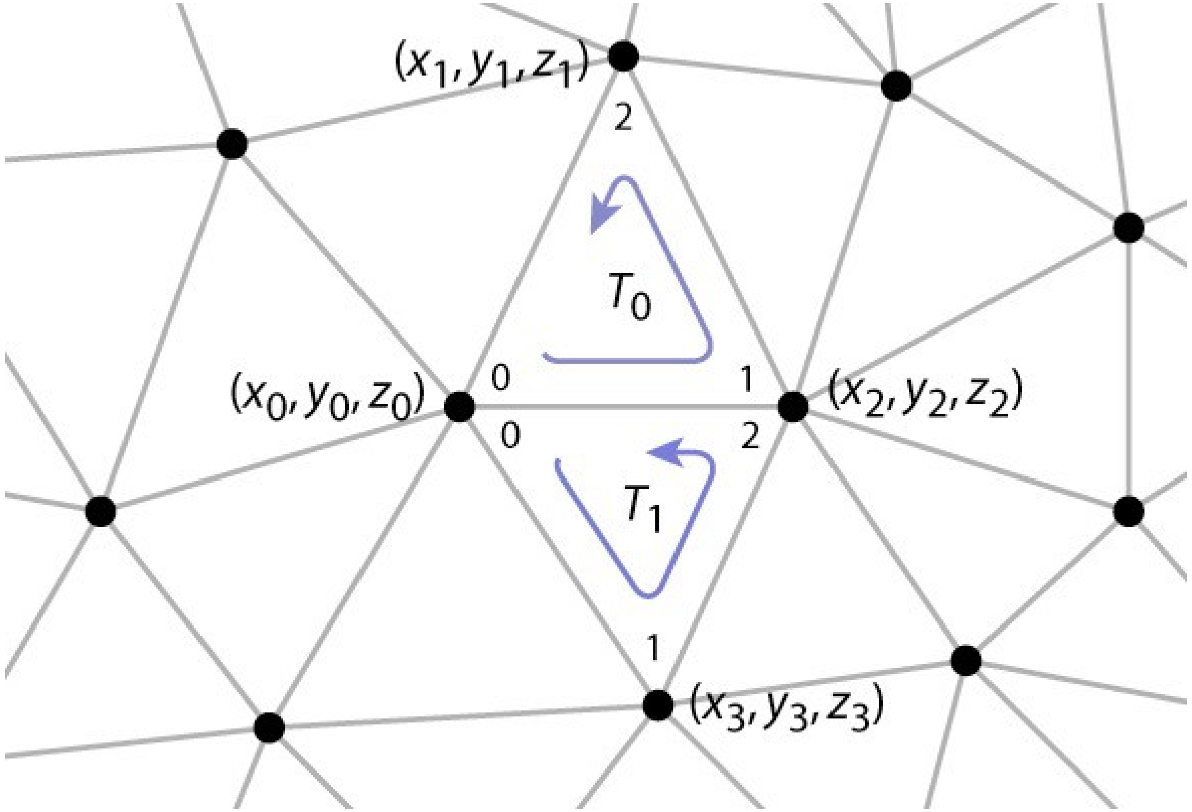
Triangle-Neighbour Data Structure

Winged-Edge Data Structure

Half-Edge Data Structure

Separate triangles

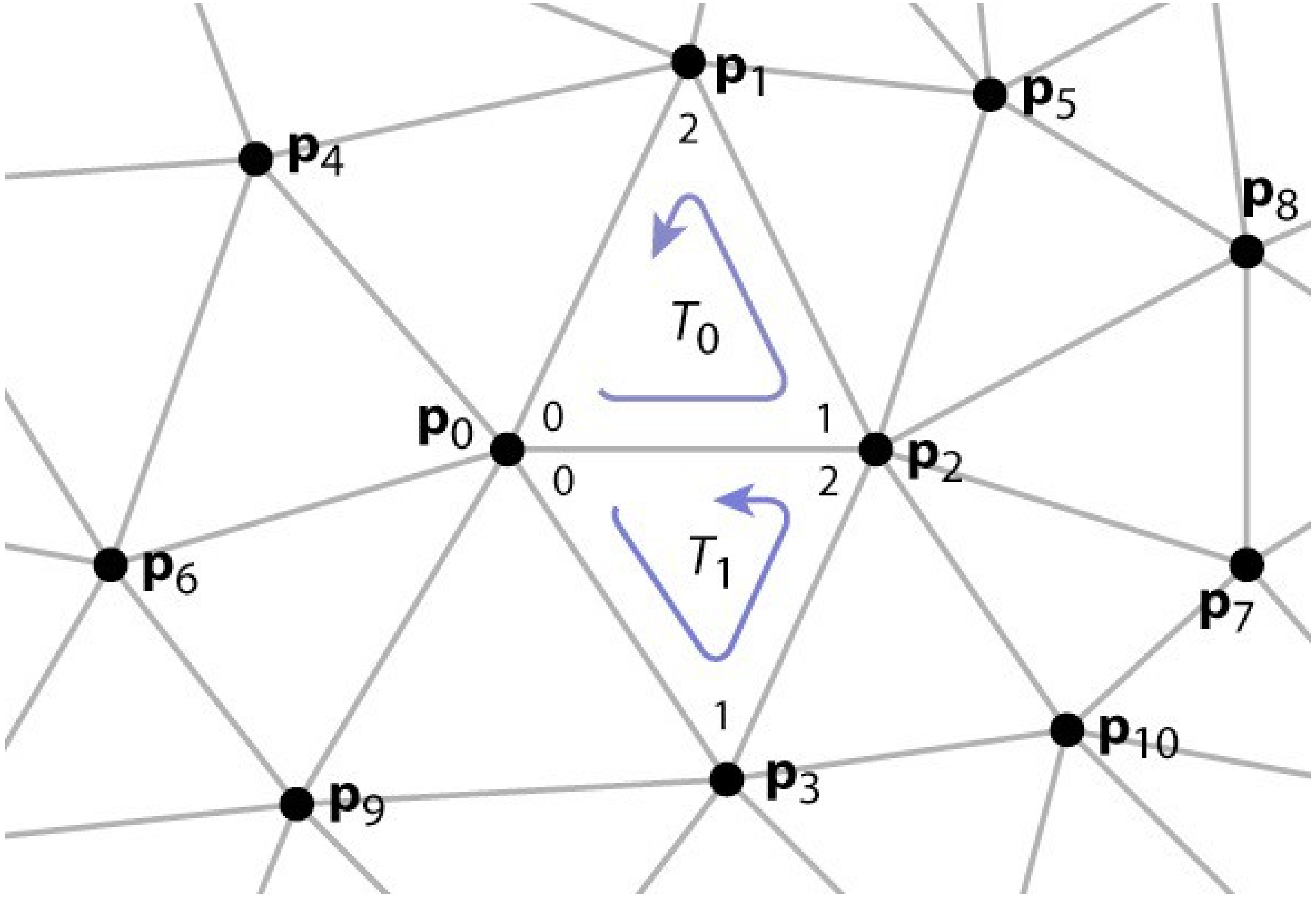
	[0]	[1]	[2]
tris[0]	x_0, y_0, z_0	x_2, y_2, z_2	x_1, y_1, z_1
tris[1]	x_0, y_0, z_0	x_3, y_3, z_3	x_2, y_2, z_2
	⋮	⋮	⋮



Indexed triangle set

verts[0]	x_0, y_0, z_0
verts[1]	x_1, y_1, z_1
	x_2, y_2, z_2
	x_3, y_3, z_3
	$\vdots$

tInd[0]	0, 2, 1
tInd[1]	0, 3, 2
	$\vdots$

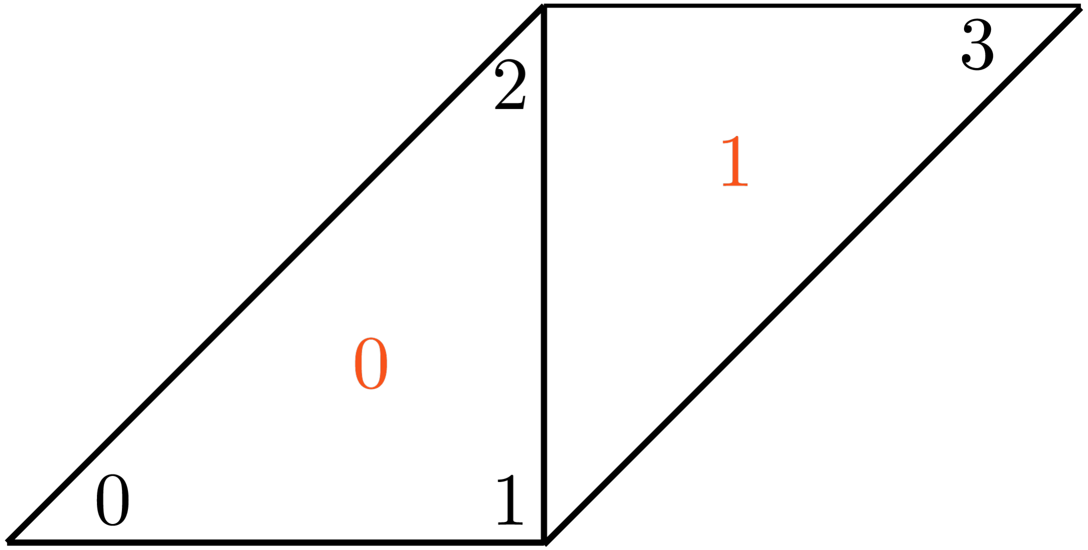


Storing Triangle Meshes

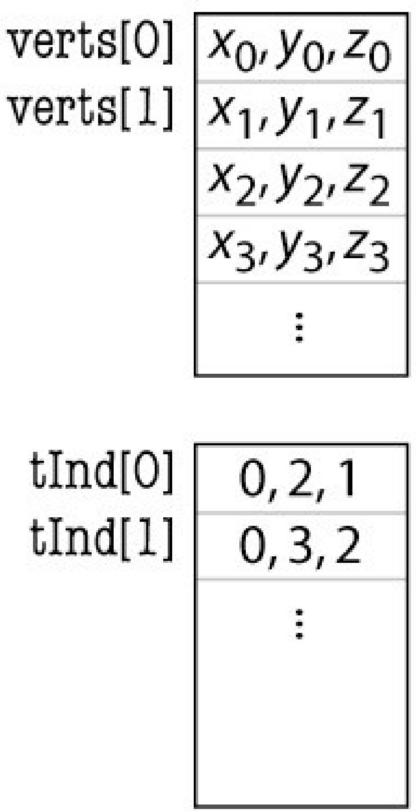
What do we care about ?

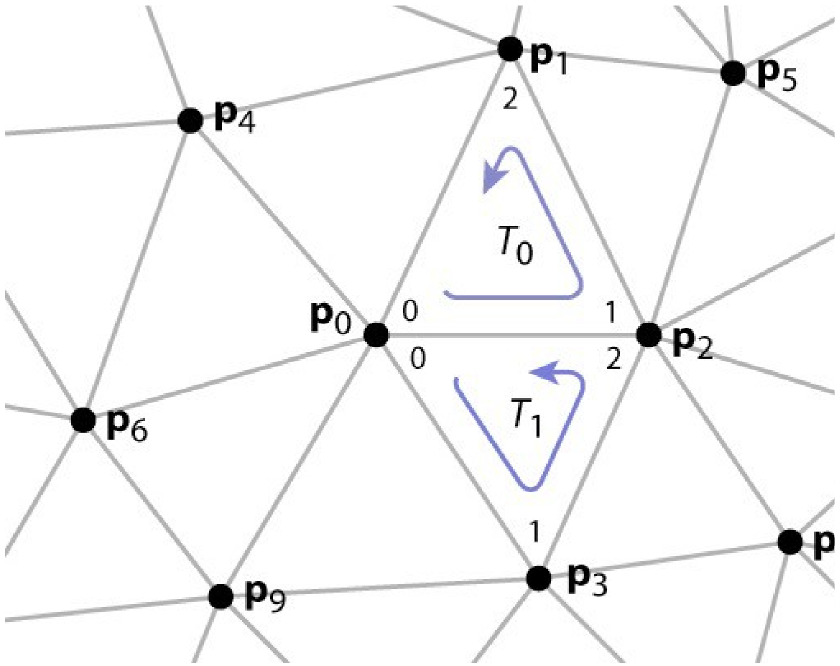
$$V = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

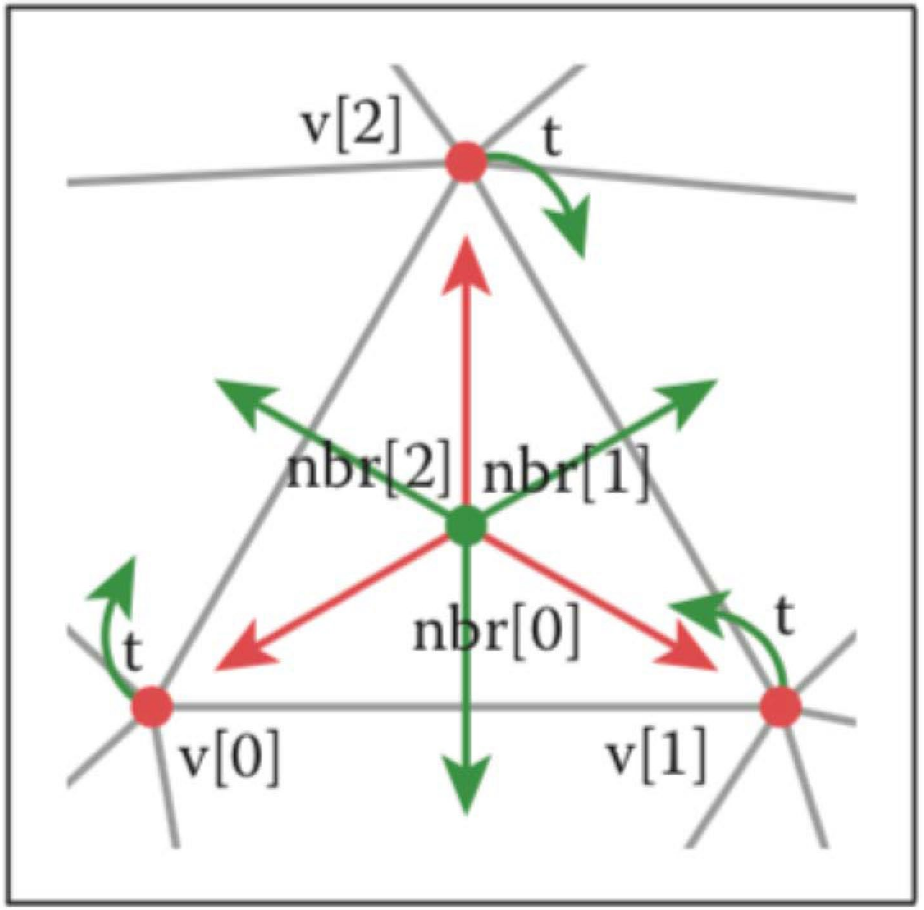
$$F = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 3 & 2 \end{pmatrix}$$



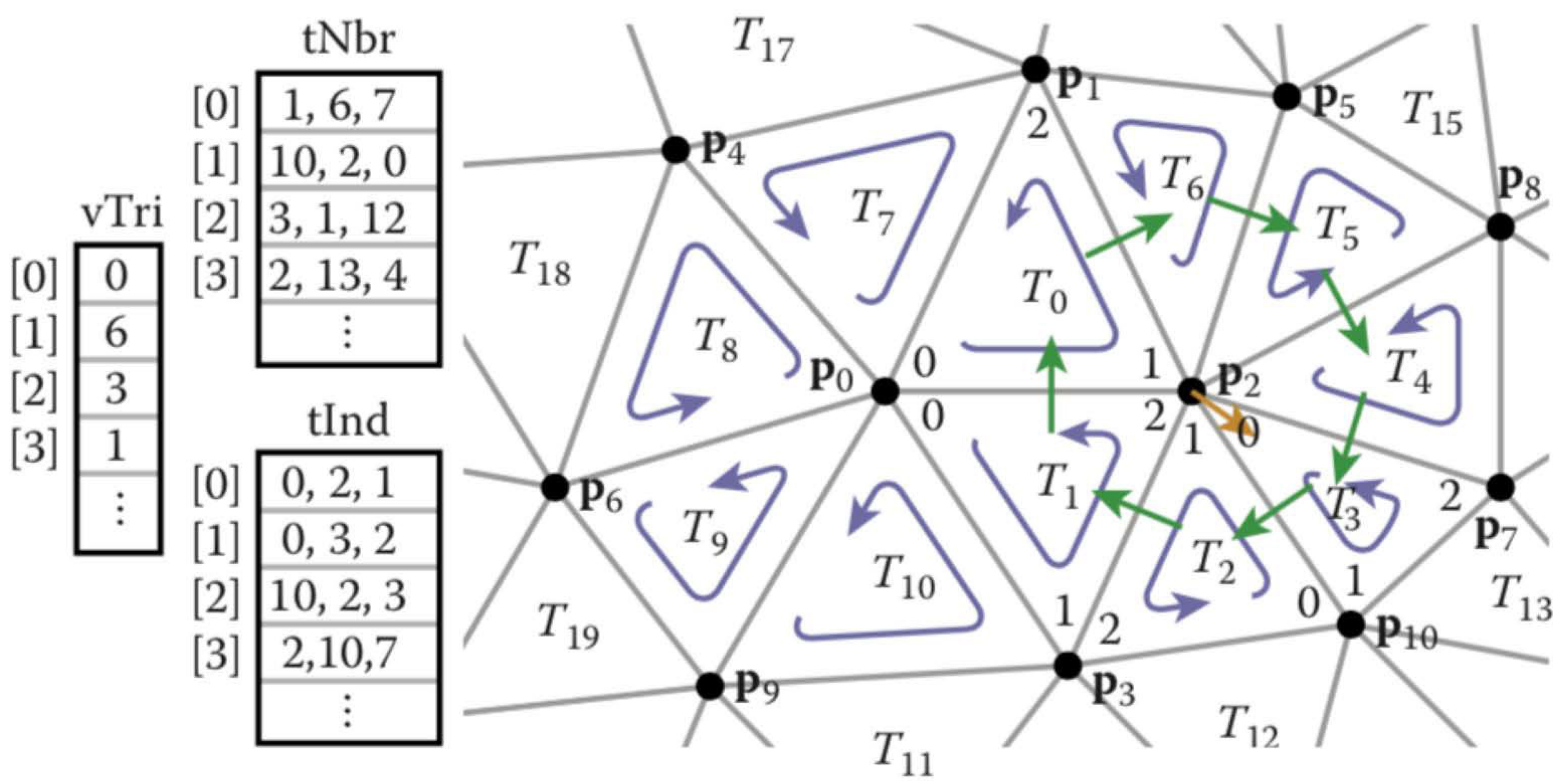
Triangle-Neighbour Data Structure





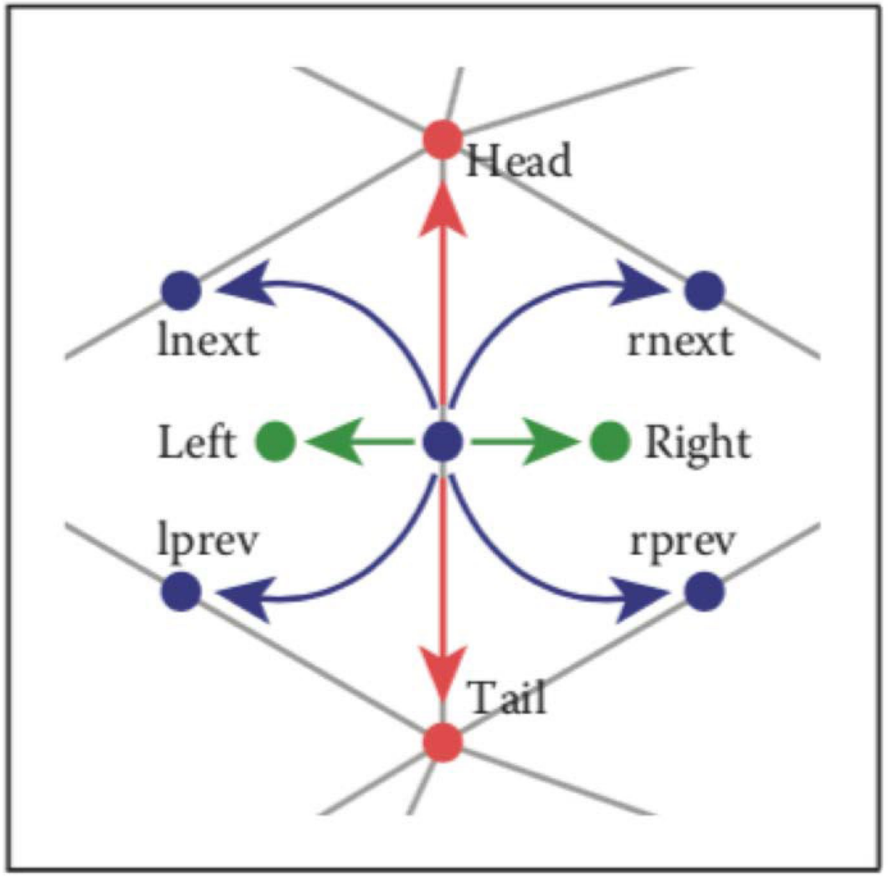


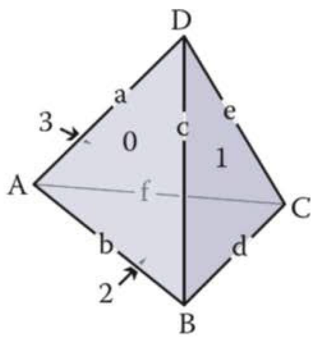
Triangle-Neighbour Data Structure



The kth entry of tNbr points to the neighboring triangle that shares vertices k and k+1

Winged-Edge Data Structure



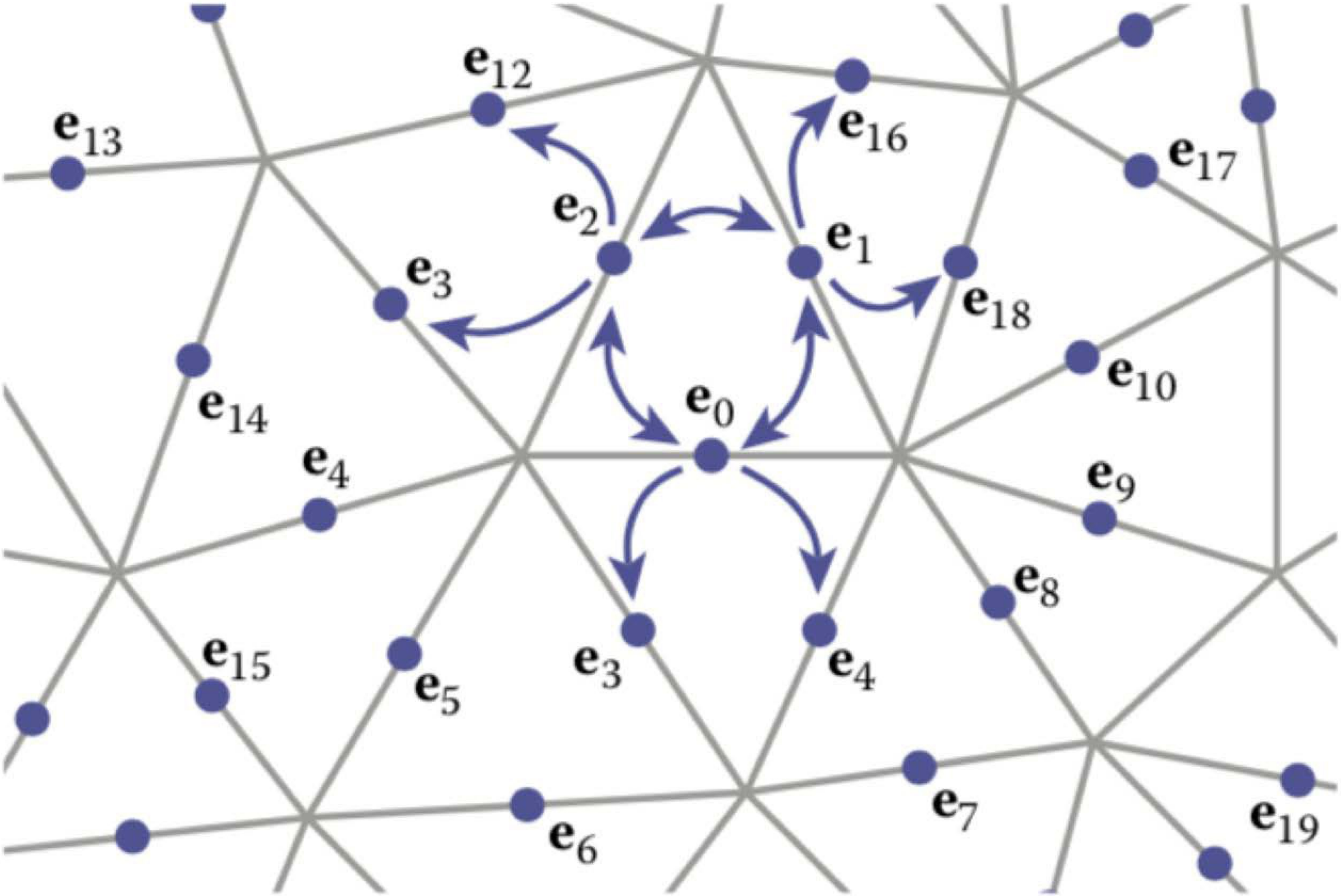


Edge	*Vertex 1*	*Vertex 2*	*Face left*	*Face right*	*Pred left*	*Succ left*	*Pred right*	*Succ right*
a	A	D	3	0	f	e	c	b
b	A	B	0	2	a	c	d	f
c	B	D	0	1	b	a	e	d
d	B	C	1	2	c	e	f	b
e	C	D	1	3	d	c	a	f
f	C	A	3	2	e	e	b	d

Vertex	*Edge*
A	a
B	d
C	d
D	e

Face	*Edge*
0	a
1	c
2	d
3	a

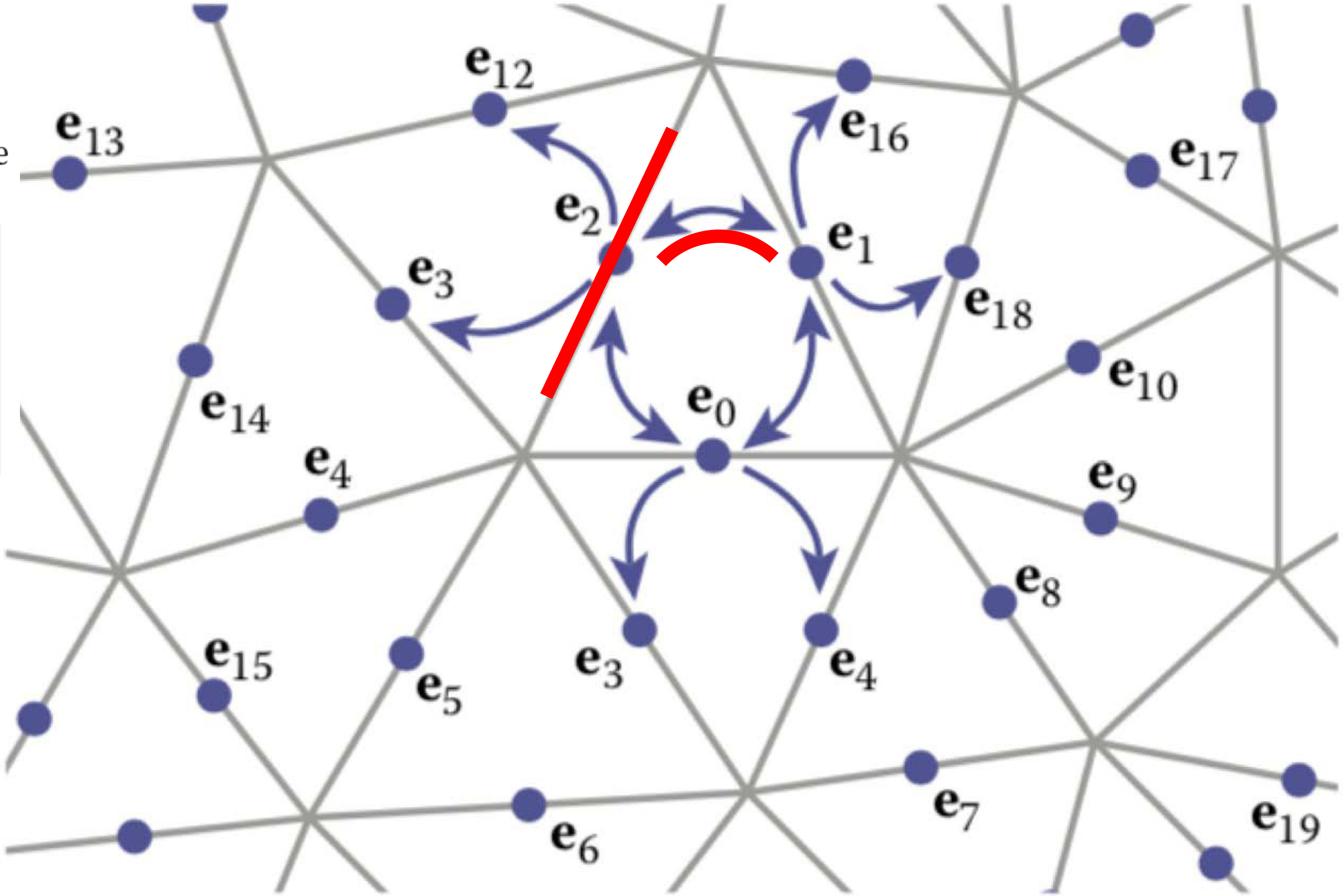
Winged-Edge Data Structure



Winged-Edge Data Structure

Winged-edge table

	ln	rp	lp	rn
[0]	1	4	2	3
[1]	18	0	16	2
[2]	12	$\boxed{1}$	3	0
		⋮		



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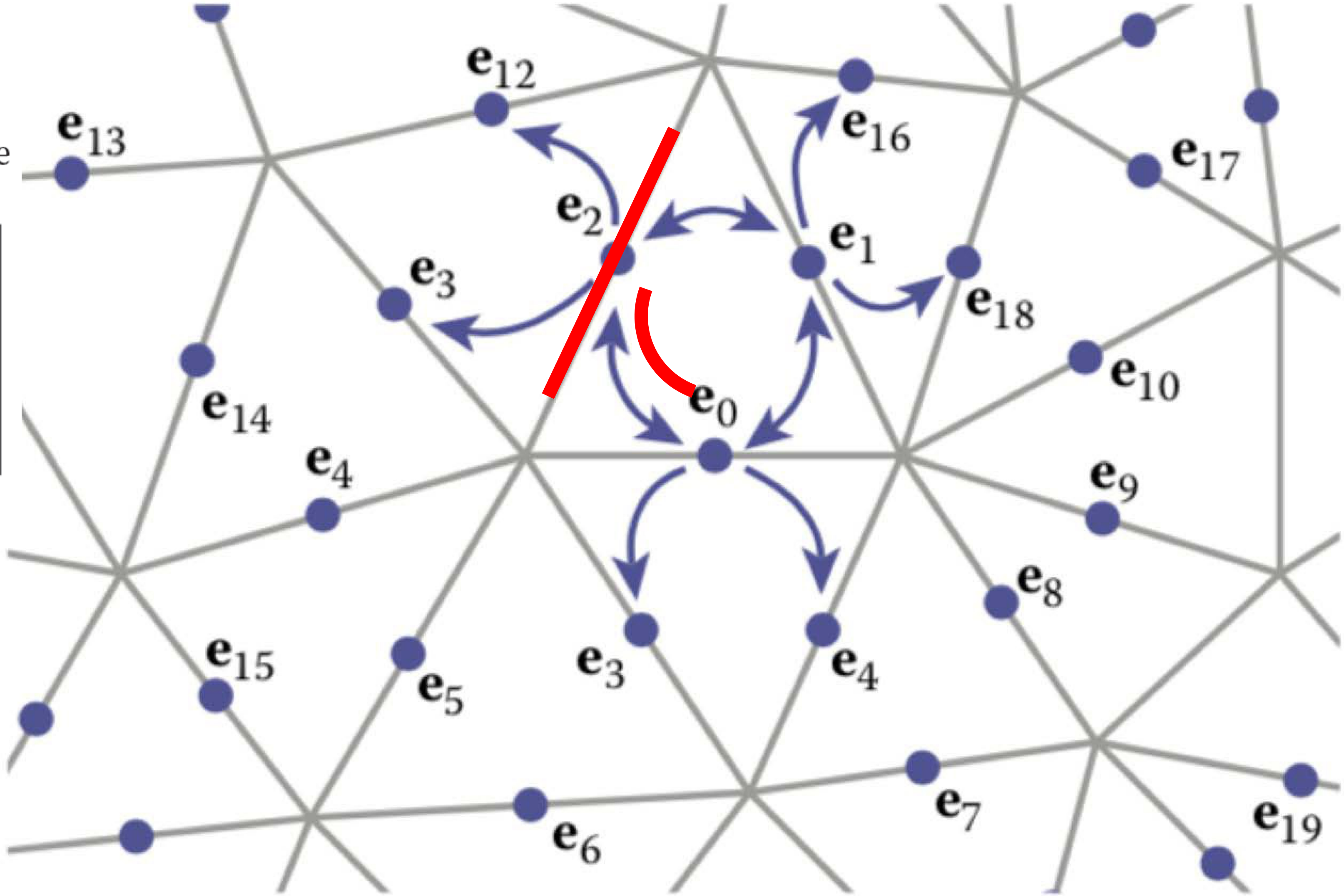
		⋮		



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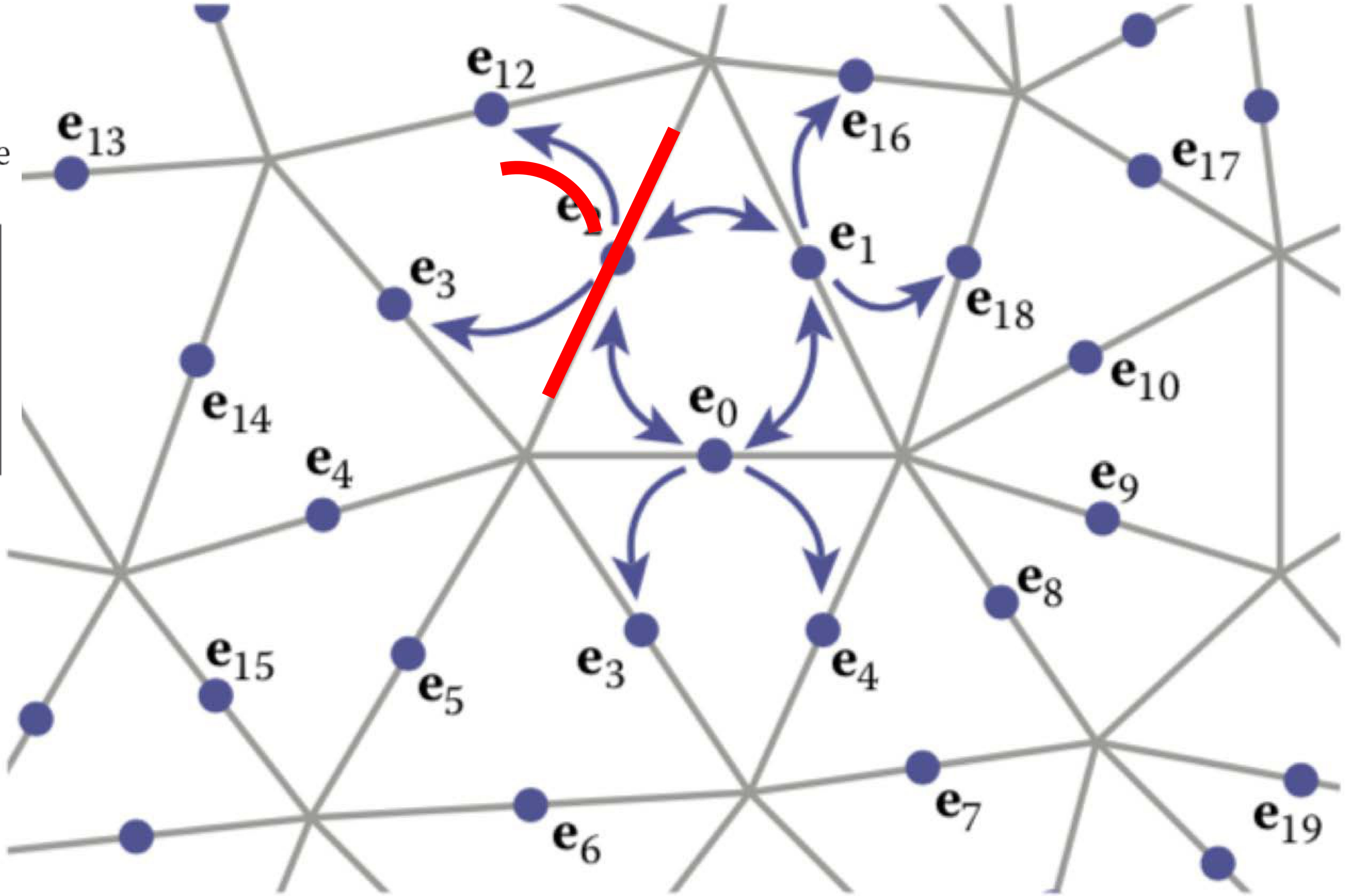
	⋮			



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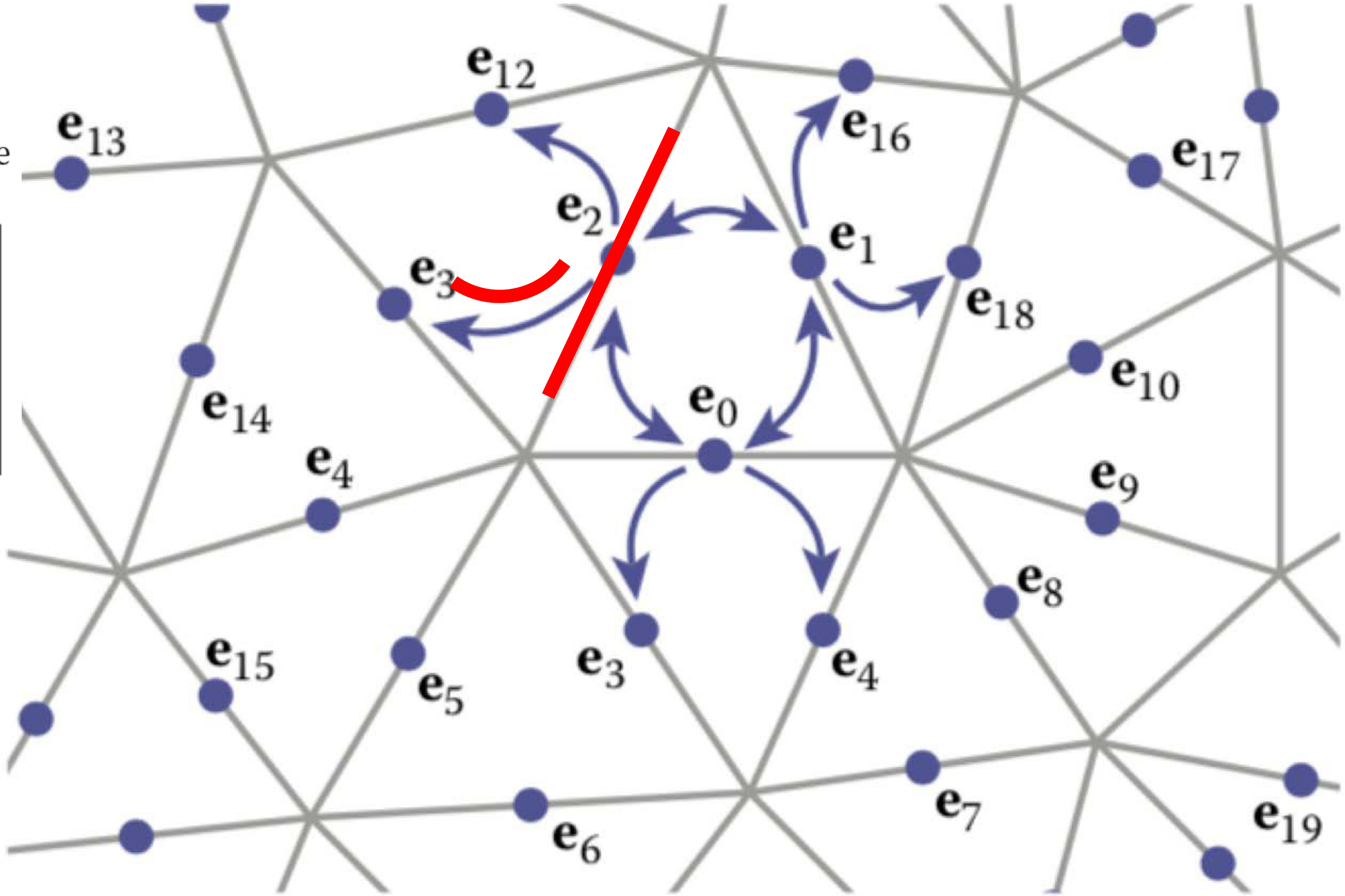

Winged-Edge Data Structure

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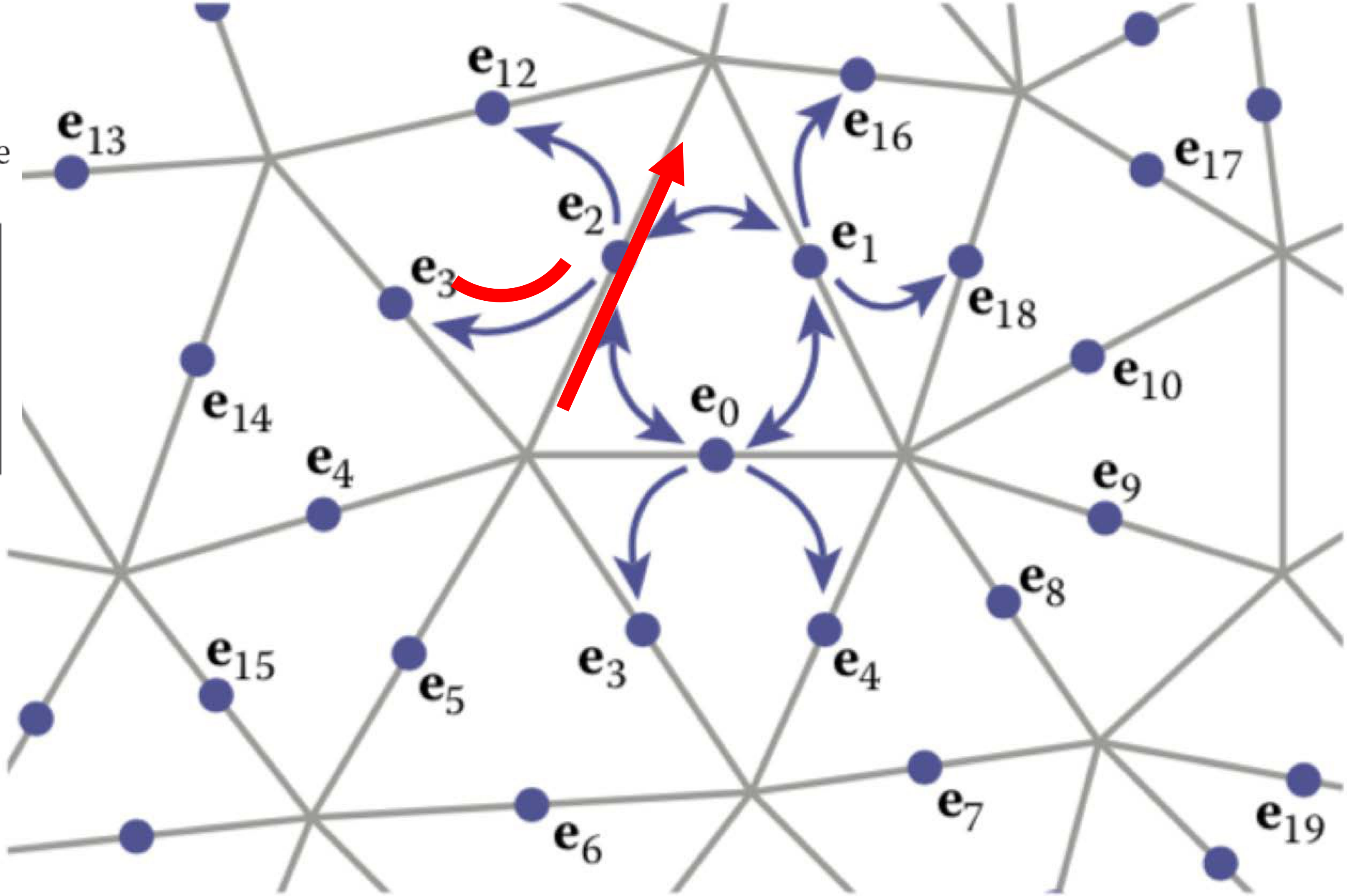
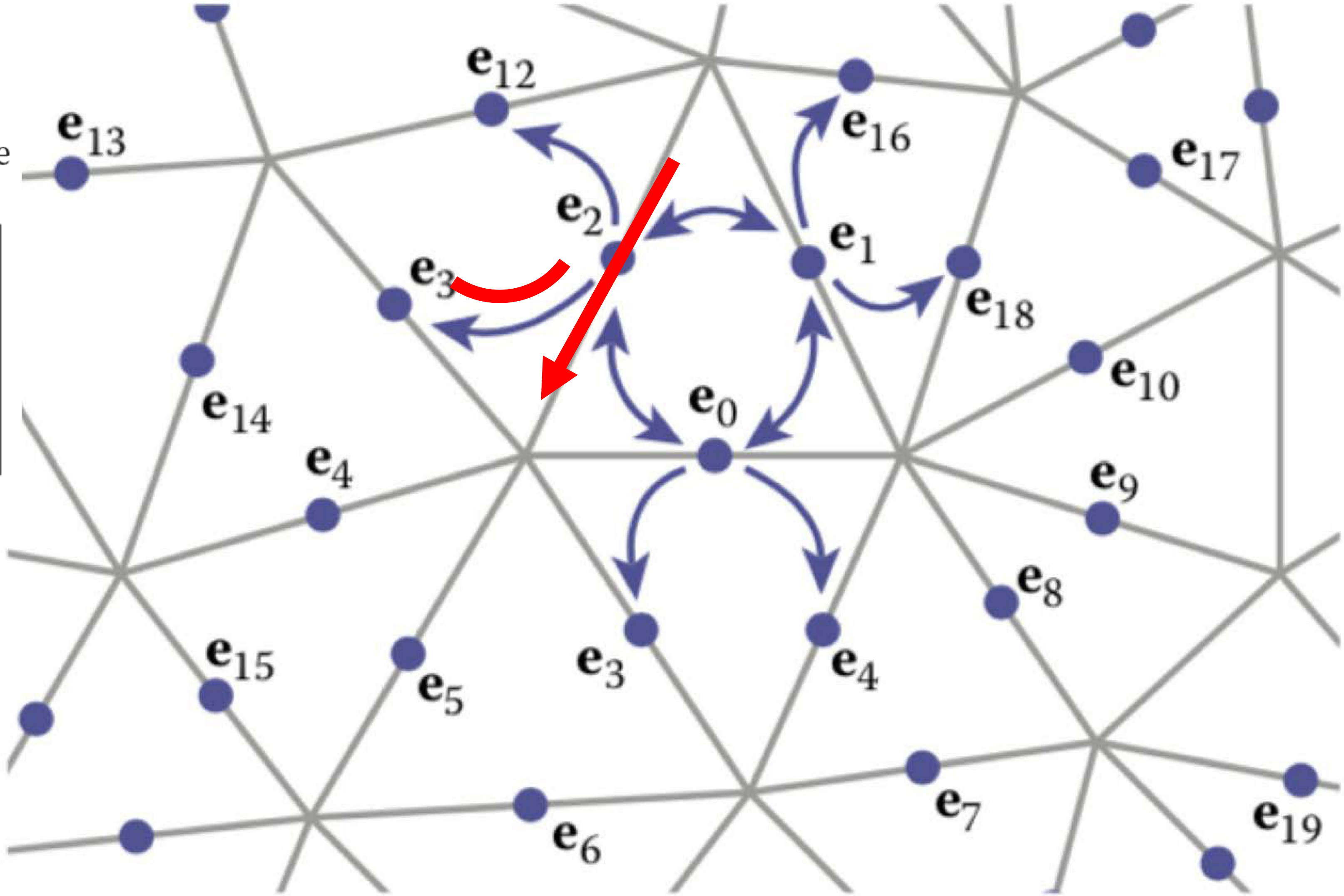
		⋮		



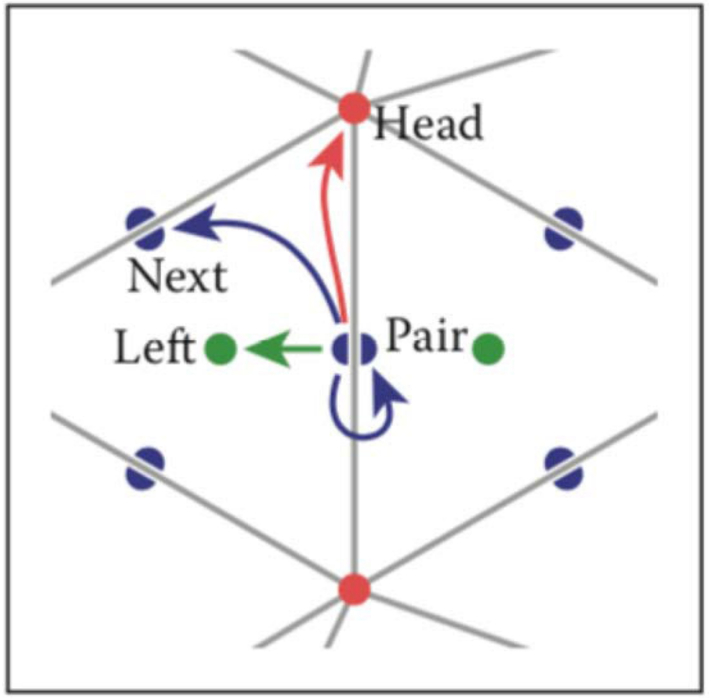
Winged-Edge Data Structure

Winged-edge table

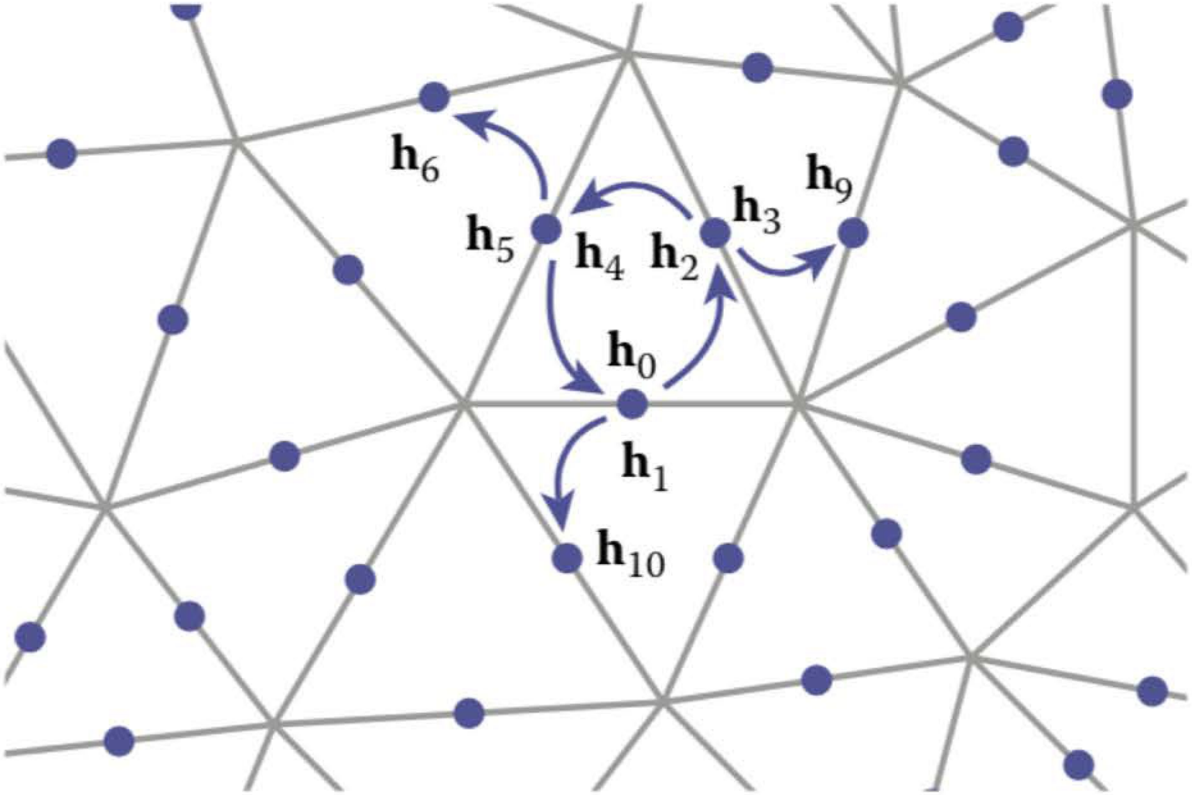
	ln	rp	lp	rn
[0]	1	4	2	3
[1]	18	0	16	2
[2]	12	1	3	0
		⋮		



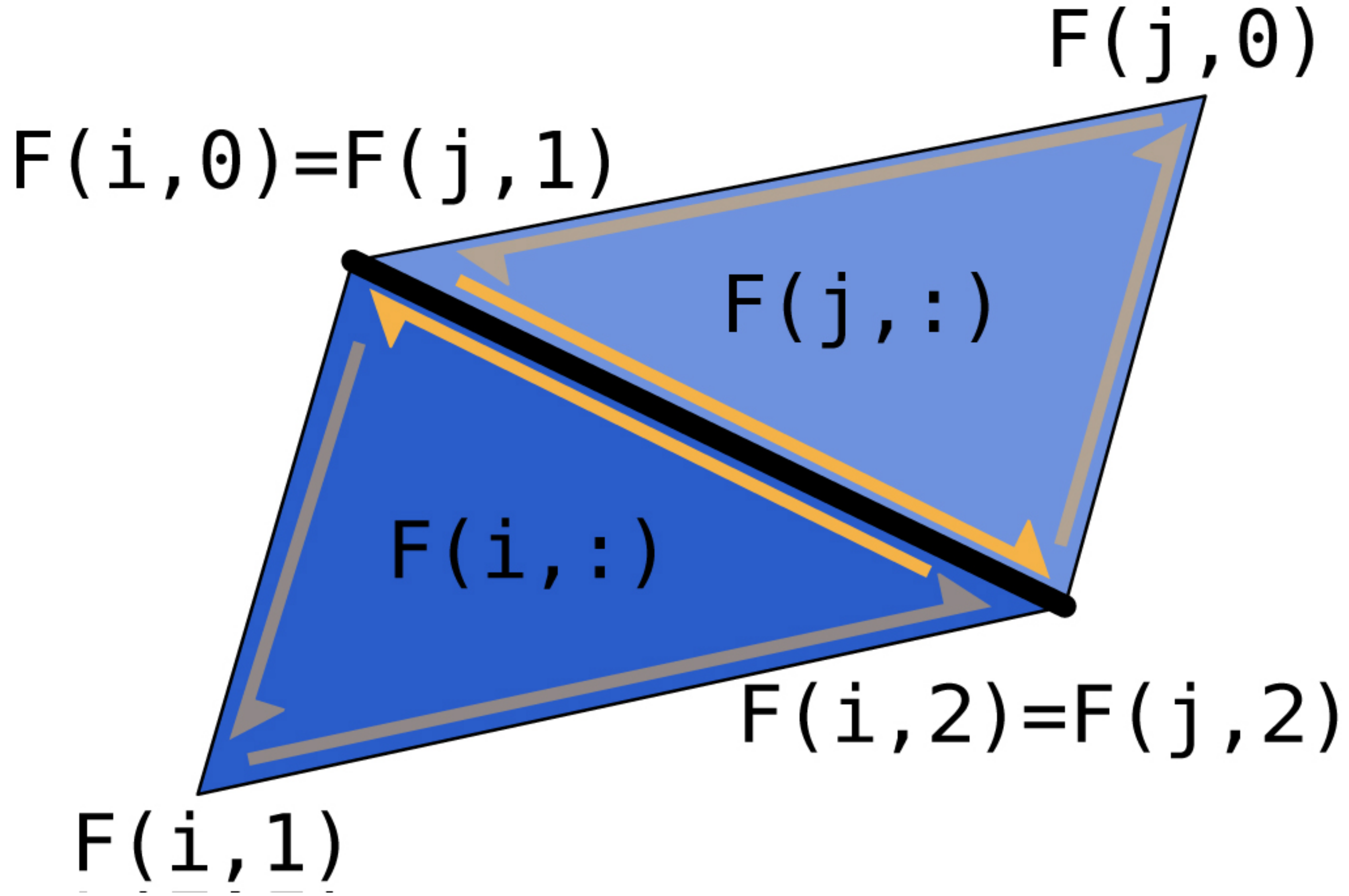
Half-Edge Data Structure



	Pair	Next
hedge[0]	1	2
hedge[1]	0	10
hedge[2]	3	4
hedge[3]	2	9
hedge[4]	5	0
hedge[5]	4	6
	⋮	

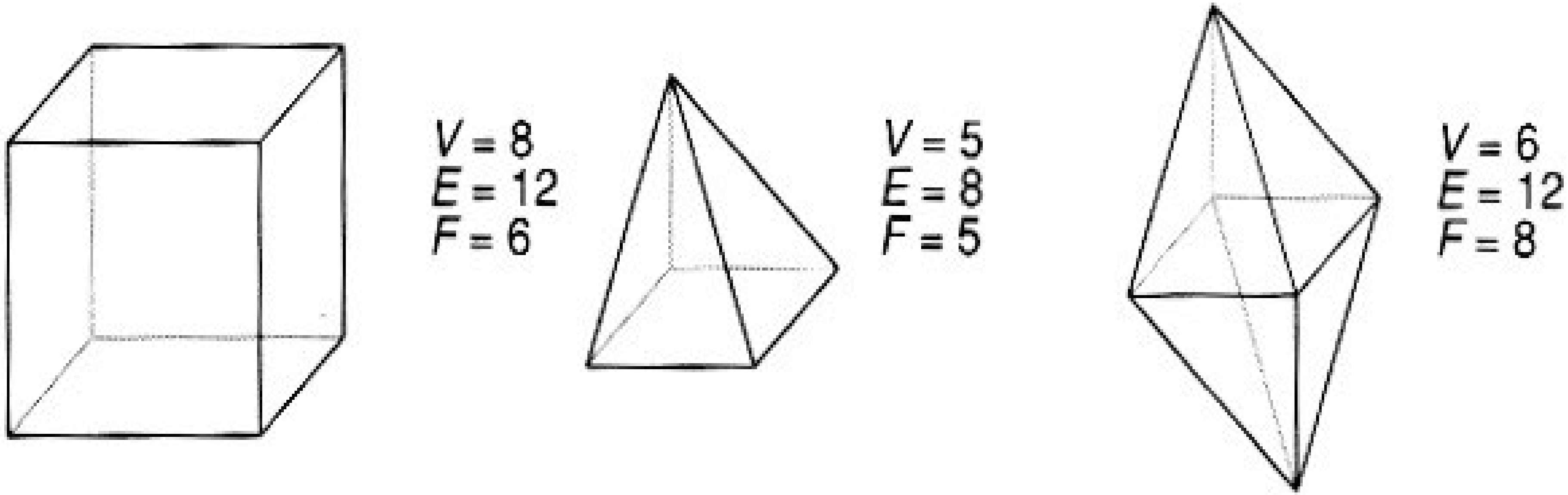


Half-Edge Data Structure



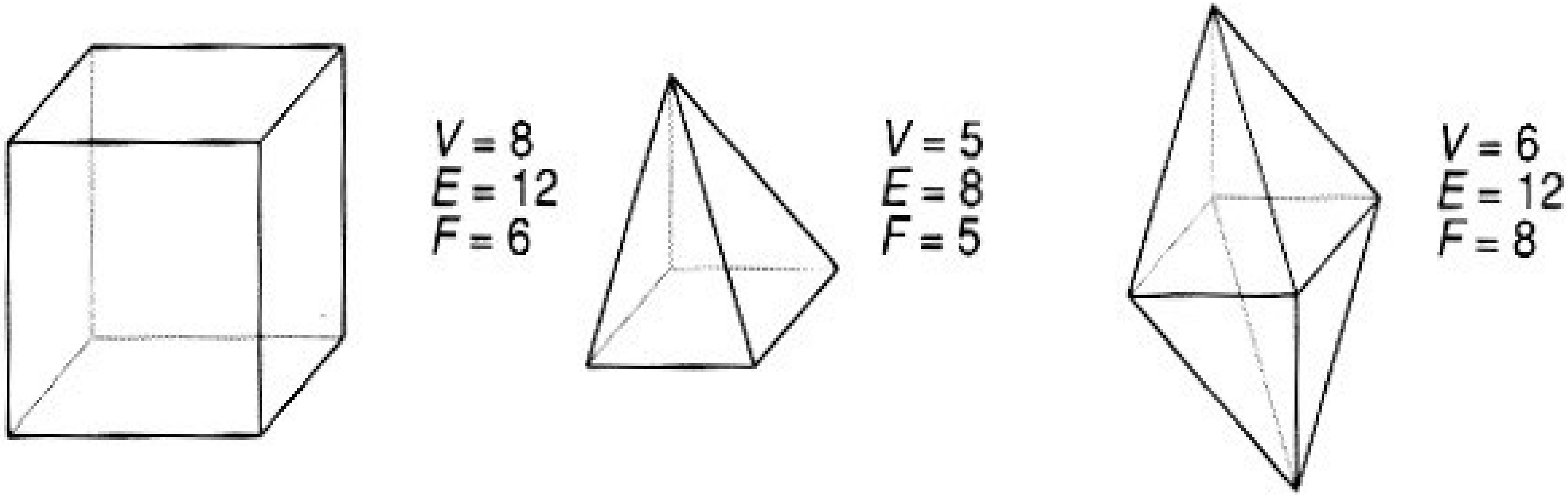
Relationships between primitive Types

What is the relationship between the number of vertices, the number of edges and the number of triangles in a mesh?



Relationships between primitive Types

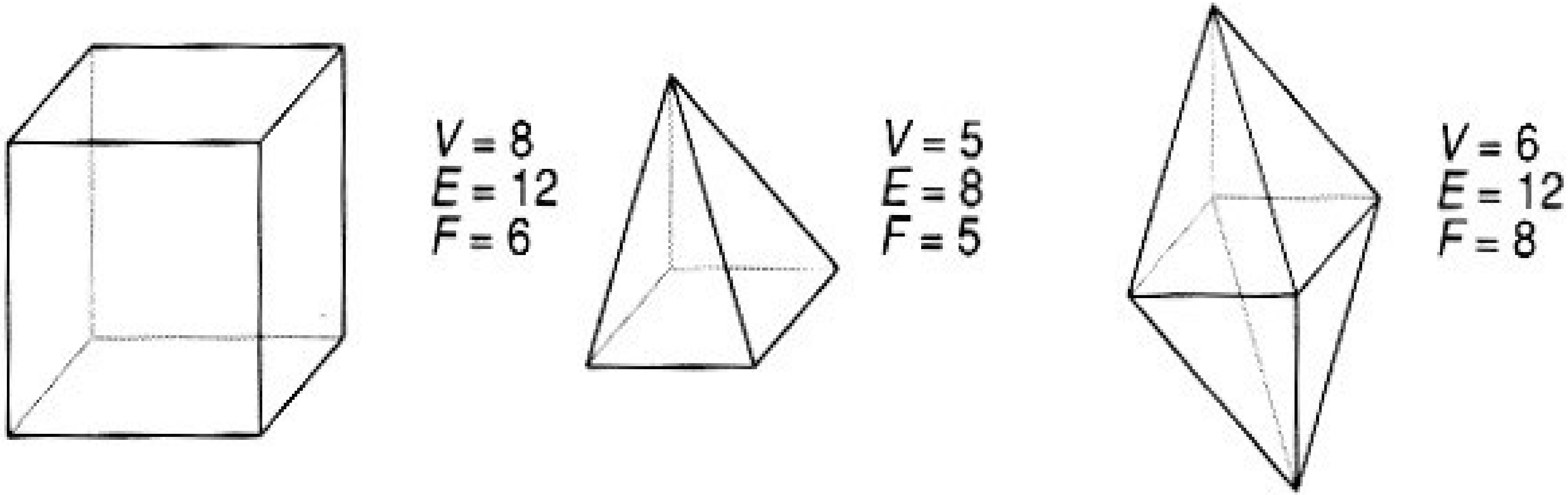
Euler's Formula: $V + F - E = 2$
(closed triangle mesh)



Relationships between primitive Types

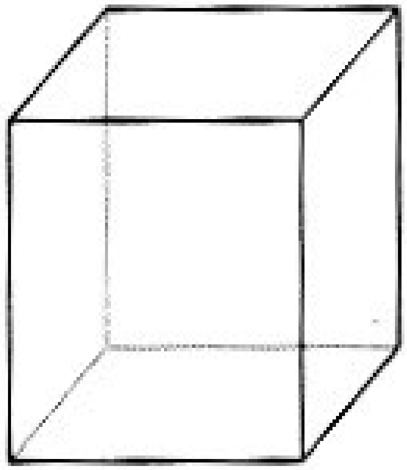
$$3F = 2E$$

(closed triangle mesh)

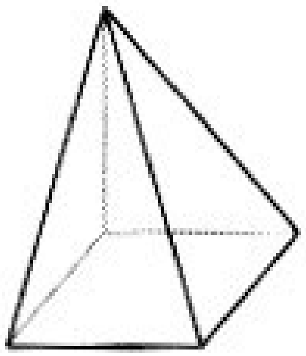


Relationships between primitive Types

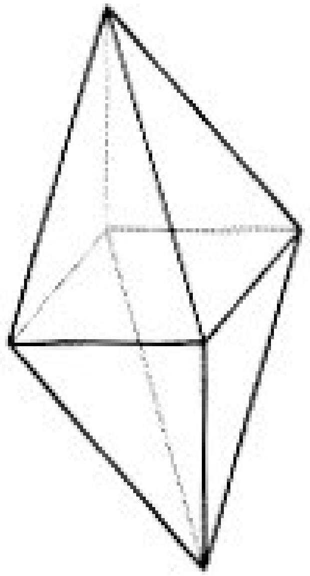
$$\text{number of half edges} = 2E$$



$V = 8$
$E = 12$
$F = 6$



$V = 5$
$E = 8$
$F = 5$

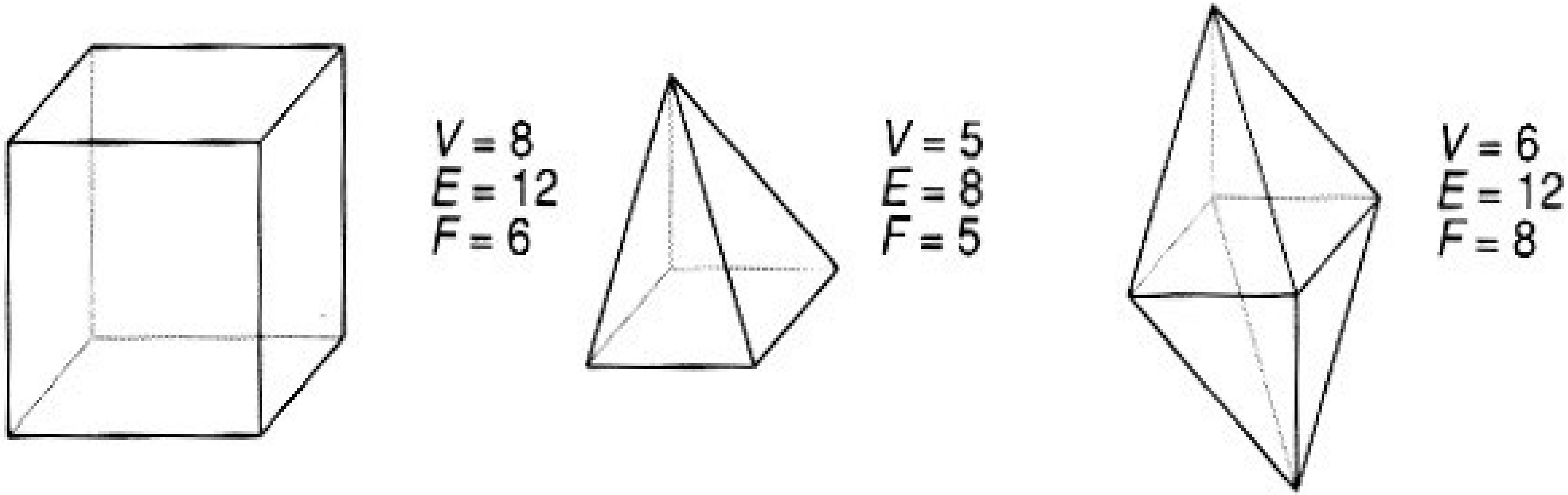


$V = 6$
$E = 12$
$F = 8$

Relationships between primitive Types

$$\text{number of half edges} = 2E$$

$$\text{number of half edges} = 3F$$



Data on meshes

Often need to store additional information besides just the geometry

Can store additional data at faces, vertices, or edges

Examples

- colours stored on faces, for faceted objects
- information about sharp creases stored at edges
- any quantity that varies continuously (without sudden changes, or discontinuities) gets stored at vertices

Key types of vertex data

Positions

- at some level this is just another piece of data
- position varies continuously between vertices

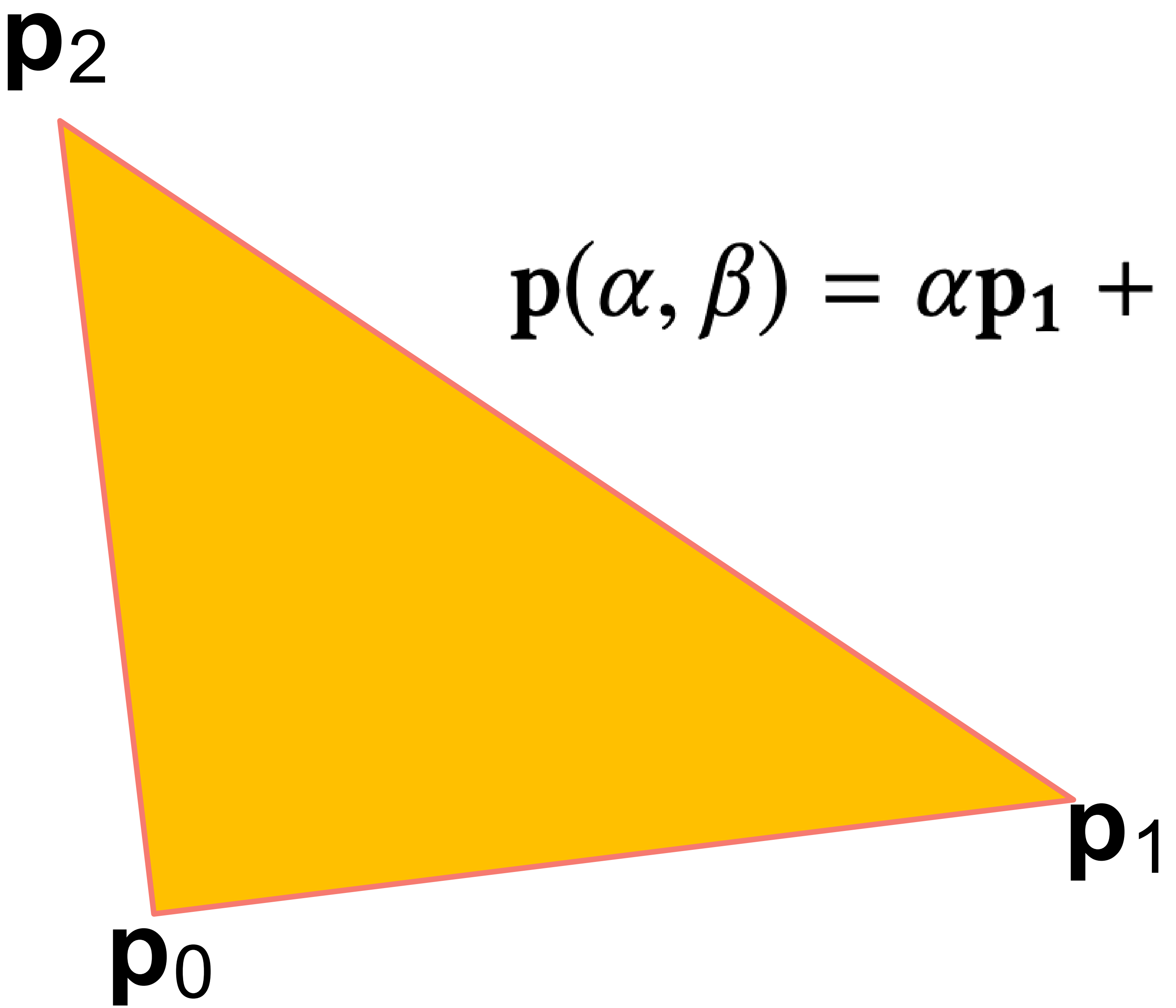
Surface normals

- when a mesh is approximating a curved surface, store normals at vertices

Texture coordinates

- 2D coordinates that tell you how to paste images on the surface

Barycentric Coordinates



$$\mathbf{p}(\alpha, \beta) = \alpha \mathbf{p_1} + \beta \mathbf{p_2} + (1 - \alpha - \beta)\mathbf{p_0}$$

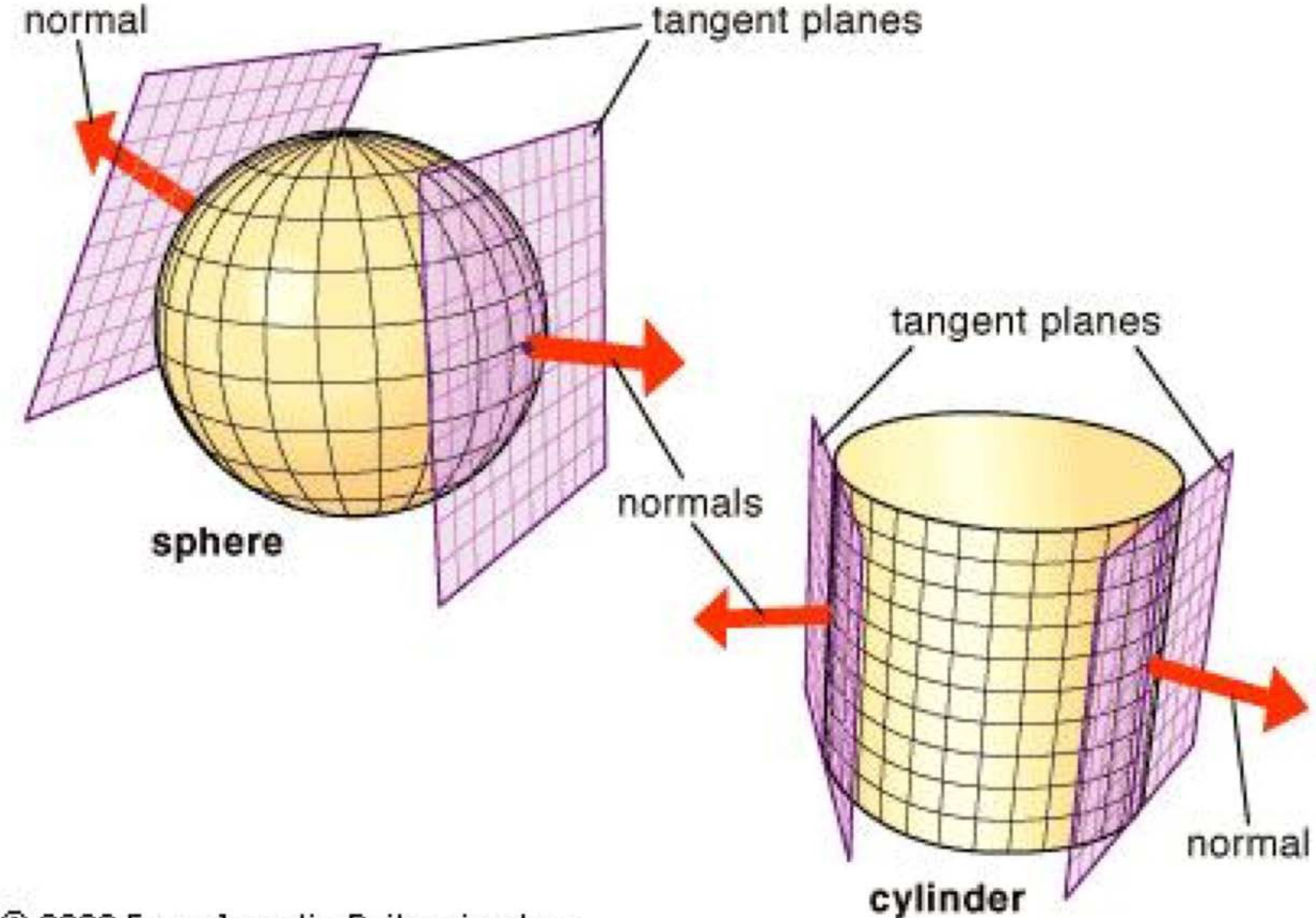
$$\alpha \geq 0$$

$$\beta \geq 0$$

$$\alpha + \beta \leq 1$$

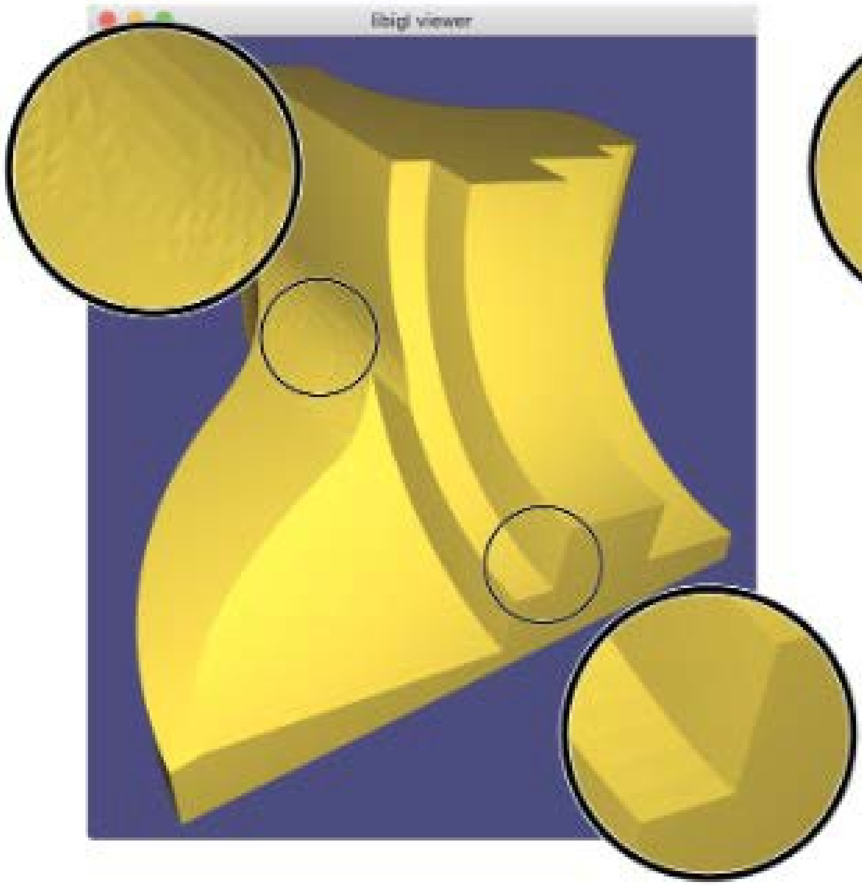
$$\gamma = 1 - \alpha - \beta$$

Normal Vectors

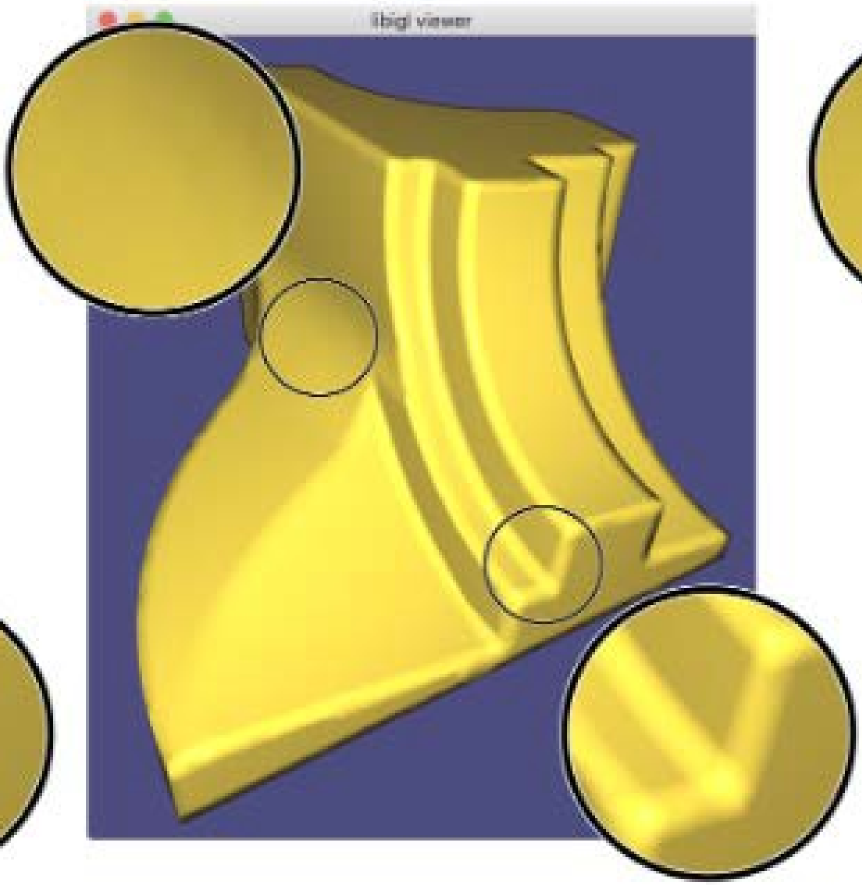


Computing Per-vertex Normals

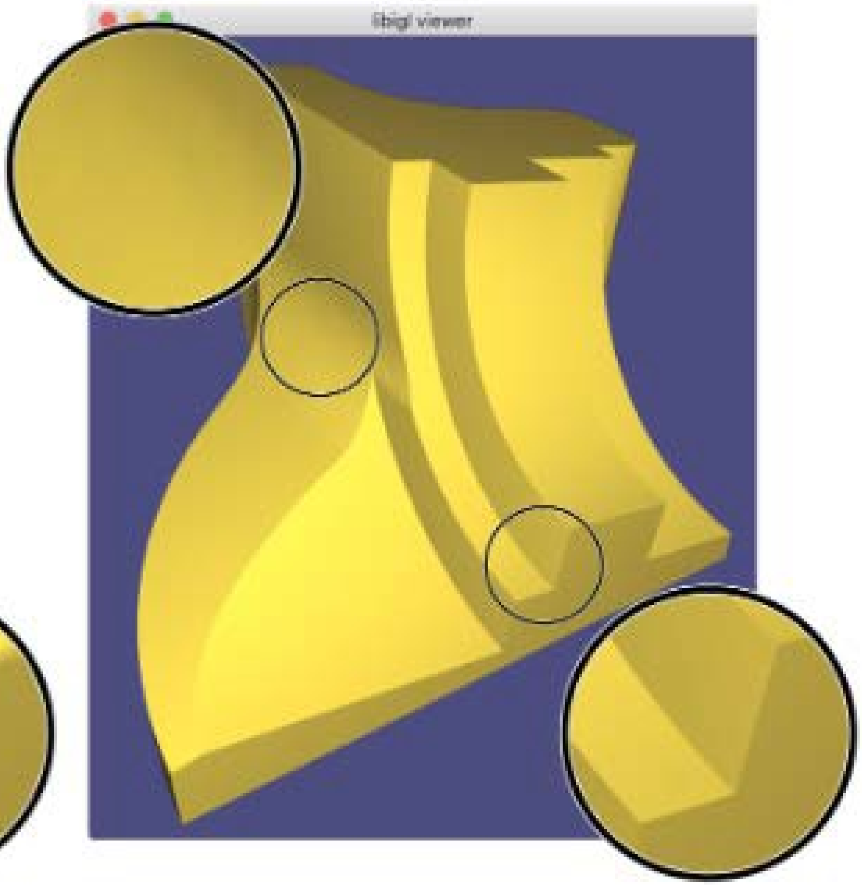
Per-Face Normals



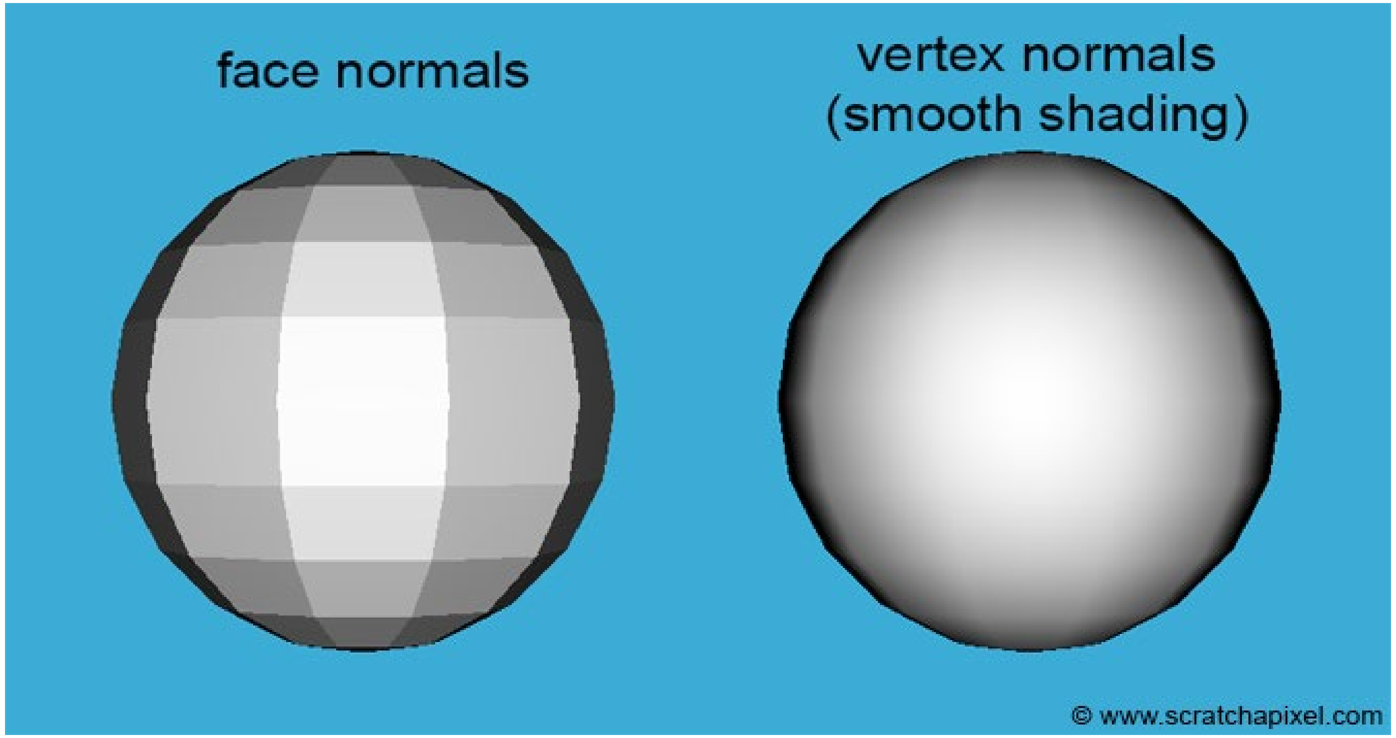
Per-Vertex Normals



Per-Corner Normals

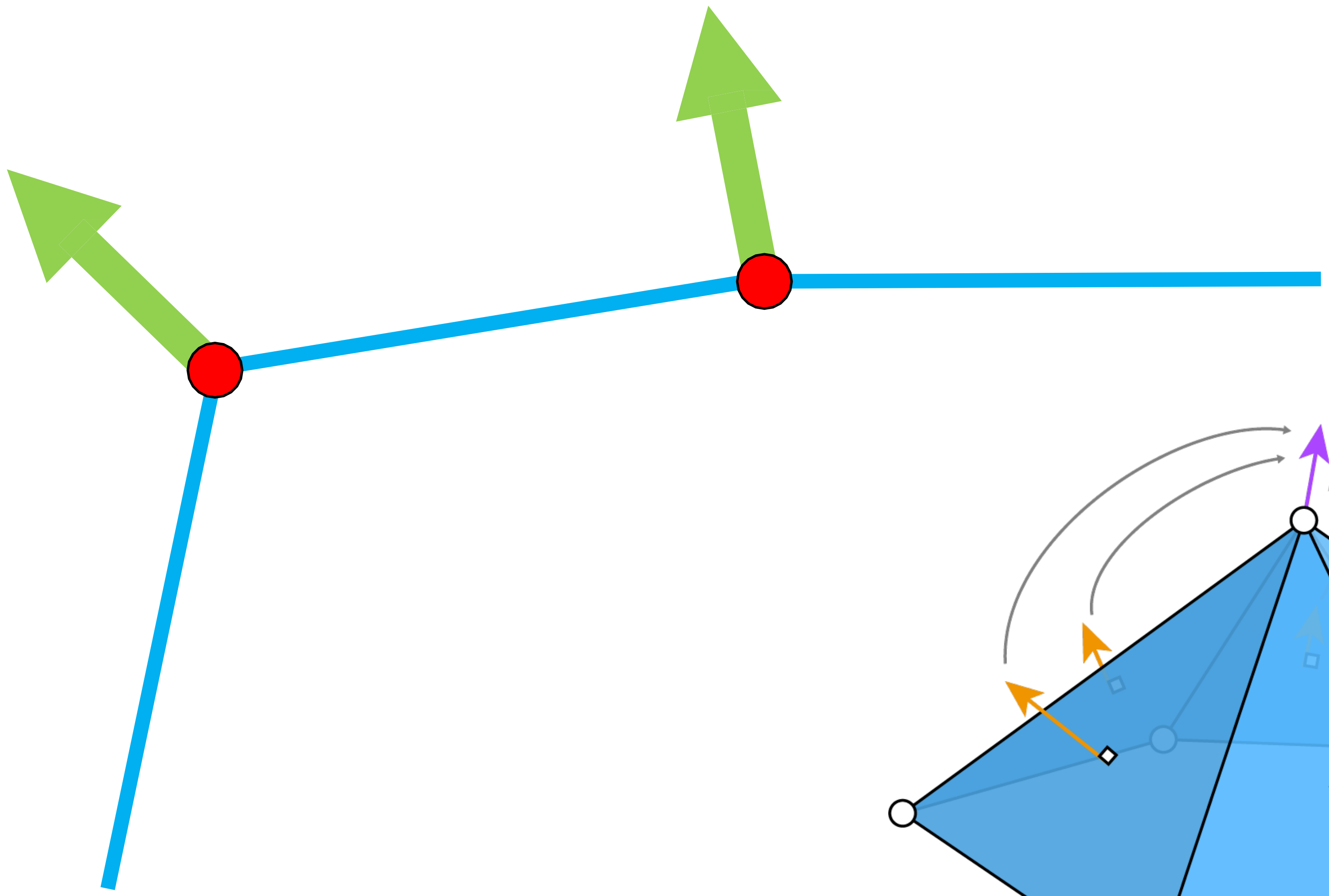


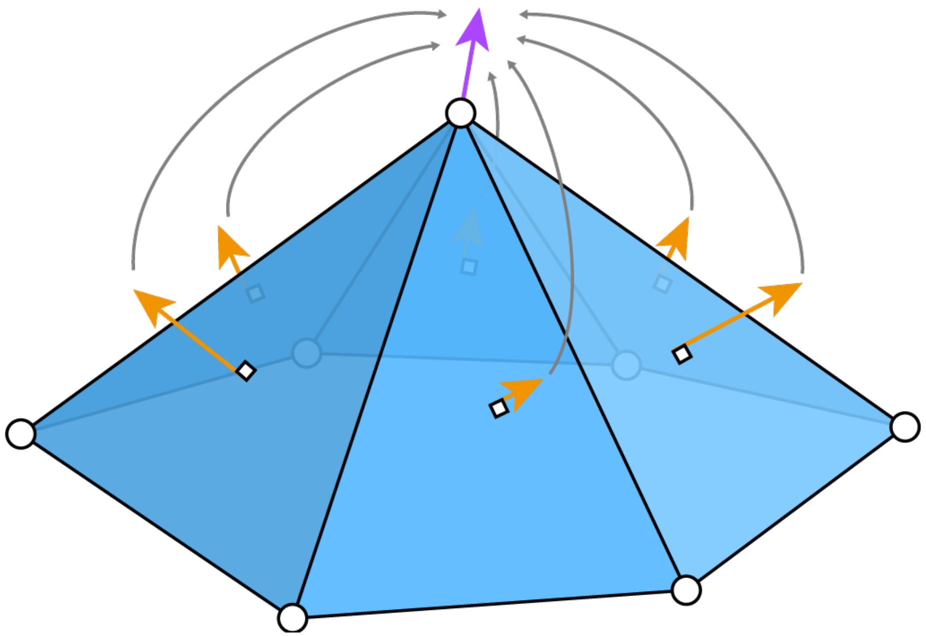
Computing Per-vertex Normals



https://www.scratchapixel.com/lessons/3d-basic-rendering/introduction-to-shading/shading-normals

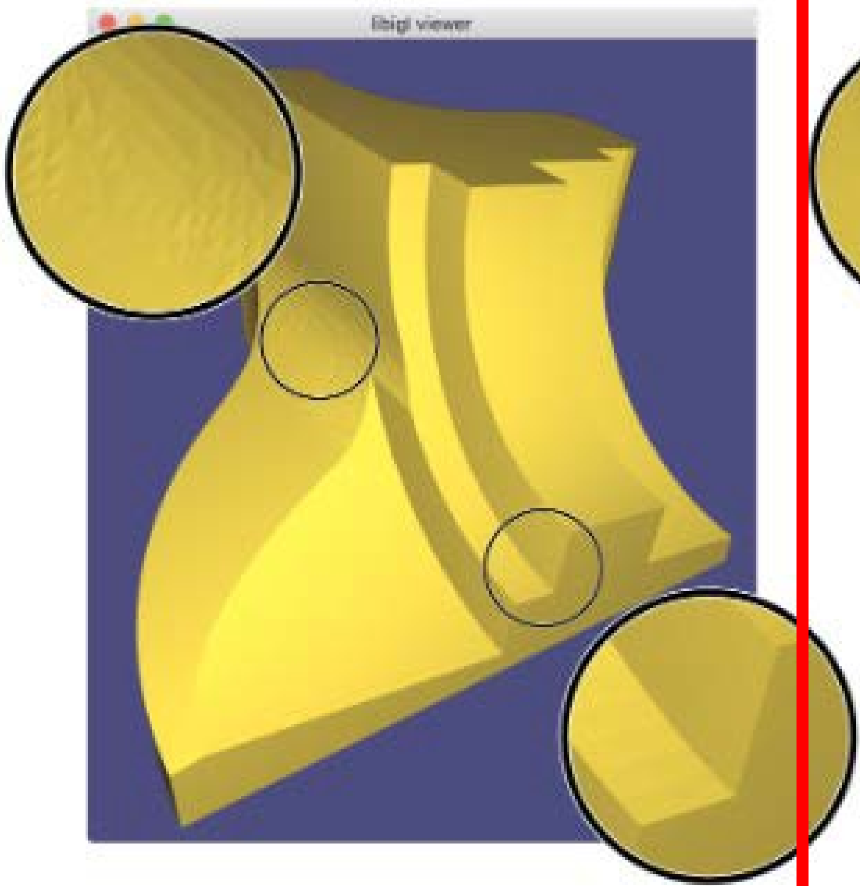
Per-Vertex Normals



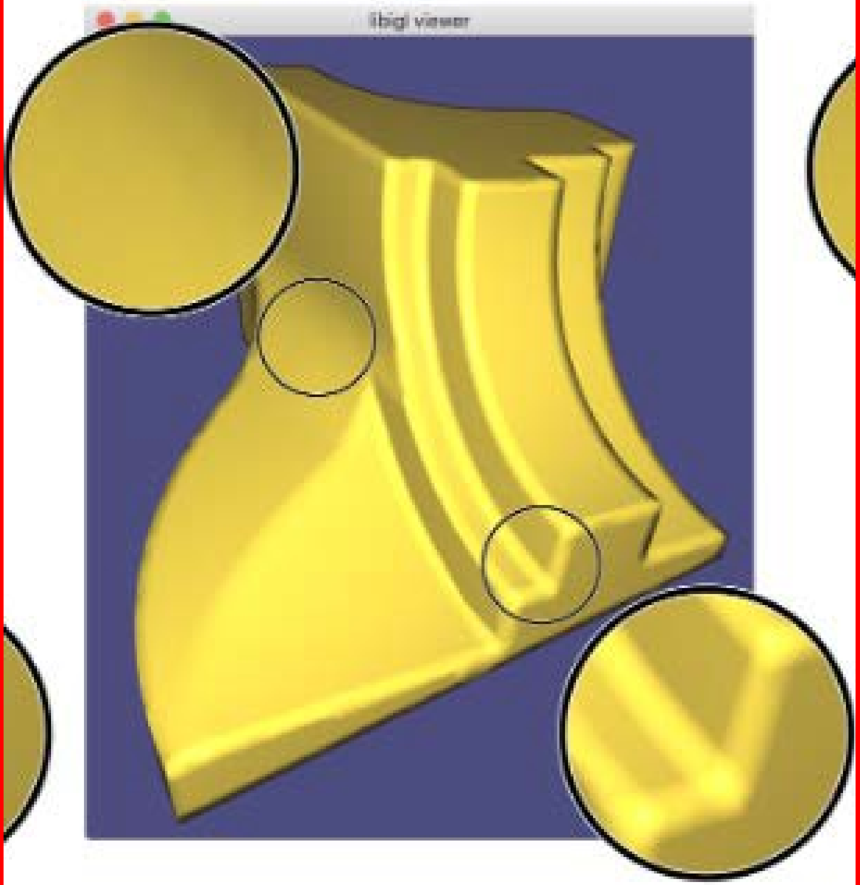


Per-Vertex Normals

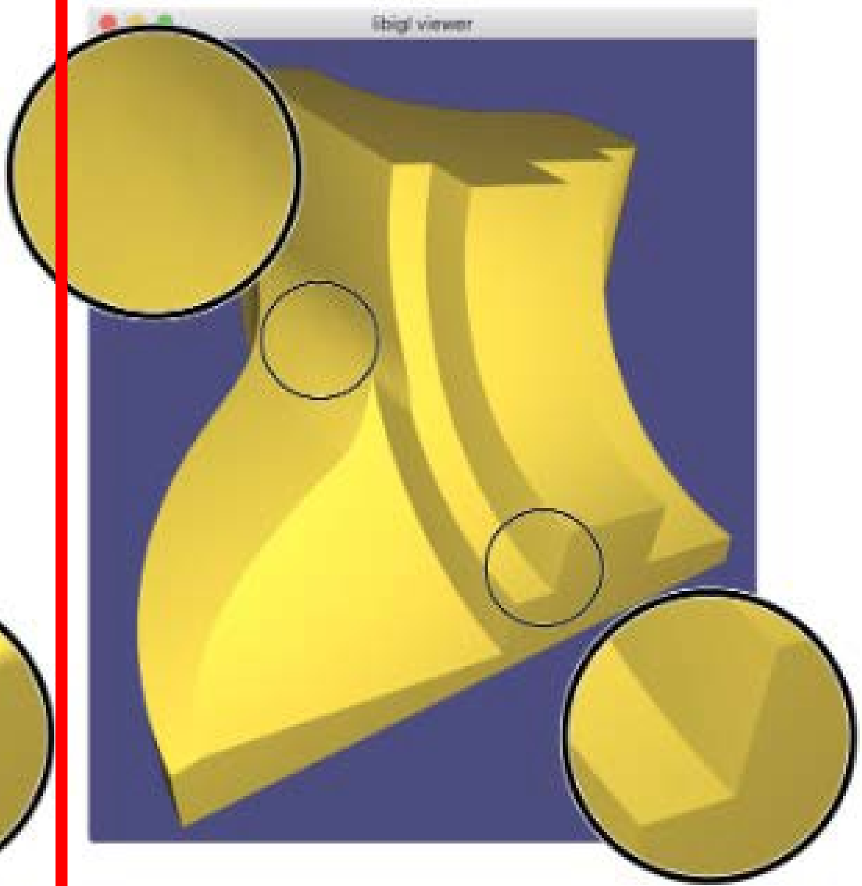
Per-Face Normals



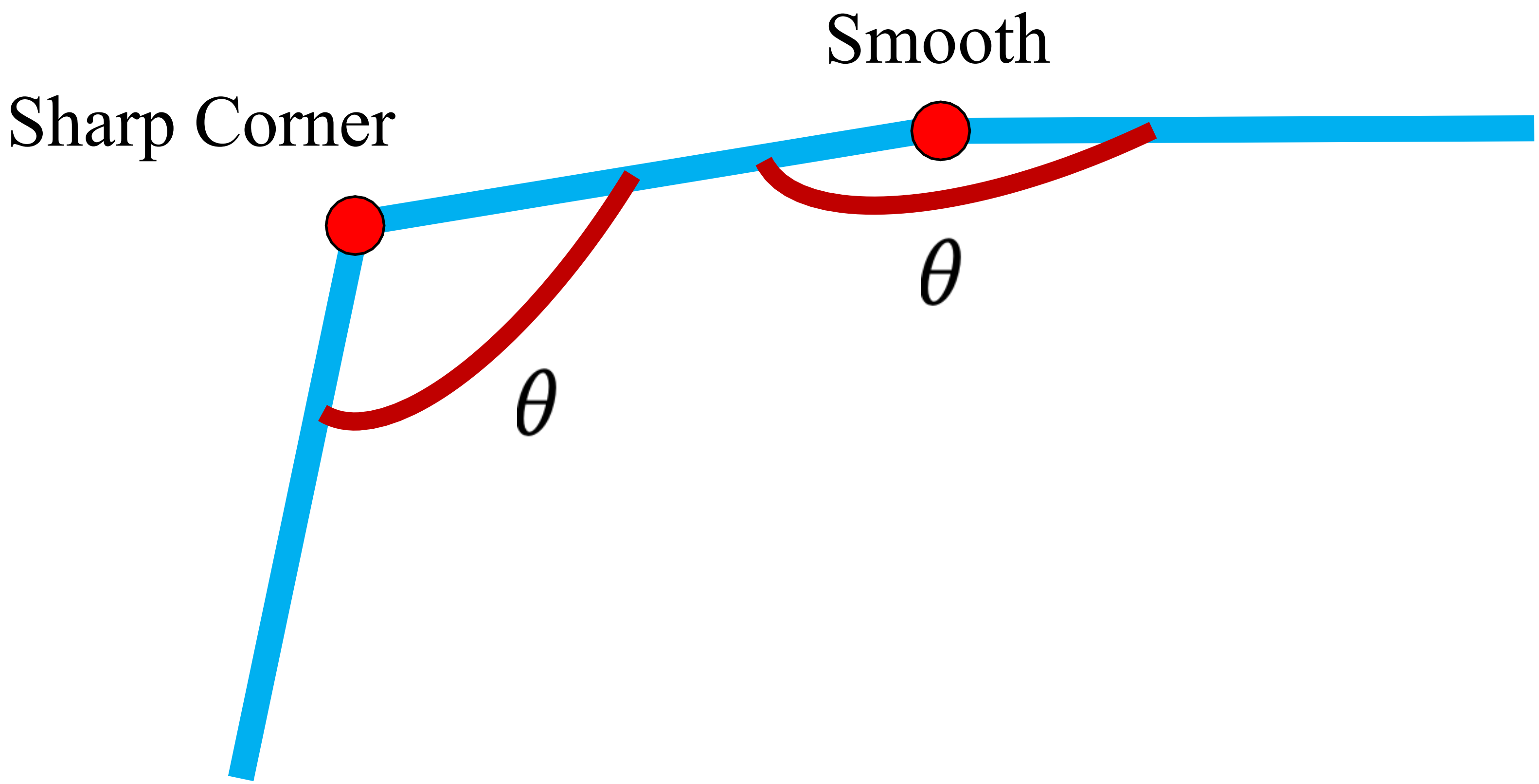
Per-Vertex Normals



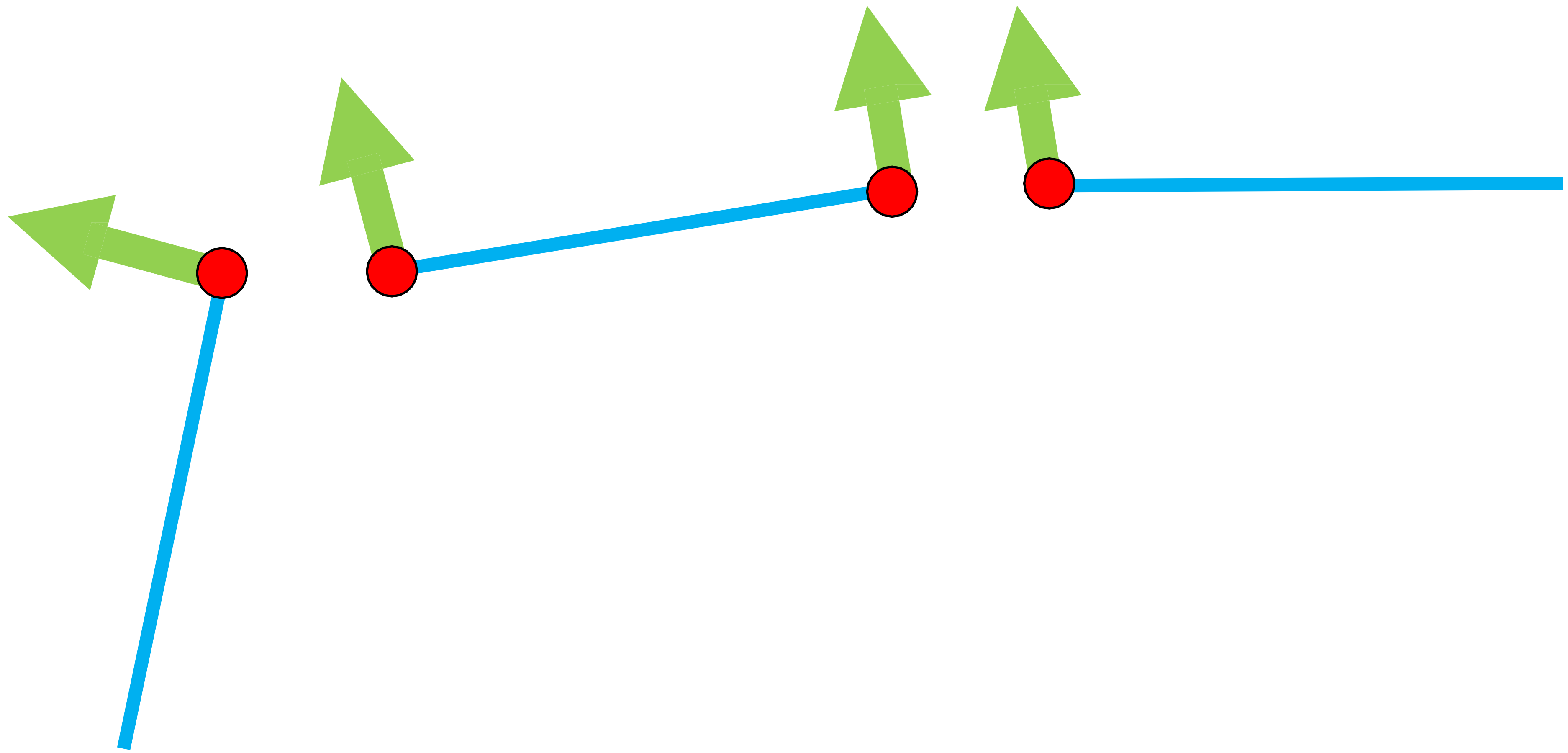
Per-Corner Normals



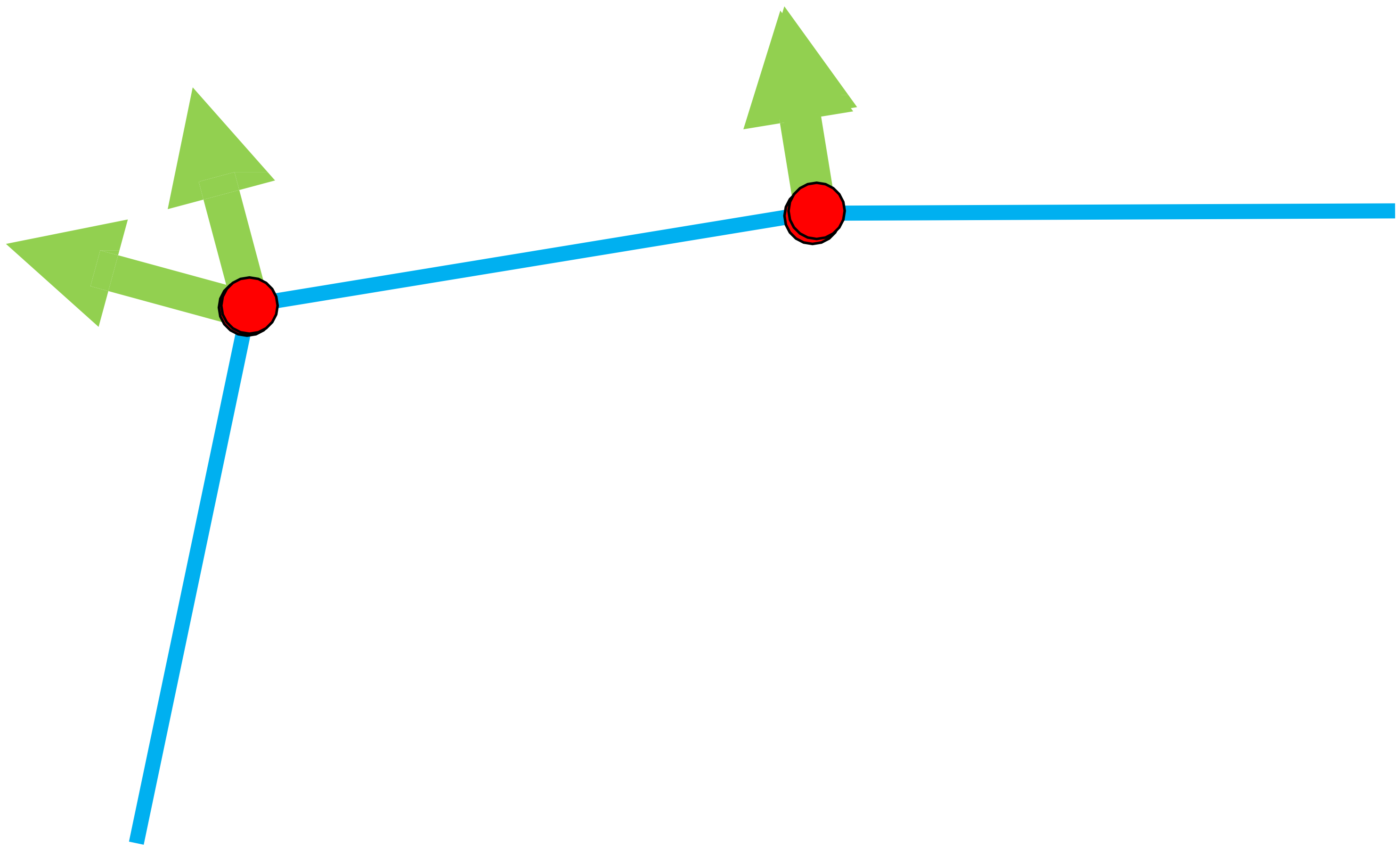
Per-Corner Normals



Per-Corner Normals



Per-Corner Normals



Where to find meshes?

Make your own!

https://www.blender.org/

https://www.autodesk.ca/en/products/maya/overview

Download them!

https://www.cs.cmu.edu/~kmcrane/Projects/ModelRepository/

https://www.turbosquid.com/

https://poly.google.com/

https://www.thingiverse.com/

https://ten-thousand-models.appspot.com/

Done for today

No class Wednesday