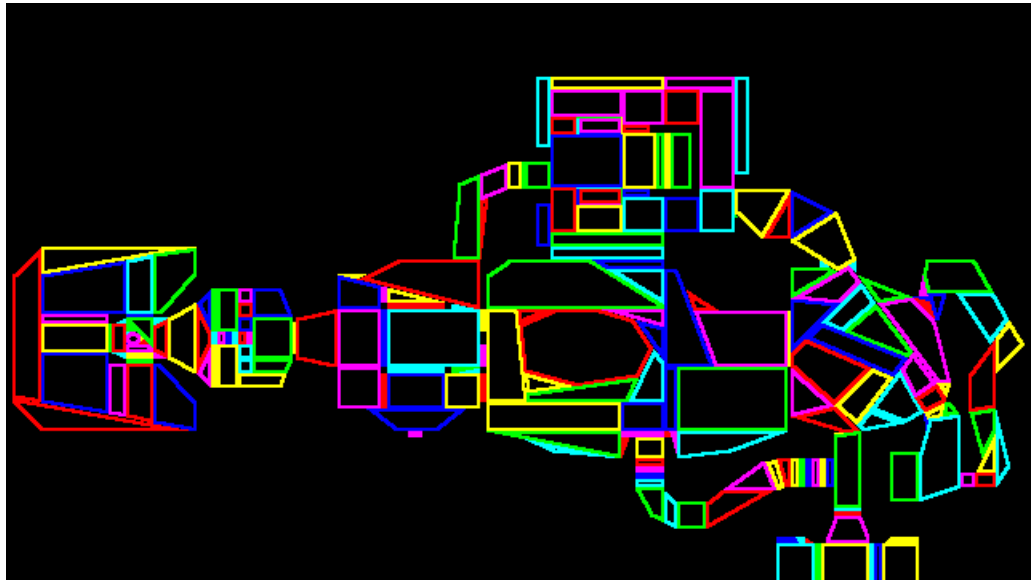
Bounding Volume Hierarchies



Some Slides/Images adapted from Marschner and Shirley and David Levin

Announcements

Assignment 1 grades soon
Assignment 3 is due on Tuesday

Assignment 4 is out soon

A4 requires OpenGL – *we officially support this on CDF only.*

Announcements

TALK TO SOMEONE RIGHT NOW

24/7 Emergency counseling services

U of T My Student Support Program (My SSP) | **1-844-451-9700.** Outside of North America, call **001-416-380-6578**.
Culturally-competent mental health and counseling services **in 146 languages** for all U of T students.

Good2Talk Student Helpline | **1-866-925-5454**
Professional counseling, information and referrals helpline for mental health, addictions and students well-being.

CONTACTS FOR DIFFERENT TYPES OF DISTRESS

Life is complicated and doesn't always go as planned. If you are in distress, we can connect you to the help you need.

Download feeling distressed (PDF) for contacts resources to support you through different kinds of distress:

- 24/7 EMERGENCY
- Mental health
- Academic
- Financial (difficulties due to unexpected circumstances)
- Housing (temporary housing crisis)
- Sexual assault/safety
- Equity offices and communities of care on campus

Contacts include on-campus and community supports during business hours and 24/7.

https://studentlife.utoronto.ca/task/support-when-you-feel-distressed/

Any Questions?

Bounding Volume Hierarchy

(Today)

Common Geometric Queries in Graphics

Bounding Volumes

- Spheres
- Boxes (AABB, OOBB)

Object-Partitioning Hierarchies Introduction

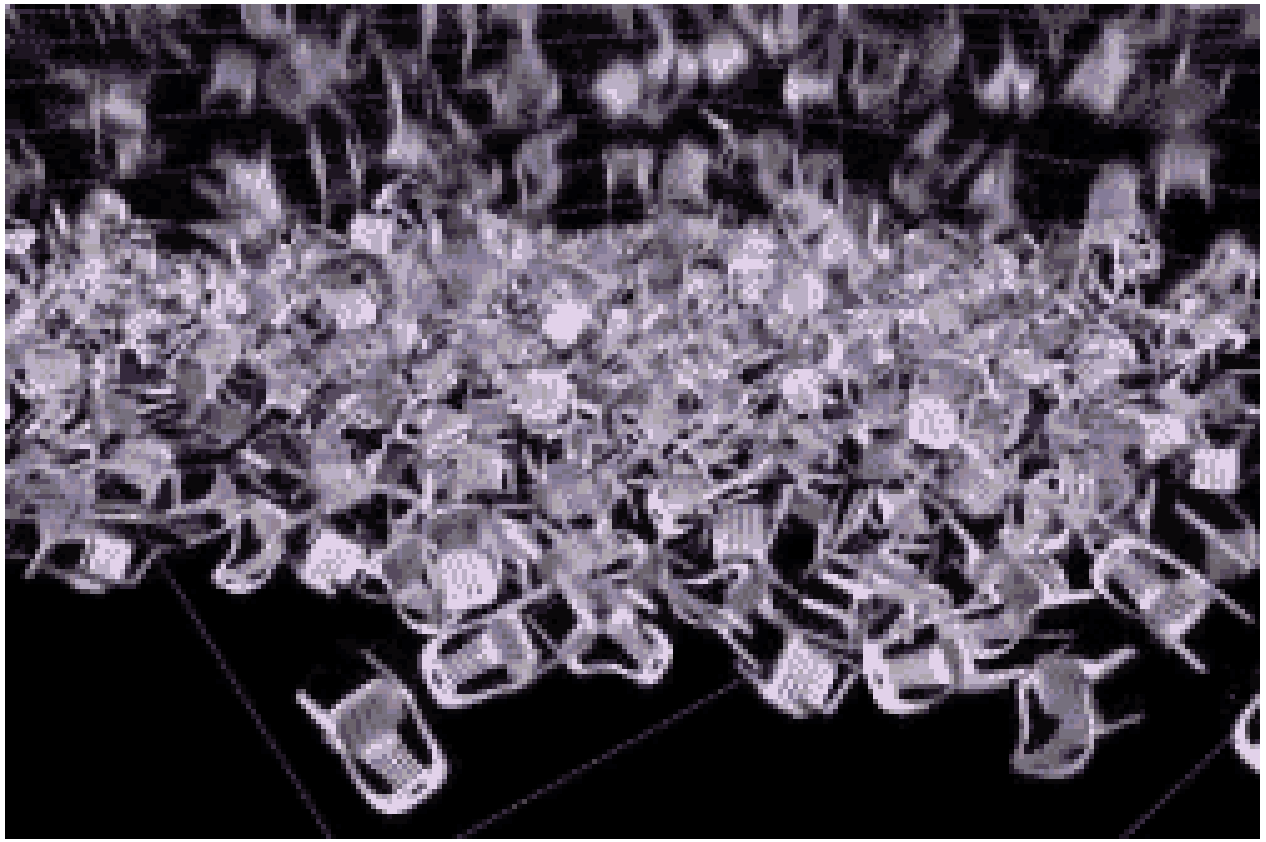
(Wednesday)

Constructing Object-Partitioning Hierarchies

- Sphere Trees
- AABB Trees

Space-Partitioning Hierarchies

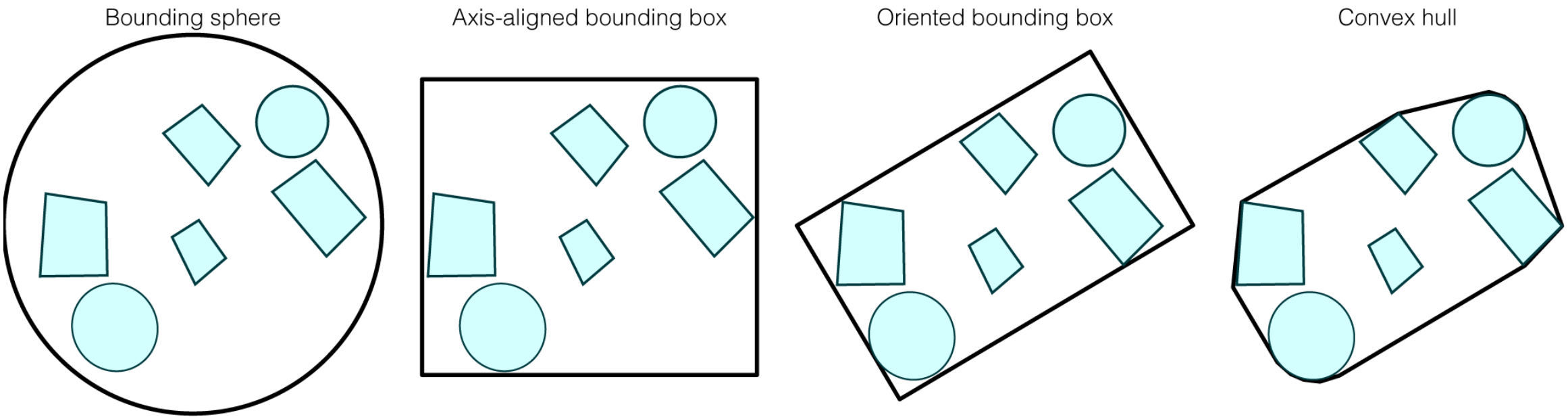
- Uniform Spatial Subdivision
- Axis-Aligned Spatial Subdivision



http://graphics.cs.cmu.edu/projects/bdtree/ **BD-Tree | James and Pai**

Bounding Volumes (BVs)

"Simple" geometry that fully encloses a **collection** of other geometry

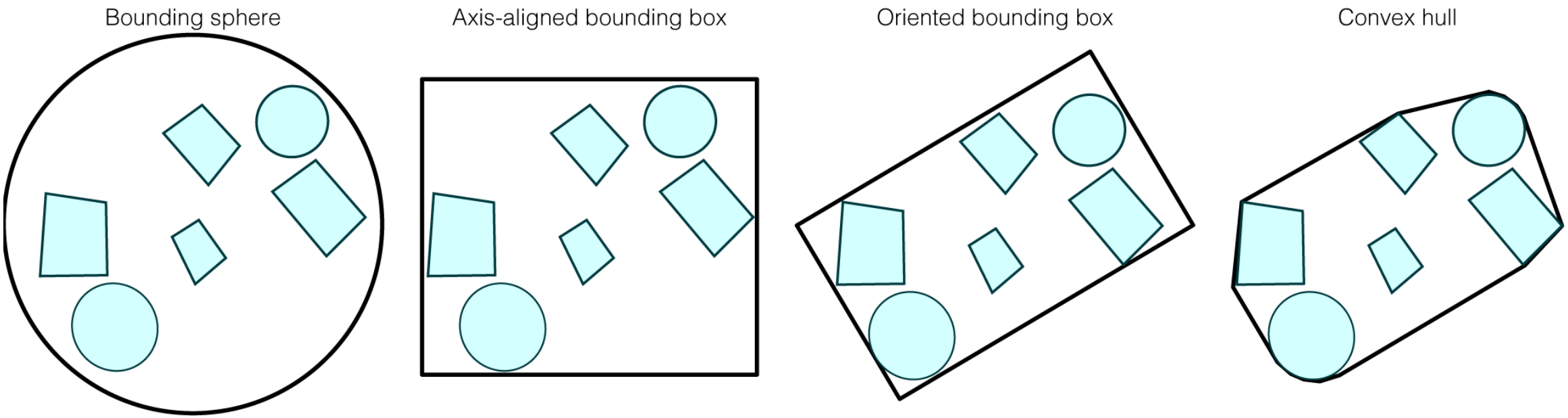


AABB

OOBB

Bounding Volumes (BVs)

"Simple" geometry that fully encloses a **collection** of other geometry



Sphere
(a lot)

AABB
(a lot)

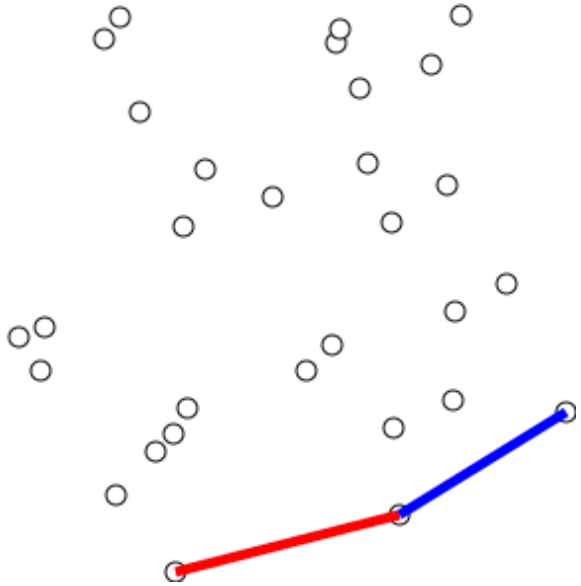
OOBB
(a little)

Convex Hull
(nope!)

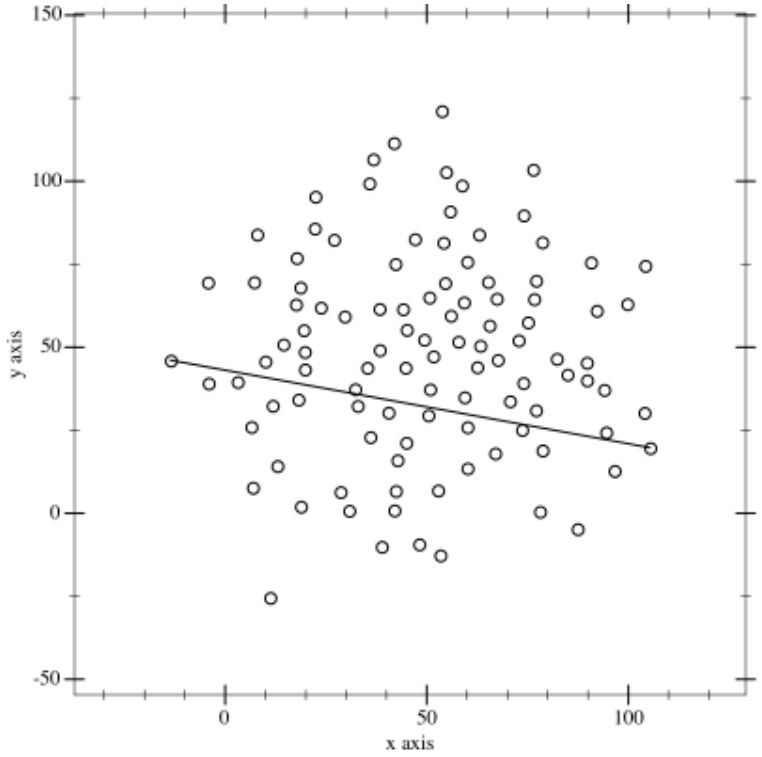
https://en.wikipedia.org/wiki/Convex_hull

Bounding Volumes (BVs)

“Simple” geometry that fully encloses a **collection** of other geometry



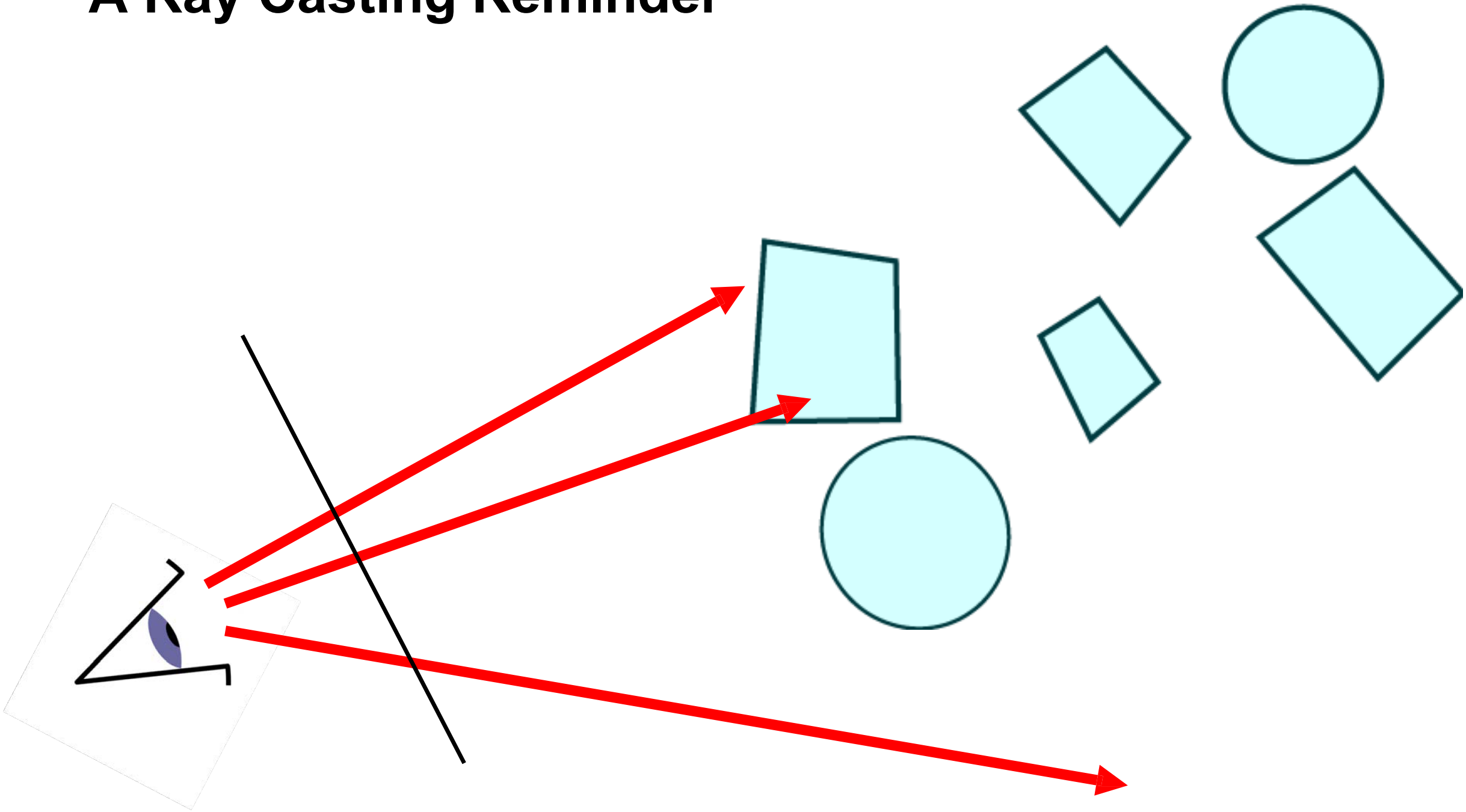
https://en.wikipedia.org/wiki/Graham_scan



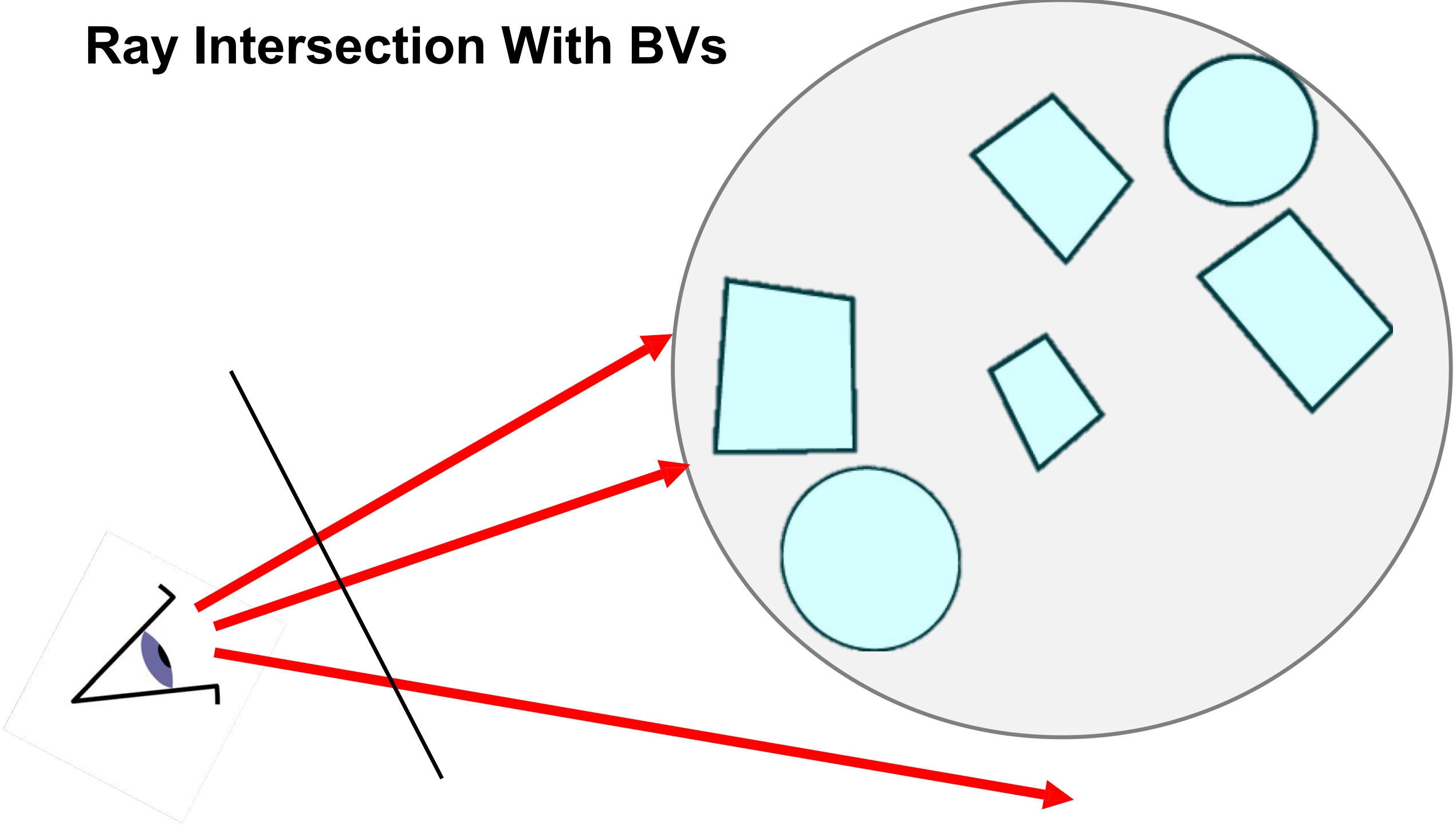
https://en.wikipedia.org/wiki/Quickhull

https://en.wikipedia.org/wiki/Convex_hull

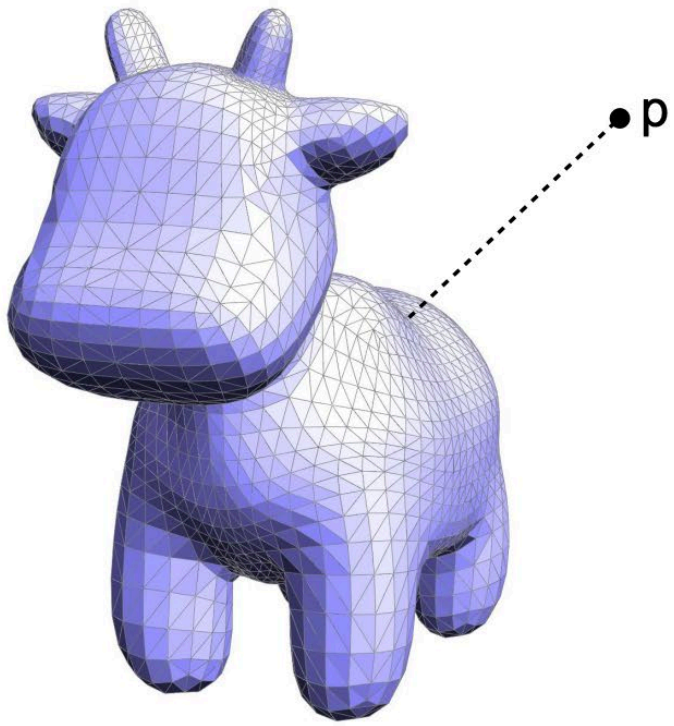
A Ray Casting Reminder



Ray Intersection With BVs



Geometric modeling and geometric queries

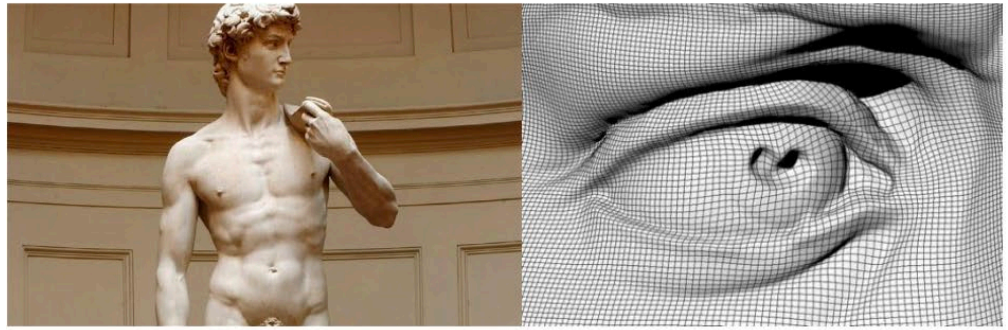


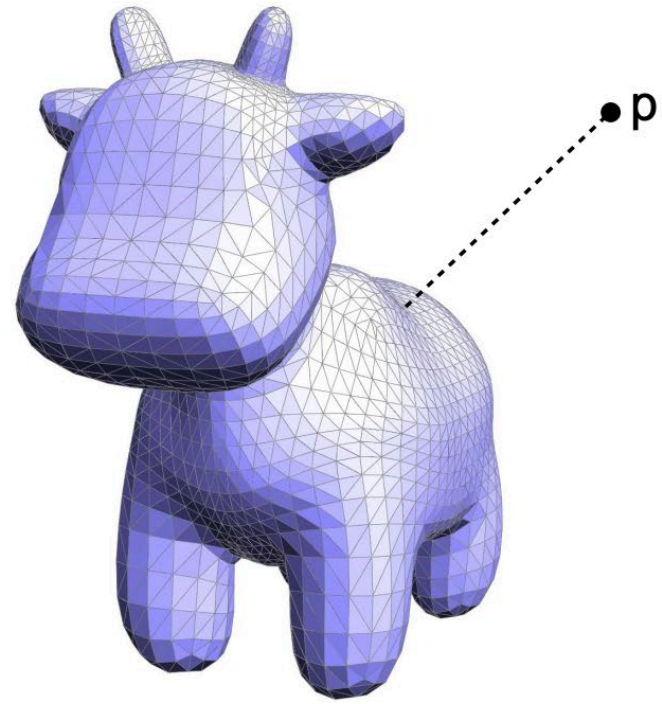
What point on the mesh is closest to **p**?

http://15462.courses.cs.cmu.edu/fall2016content/lectures/12_accelstructures/12_accelstructures_slides.pdf

What point on the mesh is closest to p?

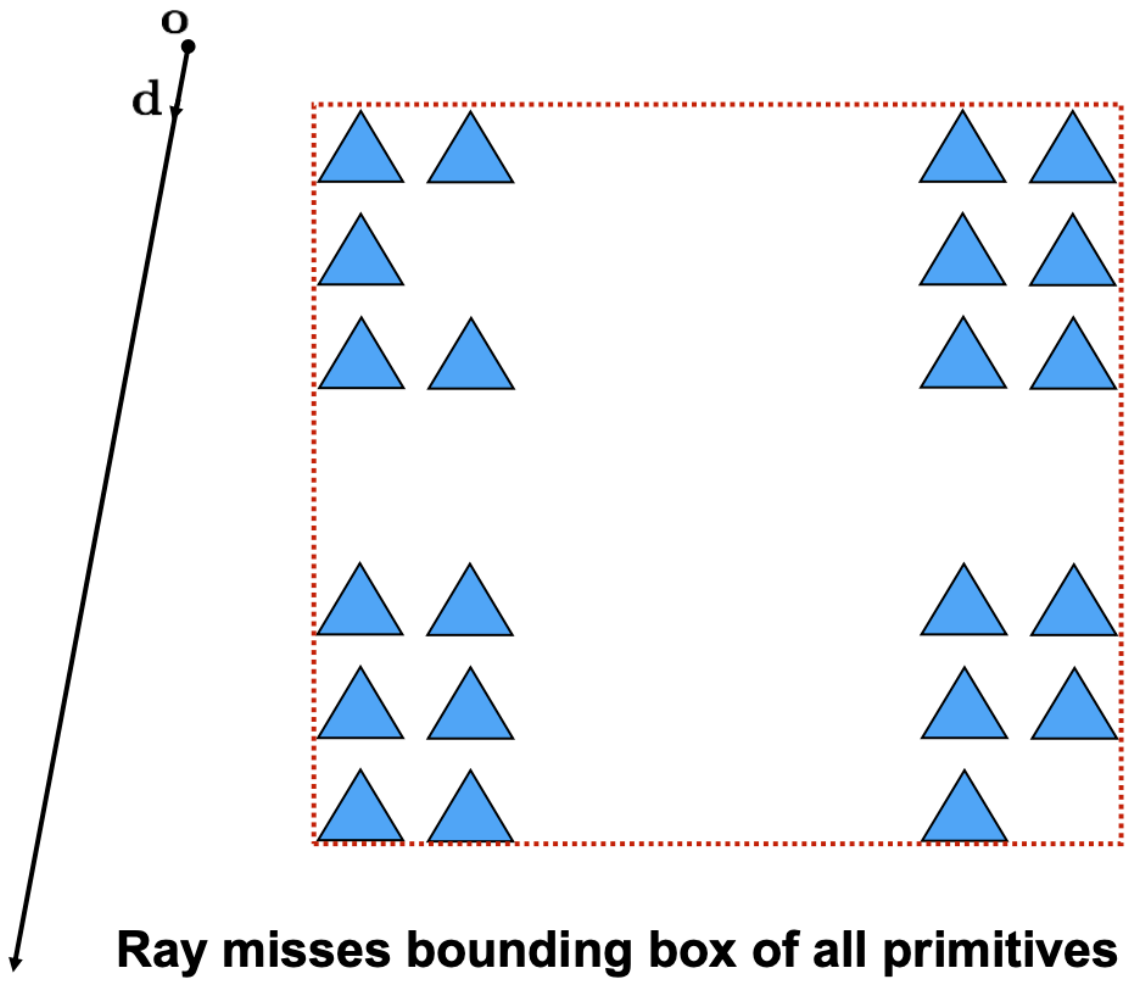
- **Conceptually easy:**
  - **loop over all triangles**
  - **compute closest point to current triangle**
  - **keep globally closest point**
- **Q: What's the cost? Does halfedge help?**
- **What if we have *billions* of faces?**





http://15462.courses.cs.cmu.edu/fall2016content/lectures/12_accelstructures/12_accelstructures_slides.pdf

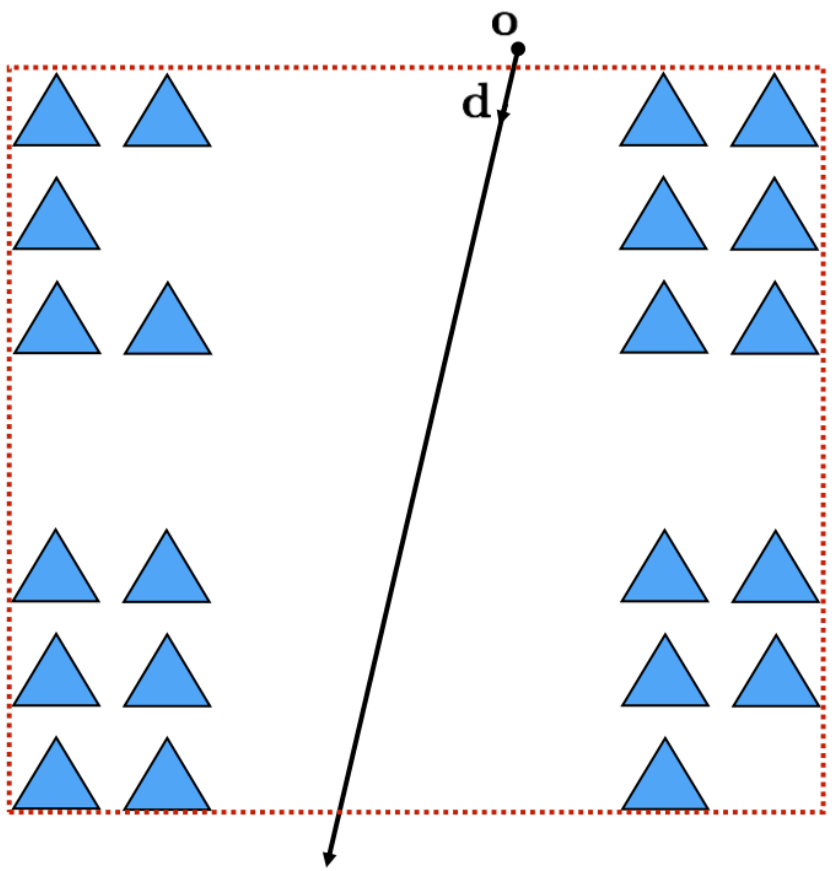
Simple case (we've seen it already)



Ray misses bounding box of all primitives in scene
O(1) cost: requires 1 ray-box test

http://15462.courses.cs.cmu.edu/fall2016content/lectures/12_accelstructures/12_accelstructures_slides.pdf

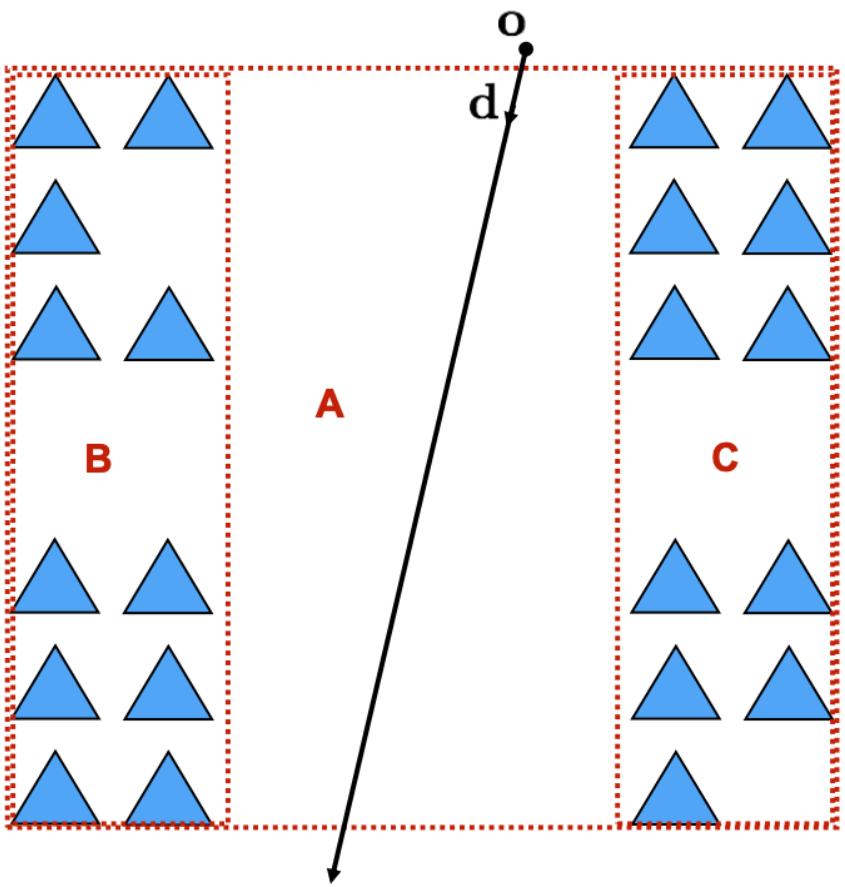
Another (should be) simple case

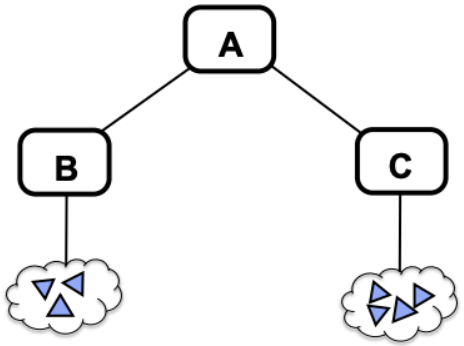


Ray hits bounding box, check all primitives
O(N) cost ☹

http://15462.courses.cs.cmu.edu/fall2016content/lectures/12_accelstructures/12_accelstructures_slides.pdf

Another (should be) simple case



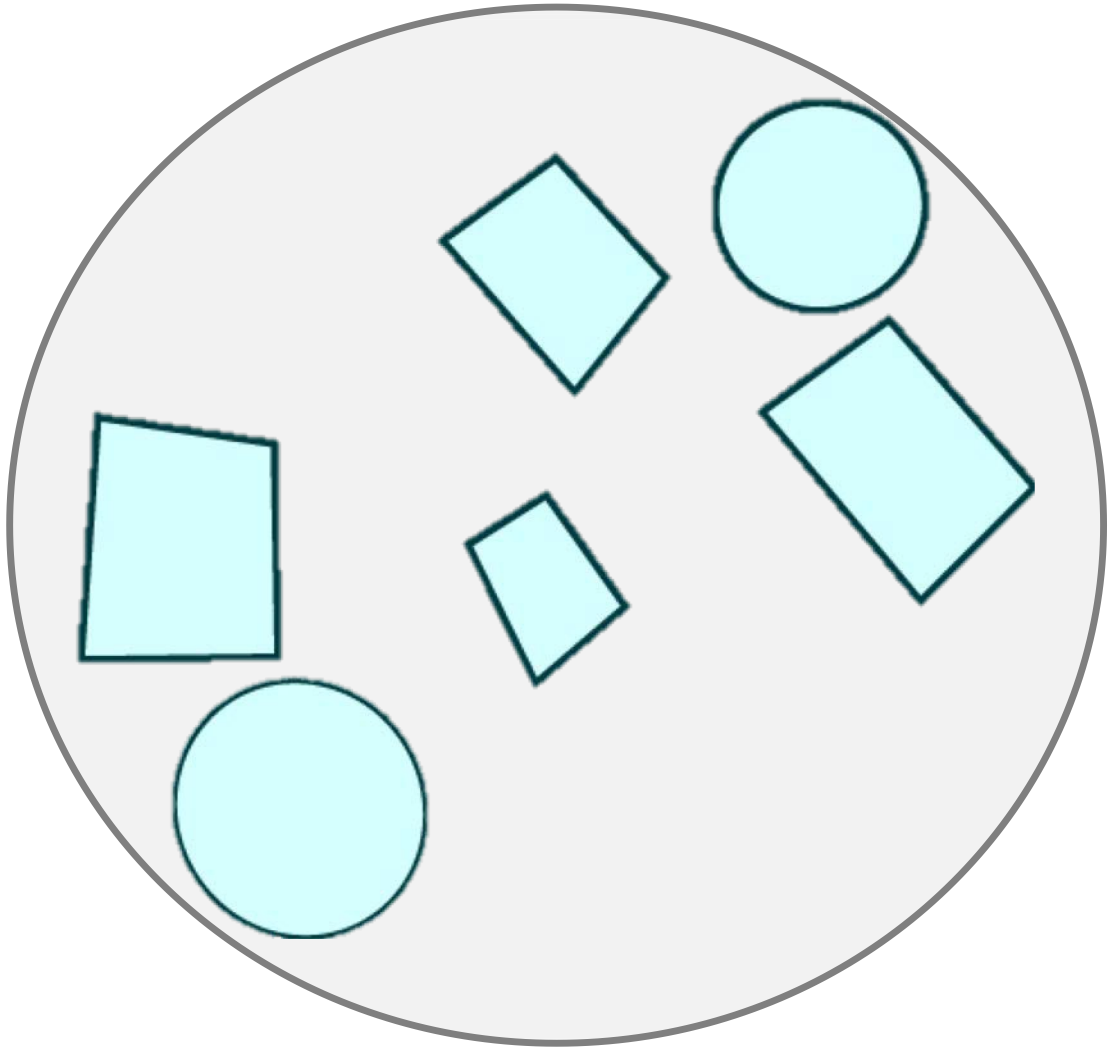


There is no reason to stop there!

http://15462.courses.cs.cmu.edu/fall2016content/lectures/12_accelstructures/12_accelstructures_slides.pdf

Building a Bounding Sphere

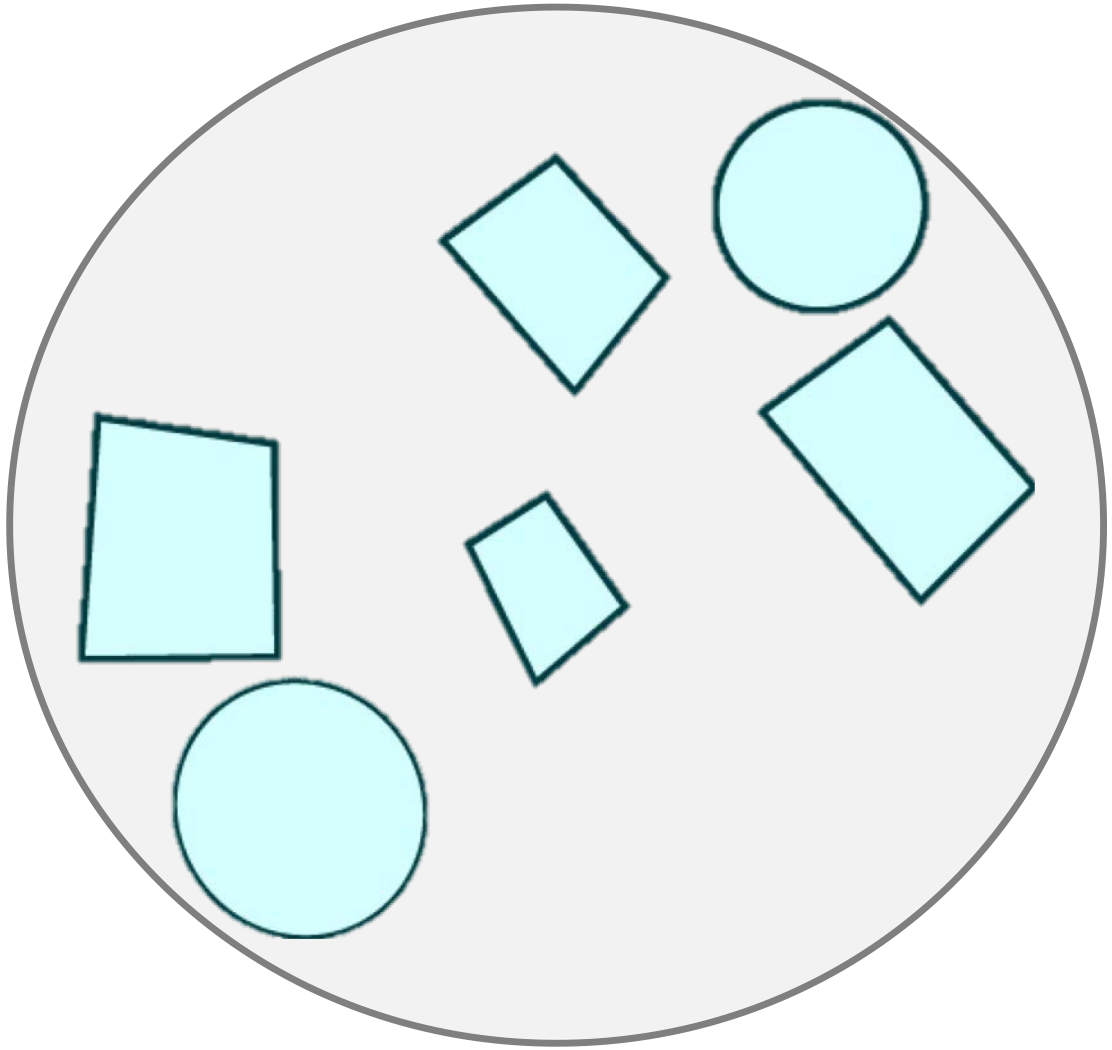
Parameters of a Sphere?



Building a Bounding Sphere

Parameters of a Sphere:

1. Center = ?
2. Radius = ?



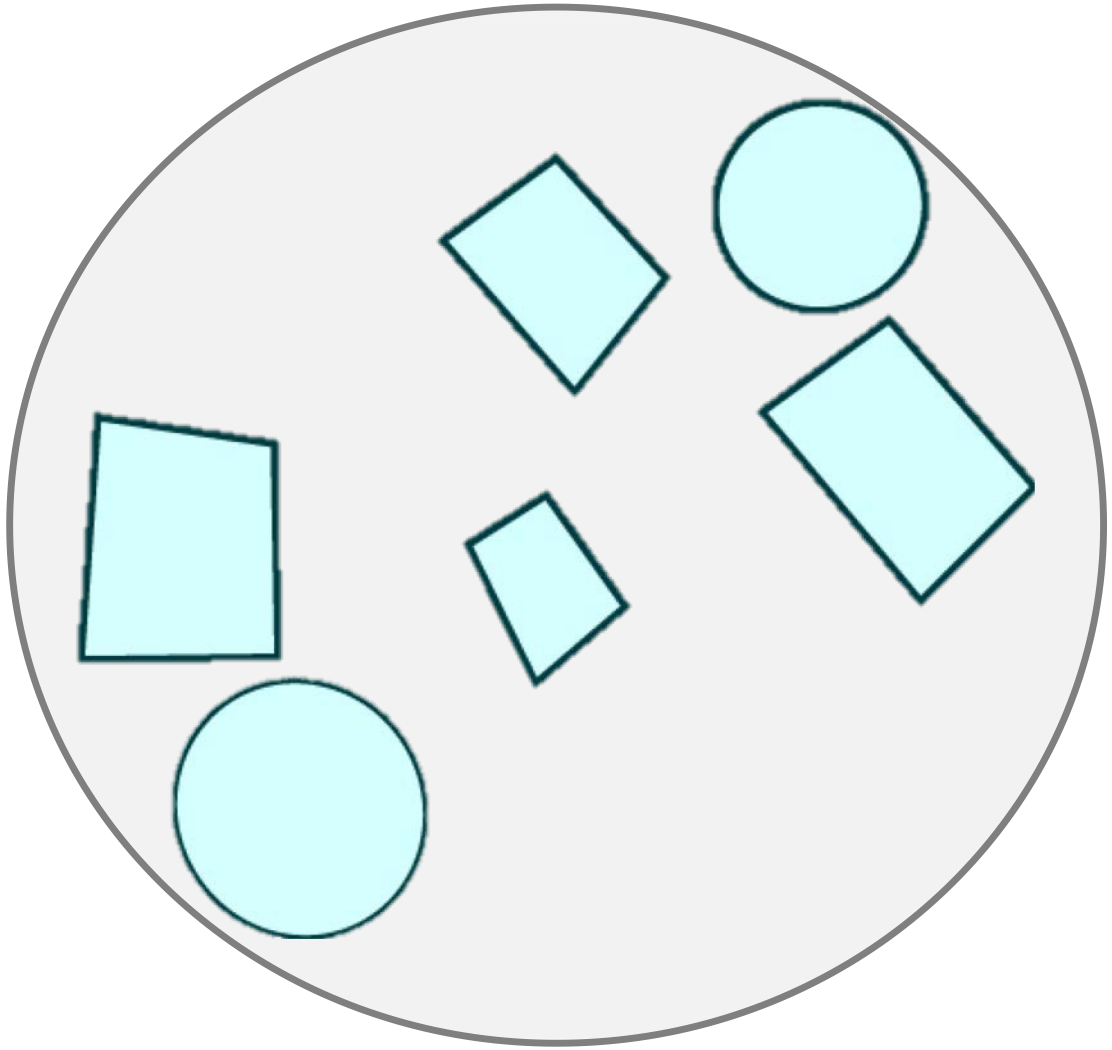
Building a Bounding Sphere

Parameters of a Sphere:

1. Center = $\mathbf{c} = \frac{1}{n}\sum_{i=1}^{n}\mathbf{v}^i$

2. Radius = $r = \max(\mathbf{v}^i - \mathbf{c})$

$$\mathbf{v}^i \in \text{Vertices}$$



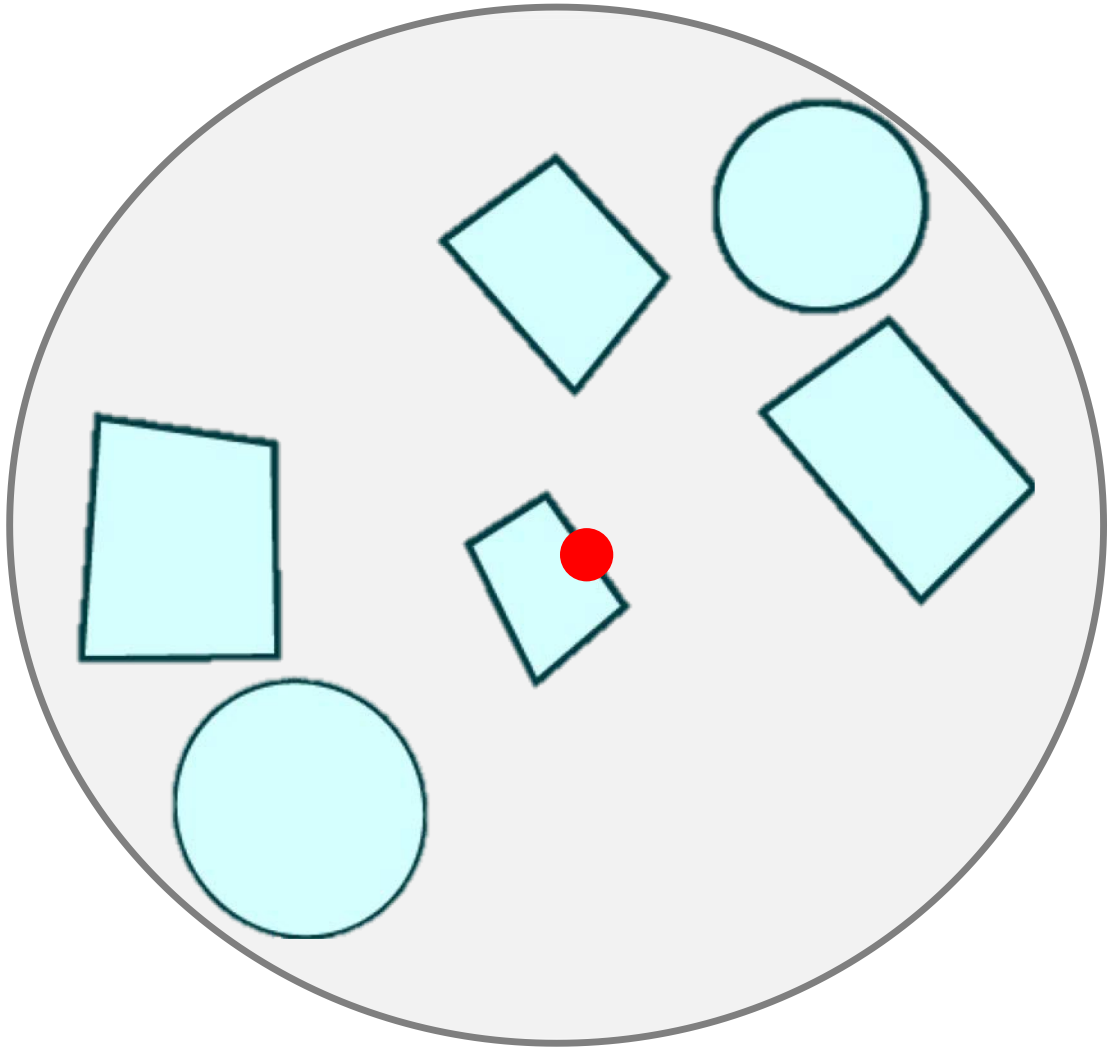
Building a Bounding Sphere

Parameters of a Sphere:

1. Center = $\mathbf{c} = \frac{1}{n}\sum_{i=1}^{n}\mathbf{v}^i$

2. Radius = $r = \max(\mathbf{v}^i - \mathbf{c})$

$$\mathbf{v}^i \in \text{Vertices}$$



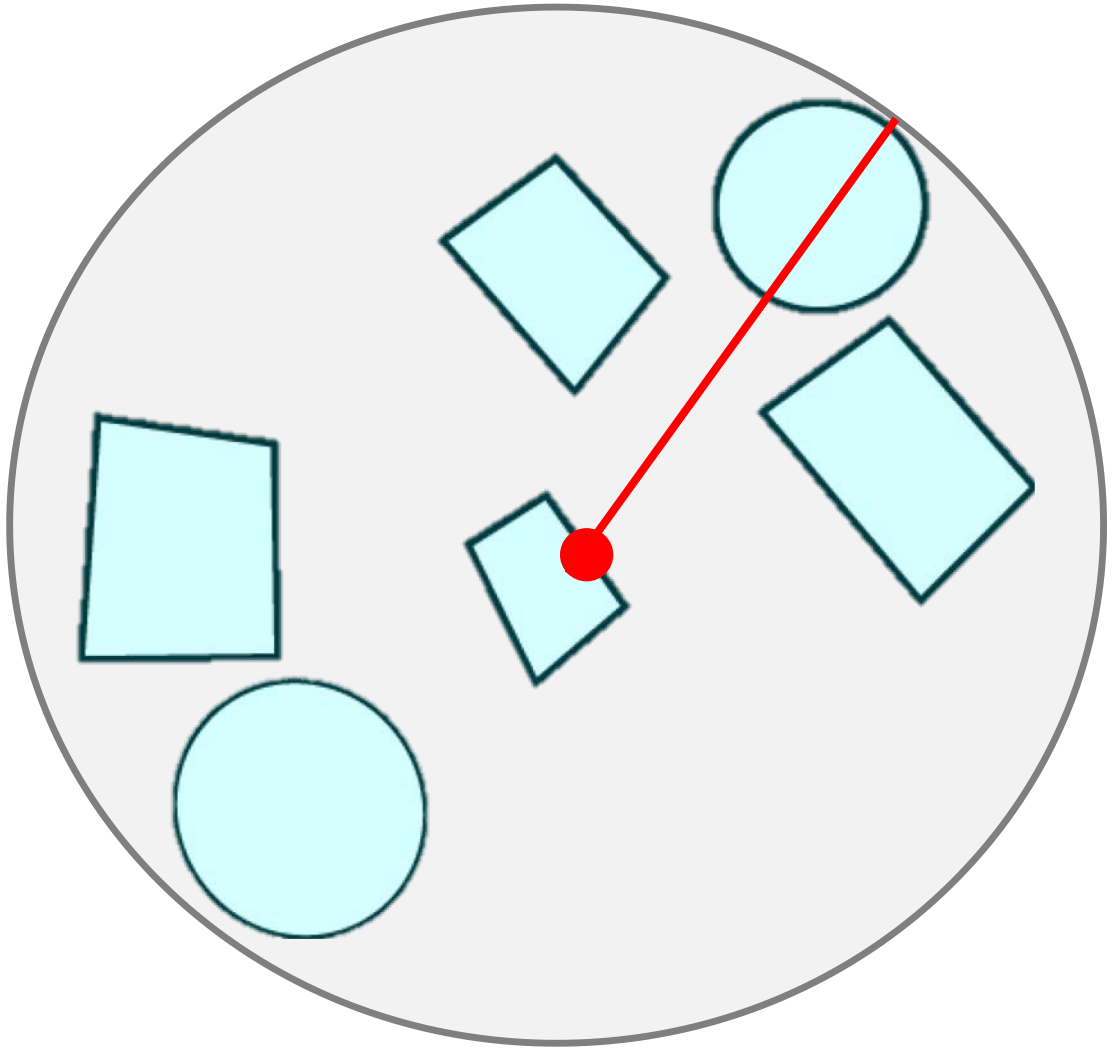
Building a Bounding Sphere

Parameters of a Sphere:

1. Center = $\mathbf{c} = \frac{1}{n}\sum_{i=1}^{n}\mathbf{v}^i$

2. Radius = $r = \max(\mathbf{v}^i - \mathbf{c})$

$$\mathbf{v}^i \in \text{Vertices}$$



Ray-Sphere Intersection

Ray-Sphere Intersection

Substitute ray equation into implicit equation for sphere

$$(\mathbf{e} + t\vec{\mathbf{d}} - \mathbf{c}) \cdot (\mathbf{e} + t\vec{\mathbf{d}} - \mathbf{c}) - r^2 = 0$$

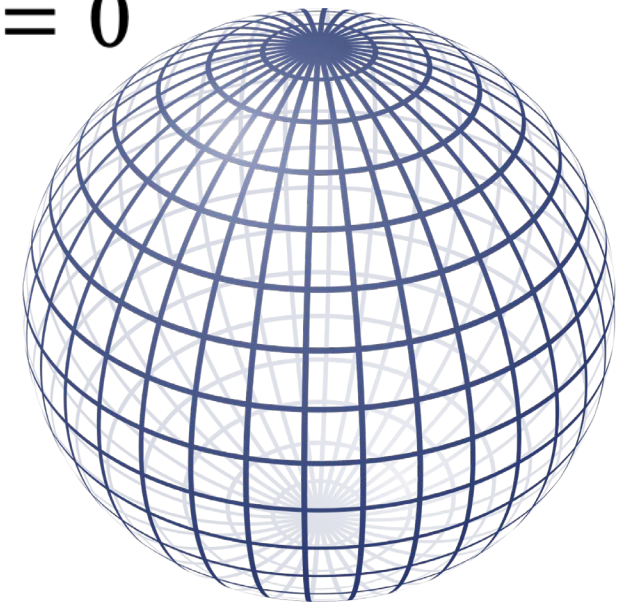
Rearrange

$$(\vec{\mathbf{d}} \cdot \vec{\mathbf{d}})t^2 + 2\vec{\mathbf{d}} \cdot (\mathbf{e} - \mathbf{c})t + (\mathbf{e} - \mathbf{c}) \cdot (\mathbf{e} - \mathbf{c}) - r^2 = 0$$

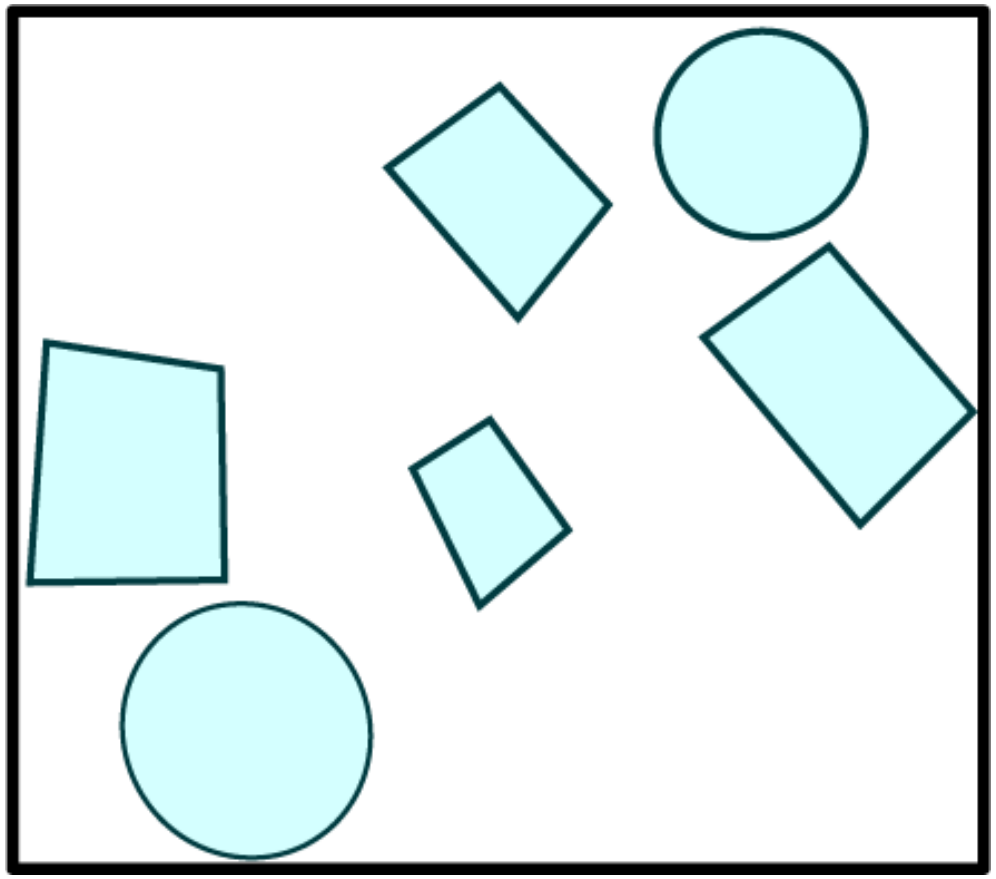
Looks familiar…

$$At^2 + Bt + C = 0$$

It's a quadratic! (can use the quadratic equation)



Building and Axis-Aligned Bounding Box (AABB)



Building and Axis-Aligned Bounding Box (AABB)

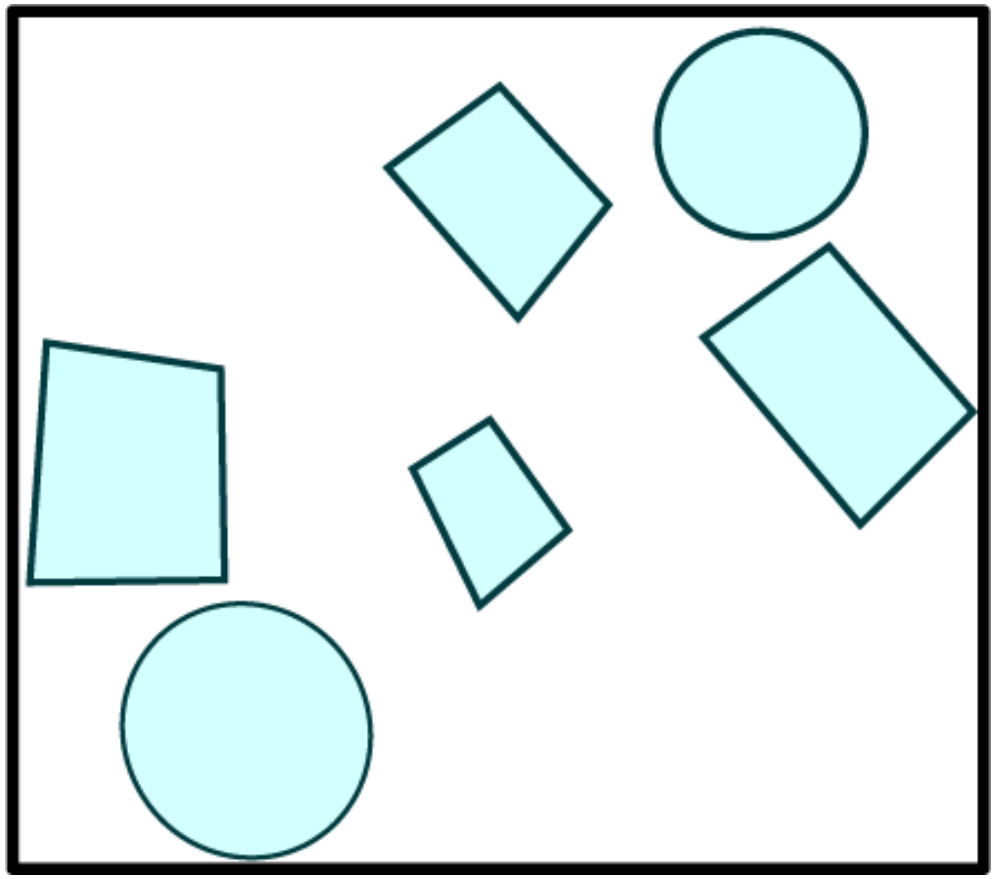
$$x_{\text{min}} = \min\left(v_x^i\right)$$

$$x_{\text{max}} = \max\left(v_x^i\right)$$

$$y_{\text{min}} = \min\left(v_y^i\right)$$

$$y_{\text{max}} = \max\left(v_y^i\right)$$

$$\mathbf{v}^i \in \text{Vertices}$$

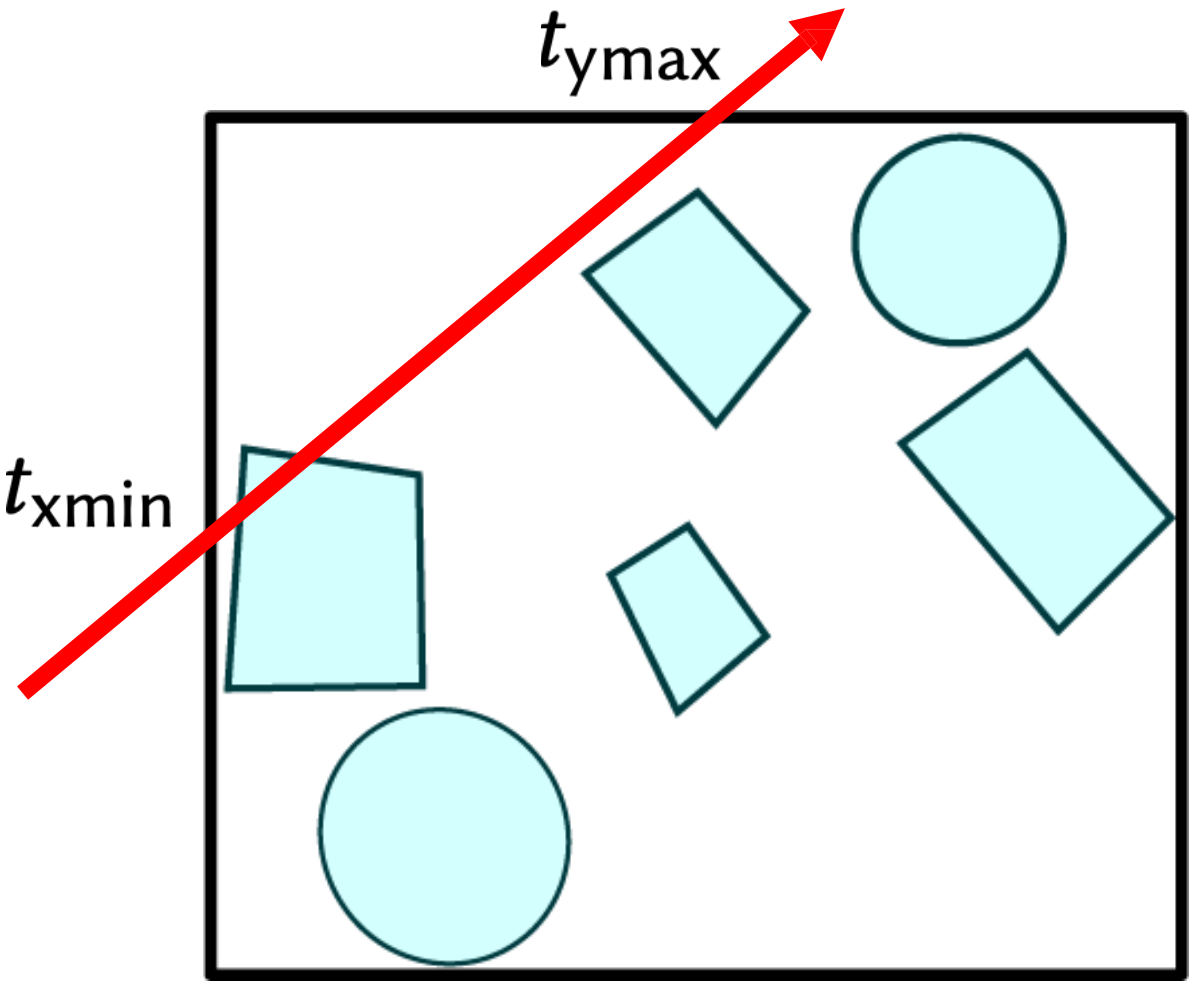


Ray-AABB Intersection

$$t_{\text{xmin}} = (x_{\text{min}} - x_e)/x_d$$
$$t_{\text{xmax}} = (x_{\text{max}} - x_e)/x_d$$
$$t_{\text{ymin}} = (y_{\text{min}} - y_e)/y_d$$
$$t_{\text{ymax}} = (y_{\text{max}} - y_e)/y_d$$

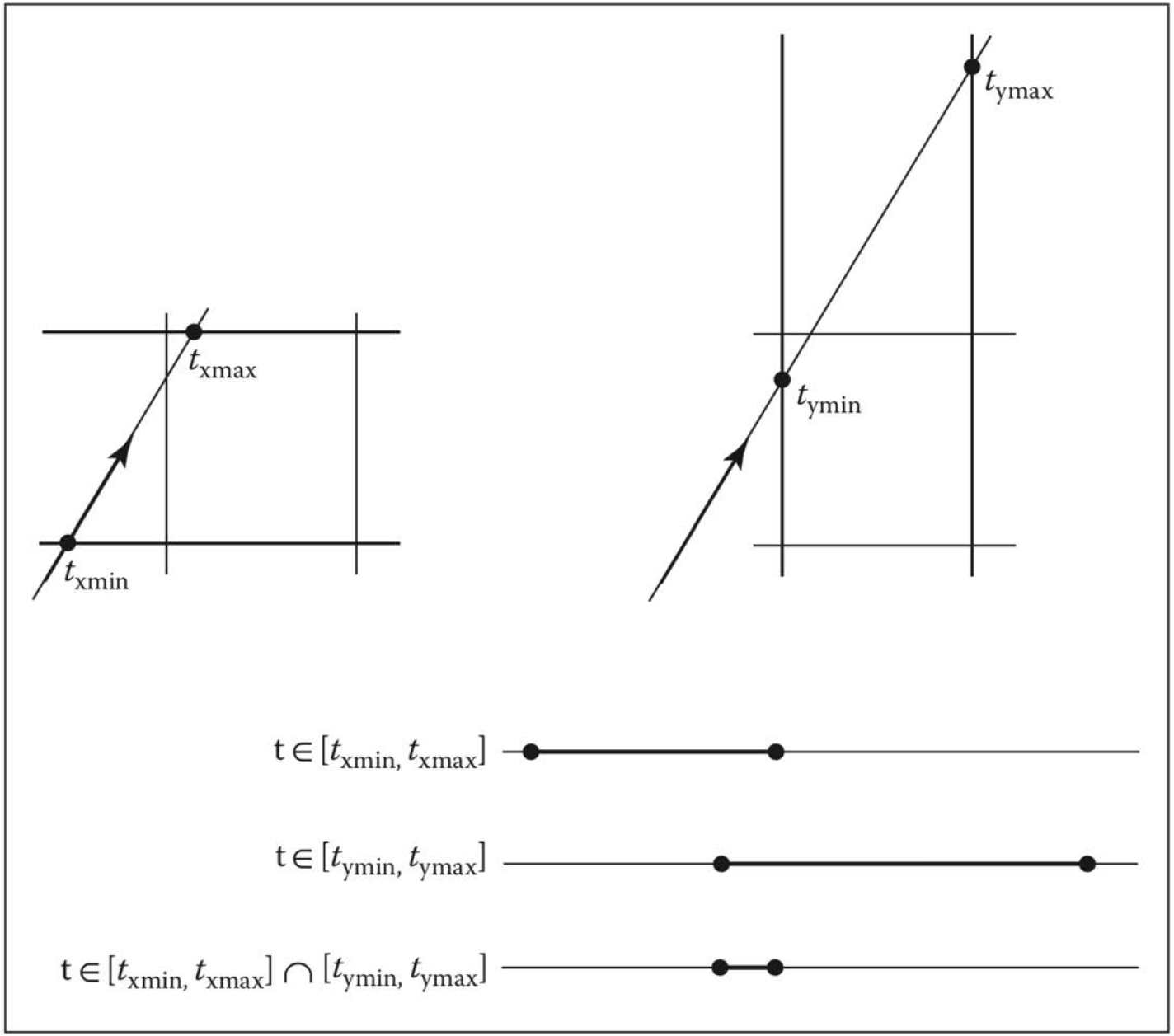
Ray-AABB Intersection

$$t_{\text{xmin}} < t_{\text{ymax}}$$



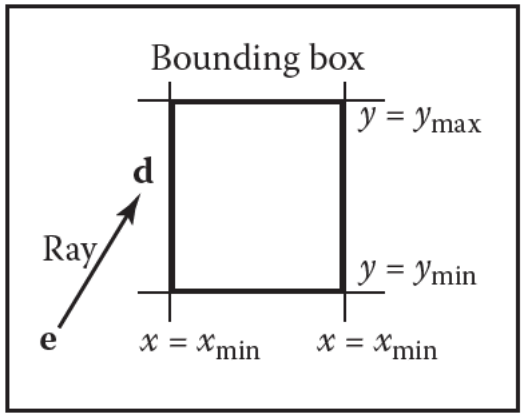
Ray-AABB Intersection

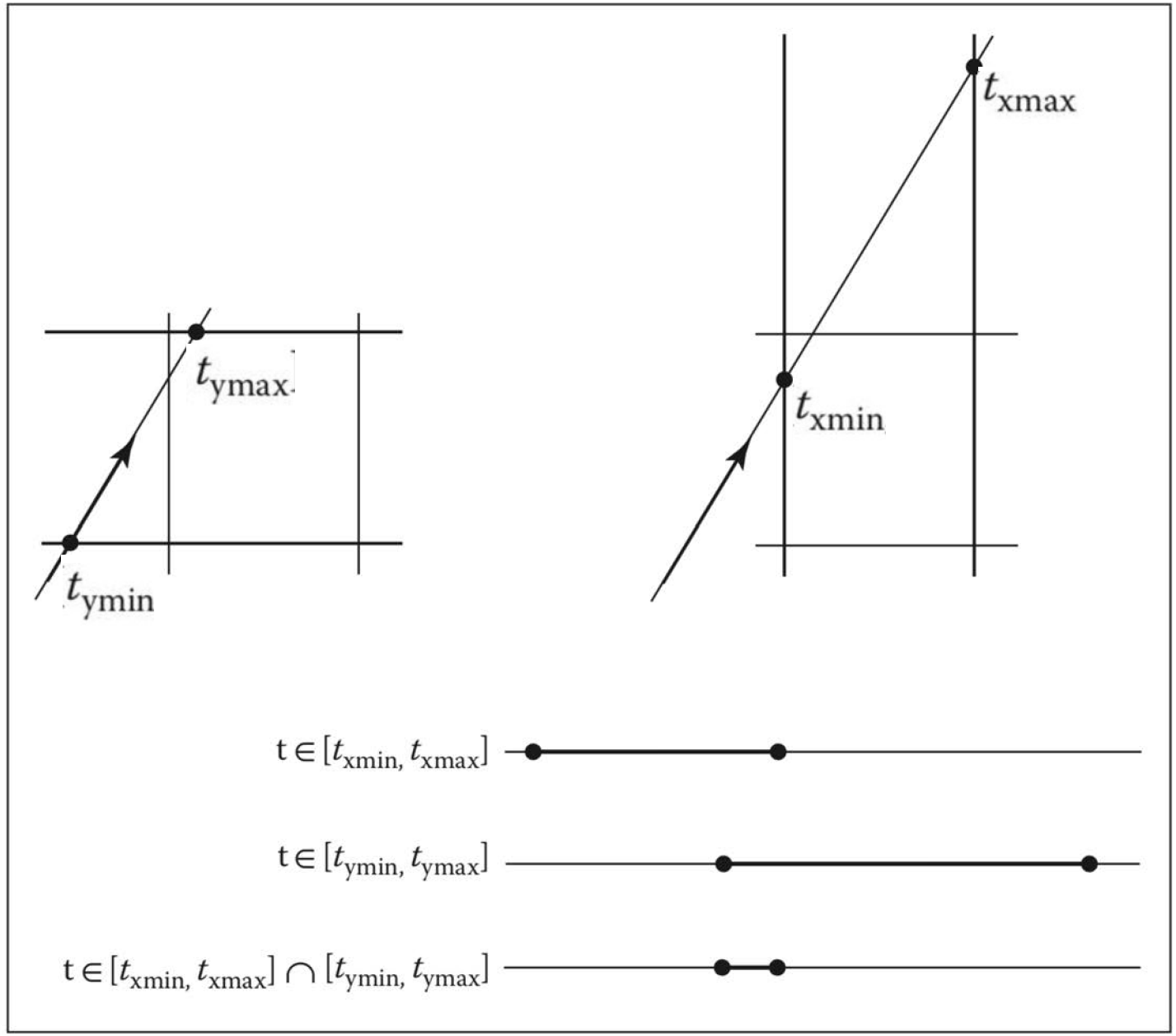
Note: I think the x pictures and y pictures are swapped



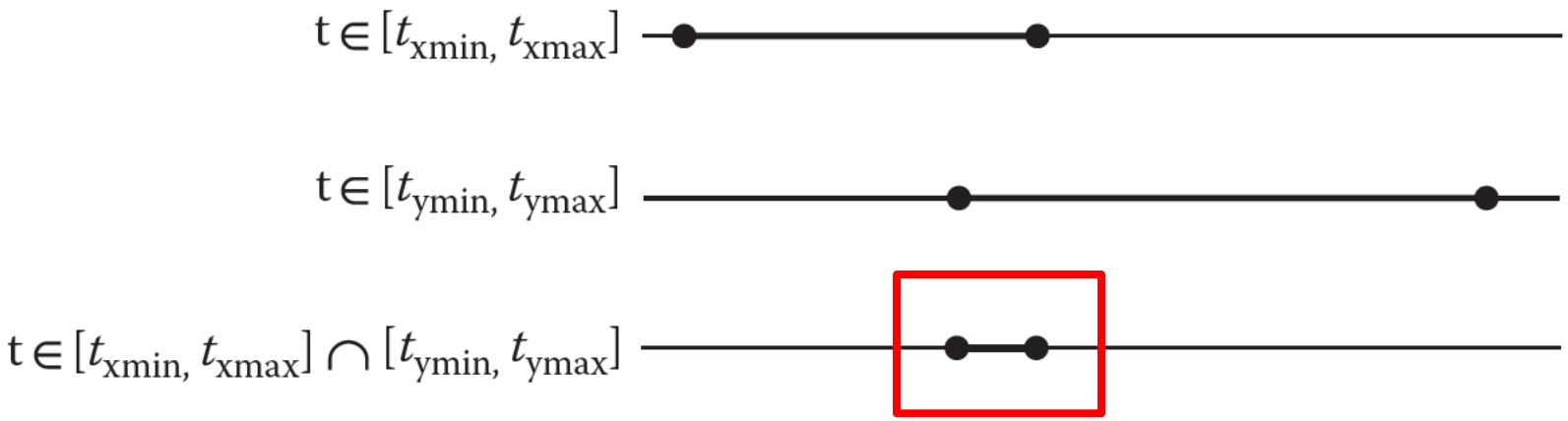
Ray-AABB Intersection

Note: I think the x pictures and y pictures are swapped

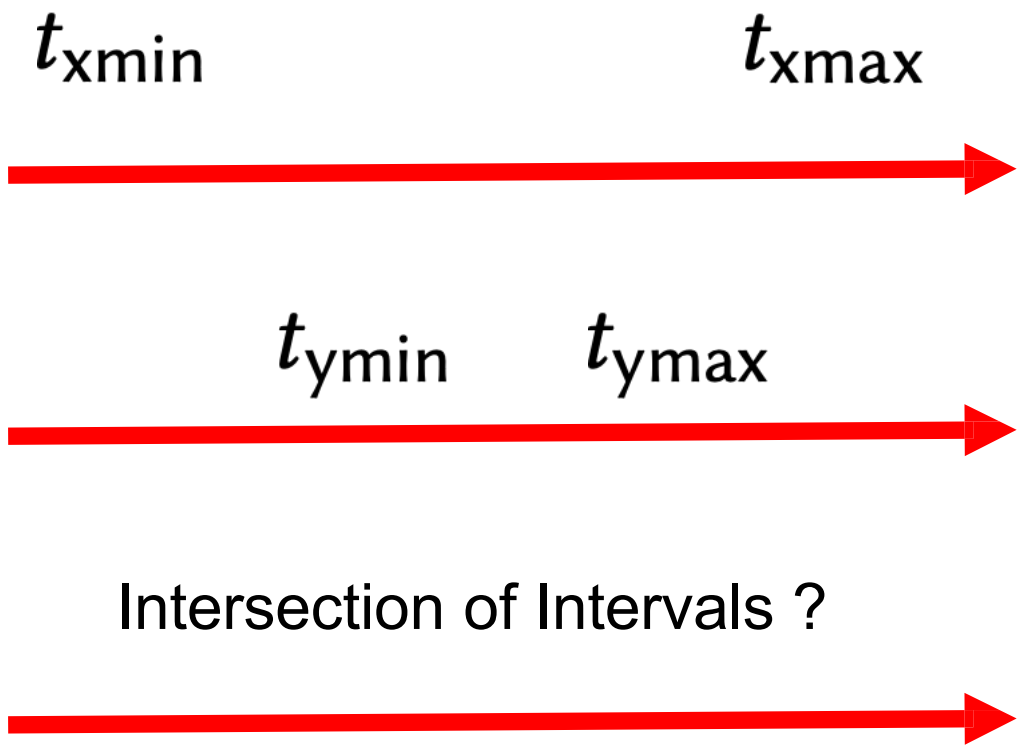




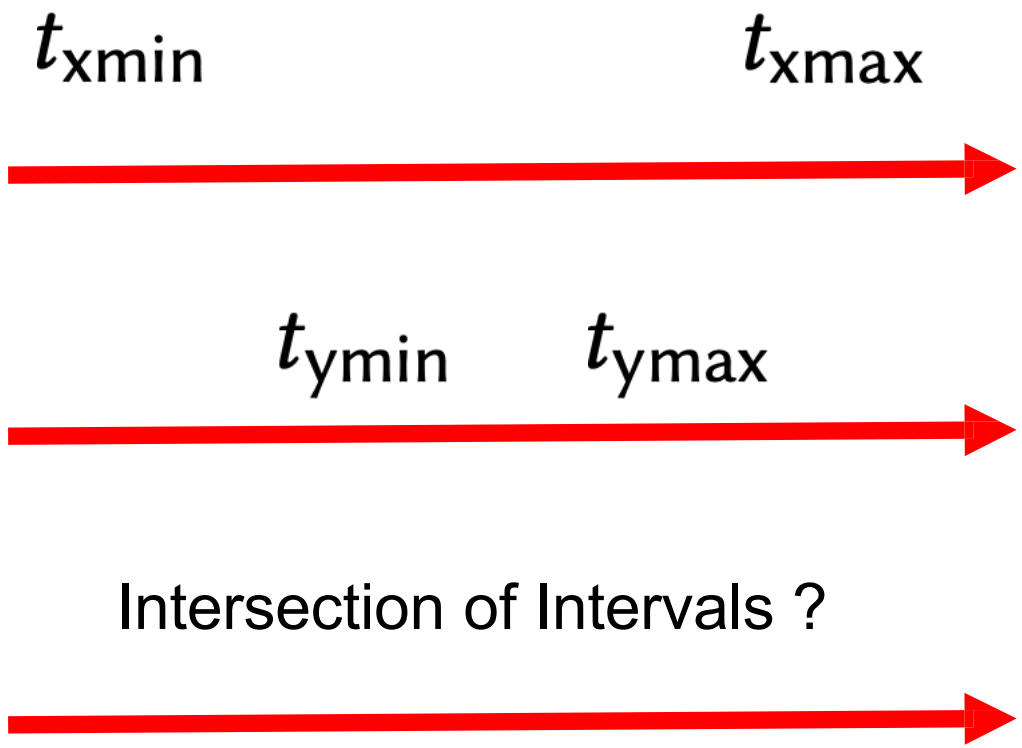
Ray-AABB Intersection



Ray-AABB Intersection

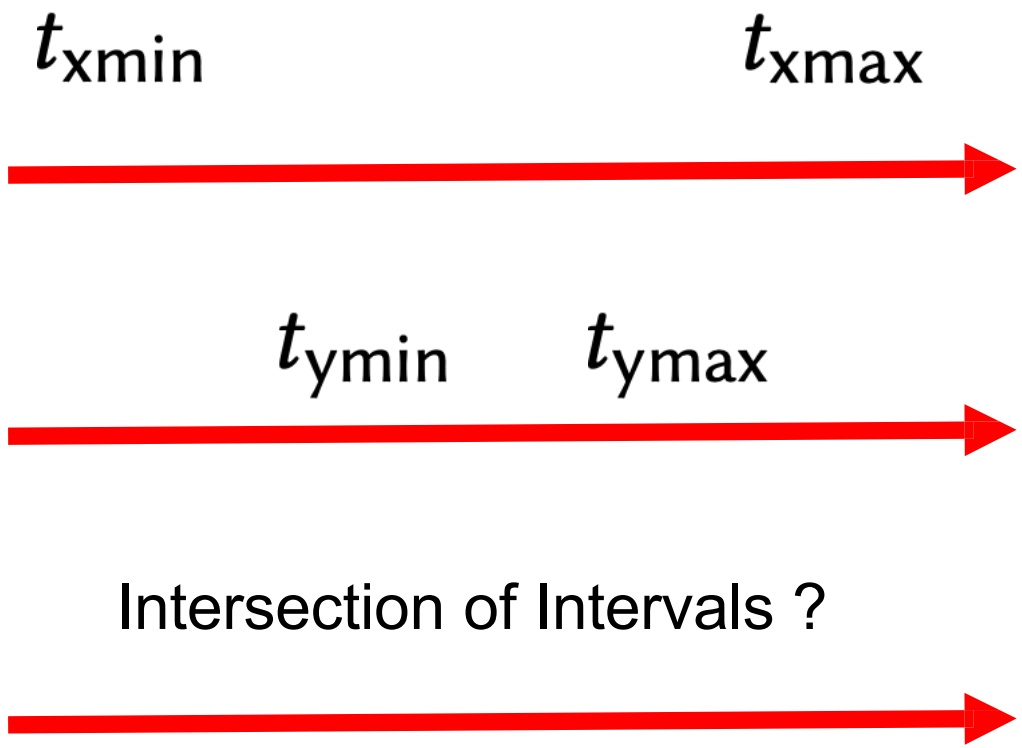


Ray-AABB Intersection



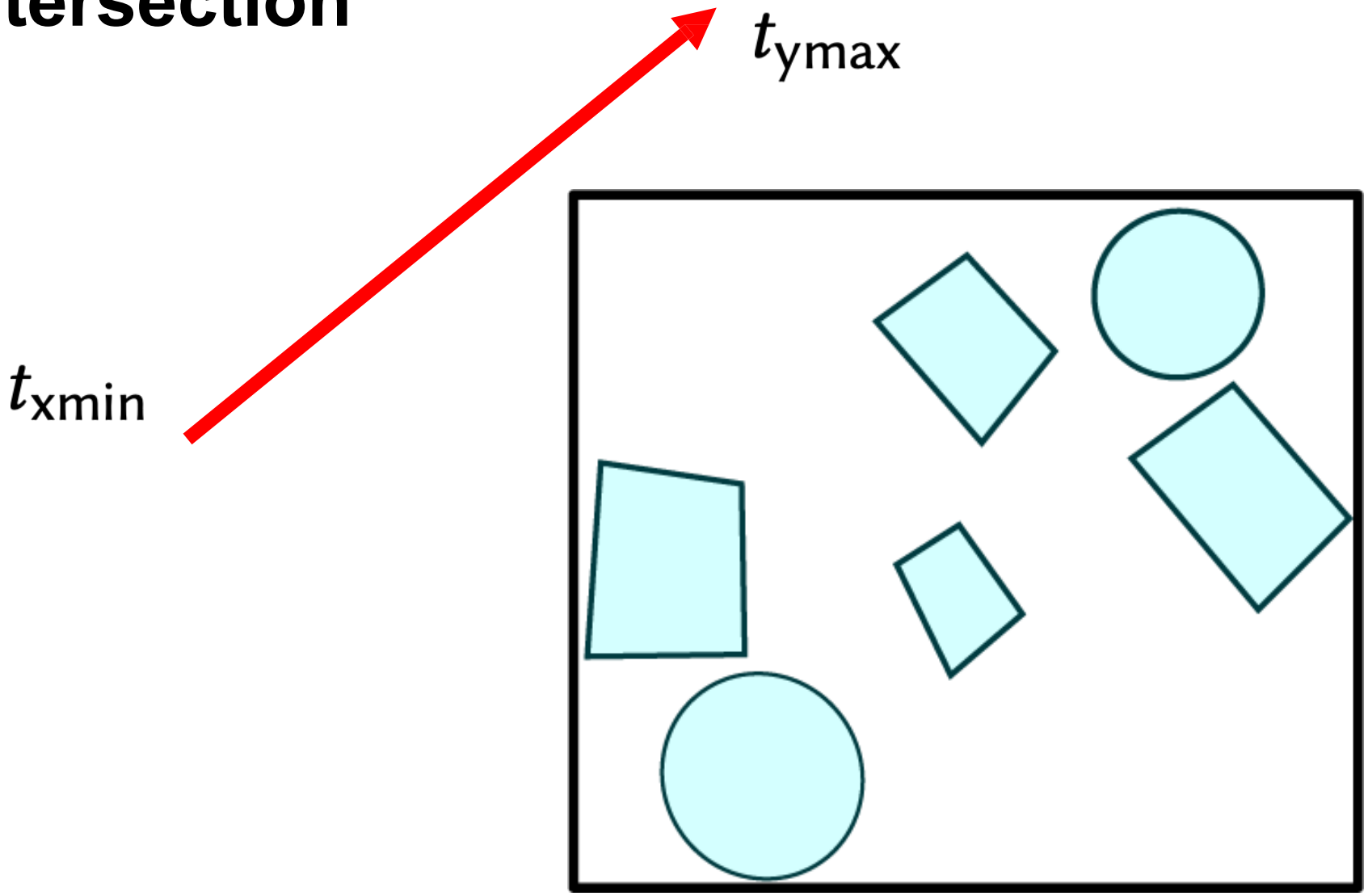
$$\max(t_{\text{xmin}}, t_{\text{ymin}}) \quad ? \quad \min(t_{\text{xmax}}, t_{\text{ymax}})$$

Ray-AABB Intersection

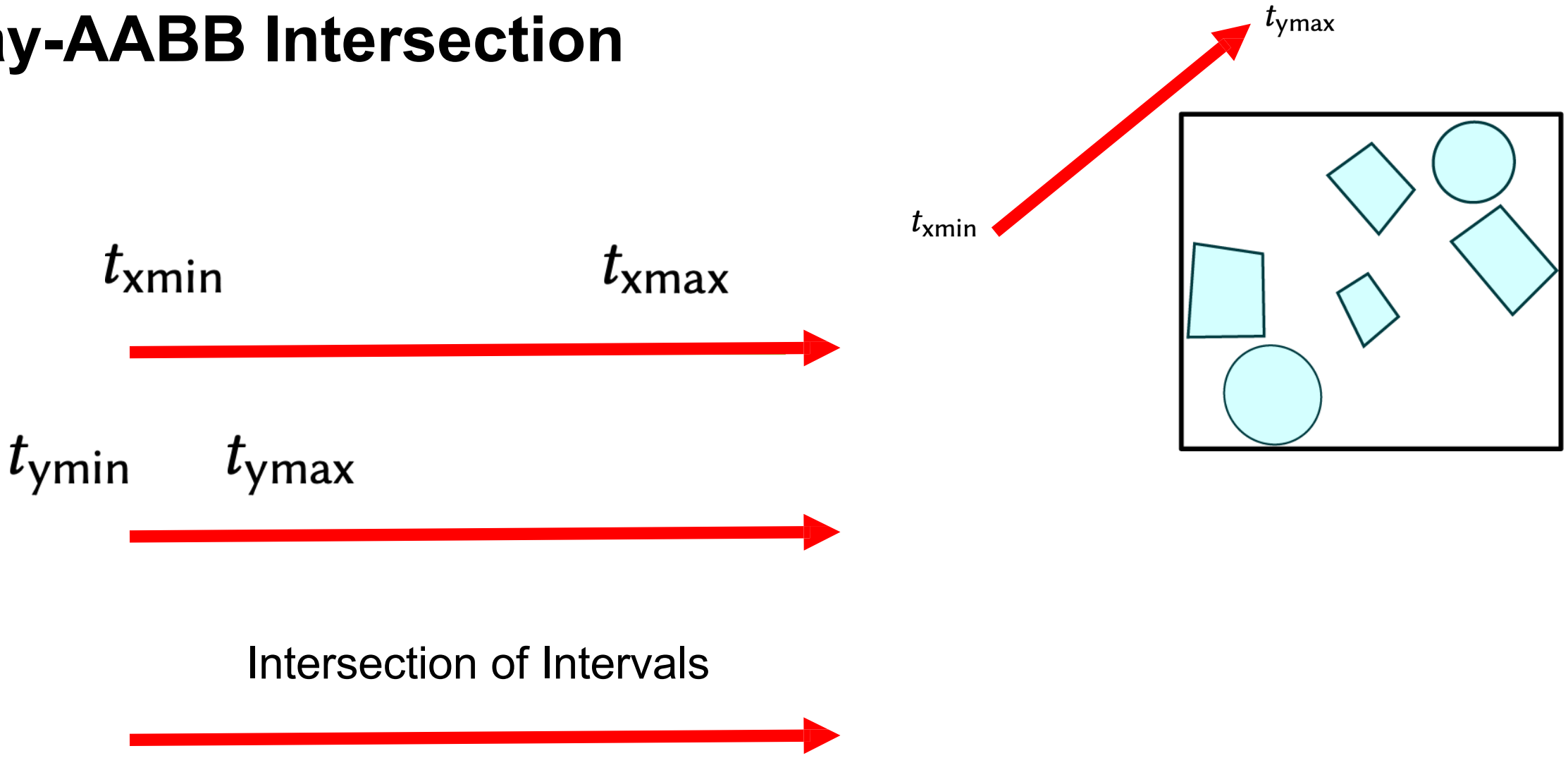


$$\max(t_{xmin}, t_{ymin}) \quad < \quad \min(t_{xmax}, t_{ymax})$$

Ray-AABB Intersection

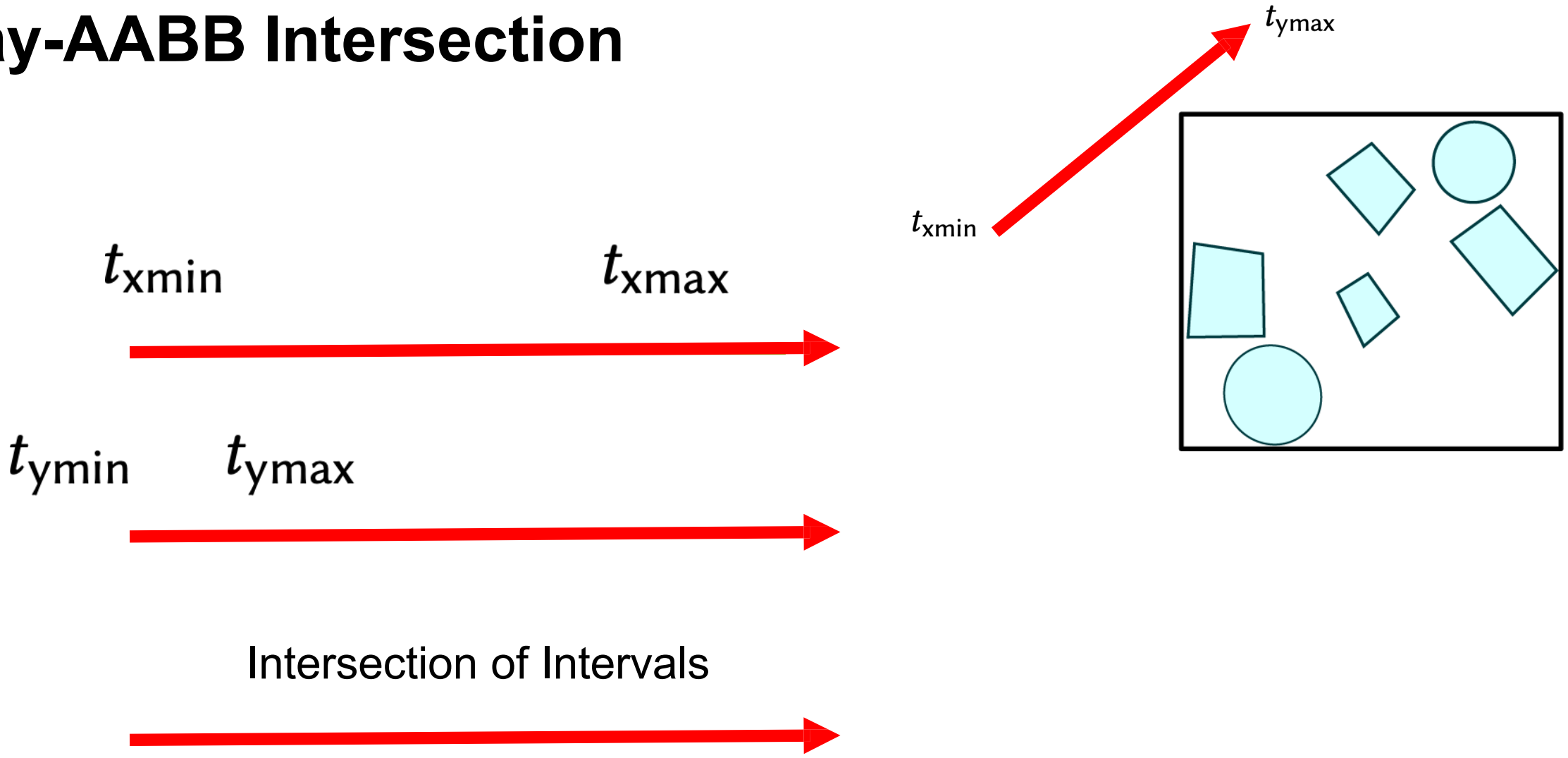


Ray-AABB Intersection



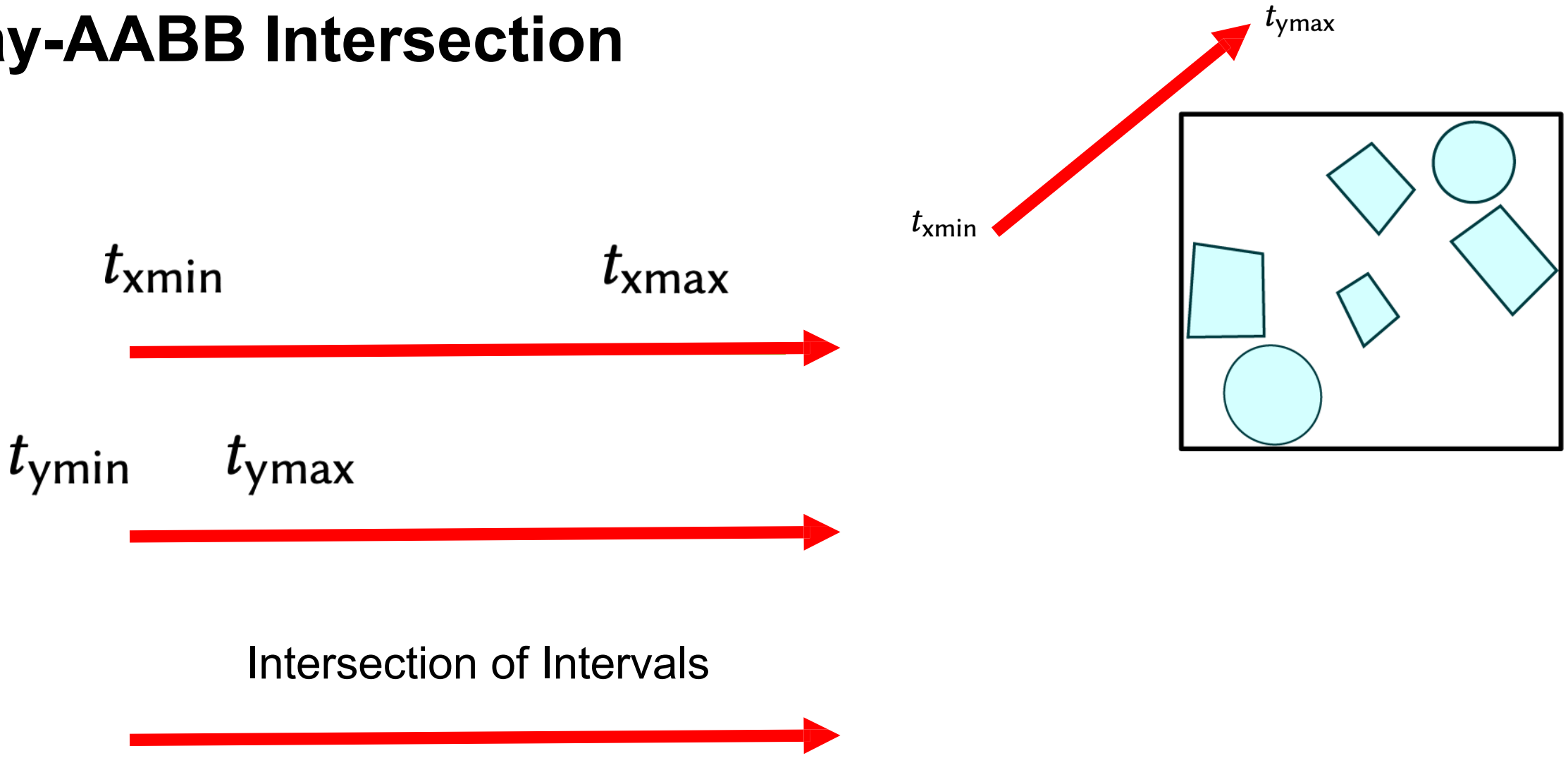
$$\max(t_{xmin}, t_{ymin}) \quad ? \quad \min(t_{xmax}, t_{ymax})$$

Ray-AABB Intersection



$$\max(t_{xmin}, t_{ymin}) \quad > \quad \min(t_{xmax}, t_{ymax})$$

Ray-AABB Intersection



$$\max(t_{xmin}, t_{ymin}) \quad > \quad \min(t_{xmax}, t_{ymax})$$ <-- CHECK

Building and Axis-Aligned Bounding Box (AABB)

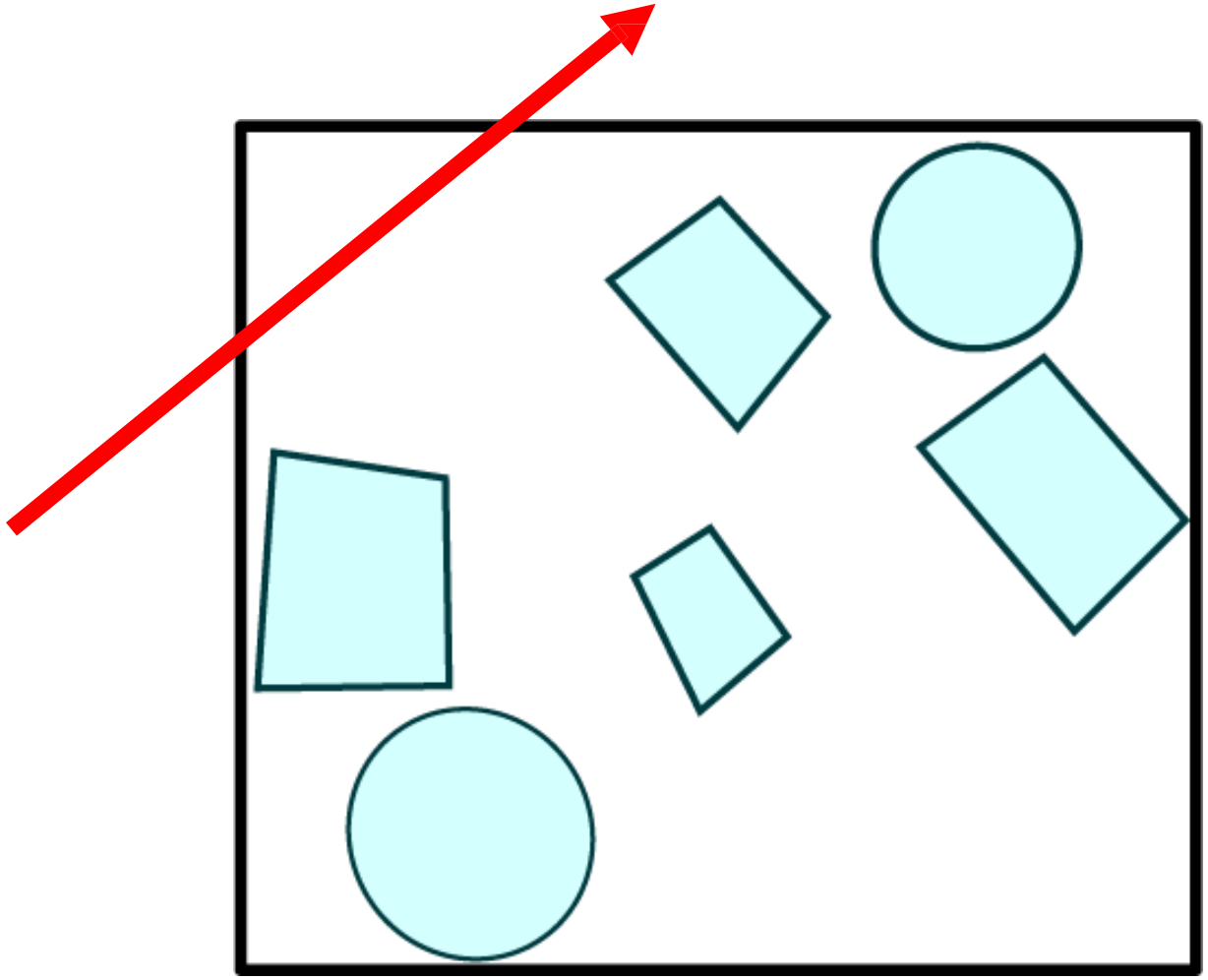
$$x_{\text{min}} = \min\left(v_x^i\right)$$

$$x_{\text{max}} = \max\left(v_x^i\right)$$

$$y_{\text{min}} = \min\left(v_y^i\right)$$

$$y_{\text{max}} = \max\left(v_y^i\right)$$

$$\mathbf{v}^i \in \text{Vertices}$$



Building and Axis-Aligned Bounding Box (AABB)

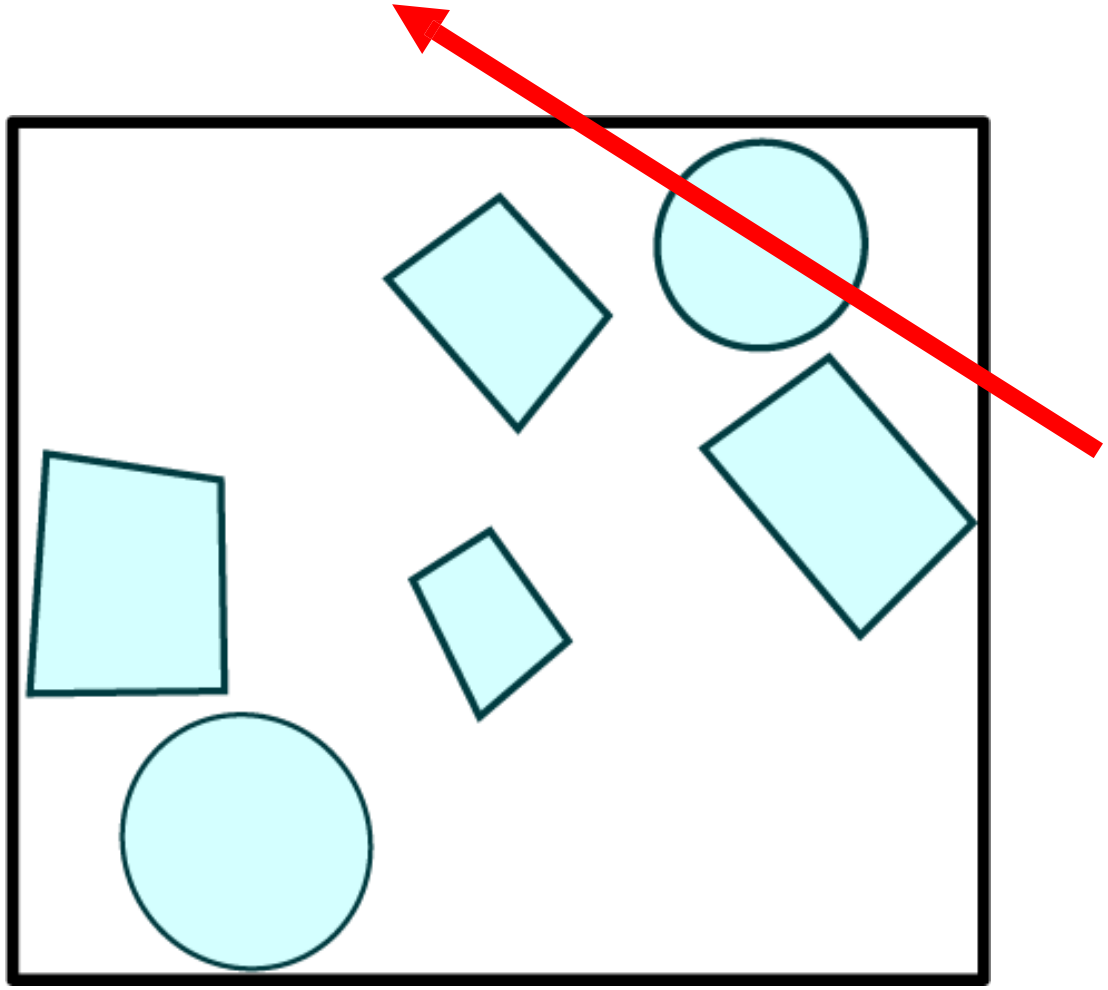
$$x_{\text{min}} = \min\left(v_x^i\right)$$

$$x_{\text{max}} = \max\left(v_x^i\right)$$

$$y_{\text{min}} = \min\left(v_y^i\right)$$

$$y_{\text{max}} = \max\left(v_y^i\right)$$

$$\mathbf{v}^i \in \text{Vertices}$$



Building and Axis-Aligned Bounding Box (AABB)

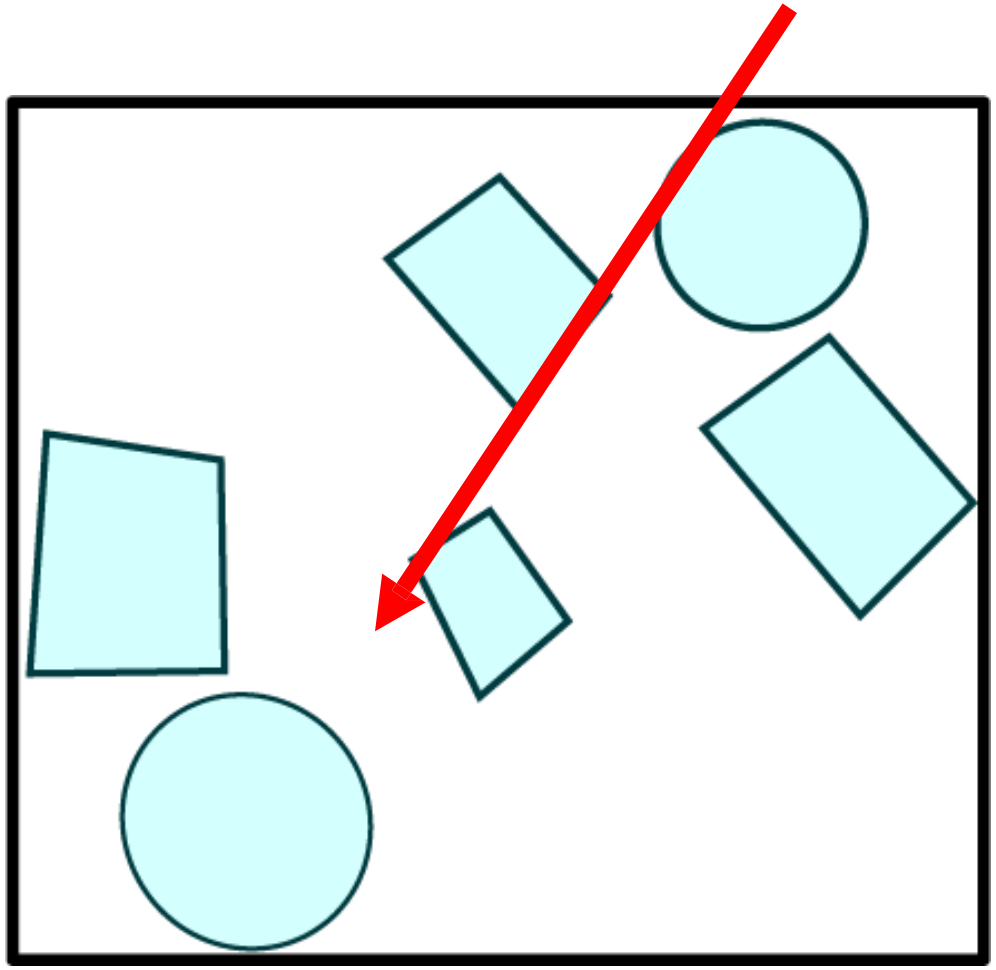
$$x_{\text{min}} = \min\left(v_x^i\right)$$

$$x_{\text{max}} = \max\left(v_x^i\right)$$

$$y_{\text{min}} = \min\left(v_y^i\right)$$

$$y_{\text{max}} = \max\left(v_y^i\right)$$

$$\mathbf{v}^i \in \text{Vertices}$$



Ray-AABB Intersection

if $(x_d \geq 0)$ **then**

$$t_{\text{xmin}} = (x_{\text{min}} - x_e)/x_d$$

$$t_{\text{xmax}} = (x_{\text{max}} - x_e)/x_d$$

else

$$t_{\text{xmin}} = (x_{\text{max}} - x_e)/x_d$$

$$t_{\text{xmax}} = (x_{\text{min}} - x_e)/x_d$$

When does this fail?

Ray-AABB Intersection

$$a = 1/x_d$$

if $(a \geq 0)$ **then**

$$t_{\text{min}} = a(x_{\text{min}} - x_e)$$

$$t_{\text{max}} = a(x_{\text{max}} - x_e)$$

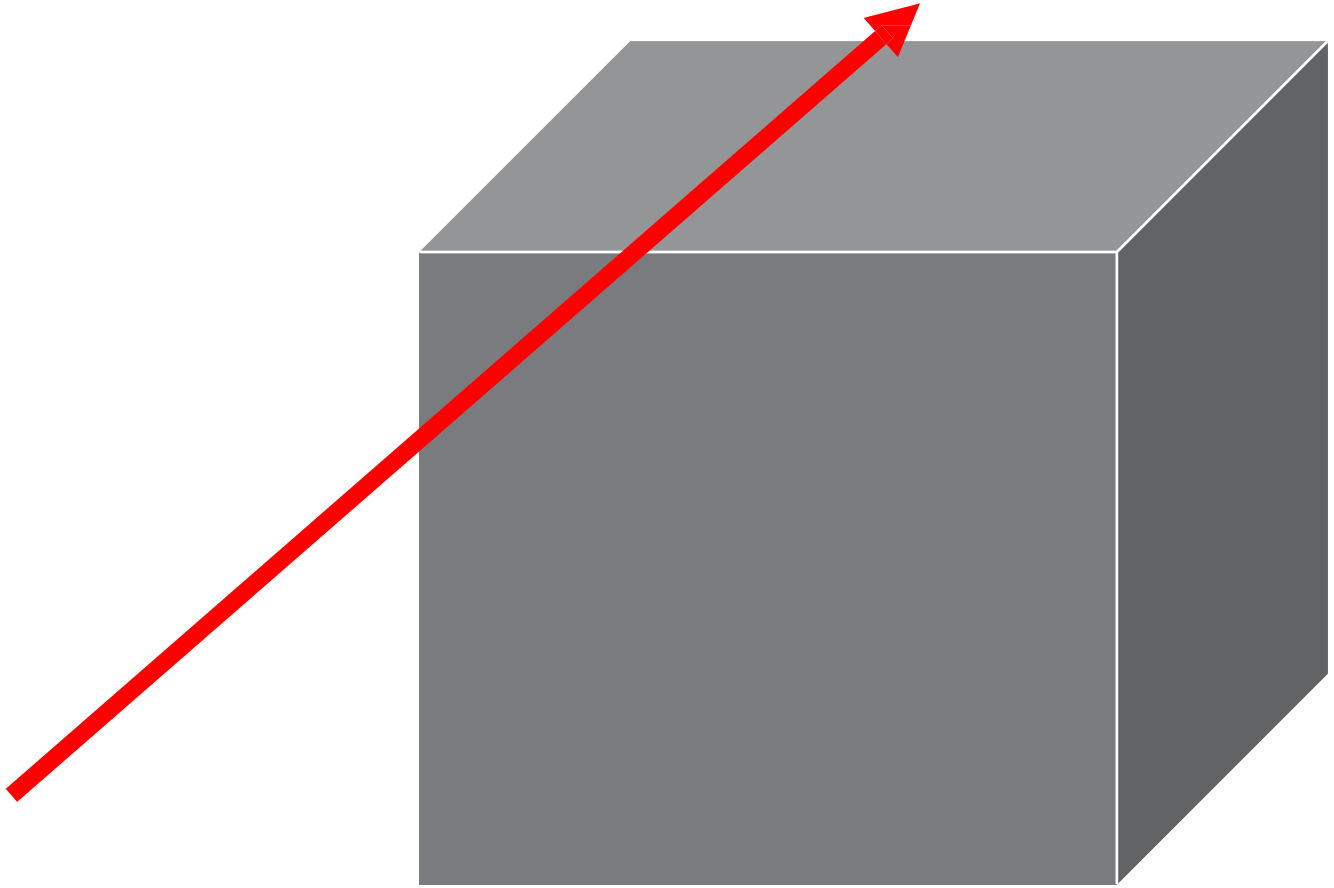
else

$$t_{\text{min}} = a(x_{\text{max}} - x_e)$$

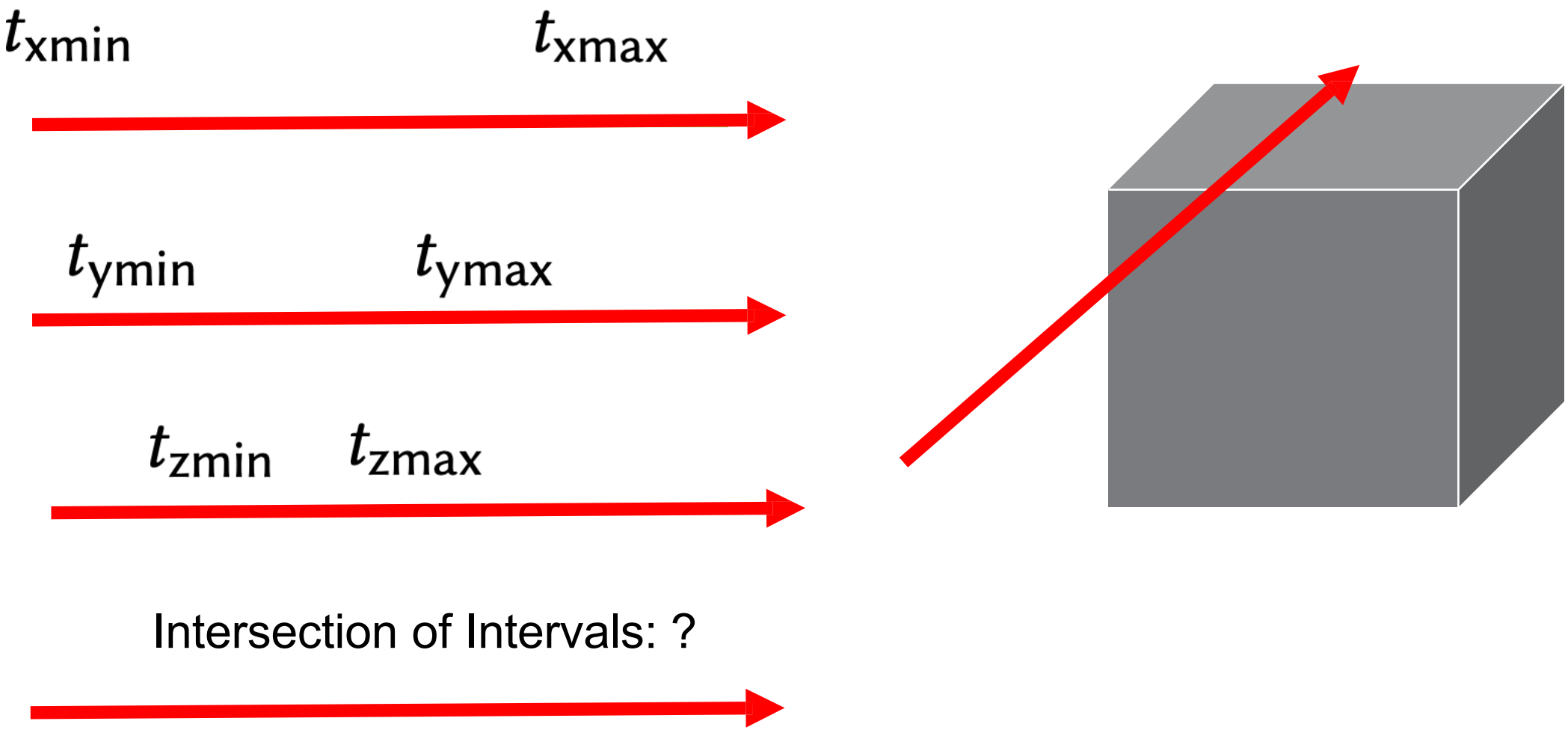
$$t_{\text{max}} = a(x_{\text{min}} - x_e)$$

Handles $x_d = -0$

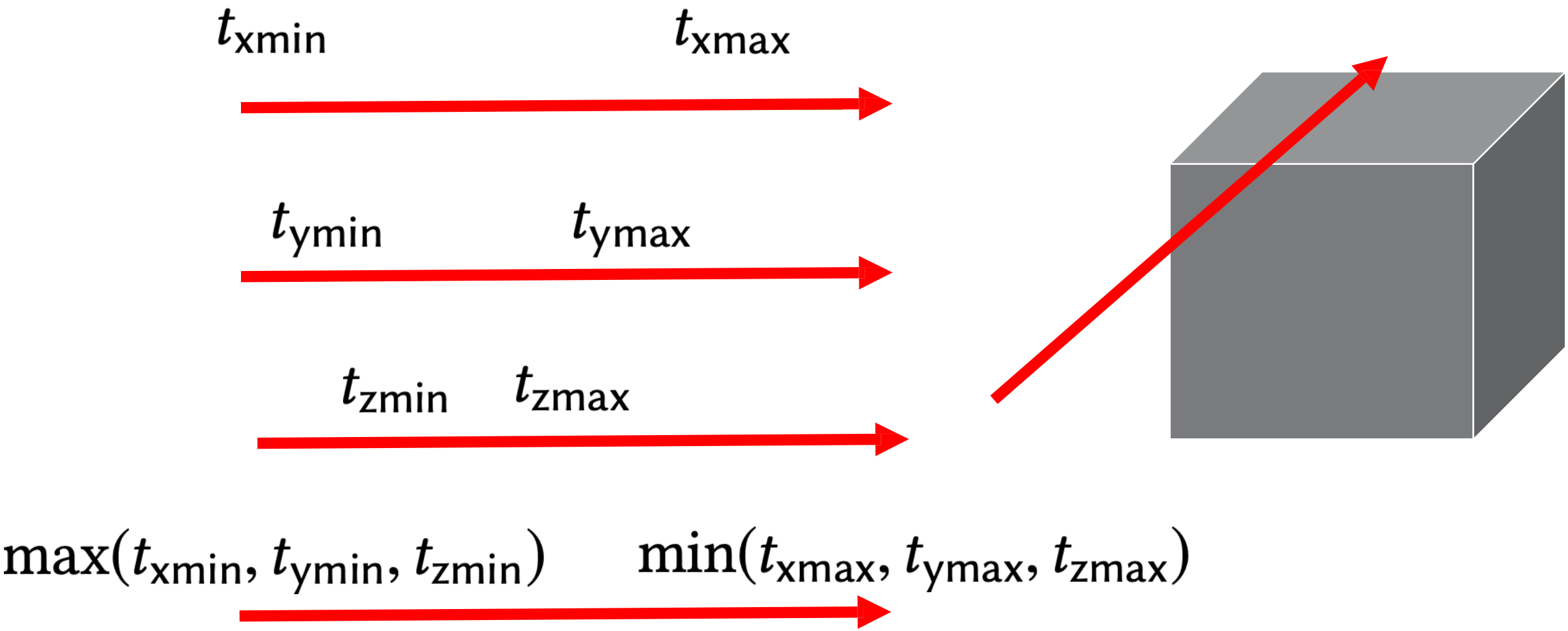
What happens in 3D ?



Ray-AABB Intersection



Ray-AABB Intersection



Ray-AABB Intersection

$$a = 1/x_d$$

if $(a \geq 0)$ **then**

$$t_{\text{min}} = a(x_{\text{min}} - x_e)$$

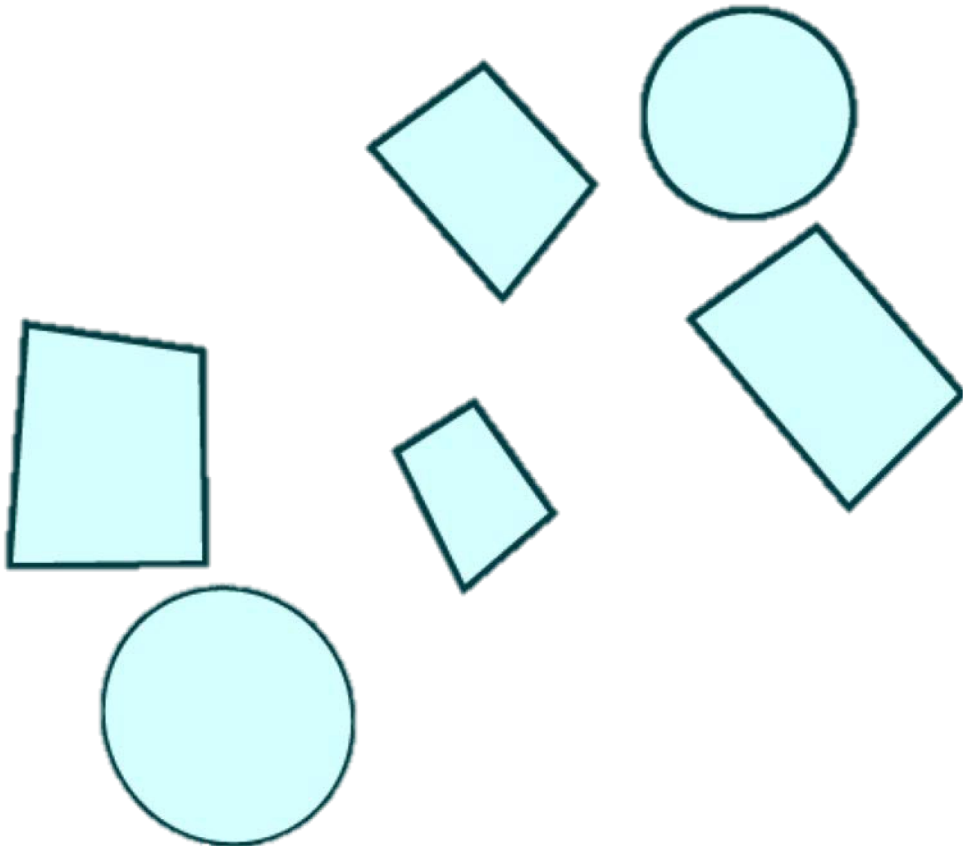
$$t_{\text{max}} = a(x_{\text{max}} - x_e)$$

else

$$t_{\text{min}} = a(x_{\text{max}} - x_e)$$

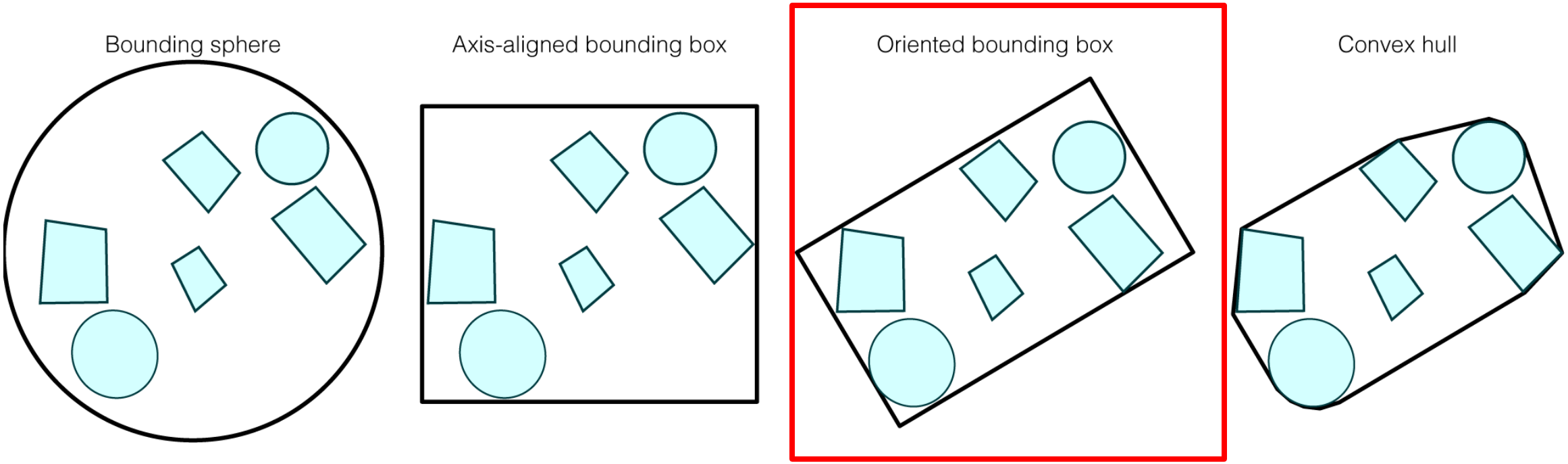
$$t_{\text{max}} = a(x_{\text{min}} - x_e)$$

Building an Object-Oriented Bounding Box (OOBB)



Bounding Volumes (BVs)

"Simple" geometry that fully encloses a **collection** of other geometry



Sphere
(a lot)

AABB
(a lot)

OOBB
(a little)

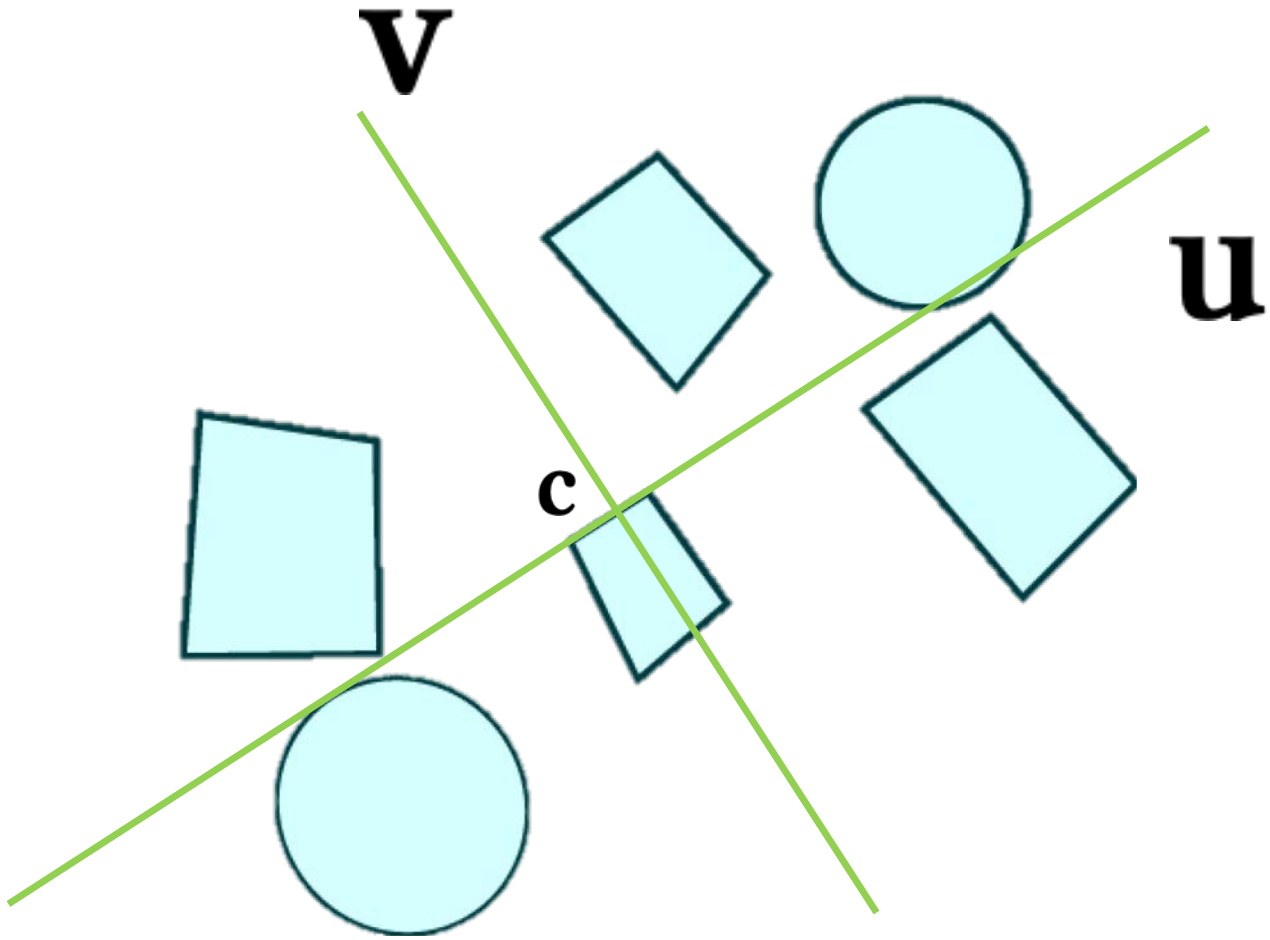
Convex Hull
(nope!)

https://en.wikipedia.org/wiki/Convex_hull

Building an Object-Oriented Bounding Box (OOBB)

$$\mathbf{c} = \frac{1}{n}\sum_{i=1}^{n}\mathbf{v}^i$$

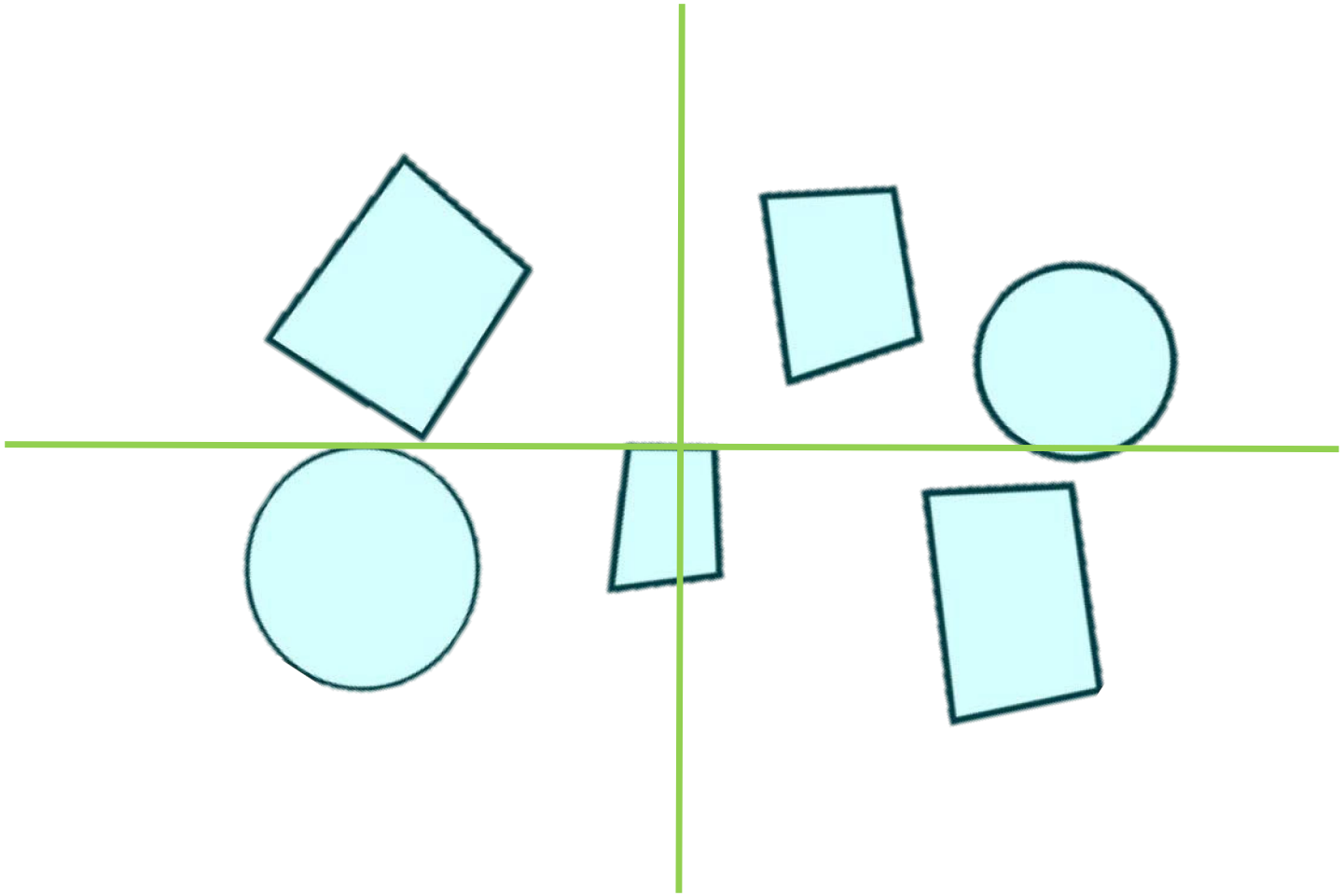
$$\begin{bmatrix}\mathbf{u} & \mathbf{v}\end{bmatrix}$$



Find directions of maximum and minimum variance

https://stats.stackexchange.com/questions/286603/understanding-direction-of-greatest-variance-in-pca

Building an Object-Oriented Bounding Box (OOBB)



Build Rotation Matrix

Collision Query with Object-Oriented Bounding Box

Spatial Data Structures

Basic Idea – asymptotic improvement in spatial queries by subdividing

Two types of subdivisions – *object-based* and *spatial*

Spatial Data Structures

Basic Idea – asymptotic improvement in spatial queries by subdividing

Two types of subdivisions – ***object-based*** and *spatial*

Spatial Data Structures

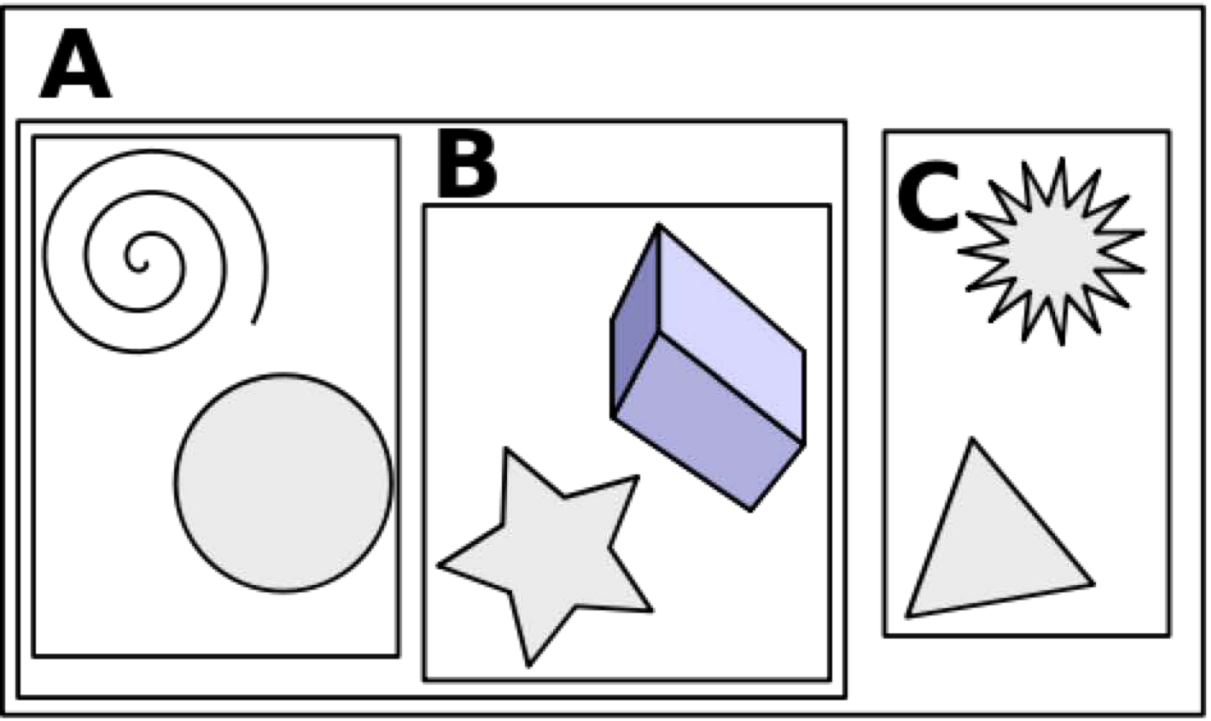
Basic Idea – asymptotic improvement in spatial queries by subdividing

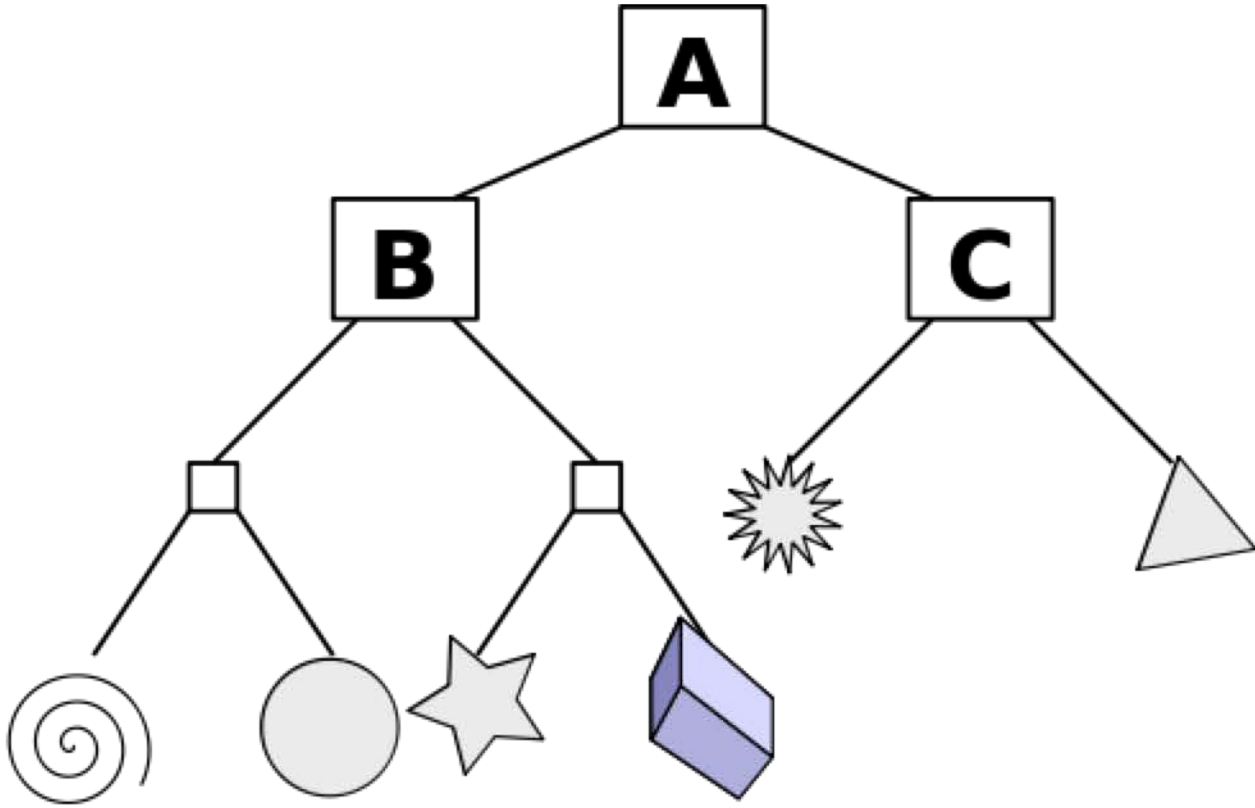
Two types of subdivisions – ***object-based*** and *spatial*

Our object-based data structures will be boundary volume hierarchies or BVHs.

BVHs are hierarchies of BVs represented by trees

Boundary Volume Hierarchy

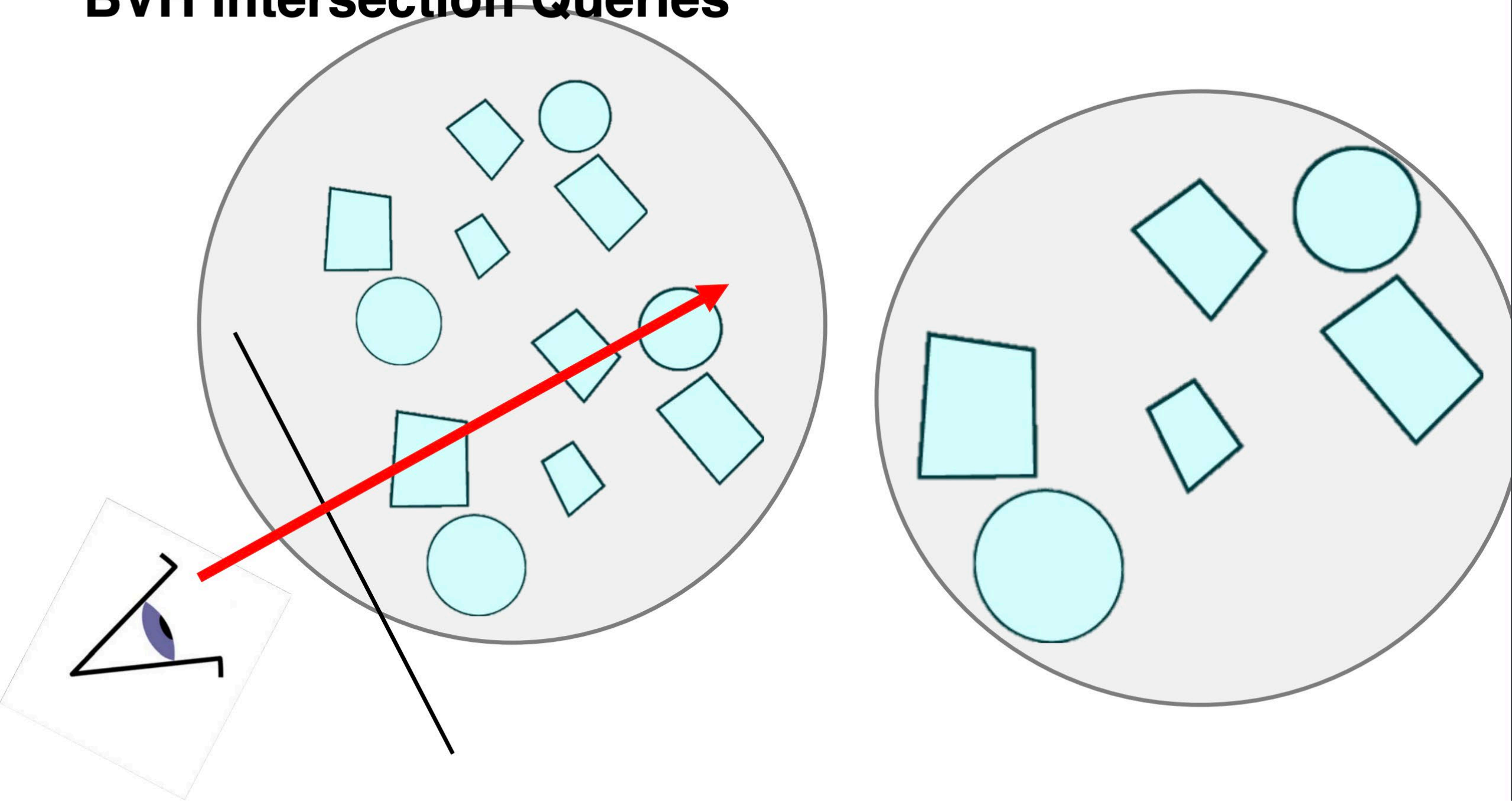




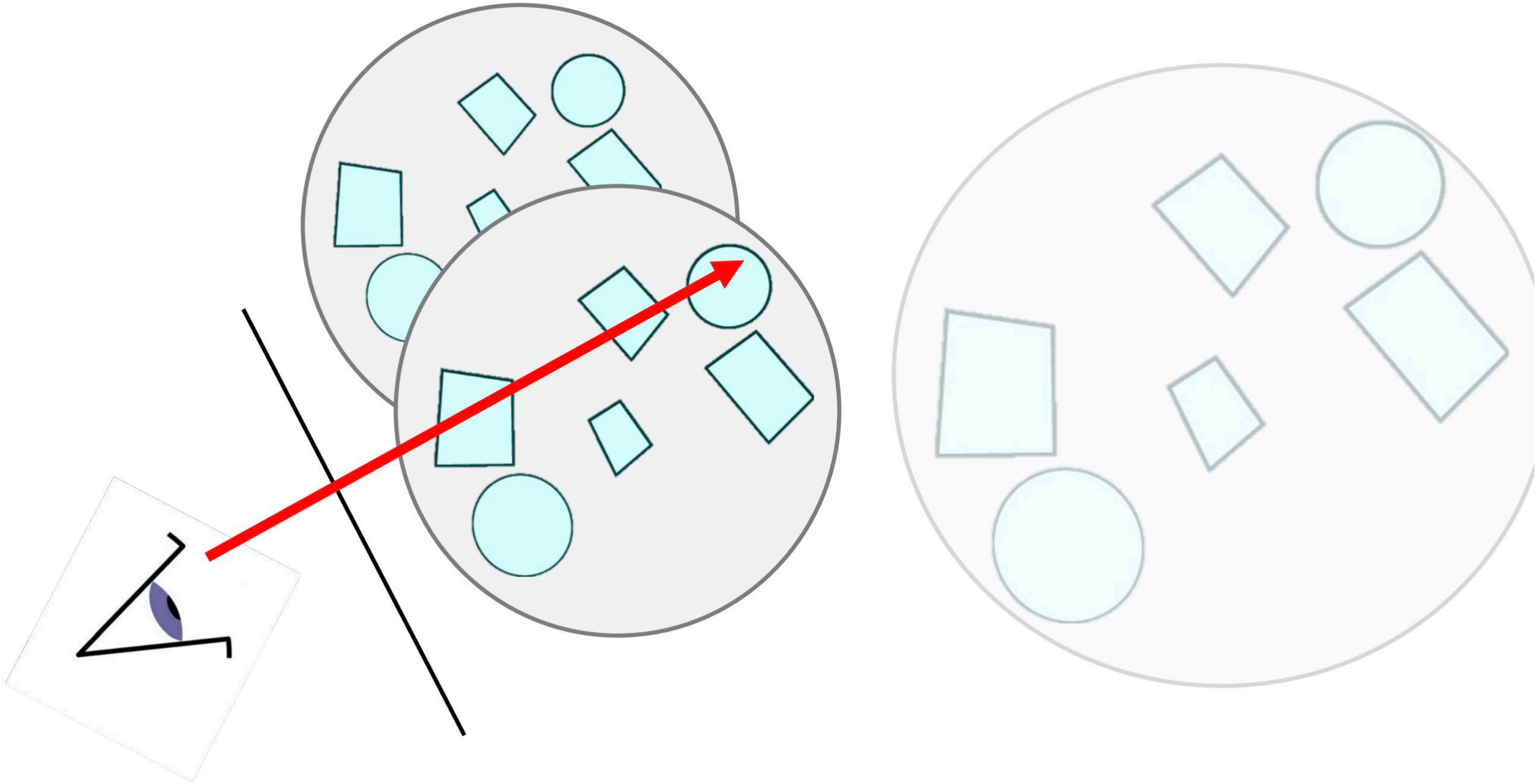
BVH Intersection Queries

```
intersect(bvNode, ray,t)
{
    if (bvNode== null || !bvNode.intersect(ray,t))
        return false;
    else
   {
      i1=intersect(bvNode.left, ray,t1);  //check left BV
      i2=intersect(bvNode.right, ray,t2); //check right BV
      if (i1 && i2) {  t=min(t1,t2); return true;  }
      if (i1) {  t=t1; return true;  }
      if (i2) {  t=t2; return true;  }
      return false;
    }
}
```

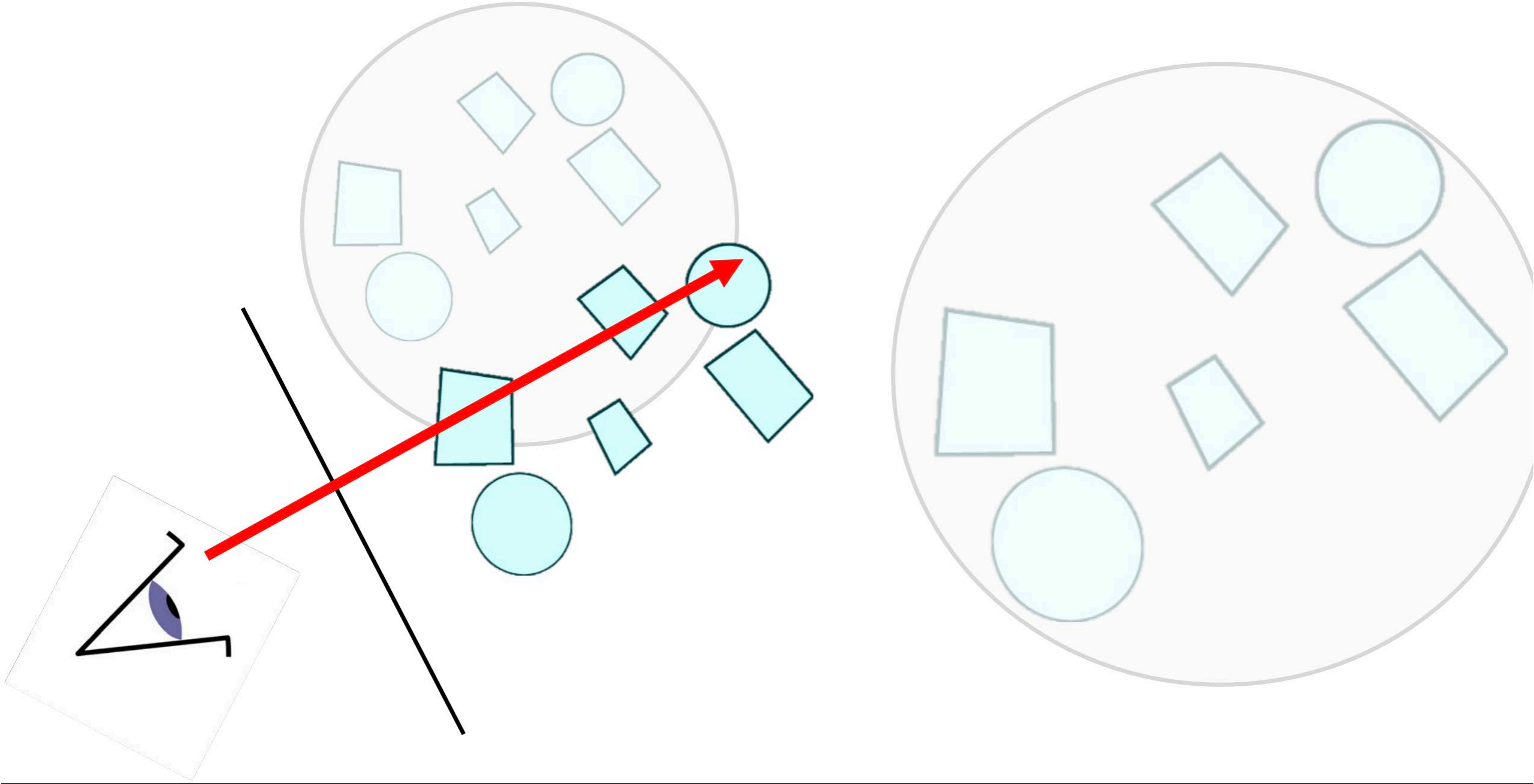
BVH Intersection Queries



BVH Intersection Queries



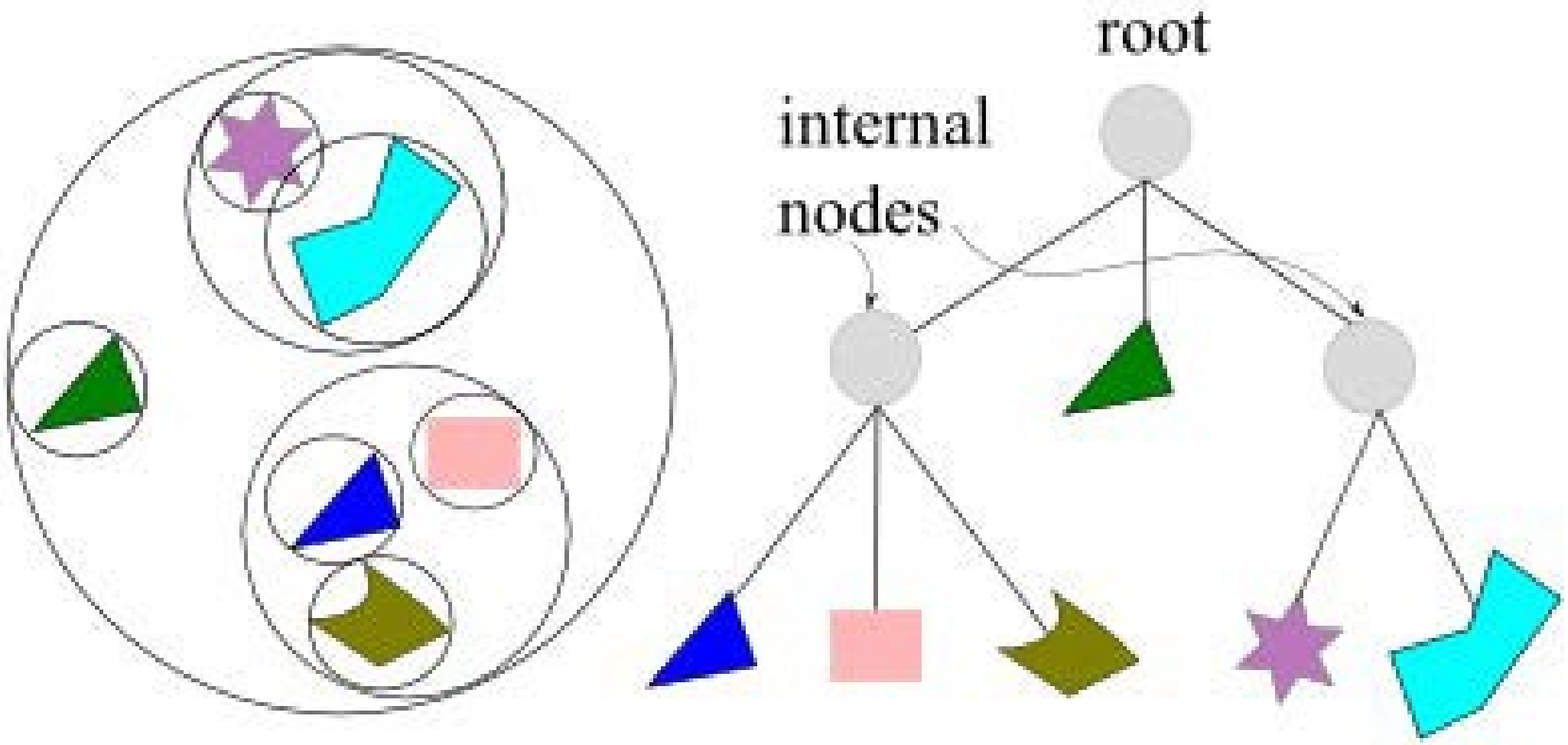
BVH Intersection Queries



BVH Distance Queries

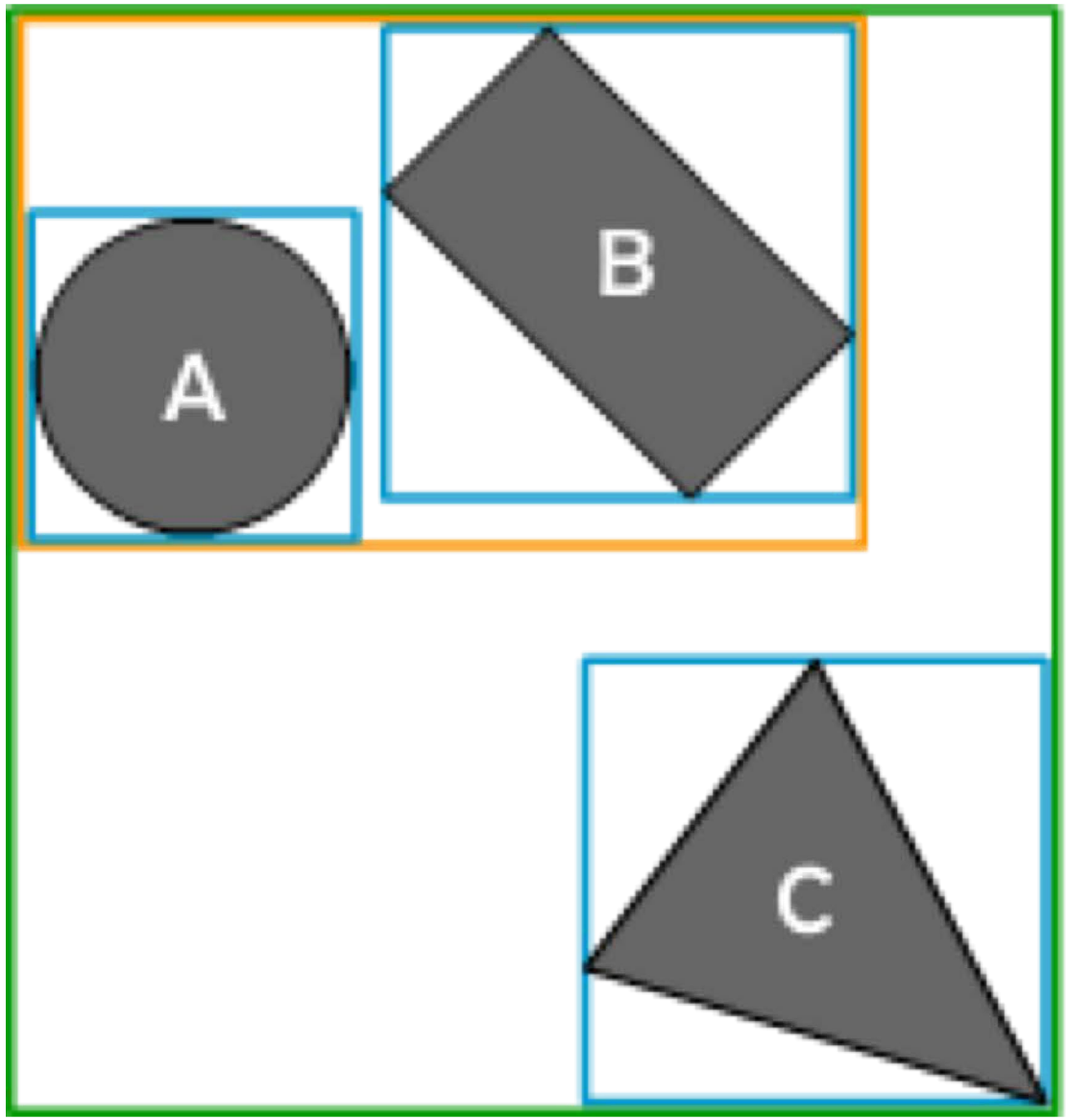
```
minDistance(bvNode, point, currentMin)
{
   d1=minDistance(bvNode.left, point, currentMin);
   d2=minDistance(bvNode.right, point, currentMin);

    if(min(d1,d2) > currentMin) {
       return currentMin
    }

    return min(d1,d2)
}
```

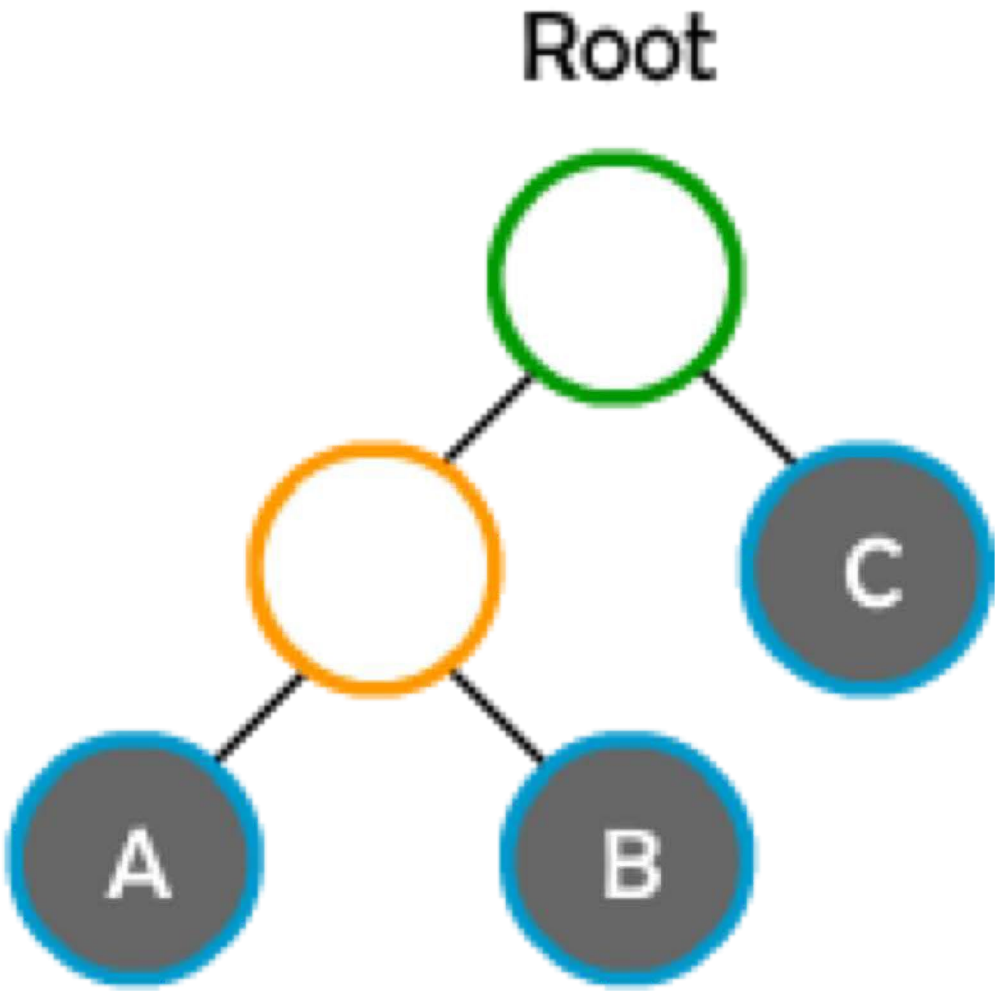
Sphere Trees



www.bogotobogo.com/Games/spatialdatastructure.php

AABB Trees





http://allenchou.net/2014/02/game-physics-broadphase-dynamic-aabb-tree/

Done for Today