

Ray Casting



Some Slides/Images adapted from Marschner and Shirley and David Levin

Announcements

Assignment 1 is due tomorrow at midnight (12 May)

Reminders:

- Tutorials are on Fridays and are used to ask questions about the concepts, assignments, and course in general
- Use the github issues page to ask questions about the assignments.
 - There will be a TA monitoring the page

Clarifications

Over operator from wikipedia

(https://en.wikipedia.org/wiki/Alpha_compositing):

$$C_o = \frac{C_a \alpha_a + C_b \alpha_b (1 - \alpha_a)}{\alpha_a + \alpha_b (1 - \alpha_a)}$$

or

$$c_o = c_a + c_b(1 - \alpha_a), \quad \alpha_o = \frac{c_o}{C_o} = \alpha_a + \alpha_b(1 - \alpha_a)$$

For further reading:

<https://keithp.com/~keithp/porterduff/p253-porter.pdf>

<https://tomforsyth1000.github.io/blog.wiki.html#%5B%5BPremultiplied+alpha%5D%5D>

Clarifications

"Normal" alpha-blending munges together two physically separate effects - the amount of light this layer of rendering lets through from behind, and the amount of light this layer adds to the image. Instead, it keeps the two related - you can't have a surface that adds a bunch of light to a scene without it also masking off what is behind it. Physically, this makes no sense - just because you add a bunch of photons into a scene, doesn't mean all the other photons are blocked. Premultiplied alpha fixes this by keeping the two concepts separate - the blocking is done by the alpha channel, the addition of light is done by the colour channel. This is not just a neat concept, it's really useful in practice.

$$C_o = \frac{C_a \alpha_a + C_b \alpha_b (1 - \alpha_a)}{\alpha_a + \alpha_b (1 - \alpha_a)}$$

or

$$c_o = c_a + c_b (1 - \alpha_a),$$

$$\alpha_o = \frac{c_o}{C_o} = \alpha_a + \alpha_b (1 - \alpha_a)$$

Any Questions?

Today: Ray Casting

The Ray Casting Algorithm

Introduction to Rays

The Camera

Ray-Object Intersection

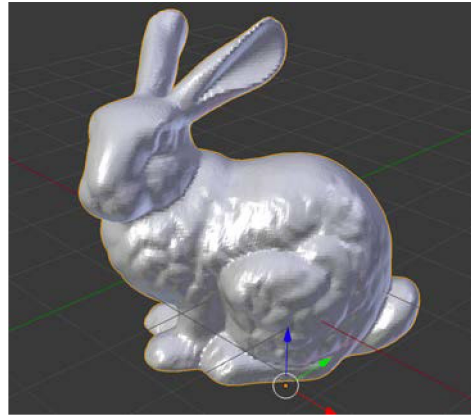
Ray-Plane Intersection

Ray-Sphere Intersection

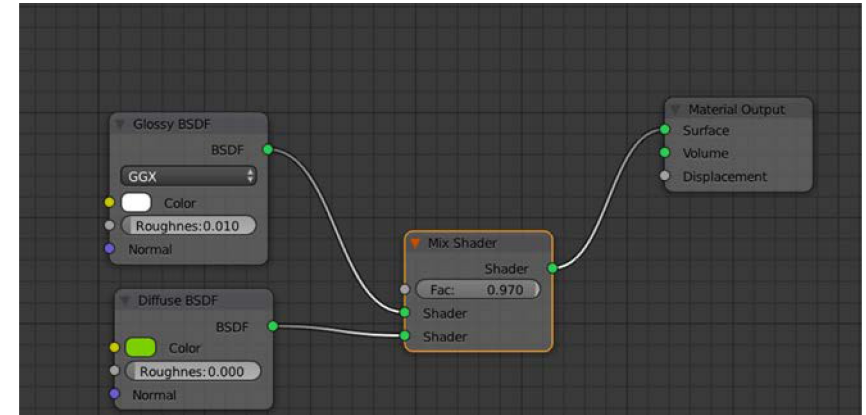
Ray-Triangle Intersection

Rendering

Input:

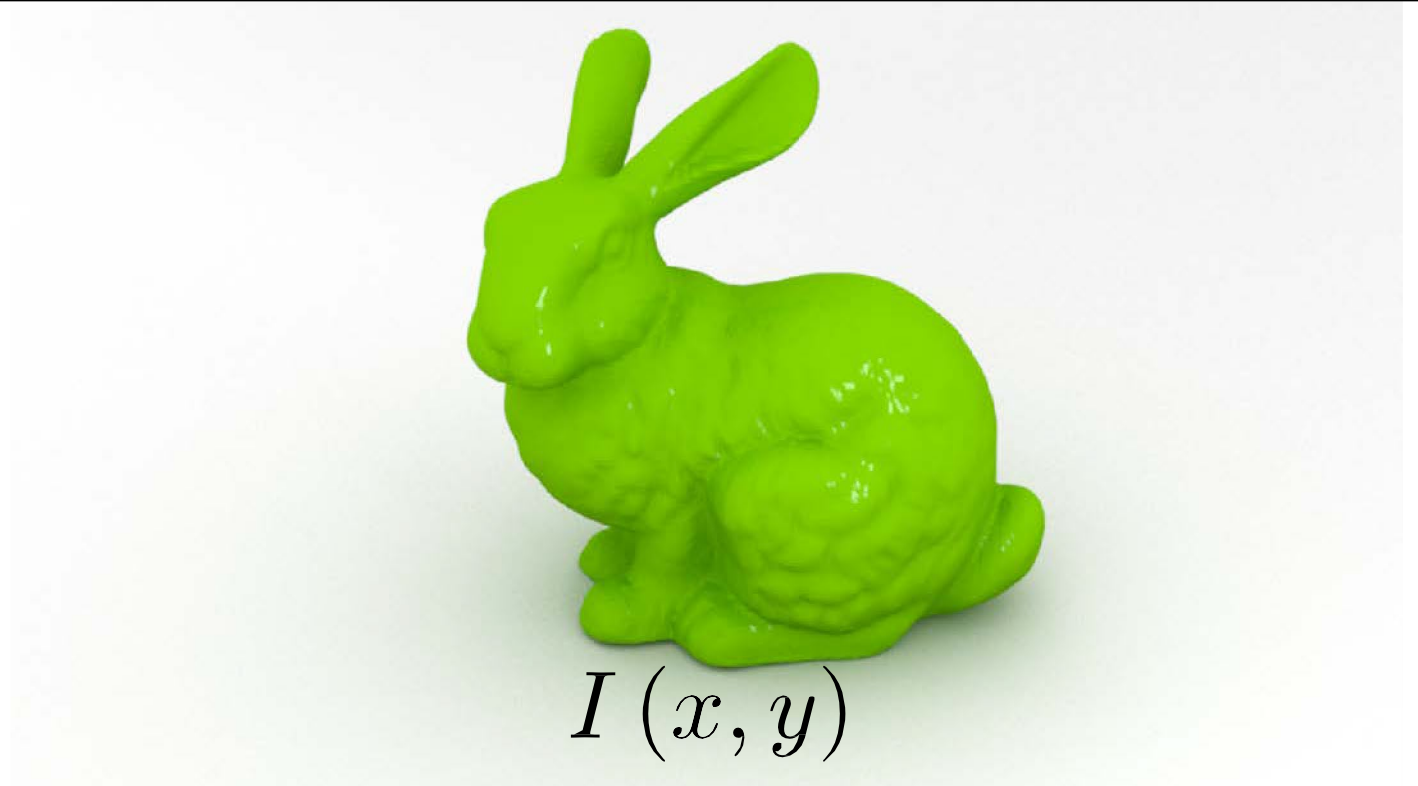


Objects



Materials

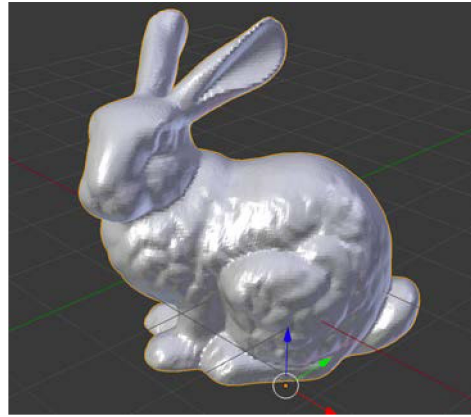
Output:



$$I(x, y)$$

Cameras

Input:



Objects



Lights

Output:



$$I(x, y)$$

WARIOR_GAMING_57

YZx-_Vulka

danielek185

zvarownik

NW N NE E
285 300 330 345 15 30 60 75

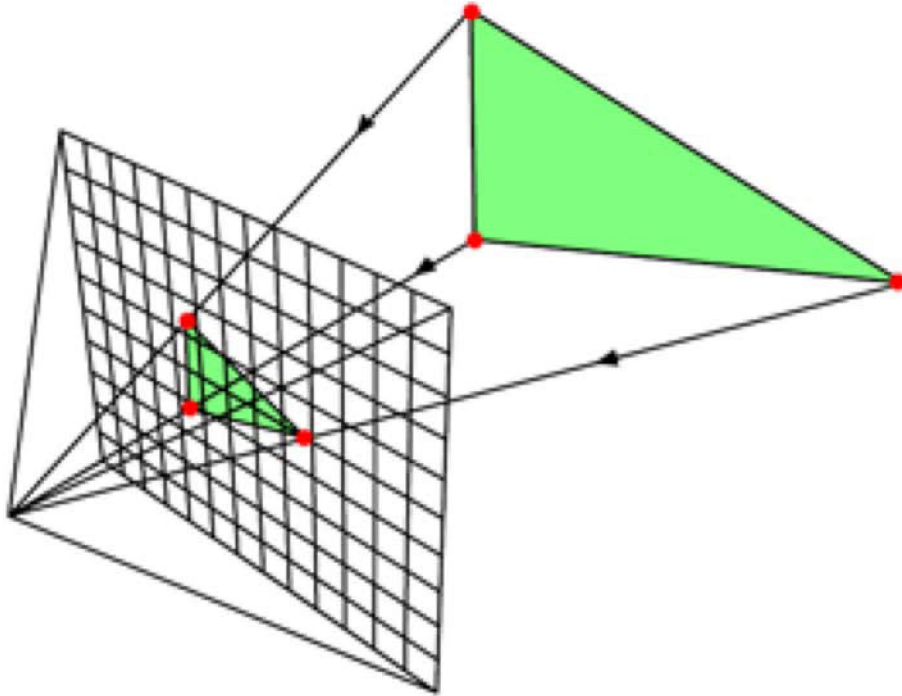
danielek185

WARIOR_GAMING_57
0:52 94 00 5
0 0 00 100
+ 100 100

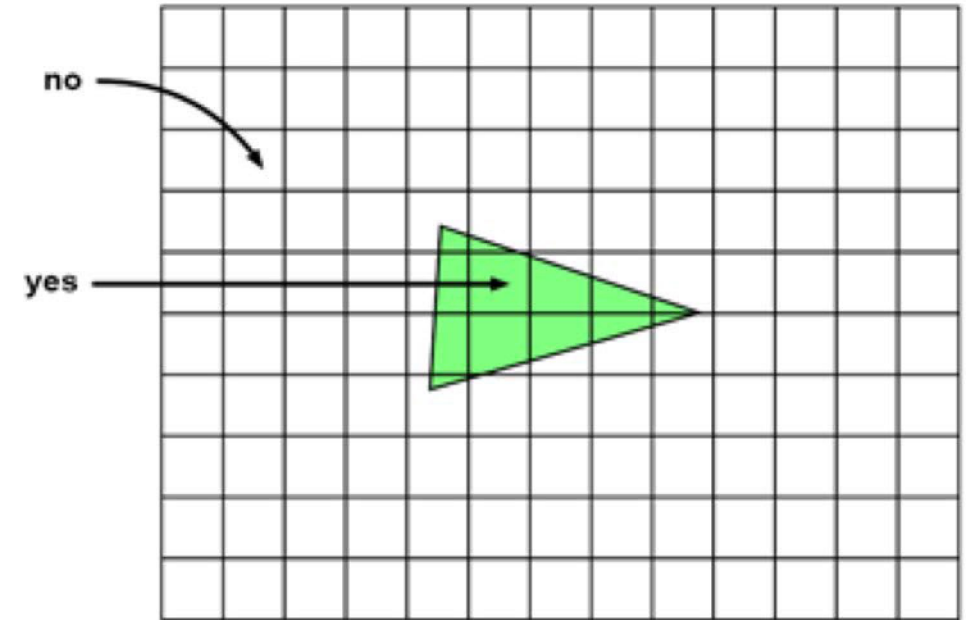
Przytrzymaj

Fortnite | Epic Games

Rasterization



© www.scratchapixel.com



1. Project Vertices to Image Plane

2. Turn on pixels inside triangle

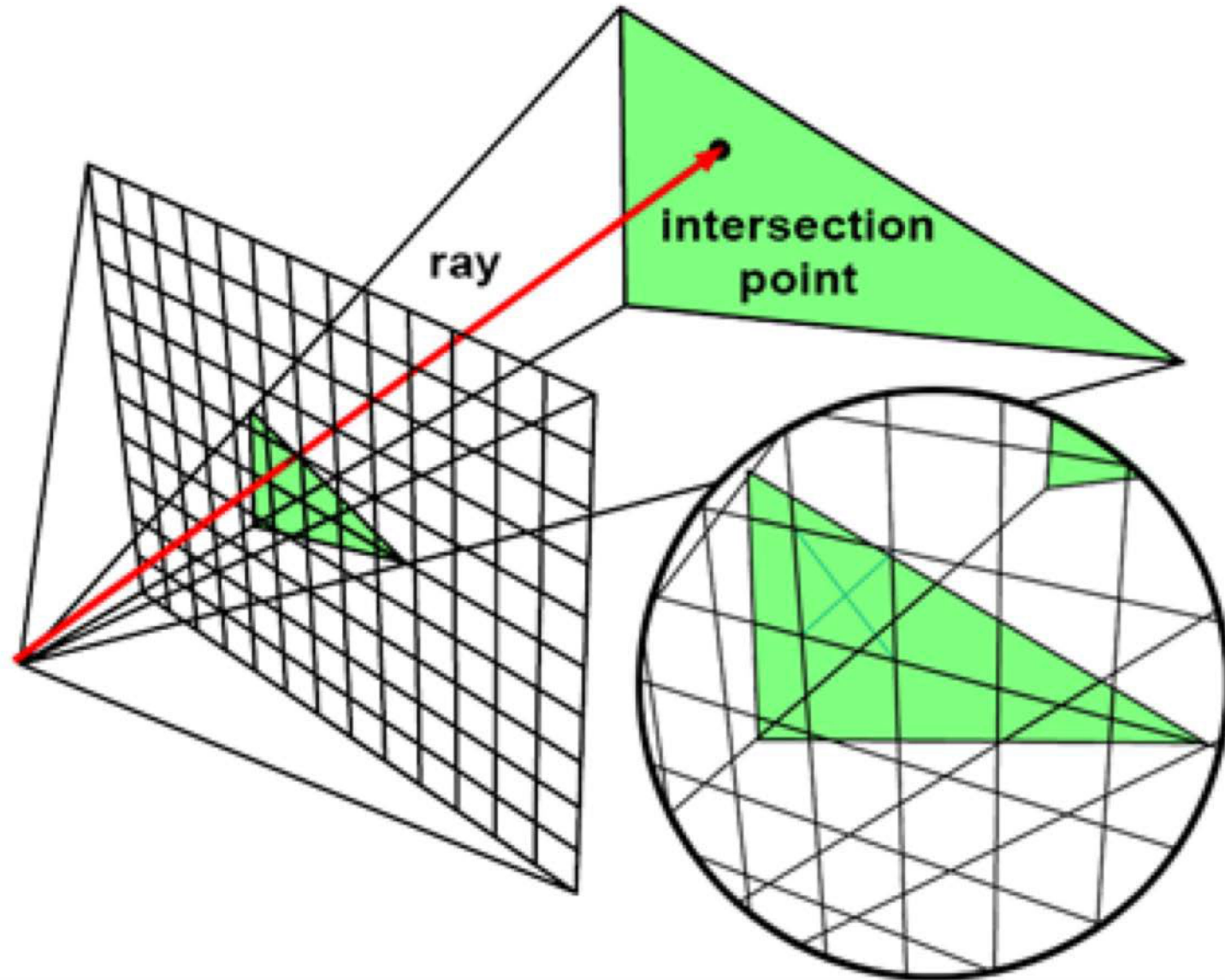
Rasterization

```
for each object in the scene {  
  for each pixel in the image  
  {  
    if (object affects pixel)  
    {  
      do something  
    }  
  }  
}
```



operations can be done
quickly on the GPU!

Ray Casting



Ray Casting

```
for each pixel in the image {  
    Generate a ray  
    for each object in the scene {  
        if (Intersect ray with  
            object) { Set pixel  
                colour  
        }  
    }  
}
```



Basic Components of Ray Casting

Ray

Camera

Intersection Tests

Basic Components of Ray Casting

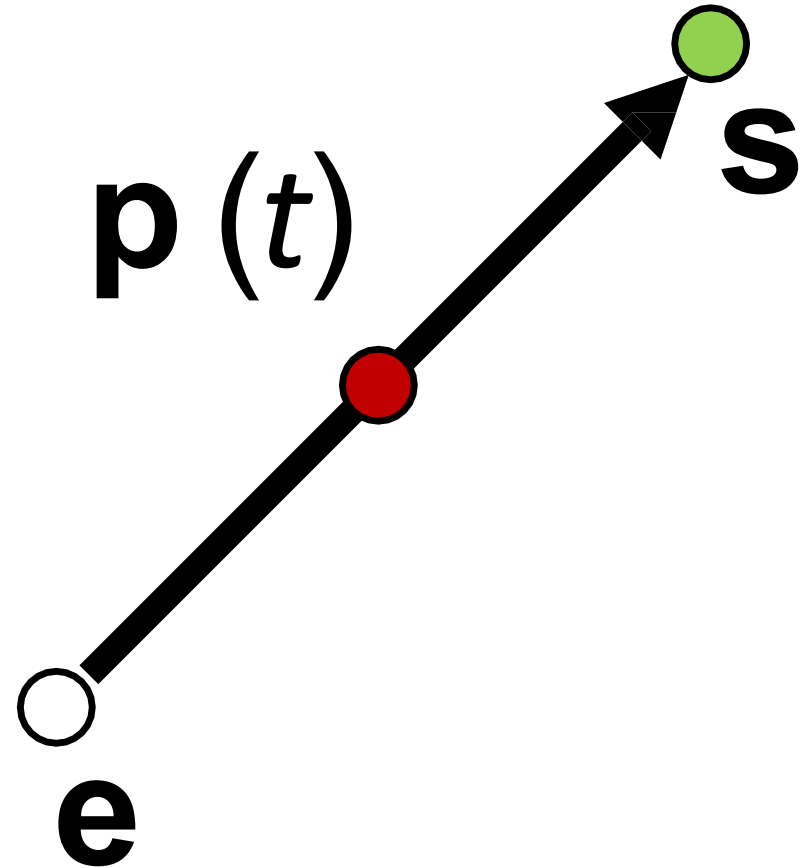
Ray

Camera

Intersection Tests

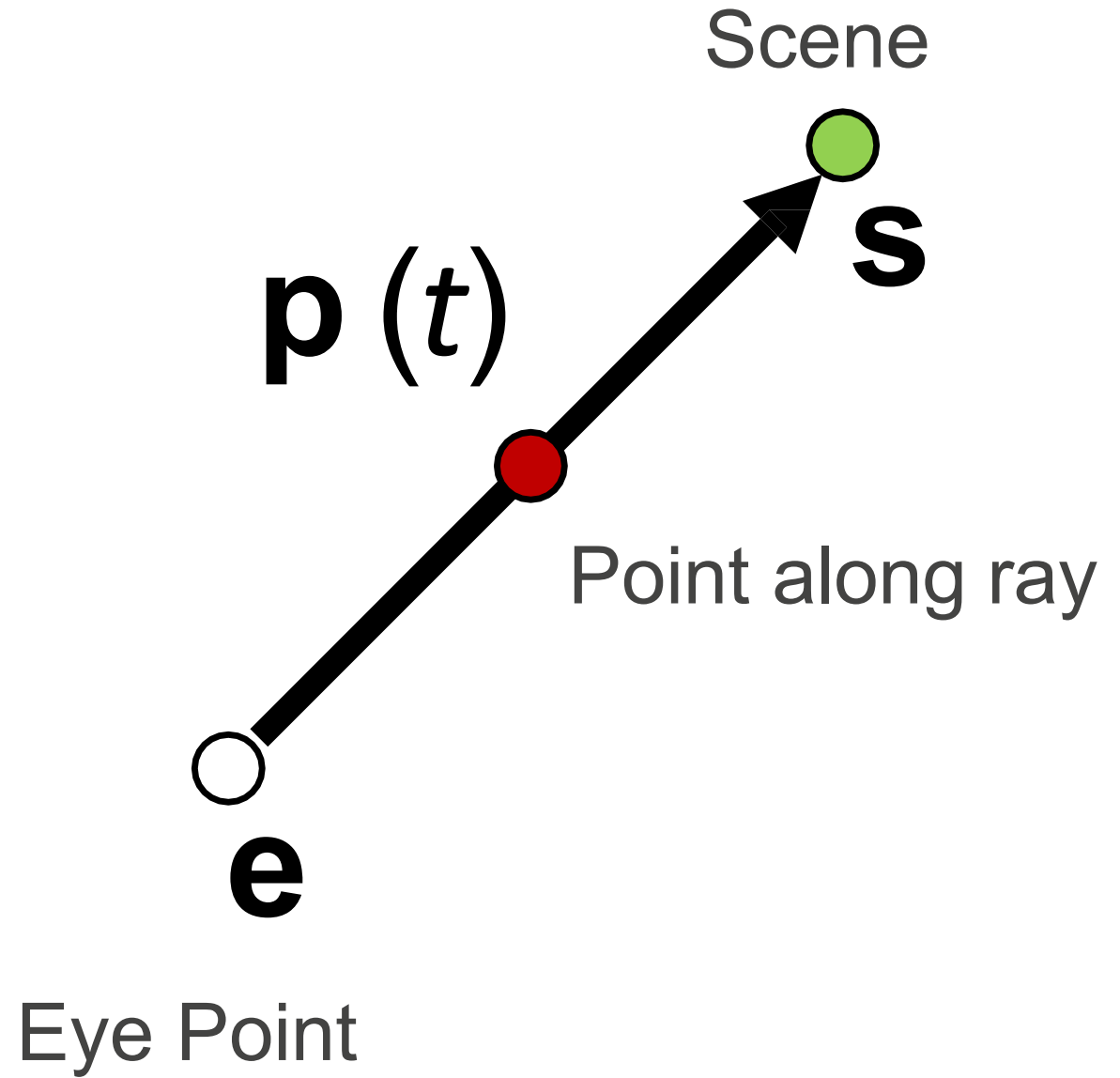
The Ray

$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$



The Ray

$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

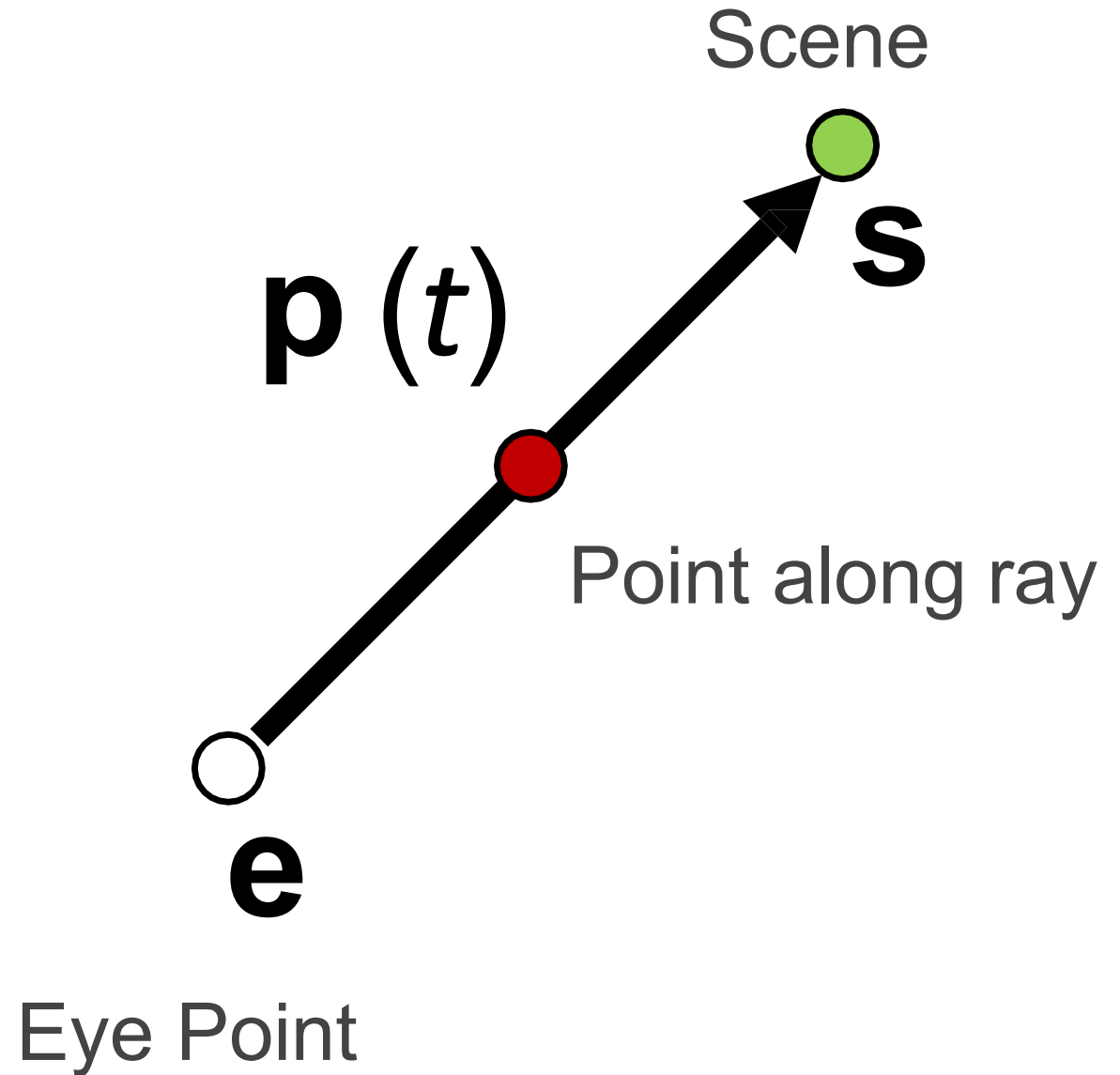


The Ray

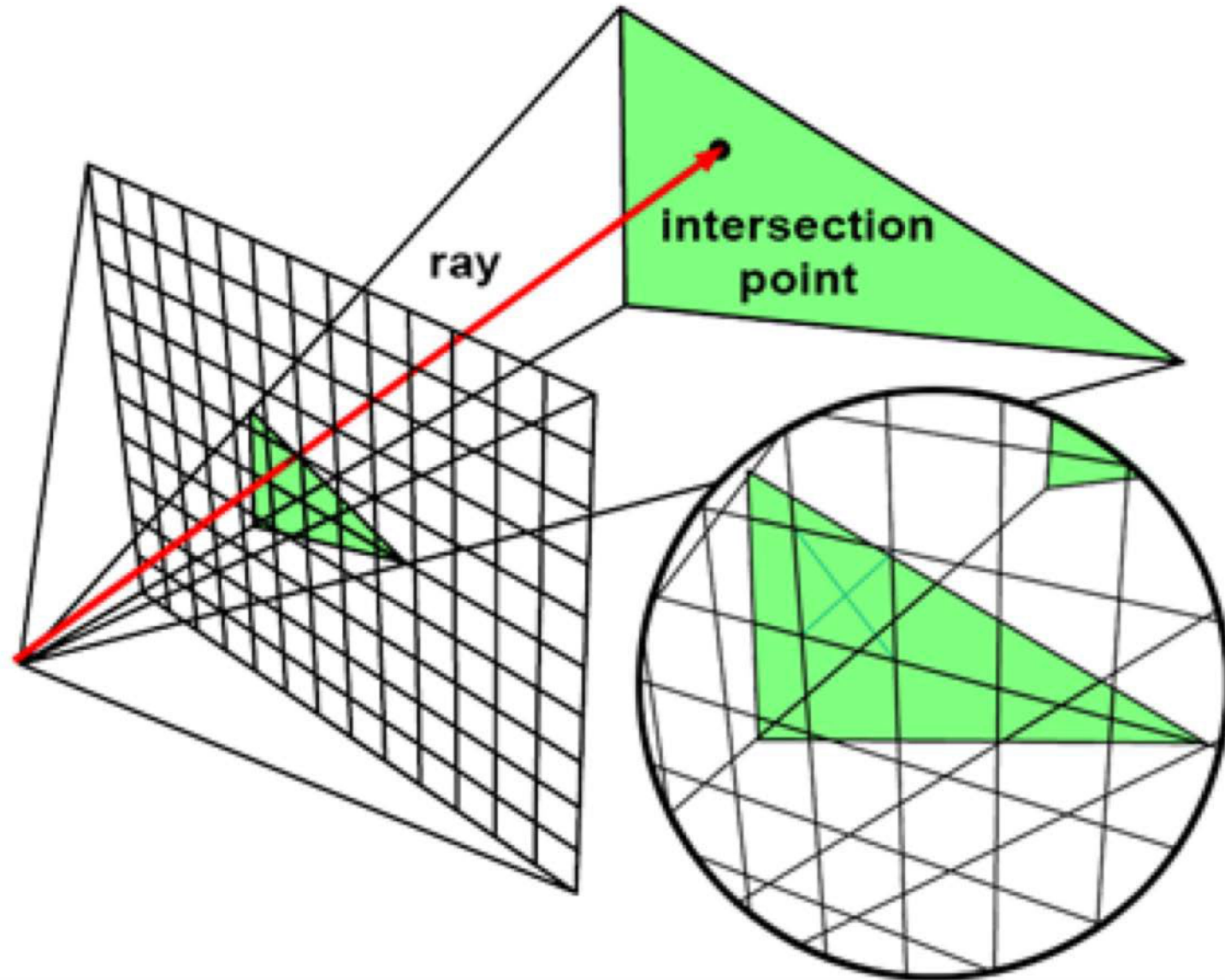
$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

$$\mathbf{p}(0) = \mathbf{e}$$

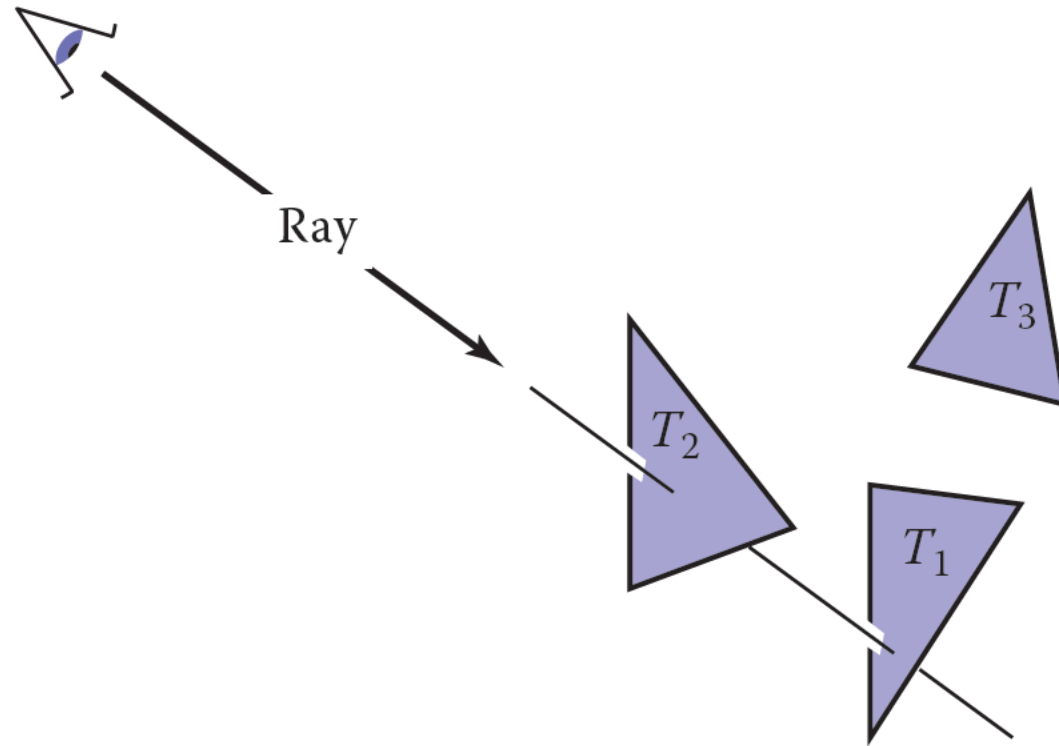
$$\mathbf{p}(1) = \mathbf{s}$$



Ray Casting



Ray Casting



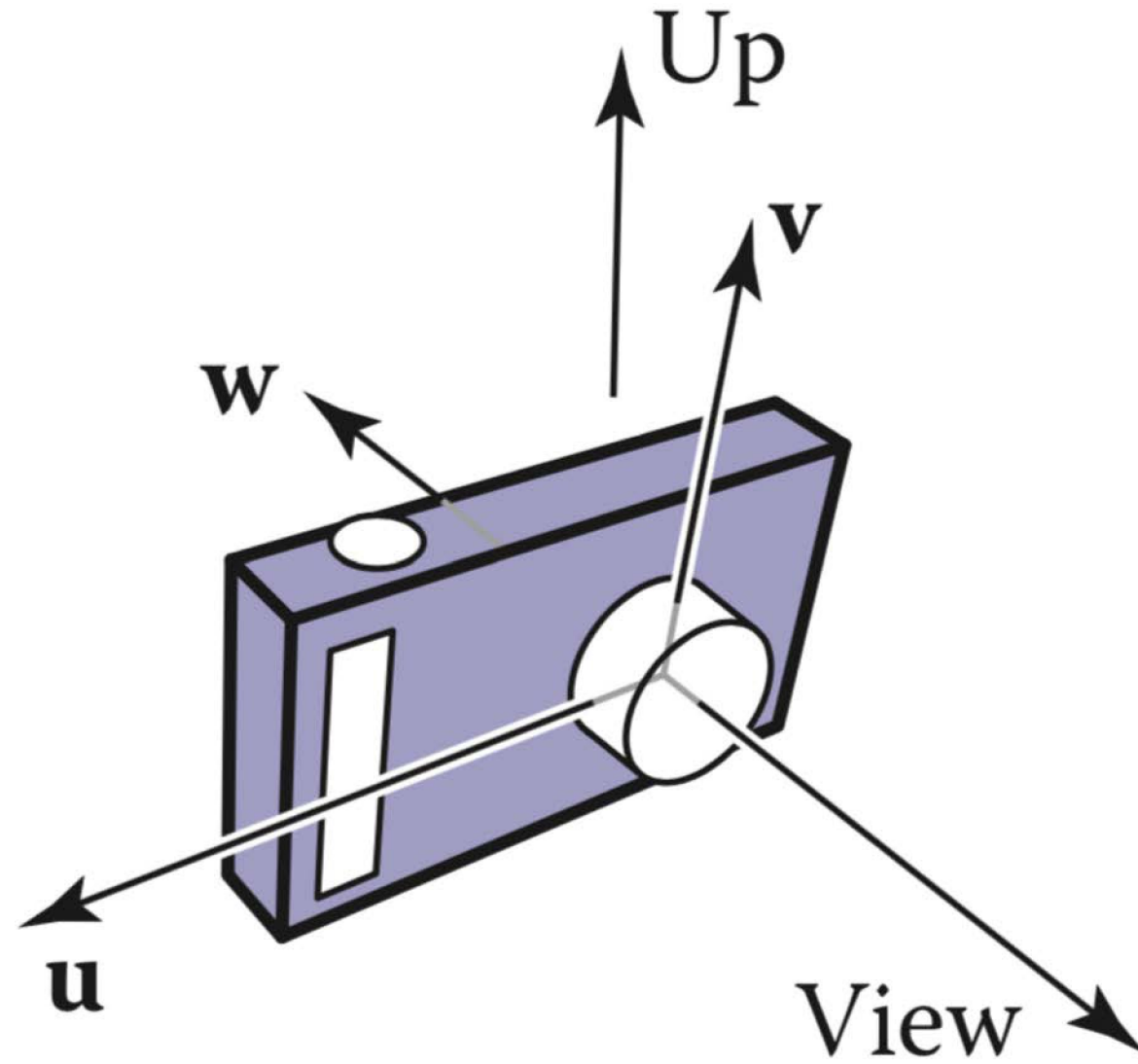
Basic Components of Ray Casting

Ray

Camera

Intersection Tests

The Camera

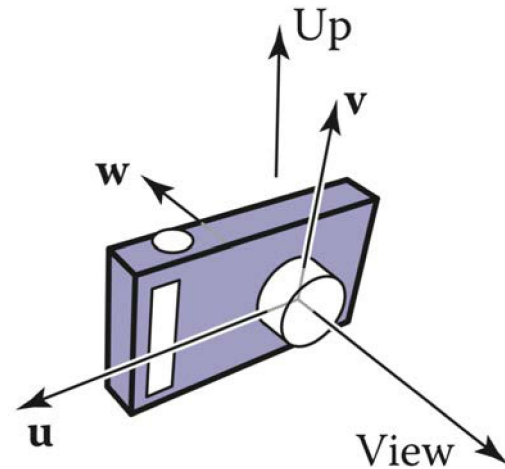


The Camera

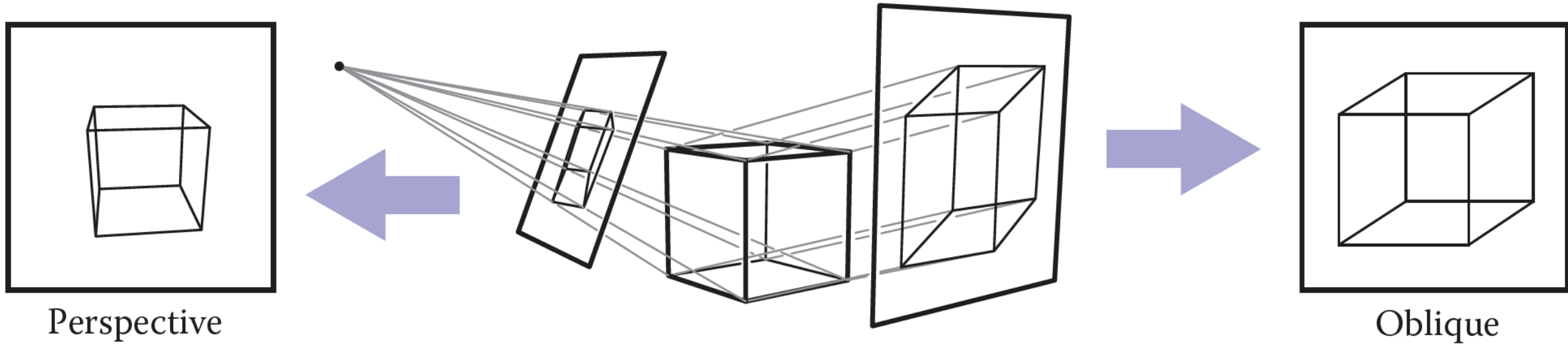
$$\mathbf{w} = -\frac{\text{View}}{\|\text{View}\|}$$

$$\mathbf{u} = \text{View} \times \text{Up}$$

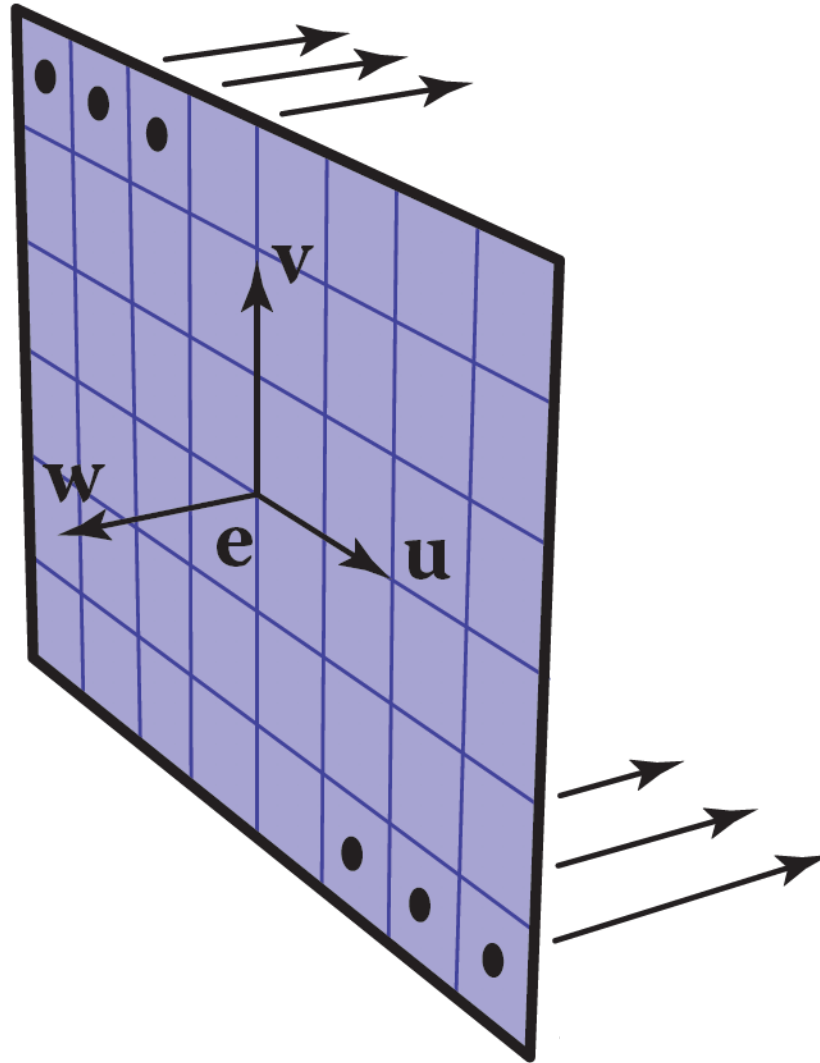
$$\mathbf{v} = \mathbf{w} \times \mathbf{u}$$



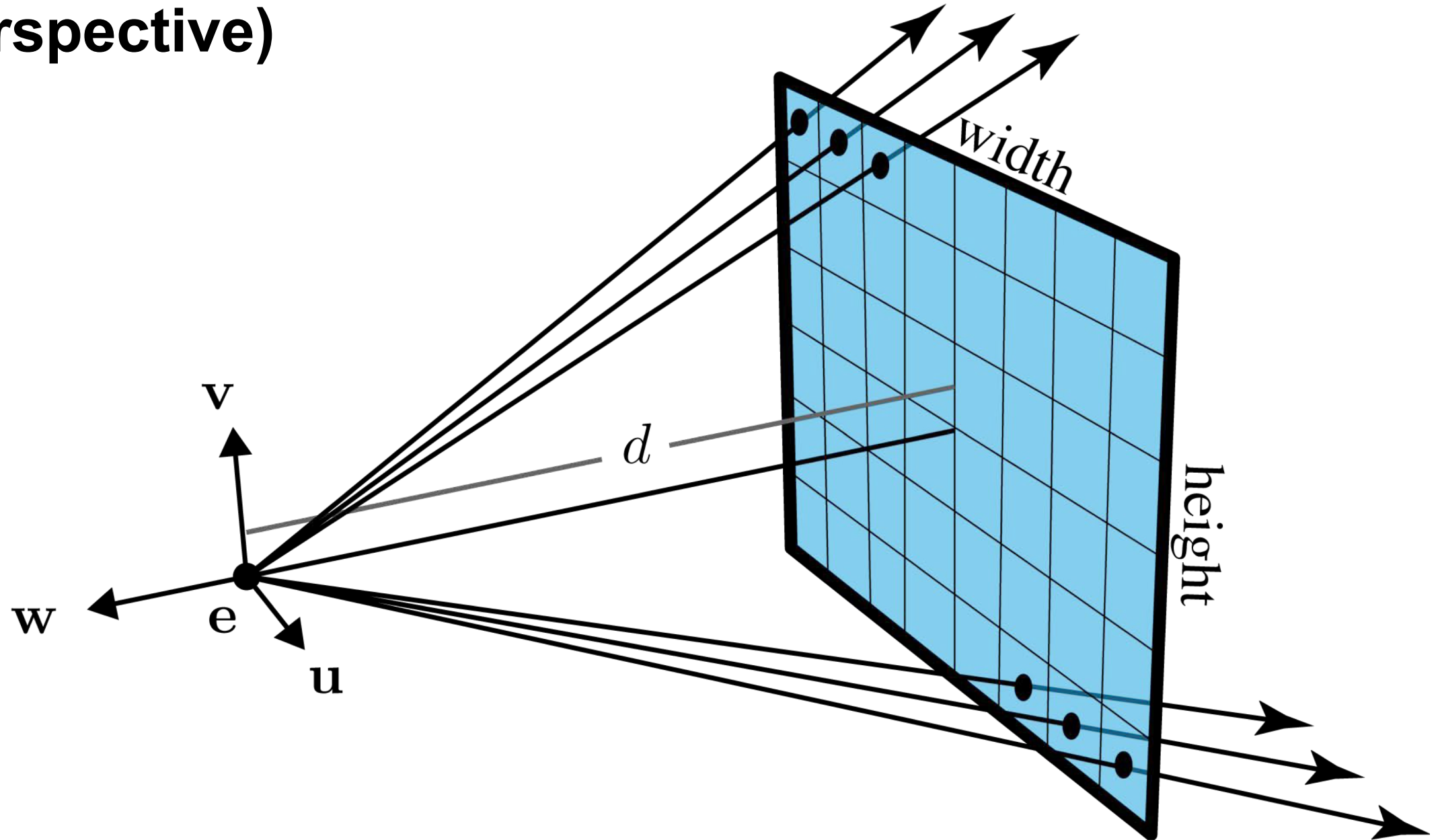
Orthographic v Perspective Projection



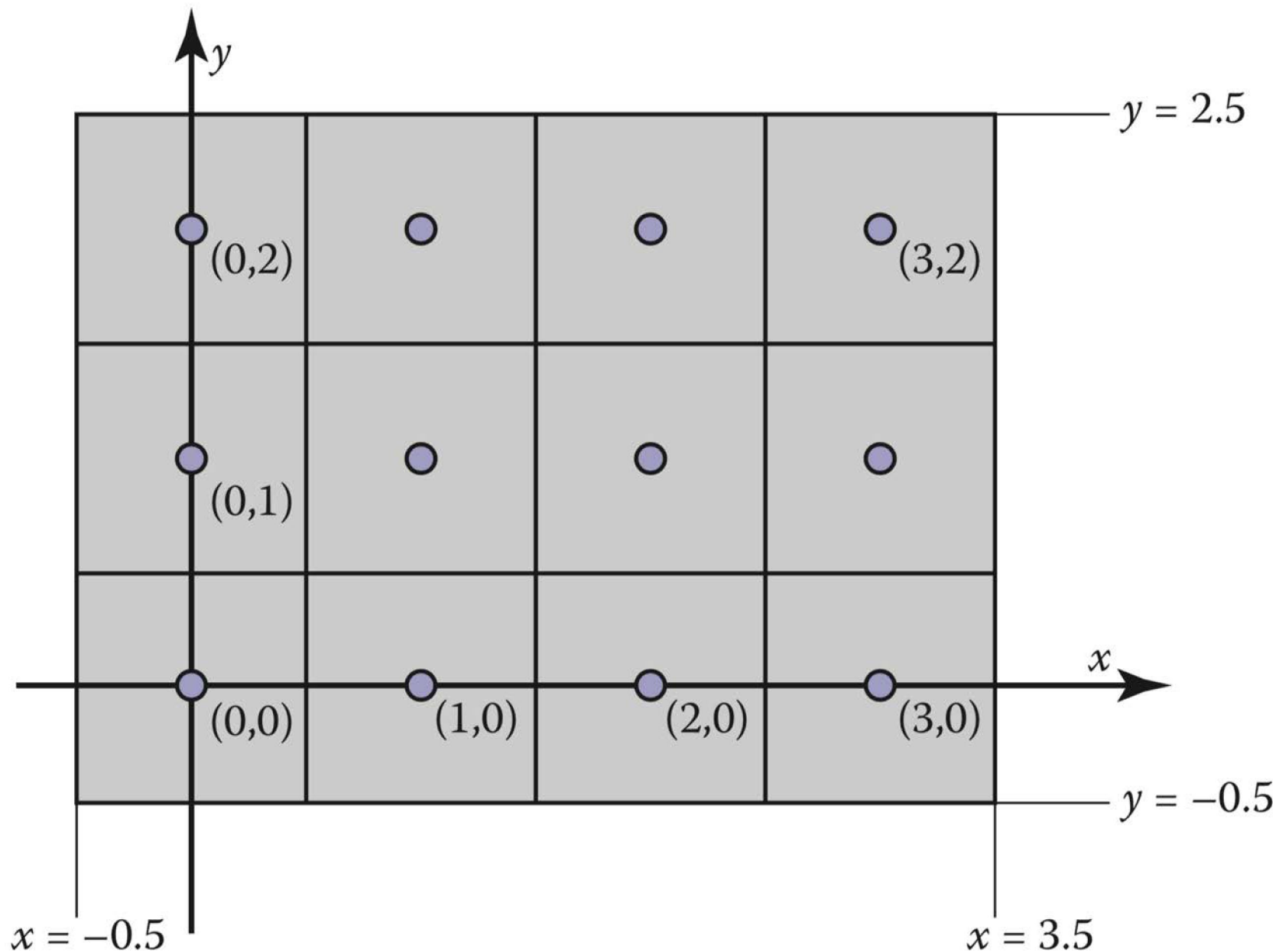
Generating Rays (Orthographic)

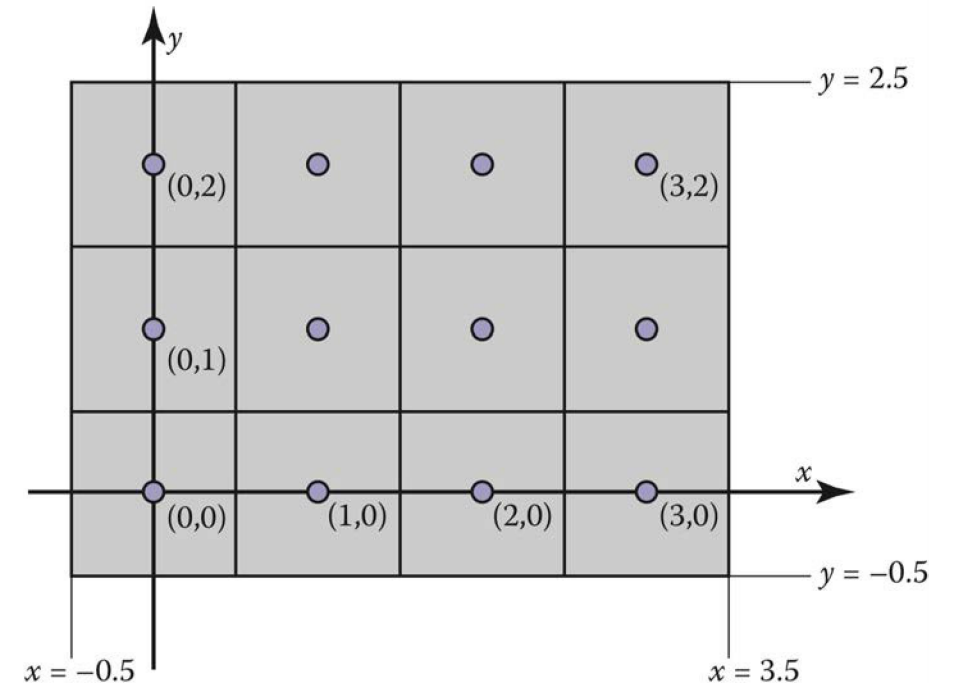
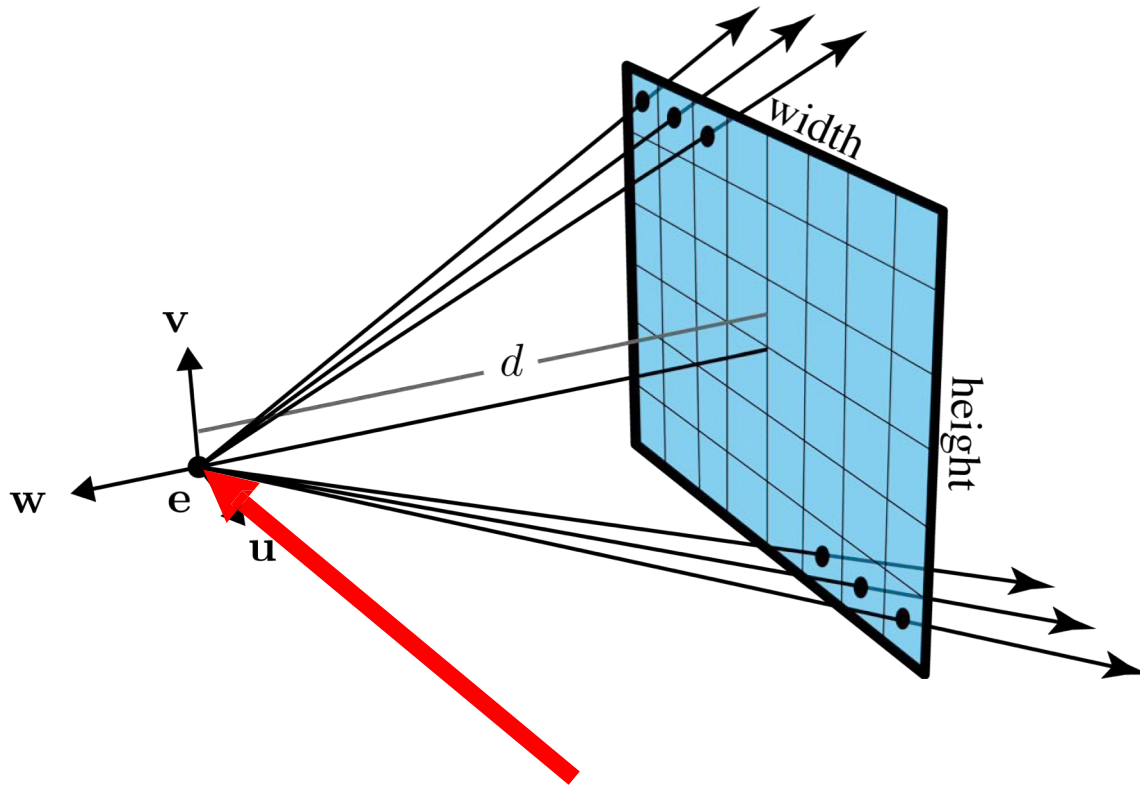


Generating Rays (Perspective)



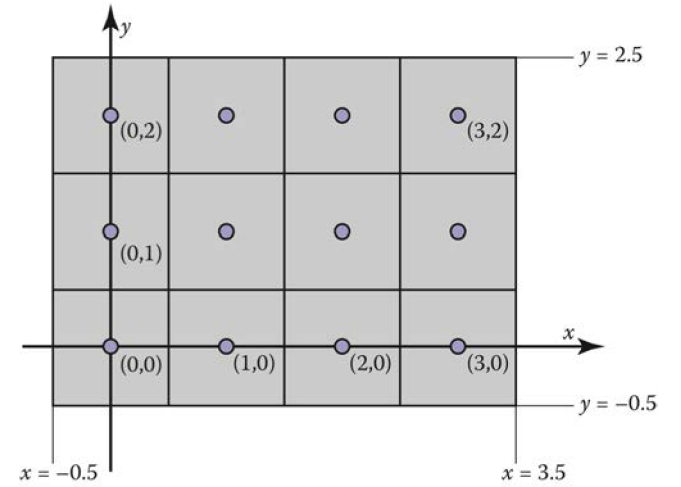
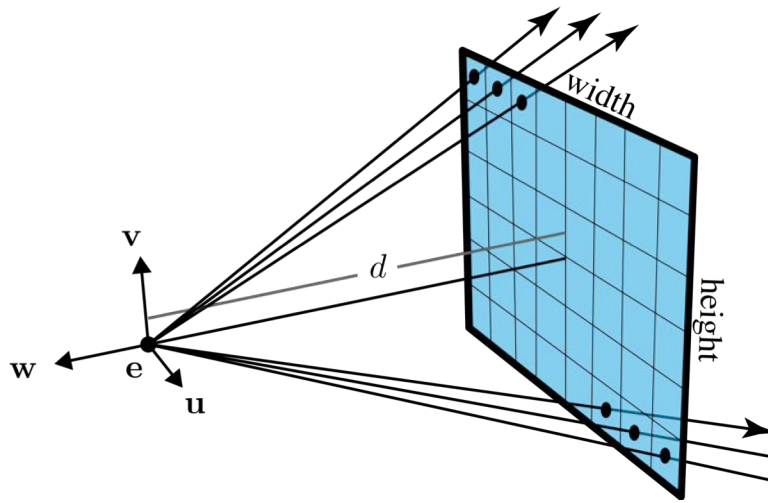
Recall: Standard Pixel Coordinate System





Origin of camera frame (the eye)

What are the coordinates for pixel (i, j) in the camera frame?



Bottom Left Corner $(i, j) : ?$

Top Right Corner $(i, j) : ?$

Bottom Left Corner $(u, v) : ?$

Top Right Corner $(u, v) : ?$

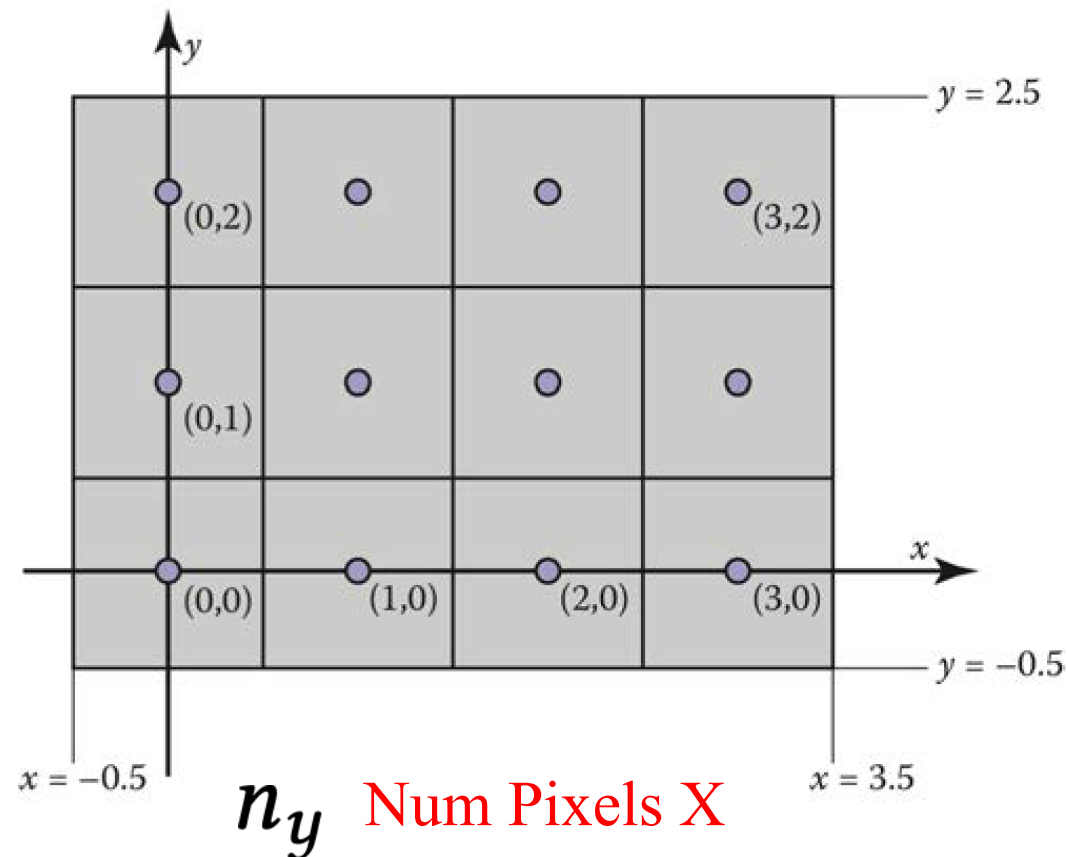
n_x Num Pixels Y

Bottom Left Corner $(i, j) : (-\frac{1}{2}, -\frac{1}{2})$

Top Right Corner $(i, j) : (n_x - \frac{1}{2}, n_y - \frac{1}{2})$

Bottom Left Corner $(u, v) :$

Top Right Corner $(u, v) :$



Physical Width of Image in 3D Space

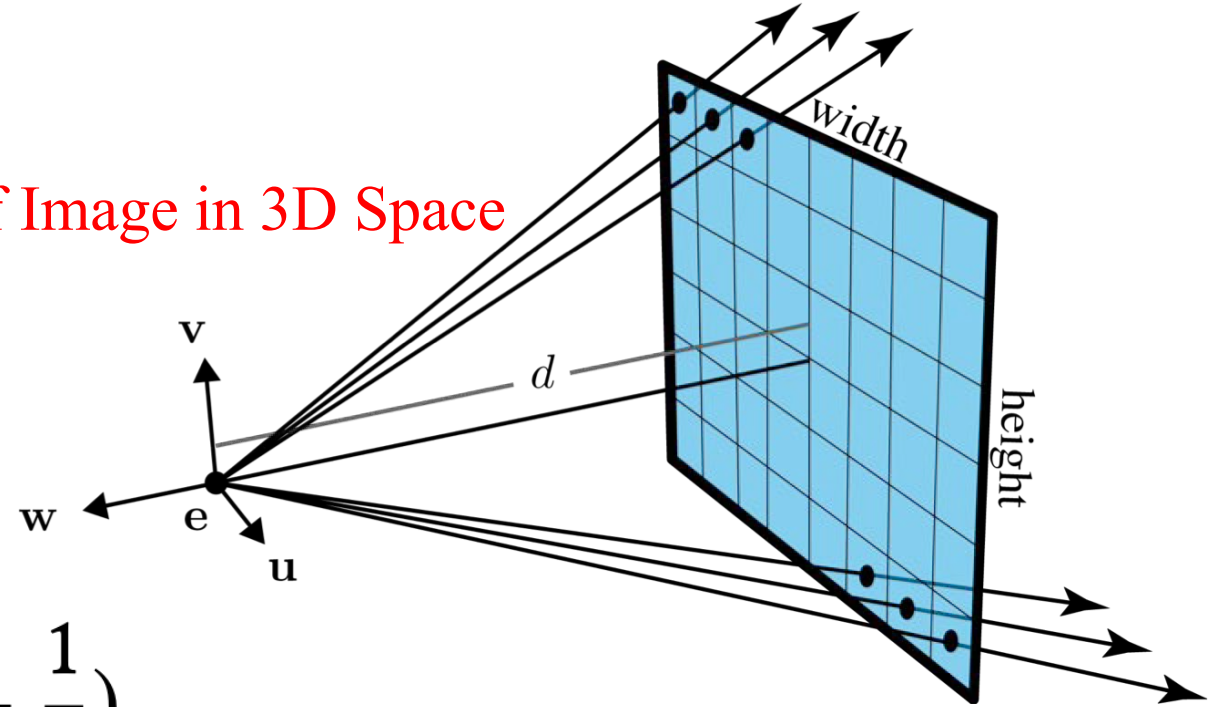
Physical Height of Image in 3D Space

Bottom Left Corner $(i, j) : (-\frac{1}{2}, -\frac{1}{2})$

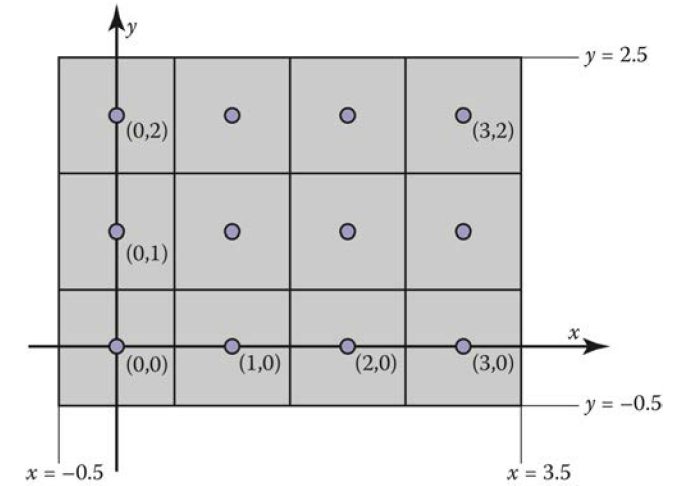
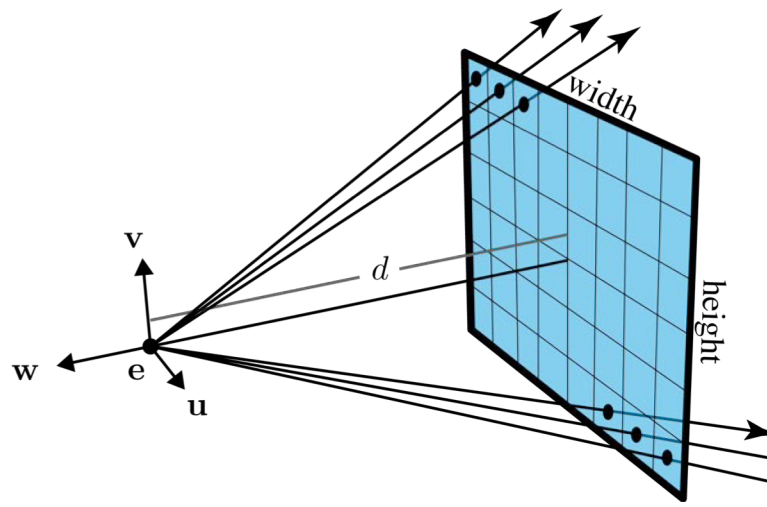
Top Right Corner $(i, j) : (n_x - \frac{1}{2}, n_y - \frac{1}{2})$

Bottom Left Corner $(u, v) : (-\frac{\text{width}}{2}, -\frac{\text{height}}{2})$

Top Right Corner $(u, v) : (\frac{\text{width}}{2}, \frac{\text{height}}{2})$



pixel at position (i, j) in the raster image has the position:



Physical Width of Image in 3D Space

$$u = \frac{\text{width}}{n_x} \cdot \left(i + \frac{1}{2} \right) - \frac{\text{width}}{2}$$

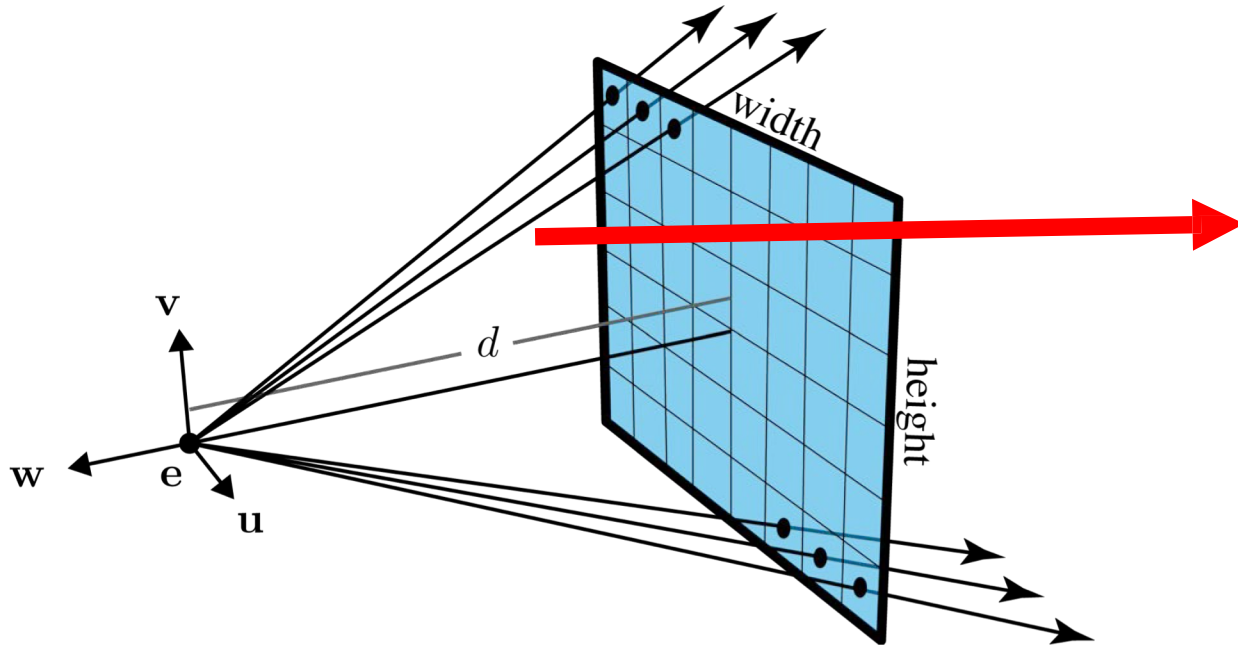
Num Pixels X

Physical Height of Image in 3D Space

$$v = \frac{\text{height}}{n_y} \cdot \left(j + \frac{1}{2} \right) - \frac{\text{height}}{2}$$

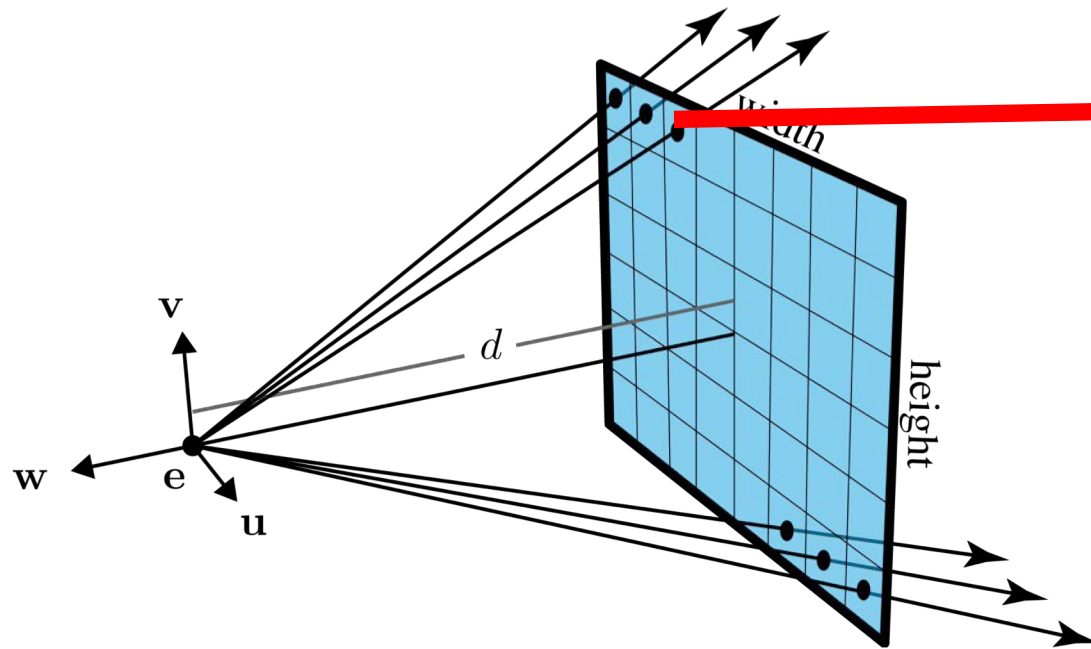
Num Pixels Y

Ray Equation in Camera Space



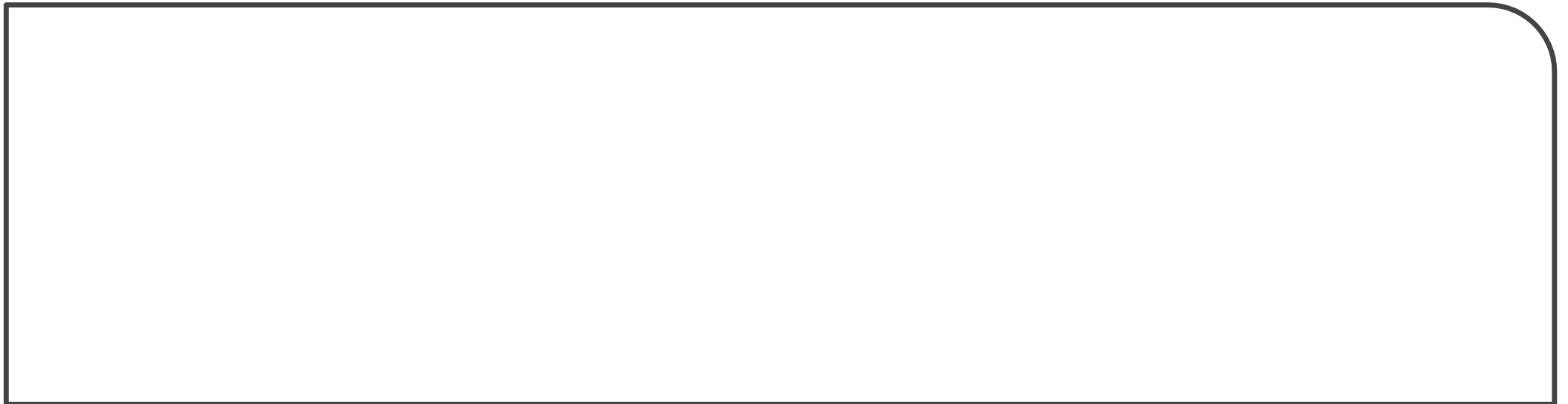
$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

Ray Equation in Camera Space

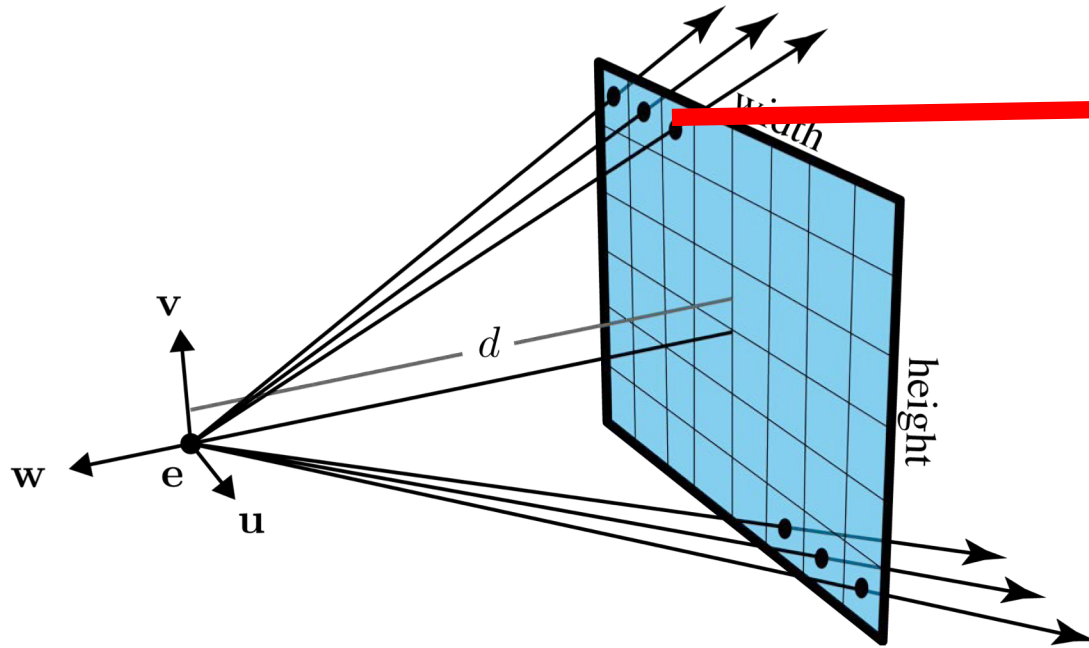


$$p(t) = e + t(s - e)$$

$$p(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + t \left(\begin{bmatrix} u(i) \\ v(j) \\ -d \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right)$$



Ray Equation in Camera Space



$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

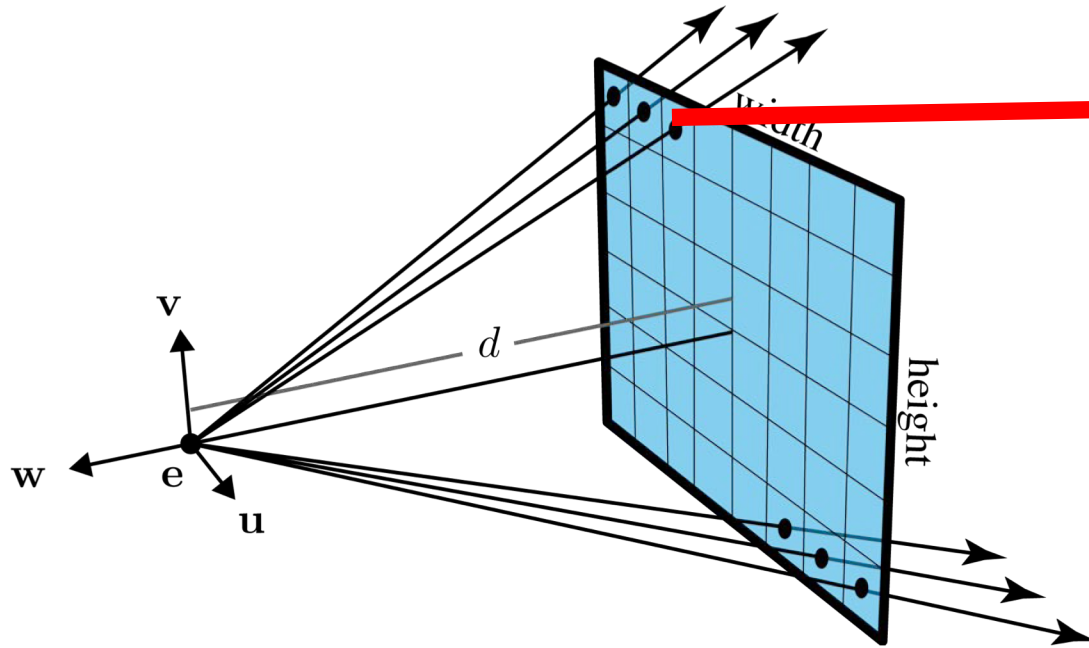
$$\mathbf{p}(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + t \left(\begin{bmatrix} u(i) \\ v(j) \\ -d \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$\mathbf{p}(t) = t \begin{bmatrix} u(i) \\ v(j) \\ -d \end{bmatrix}$$

$$u = \frac{\text{width}}{n_x} \cdot \left(i + \frac{1}{2} \right) - \frac{\text{width}}{2}$$

$$v = \frac{\text{height}}{n_y} \cdot \left(j + \frac{1}{2} \right) - \frac{\text{height}}{2}$$

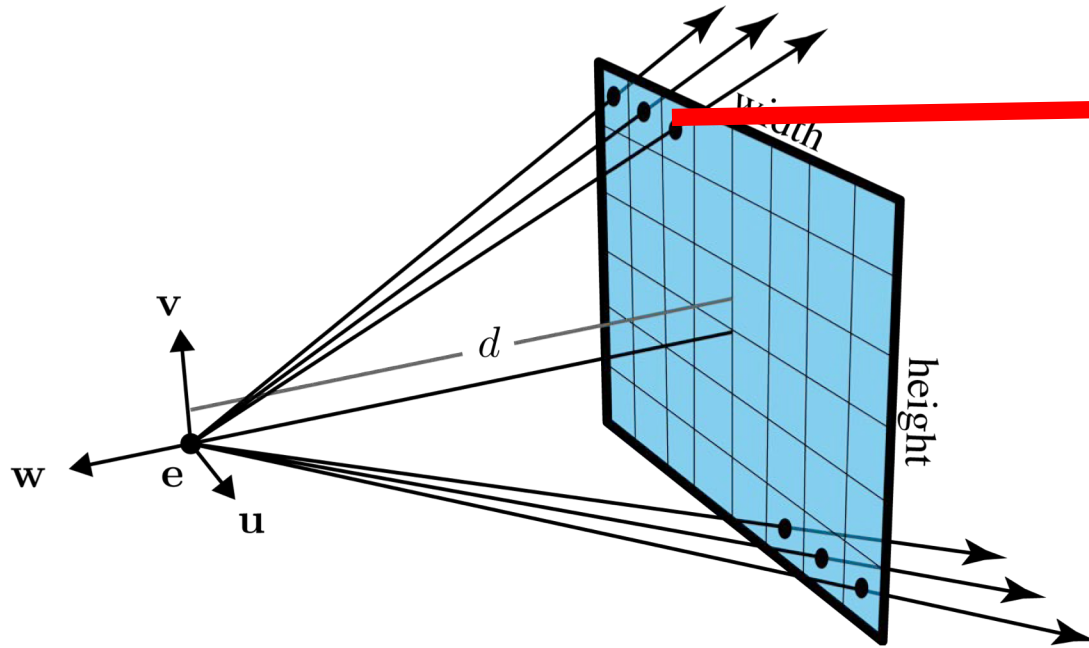
Ray Equation in World Space



$$p(t) = e + t(s - e)$$

$$p(t) = e + t(u(i)u + v(j)v + -d w)$$

Ray Equation in World Space



$$p(t) = e + t(s - e)$$

$$p(t) = e + t \begin{bmatrix} \mathbf{u} & \mathbf{v} & \mathbf{w} \end{bmatrix} \begin{bmatrix} u(i) \\ v(j) \\ -d \end{bmatrix}$$

Camera Transformation Matrix

Ray Casting

```
for each pixel in the image {  
    Generate a ray  
    for each object in the scene {  
        if (Intersect ray with  
            object) { Set pixel  
                colour  
        }  
    }  
}
```

Next class

Ray Intersections