Ray Casting

Some Slides/Images adapted from Marschner and Shirley and David Levin

Today: Ray Casting

Ray Casting and Change of Coordinates Review

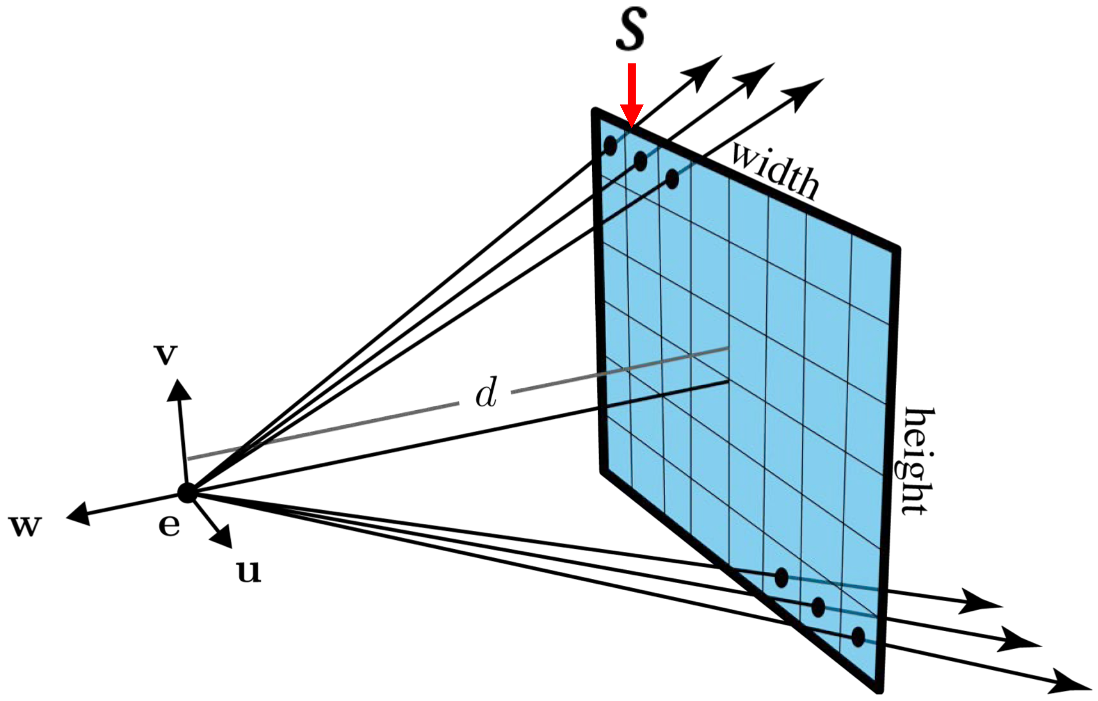
Ray-Object Intersection

- Ray-Plane Intersection
- Ray-Sphere Intersection
- Ray-Triangle Intersection

Ray Casting

```
for each pixel in the image {
    Generate a ray
    for each object in the scene {
        if (Intersect ray with
            object){   Set pixel
            colour
        }
    }
}
```

Ray Equation



$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

Example – Pixel Space

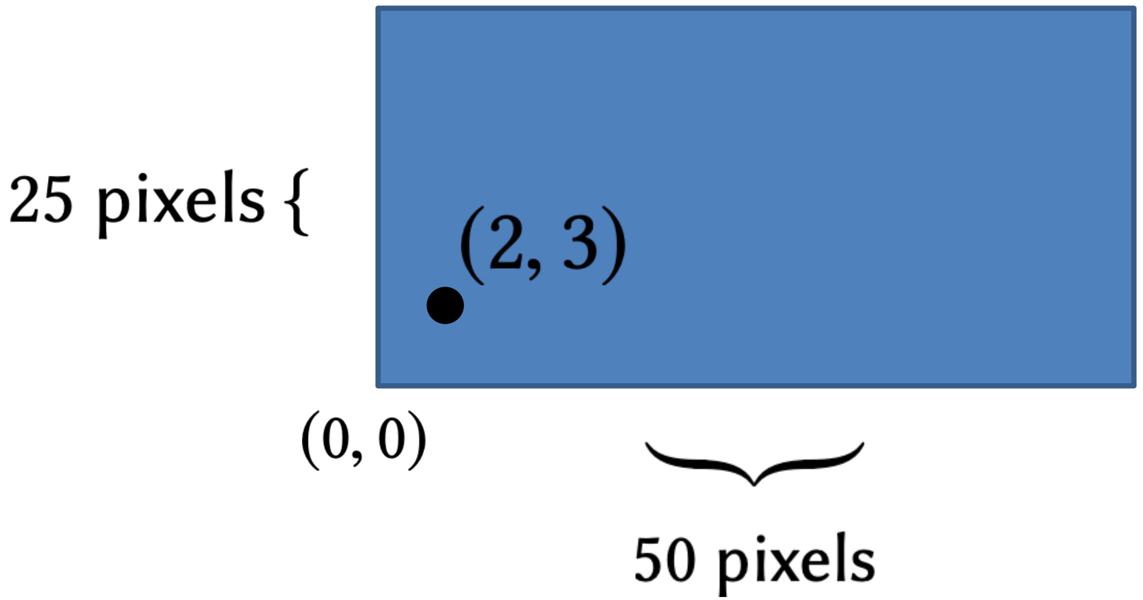
$$\text{width} = 2$$

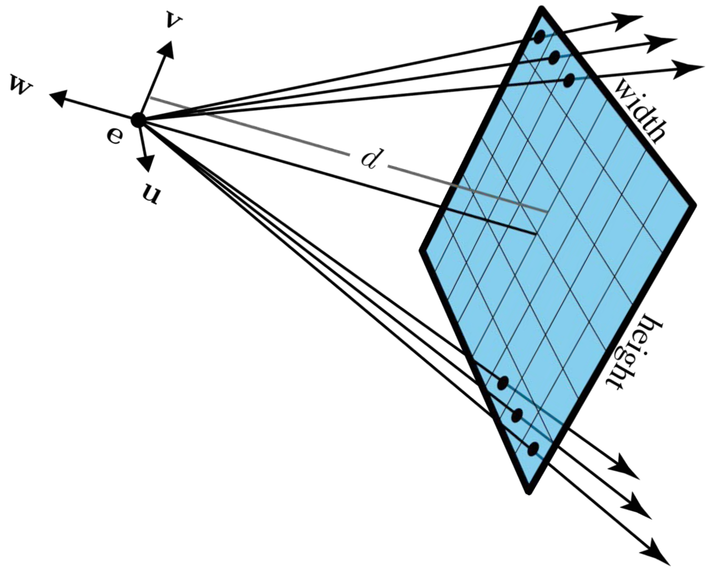
$$\text{height} = 1$$

$$n_x = 50$$

$$n_y = 25$$

Pixel space





Example – Pixel Space

$$\text{width} = 2$$

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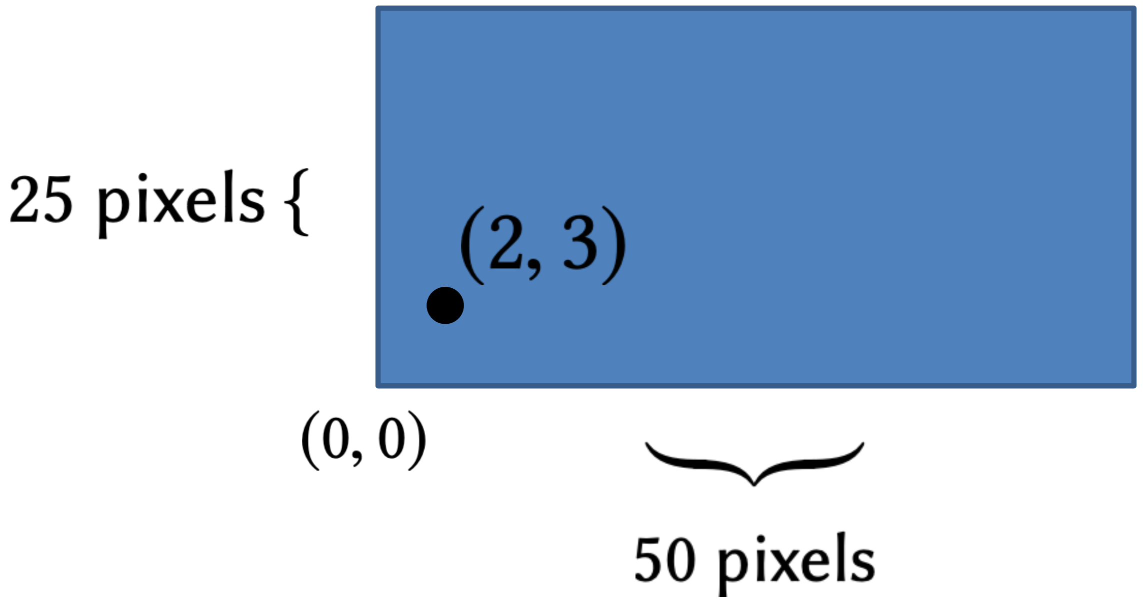
$$n_x = 50$$

$$n_y = 25$$

Pixel space

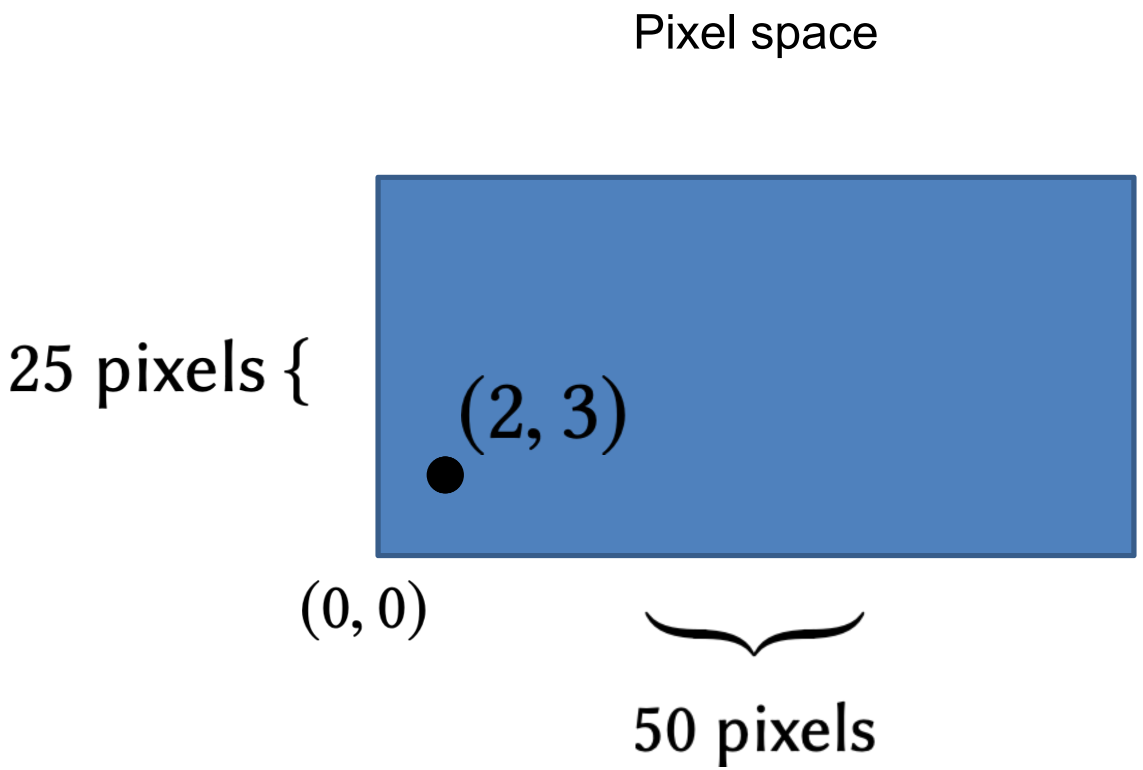


$$u = \frac{\text{width}}{n_x} \cdot \left(i + \frac{1}{2}\right) - \frac{\text{width}}{2}$$

$$v = \frac{\text{height}}{n_y} \cdot \left(j + \frac{1}{2}\right) - \frac{\text{height}}{2}$$

Example – Pixel Space

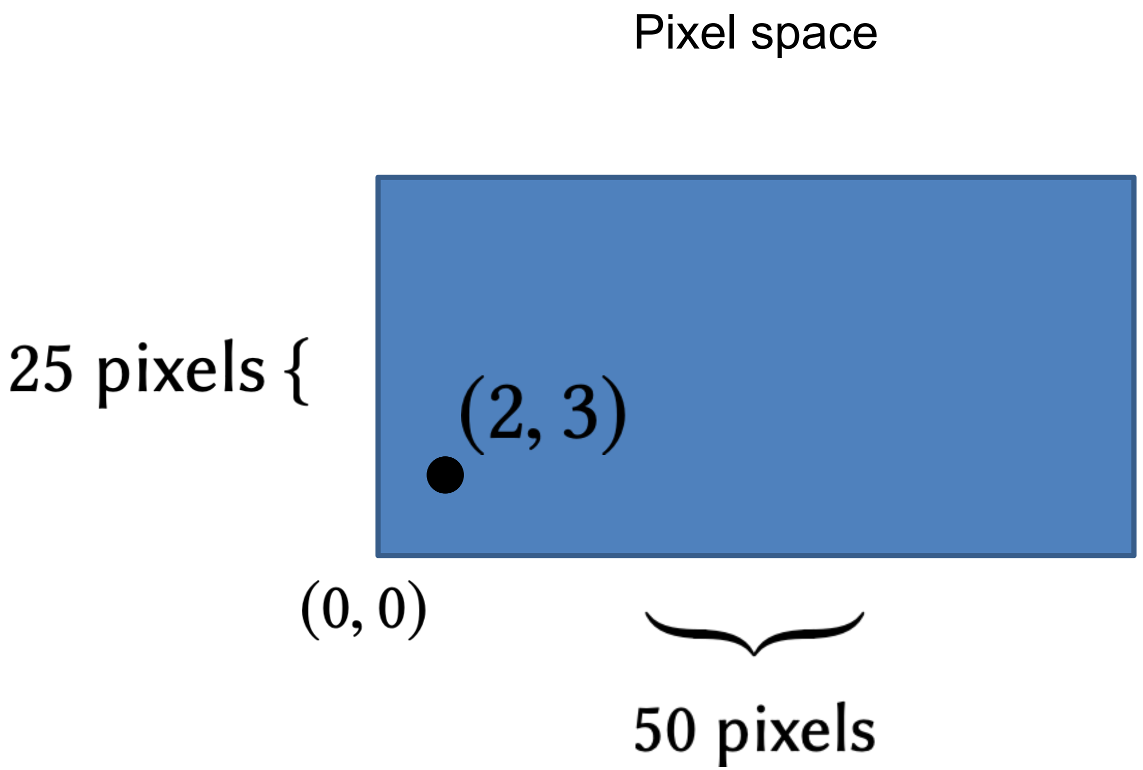
$$\text{width} = 2$$
$$\text{height} = 1$$
$$n_x = 50$$
$$n_y = 25$$



$$u = \frac{2}{50} \cdot \left(2 + \frac{1}{2}\right) - \frac{2}{2}$$

$$v = \frac{1}{25} \cdot \left(3 + \frac{1}{2}\right) - \frac{1}{2}$$

Example – Pixel Space



$$\text{width} = 2$$

$$\text{height} = 1$$

$$n_x = 50$$

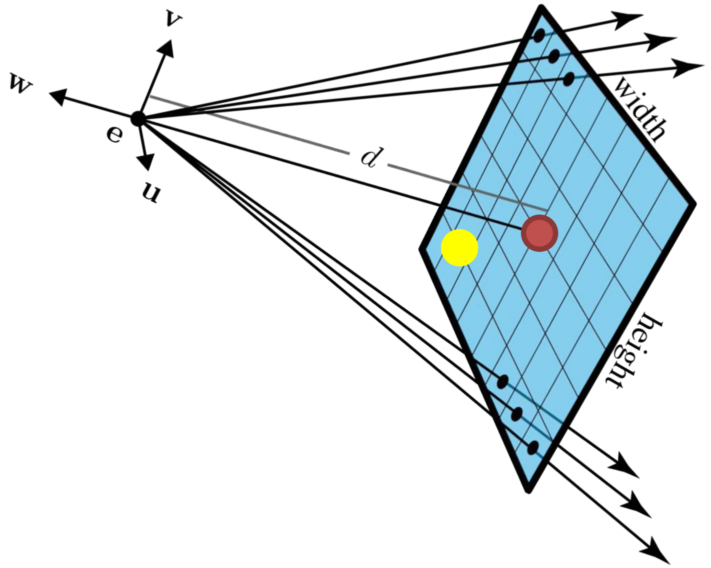
$$n_y = 25$$

$$u = -0.9$$

$$v = -0.36$$

Example – Pixel Space

Camera space



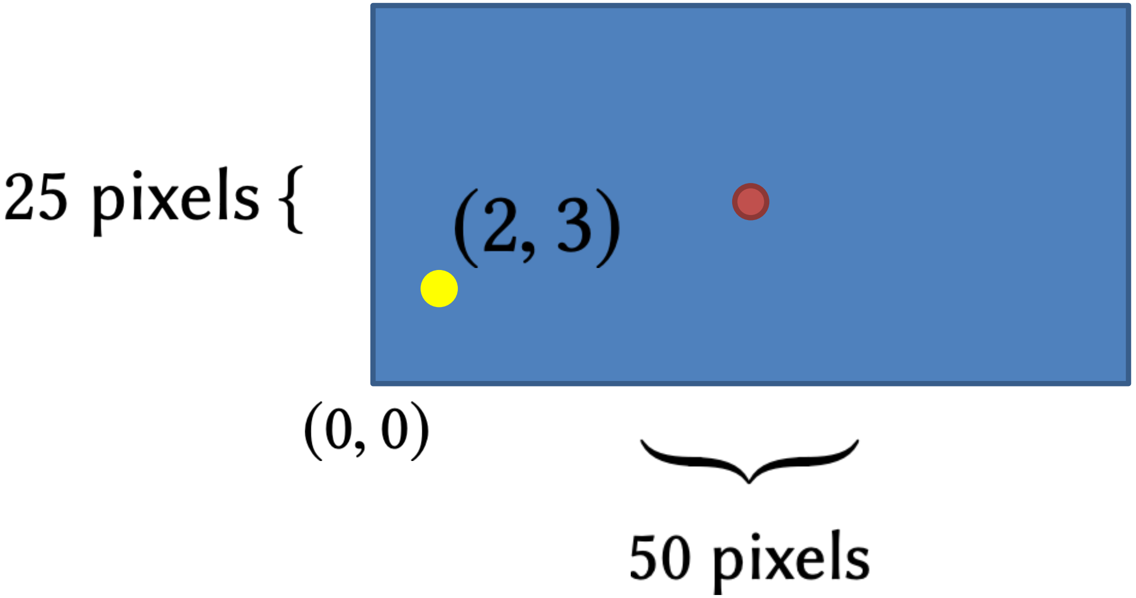
$$\text{width} = 2$$

$$\text{height} = 1$$

$$n_x = 50$$

$$n_y = 25$$

Pixel space

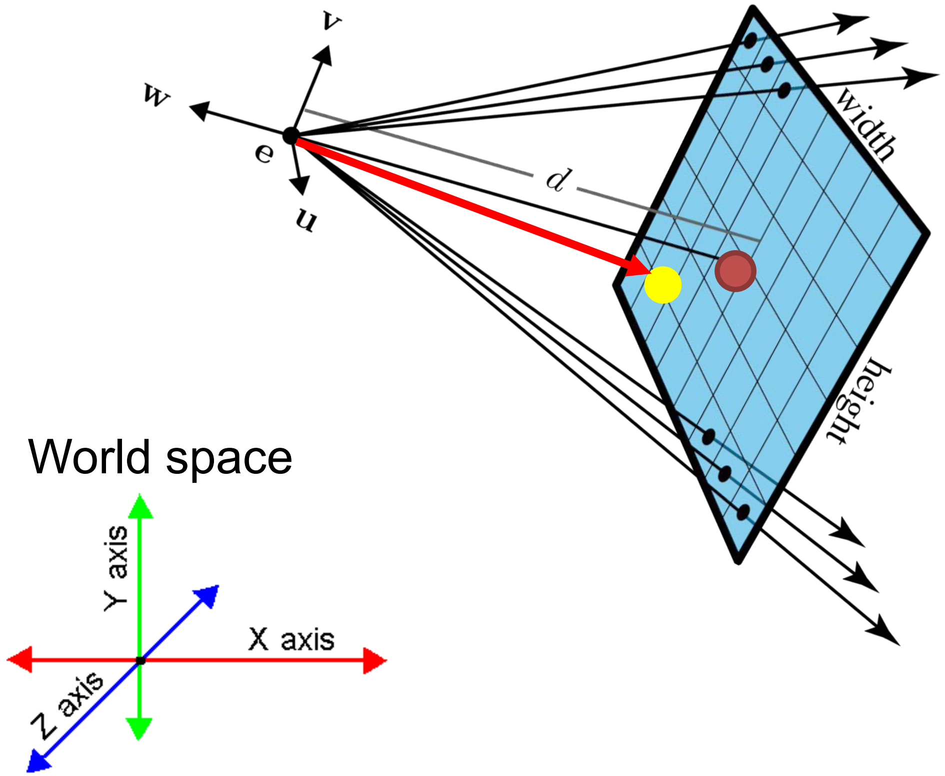


$$u = -0.9$$

$$v = -0.36$$

Example – Camera Space

Camera space



$$\mathbf{u}, \mathbf{v}, \mathbf{w}$$

are the basis vectors for camera space

$$d = 10$$

d is the distance from the viewpoint to the image plane (focal length)

$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

our ray equation

Example – Camera Space

$d = 10$

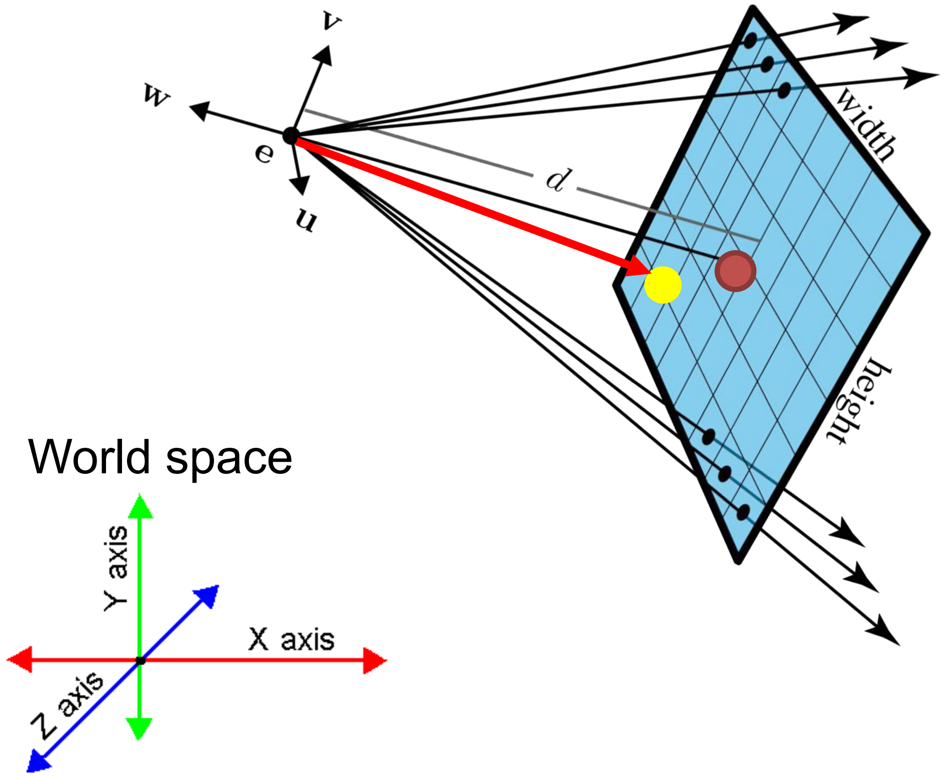
$$\mathbf{u}, \mathbf{v}, \mathbf{w}$$

are the basis vectors for camera space

$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

our ray equation

Camera space



$$\mathbf{p}(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + t\left(\begin{bmatrix} u(i) \\ v(j) \\ -d \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right)$$

Example – Camera Space

$$d = 10$$

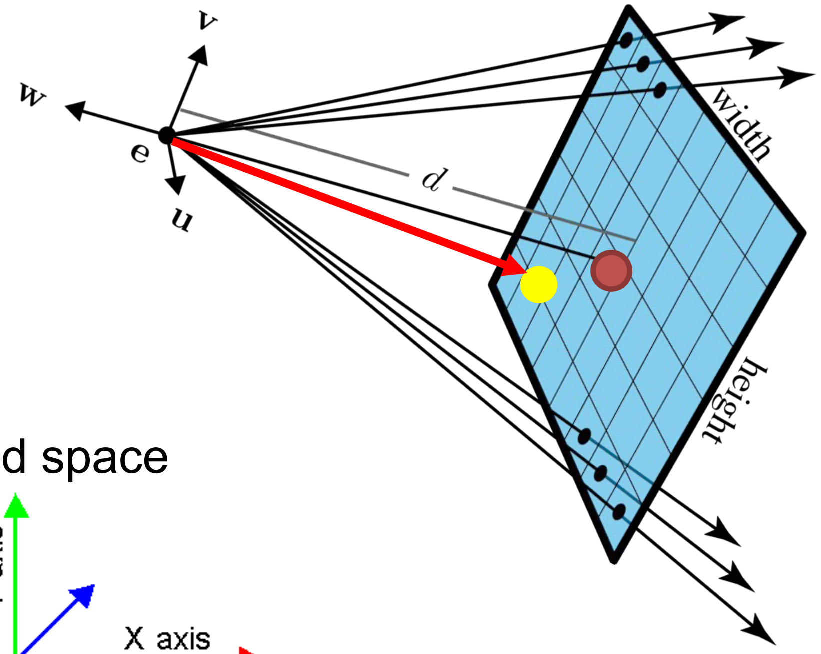
$$\mathbf{u}, \mathbf{v}, \mathbf{w}$$

are the basis vectors for camera space

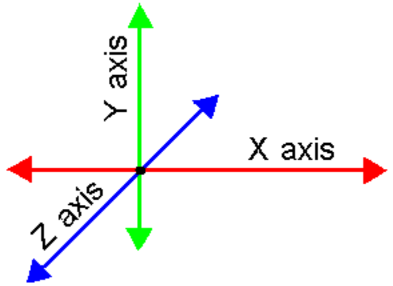
$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

our ray equation

Camera space



World space



for (i,j) = (2,3)

$$u(2) = -0.9, v(3) = -0.36$$

$$\mathbf{p}(t) = t \begin{bmatrix} -0.9 \\ -0.36 \\ -10 \end{bmatrix}$$

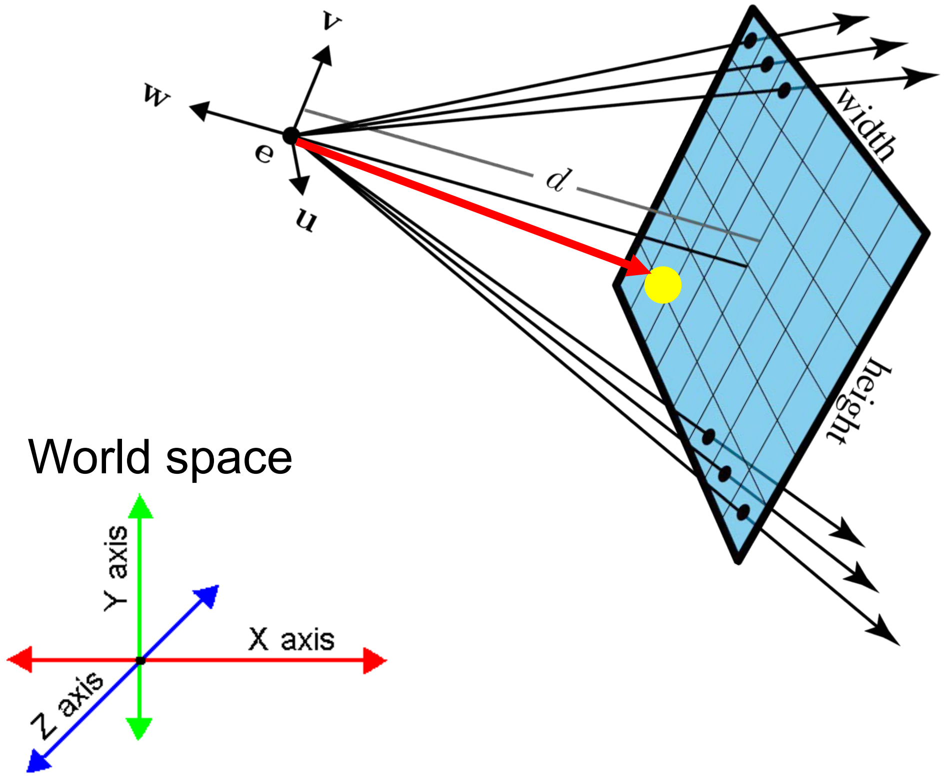
Example – World Space

$$\mathbf{u}, \mathbf{v}, \mathbf{w}$$

are the basis vectors for camera space

$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

our ray equation



$$\mathbf{e} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 0 \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\mathbf{p}(t) = \mathbf{e} + t\left(u(i)\mathbf{u} + v(j)\mathbf{v} + -d\mathbf{w}\right)$$

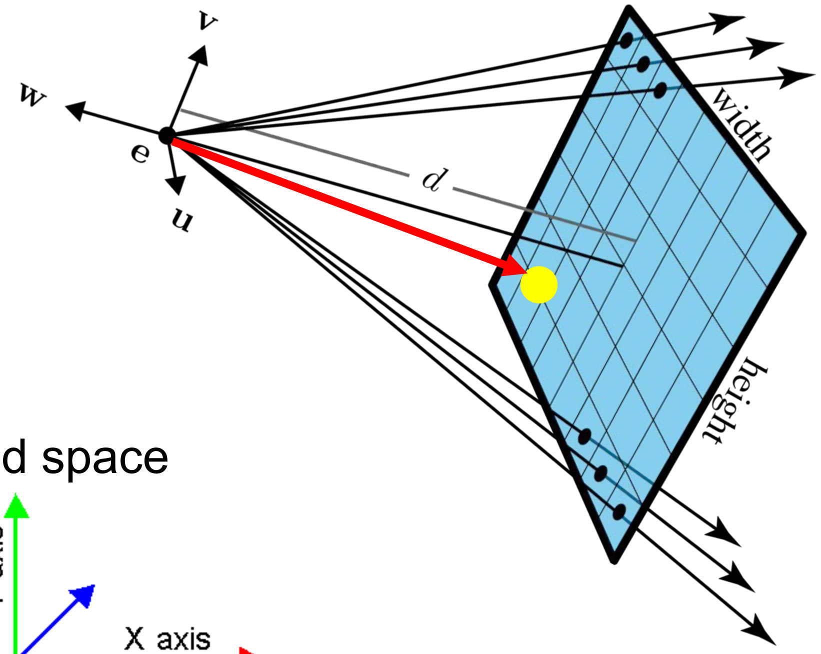
Example – World Space

$$\mathbf{u}, \mathbf{v}, \mathbf{w}$$

are the basis vectors for camera space

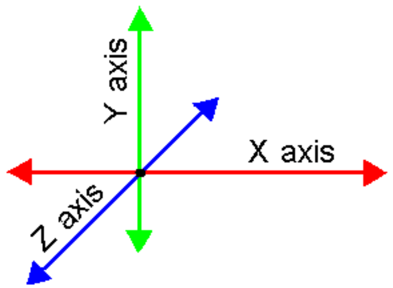
$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

our ray equation



$$\mathbf{e} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 0 \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

World space



$$\mathbf{p}(t) = \mathbf{e} + t \begin{bmatrix} \mathbf{u} & \mathbf{v} & \mathbf{w} \end{bmatrix} \begin{bmatrix} u(i) \\ v(j) \\ -d \end{bmatrix}$$

Example – World Space

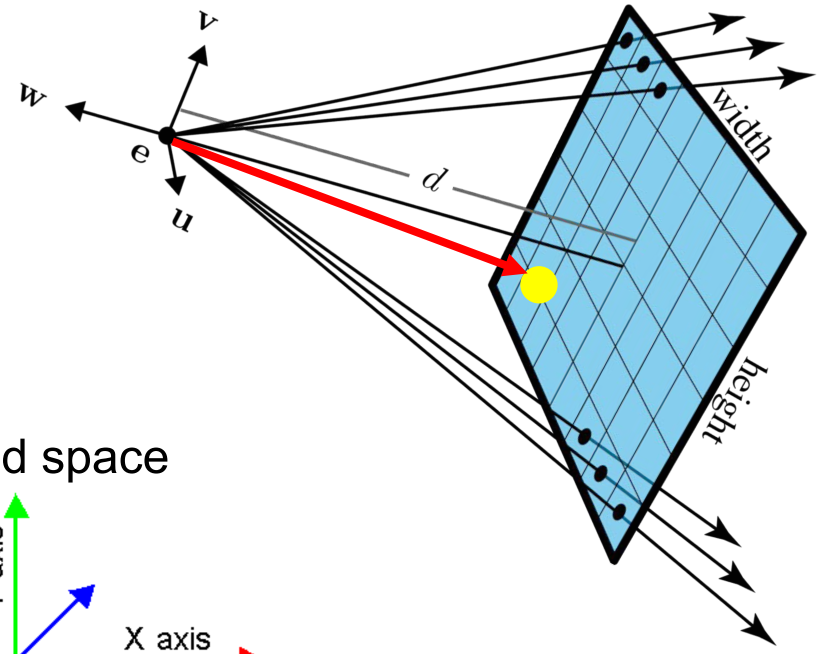
$$\mathbf{e} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 0 \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\mathbf{u}, \mathbf{v}, \mathbf{w}$$

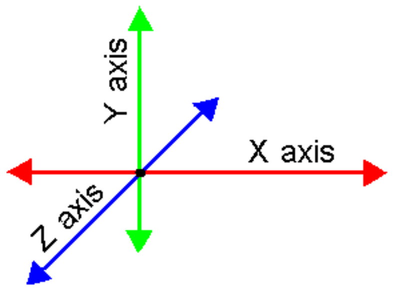
are the basis vectors for camera space

$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

our ray equation



World space



$$\mathbf{p}(t) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} -0.9 \\ -0.36 \\ -10 \end{bmatrix}$$

https://mathworld.wolfram.com/ChangeofCoordinatesMatrix.html

Basic Components of Ray Casting

Ray

Camera

Intersection Tests

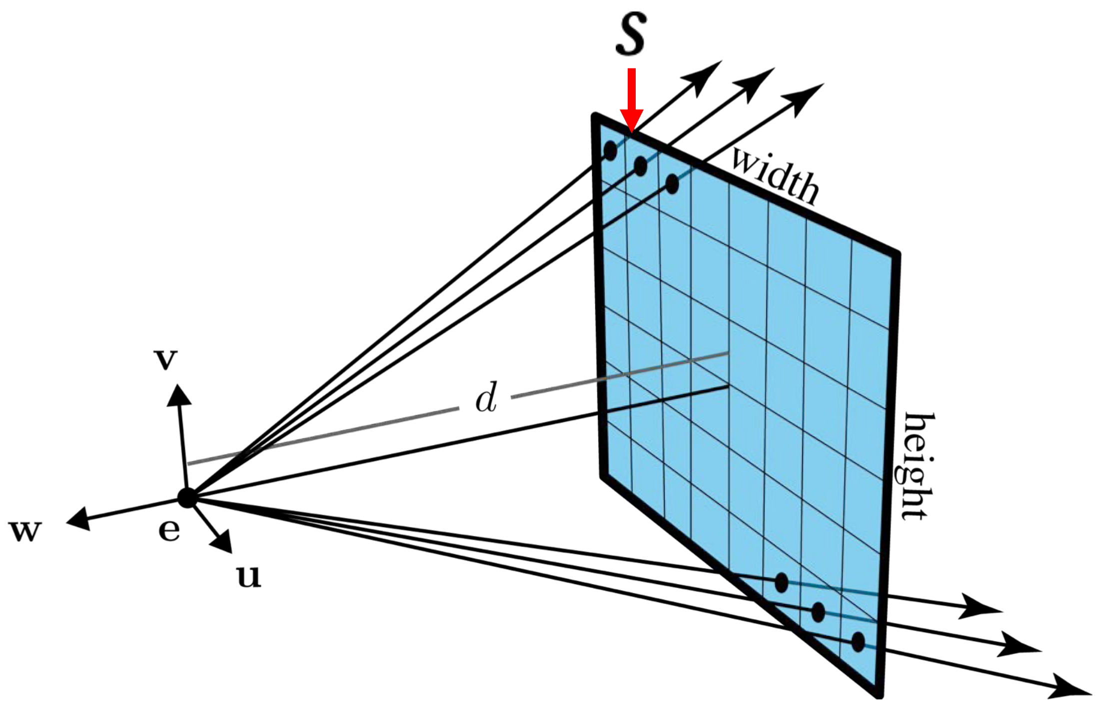
Intersection Tests

Plane

Sphere

Triangle

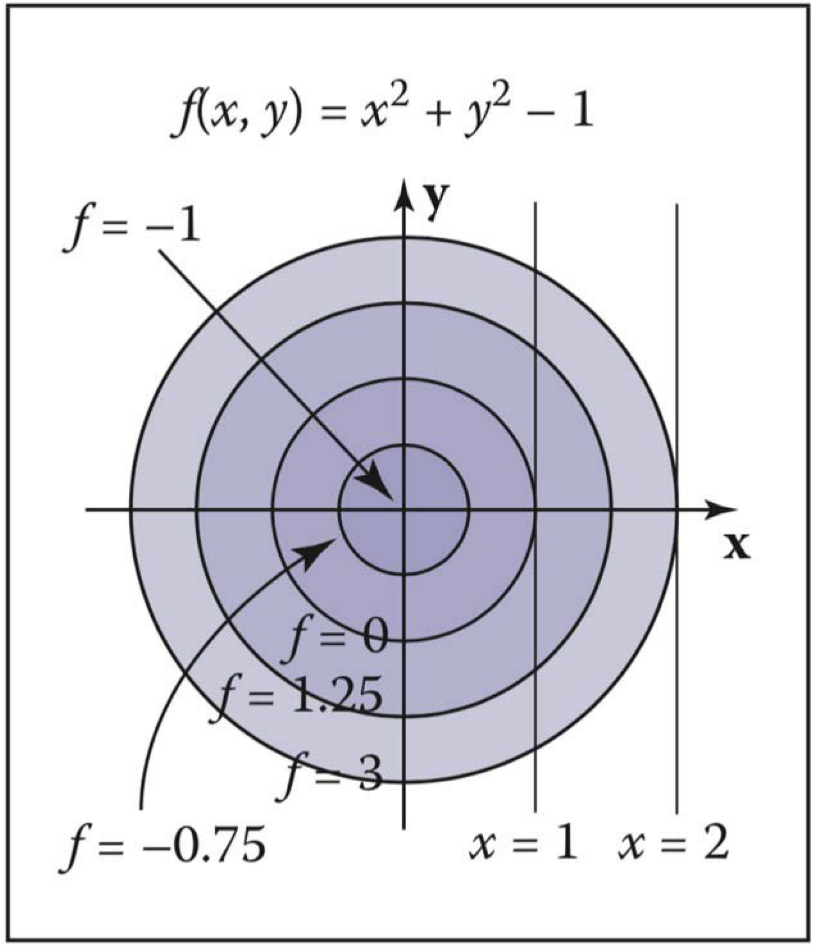
Ray Equation



$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

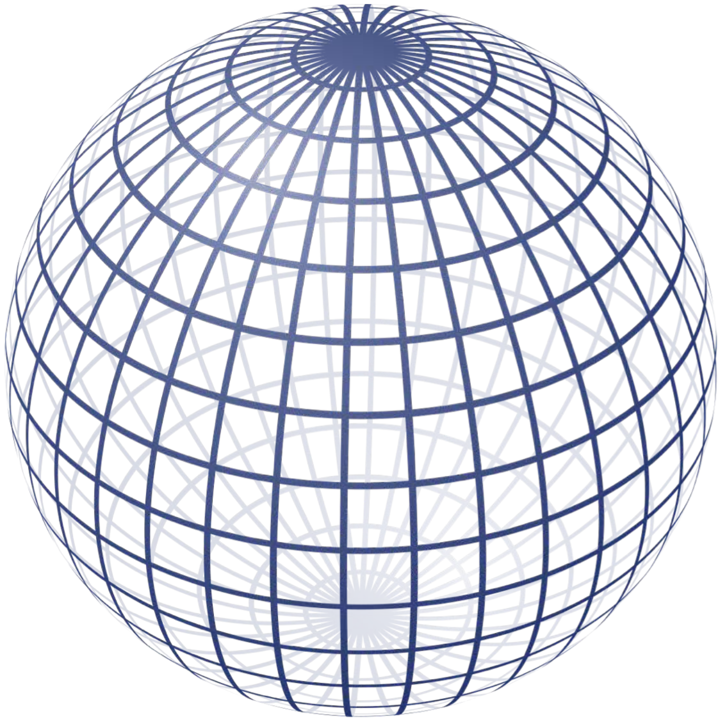
Aside: Types of Surface

Implicit Surface



Parametric Surface

$$x = r\cos\phi\sin\theta,$$
$$y = r\sin\phi\sin\theta,$$
$$z = r\cos\theta.$$



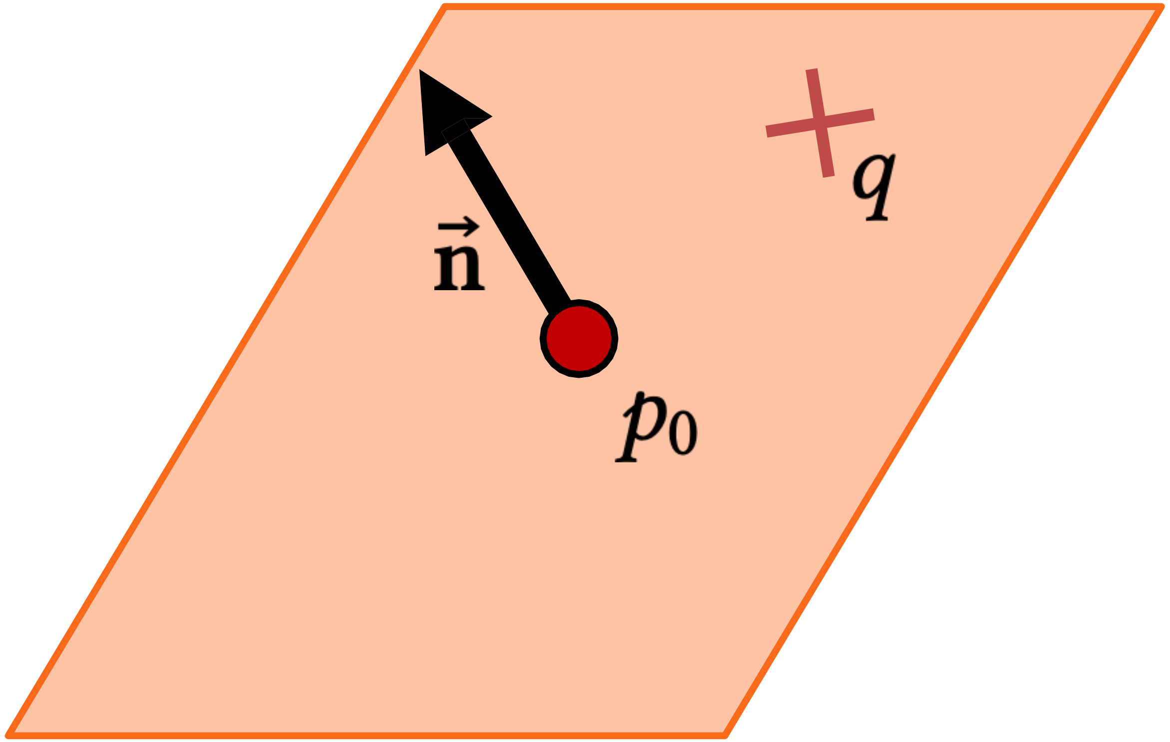
Intersection Tests

Plane

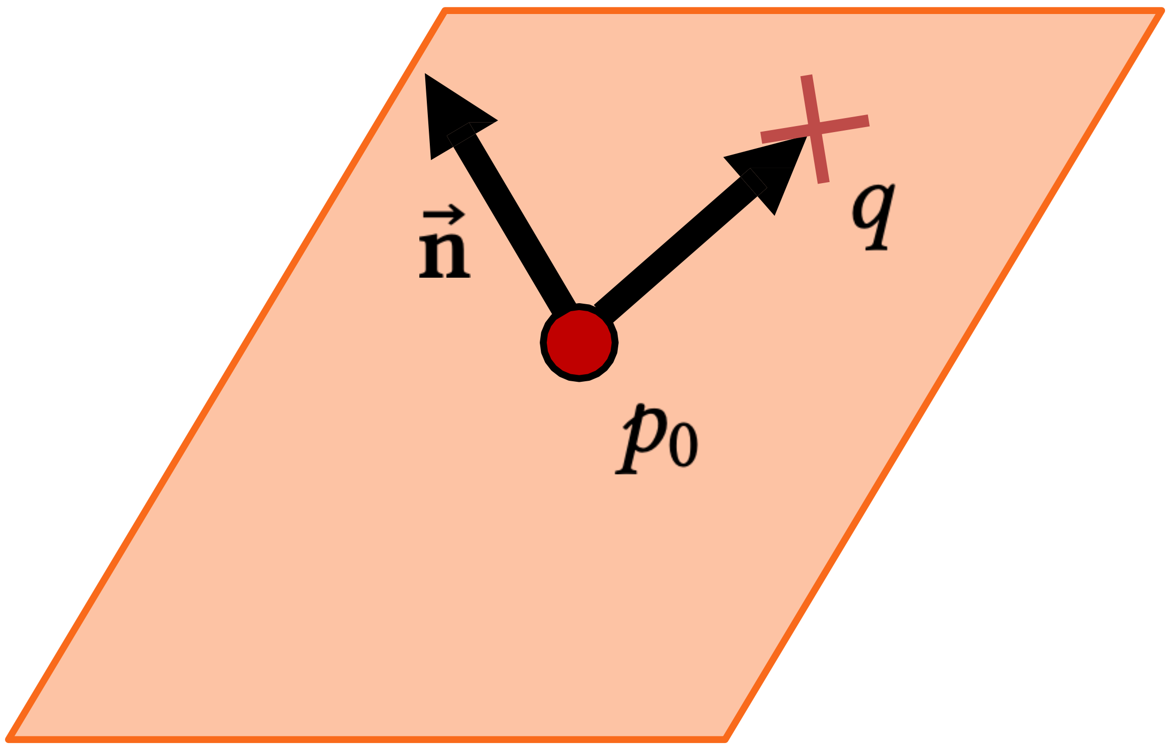
Sphere

Triangle

Plane Equation

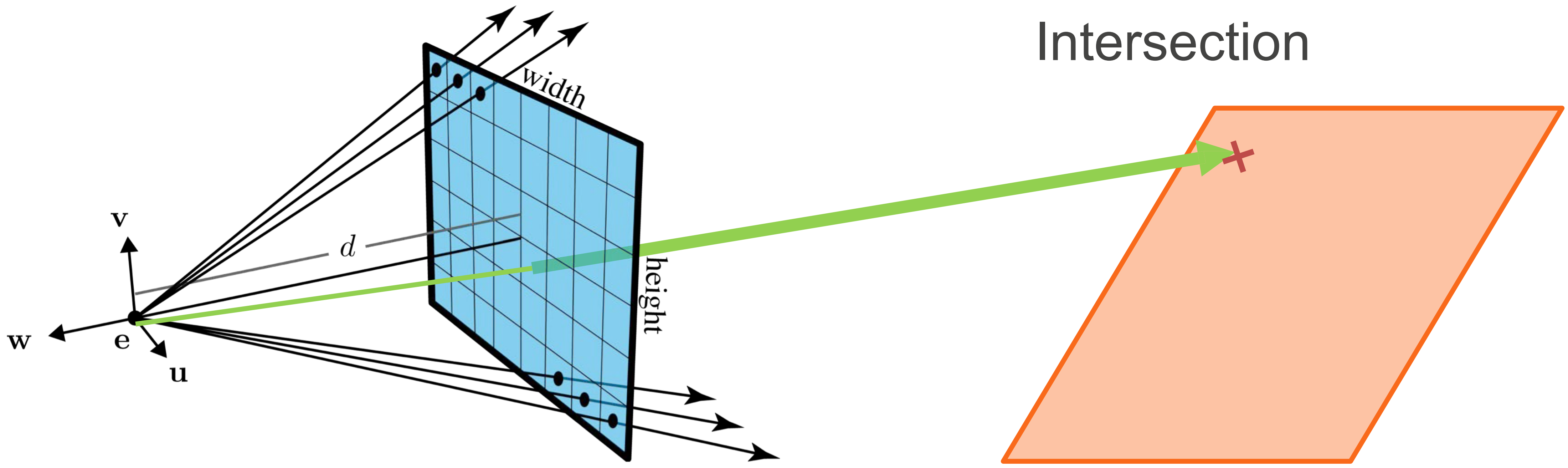


Plane Equation



$$\vec{\mathbf{n}} \cdot (q - p_0) = 0$$

Ray-Plane Intersection

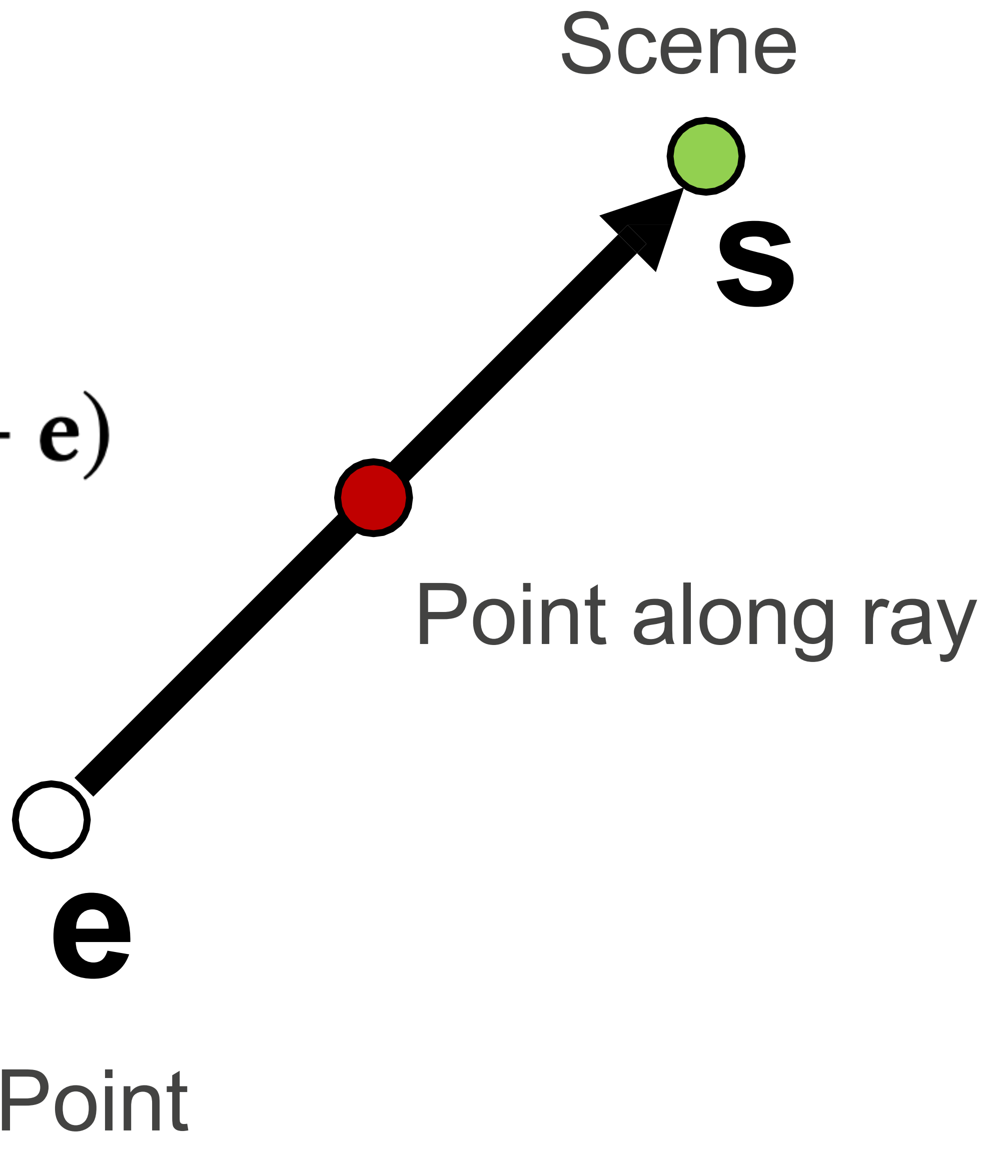


Ray-Plane Intersection

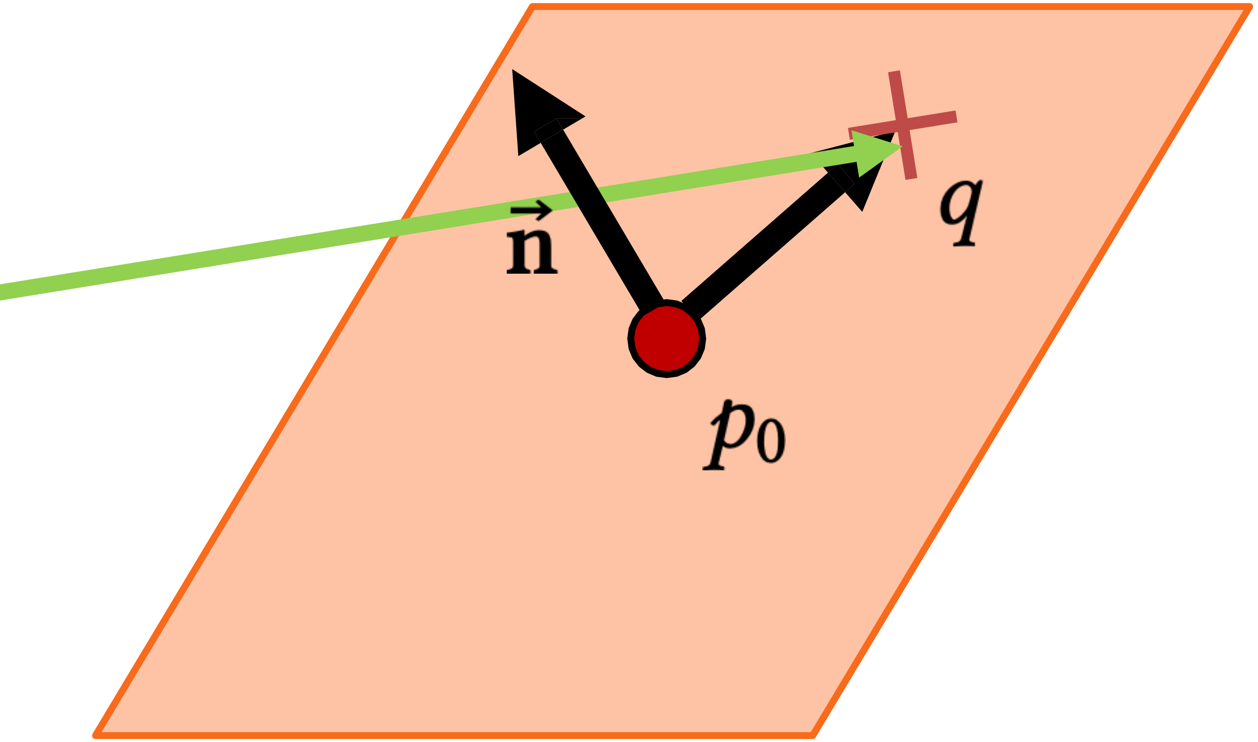
Ray equation

$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

$$\mathbf{p}(t) = \mathbf{e} + t\vec{\mathbf{d}}$$



Plane Equation

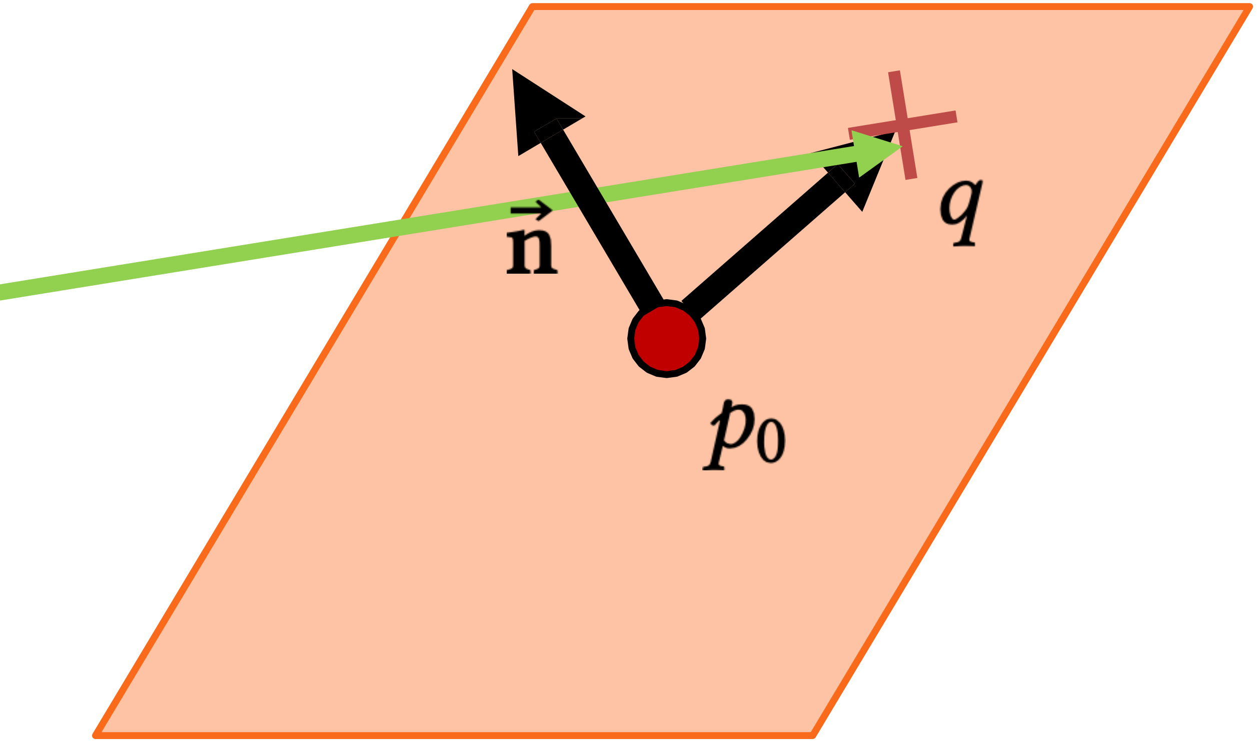


Plane equation
Substitute ray equation into it

$$\vec{\mathbf{n}} \cdot (\mathbf{p}(t) - p_0) = 0$$

$$\vec{\mathbf{n}} \cdot ((\mathbf{e} + t\vec{\mathbf{d}}) - p_0) = 0$$

Plane Equation



Plane equation
Substitute ray equation into it

$$\vec{\mathbf{n}} \cdot (\mathbf{p}(t) - p_0) = 0$$

$$\vec{\mathbf{n}} \cdot ((\mathbf{e} + t\vec{\mathbf{d}}) - p_0) = 0$$

Solve for t

$$\boxed{t = \frac{-\vec{\mathbf{n}} \cdot (\mathbf{e} - p_0)}{\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}}}$$

Intersection Tests

Plane

Sphere

Triangle

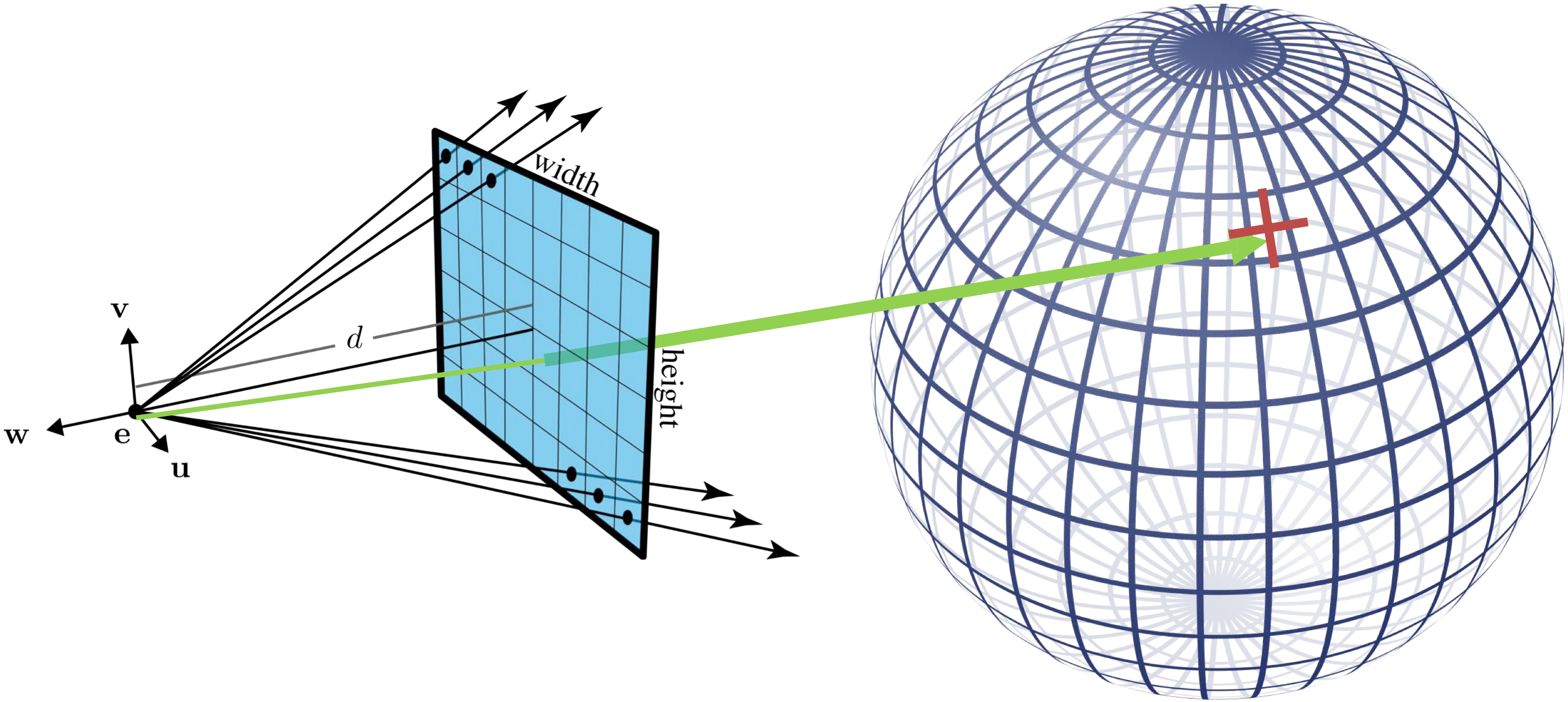
Intersection Tests

Plane

Sphere

Triangle

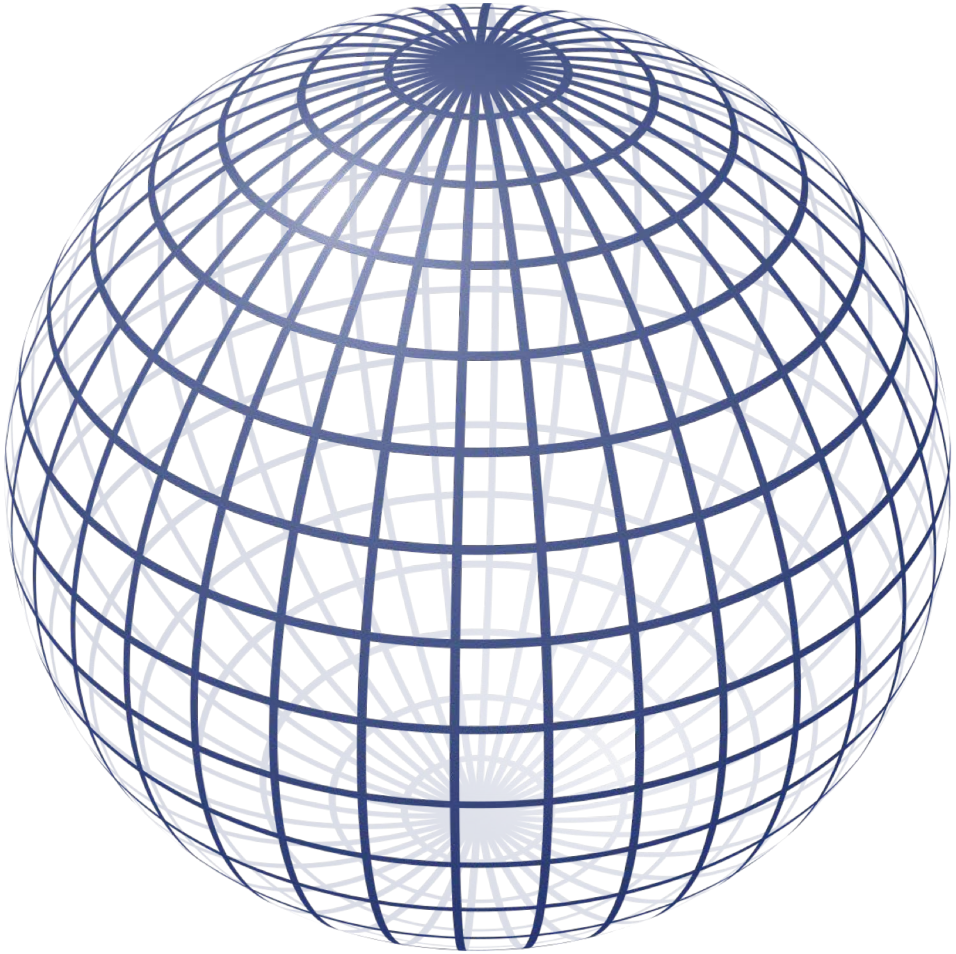
Ray-Sphere Intersection



Implicit Equation of a Sphere

$$(q - \mathbf{c}) \cdot (q - \mathbf{c}) - r^2 = 0$$

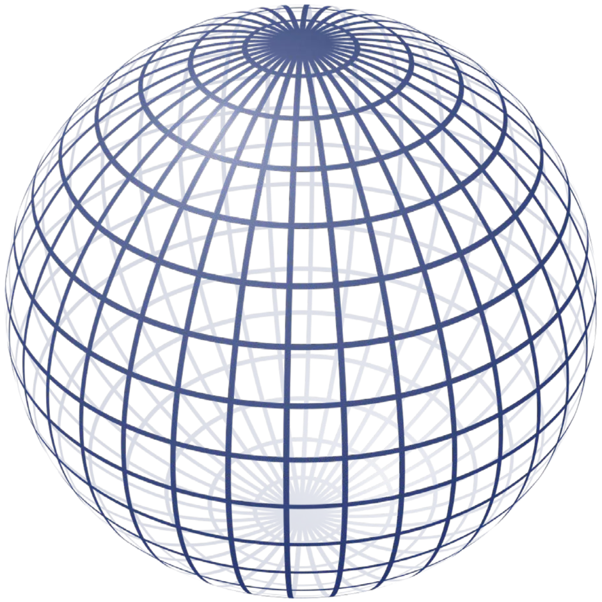
*Sphere centered at **c** with radius r*



Ray-Sphere Intersection

Substitute ray equation into implicit equation for sphere

$$(\mathbf{e} + t\vec{\mathbf{d}} - \mathbf{c}) \cdot (\mathbf{e} + t\vec{\mathbf{d}} - \mathbf{c}) - r^2 = 0$$



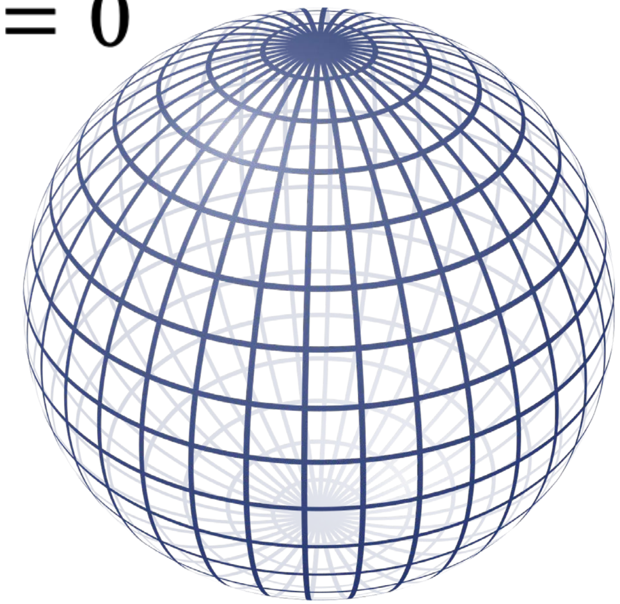
Ray-Sphere Intersection

Substitute ray equation into implicit equation for sphere

$$(\mathbf{e} + t\vec{\mathbf{d}} - \mathbf{c}) \cdot (\mathbf{e} + t\vec{\mathbf{d}} - \mathbf{c}) - r^2 = 0$$

Rearrange

$$(\vec{\mathbf{d}} \cdot \vec{\mathbf{d}})t^2 + 2\vec{\mathbf{d}} \cdot (\mathbf{e} - \mathbf{c})t + (\mathbf{e} - \mathbf{c}) \cdot (\mathbf{e} - \mathbf{c}) - r^2 = 0$$



Ray-Sphere Intersection

Substitute ray equation into implicit equation for sphere

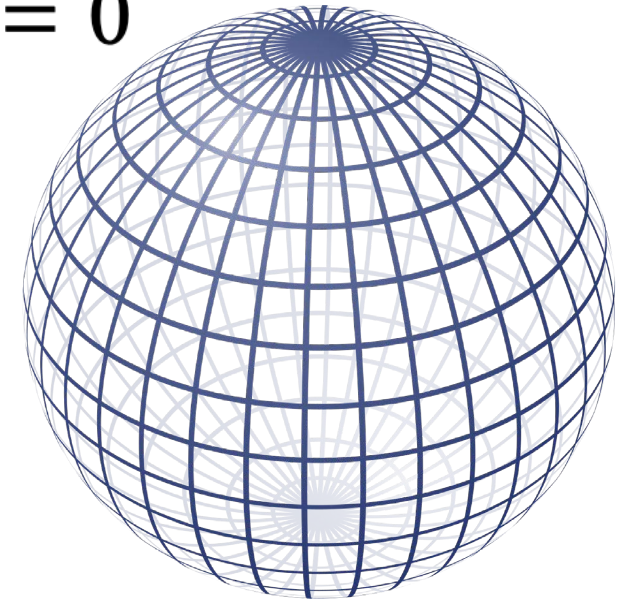
$$(\mathbf{e} + t\vec{\mathbf{d}} - \mathbf{c}) \cdot (\mathbf{e} + t\vec{\mathbf{d}} - \mathbf{c}) - r^2 = 0$$

Rearrange

$$(\vec{\mathbf{d}} \cdot \vec{\mathbf{d}})t^2 + 2\vec{\mathbf{d}} \cdot (\mathbf{e} - \mathbf{c})t + (\mathbf{e} - \mathbf{c}) \cdot (\mathbf{e} - \mathbf{c}) - r^2 = 0$$

Looks familiar…

$$At^2 + Bt + C = 0$$



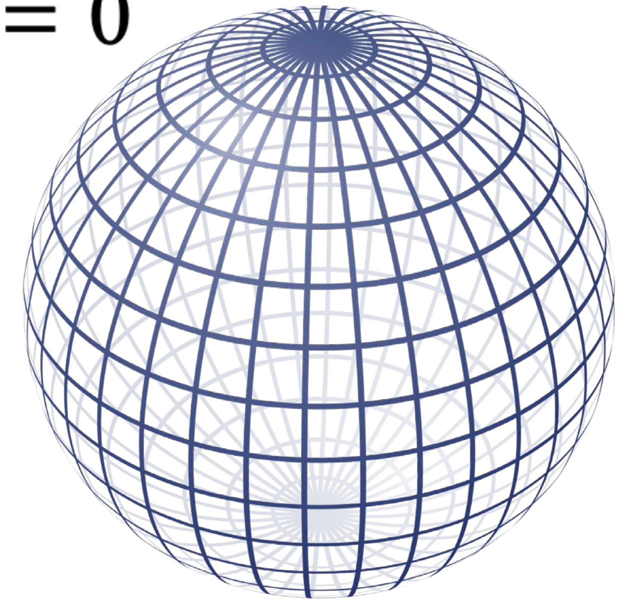
Ray-Sphere Intersection

Substitute ray equation into implicit equation for sphere

$$(\mathbf{e} + t\vec{\mathbf{d}} - \mathbf{c}) \cdot (\mathbf{e} + t\vec{\mathbf{d}} - \mathbf{c}) - r^2 = 0$$

Rearrange

$$(\vec{\mathbf{d}} \cdot \vec{\mathbf{d}})t^2 + 2\vec{\mathbf{d}} \cdot (\mathbf{e} - \mathbf{c})t + (\mathbf{e} - \mathbf{c}) \cdot (\mathbf{e} - \mathbf{c}) - r^2 = 0$$



Looks familiar…

$$At^2 + Bt + C = 0$$

It's a quadratic! (can use the quadratic equation)

Hint for the homework: the discriminant tells us what kinds of roots the equation has.

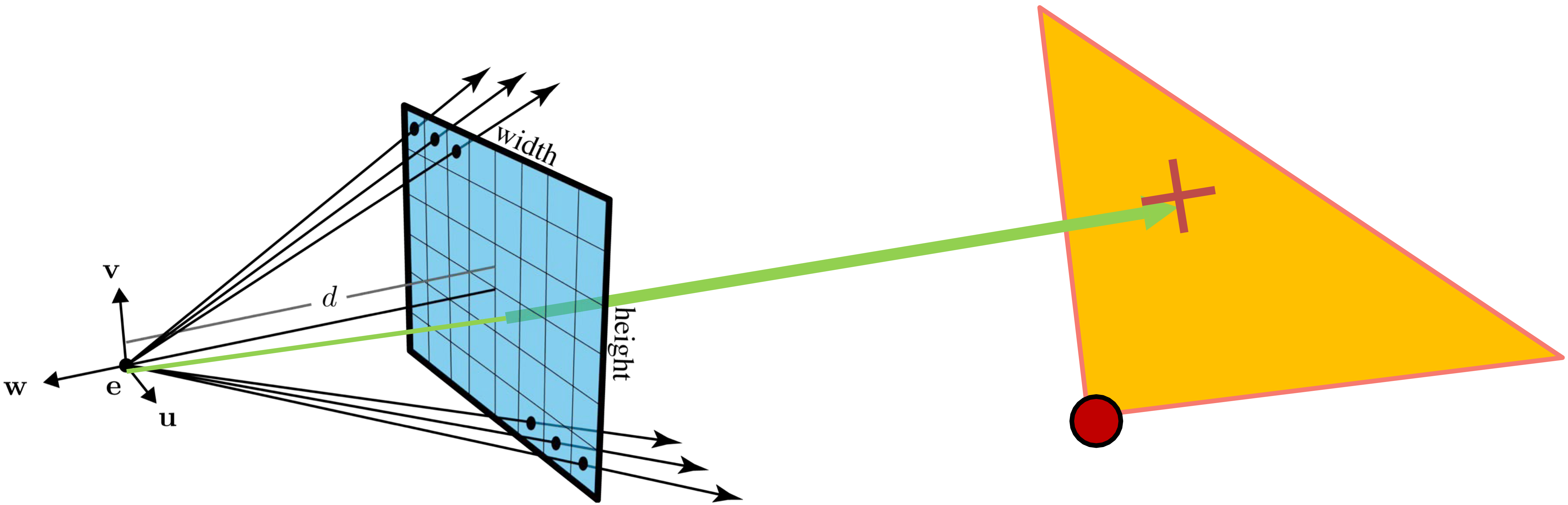
Intersection Tests

Plane

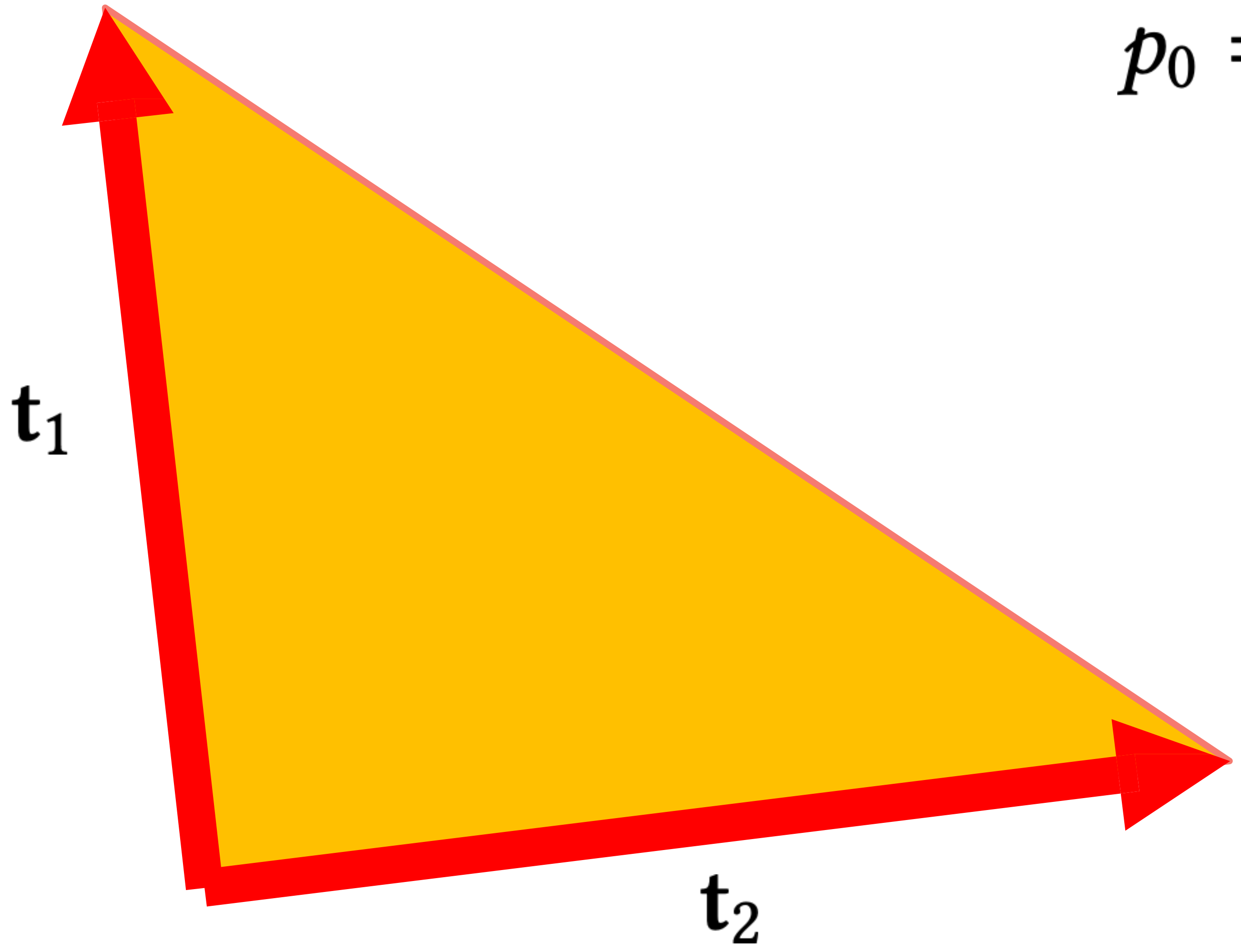
Sphere

Triangle

Ray-Triangle Intersection

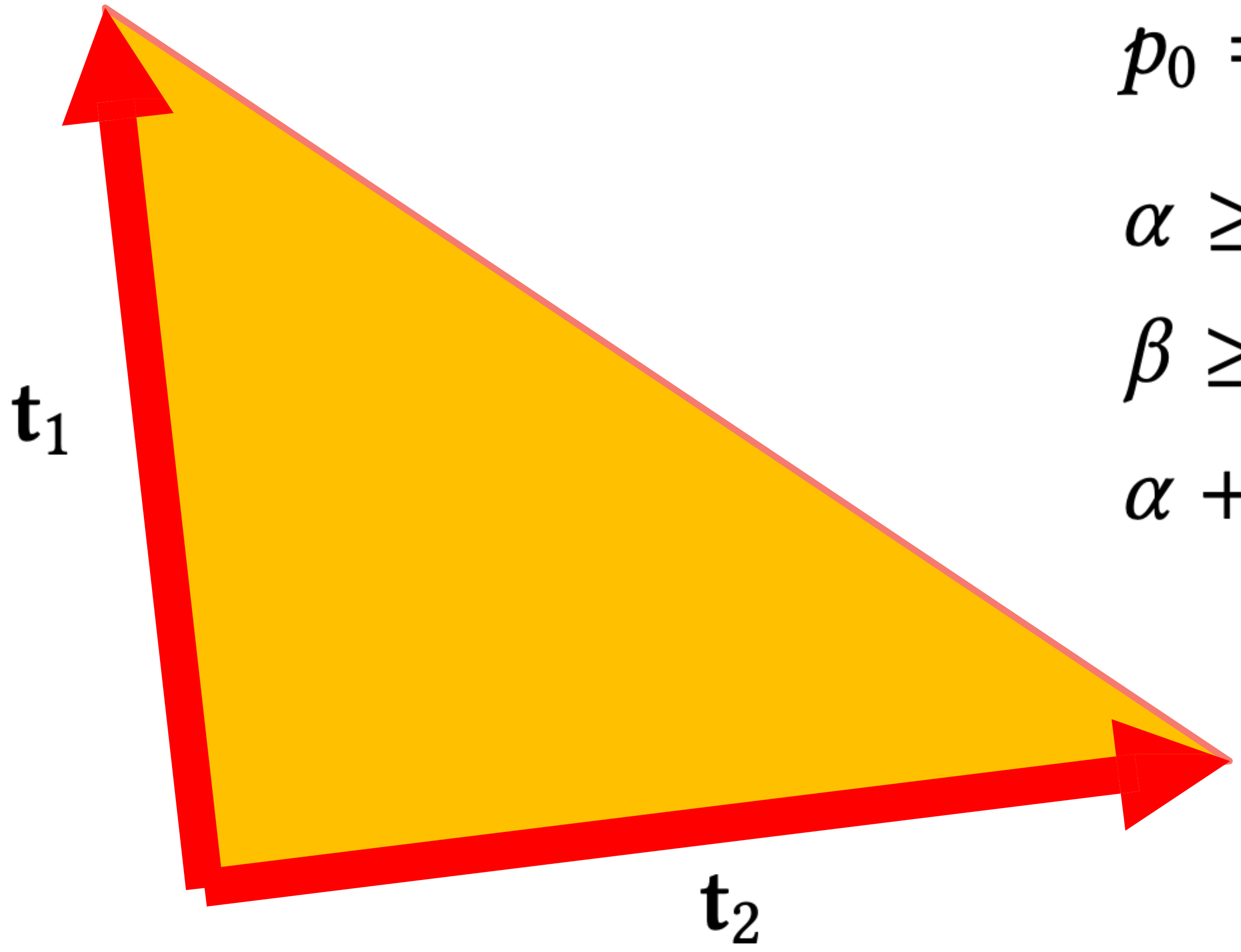


Equations for a Triangle



$$p_0 = \alpha \mathbf{t}_1 + \beta \mathbf{t}_2$$

Equations for a Triangle



$$p_0 = \alpha\mathbf{t}_1 + \beta\mathbf{t}_2$$

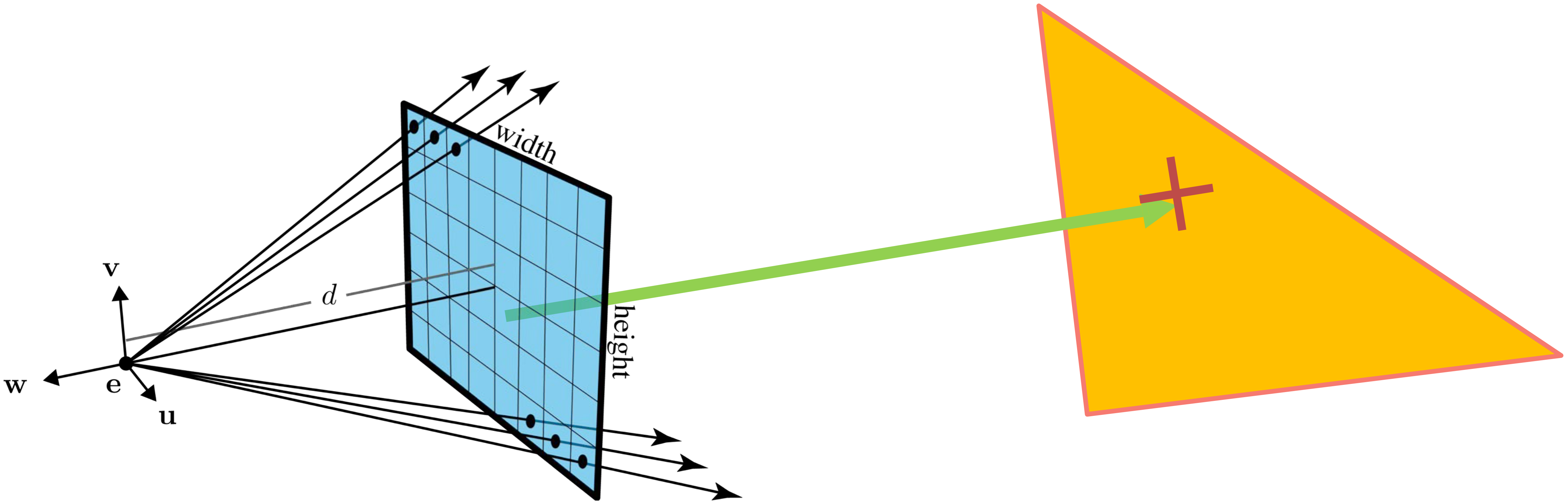
$$\alpha \geq 0$$

$$\beta \geq 0$$

$$\alpha + \beta \leq 1$$

Intersection with a Triangle (Parametric Surface)

Check via equating point on surface with point on ray



Intersection with a Triangle (Parametric Surface)

Check via equating point on surface with point on ray

$$\mathbf{p}(t) = \alpha \mathbf{t}_1 + \beta \mathbf{t}_2$$

$$\mathbf{e} + t\vec{\mathbf{d}} = \alpha \mathbf{t}_1 + \beta \mathbf{t}_2$$

$$\mathbf{e} = \alpha \mathbf{t}_1 + \beta \mathbf{t}_2 - t\vec{\mathbf{d}}$$

Intersection with a Triangle (Parametric Surface)

Check via equating point on surface with point on ray

$$\mathbf{e} = \alpha \mathbf{t}_1 + \beta \mathbf{t}_2 - t\vec{\mathbf{d}}$$

$$\mathbf{e} = \begin{bmatrix} \mathbf{t}_1 & \mathbf{t}_2 & -\mathbf{d} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ t \end{bmatrix}$$

Check values of α, β, t

Ray Casting

```
for each pixel in the image {
    Generate a ray
    for each object in the scene {
        if (Intersect ray with
            object) {  Set pixel
            colour
        }
    }
}
```

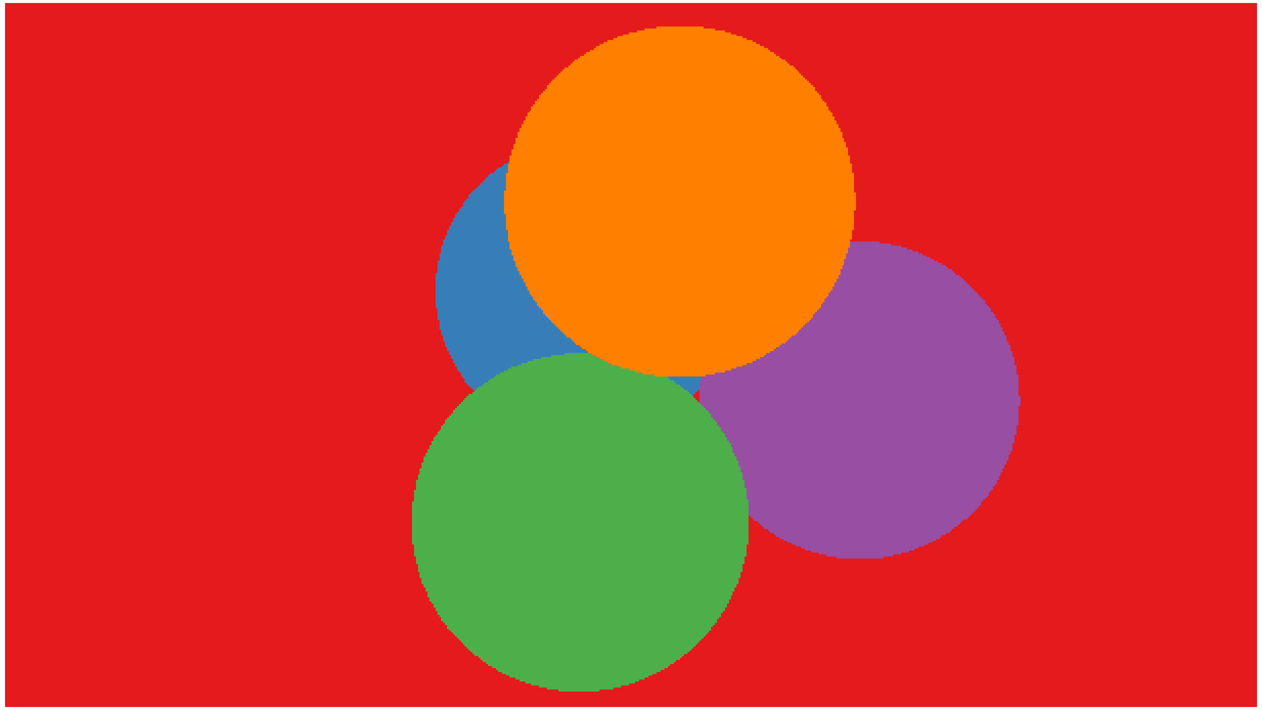
Output Type

Object ID

Surface Normal

Depth

Output Type



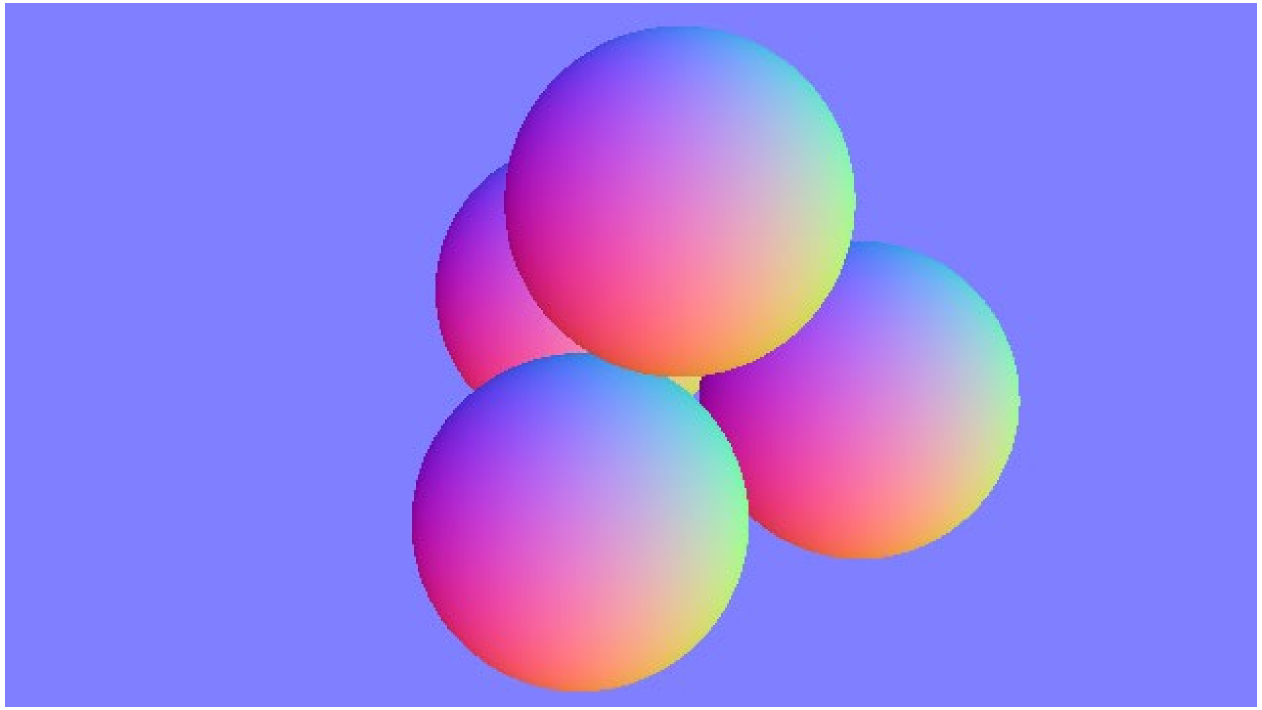
Output Type

Object ID

Surface Normal

Depth

Output Type



Output Type

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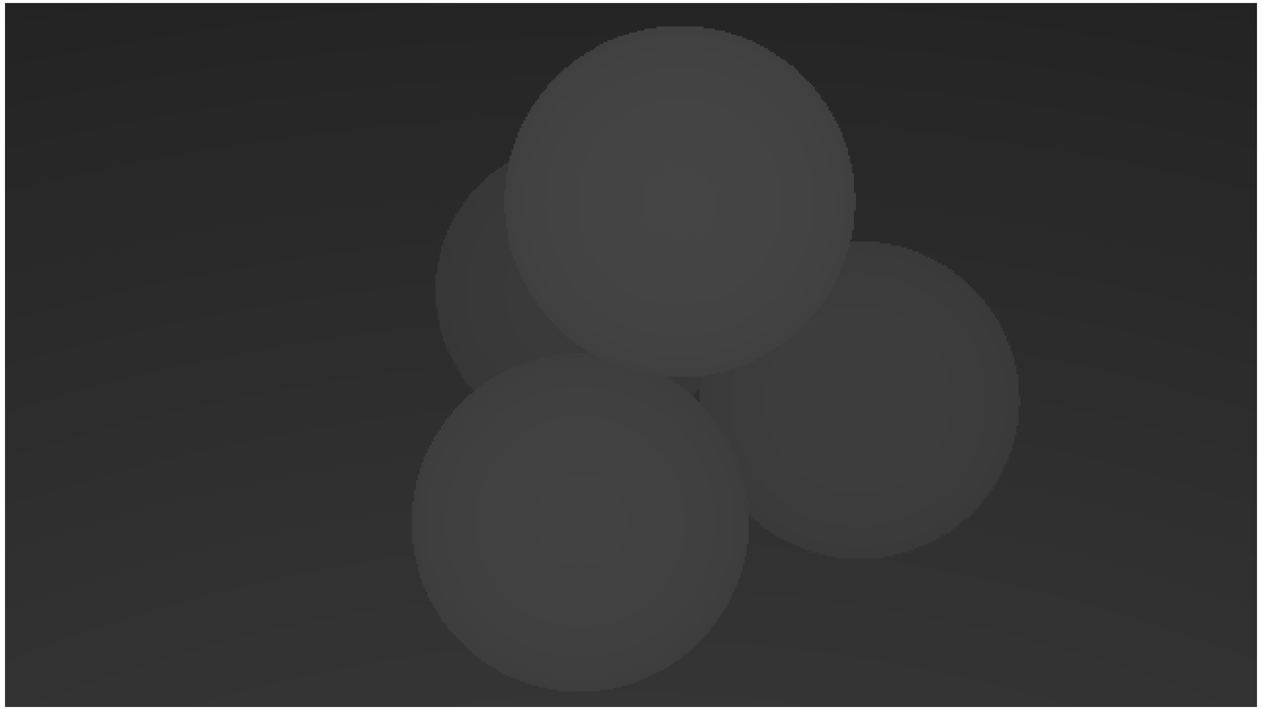

Ray Casting

```
for each pixel in the image {
    Generate a ray
    for each object in the scene {
        if (Intersect ray with
            object) {   Set pixel
            colour
        }
    }
}
```

Done for Today

Questions?

Assignment 2 due 19 May

Tutorial Friday at 1pm EDT for questions on Assignment 2