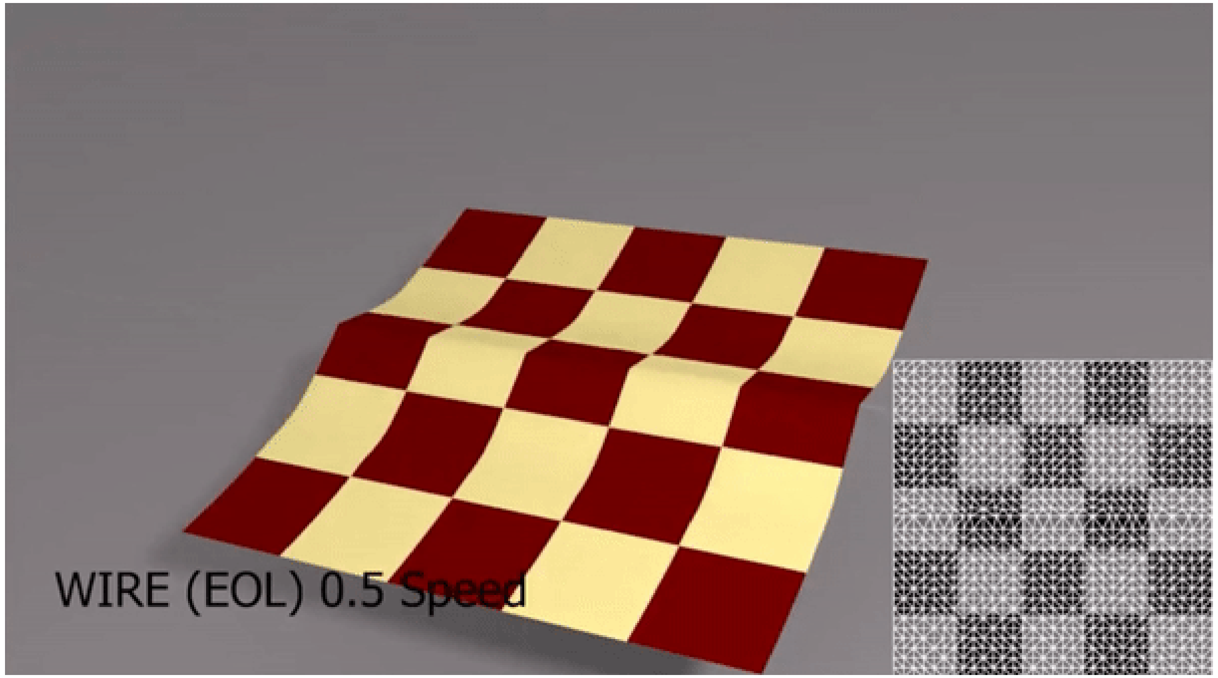
Physics Based Animation: Mass-Spring Systems



Some Slides/Images adapted from Marschner and Shirley and David Levin

Announcements

Assignment due Sunday 2 August

Office hours on Wednesday after lecture

No class next Monday 3 August

Bonus Assignment

Goal: make the coolest image or video using the tools we learned in the course

Turn in:

- the image or video in a zip file
- README describing
  - why you decided to make what you did
  - your process
  - what you tried (what worked and did not work)

Will be scored 0-5. Add this number to your final mark. e.g. you have a 78% in the course and get 3 points on the bonus assignment. Your mark is now 81%.

Bonus Assignment Scale

0: don’t do it. No penalty, this is optional

1-2: easily extend an assignment. E.g. add some planets to A6, add some models to a scene rendered with A3

2-3: add new functionality to an assignment. E.g. new noise function for A6, add refraction to ray tracer from A3.

3-4: creatively combine assignments and techniques. E.g. keyframe a ray traced animation (A3+A7).

4-5: do something really creative or bring something from outside the course. E.g. implement a research paper

These are only examples. If you have an idea but are unsure how much it could be worth, email me. Marked on ***idea*** + ***effort*** generously.

Any Questions?

Physics-Based Animation

Today:

Newton’s Laws of Motion
The Mass-Spring System
Time Integration
Implicit Integration via Optimization

Wednesday:

A Local-Global Solver for Fast-Mass Springs
Fixed Points
Dense and Sparse Matrices

Physics-Based Animation

http://faculty.cs.tamu.edu/sueda/projects/eol-cloth/

Newton’s Laws: You know em and love em

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1. Every object will remain at rest or in uniform motion in a straight line unless compelled to change its state by the action of an external force

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2. The force acting on an object is equal to the time rate-of-change of the momentum

Newton’s Laws: You know em and love em

1. Every object will remain at rest or in uniform motion in a straight line unless compelled to change its state by the action of an external force

2. The force acting on an object is equal to the time rate-of-change of the momentum

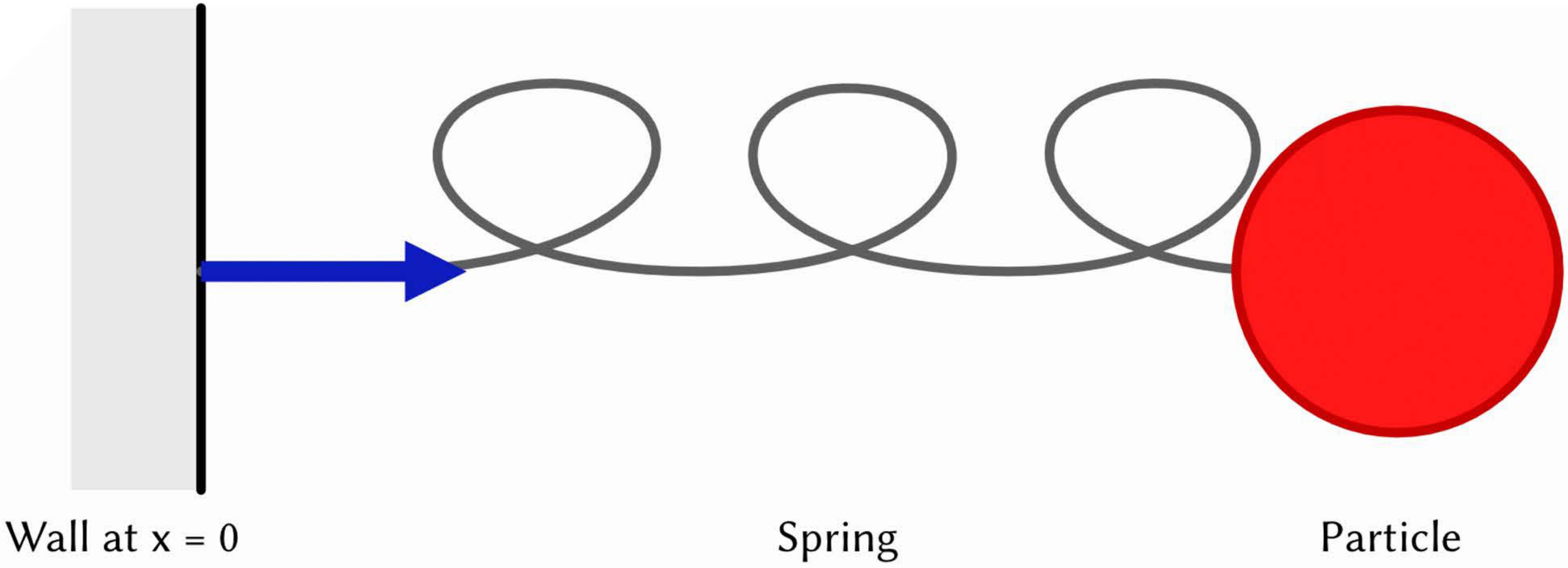
3. For every action there is an equal and opposite reaction

Good ole Newton's Second Law

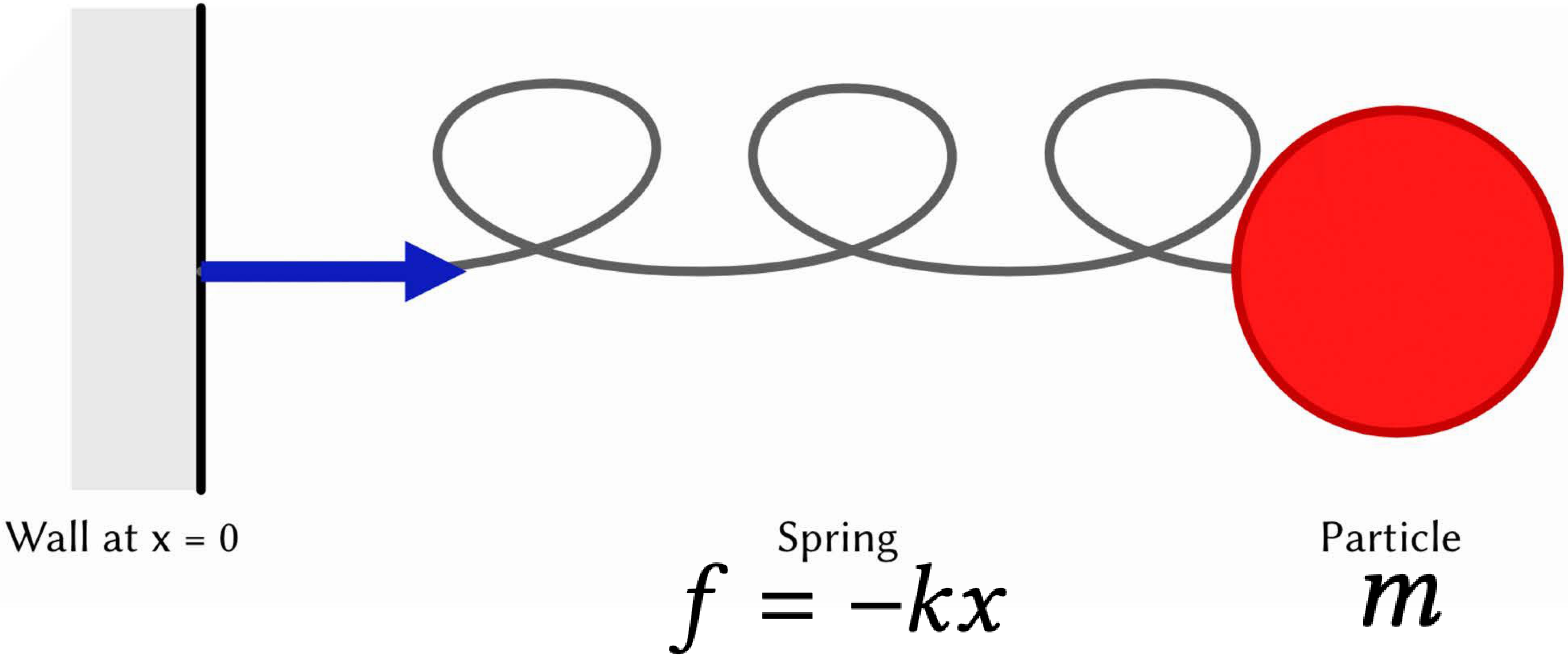
Acceleration

force $f = ma$

Mass



Wall at x = 0
Spring
Particle


Wall at x = 0
Spring
$f = -kx$
Particle
m

Good ole Newton's Second Law

Acceleration

force $$f = ma$$

Mass

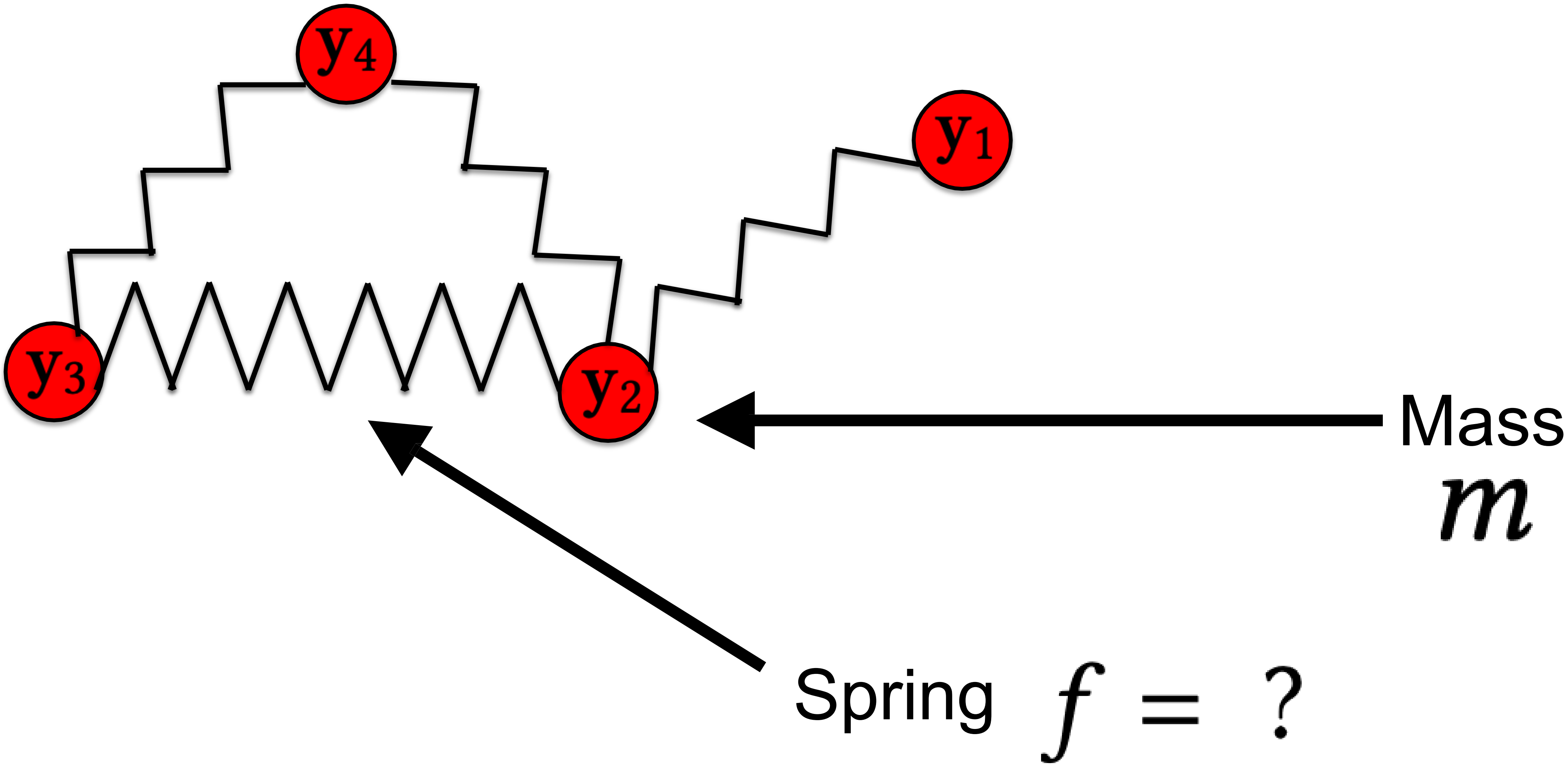
Good ole Newton's Second Law

Acceleration

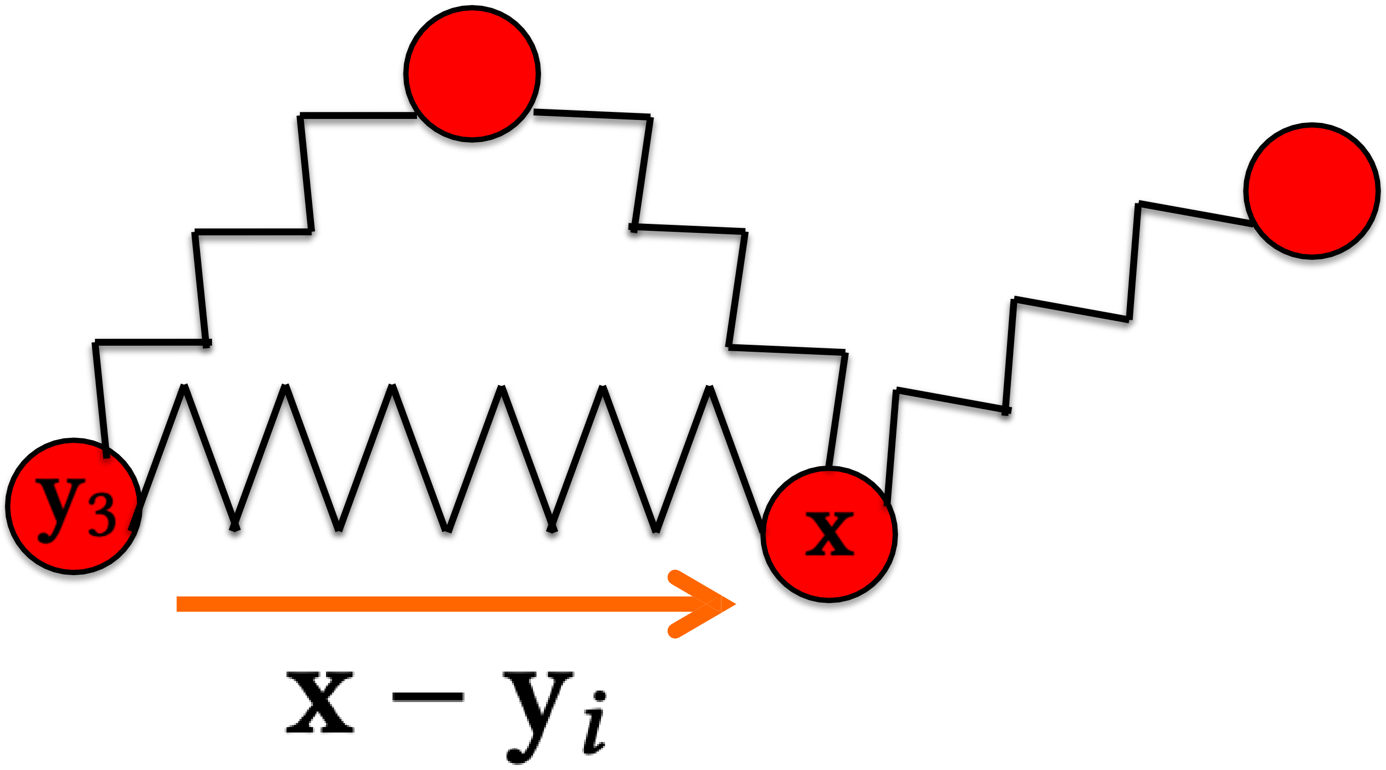
force $\mathbf{f} = m\mathbf{a}$

Mass

The Mass-Spring System



The Mass-Spring System



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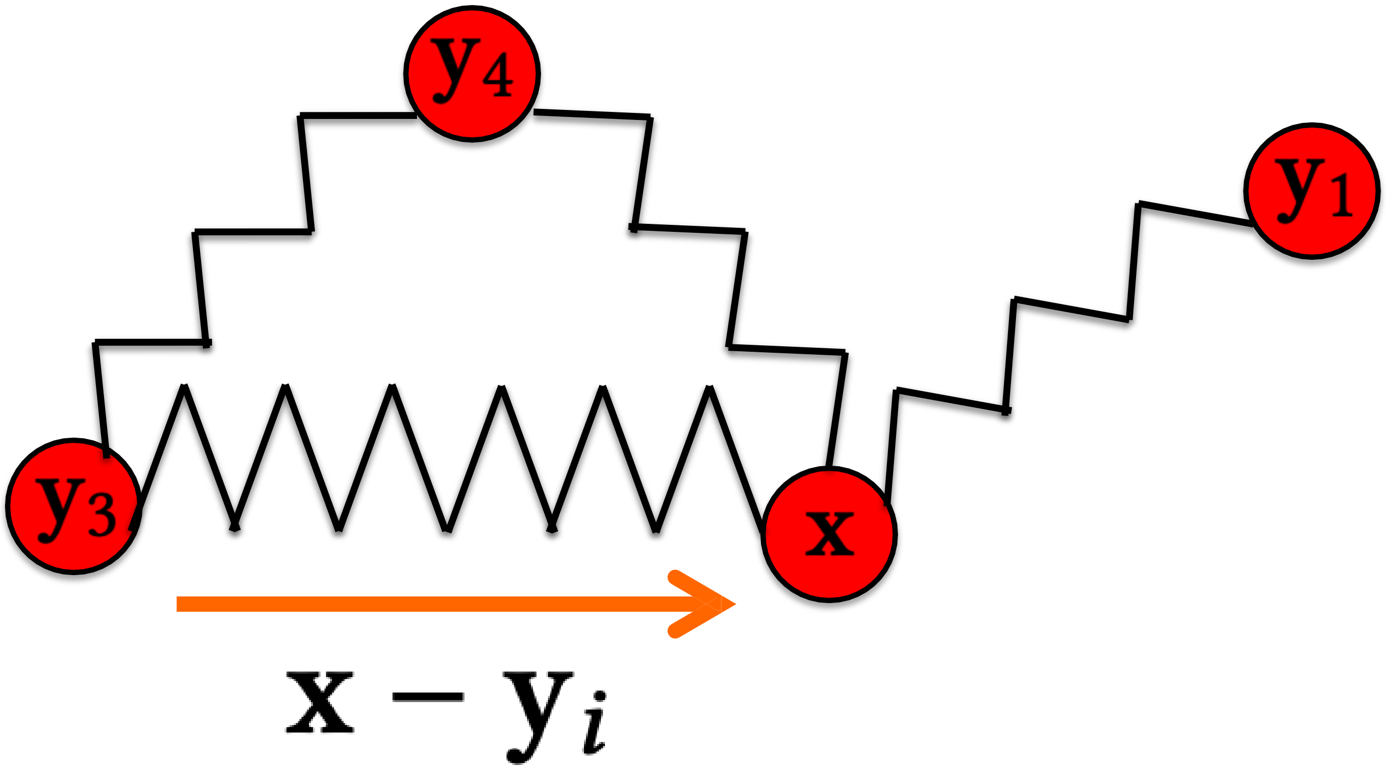
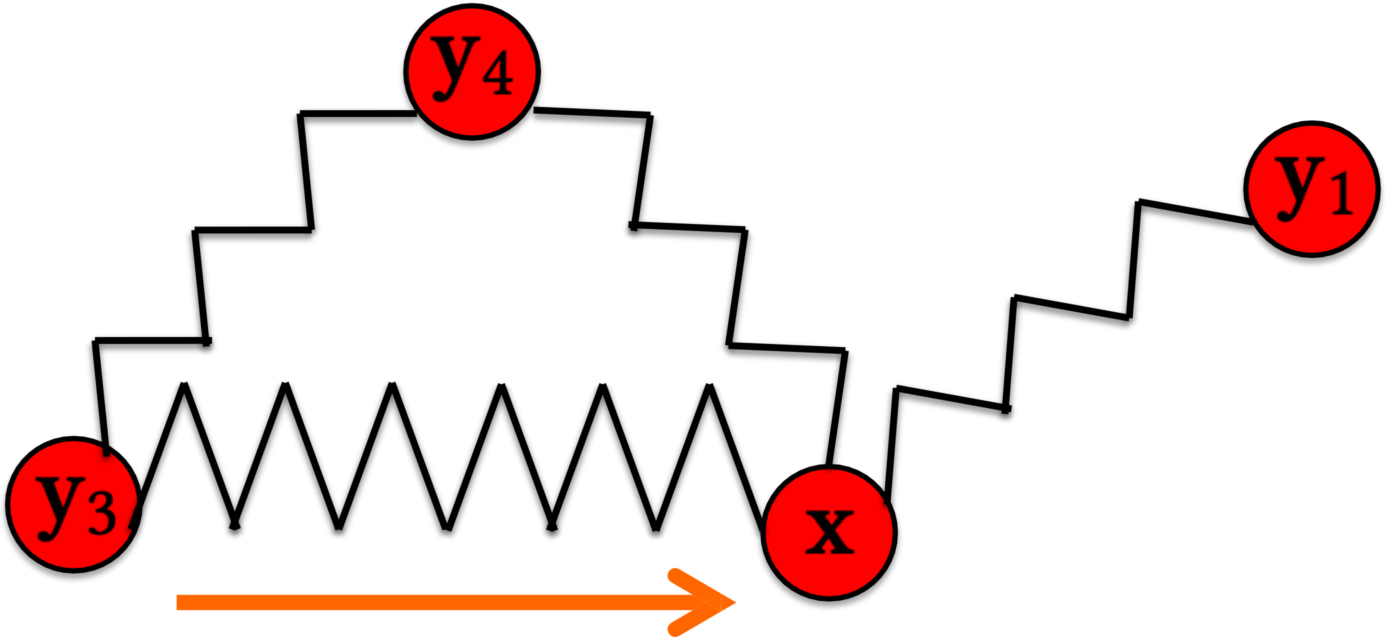


$$\mathbf{f}_x = -k(\|\mathbf{x} - \mathbf{y}_i\| - r_{ix})\frac{\mathbf{x} - \mathbf{y}_i}{\|\mathbf{x} - \mathbf{y}_i\|}$$

The Mass-Spring System



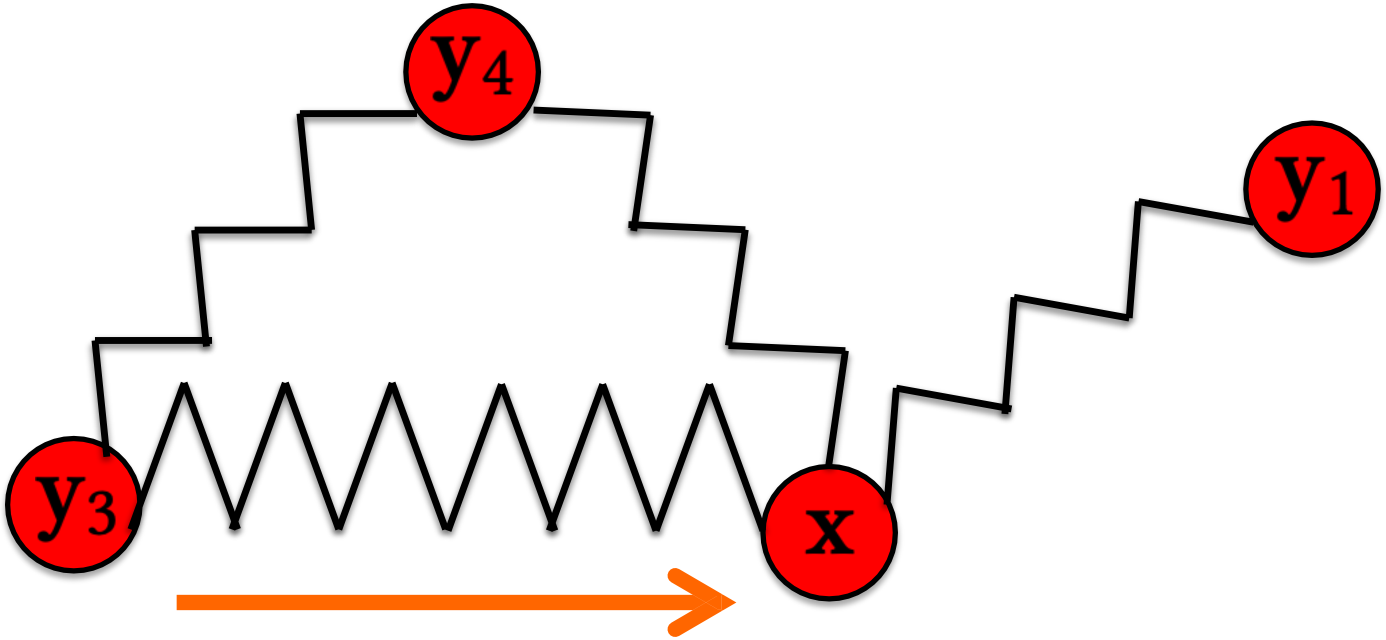
$$\mathbf{x} - \mathbf{y}_i$$

Direction of Spring

Rest Length

$$\mathbf{f}_x = -k\underbrace{(\|\mathbf{x}-\mathbf{y}_i\| - r_{ix})}_{\text{Change in Length}}\frac{\mathbf{x}-\mathbf{y}_i}{\|\mathbf{x}-\mathbf{y}_i\|}$$

The Mass-Spring System



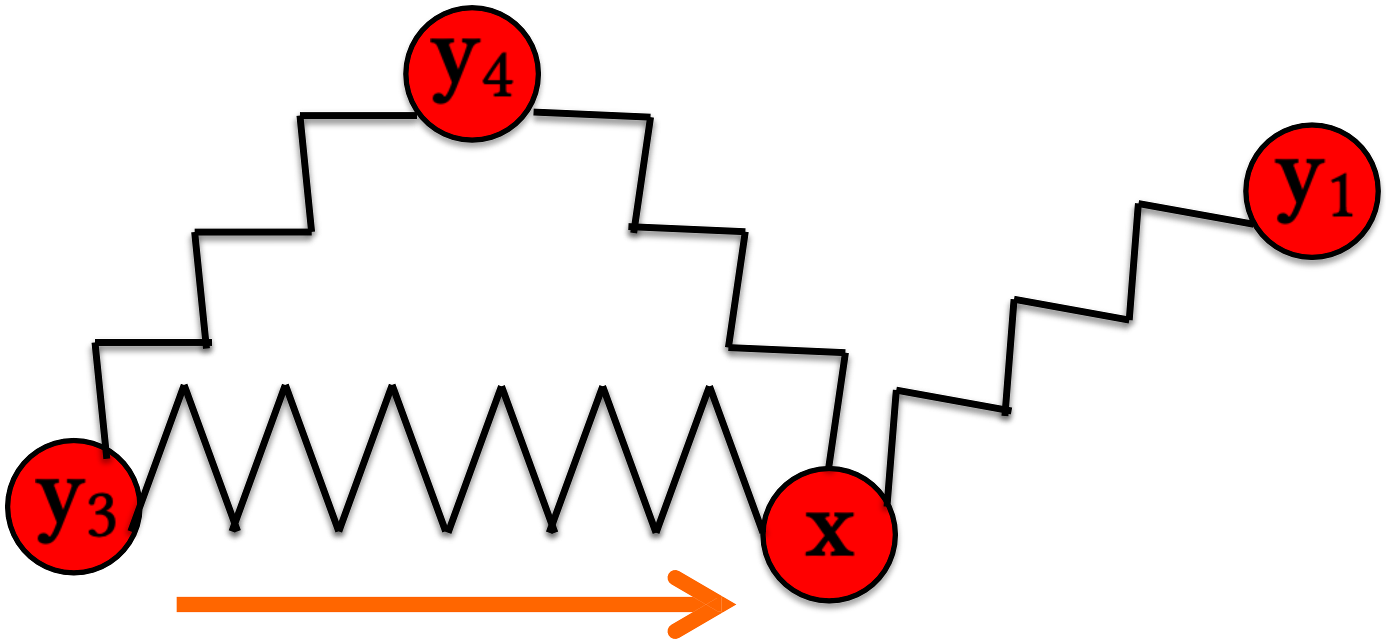
$$\mathbf{x} - \mathbf{y}_i$$

Direction of Spring

Rest Length

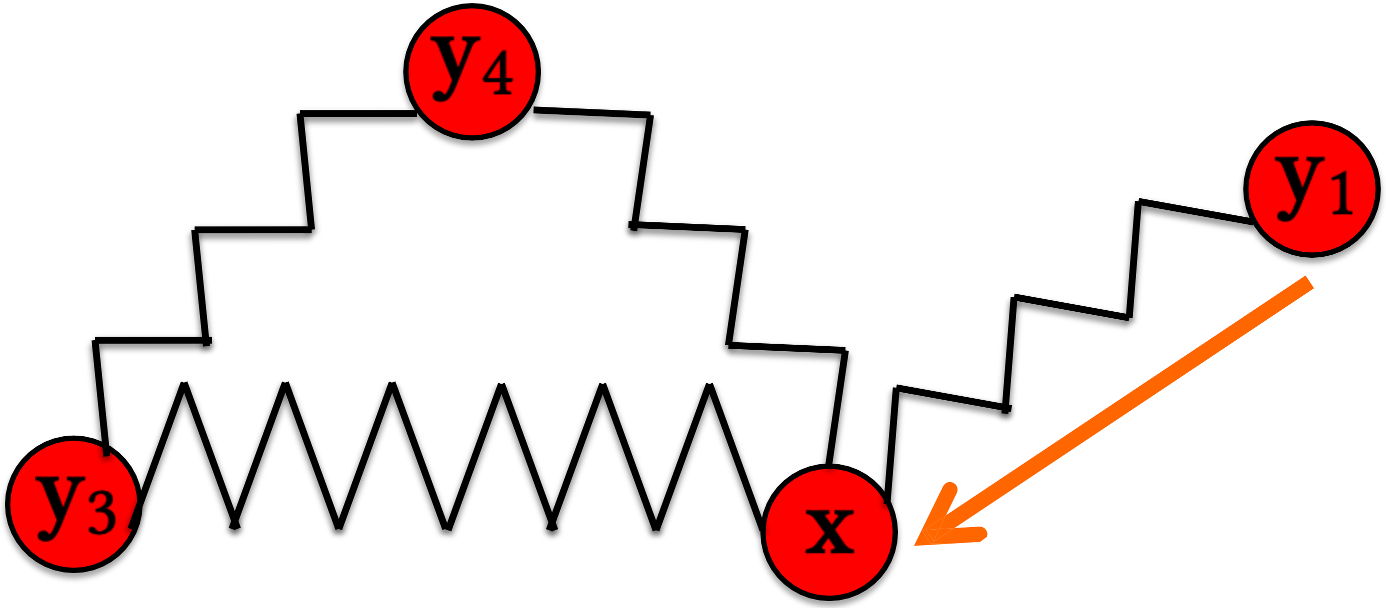
$$\boxed{\mathbf{f}_x(\mathbf{y}_i)} = -k\underbrace{(\|\mathbf{x}-\mathbf{y}_i\| - r_{ix})}_{\text{Change in Length}}\frac{\mathbf{x}-\mathbf{y}_i}{\|\mathbf{x}-\mathbf{y}_i\|}$$

Newton's Second Law for Each Particle



$$m_x \mathbf{a}_x = \sum_i \mathbf{f}_x(\mathbf{y}_i)$$

Newton's Second Law for Each Particle



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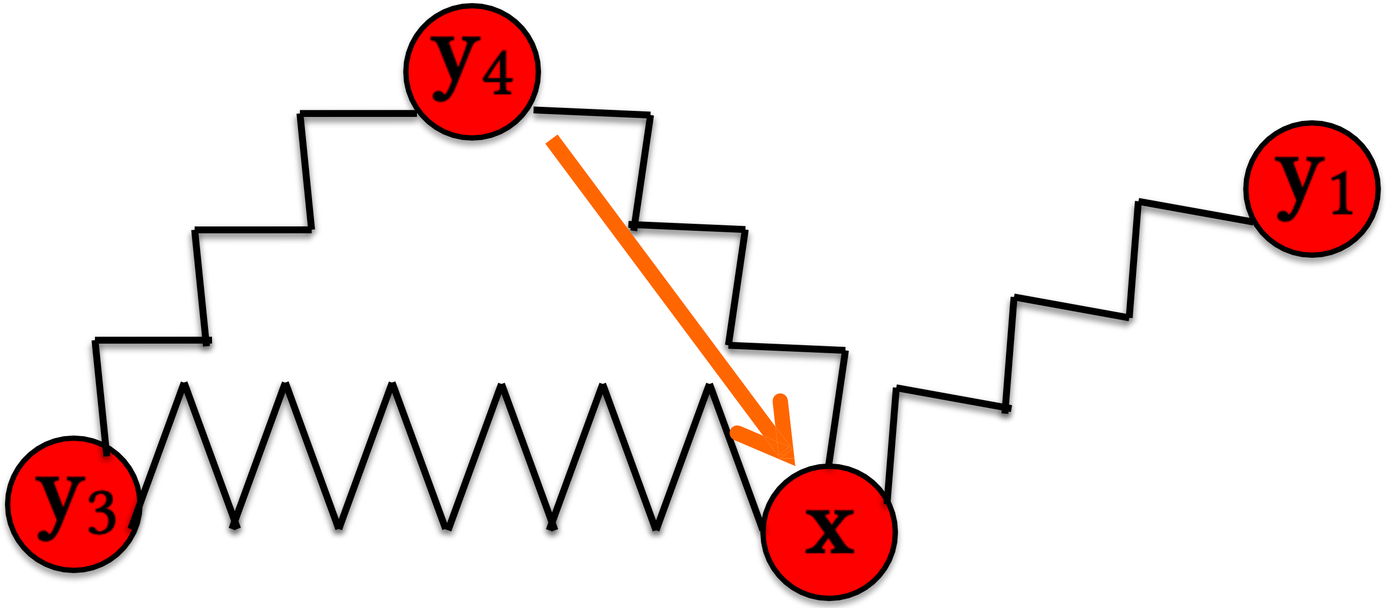
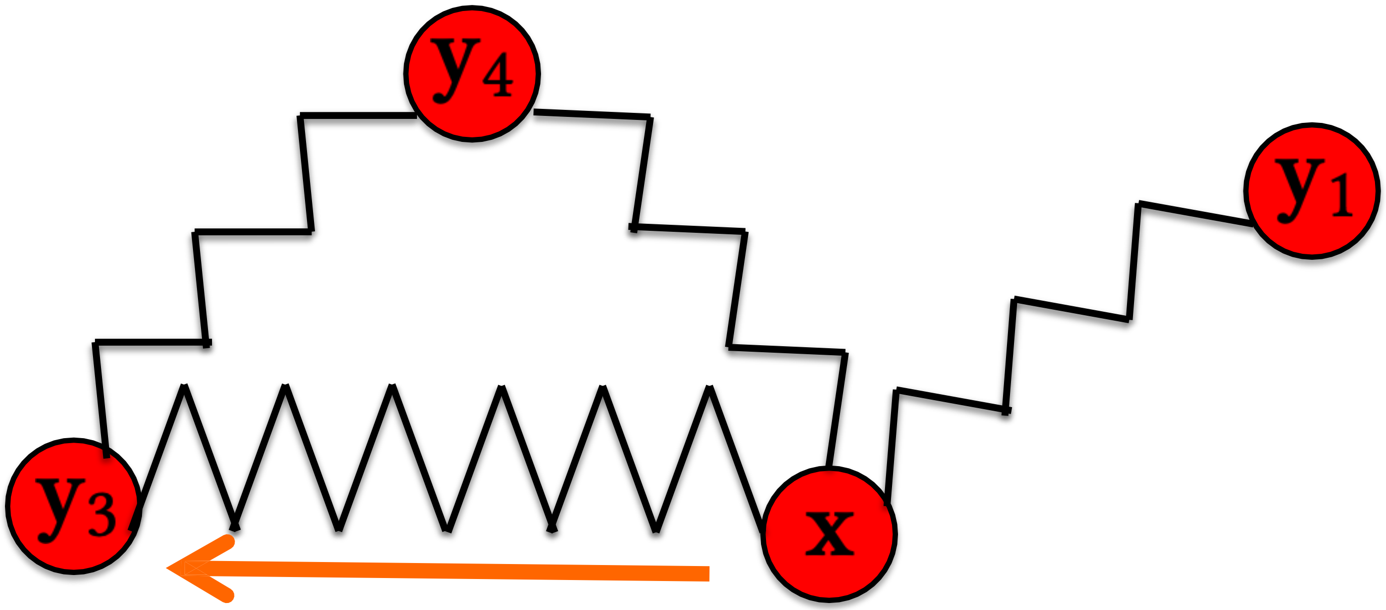


$$m_x \mathbf{a}_x = \sum_i \mathbf{f}_x(\mathbf{y}_i)$$

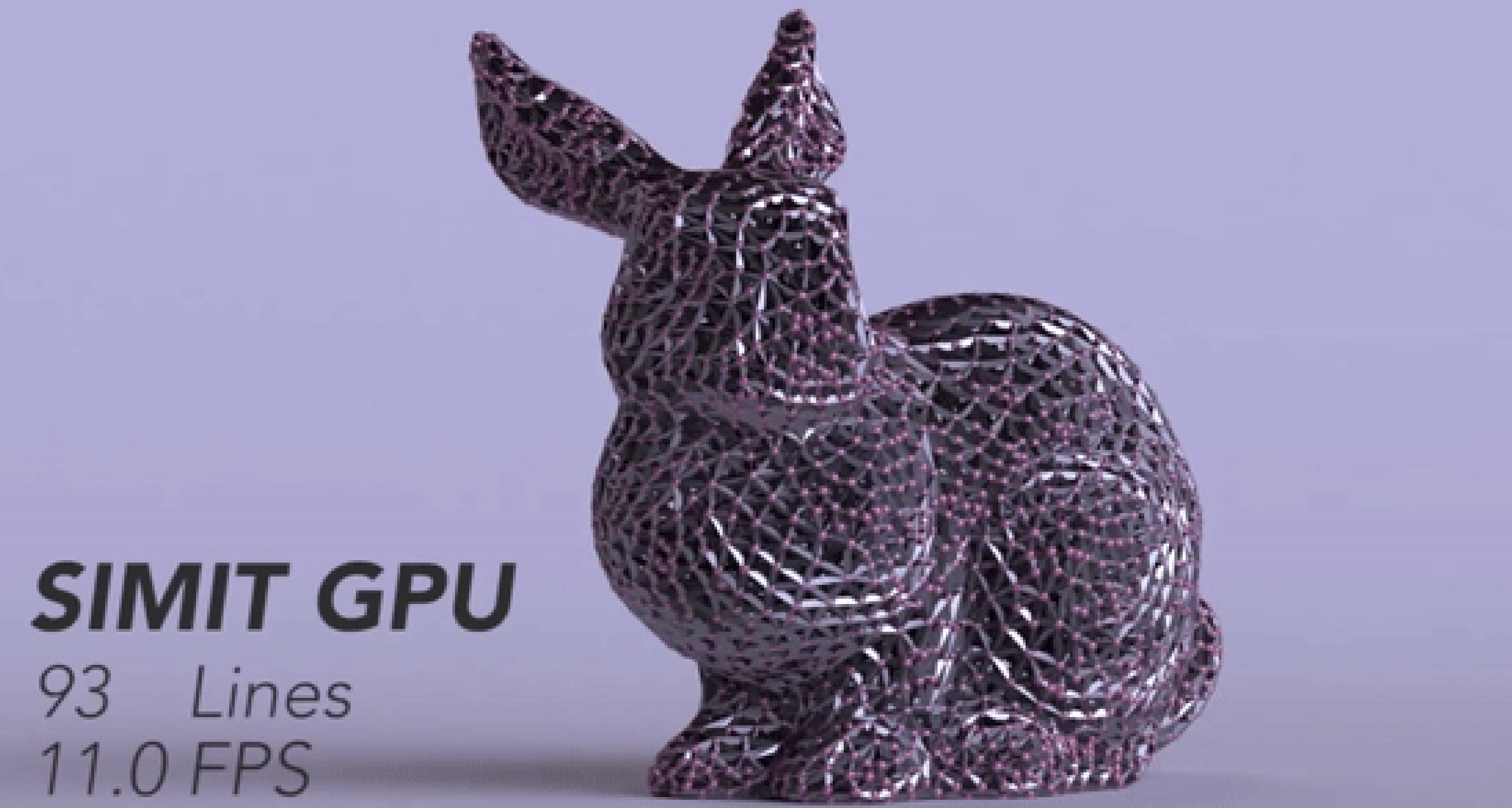
Newton's Second Law for Each Particle



$$m_x \mathbf{a}_x = \sum_i \mathbf{f}_x(\mathbf{y}_i)$$

One equation for each particle.
We will solve them all together.

https://www.youtube.com/watch?v=tFIG3Ygpews



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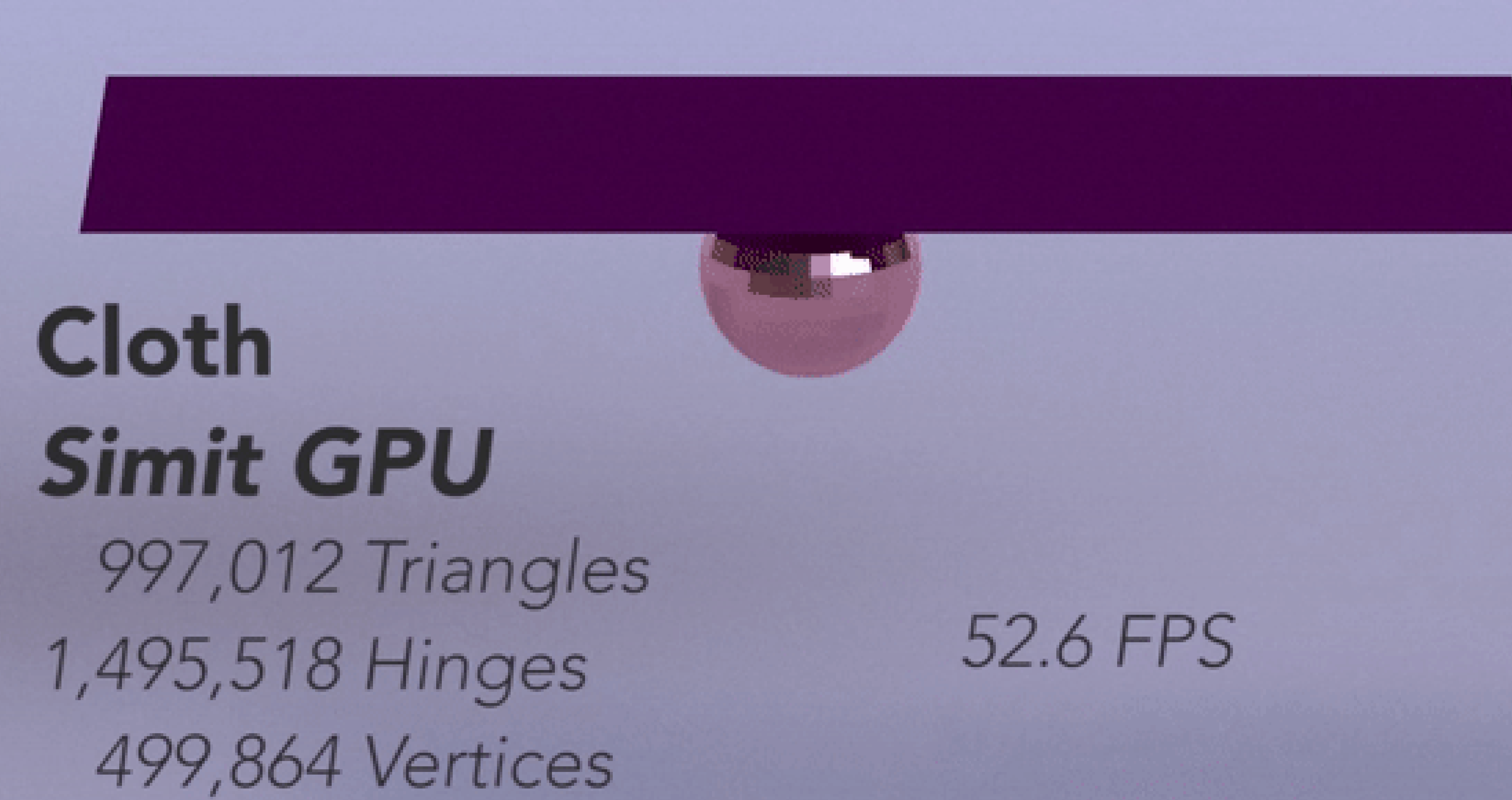
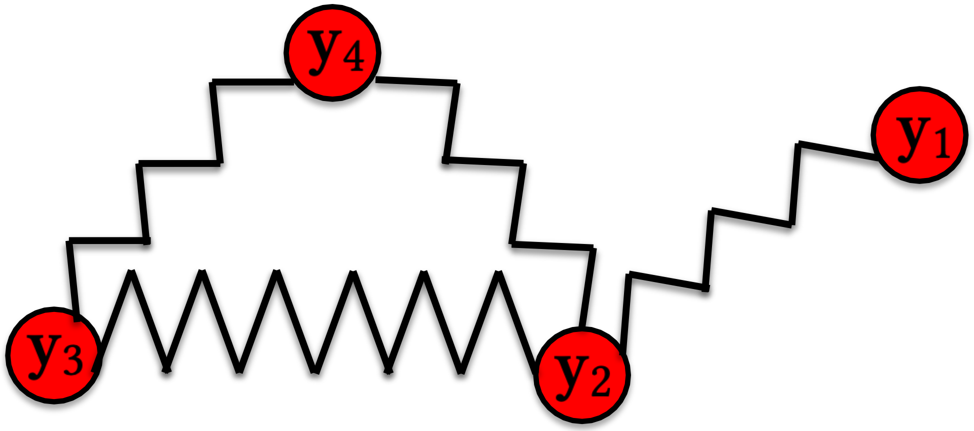


Newton's Second Law: System of Equations



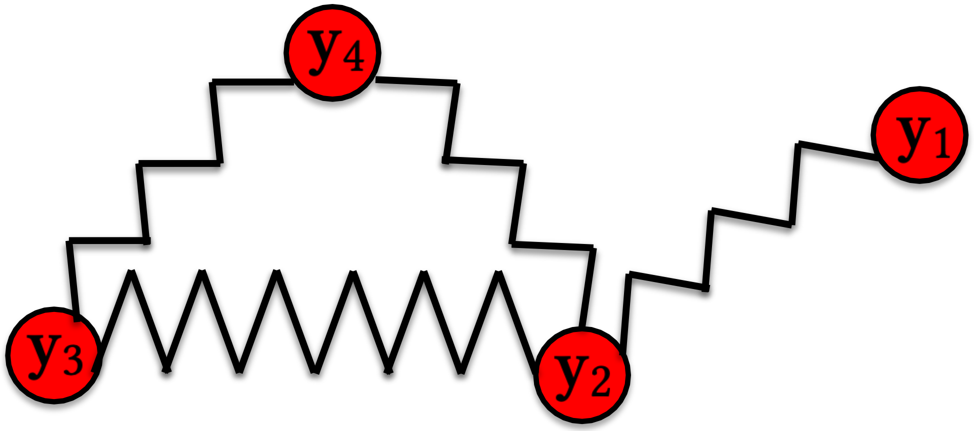
$$m_1 \mathbf{a}_1 = \sum_i \mathbf{f}_1(\mathbf{y}_i)$$

$$m_2 \mathbf{a}_2 = \sum_i \mathbf{f}_2(\mathbf{y}_i)$$

$$m_3 \mathbf{a}_3 = \sum_i \mathbf{f}_3(\mathbf{y}_i)$$

$$m_4 \mathbf{a}_4 = \sum_i \mathbf{f}_4(\mathbf{y}_i)$$

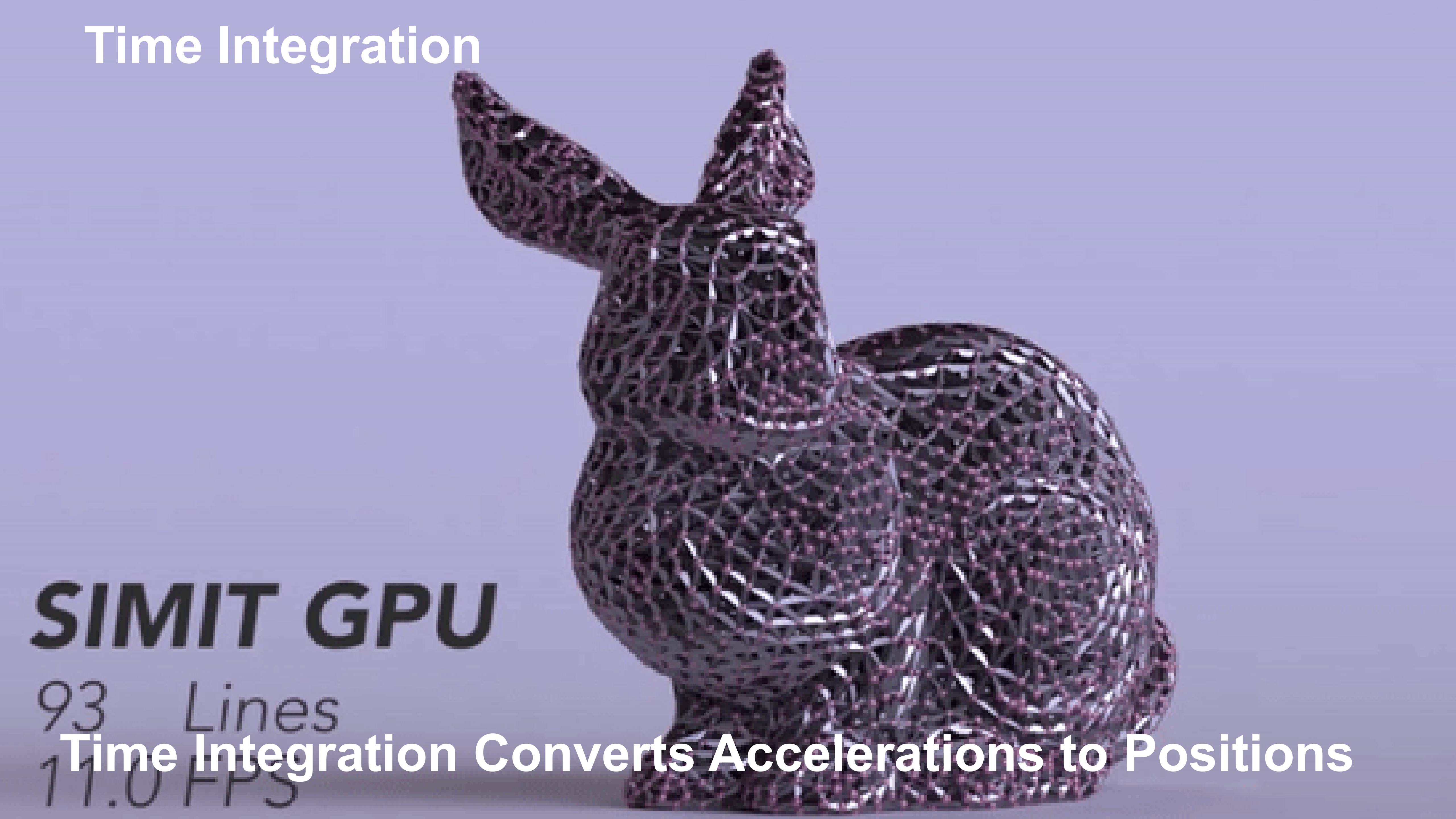
Newton's Second Law: System of Equations



$$\begin{pmatrix} m_1 \cdot I & 0 & 0 & 0 \\ 0 & m_2 \cdot I & 0 & 0 \\ 0 & 0 & m_3 \cdot I & 0 \\ 0 & 0 & 0 & m_4 \cdot I \end{pmatrix} \begin{pmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \mathbf{a}_3 \\ \mathbf{a}_4 \end{pmatrix} = \begin{pmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \mathbf{f}_3 \\ \mathbf{f}_4 \end{pmatrix}$$

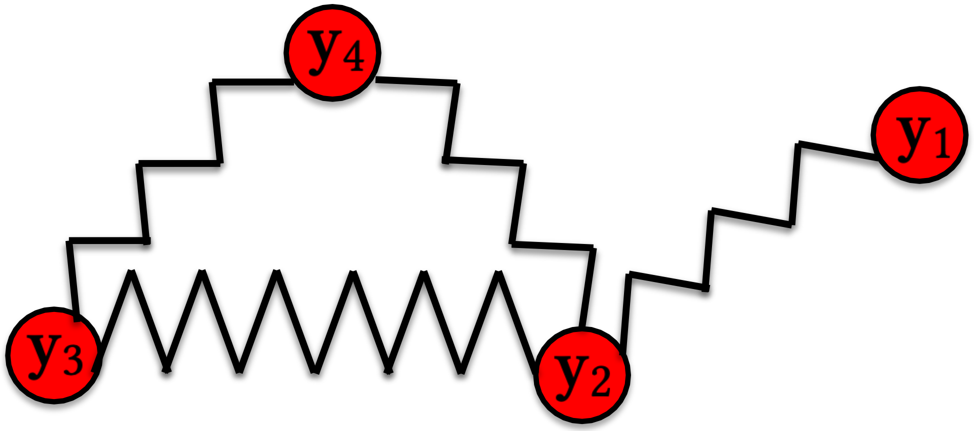
Mass Matrix

Time Integration



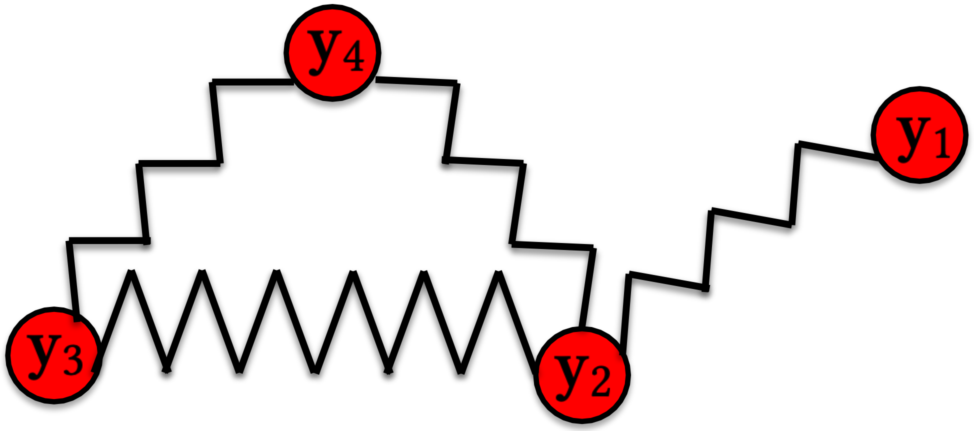
Time Integration Converts Accelerations to Positions

Newton's Second Law: System of Equations



$$\underbrace{\begin{pmatrix} m_1 \cdot I & 0 & 0 & 0 \\ 0 & m_2 \cdot I & 0 & 0 \\ 0 & 0 & m_3 \cdot I & 0 \\ 0 & 0 & 0 & m_4 \cdot I \end{pmatrix}}_{\text{Mass Matrix}} \underbrace{\begin{pmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \mathbf{a}_3 \\ \mathbf{a}_4 \end{pmatrix}}_{\mathbf{a}(t)} = \underbrace{\begin{pmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \mathbf{f}_3 \\ \mathbf{f}_4 \end{pmatrix}}_{\mathbf{f}(t)}$$

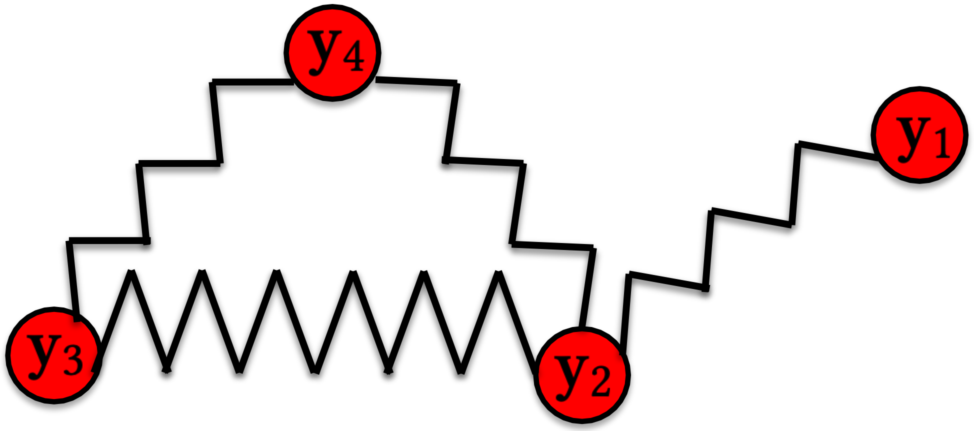
Time Integration



$$M\mathbf{a}(t) = \mathbf{f}(\mathbf{y}(t))$$

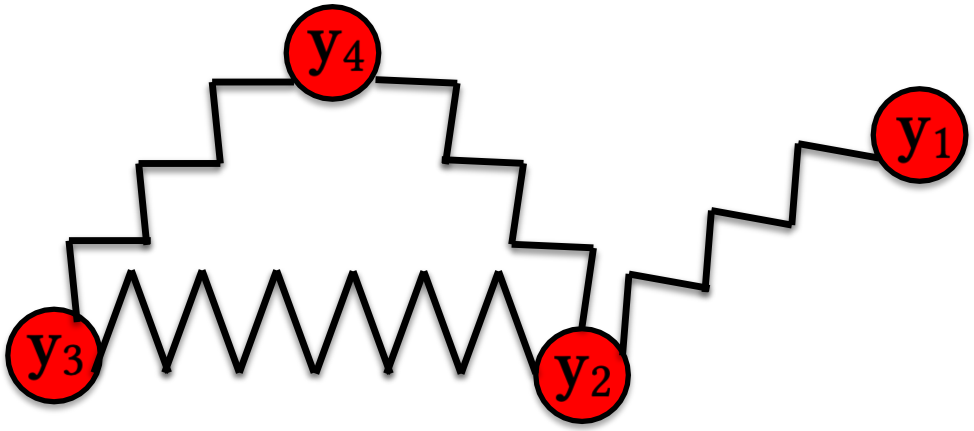
Mass Matrix

Time Integration



$$M\frac{d^2\mathbf{y}(t)}{dt^2} = \mathbf{f}(\mathbf{y}(t))$$

Time Integration



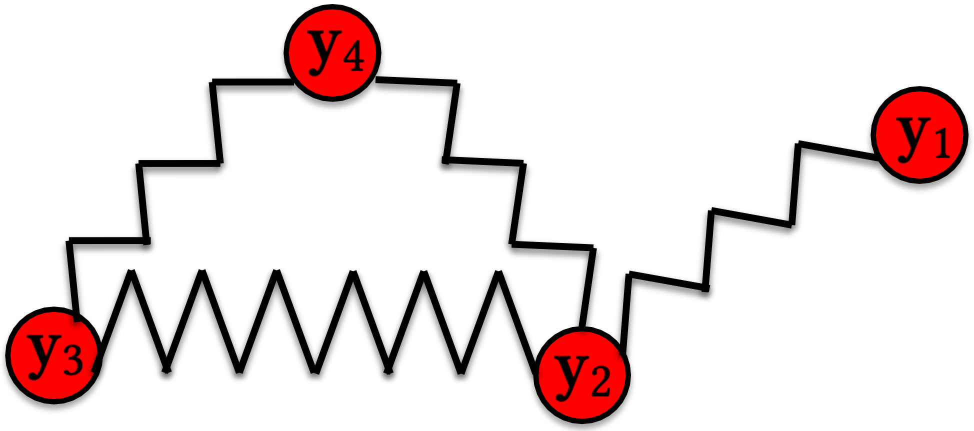
$$M\frac{d^2\mathbf{y}(t)}{dt^2} = \mathbf{f}(\mathbf{y}(t))$$

Use Finite Differences!

$$\frac{d^2\mathbf{y}(t)}{dt^2} \approx \frac{\mathbf{y}^{t+1} - 2\mathbf{y}^t + \mathbf{y}^{t-1}}{\Delta t^2}$$

https://en.wikipedia.org/wiki/Finite_difference#Higher-order_differences
https://en.wikipedia.org/wiki/Finite_difference_coefficient#Central_finite_difference

Time Integration



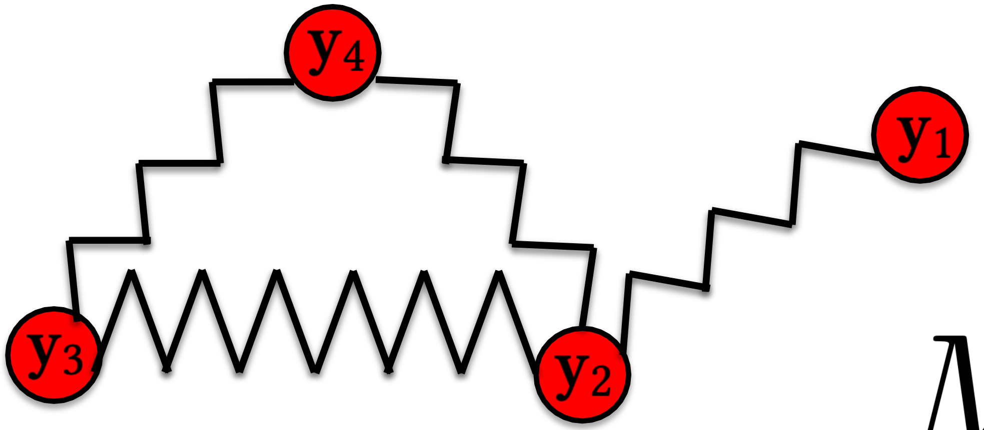
Need to Discretize!

$$M\frac{d^2\mathbf{y}(t)}{dt^2} = \boxed{\mathbf{f}(\mathbf{y}(t))}$$

Use Finite Differences!

$$\frac{d^2\mathbf{y}(t)}{dt^2} \approx \frac{\mathbf{y}^{t+1} - 2\mathbf{y}^{t} + \mathbf{y}^{t-1}}{\Delta t^2}$$

Implicit Time Integration

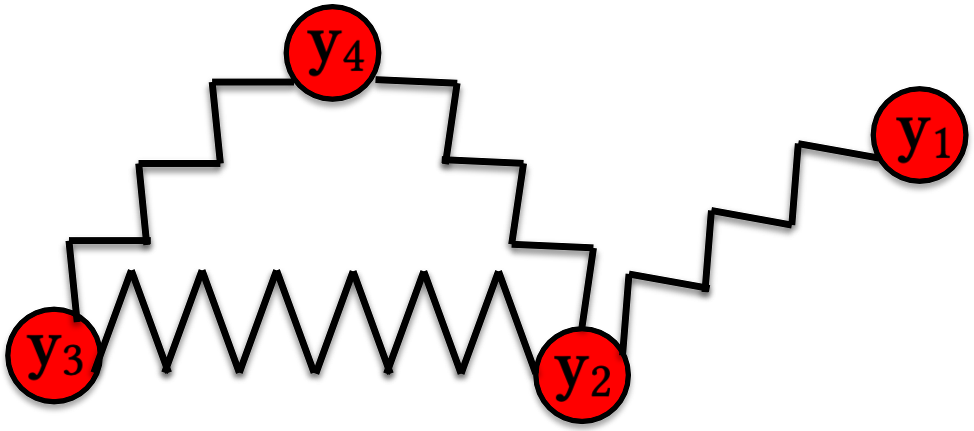


$$M\frac{d^2\mathbf{y}}{dt^2}(t) = \mathbf{f}\left(\mathbf{y}^{t+1}\right)$$

Use Finite Differences!

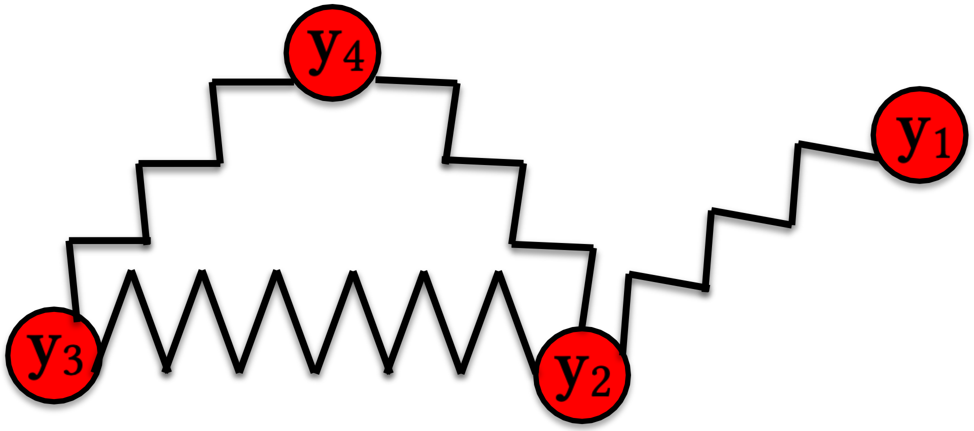
$$\frac{d^2\mathbf{y}(t)}{dt^2} \approx \frac{\mathbf{y}^{t+1} - 2\mathbf{y}^{t} + \mathbf{y}^{t-1}}{\Delta t^2}$$

Implicit Time Integration



$$M\left(\frac{\mathbf{y}^{t+1}-2\mathbf{y}^{t}+\mathbf{y}^{t-1}}{\Delta t^2}\right)=\mathbf{f}(\mathbf{y}^{t+1})$$

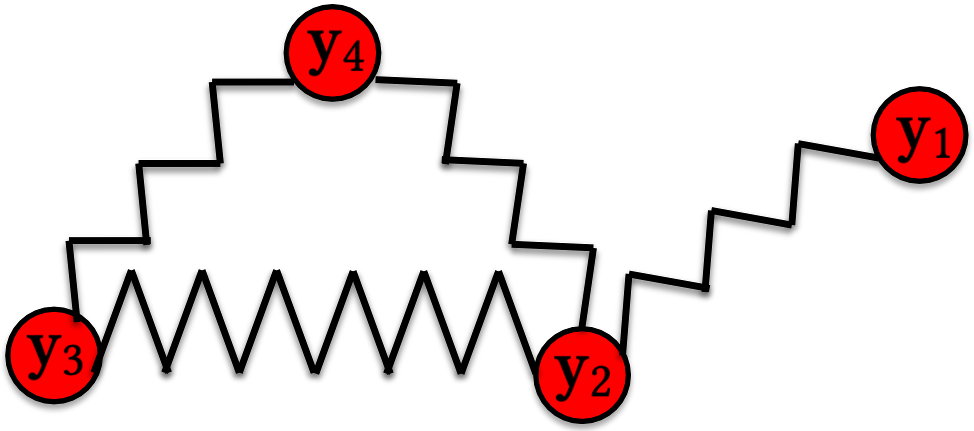
Implicit Time Integration



$$M\left(\frac{\mathbf{y}^{t+1}-2\mathbf{y}^{t}+\mathbf{y}^{t-1}}{\Delta t^2}\right)=\mathbf{f}(\mathbf{y}^{t+1})$$

$$M\mathbf{y}^{t+1}=M\left(2\mathbf{y}^{t}-\mathbf{y}^{t-1}\right)+\Delta t^2\mathbf{f}\left(\mathbf{y}^{t+1}\right)$$

Implicit Time Integration

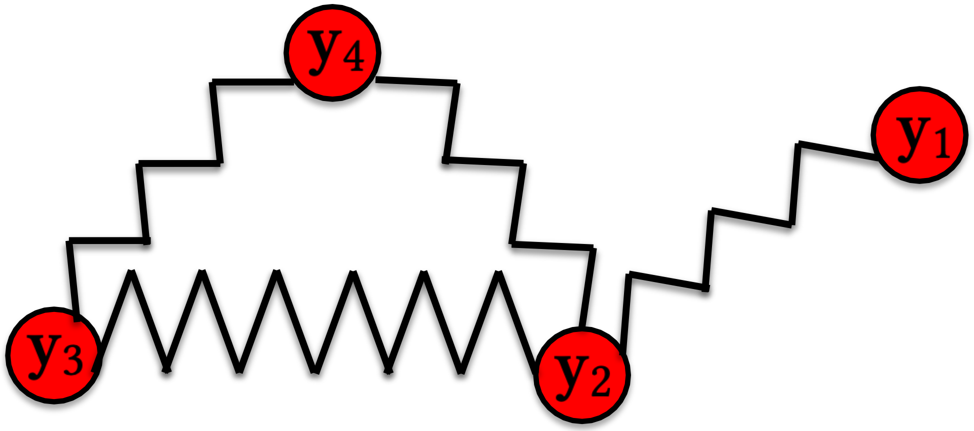


$$M\mathbf{y}^{t+1} = M\left(2\mathbf{y}^{t} - \mathbf{y}^{t-1}\right) + \Delta t^2 \mathbf{f}\left(\mathbf{y}^{t+1}\right)$$

Goal: Solve for $\mathbf{y}^{t+1}$

https://en.wikipedia.org/wiki/Explicit_and_implicit_methods
https://www.flow3d.com/resources/cfd-101/numerical-issues/implicit-versus-explicit-numerical-methods/

Implicit Time Integration



$$M\mathbf{y}^{t+1} - M(2\mathbf{y}^t - \mathbf{y}^{t-1}) - \Delta t^2 \mathbf{f}(\mathbf{y}^{t+1}) = 0$$

How to find when some equation = 0?

Goal: Solve for $\mathbf{y}^{t+1}$

Implicit Integration as Optimization

If we can find a function E(*q*) such that:

$$\nabla_{\mathbf{q}} E\left(\mathbf{y}^{t+1}\right) = 0$$

Implicit Integration as Optimization

If we can find a function E(*q*) such that:

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then, rather than solve

$$M\mathbf{y}^{t+1} - M(2\mathbf{y}^{t} - \mathbf{y}^{t-1}) - \Delta t^{2}\mathbf{f}(\mathbf{y}^{t+1}) = 0$$

Implicit Integration as Optimization

If we can find a function E(*q*) such that:

$$\nabla_{\mathbf{q}} E\left(\mathbf{y}^{t+1}\right) = 0$$

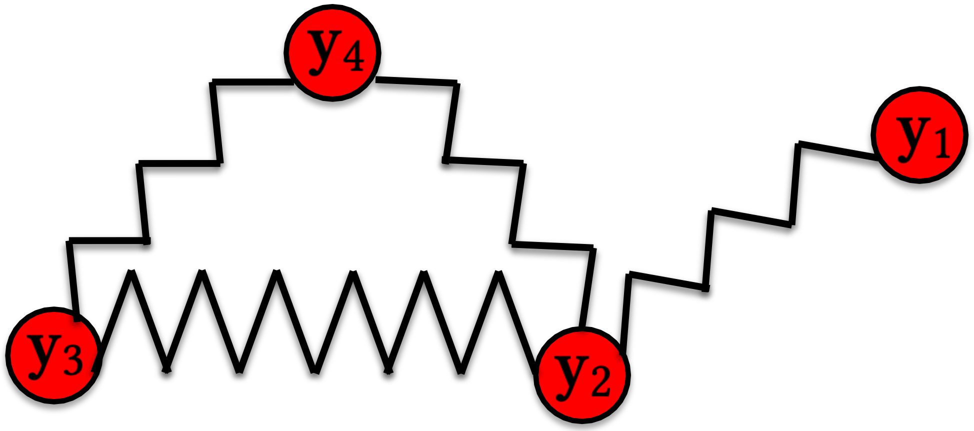
then, rather than solve

$$M\mathbf{y}^{t+1} - M(2\mathbf{y}^{t} - \mathbf{y}^{t-1}) - \Delta t^2 \mathbf{f}(\mathbf{y}^{t+1}) = 0$$

we can solve instead

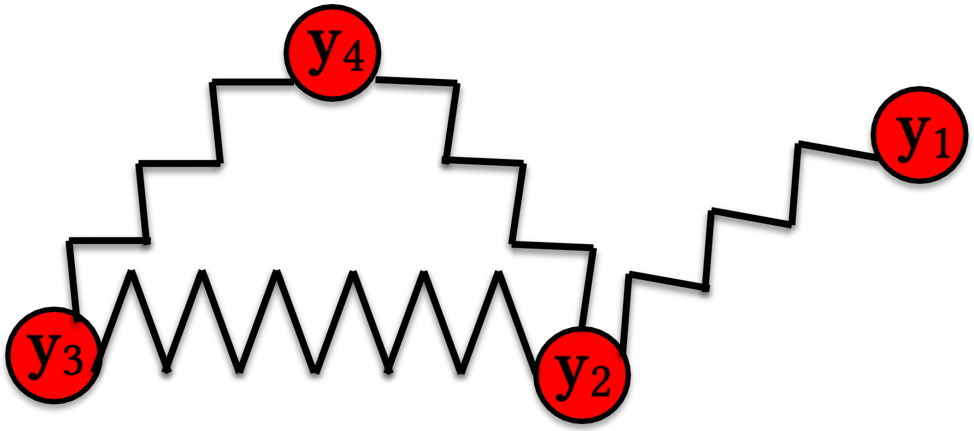
$$\mathbf{y}^{t+1} = \arg\min_{\mathbf{q}} E\left(\mathbf{q}\right)$$

Implicit Integration as Optimization



$$\underbrace{M\mathbf{y}^{t+1} - M(2\mathbf{y}^{t} - \mathbf{y}^{t-1})}_{\text{find } \mathbf{E}_1(\mathbf{y}^{t+1})} - \Delta t^2 \underbrace{\mathbf{f}(\mathbf{y}^{t+1})}_{\text{find } \mathbf{E}_2(\mathbf{y}^{t+1})} = 0$$

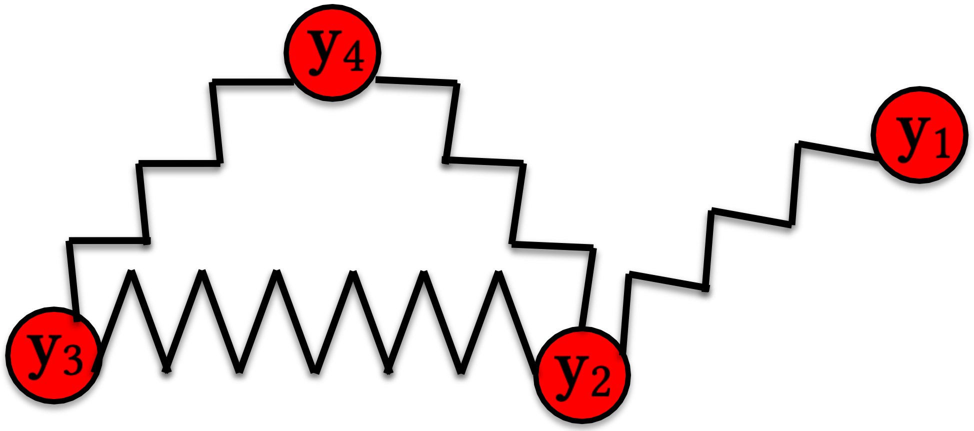
Implicit Integration as Optimization: E_1



$$\mathbf{E}_1\left(\mathbf{y}^{t+1}\right) = \frac{1}{2}\left(\mathbf{y}^{t+1}\right)^T M\mathbf{y}^{t+1} - \left(\mathbf{y}^{t+1}\right)^T M\mathbf{b}$$

$$\mathbf{b} = 2\mathbf{y}^t - \mathbf{y}^{t-1}$$

Implicit Integration as Optimization



$$\underbrace{M\mathbf{y}^{t+1} - M(2\mathbf{y}^t - \mathbf{y}^{t-1})}_{\text{find } \mathbf{E}_1\left(\mathbf{y}^{t+1}\right)} - \Delta t^2 \underbrace{\mathbf{f}(\mathbf{y}^{t+1})}_{\text{find } \mathbf{E}_2\left(\mathbf{y}^{t+1}\right)} = 0$$

Potential energy : E_2

We are going to introduce a special type of energy called potential energy

If $\mathbf{E}_2$(q) is a potential energy then

$$\nabla_{\mathbf{q}} E_2 = -\mathbf{f}\left(\mathbf{q}\right)$$

Done for Today