Meshes: Part 2

Some Slides/Images adapted from Marschner and Shirley and David Levin

Announcements

- Assignment 5 due Sunday (21/06)
- Office Hours immediately following class for A5
  - TAs will be active on Piazza
- Mid-course evaluations
- No class until 6 July
  - Midterm study guide coming next week

Meshes

Last week:

Types of Surfaces

Triangles

Data Structures for Triangle Meshes

Normals for Meshes

Today:

Review + our data structure

Normals for Meshes

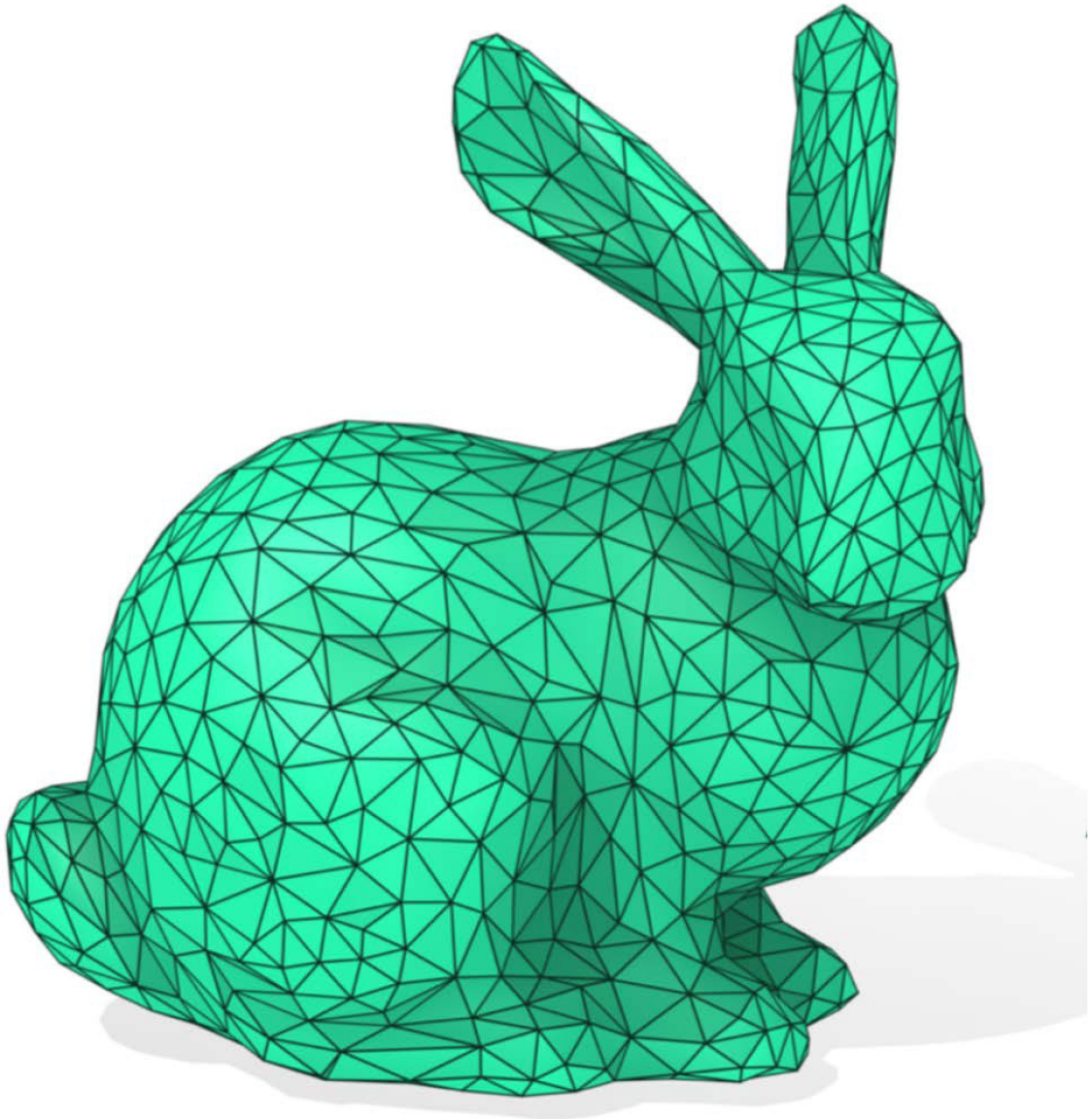
Quad Meshes

Texture Mapping

Subdivision Surfaces

Any Questions?

What makes a “good” mesh?



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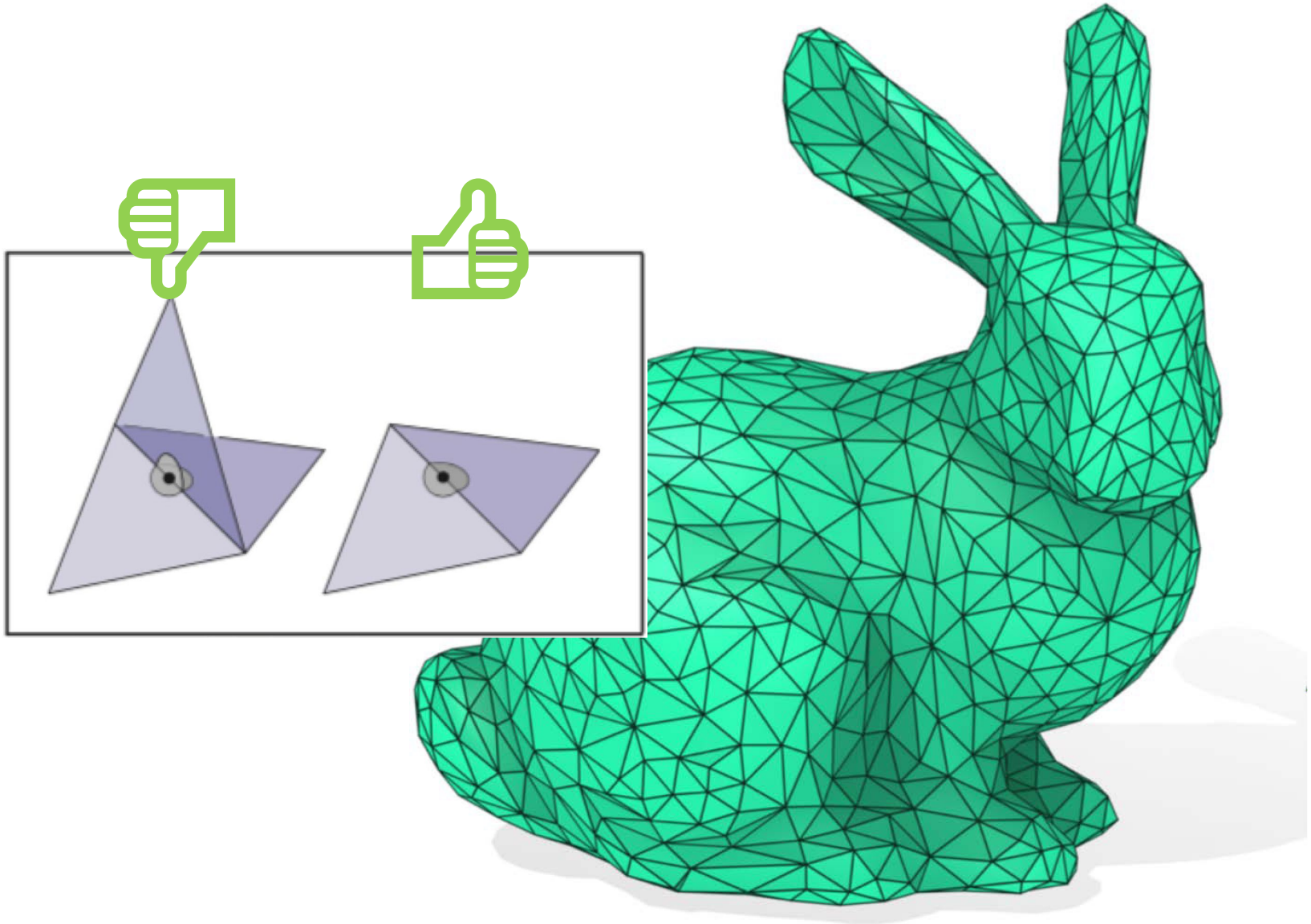

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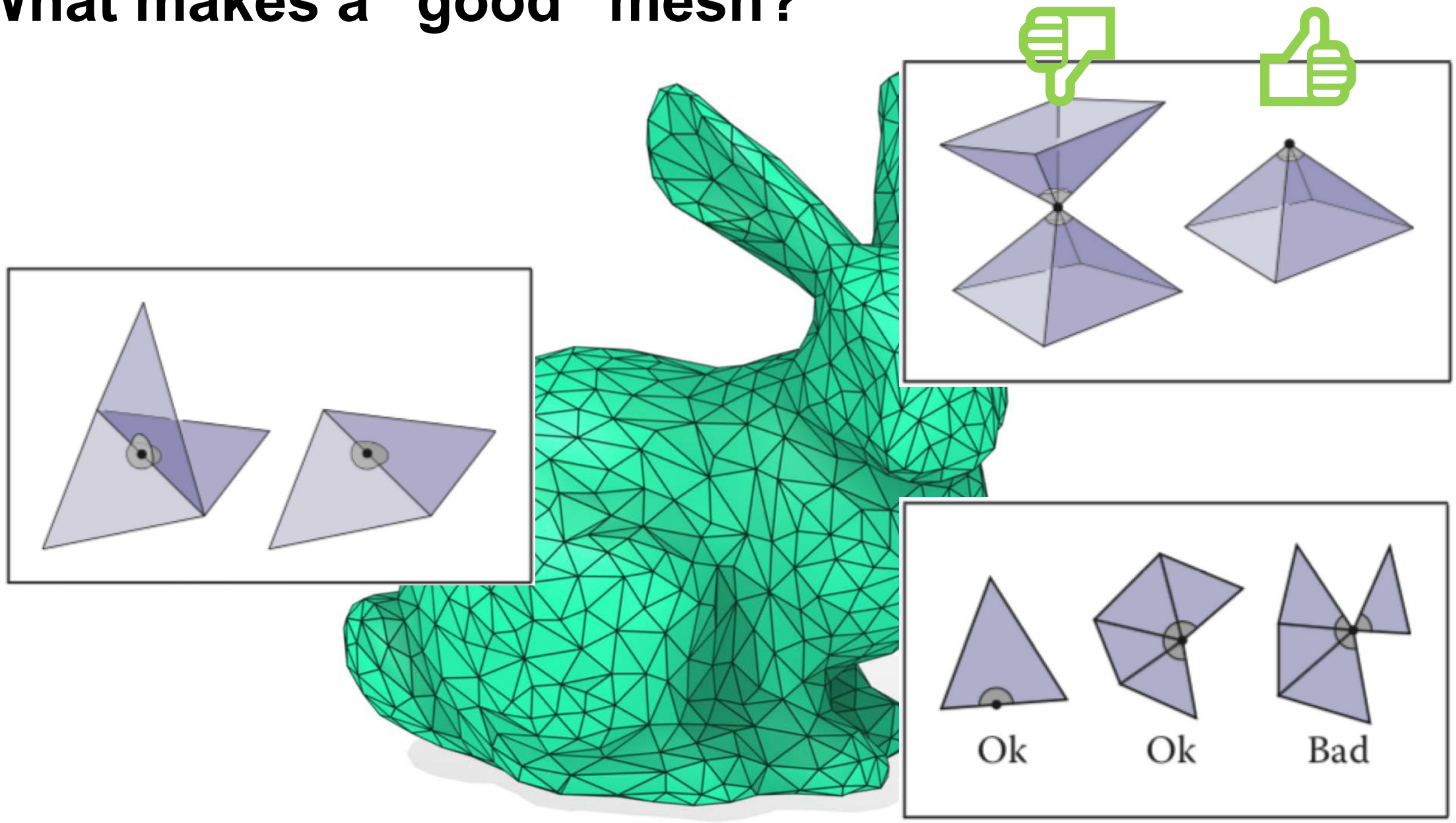
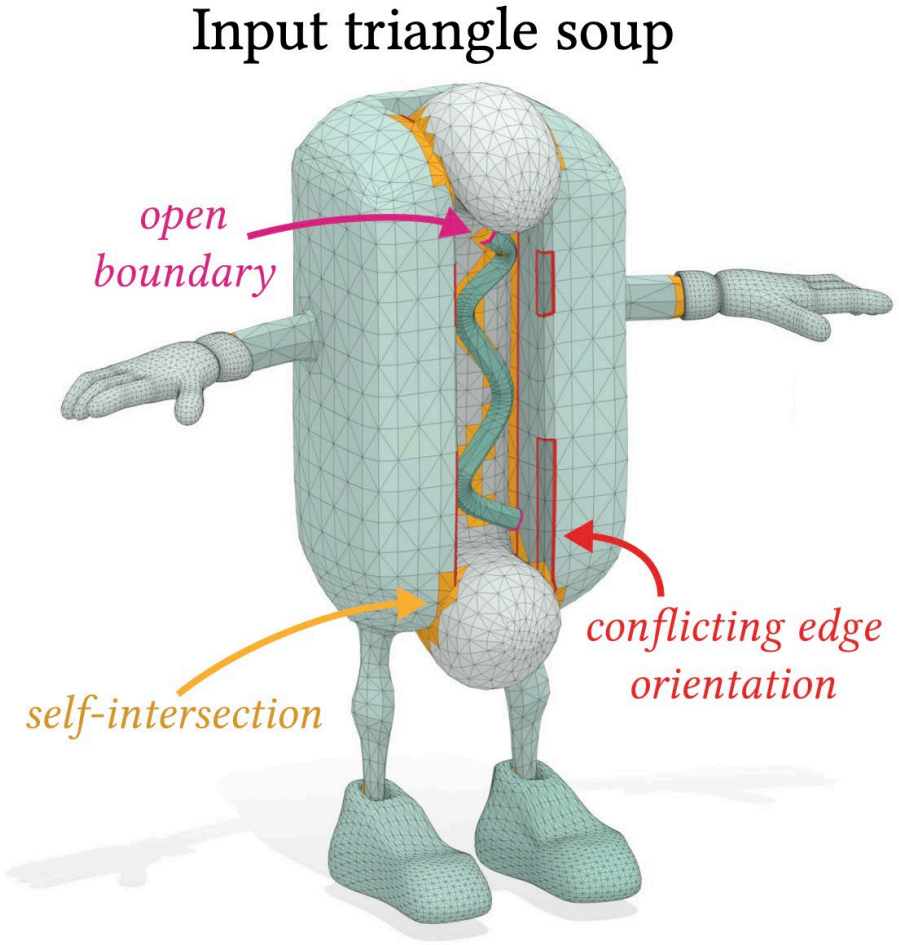


Watertight

Watertight meshes have no holes



https://www.dgp.toronto.edu/projects/fast-winding-numbers/

Manifold

A *2-manifold* is a surface for which the neighbourhood around any point can be flattened onto the plane

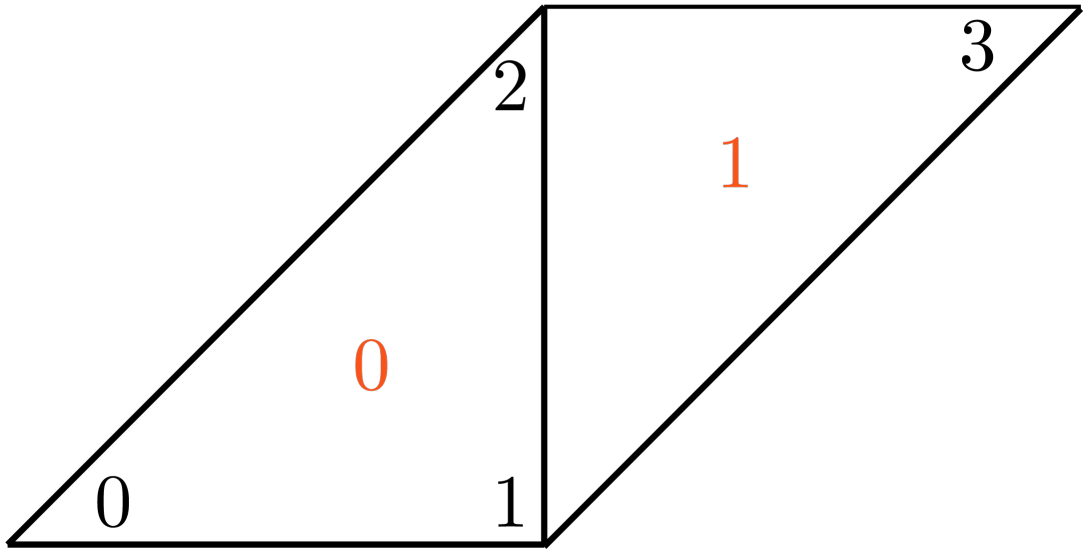
Manifold with boundaries

https://www.dgp.toronto.edu/projects/cubic-stylization/

Storing Triangle Meshes

What do we care about ?

$$V = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{pmatrix} \qquad F = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 3 & 2 \end{pmatrix}$$



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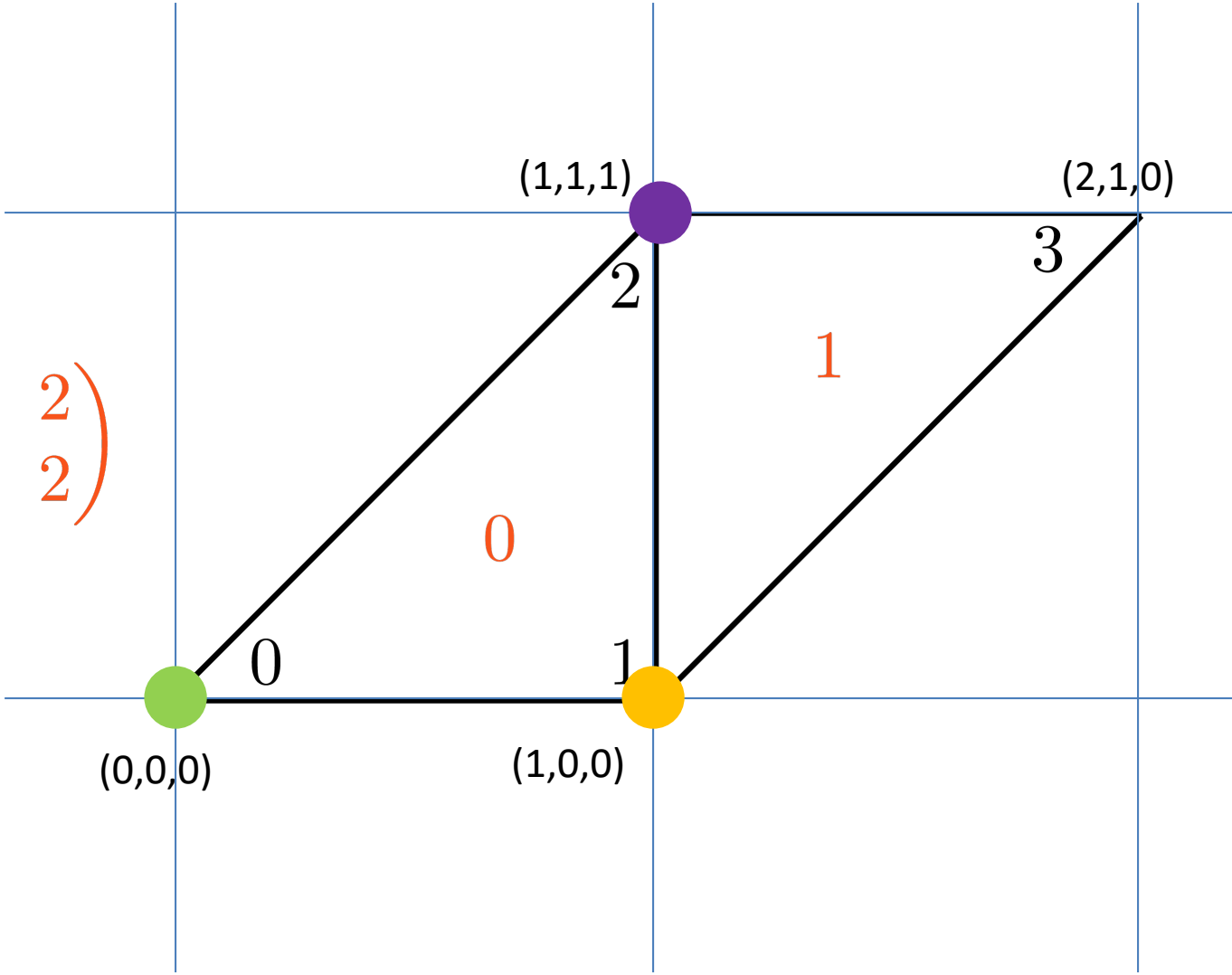

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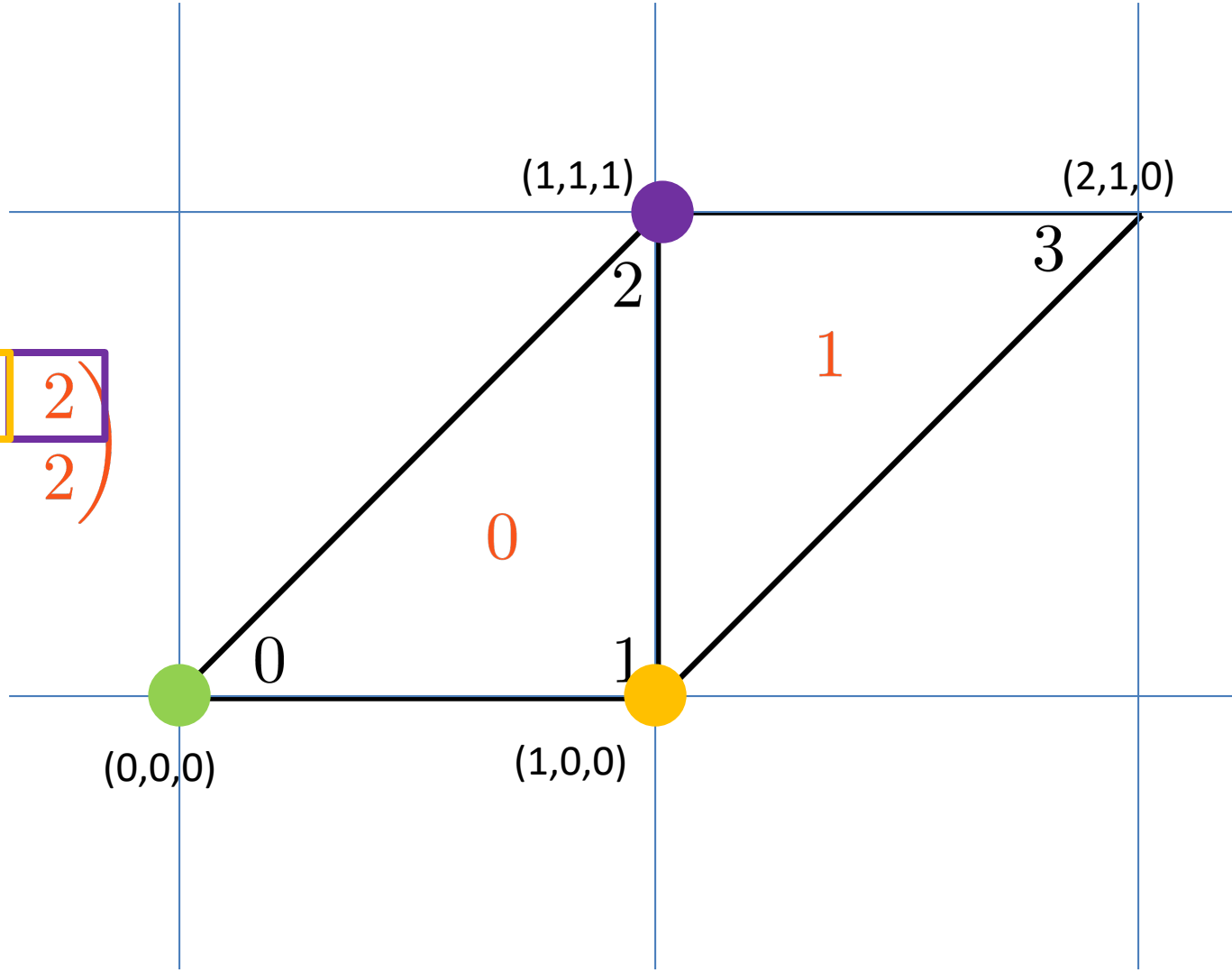
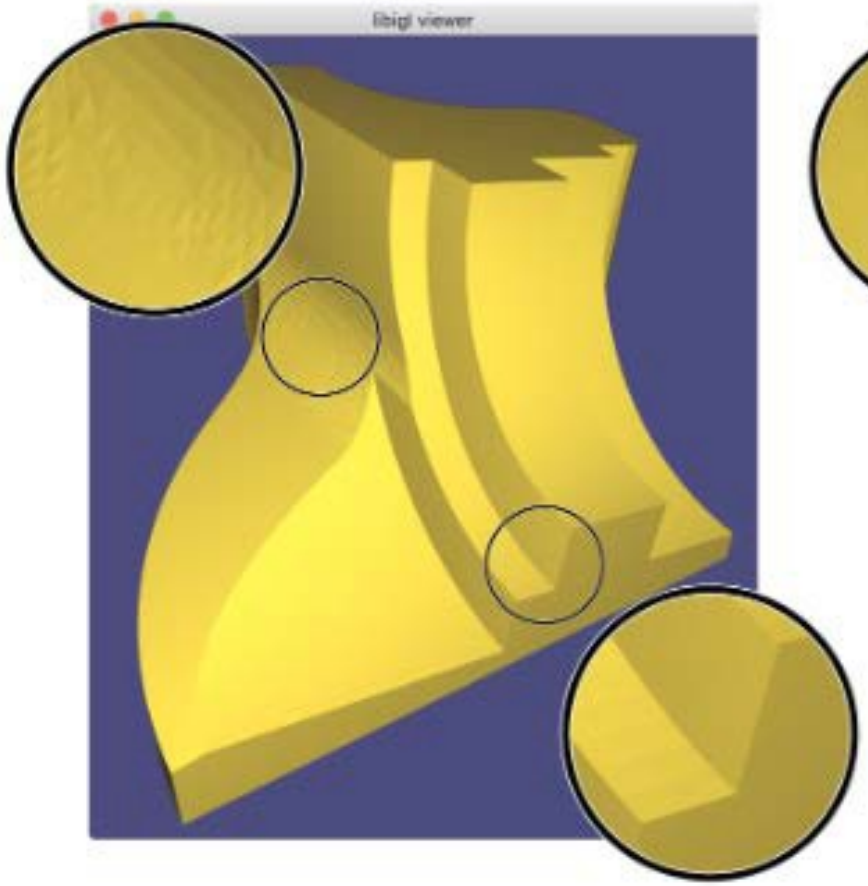
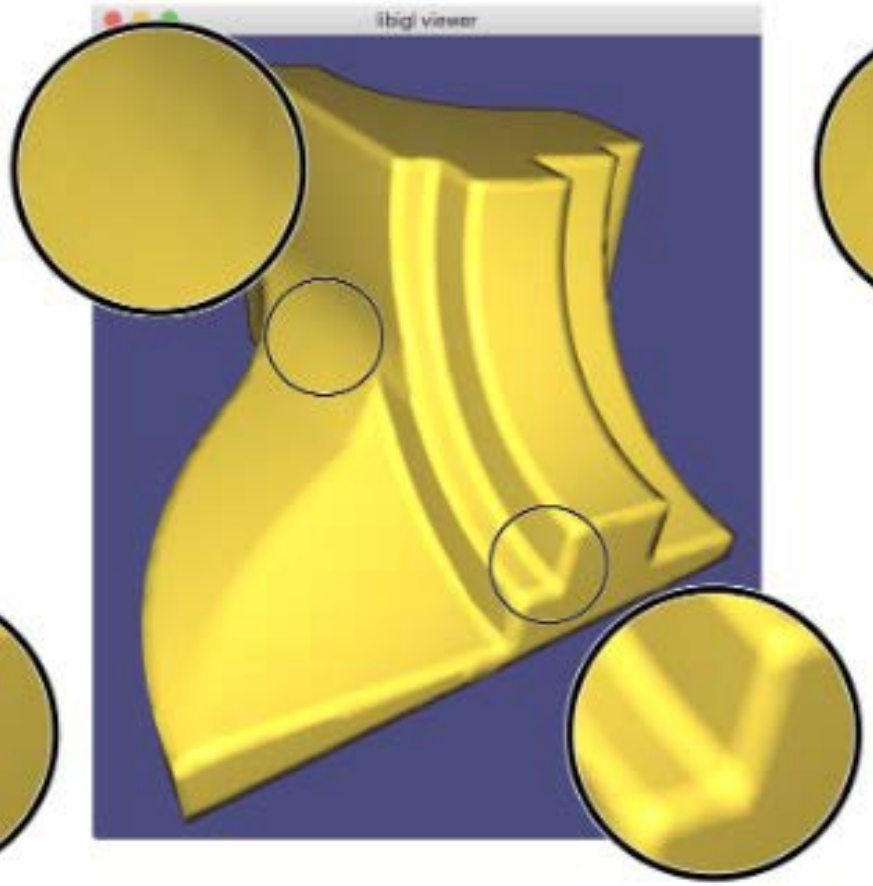


Per-Vertex Normals

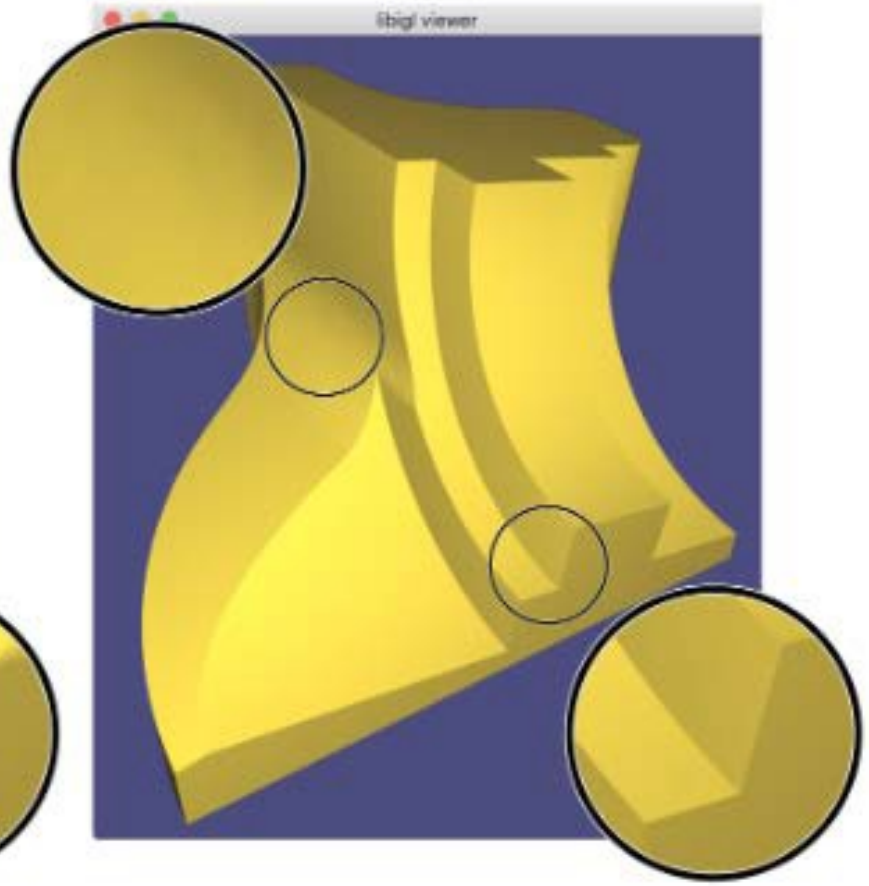
Per-Face Normals



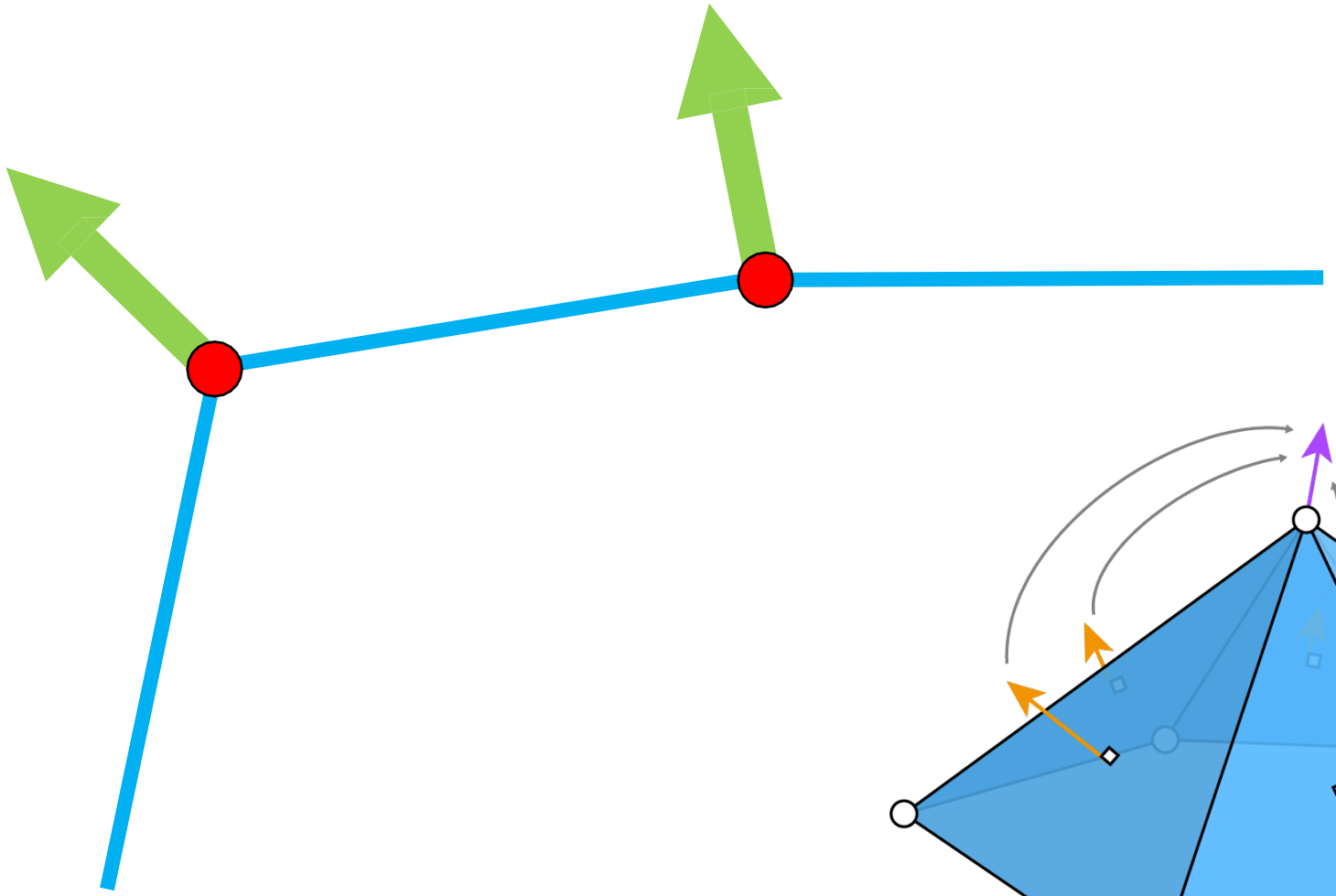
Per-Vertex Normals

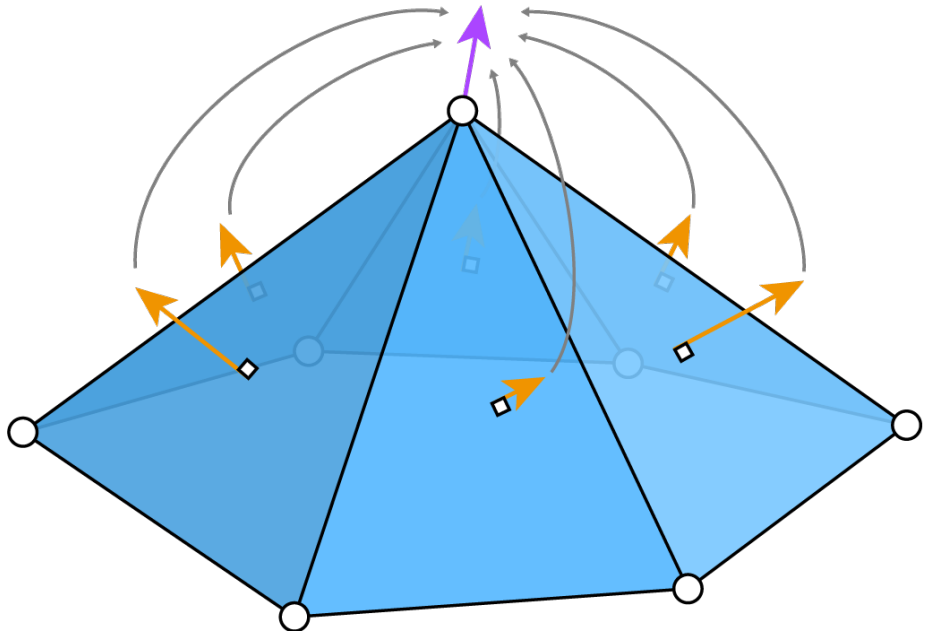


Per-Corner Normals



Per-Vertex Normals





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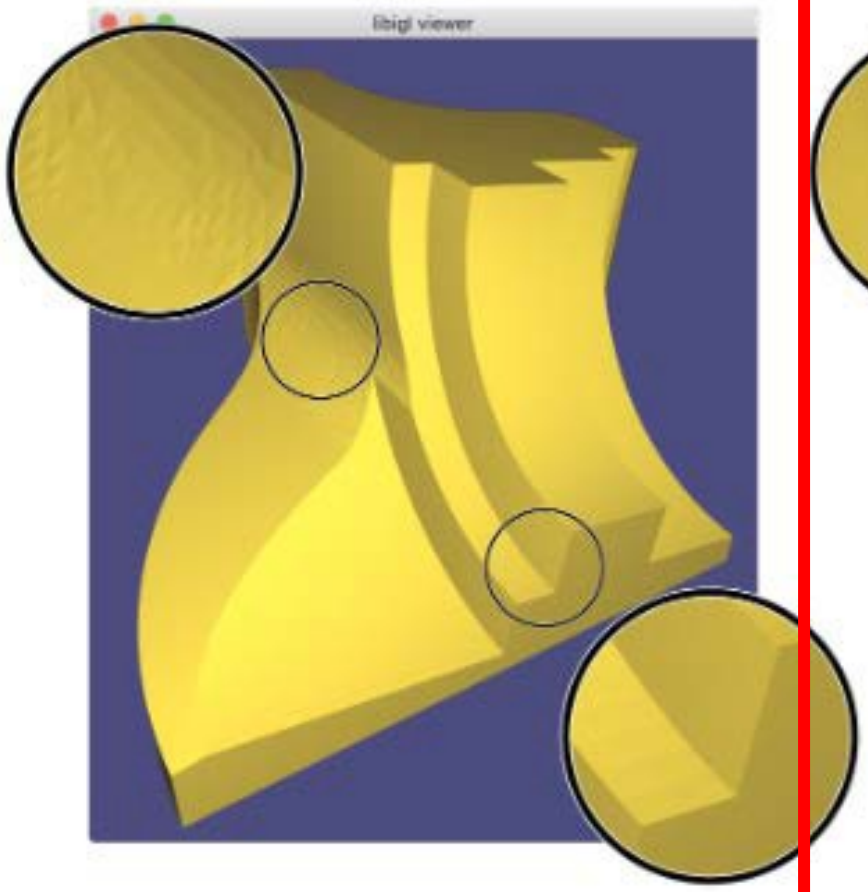

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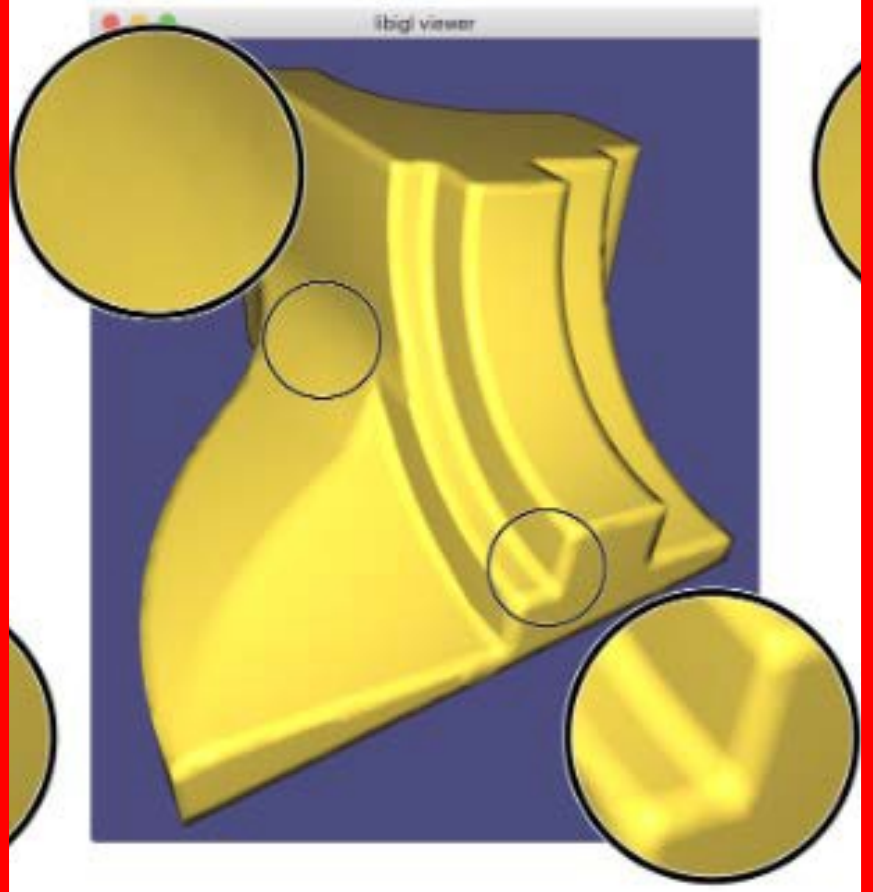

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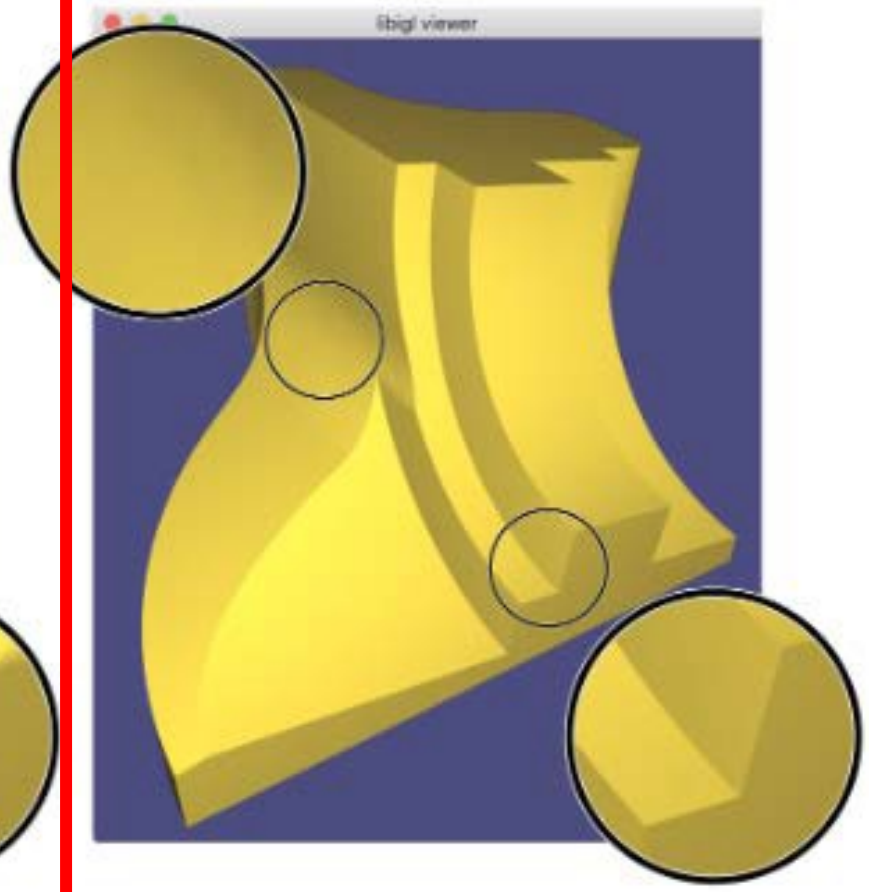

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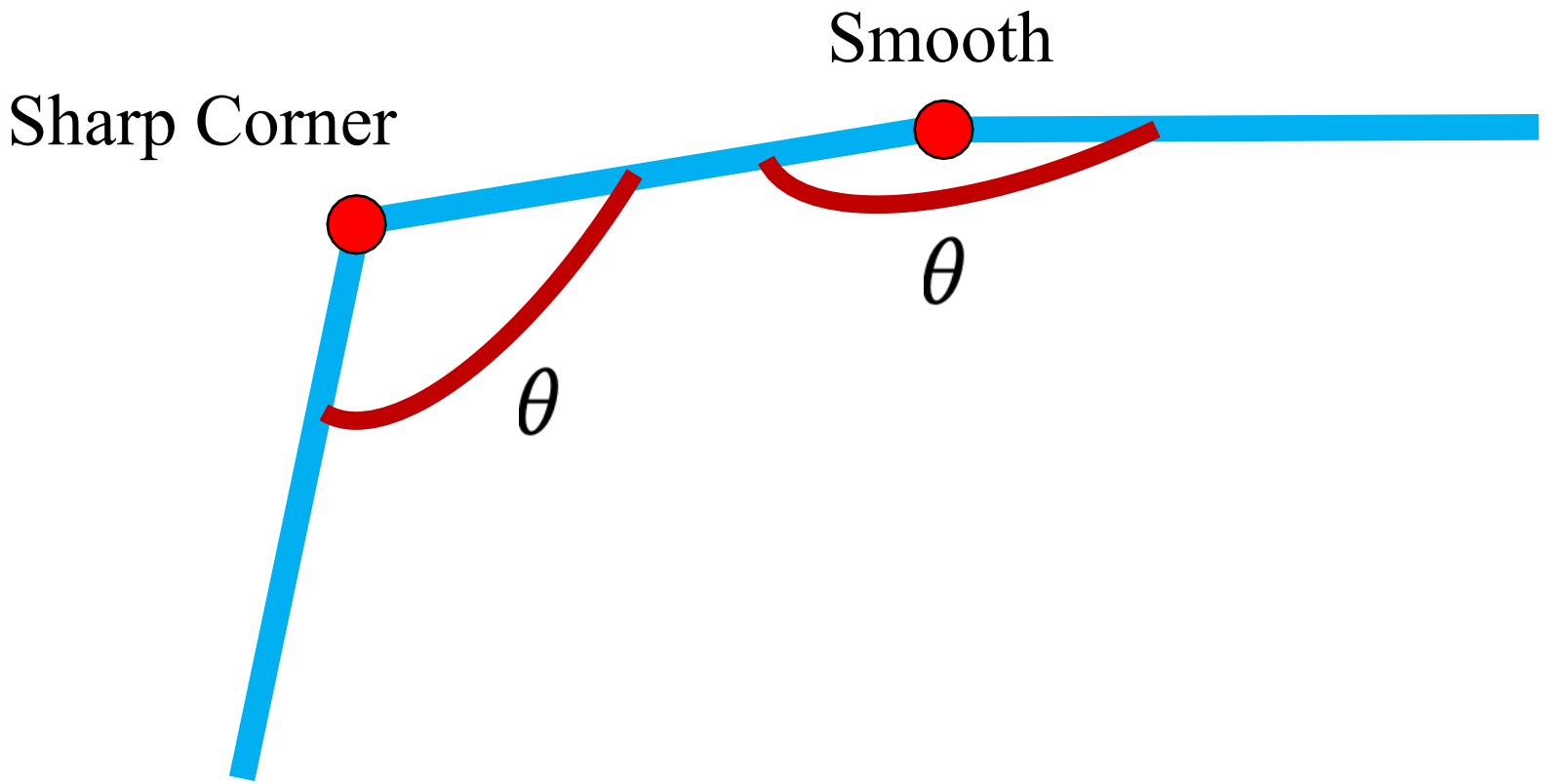

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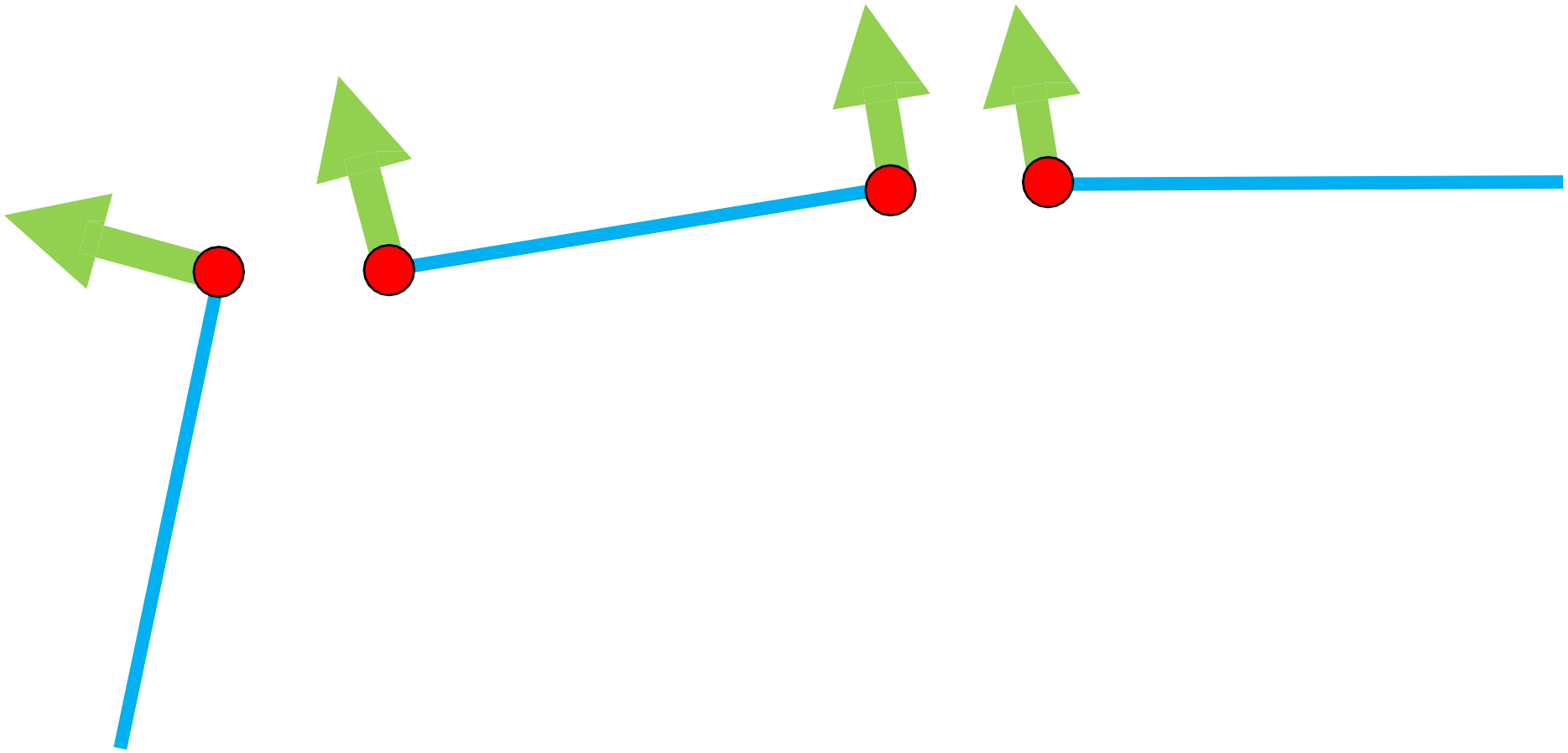

Per-Vertex Normals

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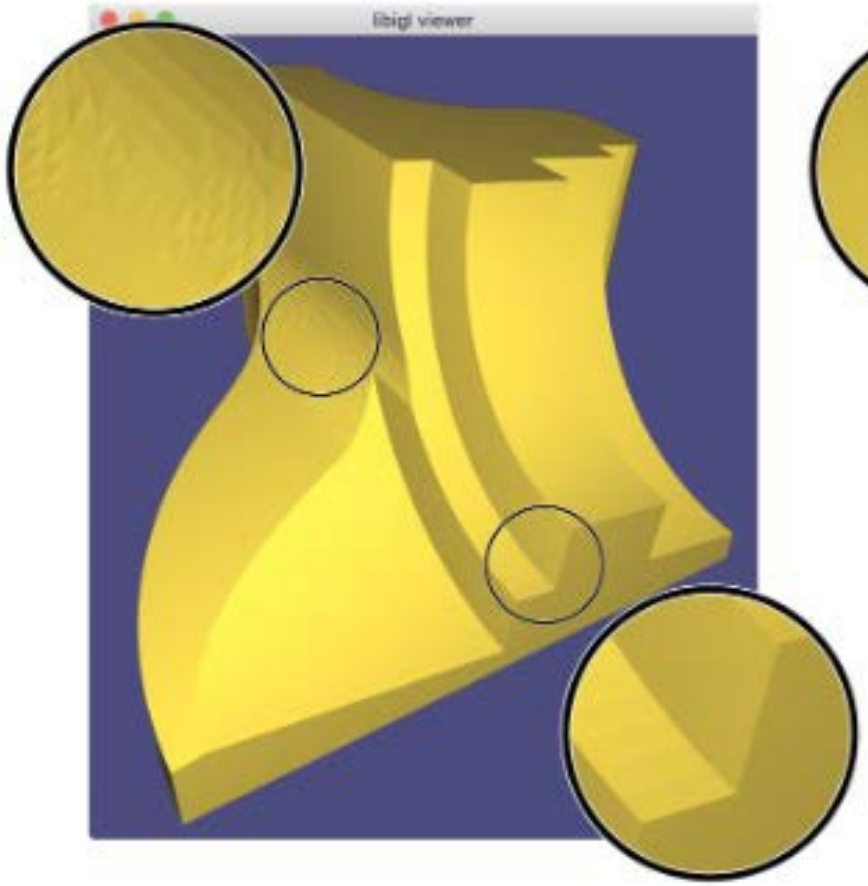

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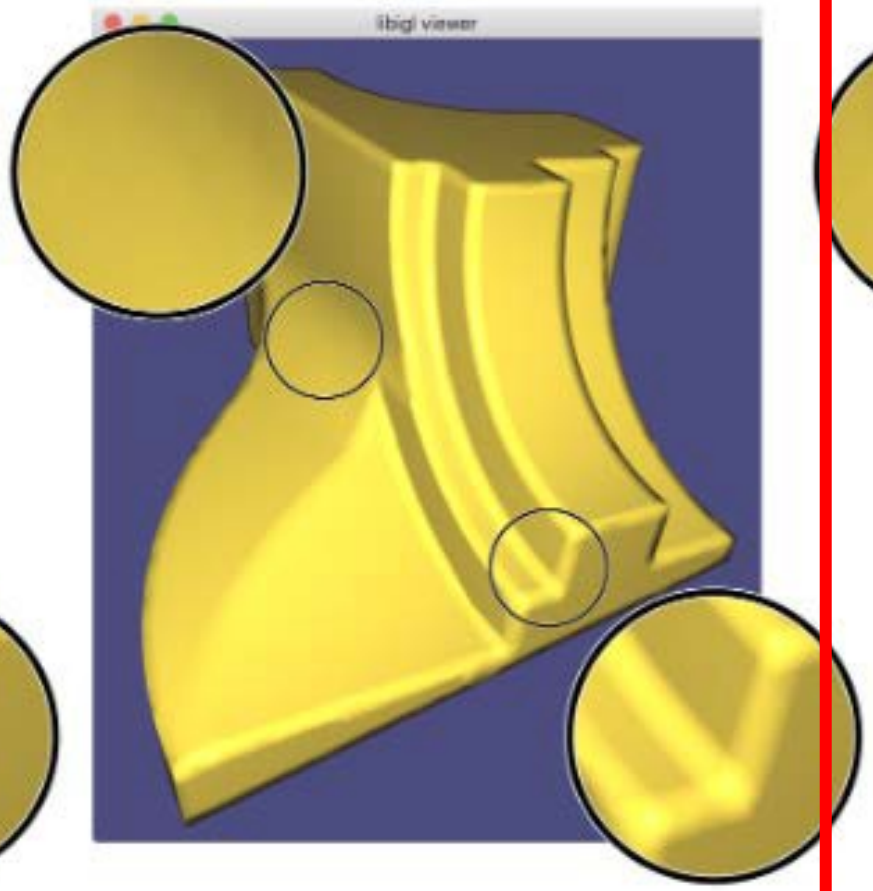

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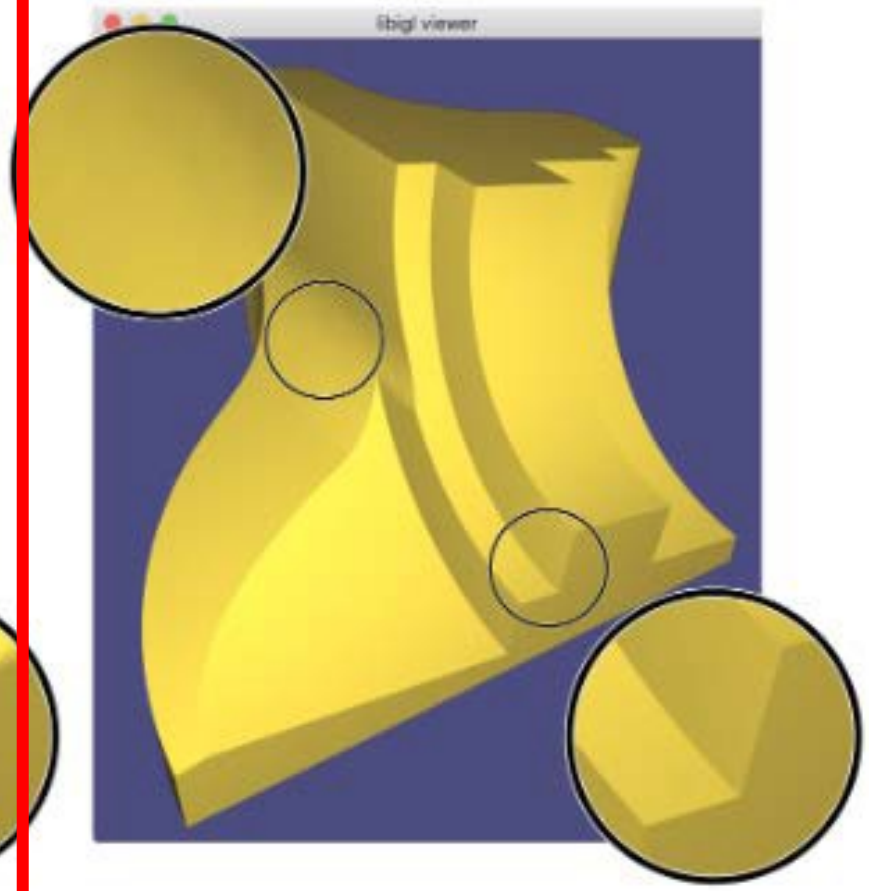

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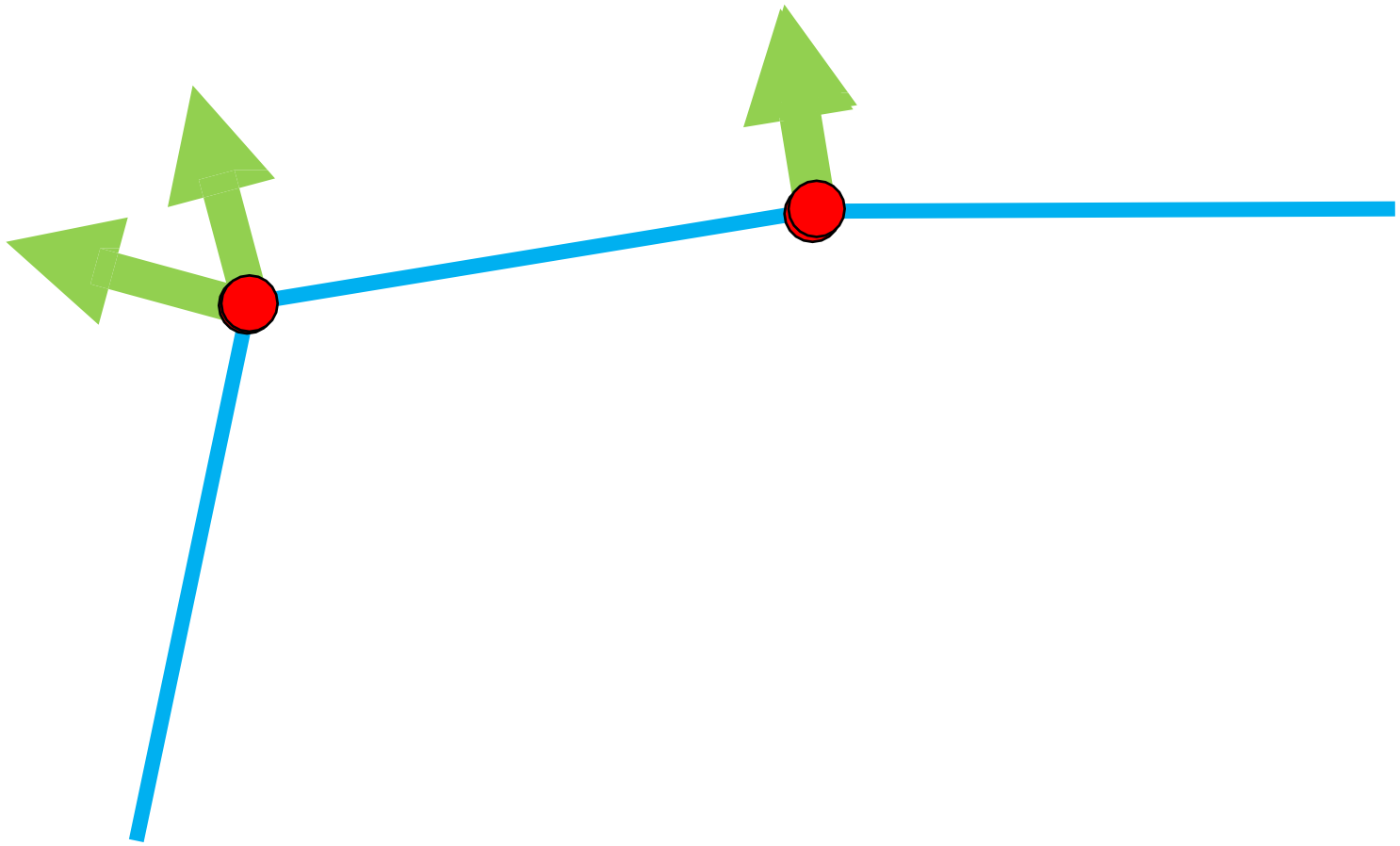
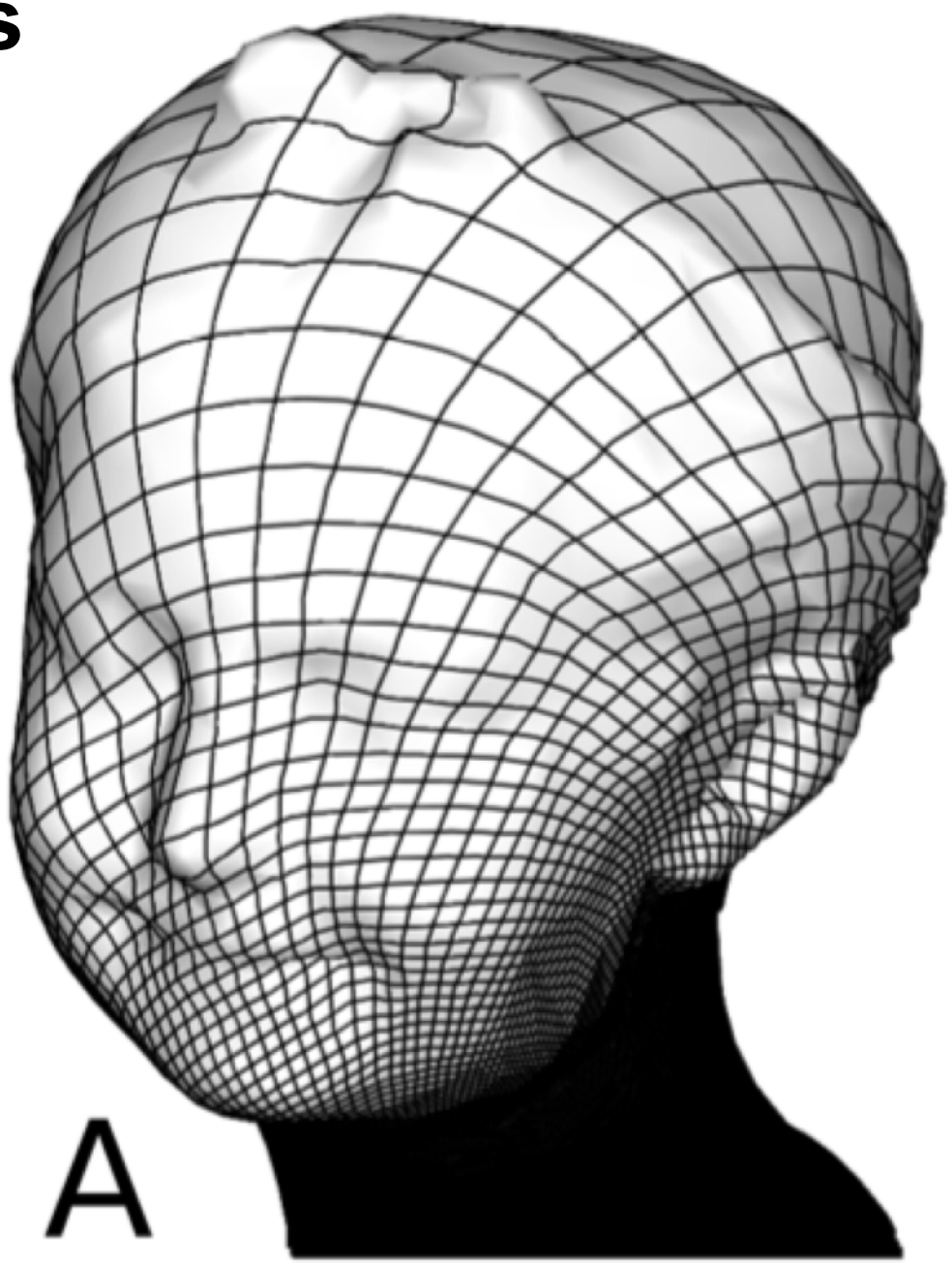
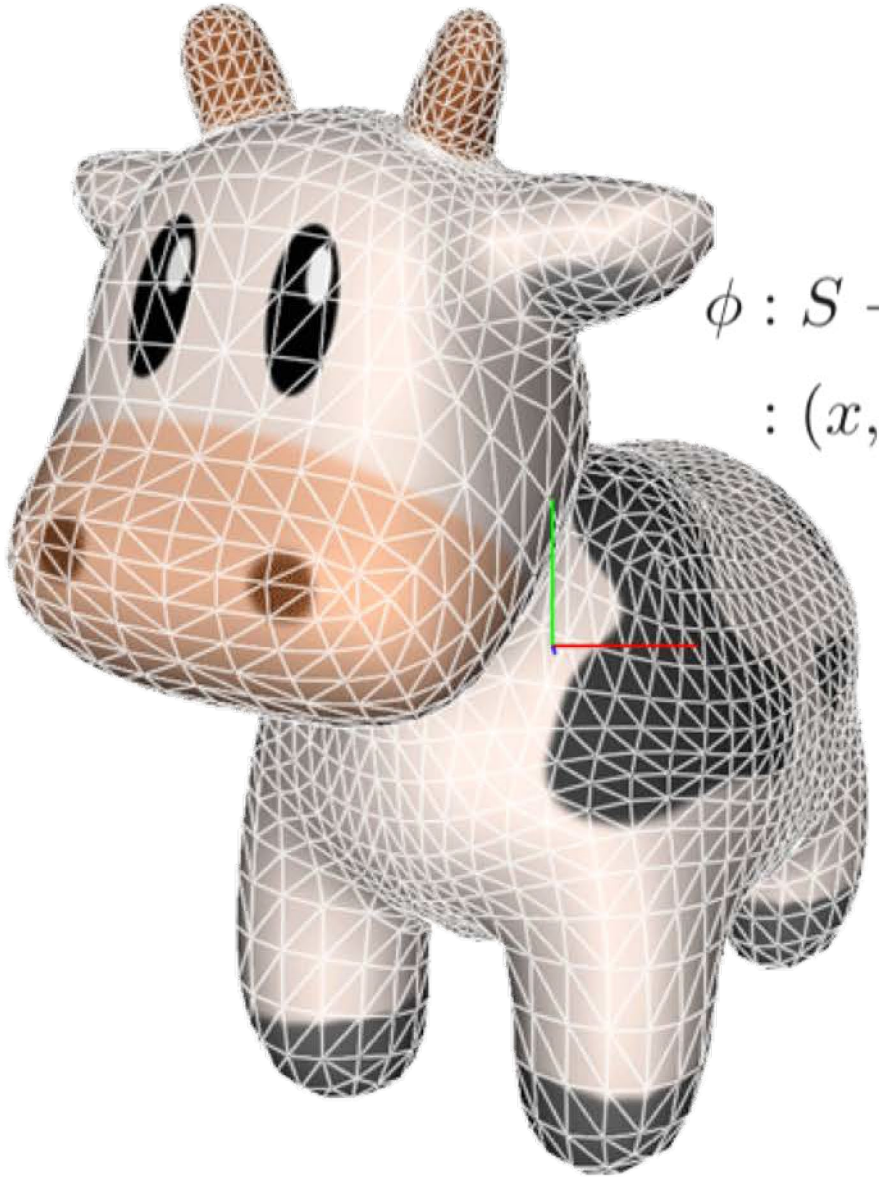


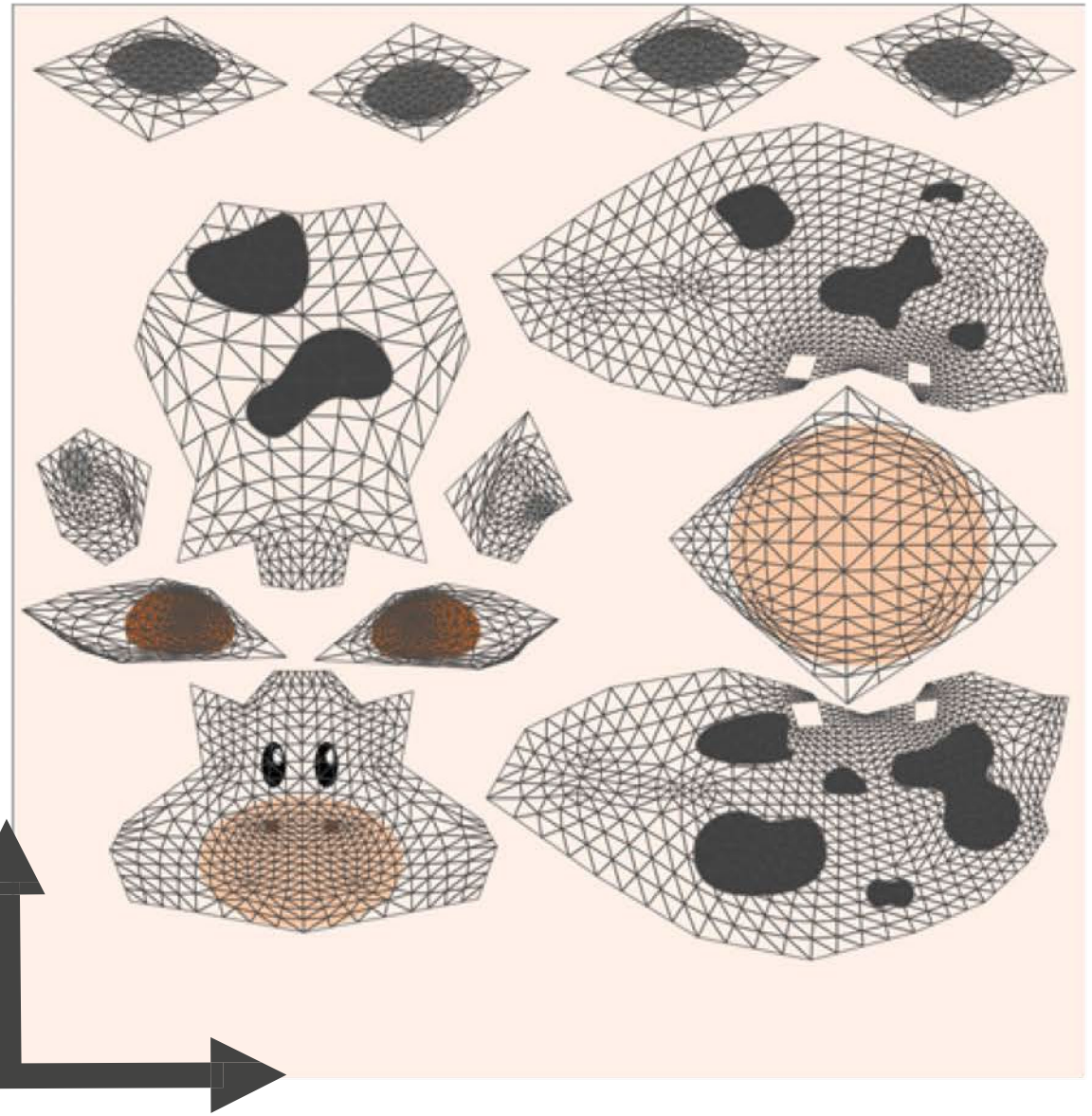
Quadrilateral (Quad) Meshes



Texture coordinates



$$\phi : S \to T$$
$$: (x, y, z) \mapsto (u, v)$$



What makes a good function?

- Bijectivity
- Size distortion
- Shape distortion
- Continuity

$$\phi : S \to T$$
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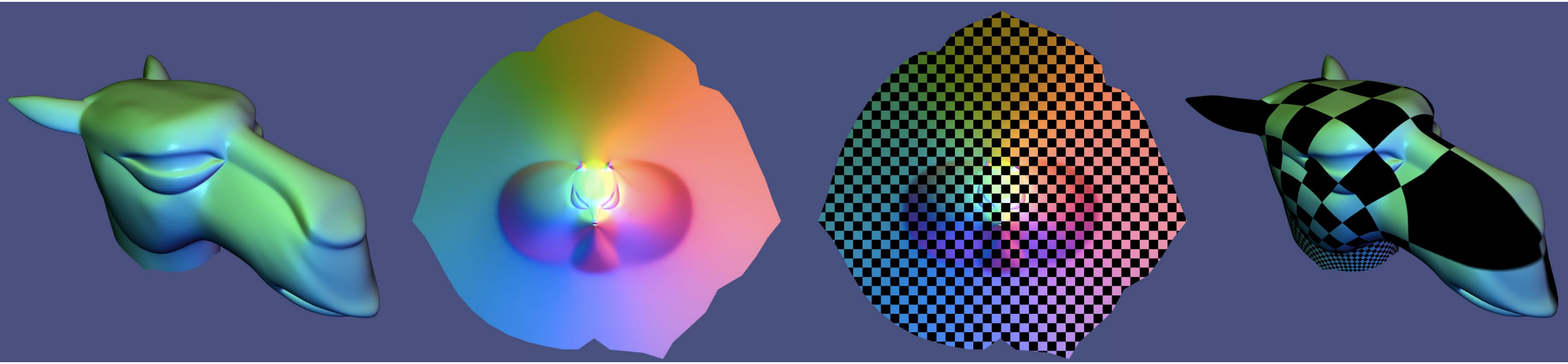
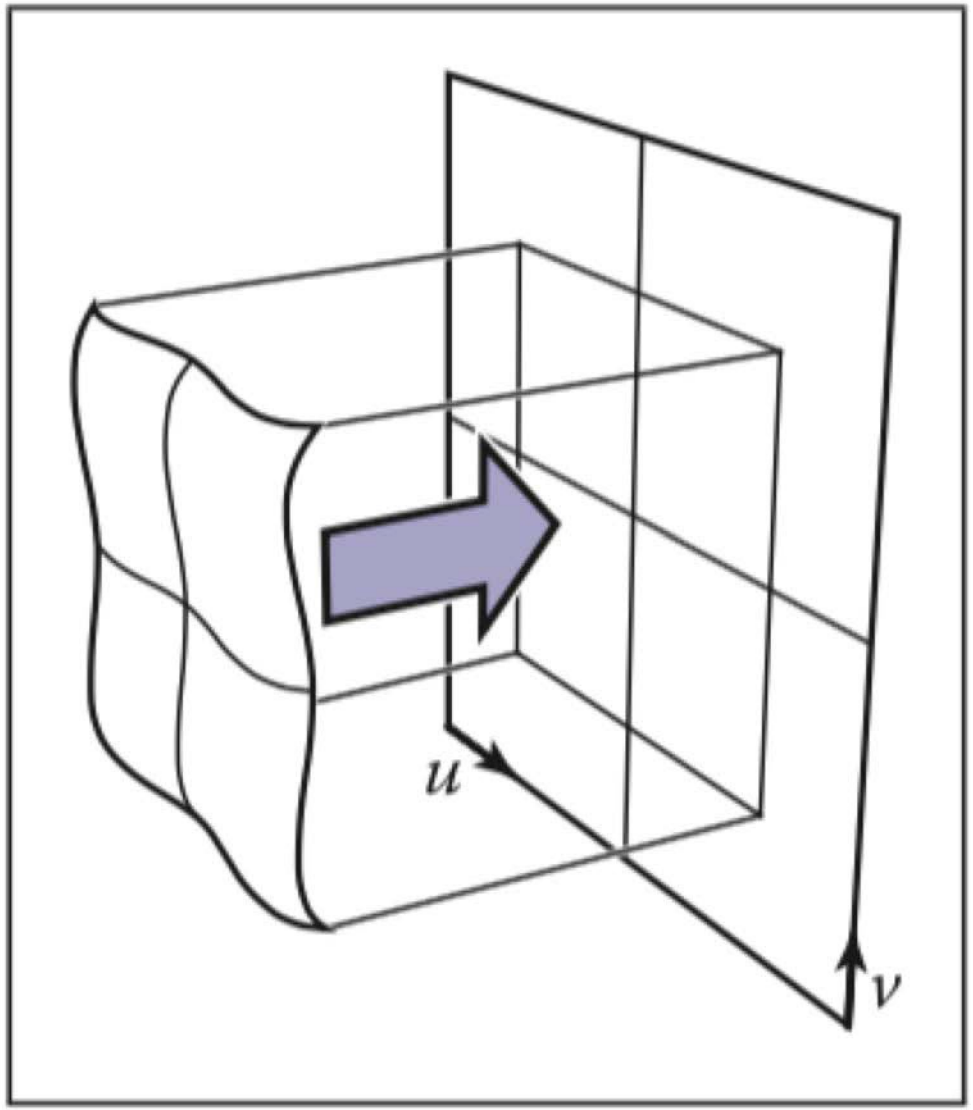


https://github.com/alecjacobson/geometry-processing-csc2520/blob/master/lecture-notes/parameterization-ryan-schmidt.pdf
https://members.loria.fr/Bruno.Levy/papers/LSCM_SIGGRAPH_2002.pdf
https://arxiv.org/pdf/1704.06873.pdf

Planar Texture Map

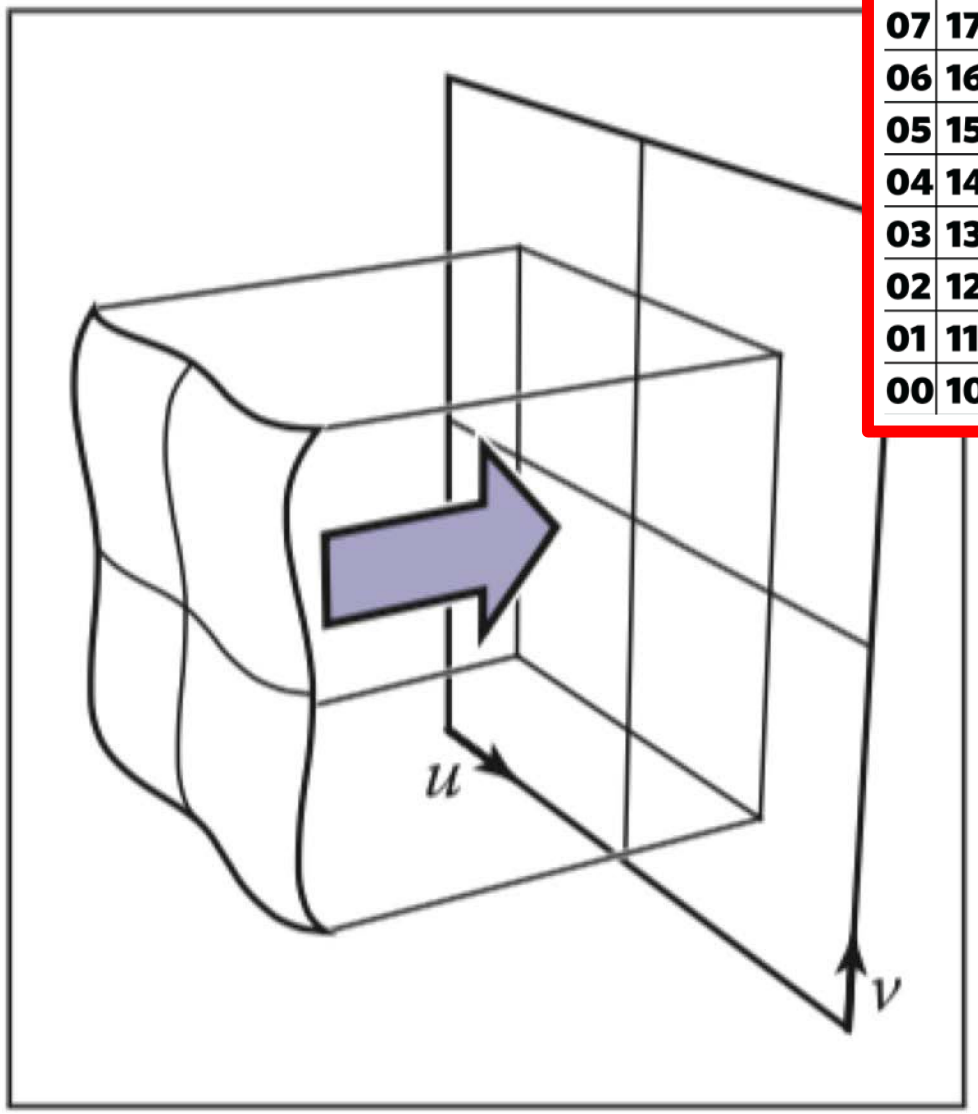


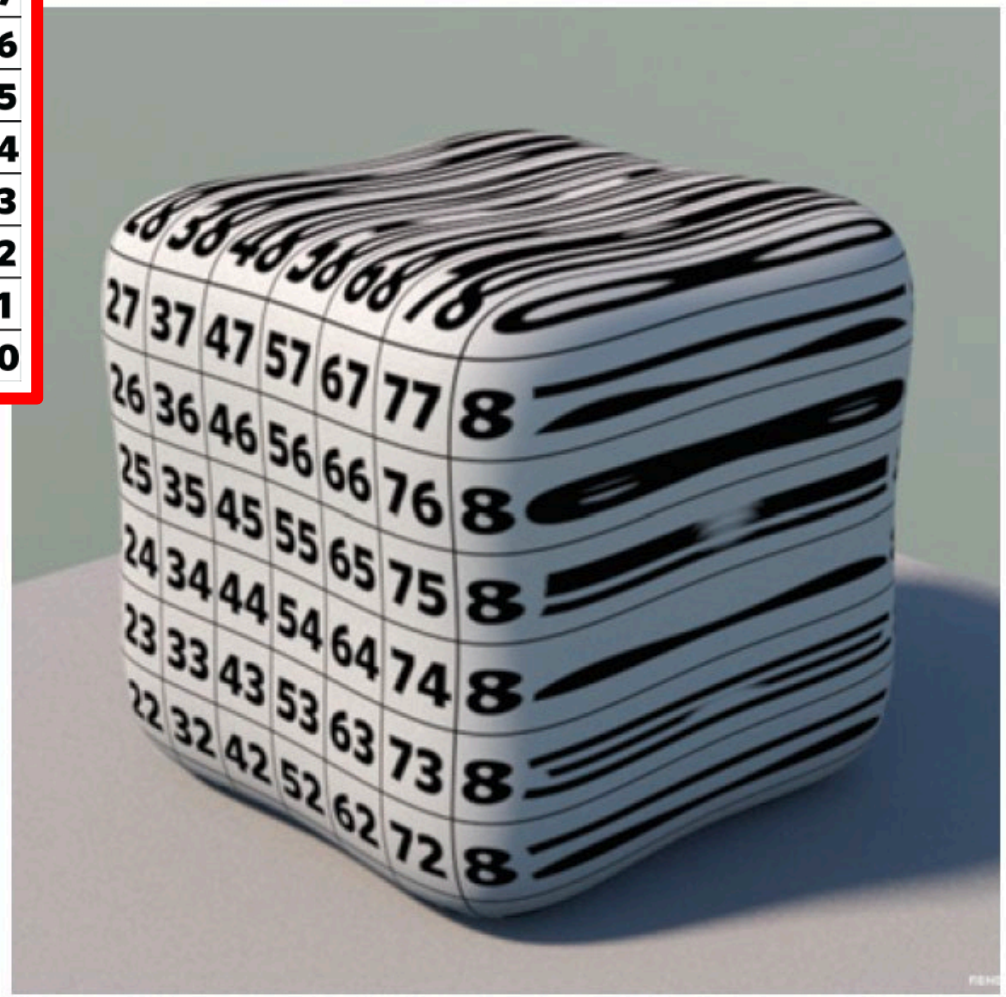
$$\phi(x, y, z) = (u, v)$$

where $$\begin{bmatrix} u \\ v \\ * \\ 1 \end{bmatrix} = M_t \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Planar Texture Map

09	19	29	39	49	59	69	79	89	99
08	18	28	38	48	58	68	78	88	98
07	17	27	37	47	57	67	77	87	97
06	16	26	36	46	56	66	76	86	96
05	15	25	35	45	55	65	75	85	95
04	14	24	34	44	54	64	74	84	94
03	13	23	33	43	53	63	73	83	93
02	12	22	32	42	52	62	72	82	92
01	11	21	31	41	51	61	71	81	91
00	10	20	30	40	50	60	70	80	90





Spherical Texture Map

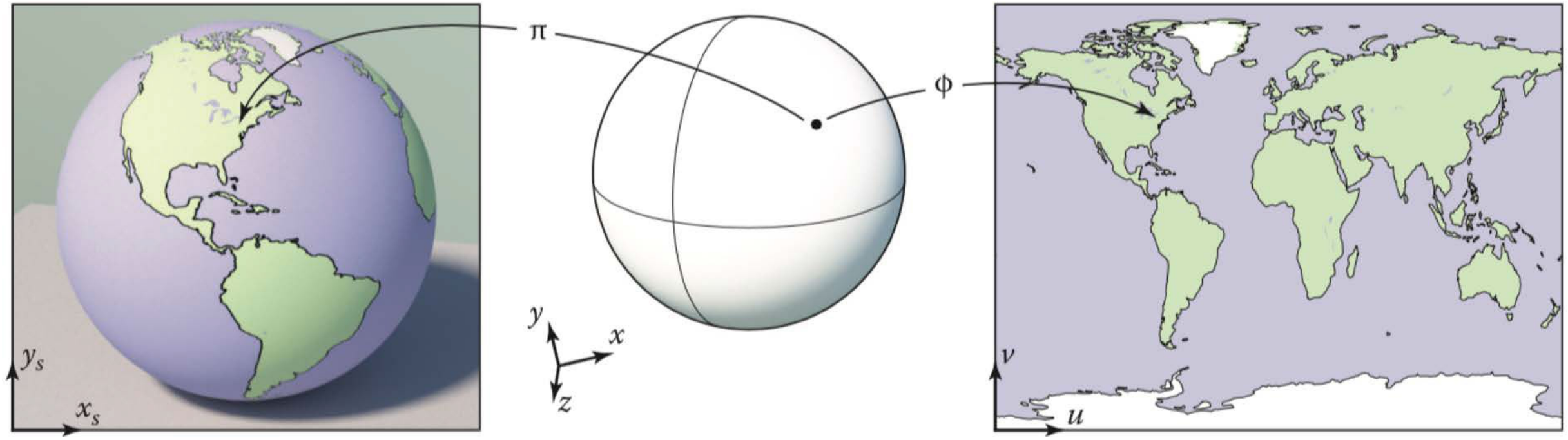
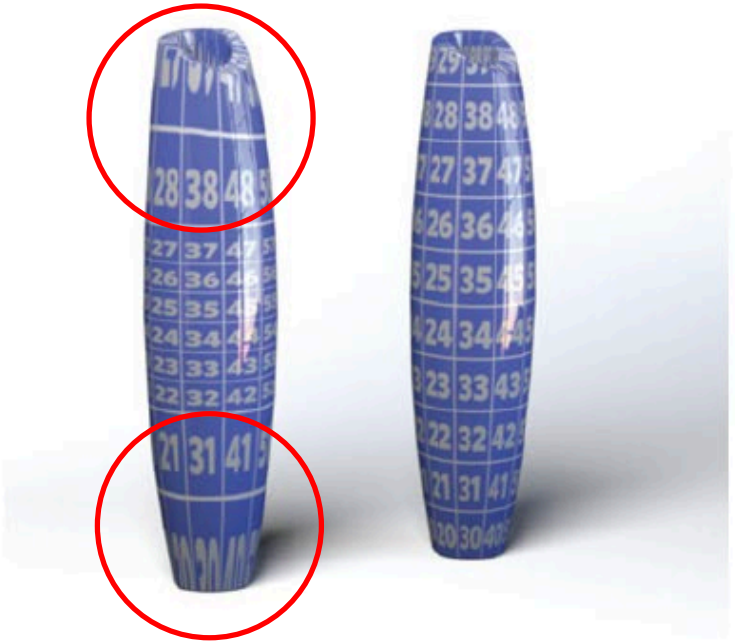


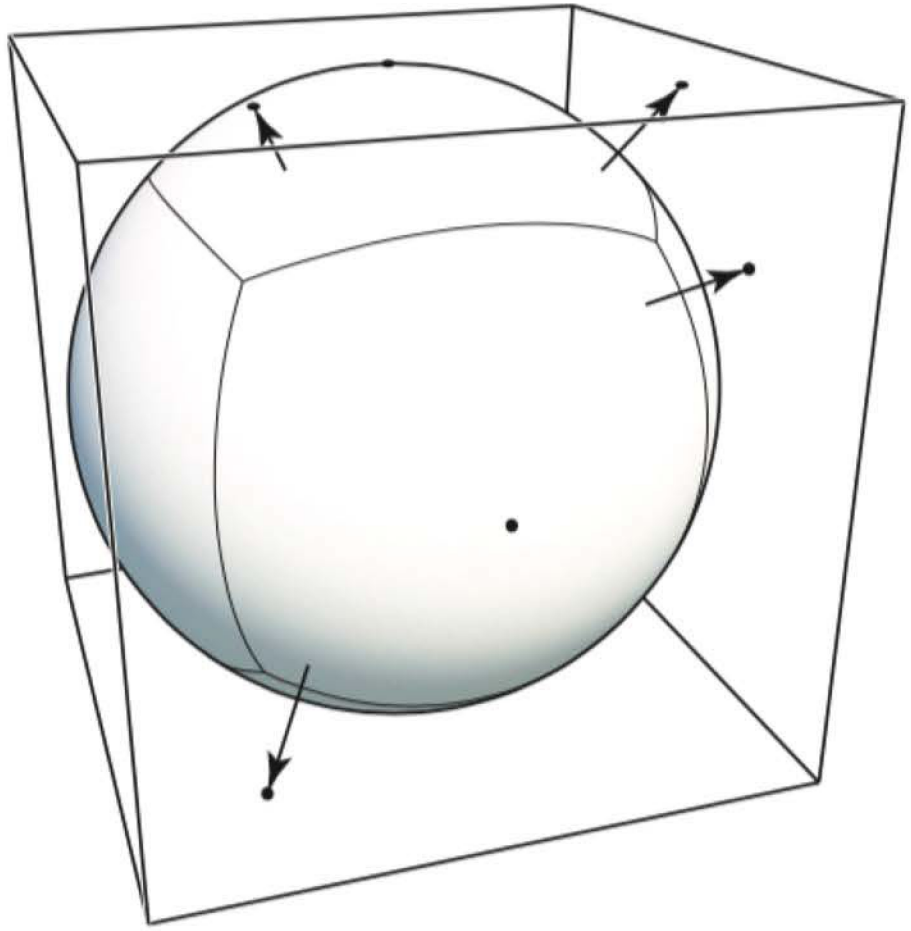
Image space | Surface S in world space | Texture space, T

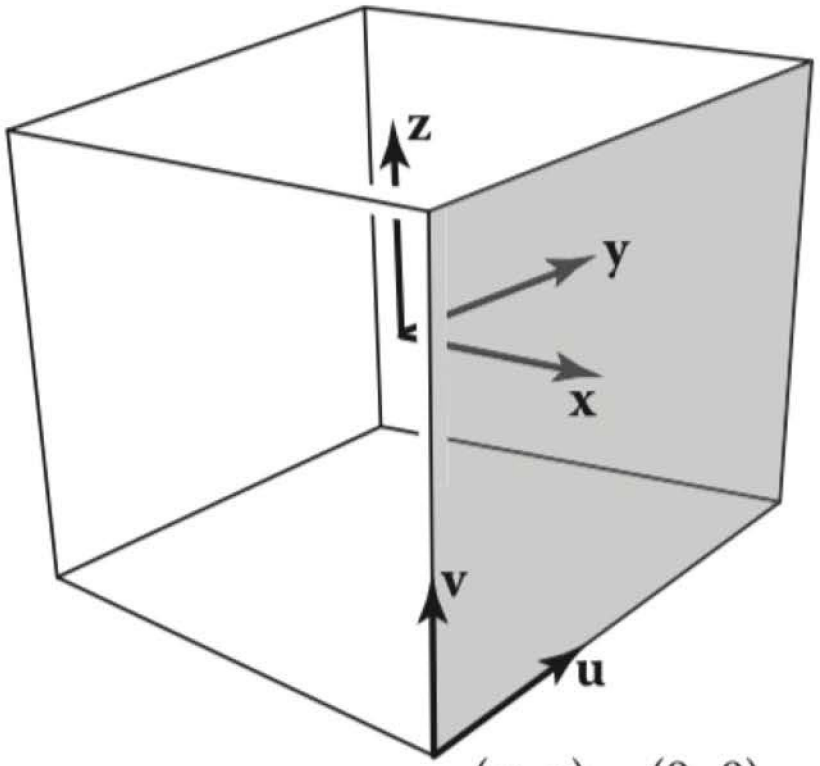
$$\phi(x, y, z) = ([\pi + \text{atan2}(y, x)]/2\pi, [\pi - \text{acos}(z/\|x\|)]/\pi)$$

Spherical Texture Map Distortion



Cube Texture Map





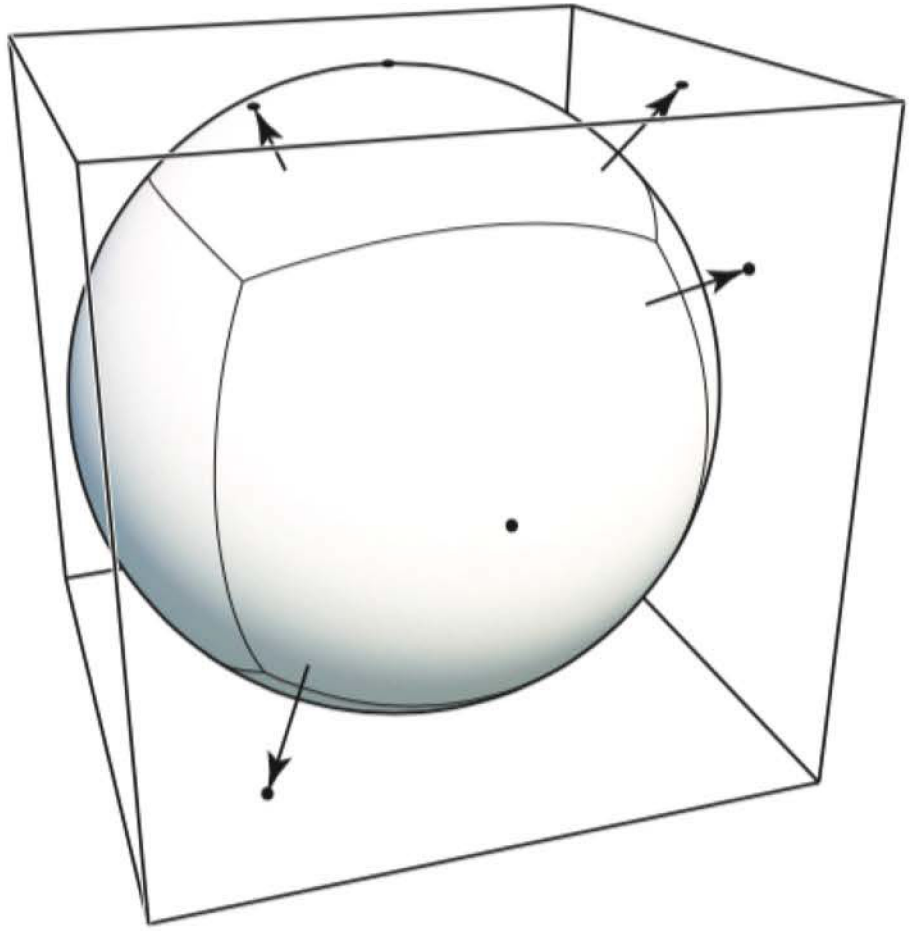
$(u, v) = (1, 1)$
$x = y = z$

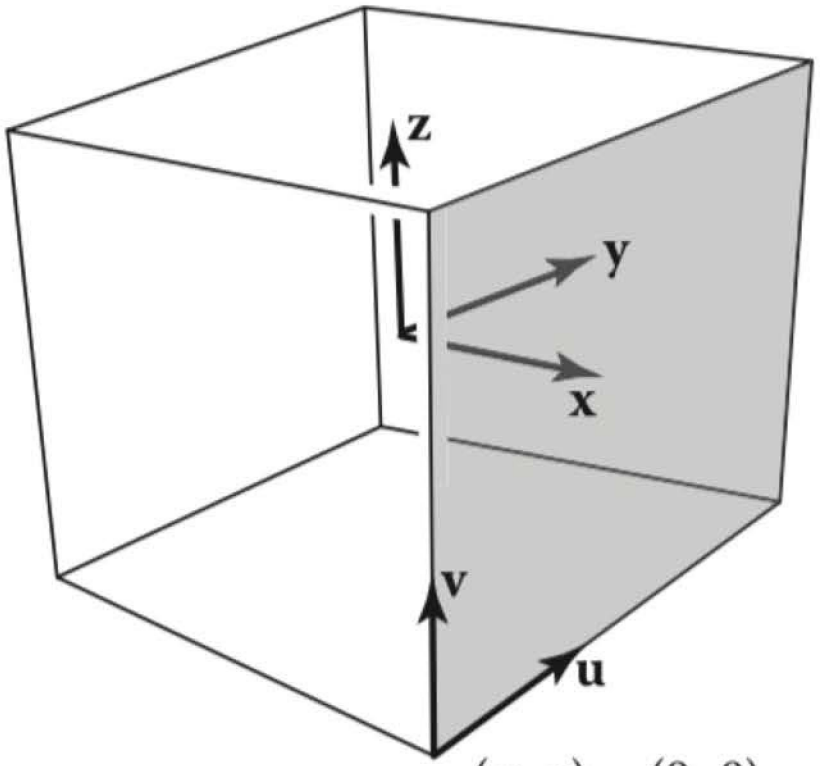
Right face has
$x > |y|$ and
$x > |z|$

$(u, v) = (0, 0)$
$x = -y = -z$

$$(x, y, z) \mapsto \left(\frac{x}{z}, \frac{y}{z}\right).$$

Cube Texture Map





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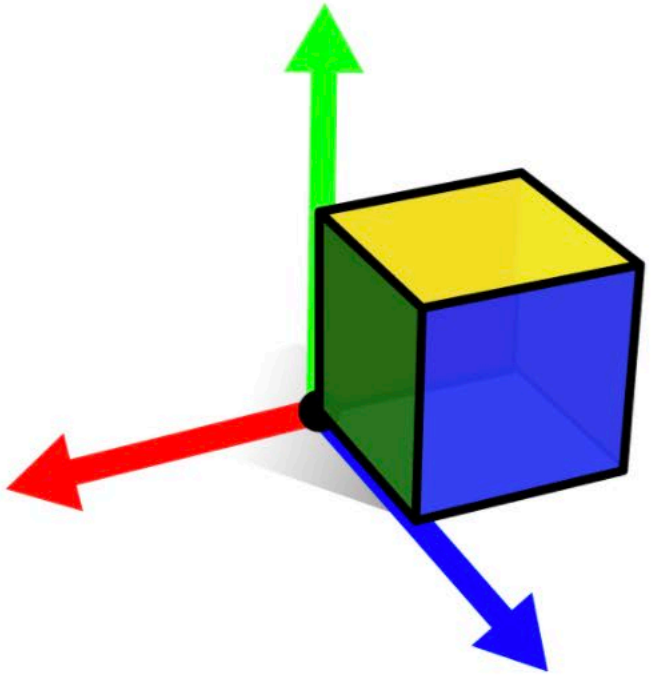
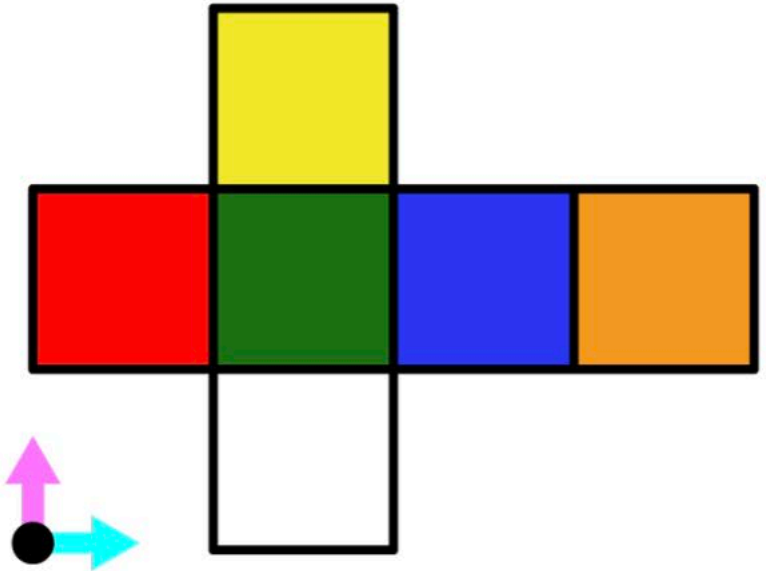
Cube Texture Map

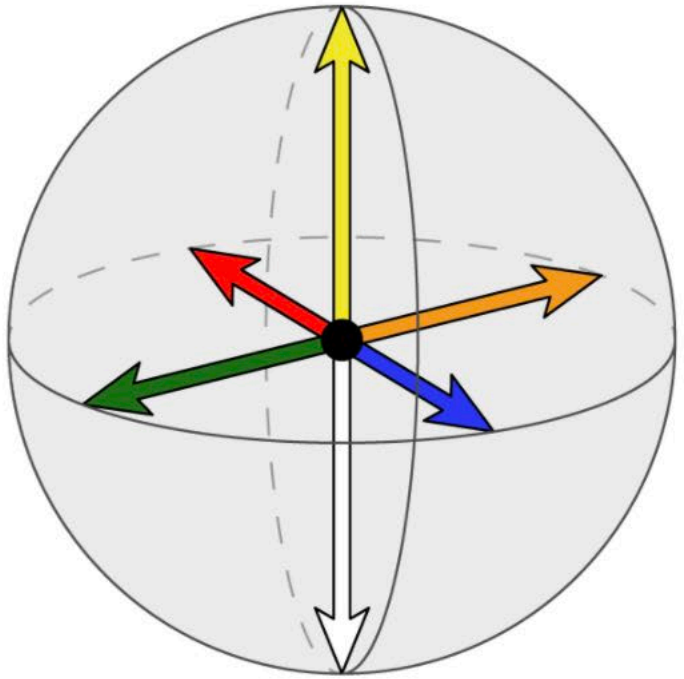
3D Vertex Positions



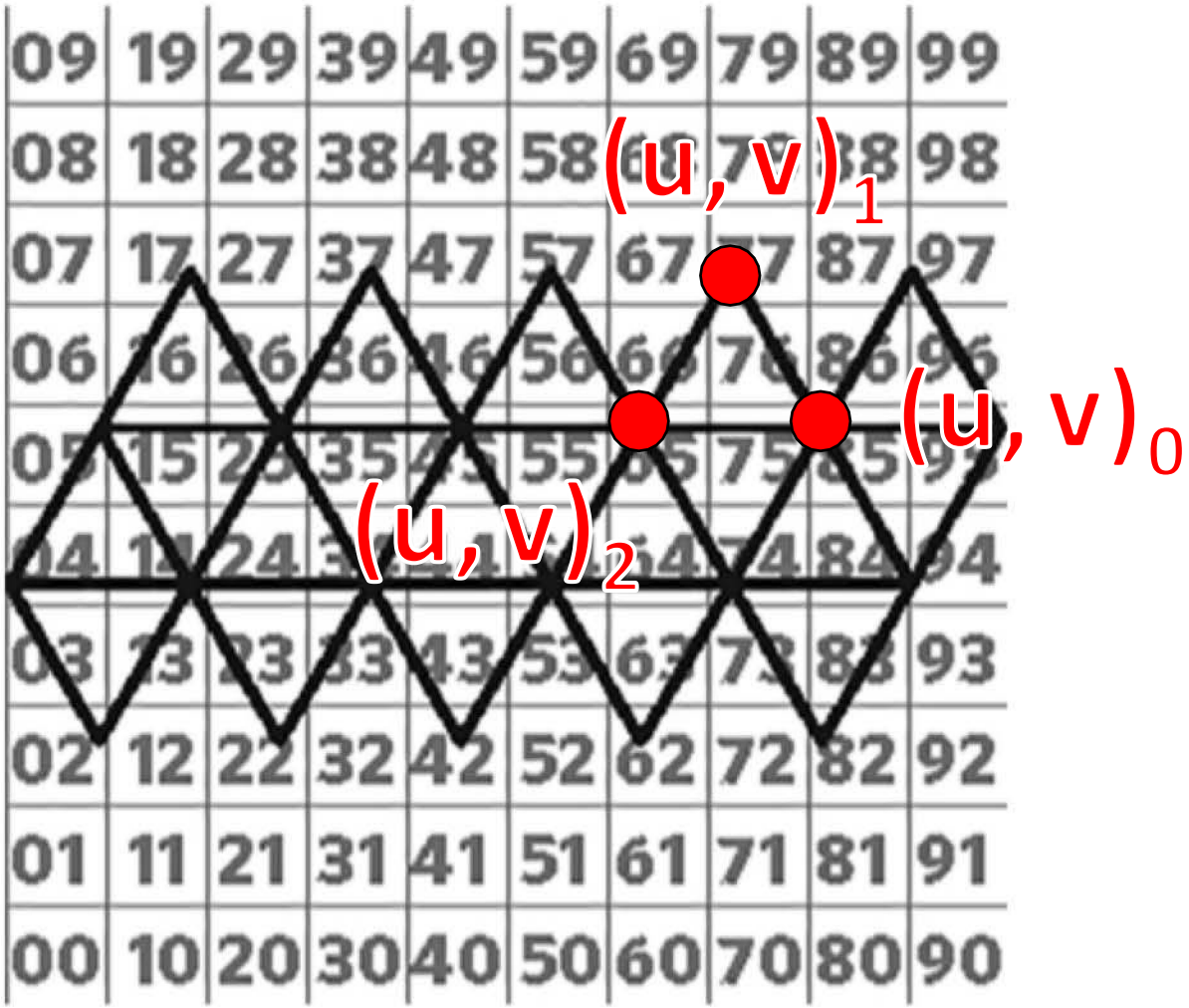
2D Parameterization Positions

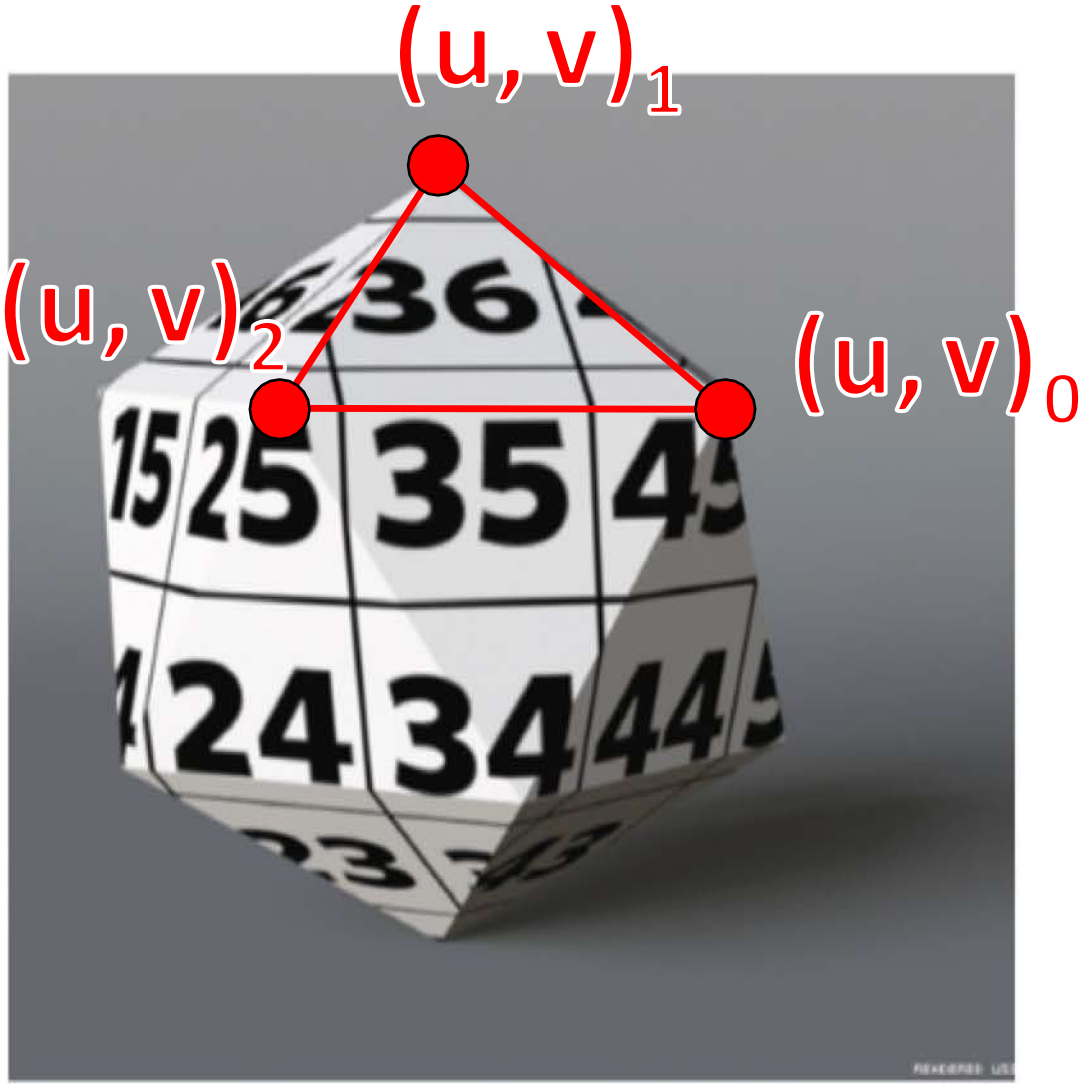


3D Normal Vectors

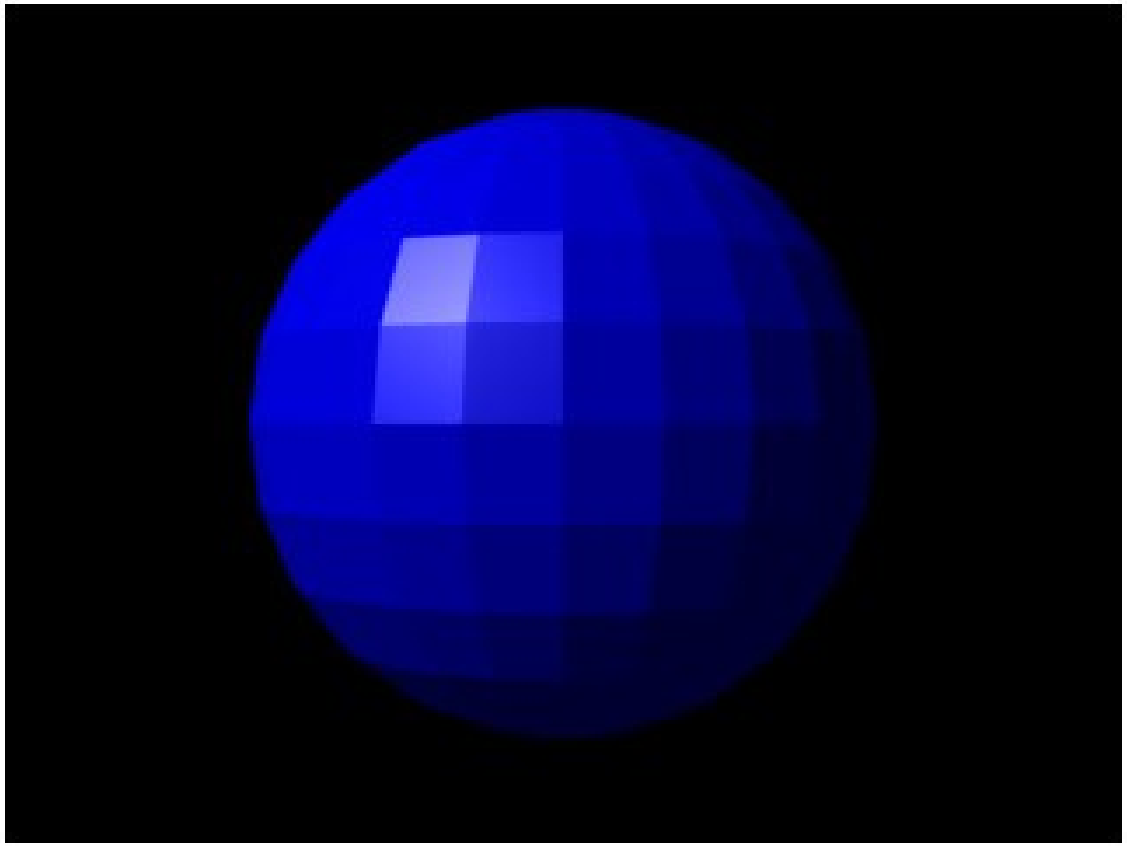


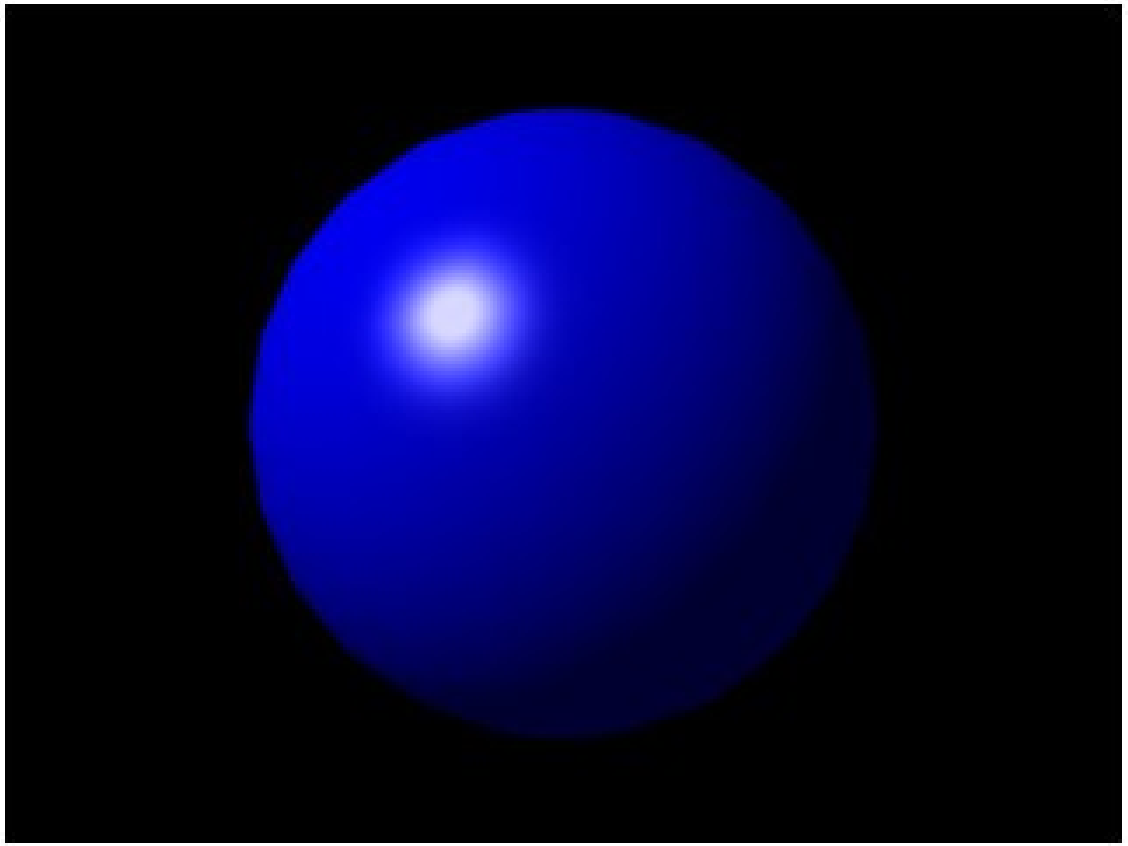
Interpolated Texture Coordinates



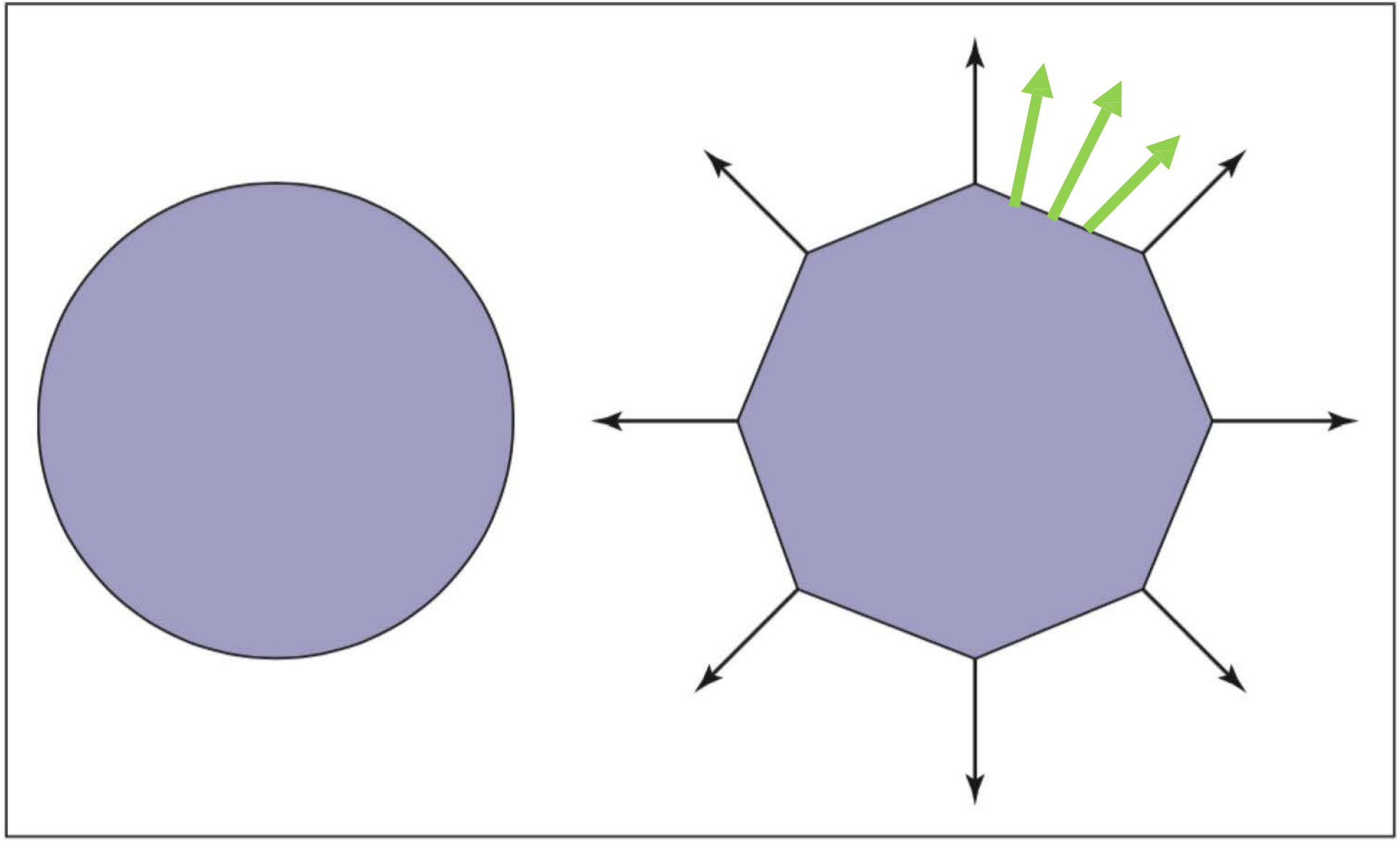


Smooth Surfaces in Computer Graphics

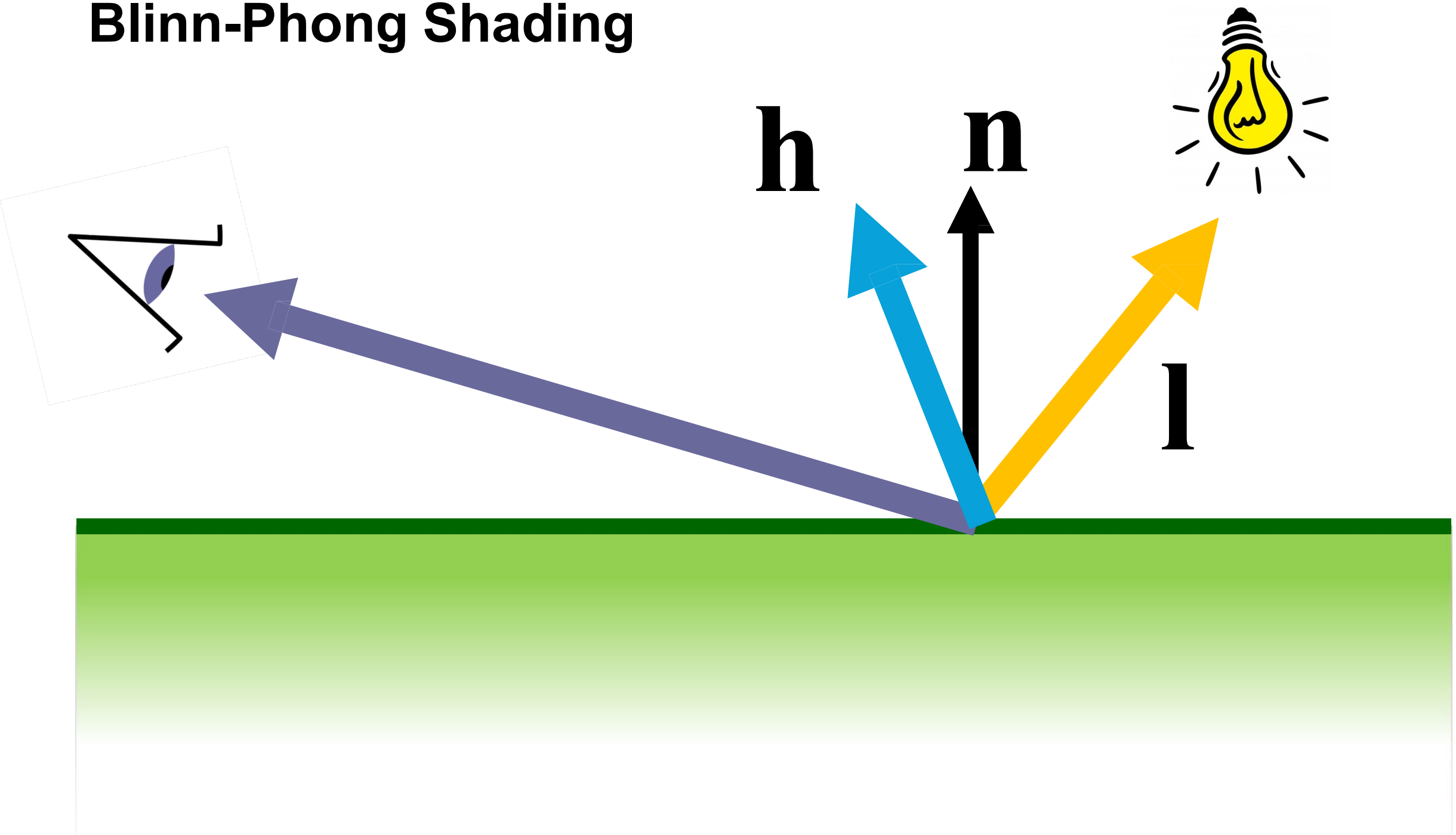




Phong Shading



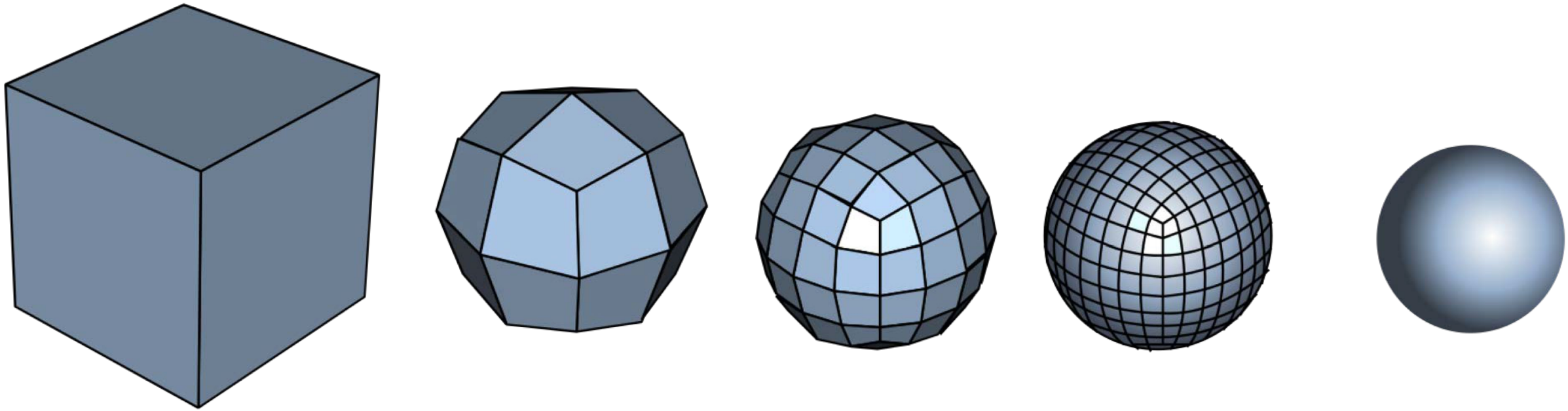
Blinn-Phong Shading



Subdivision Surfaces

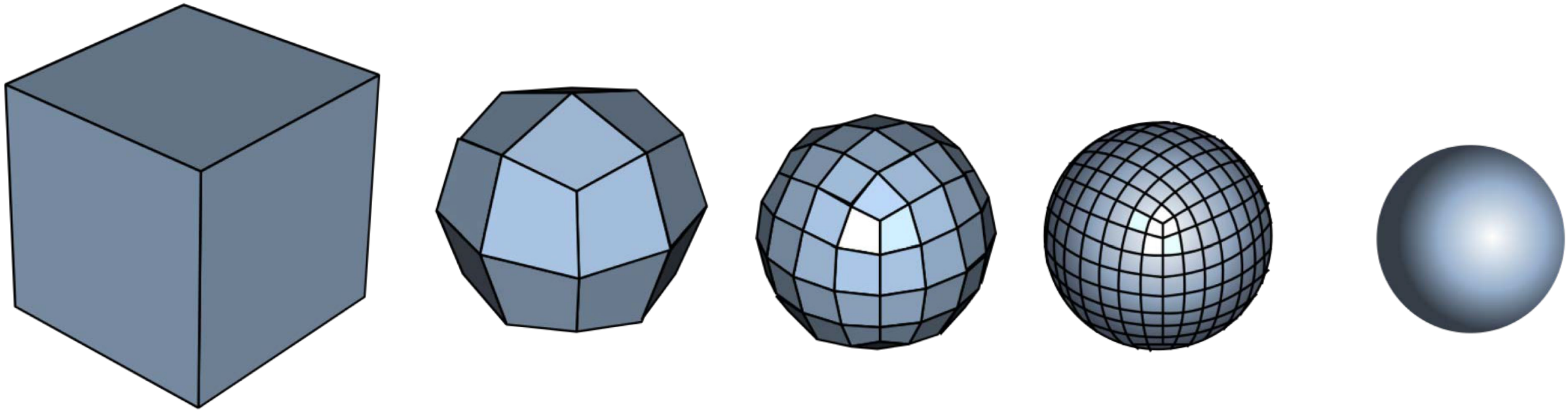
Recursive refinement of polygonal mesh

Results in a smooth “limit surface”



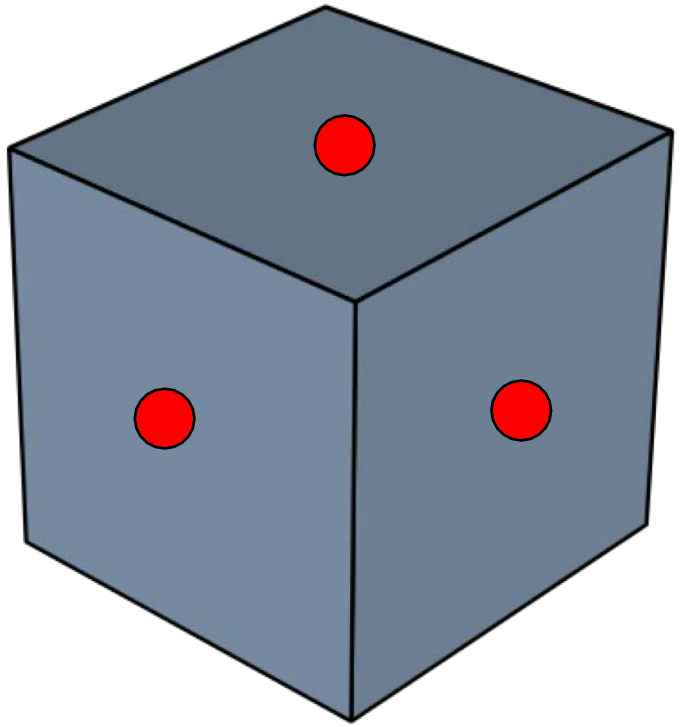
Catmull-Clark Subdivision

Particular type of subdivision scheme.



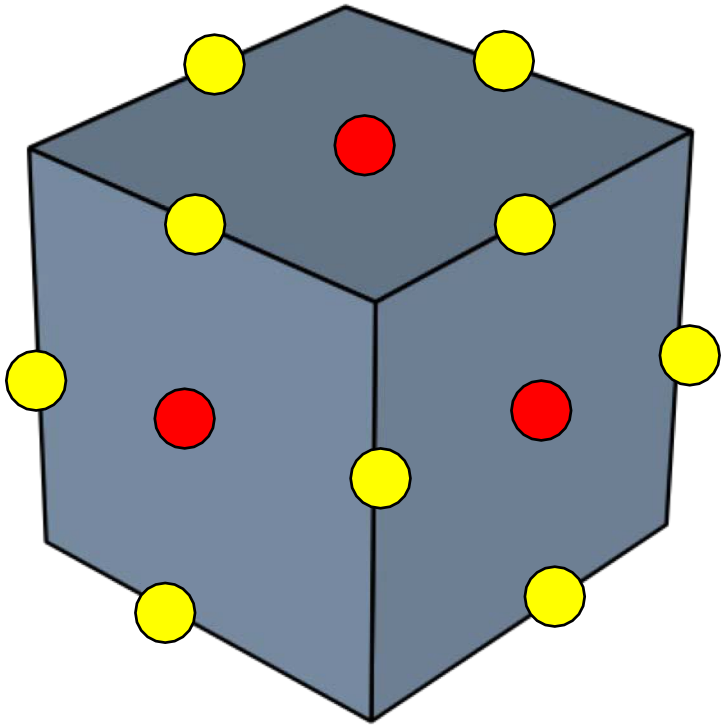
Catmull-Clark Subdivision

Step 1: Set the face point for each facet to be the average of its vertices



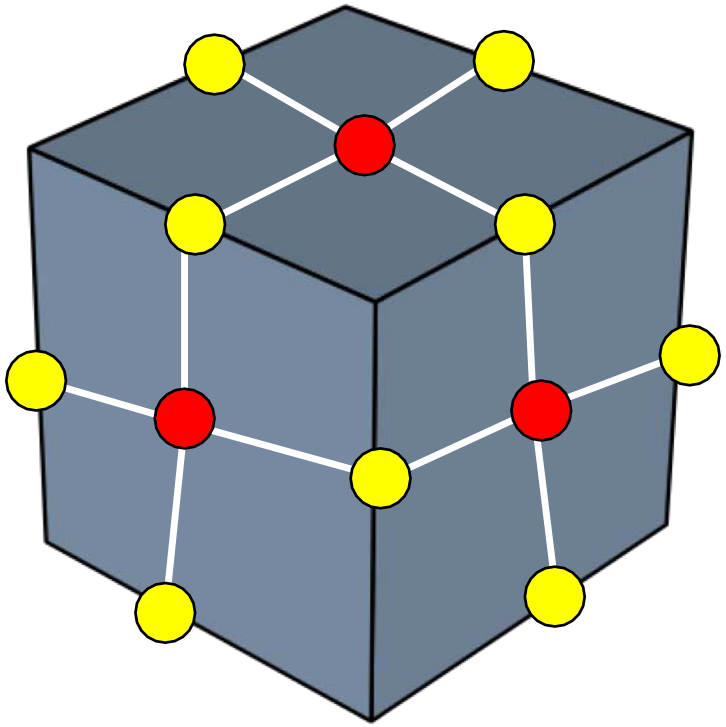
Catmull-Clark Subdivision

Step 2: Add edge points – average of two neighbouring face points and edge end points



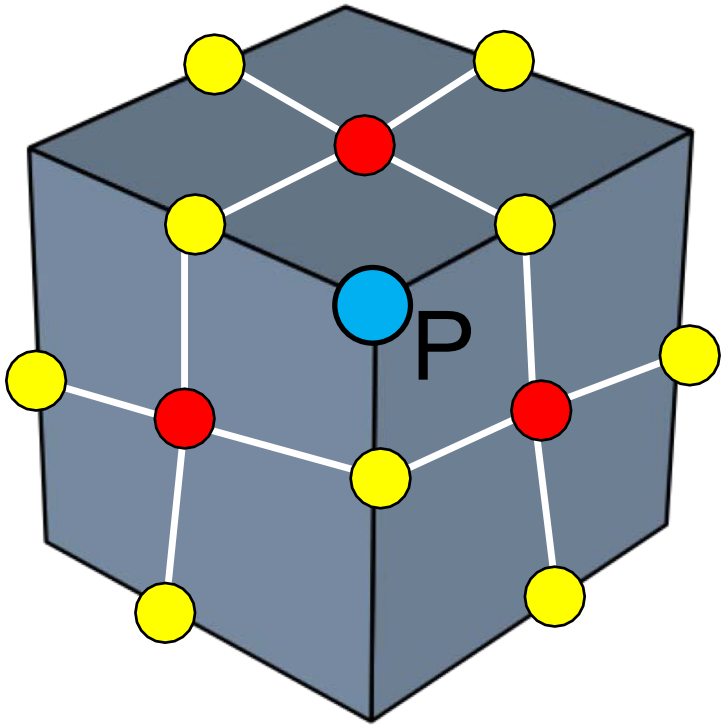
Catmull-Clark Subdivision

Step 3: Add edges between face points and edge points



Catmull-Clark Subdivision

Step 4: Move each original vertex according to new position given by:



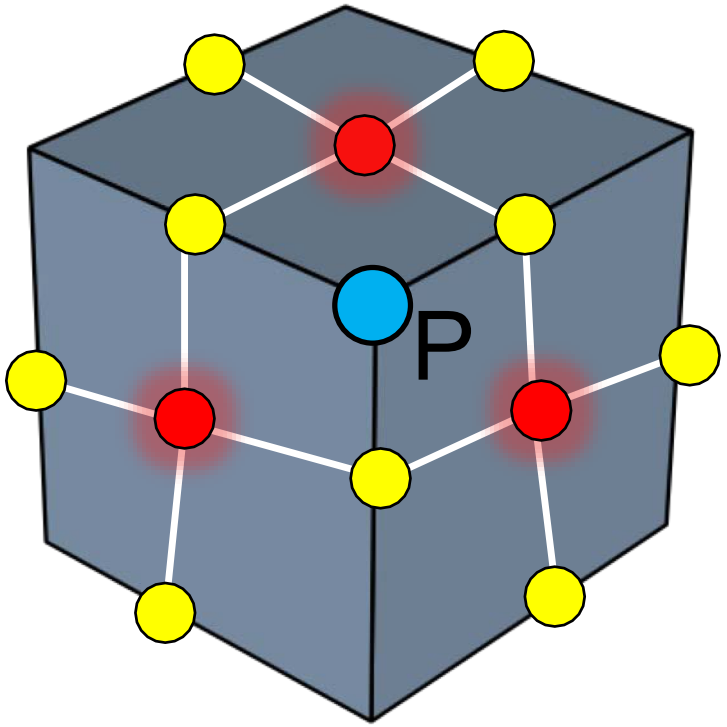
$$\frac{F + 2R + (n-3)P}{n}$$

F : Average of all n created face points adjacent to P

R : Average of all original edge midpoints touching P

Catmull-Clark Subdivision

Step 4: Move each original vertex according to new position given by:



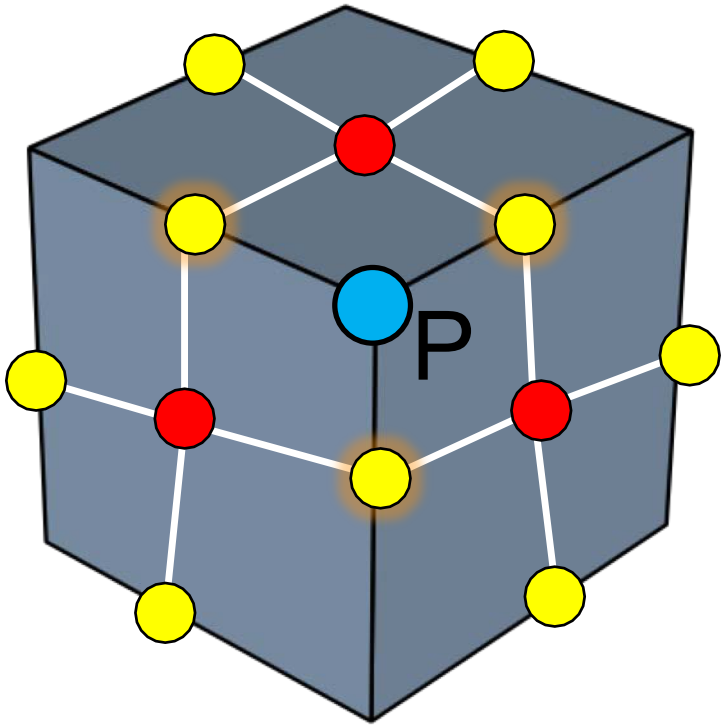
$$\frac{F + 2R + (n-3)P}{n}$$

F : Average of all n created face points adjacent to P

R : Average of all original edge midpoints touching P

Catmull-Clark Subdivision

Step 4: Move each original vertex according to new position given by:



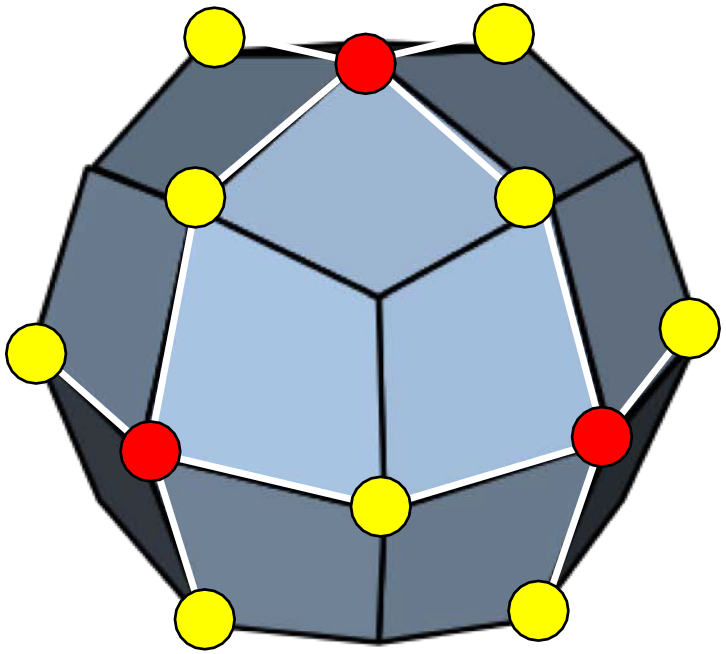
$$\frac{F + 2R + (n-3)P}{n}$$

F : Average of all n created face points adjacent to P

R : Average of all original edge midpoints touching P

Catmull-Clark Subdivision

Step 4: Move each original vertex according to new position given by:



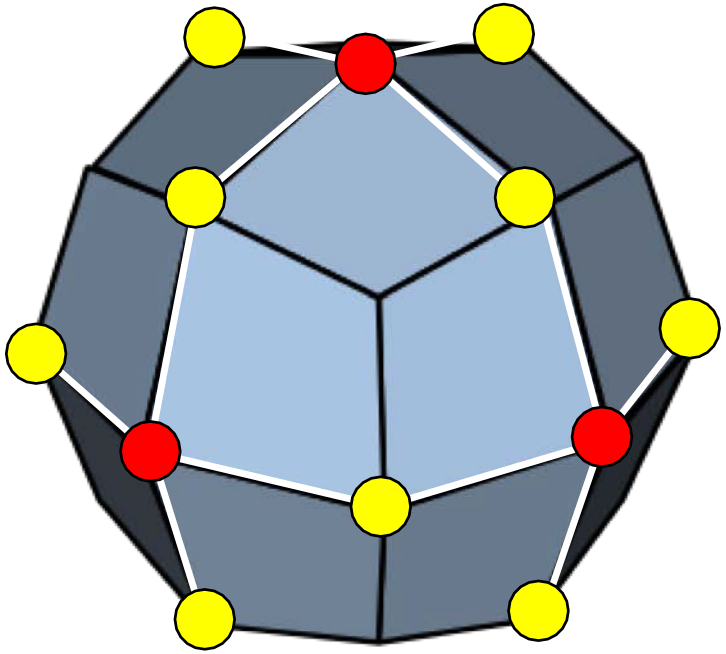
$$\frac{F + 2R + (n-3)P}{n}$$

F : Average of all n created face points adjacent to P

R : Average of all original edge midpoints touching P

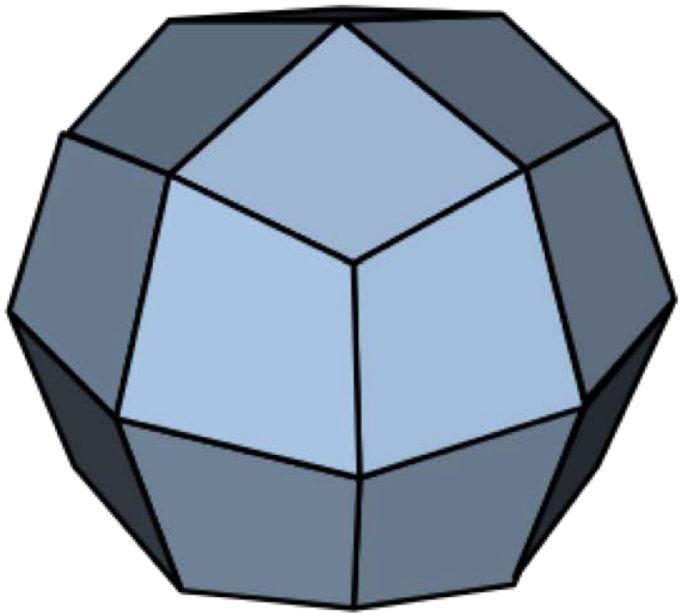
Catmull-Clark Subdivision

Step 5: Connect up original points to make facets



Catmull-Clark Subdivision

Repeat



Subdivision Surfaces in Action

http://graphics.pixar.com/opensubdiv/

Subdivision Surfaces in Action

Figure 5: Geri's hand as a piecewise smooth Catmull-Clark surface. Infinitely sharp creases are used between the skin and the finger nails.

https://www.youtube.com/watch?v=9IYRC7g2ICg
https://graphics.pixar.com/library/Geri/paper.pdf

Done