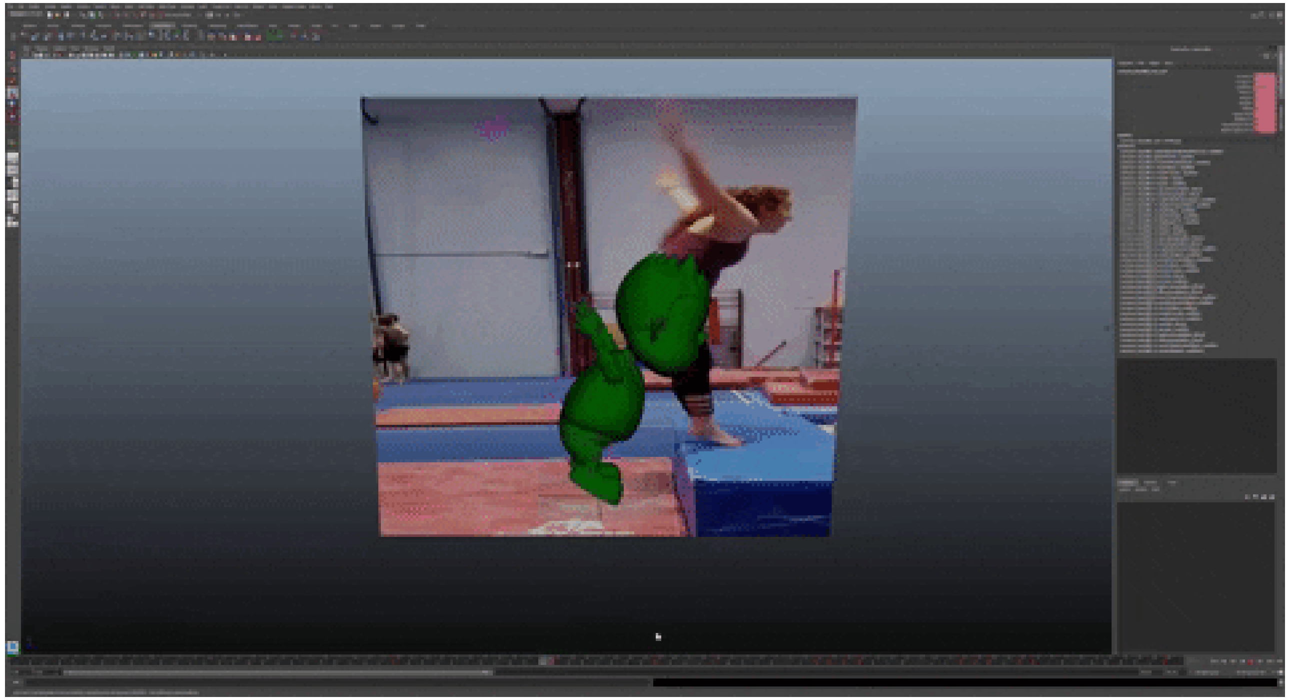
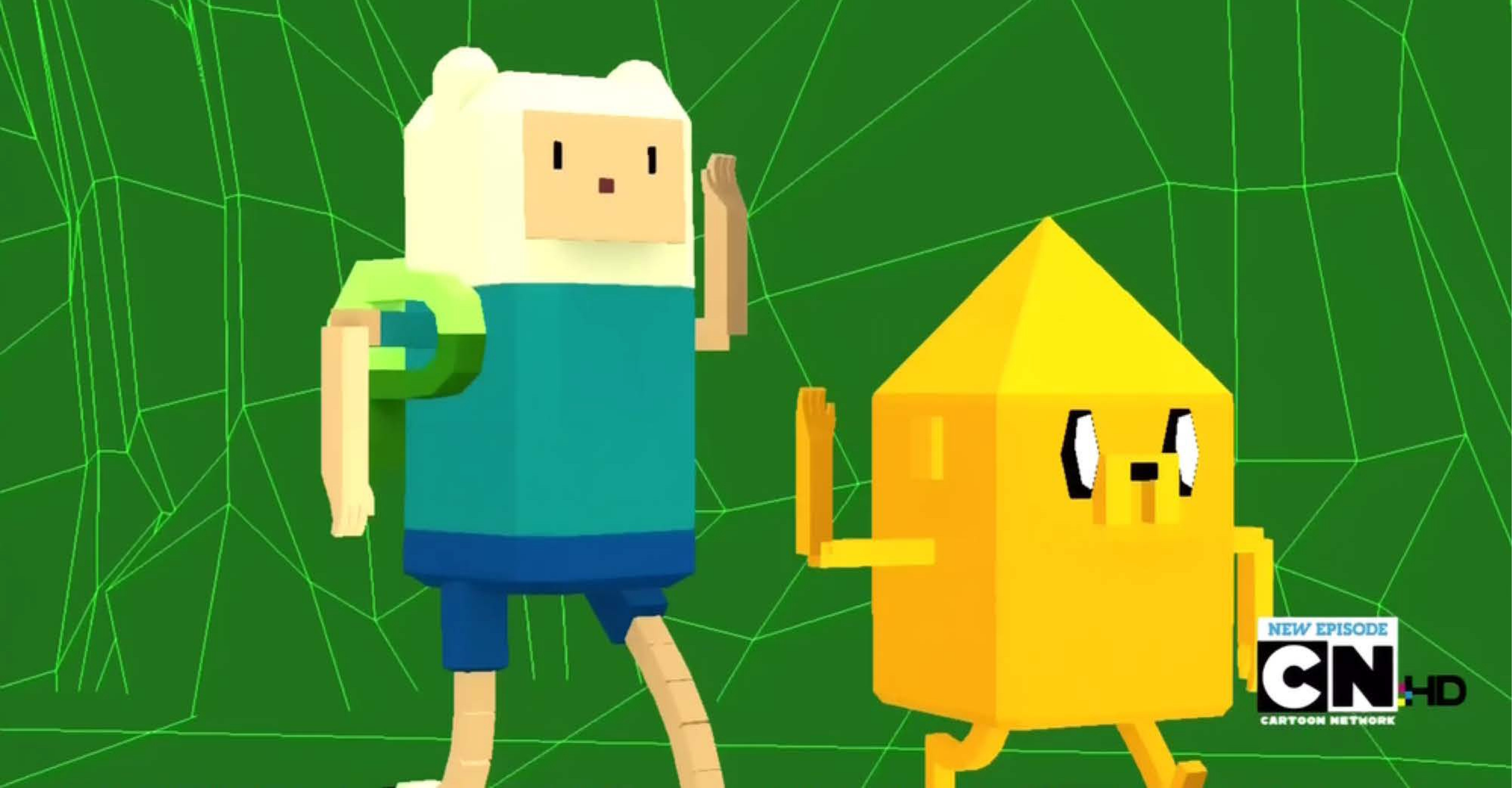
Kinematics



Some Slides/Images adapted from Marschner and Shirley and David Levin

Animation and Kinematics



Announcements

Marks out for A1-A4 and midterm

You know 48% of your final grade

Drop date is today

TAs are handling midterm remark requests this week

A6 due date moved to Sunday 26 July :)

Any Questions ?

Animation and Kinematics

Today:

Animation in Computer Graphics

Skinning for Mesh Deformation

Forward Kinematics

Keyframe Animation

Wednesday:

Keyframe Animation + Splines

Inverse Kinematics

“Core” Areas of Computer Graphics

Modeling/Geometry

Rendering

Animation

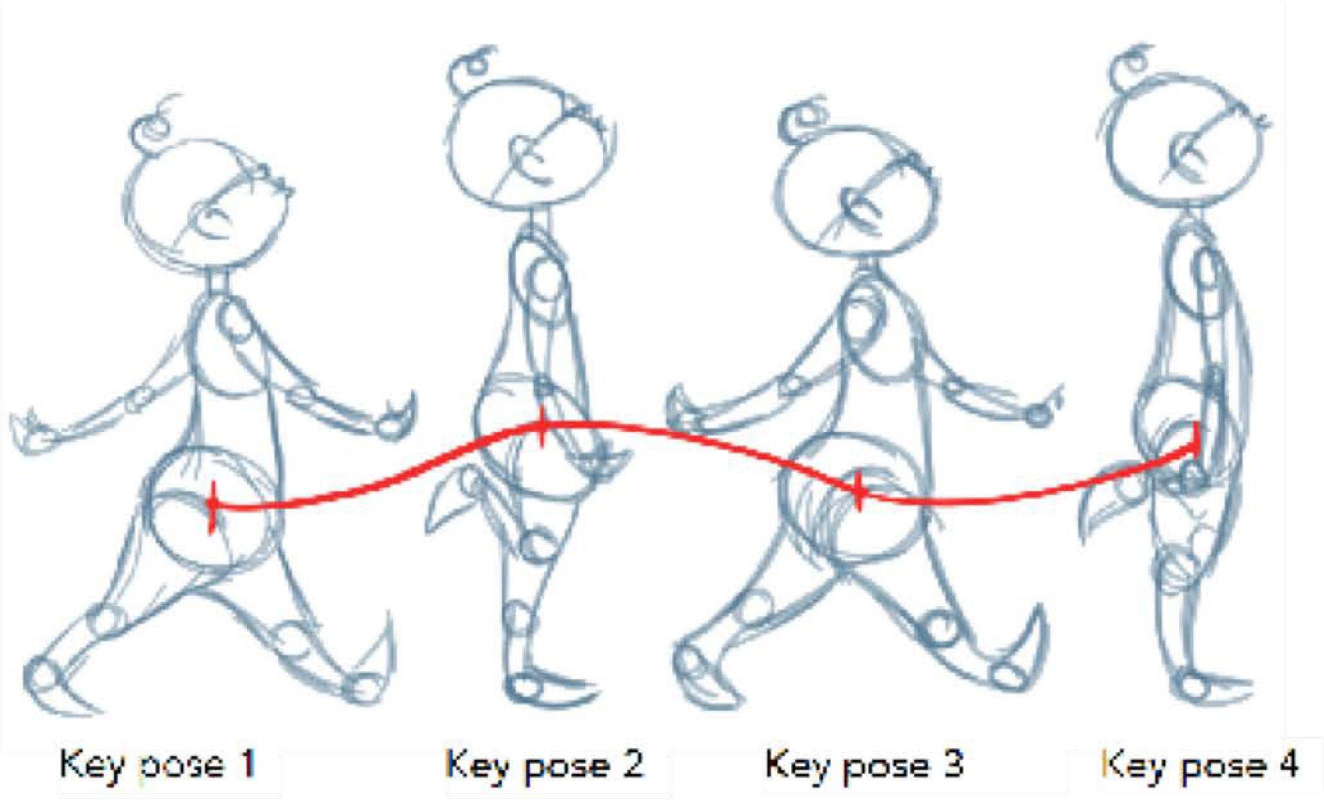
Animation How does one make digital models move?

Motion Capture

Animation How does one make digital models move?

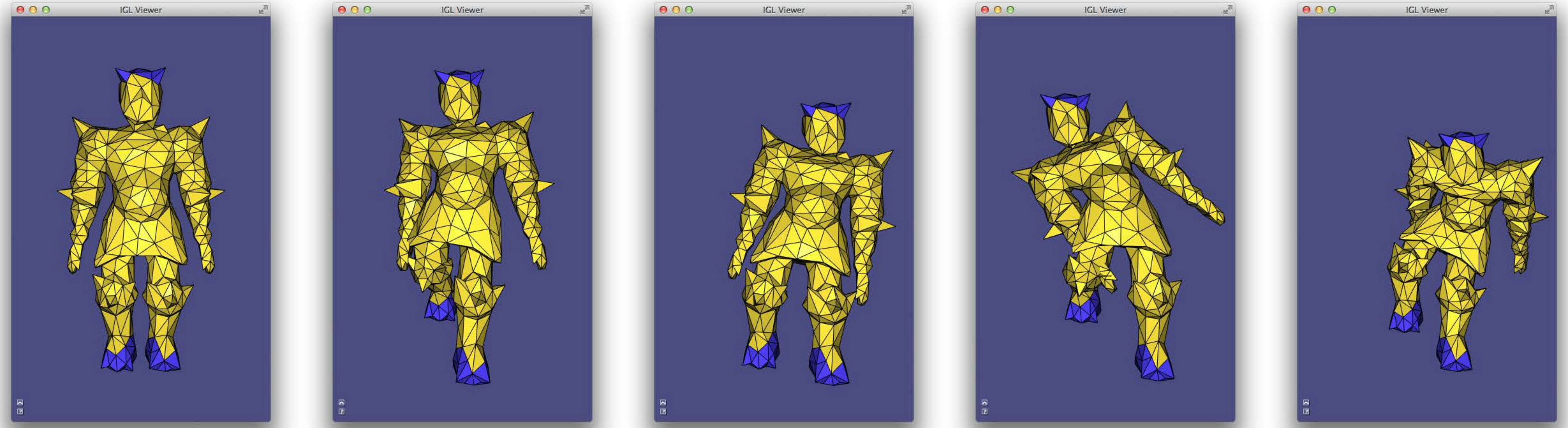
Physics Simulation

Animation How does one make digital models move?



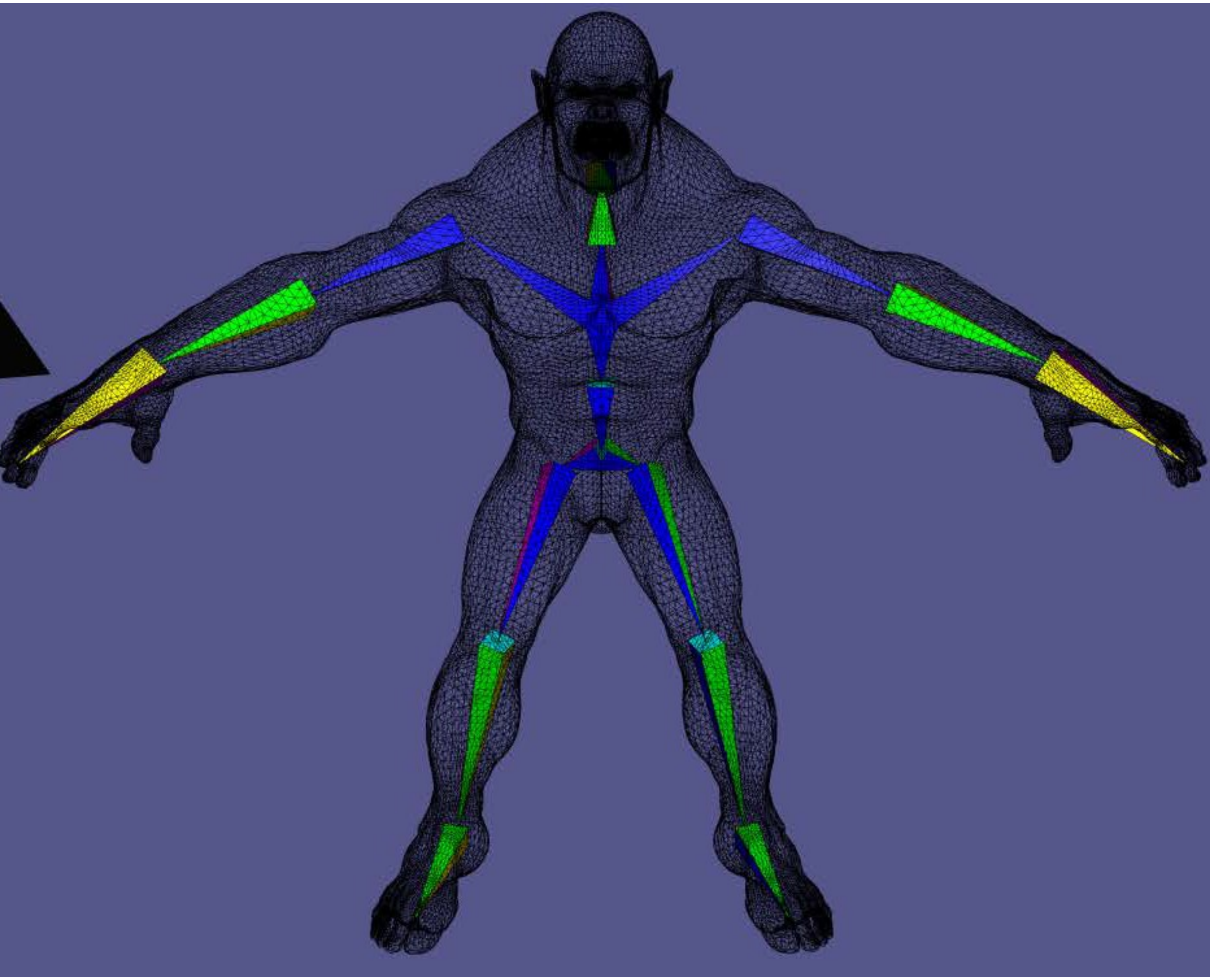
"Draw" Important Poses, interpolate in-between

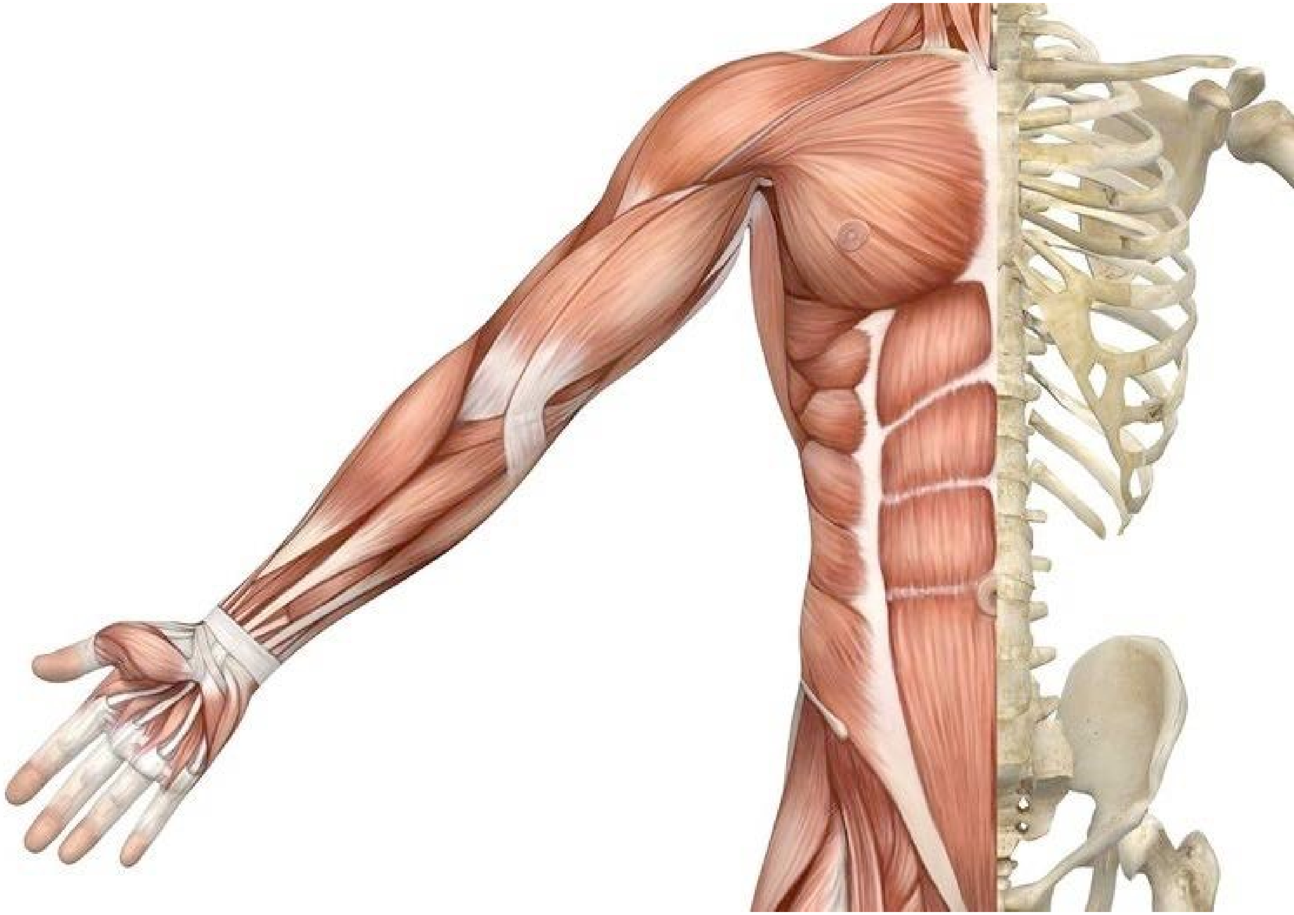
How Can We Specify an Animation



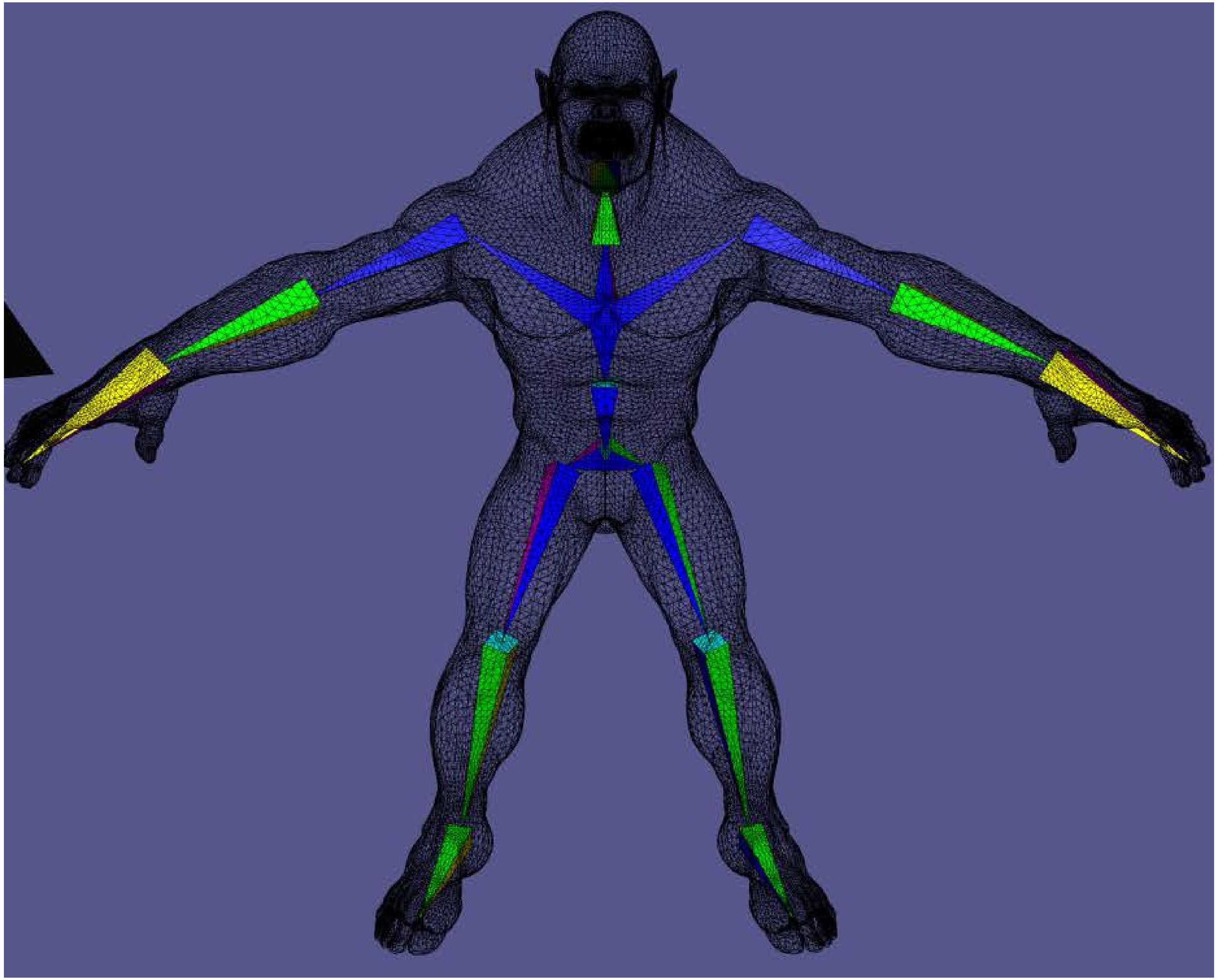
Per-Vertex?

Bones

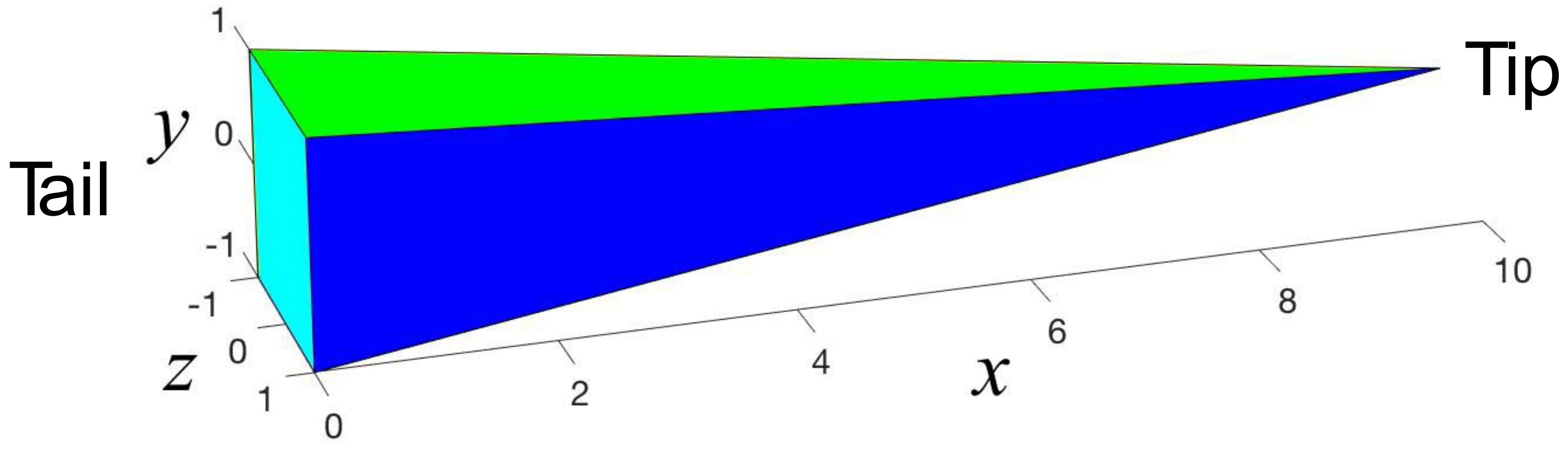




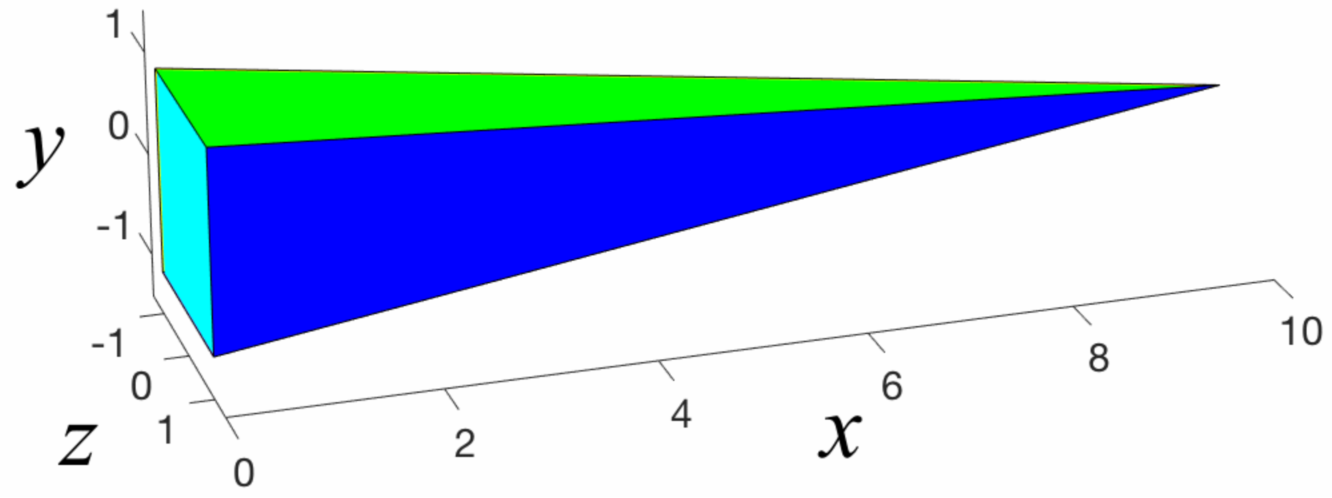
Skinning



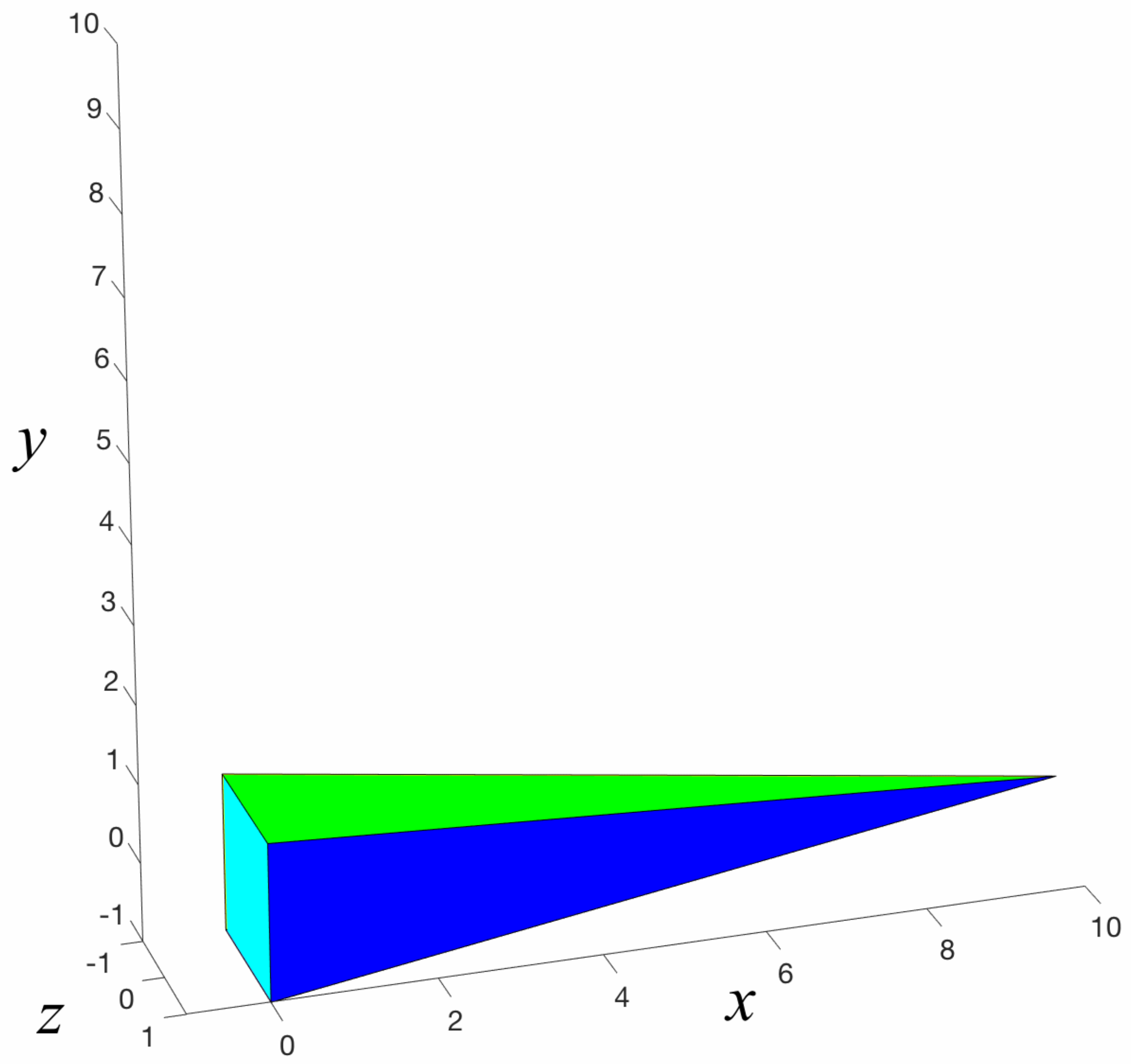
Bone of length $\ell = 10$



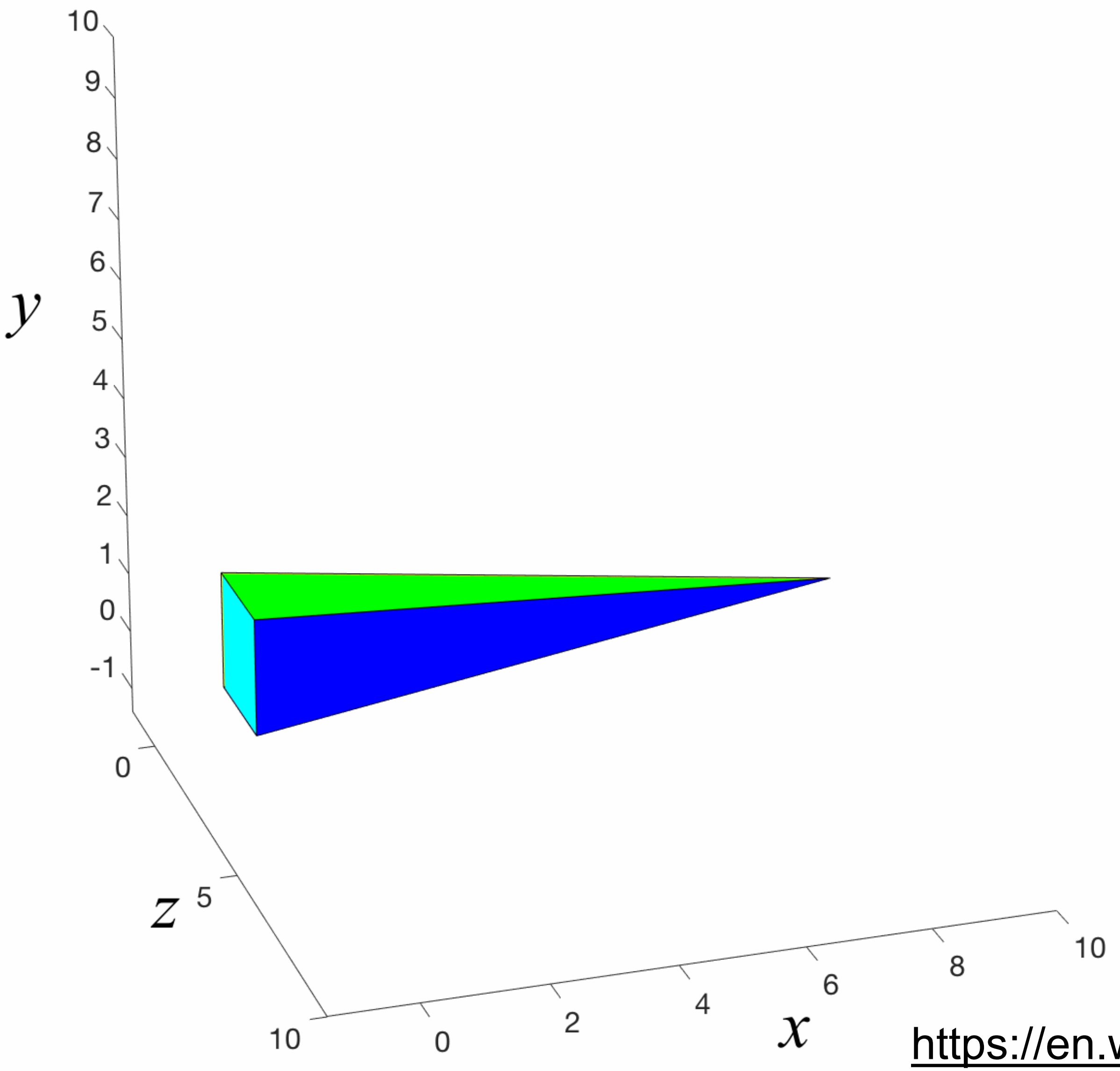
Twisting around x axis: $\theta_1 = 0^\circ$



Bending around z axis: $\theta_2 = 0^\circ$



Twist-bend-twist: $(\theta_1, \theta_2, \theta_3) = (0°, 0°, 0°)$



https://en.wikipedia.org/wiki/Euler_angles

https://mathworld.wolfram.com/EulerAngles.html

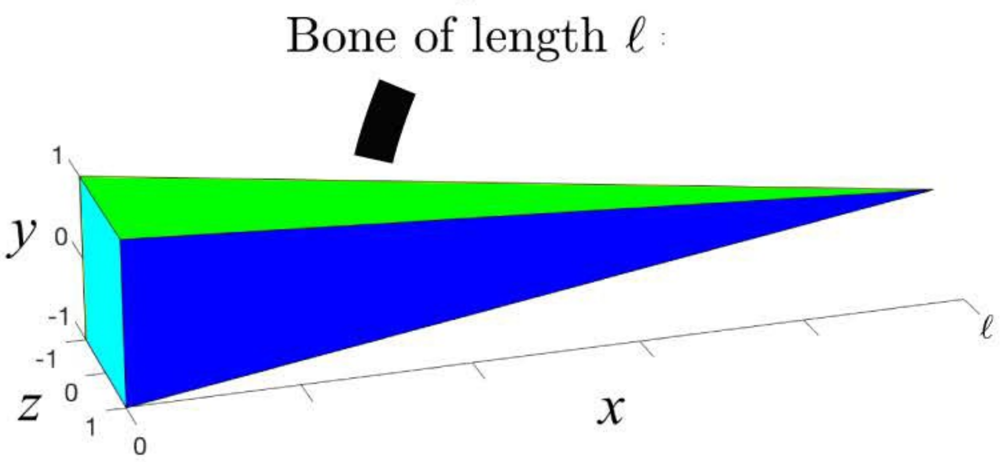
Bones in the Rest Pose

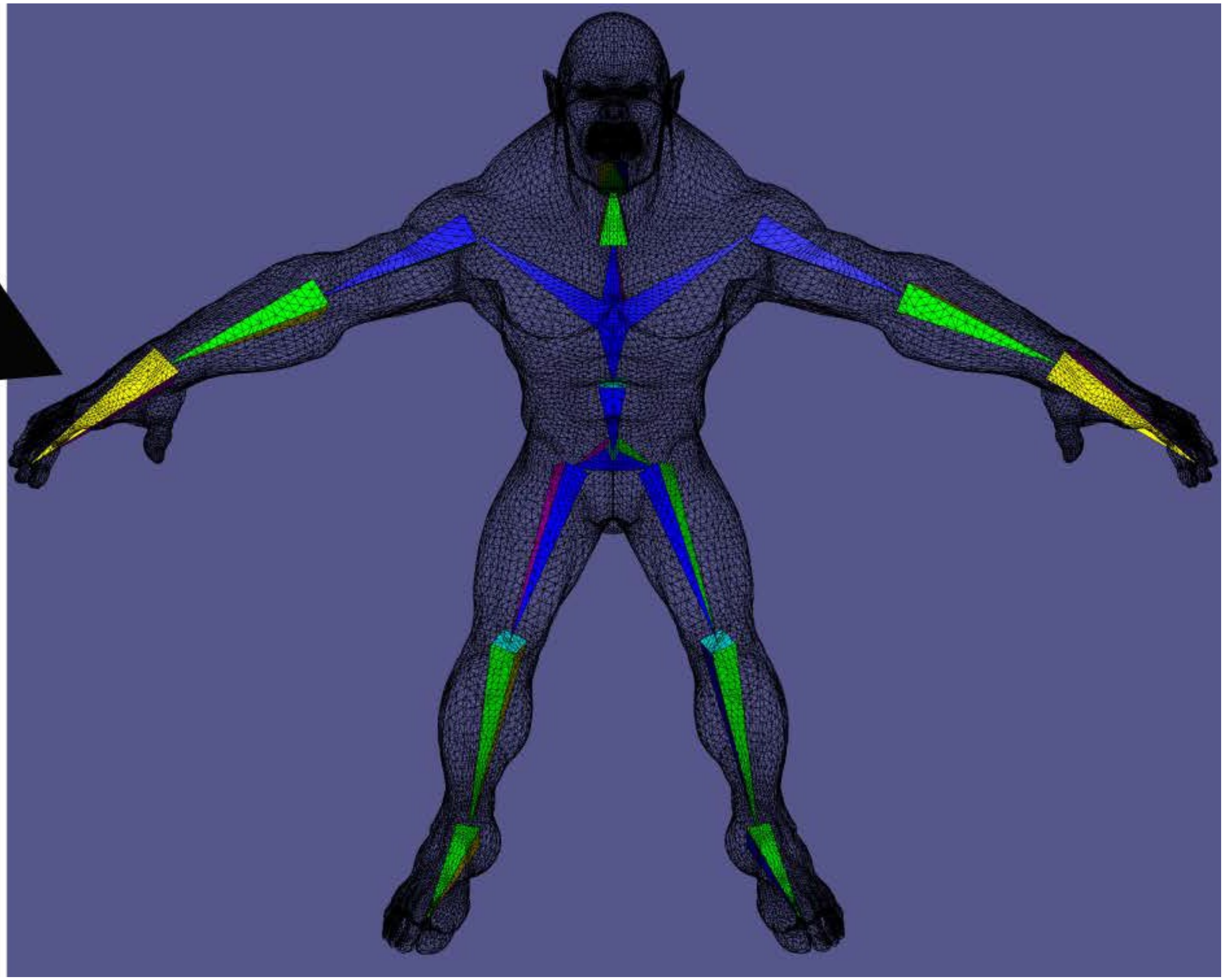
T = Transformation

R = Rotation

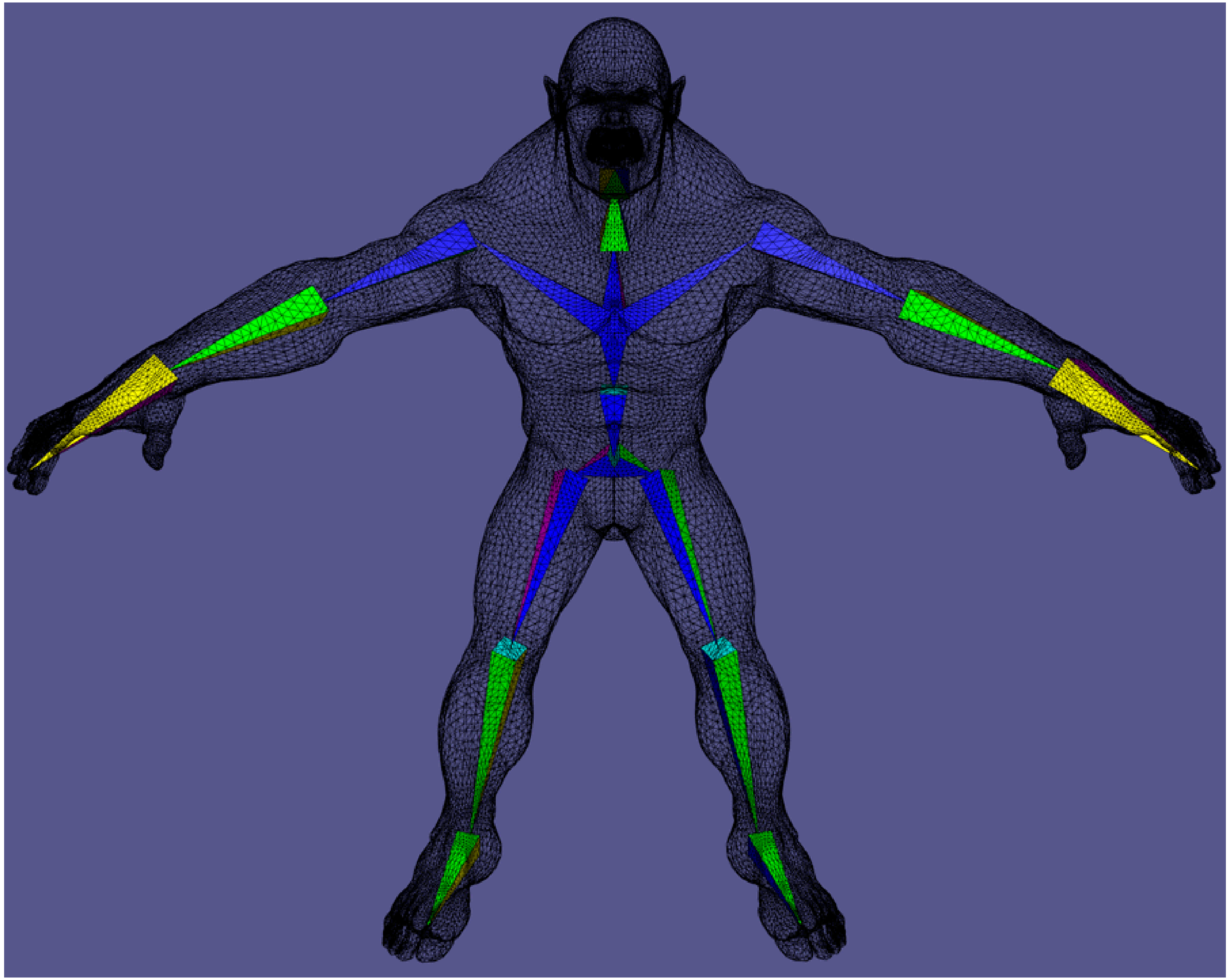
t = translation

$$\widehat{\mathbf{T}} = (\widehat{\mathbf{R}} \quad \hat{\mathbf{t}}) \in \mathbb{R}^{3\times 4}$$





Posing a Bone



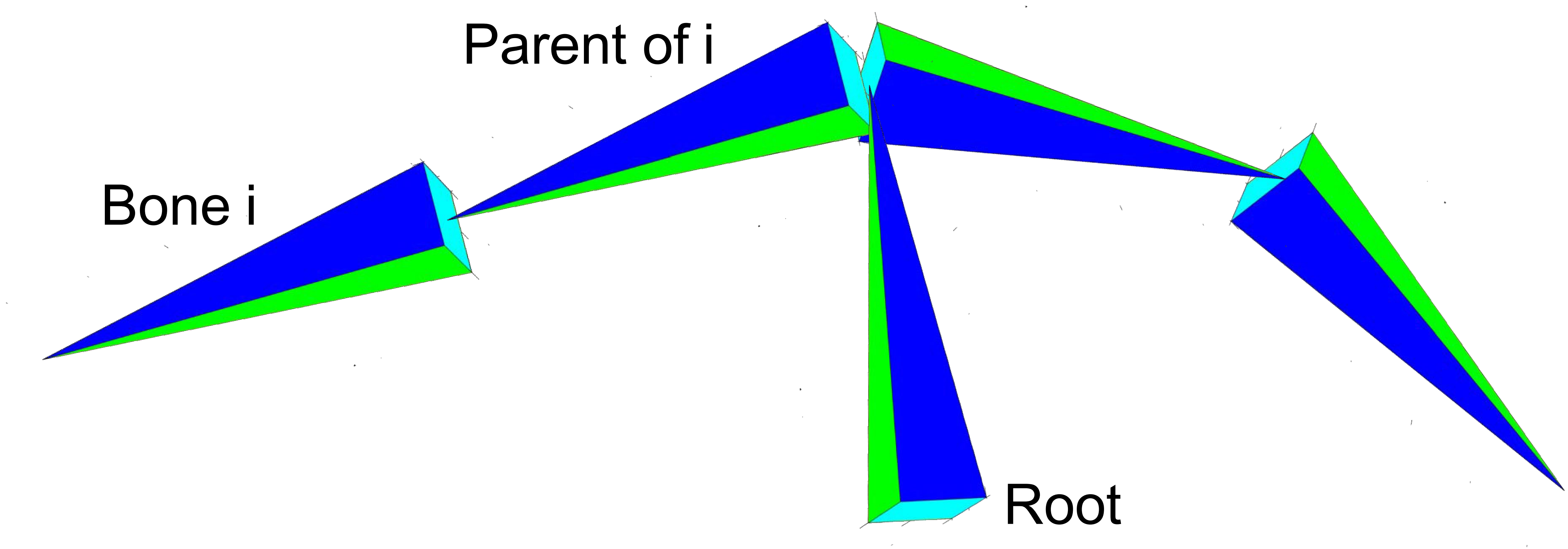
Forward Kinematics

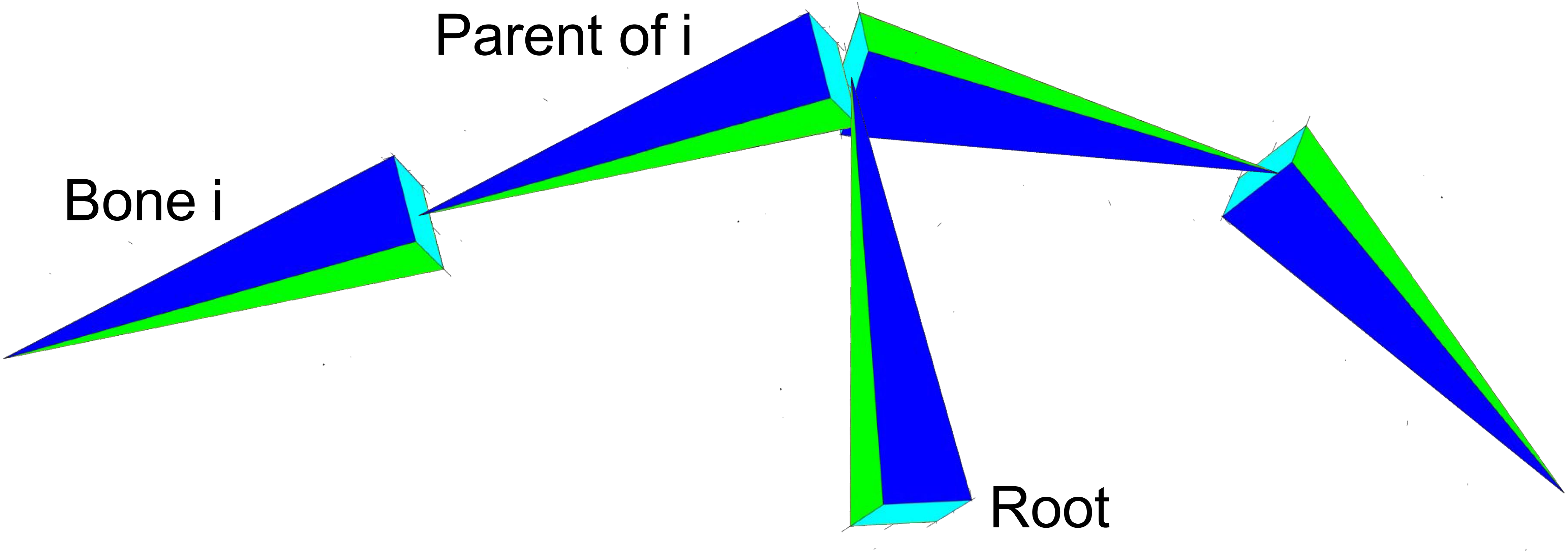
Kinematics – study of motion without consideration of what causes that motion

Forward Kinematics – Generate motion by setting all the bone positions by hand

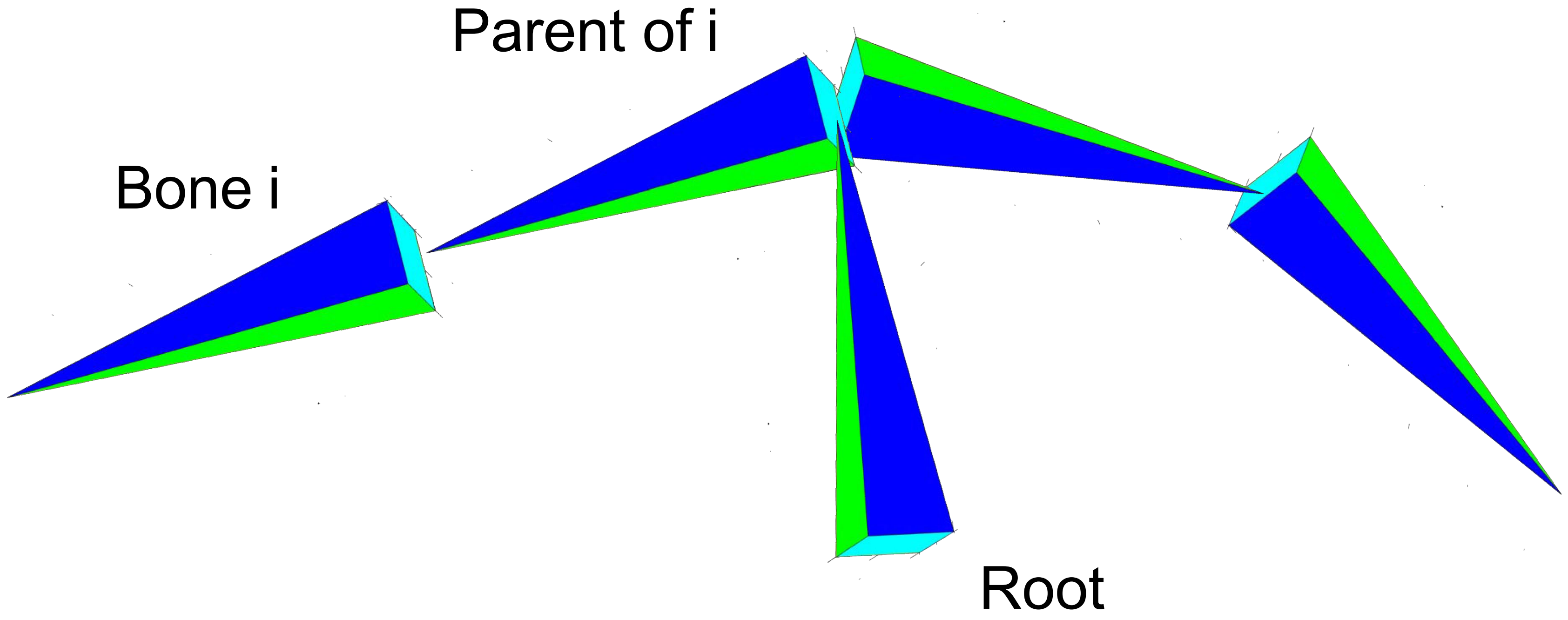
To do this we need a more rigourous understanding of how our bone motions are represented

https://en.wikipedia.org/wiki/Kinematics


Parent of i
Bone i
Root



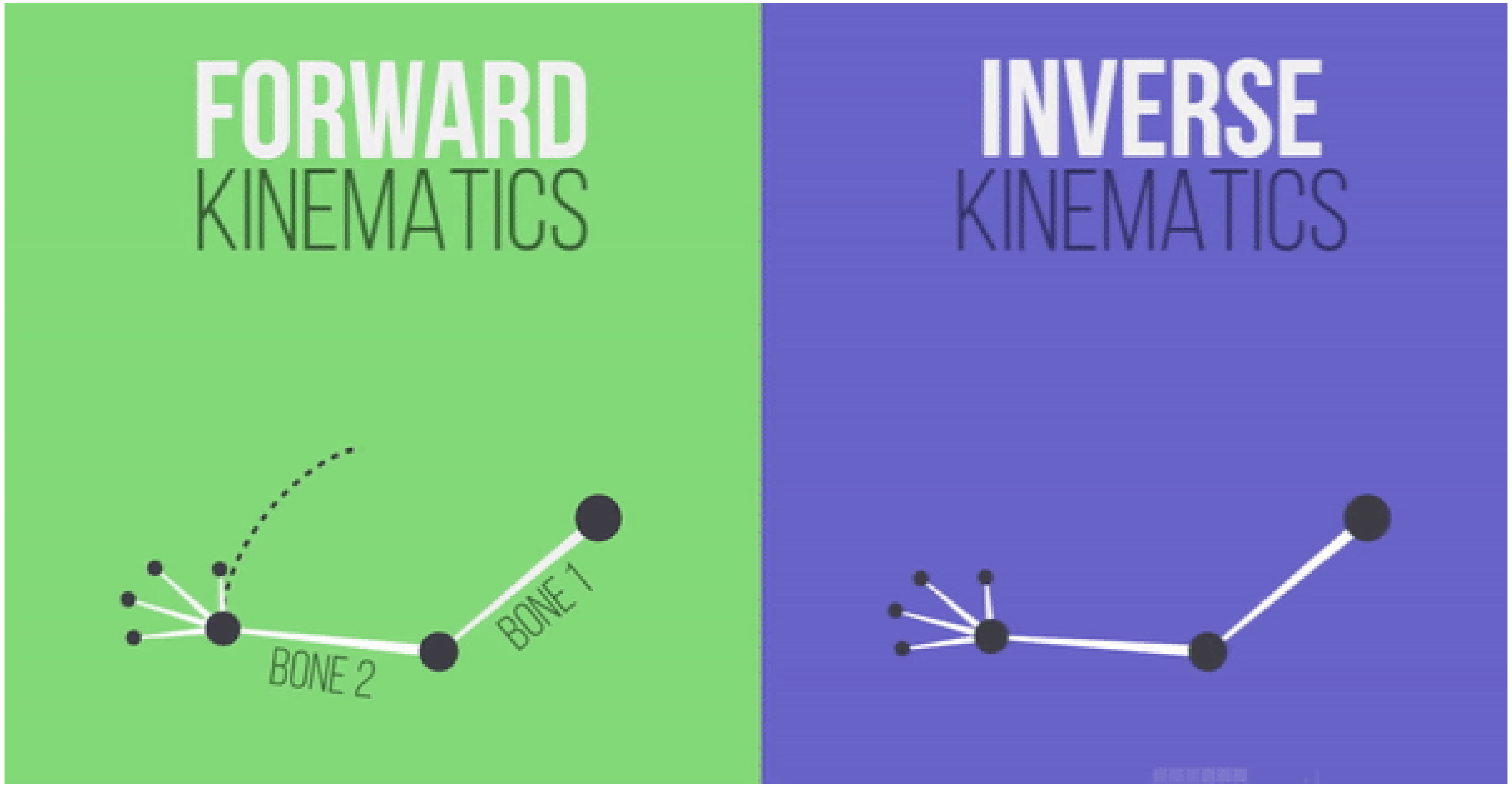
We will express bone transformations incrementally from their parent



$$\mathbf{x} = T\hat{\mathbf{x}}$$

$$T = (R \quad \hat{t}) \in \mathbb{R}^{3\times 4}$$

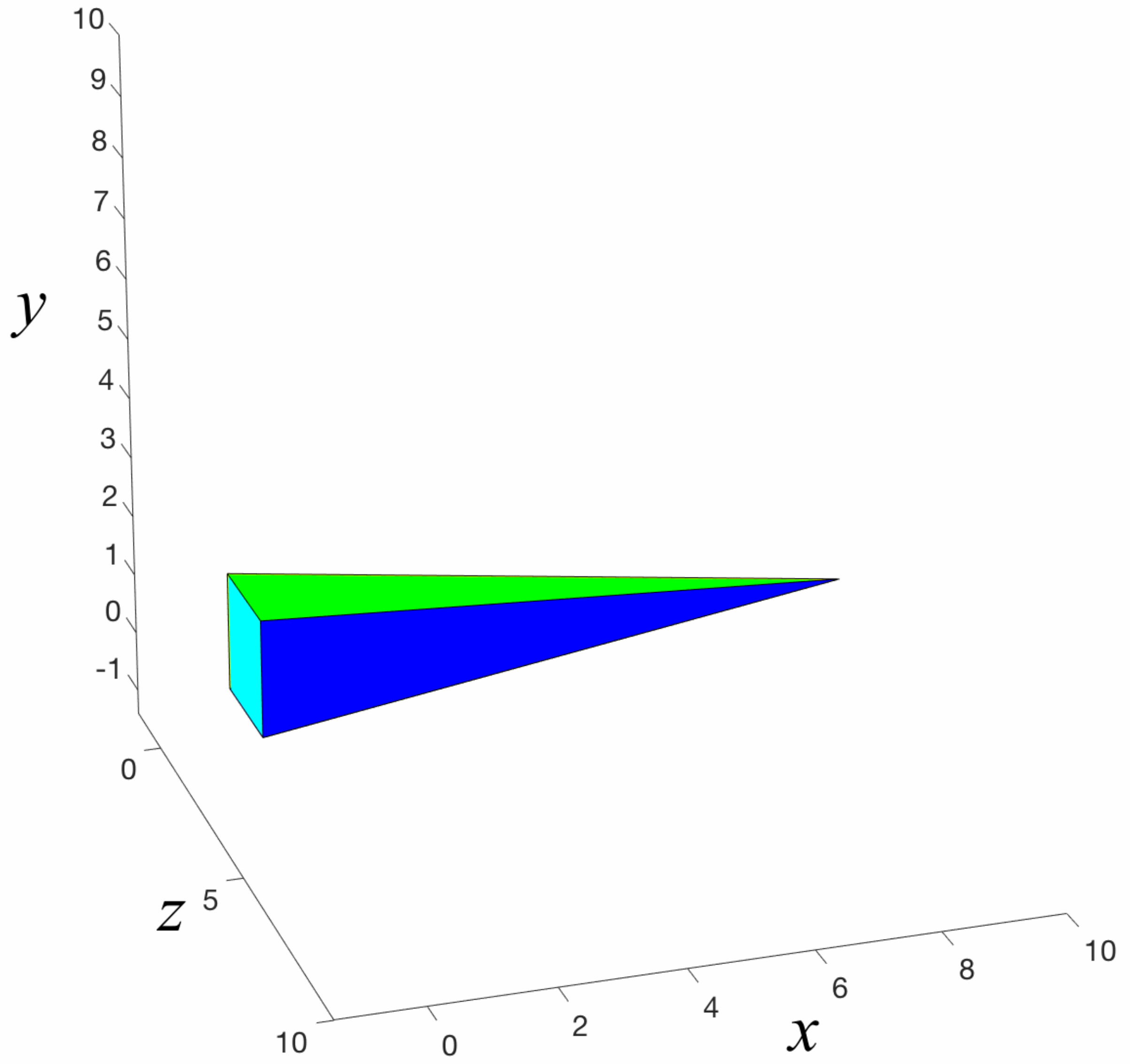
Forward Kinematics v Inverse Kinematics



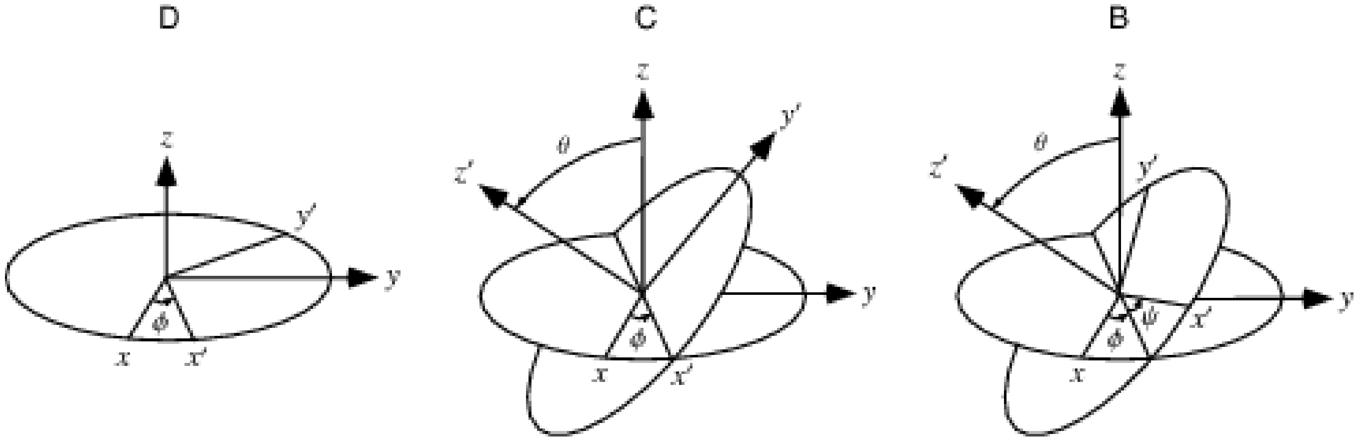
https://www.youtube.com/watch?v=0a9qIj7kwiA

Euler Angles

Twist-bend-twist: $(\theta_1, \theta_2, \theta_3) = (0^\circ, 0^\circ, 0^\circ)$



Euler Angles



The first rotation is by an angle ϕ about the z-axis using D.

$$A = BCD$$

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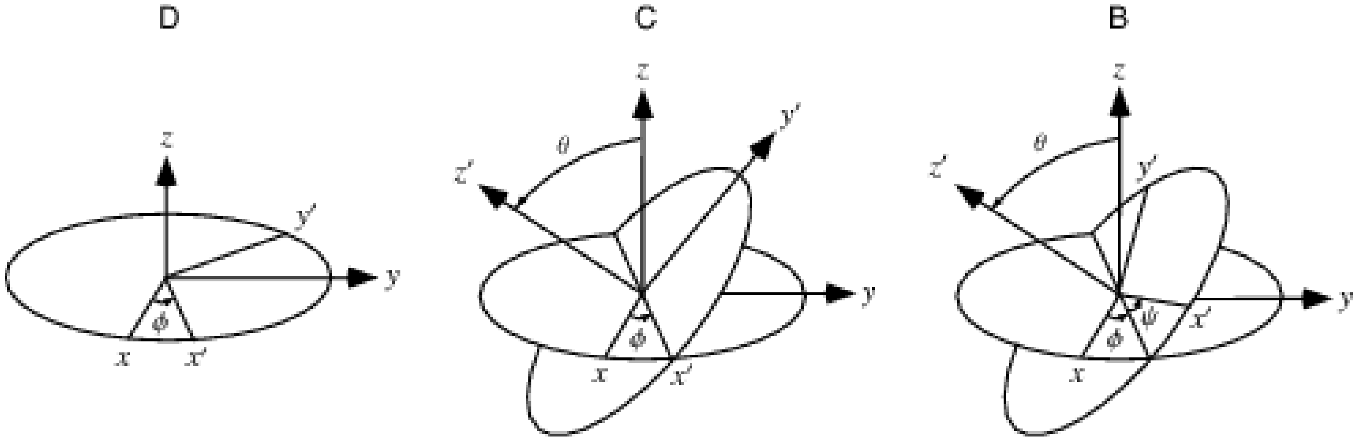
Euler Angles



$$A = BCD$$

The first rotation is by an angle ϕ about the z-axis using D.

The second rotation is by an angle θ about the x' axis using C.

https://en.wikipedia.org/wiki/Euler_angles

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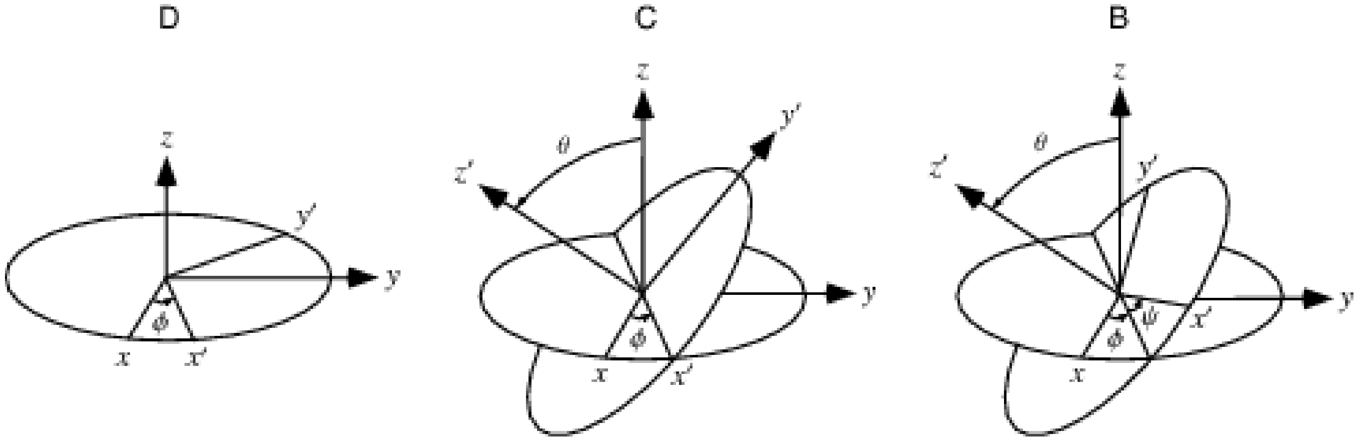
Euler Angles



$$A = BCD$$

The first rotation is by an angle ϕ about the z-axis using D.

The second rotation is by an angle θ ∊ [0, π]about the x' axis using C.

The third rotation is by an angle ψ about the z' axis using B.

https://en.wikipedia.org/wiki/Euler_angles

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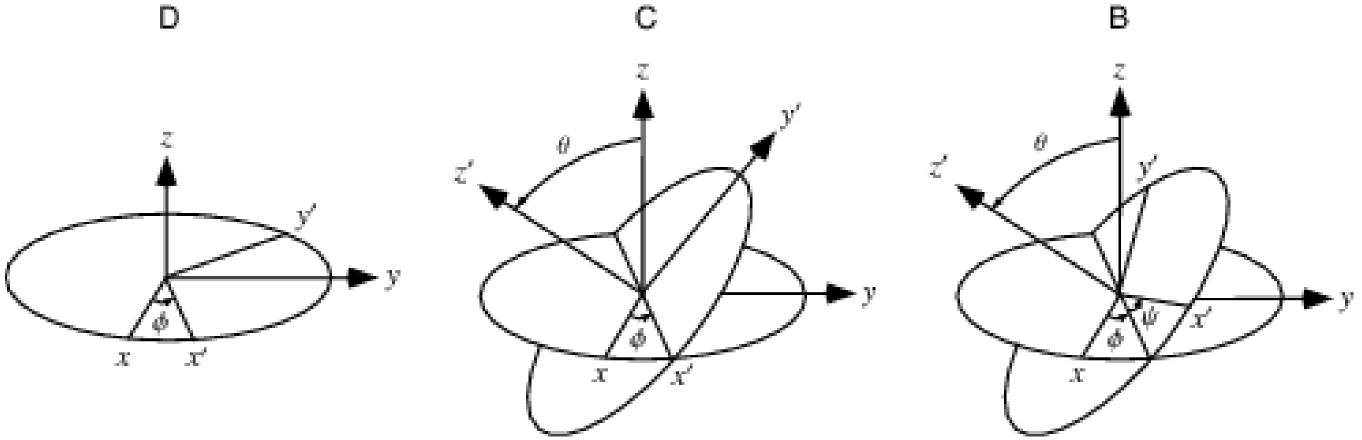
Euler Angles



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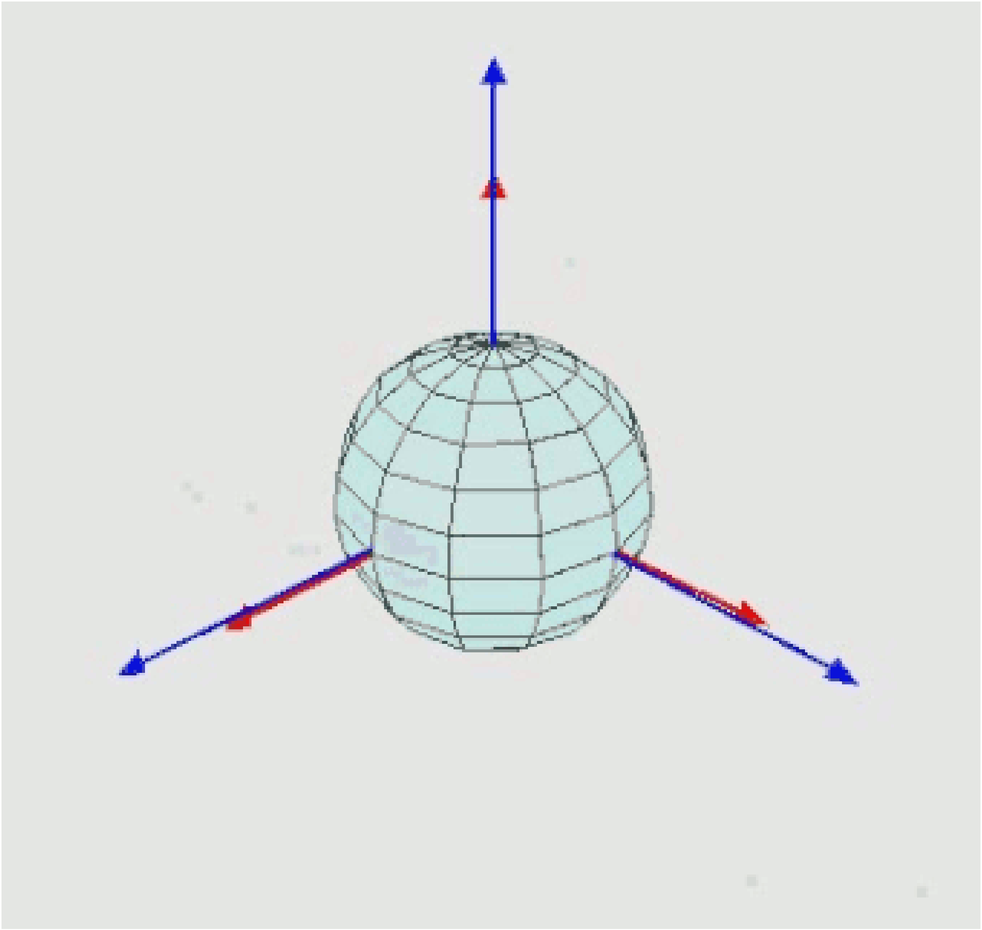

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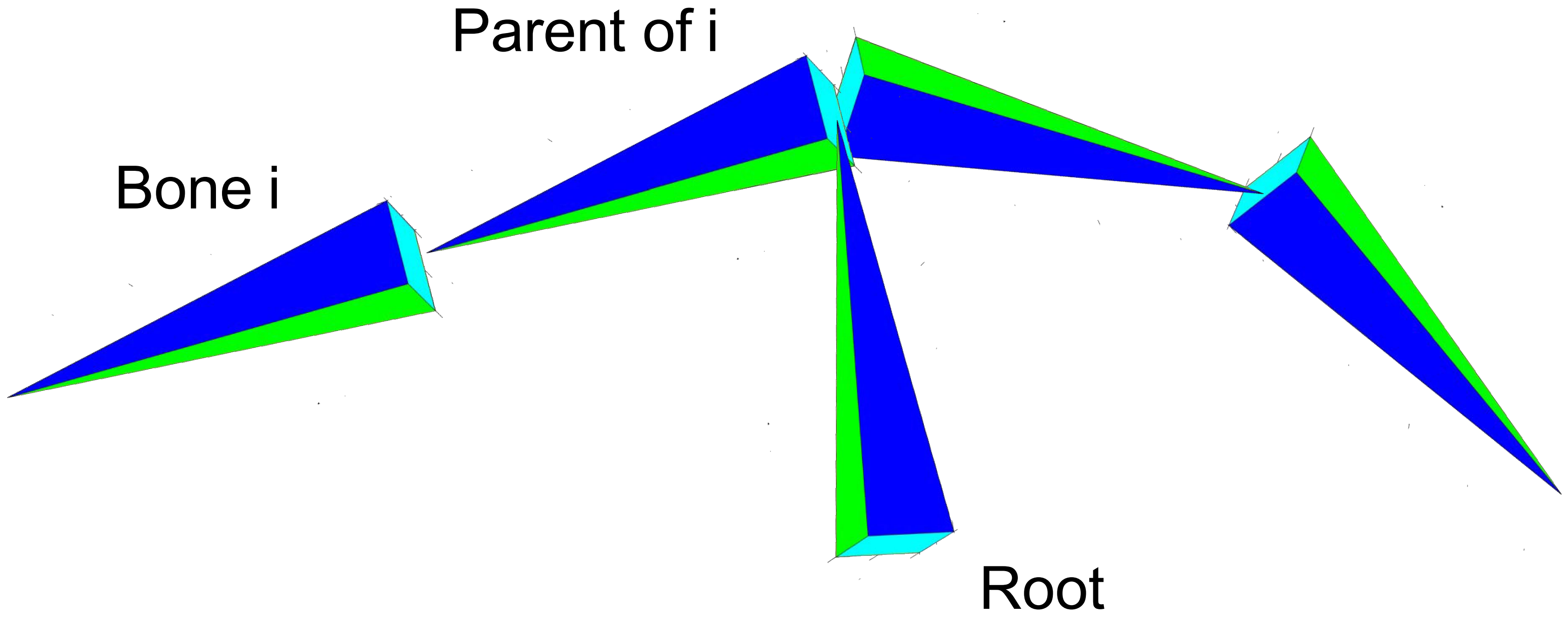
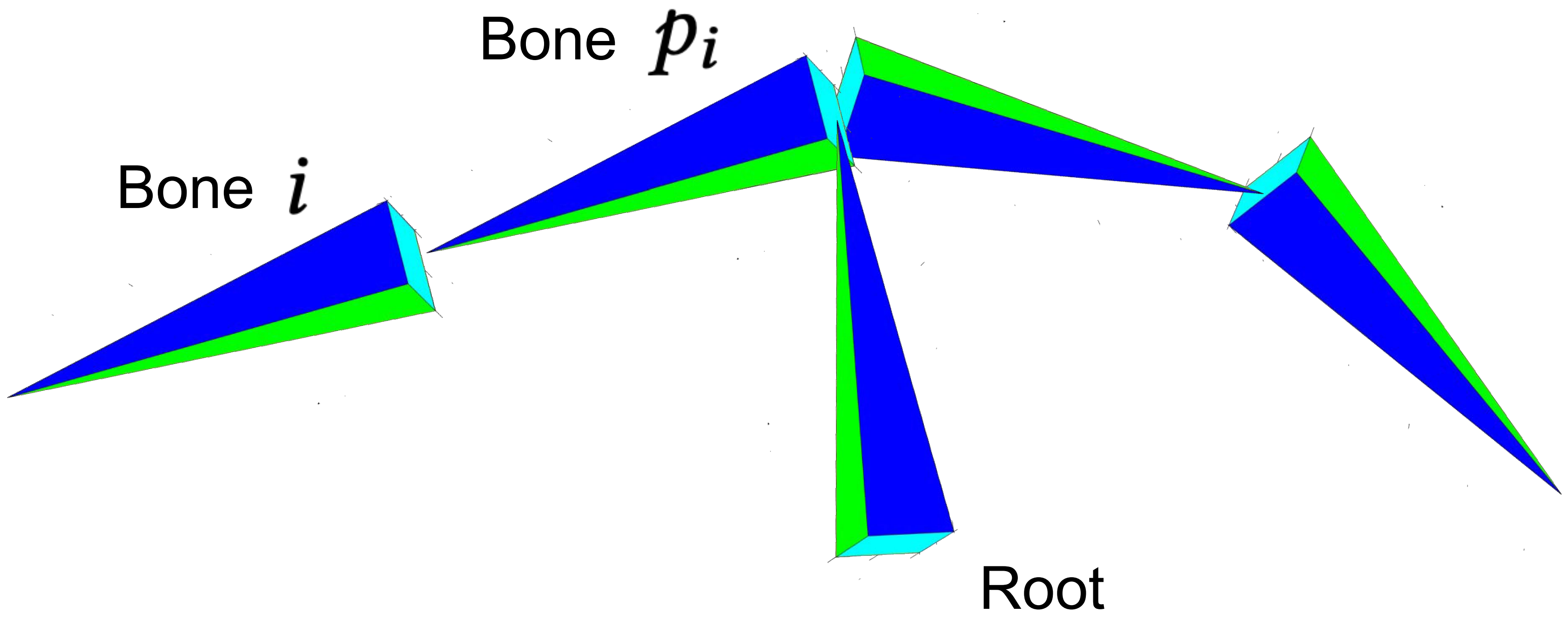


$$\mathbf{x} = T\hat{\mathbf{x}}$$

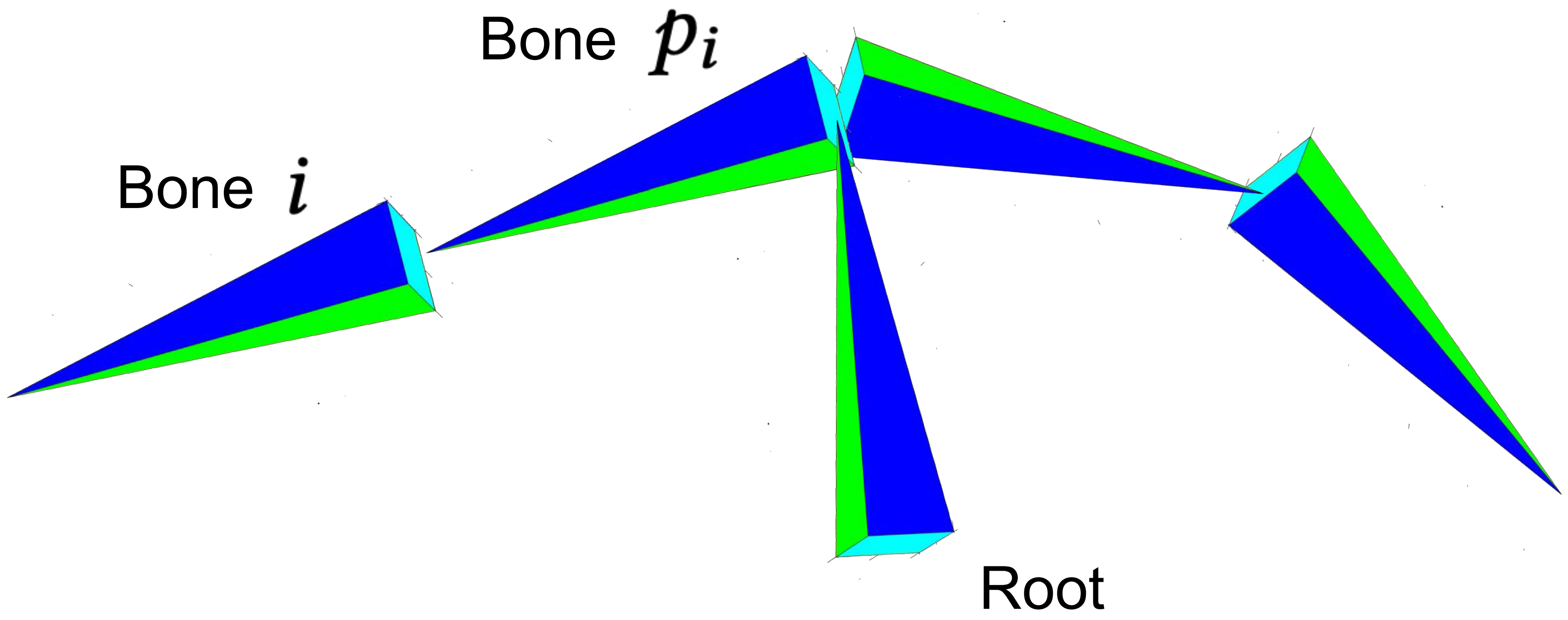
$$T = \begin{pmatrix} R & \hat{t} \end{pmatrix} \in \mathbb{R}^{3\times 4}$$



$$T = (R \quad \hat{t}) \in \mathbb{R}^{3\times4}$$

Need to determine $T_i \in \mathbb{R}^{3\times4}$

aggregate *relative rotations* $\bar{R}_i \in \mathbb{R}^{3\times3}$ ← Computed recursively!



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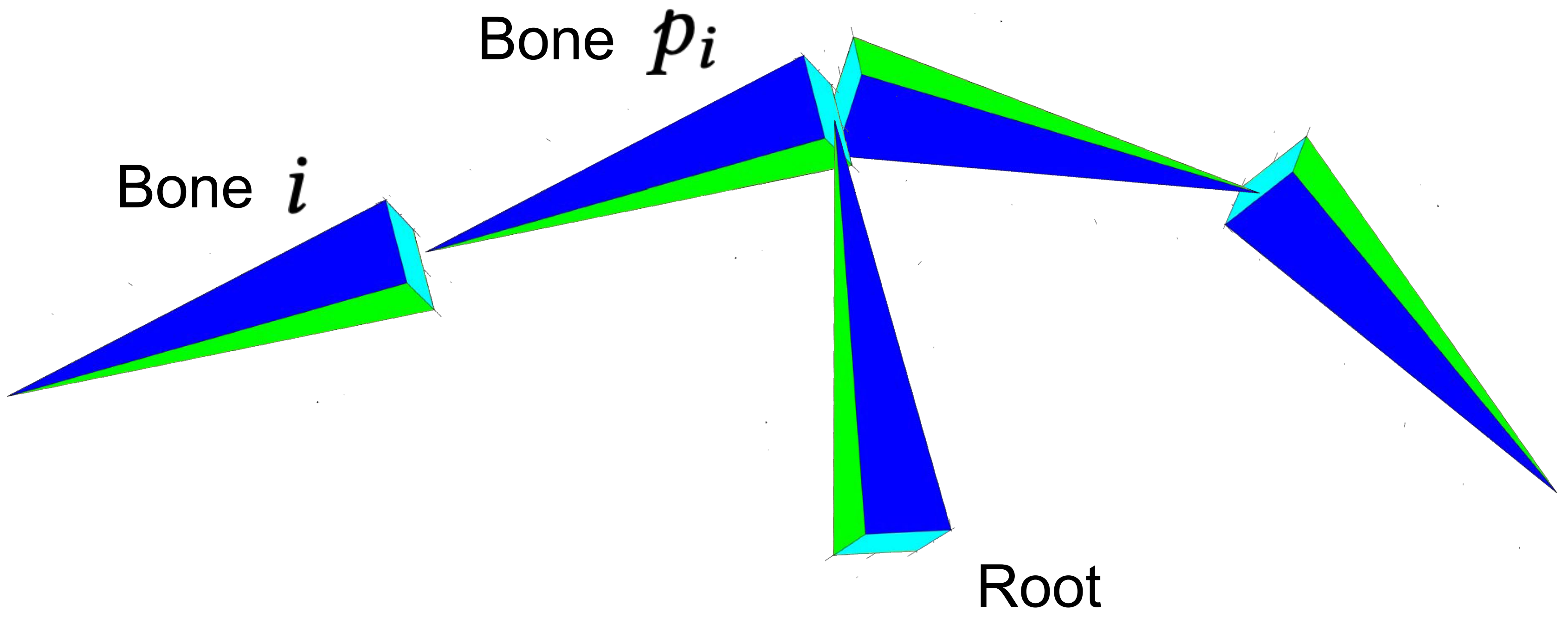

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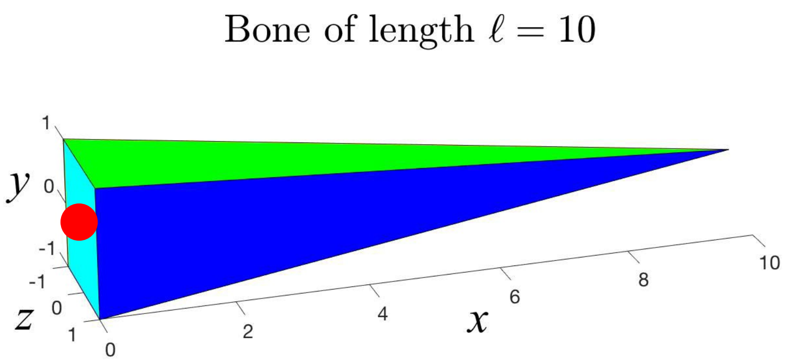

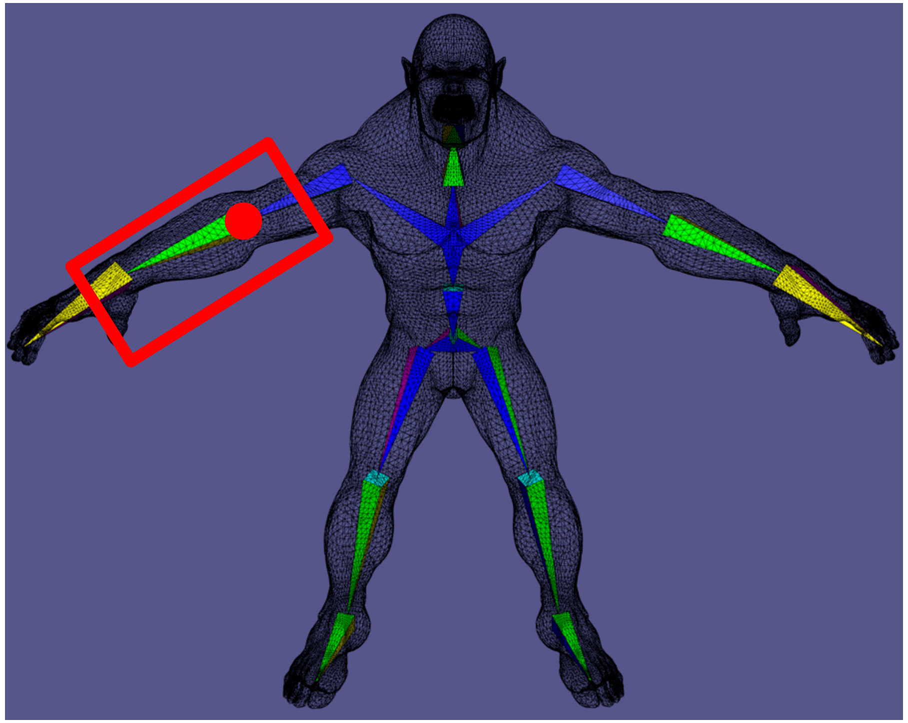

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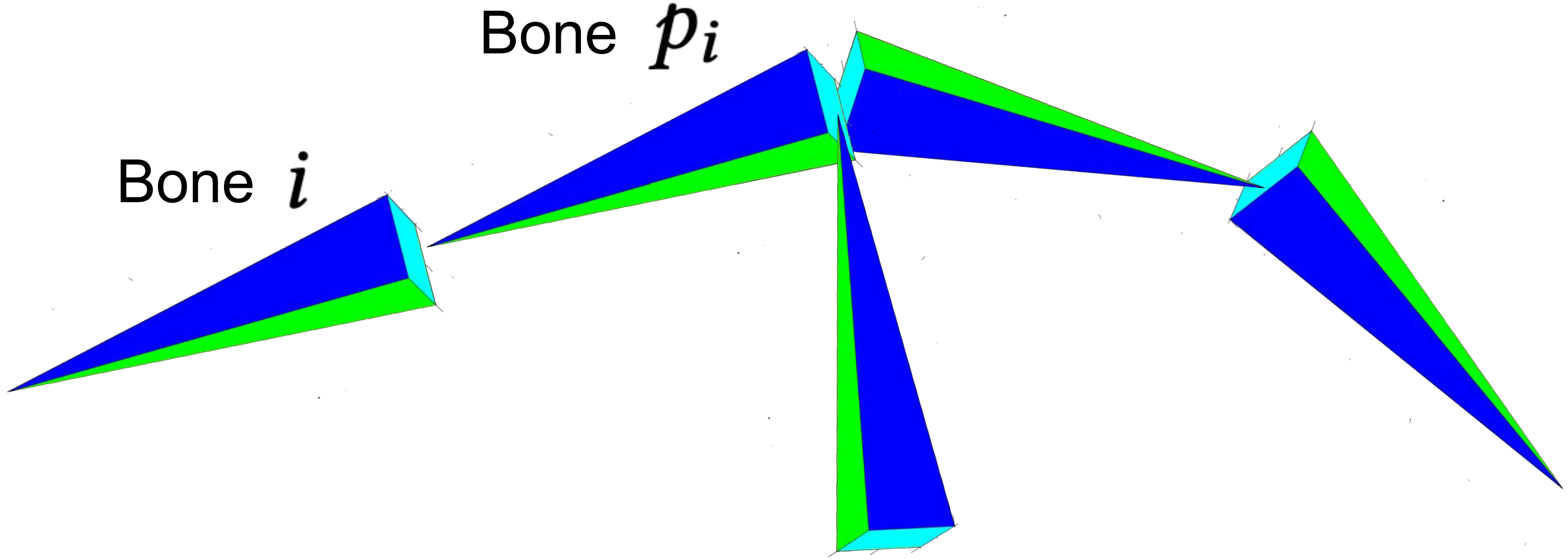

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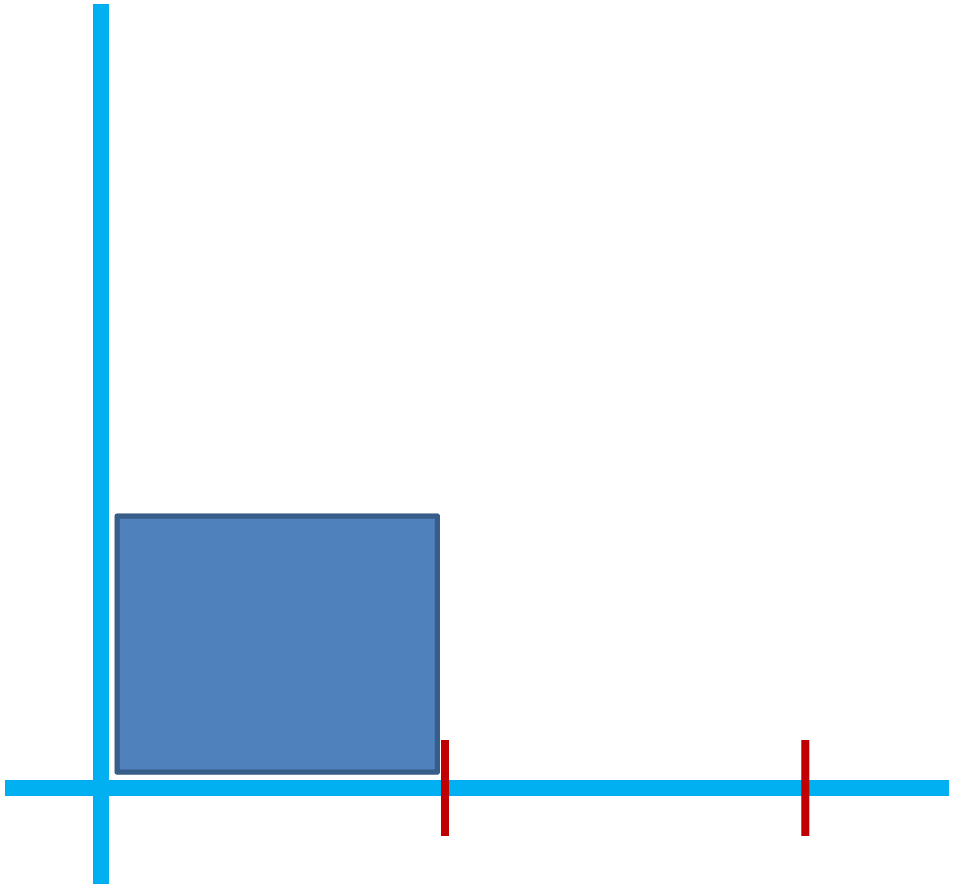
Order of operations

rotate 45 degrees
then translate by 1



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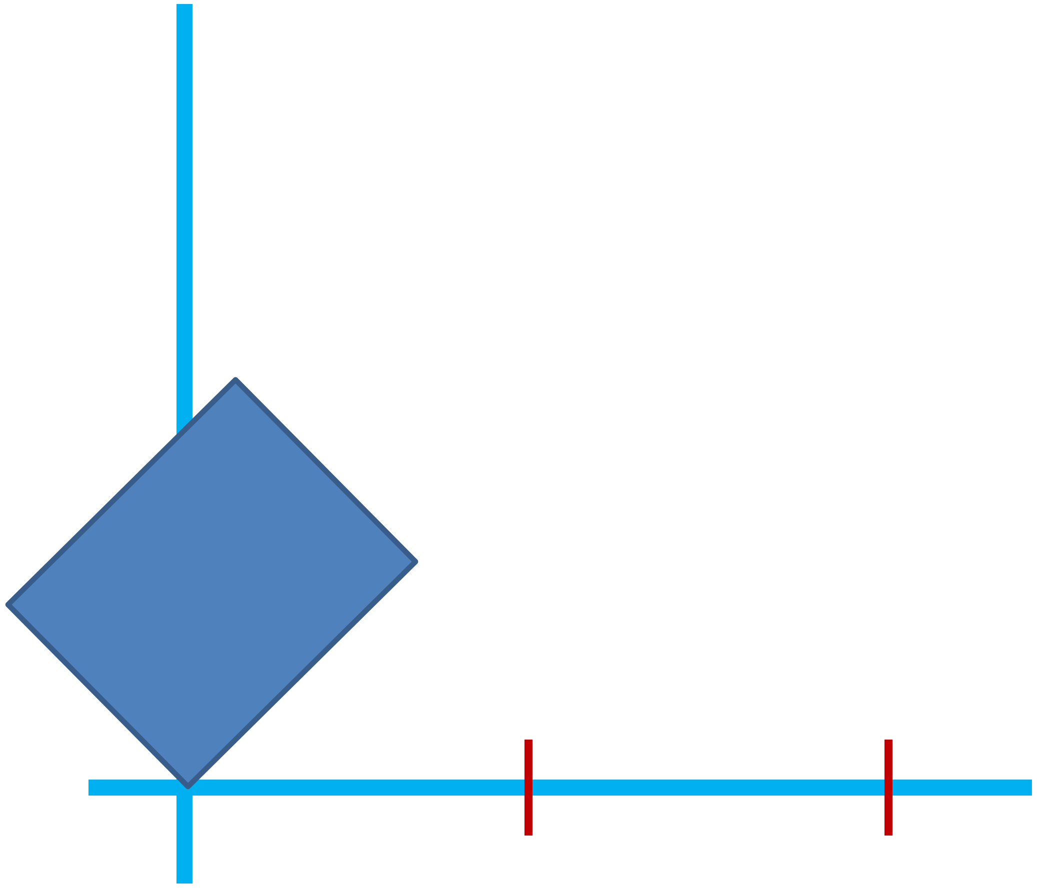
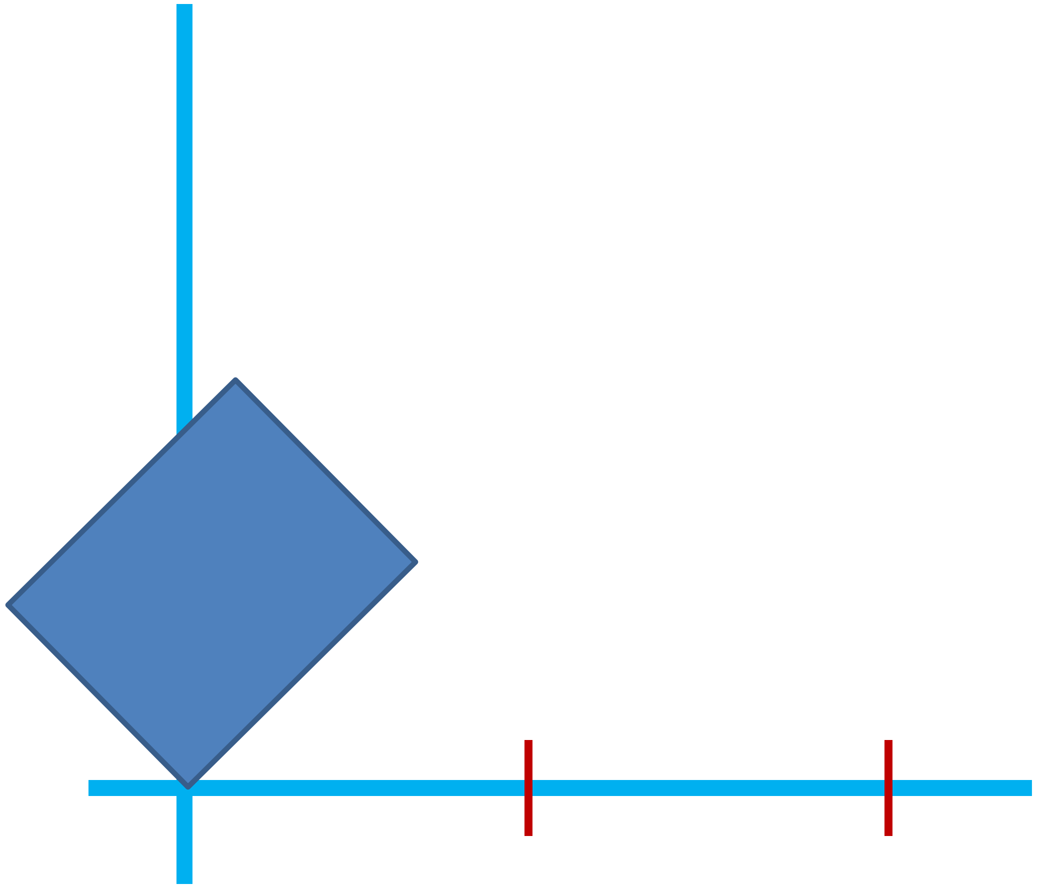


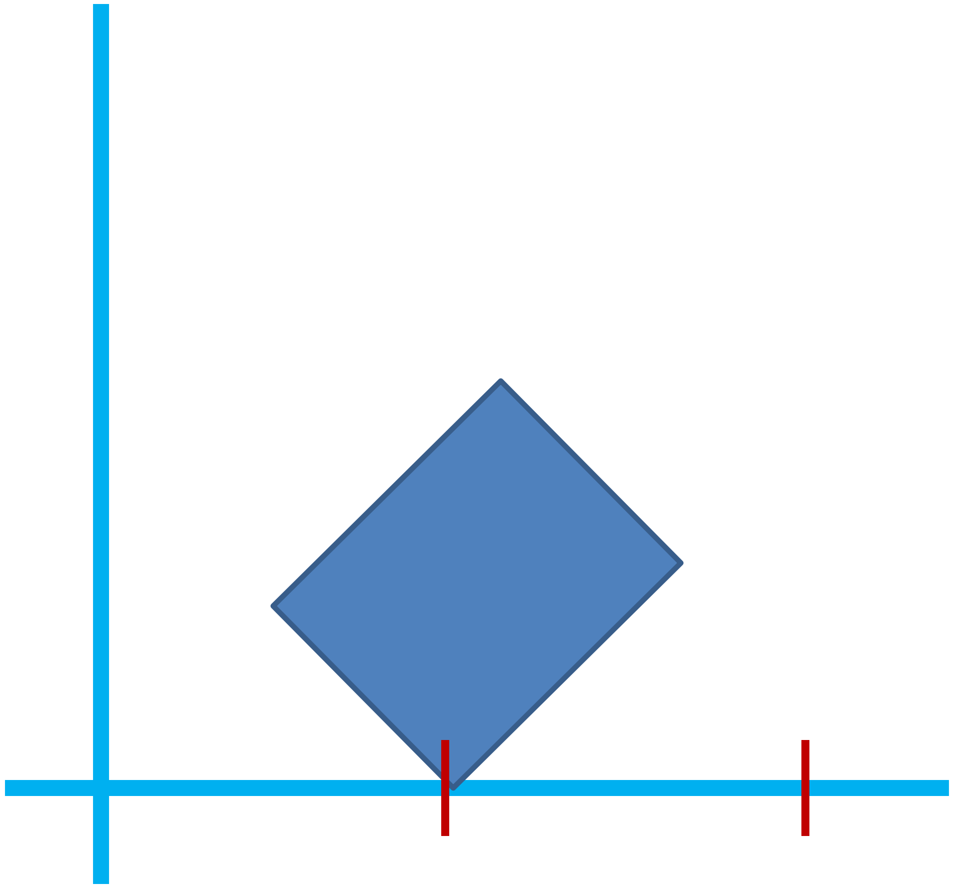
Order of operations



rotate 45 degrees
then translate by 1

rotation matrices
rotate about the
origin!

Order of operations



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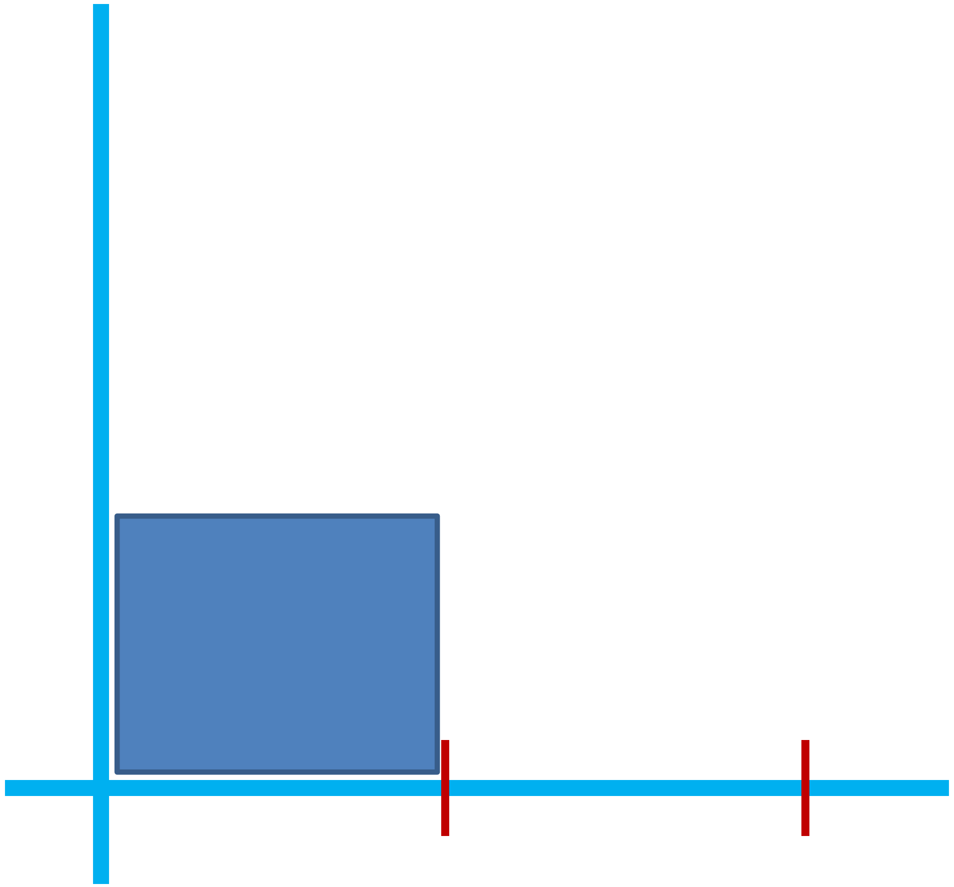

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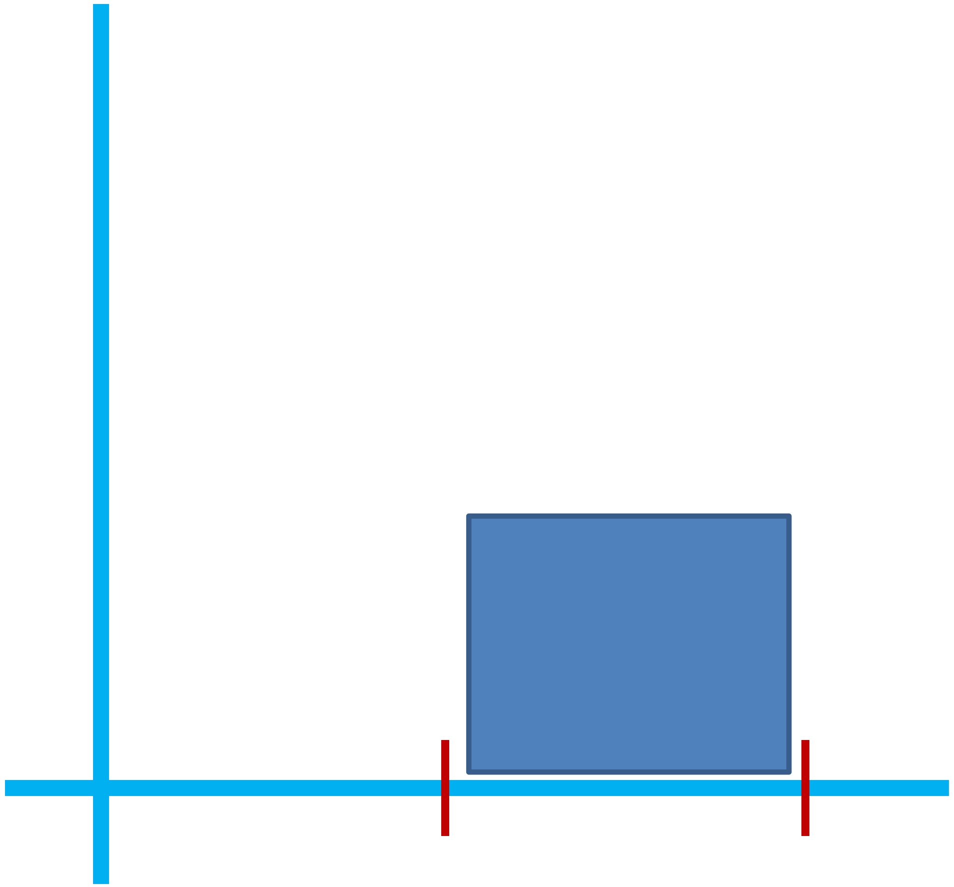

Order of operations

translate by 1 then
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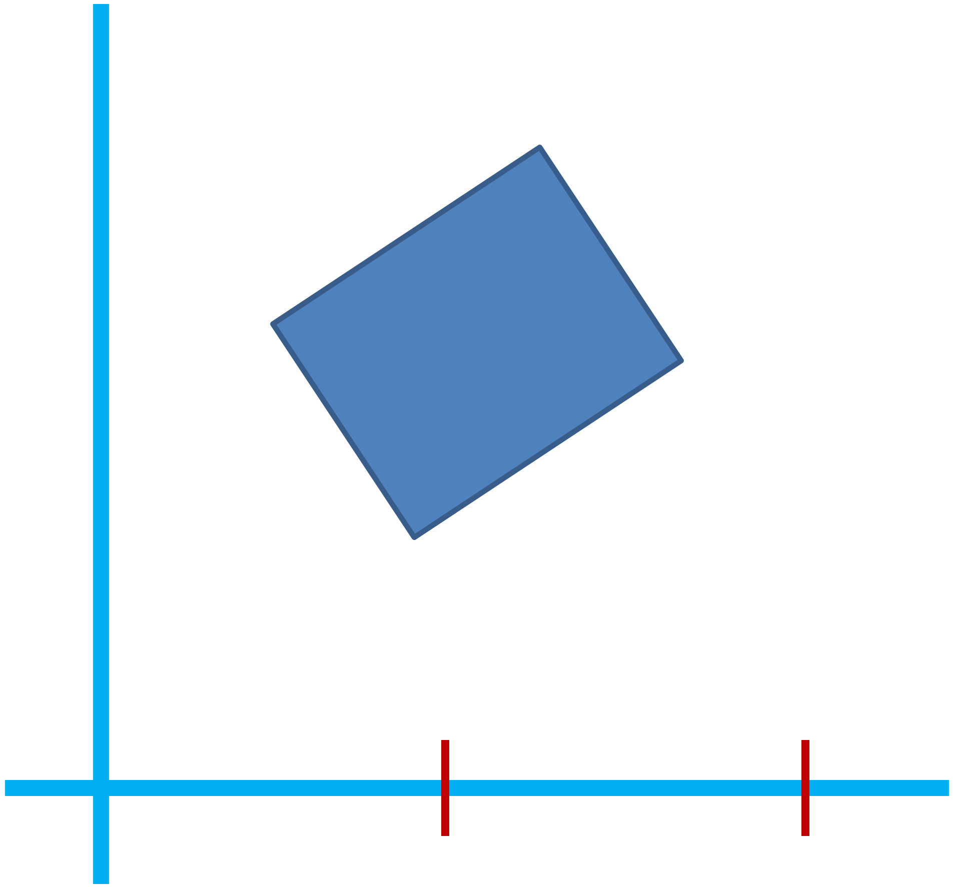
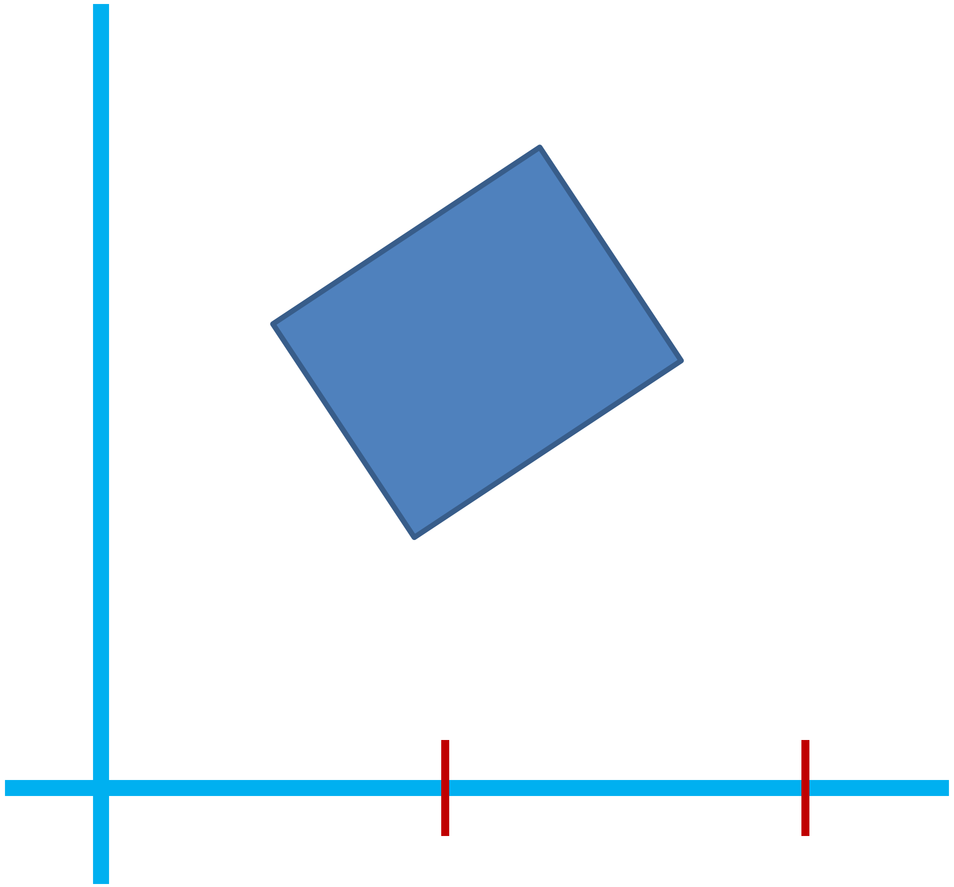
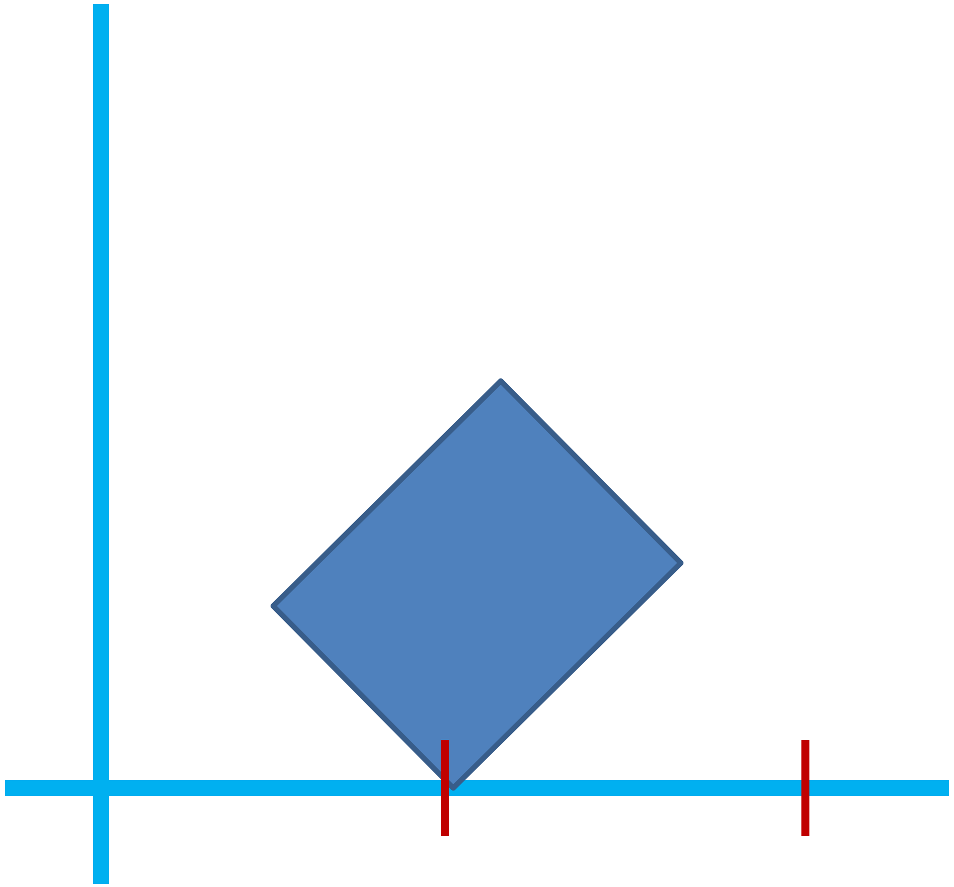
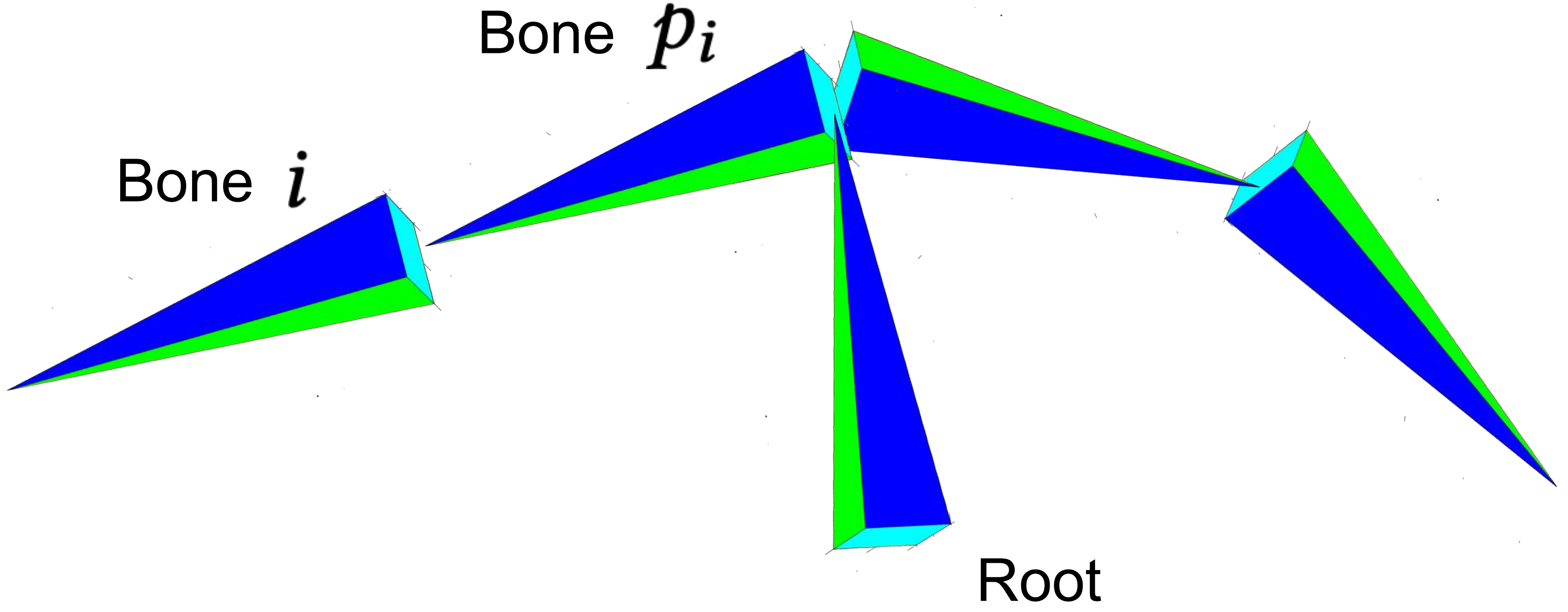


Order of operations

Order matters!







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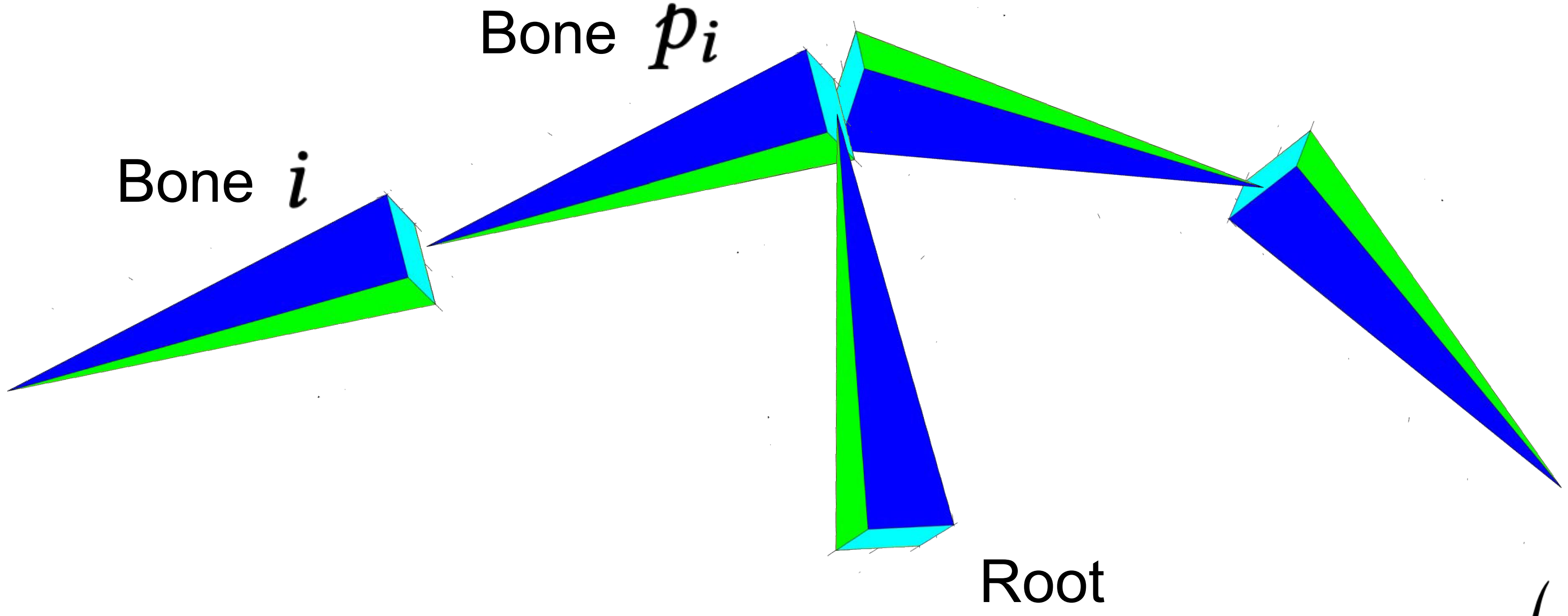
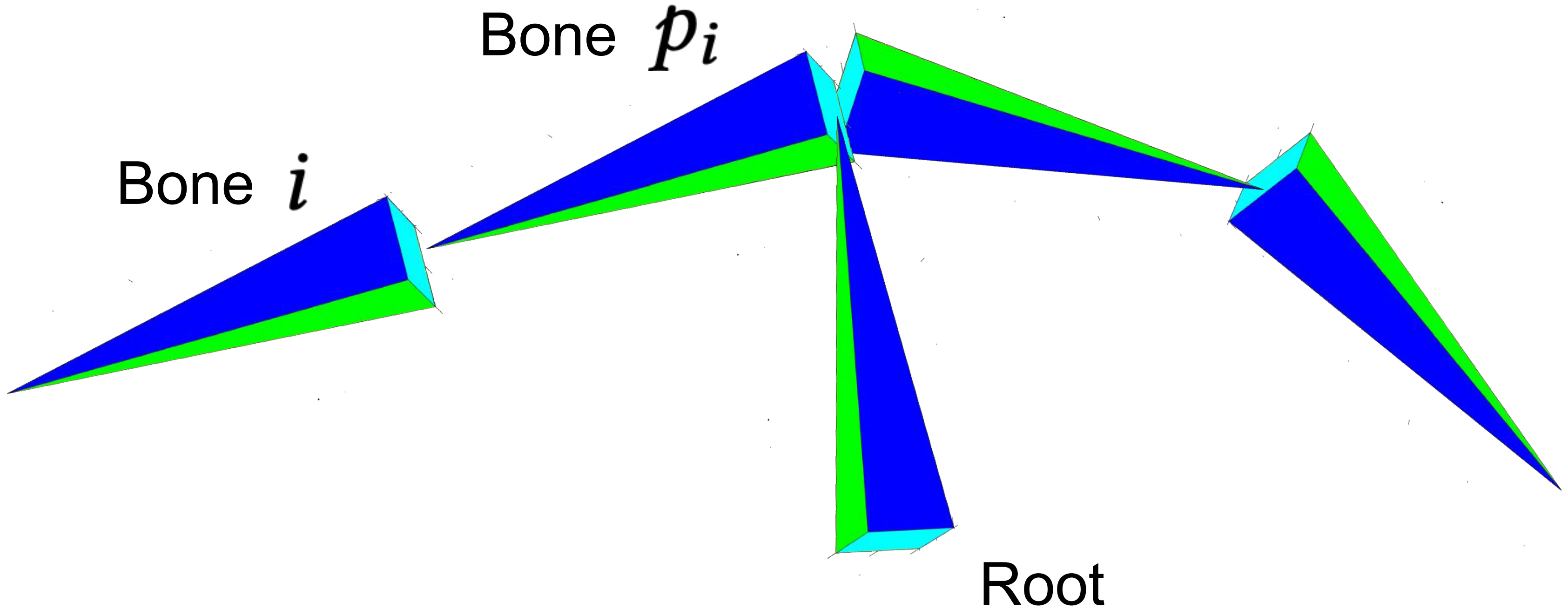


$$\mathbf{T}_i = \mathbf{T}_{p_i} \begin{pmatrix} \widehat{\mathbf{T}}_i \\ 0\ 0\ 0\ 1 \end{pmatrix} \begin{pmatrix} \overline{\mathbf{R}}_i & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ 0\ 0\ 0 & 1 \end{pmatrix} \begin{pmatrix} \widehat{\mathbf{T}}_i \\ 0\ 0\ 0\ 1 \end{pmatrix}^{-1}$$

Need to determine $T_i \in \mathbb{R}^{3\times 4}$



$$\mathbf{T}_i = \mathbf{T}_{p_i} \widehat{\mathbf{T}}_i \begin{pmatrix} \mathbf{R}_x(\theta_{i3}) & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ 0\ 0\ 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{R}_z(\theta_{i2}) & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ 0\ 0\ 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{R}_x(\theta_{i1}) & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ 0\ 0\ 0 & 1 \end{pmatrix} \widehat{\mathbf{T}}_i^{-1}$$

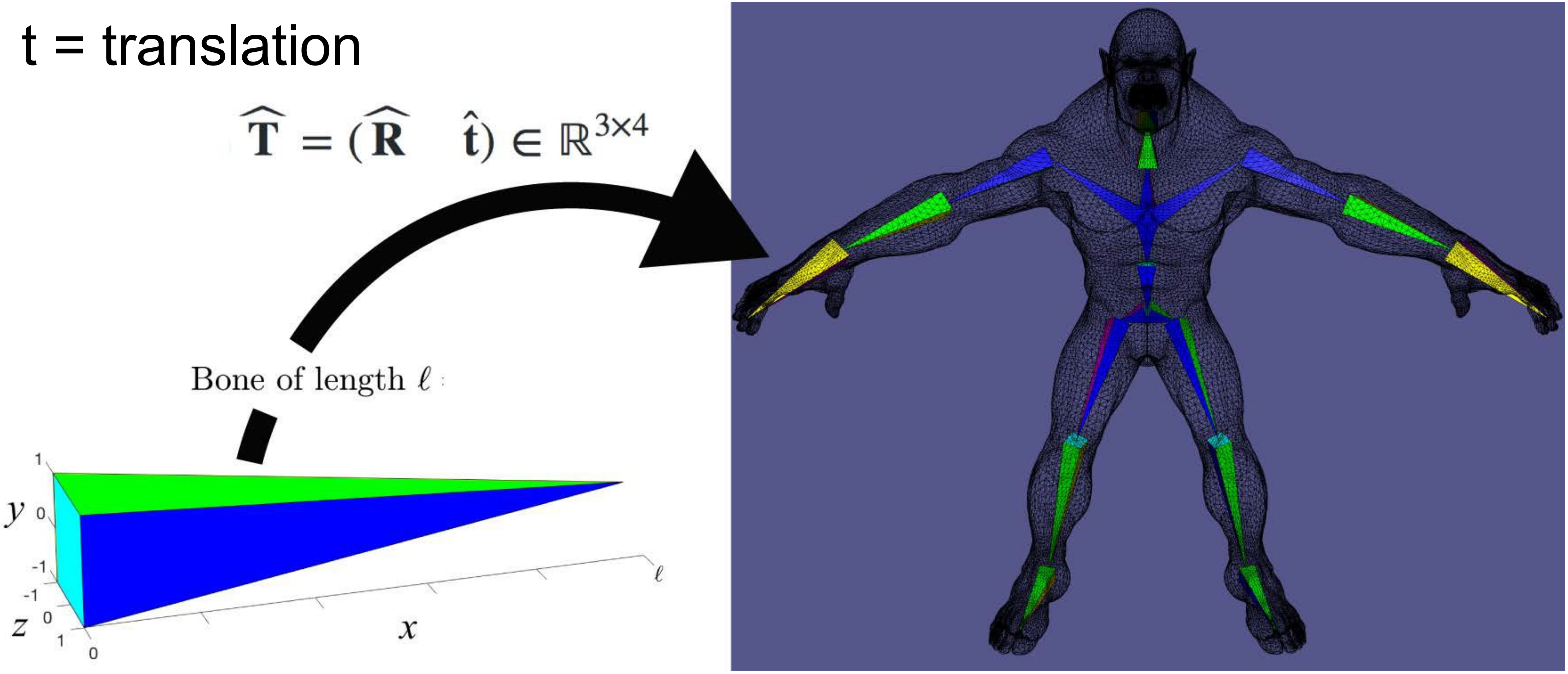
Connecting Bones to Vertices

T = Transformation

R = Rotation

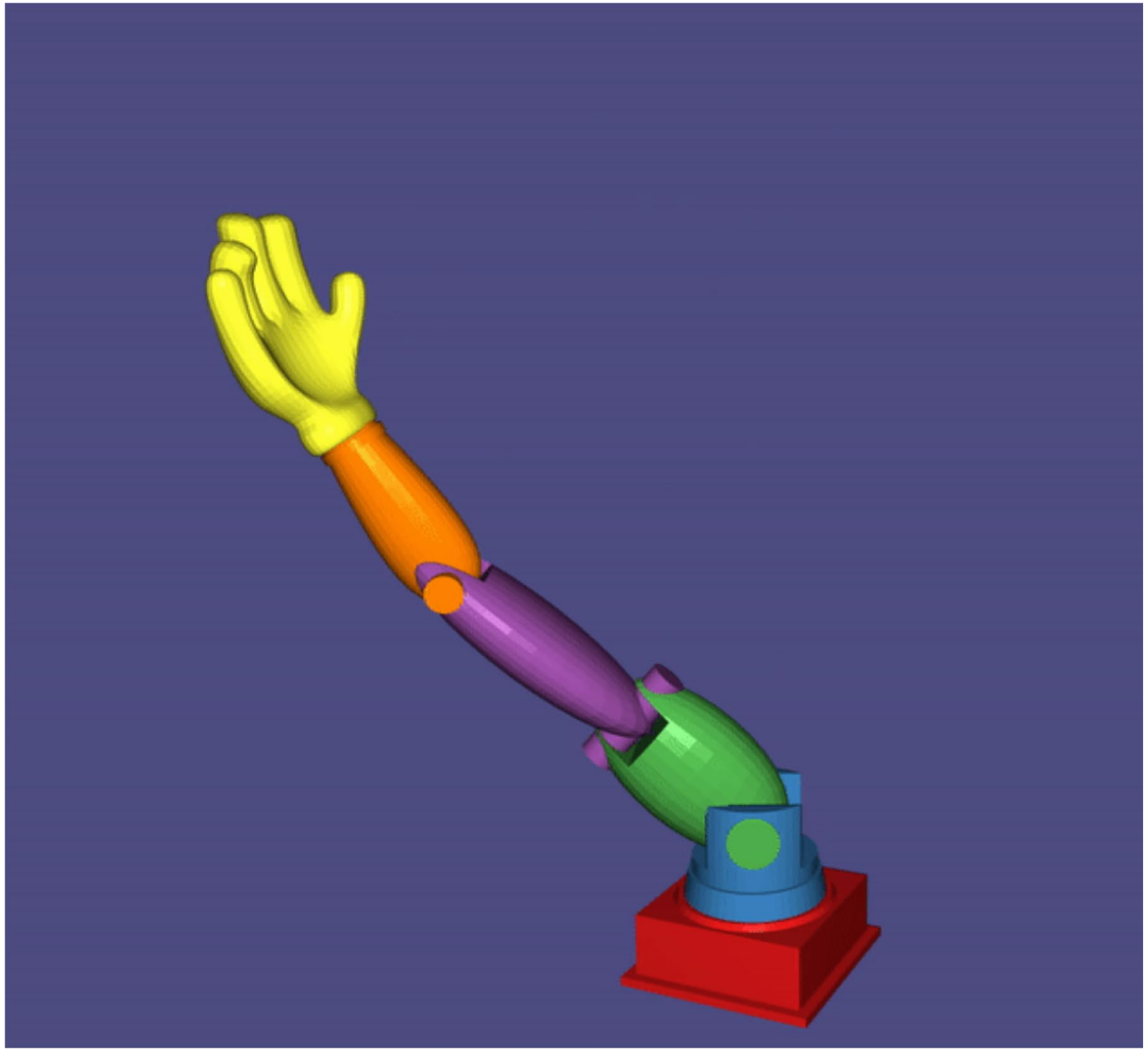
t = translation

$$\widehat{\mathbf{T}} = (\widehat{\mathbf{R}} \quad \hat{\mathbf{t}}) \in \mathbb{R}^{3\times 4}$$



Rigid "Skinning"

Idea: Attach each vertex to a single bone



$$\mathbf{v}_j = \mathrm{T}_i \hat{\mathbf{v}}_j$$

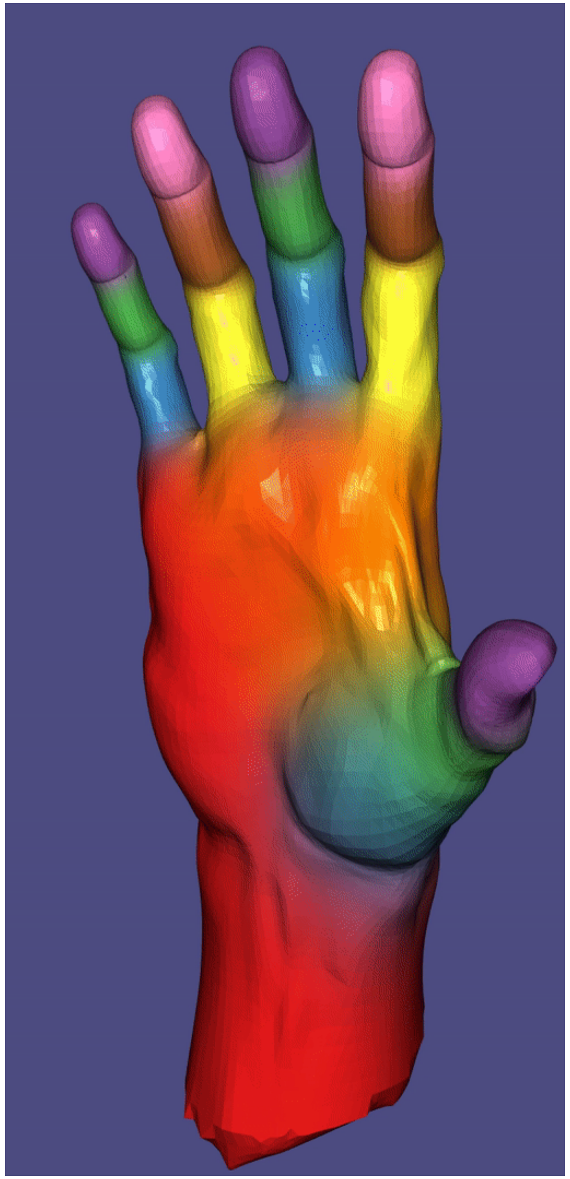
Deformable Skinning

Rigid Skinning is fine for mechanical things, but for smoother deformations we need to try something else

Rather than attach each vertex to a single bone, we attach each vertex to multiple bones and ***blend*** their transformations

If this blending is linear in the transformations, we call it linear blend skinning.

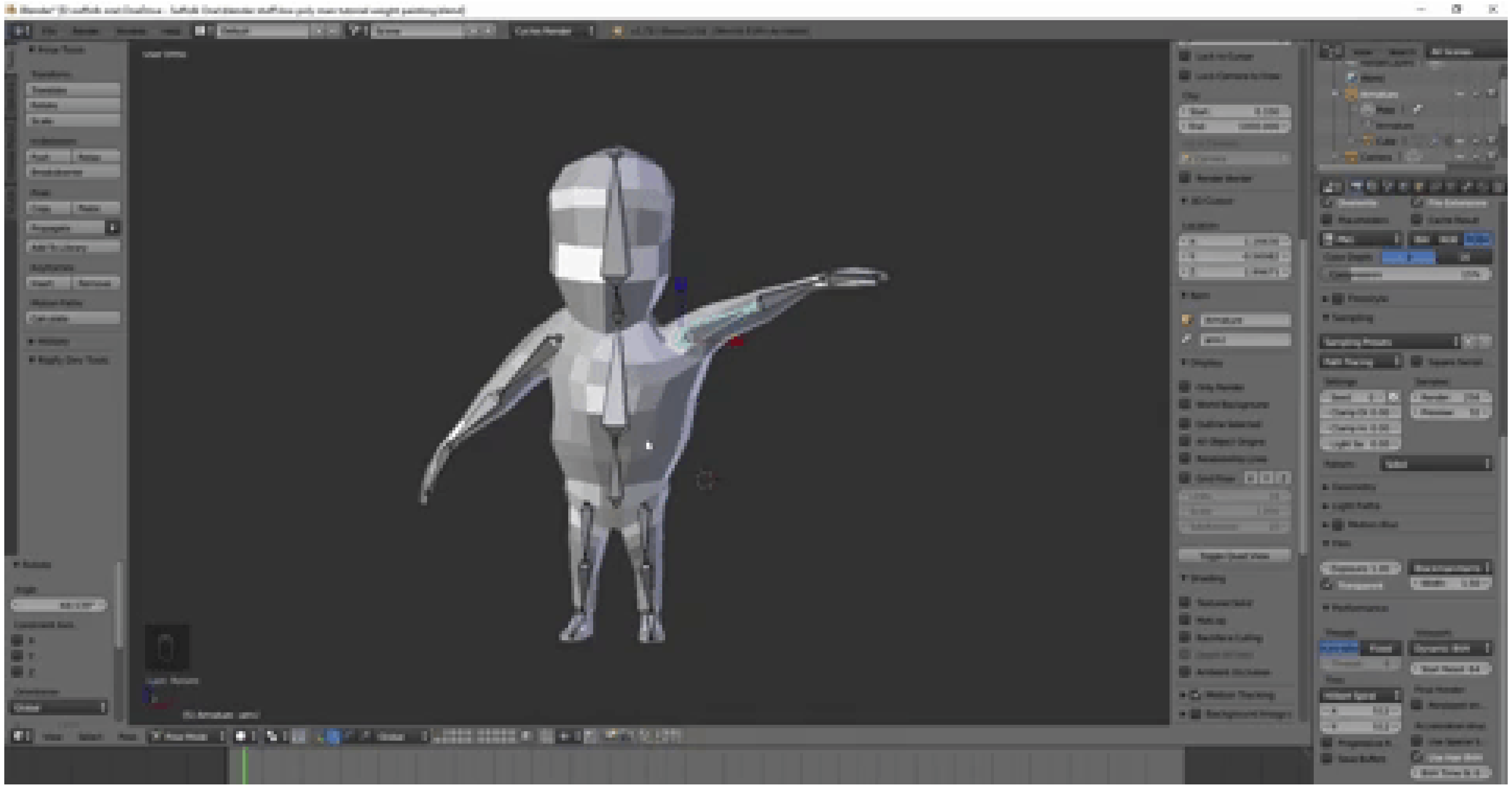
Linear Blend Skinning



$$\mathbf{v}_j = \sum_{i=1}^{\#\text{bones}} w_{ij} \mathbf{T}_i \hat{\mathbf{v}}_j$$

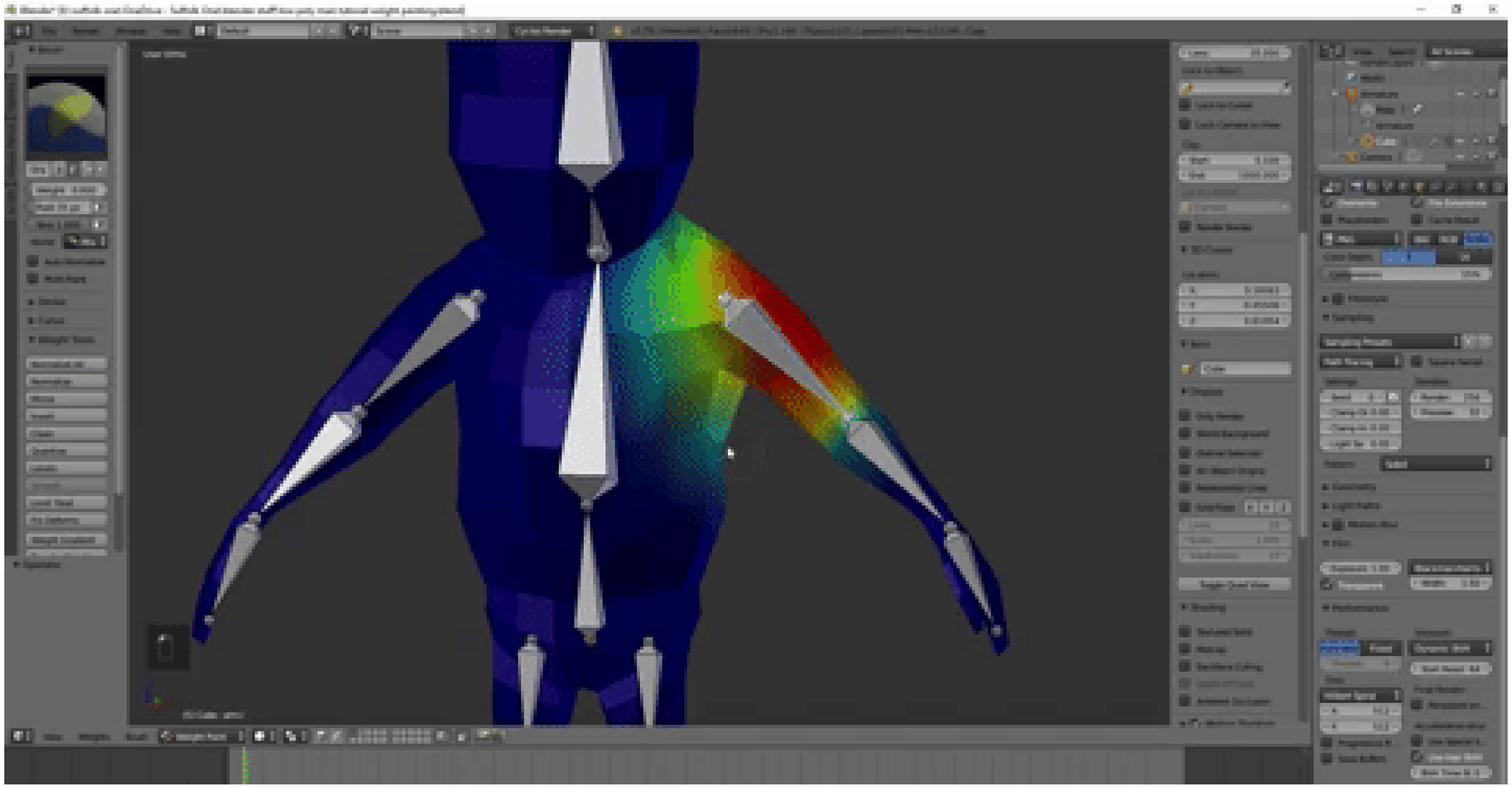
Rigid vs Linear Blend Skinning

Weight Painting



https://www.youtube.com/watch?v=Tl4qTgwQwYw

Weight Painting



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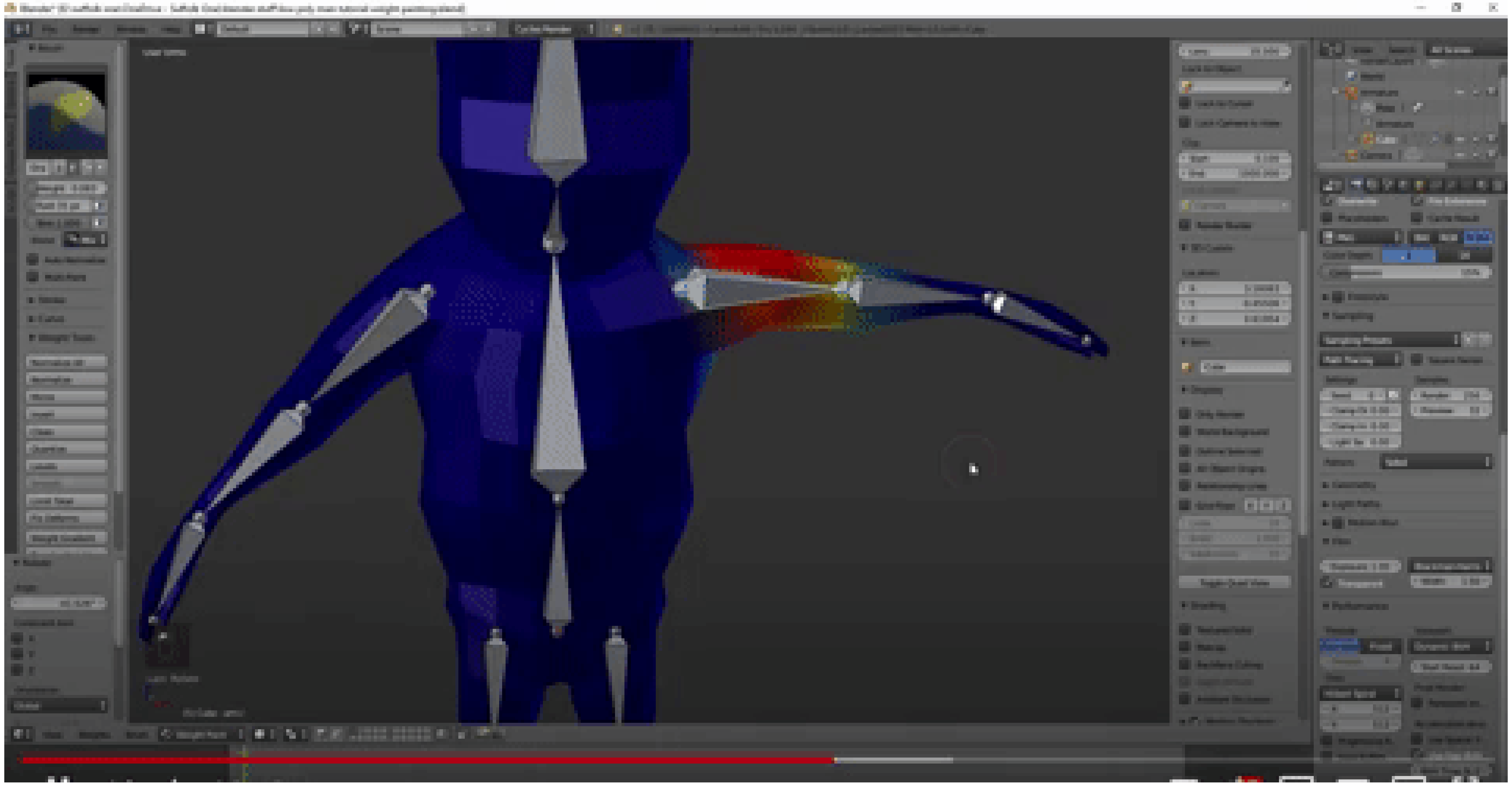
Weight Painting

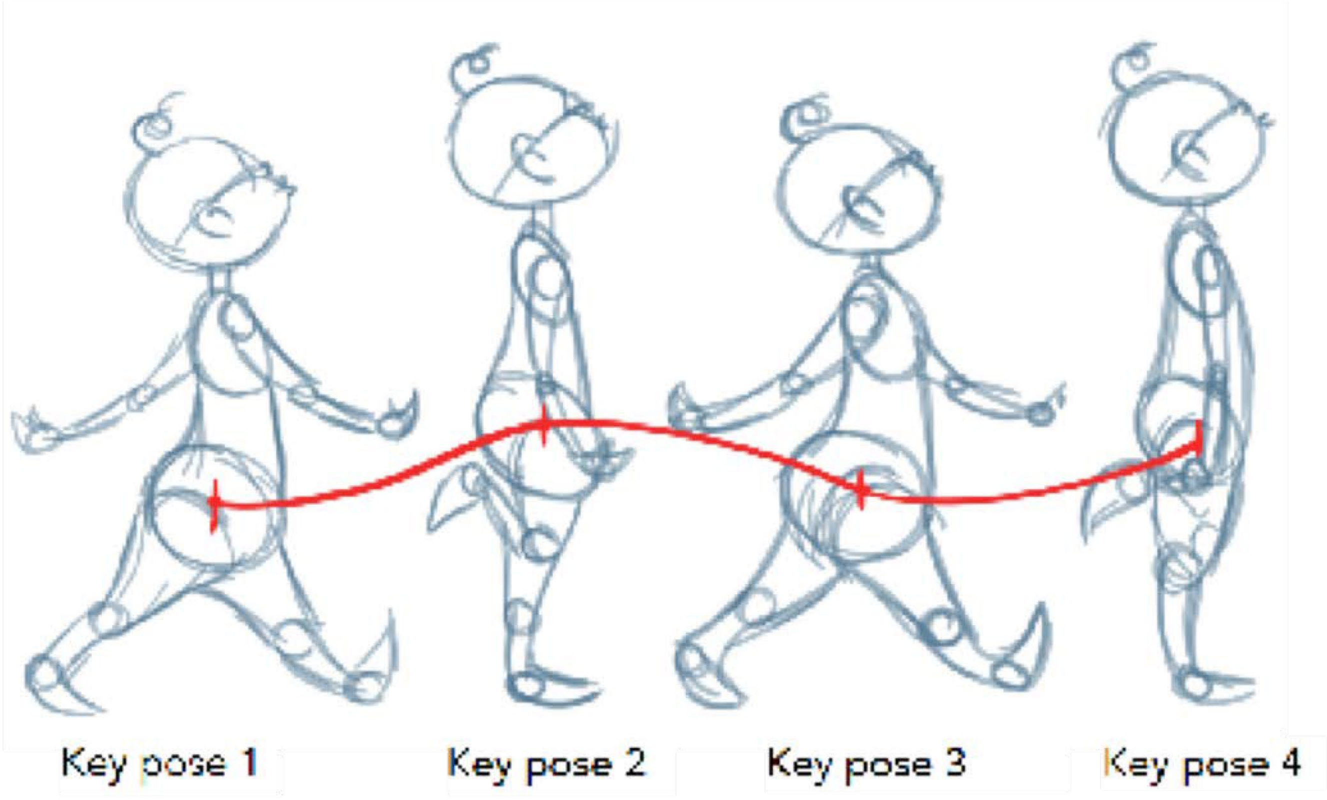


https://www.youtube.com/watch?v=Tl4qTgwQwYw

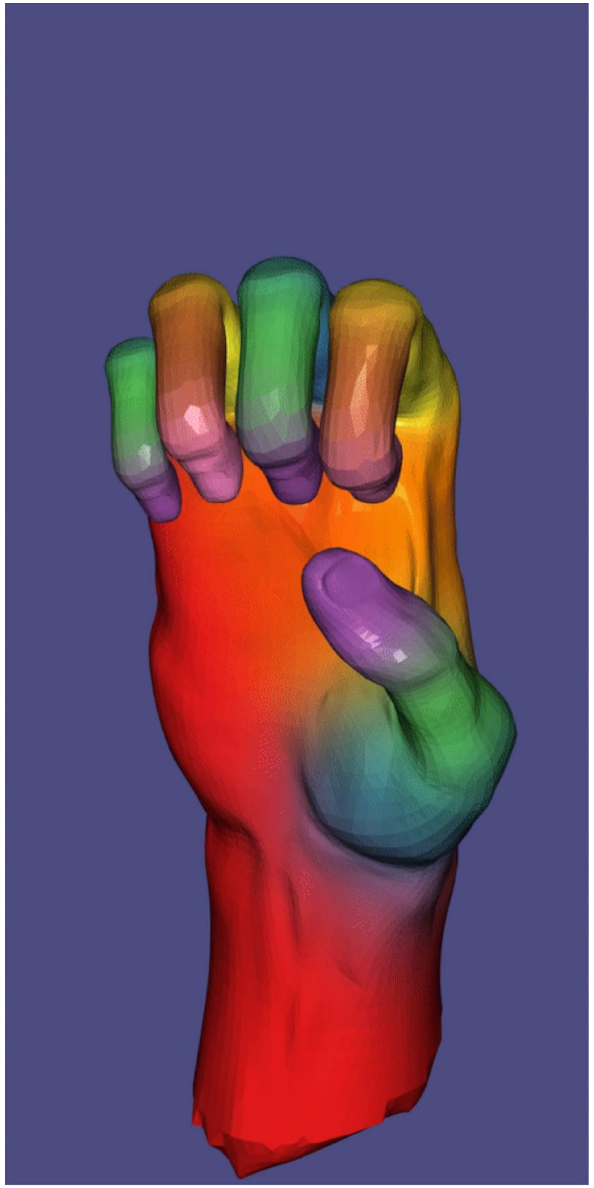
Animation via Keyframing

We can pose objects now!

How do we generate an animation ?



Specifying Keyframes



Time = 0

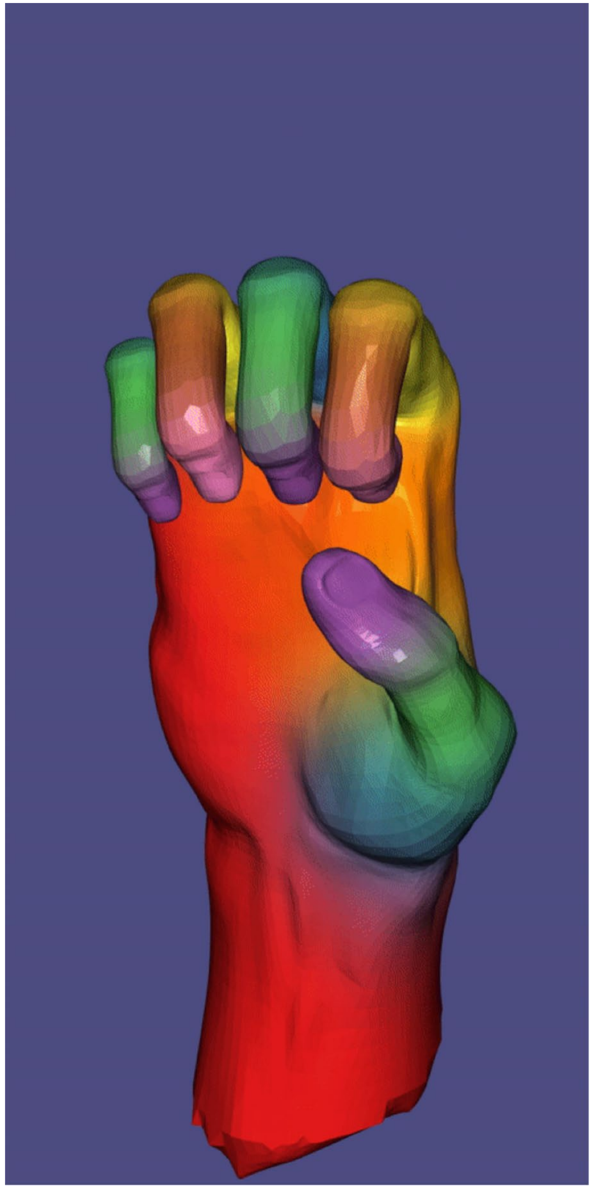
Time = 10s

Poses are generated by specifying rotations of bones

Each pose can be represented as

$$\left(t, \begin{bmatrix} \theta_{i1} \\ \theta_{i2} \\ \theta_{i3} \end{bmatrix}\right)$$

Specifying Keyframes

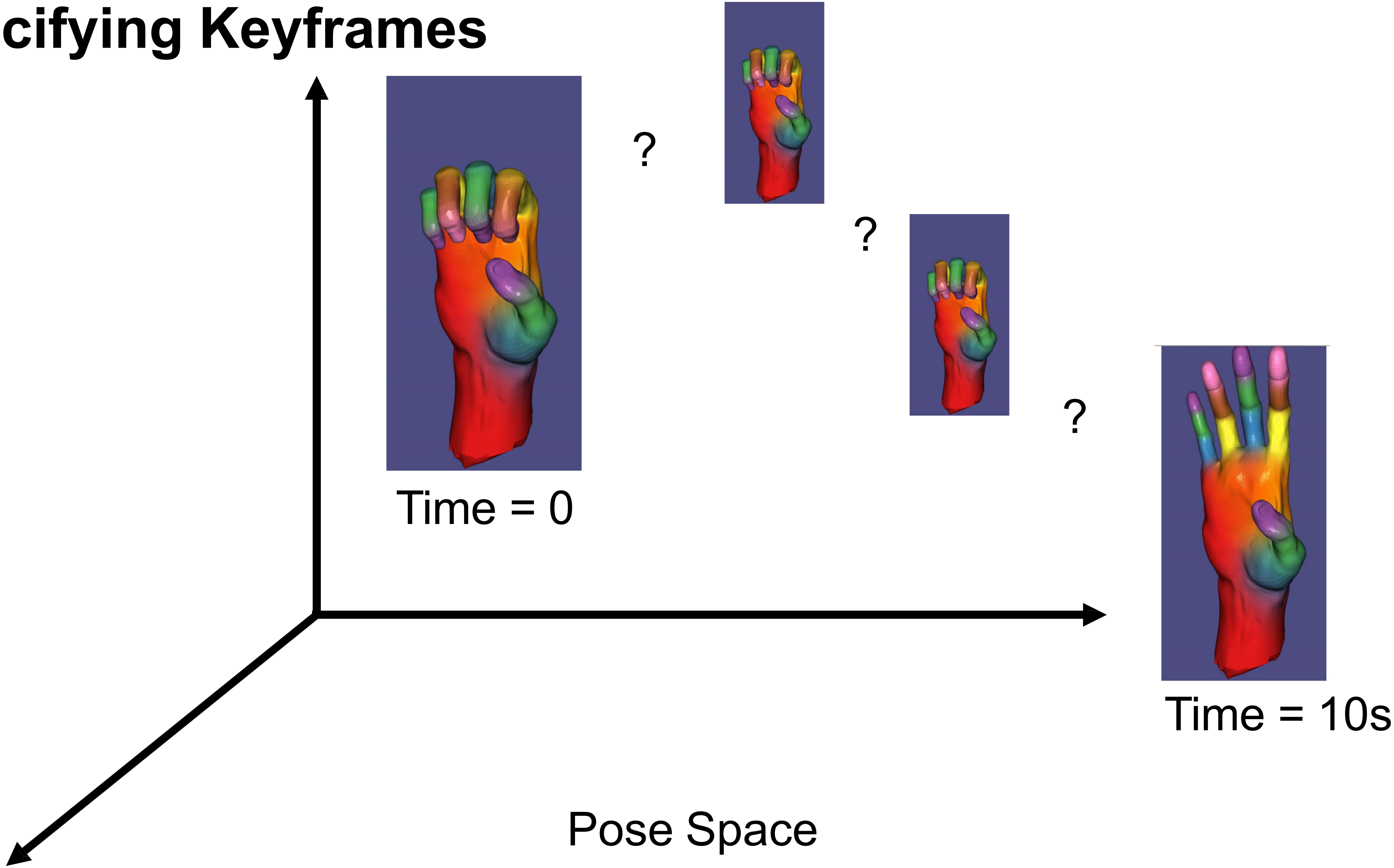


Time = 0

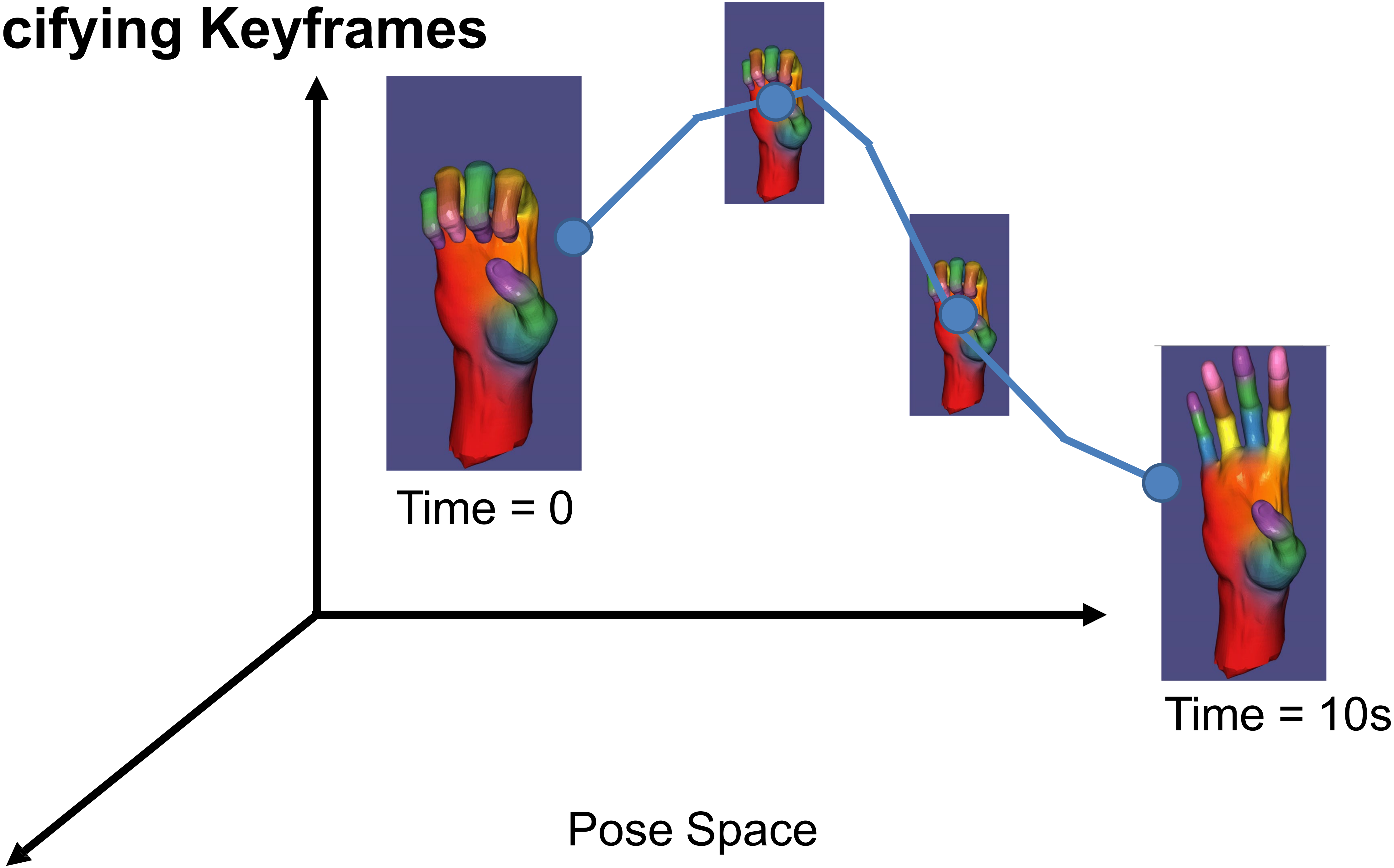
????????????

Time = 10s

Specifying Keyframes



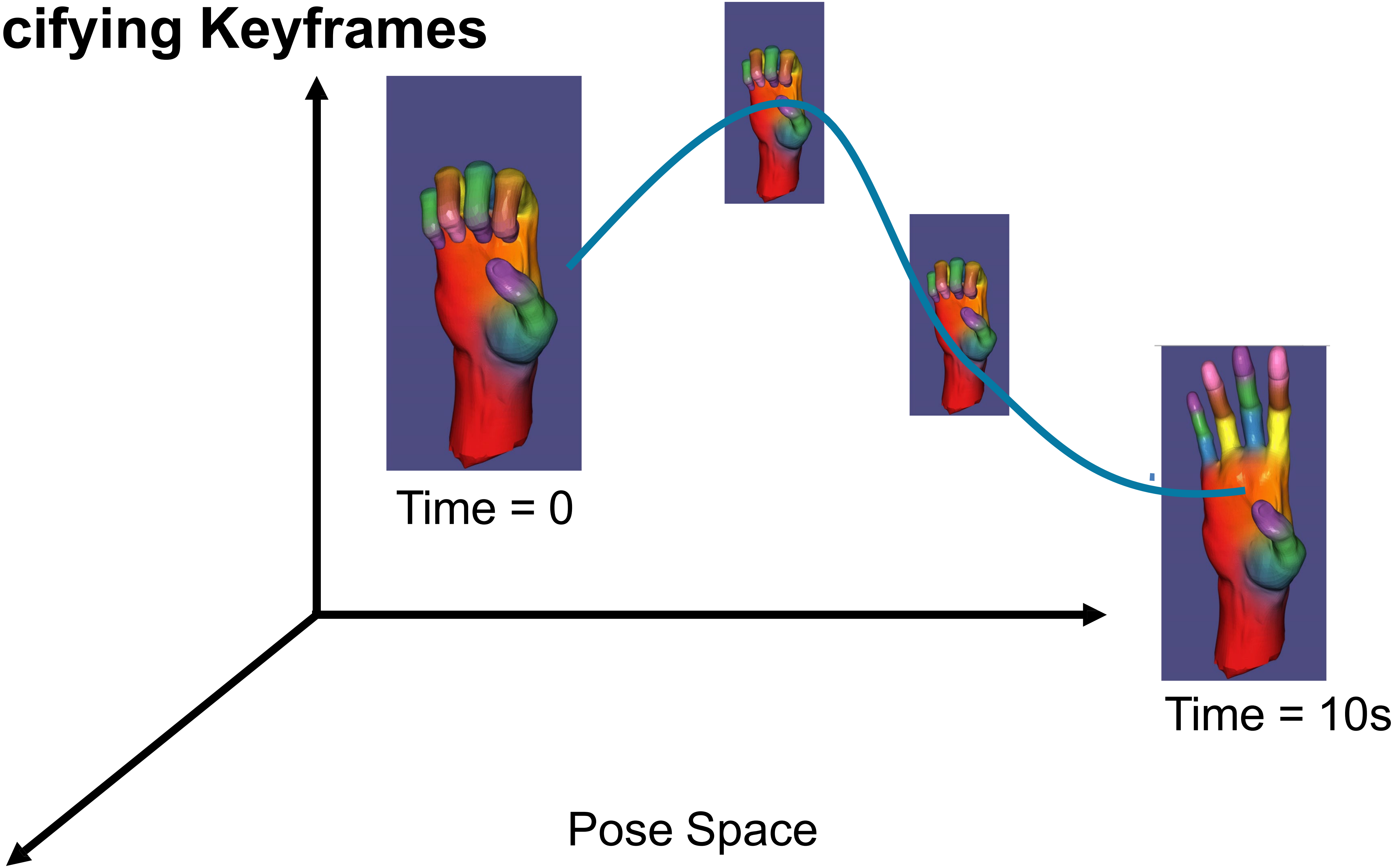
Specifying Keyframes



Specifying Keyframes

https://en.wikipedia.org/wiki/Twelve_basic_principles_of_animation#Slow_in_and_slow_out
https://www.youtube.com/watch?v=fQBFsTqbKhY

Specifying Keyframes



Interpolating Keyframes

$\theta = \mathbf{c}(t)$ is a curve in the pose space

How could we construct such a curve from keyframes ?

…

We'll cover this on Wednesday!

Done for Today