Midterm

SDS321

Spring 2016

You may use a two (2 sided) pages of notes, and you may use a calculator.

This exam consists of five questions, containing multiple sub-questions. The assigned points are noted next to each question; the total number of points is 25. You have 75 minutes to answer the questions.

Please answer all problems in the space provided on the exam. Use extra pages if needed. Of course, please put your name on extra pages.

Read each question carefully, show your work and clearly present your answers. Note, the exam is printed two-sided - please don't forget the problems on the even pages!

Good Luck!

Name: _____

UTeid: _____

1. (5 pts) 60% of the students at a certain school take neither calculus nor statistics. 20% percent takes calculus and 30% takes statistics. If one of the students is chosen randomly, what is the probability that this student takes

   (a) (2 pts) calculus and statistics?

   (b) (2 pts) Are the events {A randomly picked student takes calculus} and {A randomly picked student takes statistics} independent?

   (c) (1 pts) either calculus or statistics (but not both)?

2. (5 pts) How many solutions are there to the equation $x_1 + x_2 + x_3 = 8$ where:

(a) (1 pt) x_1, x_2 and x_3 are non-negative integers, i.e. $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$?

(b) (3 pts) x_1, x_2 and x_3 are non-negative integers such that $x_1 > 2$?

(c) (1 pt) x_1, x_2 and x_3 are non-negative integers such that $x_1 \leq 2$? *Hint: no further calculation necessary.*

3. (5 pts) The joint probability distribution $P_{X,Y}(x,y)$ of two discrete r.v's X and Y is as follows, with $P(X = 1, Y = 1) = a$, $P(X = 1, Y = 2) = 0.3$, etc.

		x
y	1	2
1	a	0.2
2	0.3	b

(a) (1 pt) Find $P(X = x)$ for each $x \in \{1, 2\}$ in terms of a and b.

(b) (2 pts) If $E[X] = 1.3$, what are the values a and b?

(c) (2 pts) Find $P(XY \geq 2)$. This should be a numeric answer, i.e. not in terms of a and b.

4. (5 pts) The probability of a royal flush in a poker hand is $p = 1/649740$. Below, let X denote the number of royal flushes in n hands.

   (a) (2 pts) Find the probability of no royal flush in $n = 1000000$ hands. You can leave your answer in terms of n and p if you want.

   (b) (2 pts) How large must n be to render the probability of having no royal flush in n hands smaller than $1/e$? That is, how large must n be for $P(X = 0) < 1/e$?

   (c) (1 pt) How large must n be to have at least two royal flushes on average. That is, how large must n be for $E[X] \geq 2$?

5. (5 pts) Each day, Alice's boss gives her 10 umbrellas to sell that day, and takes back any she doesn't sell. She gets a bonus on a day if and only if she sells at least one umbrella that day. On a rainy day Alice sells each umbrella independently with probability 1. On a sunny day Alice sells each umbrella independently with probability 0.2. The probability that it would rain on a particular day is 0.1.

(a) (1 pt) If it's a rainy day, find the probability that Alice gets a bonus.

(b) (1 pt) If it's a sunny day, find the probability that Alice gets a bonus.

(c) (2 pts) Given that Alice got a bonus, what is the probability that it rained that day?

(d) (1 pt) What is the expected number of bonuses Alice gets in a week?