THE UNIVERSITY OF TEXAS AT AUSTIN
Department of Statistics and Data Sciences
College of Natural Sciences

SDS 385: Stat Models for Big Data

Lecture 9: KD trees

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`https://psarkar.github.io/teaching`

Background

- Has a long history–invented in 1970 by Jon Bentley
- k represents the number of dimensions
- Idea is to partition the data spatially, by using only one dimension at any level.
- While searching, this helps pruning most of the search space.

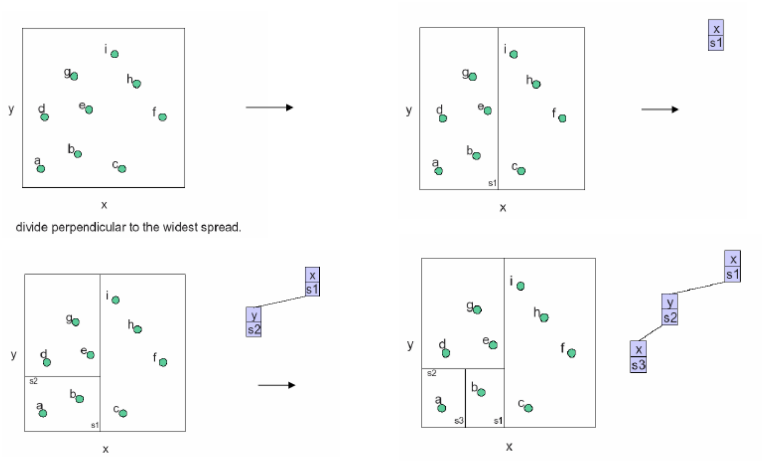
General idea

- Cycle through the dimensions for each level
- Call this cut-dim (cutting dimension)
- Node in tree contains $P = (x, y)$
- So, to find a point, only need to compare the cutting dimension.

Construct

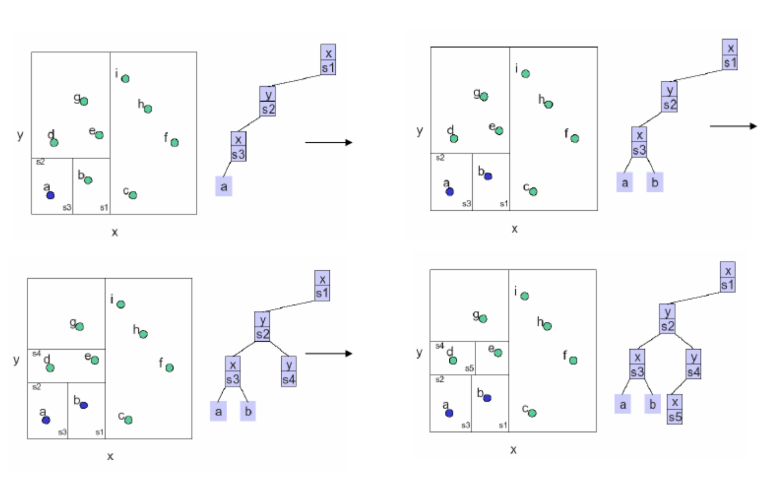
- If there is one point, just form a leaf node
- Otherwise divide the points in half along the cutting axis
    - Find the axis with the widest spread
    - divide in alternative/round robin fashion
- recursively build kdtrees from each half
- Complexity $dn \log n$

Insert



divide perpendicular to the widest spread.

Insert



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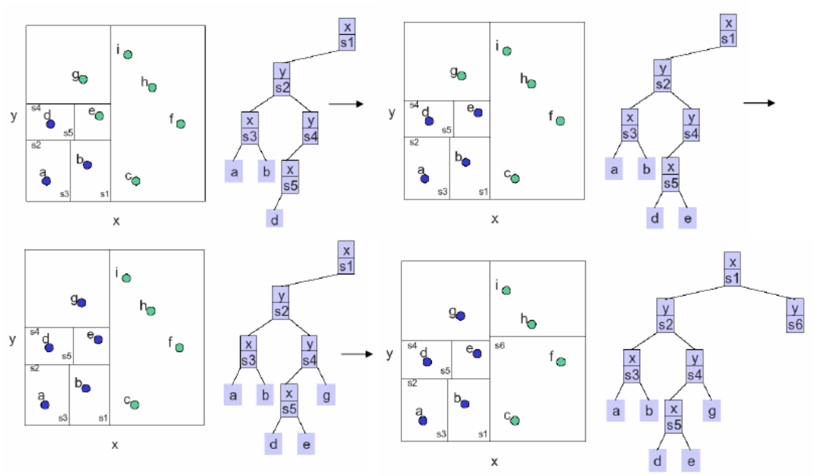
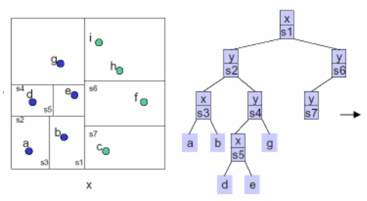
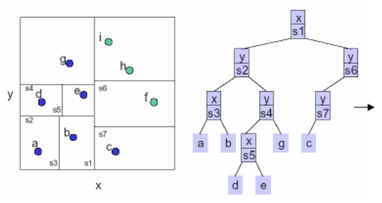
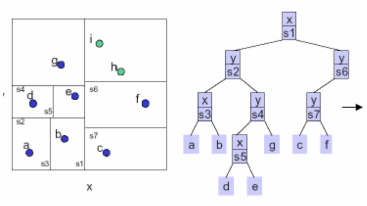
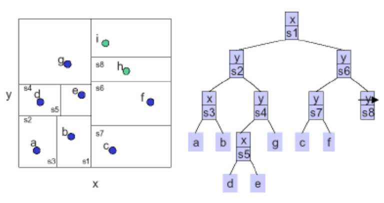


Insert

Find point with the smallest element in dimension a

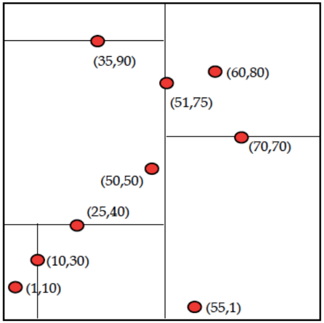
- If cutdim at current node equals a,
    - the min cannot be in the right subtree
    - recurse on the left subtree

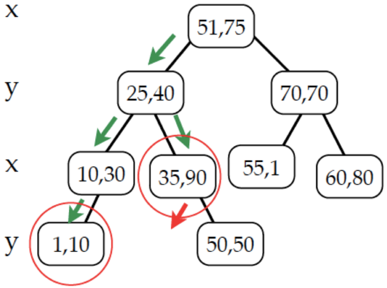
Base case: if there are no left children, stop and return current point.

- Otherwise
    - the min could be in either
    - recurse on both left and right subtrees

Find point with the smallest element in dimension x

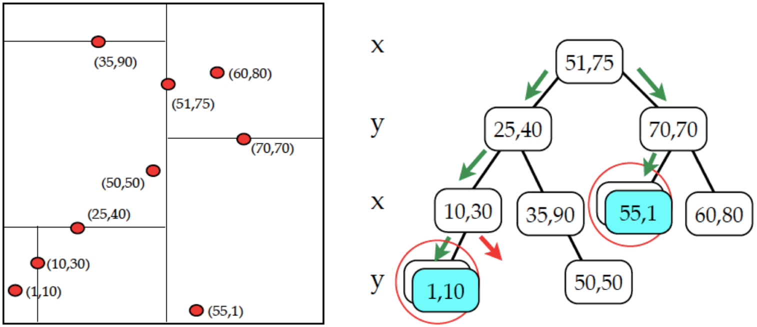
FindMin(x-dimension):





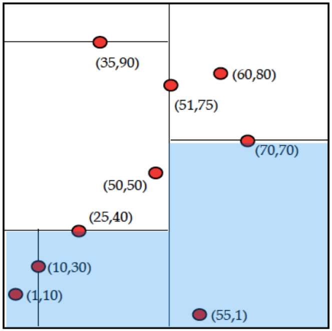
Find point with the smallest element in dimension y

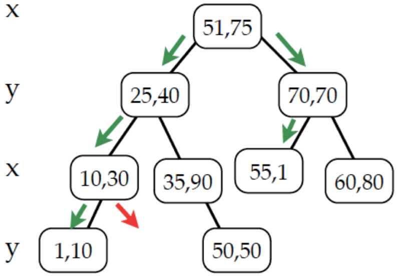
FindMin(y-dimension):



Find point with the smallest element in dimension y

FindMin(y-dimension): space searched





Nearest neighbor queries

- Given point Q, find the closest point R
- Have to be careful, because its possible that two points are far away in the tree but close in the Eucidean space.
- For each node store a bounding box

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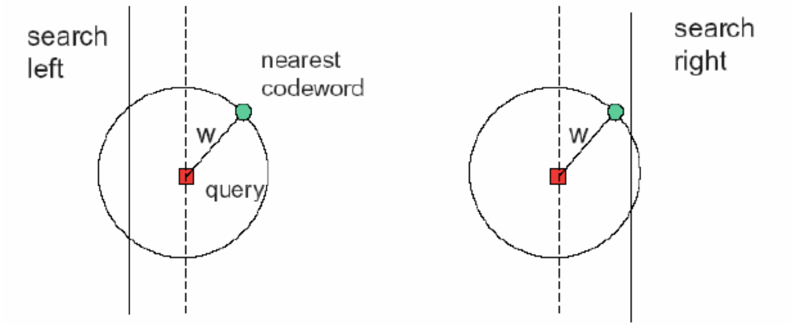
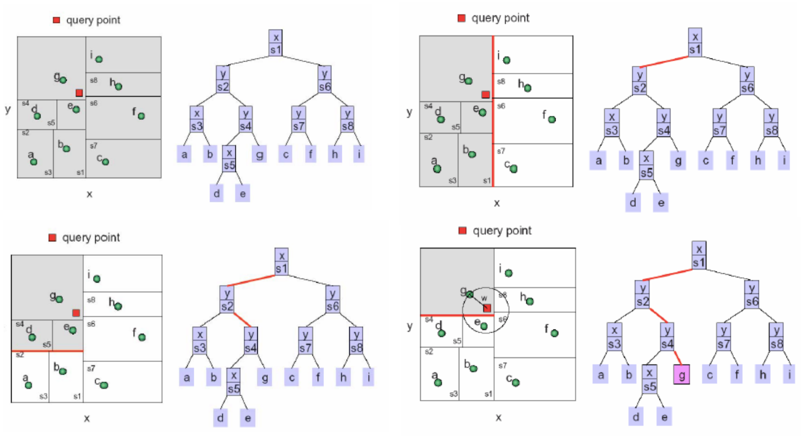
- For each node store a bounding box
- Remember the closest point to Q seen so far (call this R')
- Prune subtrees where bounding boxes cannot contain R'

Nearest neighbor queries

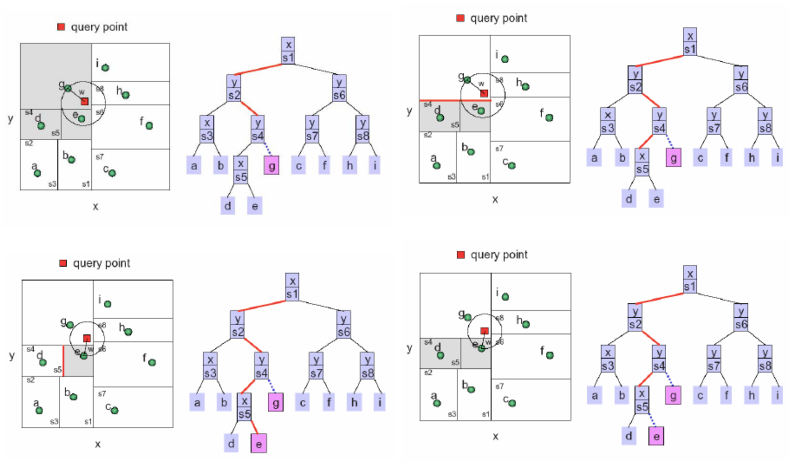


- If circle overlaps with left subtree, search left subtree
- If circle overlaps with right subtree search right subtree
- Has been shown to work in about $O(\log n)$ time.

NN search



NN search


query point
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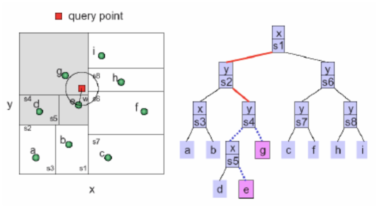
NN search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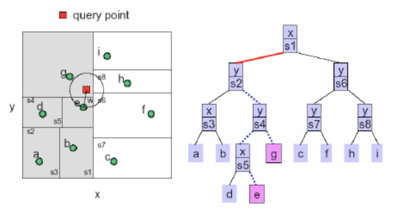
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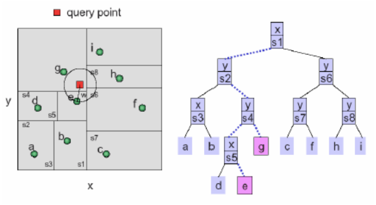
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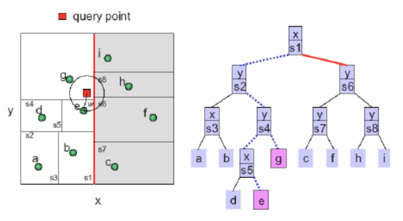
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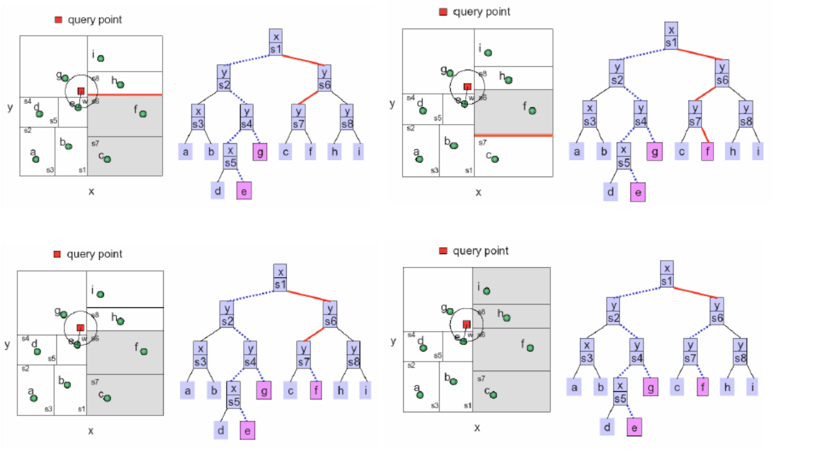
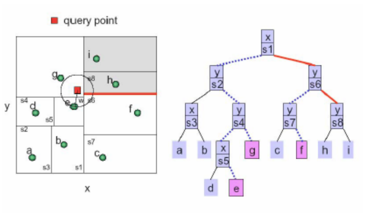
NN search



NN search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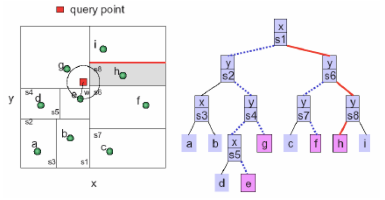
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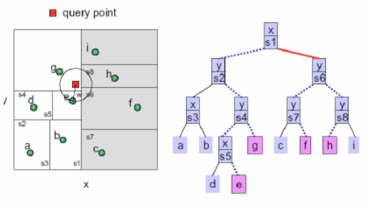
query point
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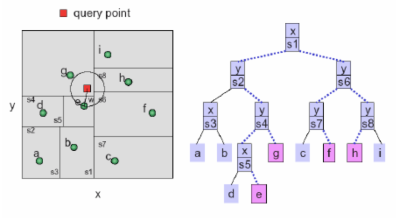
a b c d e f g h i w


query point
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s1 s2 s3 s4 s5 s6 s7 s8
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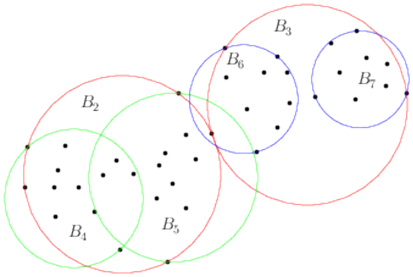
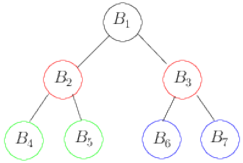
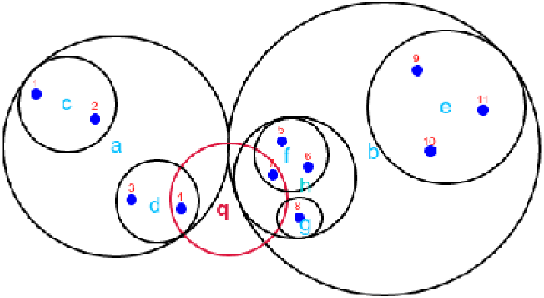
Ball trees

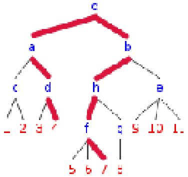




Ball tree search



(a)



(b)

Acknowledgment

- The kdtrees animations were borrowed from
  - Thinh Nguyen's slides
  - Carl Kingsford's slides
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