Homework Assignment 3

Due by 5pm via Canvas, Feb 9

SDS 321 Intro to Probability and Statistics

1. (1+1 pts) How many different letter arrangements can be made from the letters?
   (a) alohomora Only two letters are non-unique: 'a' appears twice, 'o' appears three $\frac{9!}{2!3!}$
   (b) dumbledore Only two letters are non-unique: 'd' appears twice, 'e' appears twice $\frac{10!}{2!2!}$

2. (1+3) I am making a quiz team of one boy and one girl from my class which has 10 girls and 15 boys. How many possible results are there if
   (a) If I pick 1 boy and 1 girl and pair them up? 10×15 since 10 ways to pick the girl and 15 ways to pick the boy.
   (b) If I pick 6 boys and 6 girls and pair them up? $\binom{10}{6}$ ways to pick the girls, $\binom{15}{6}$ ways to pick the boys, and then the first boy can be paired with all 6 girls; the second to the remaining 5, and so on. So $\binom{10}{6}\binom{15}{6}6!$

3. (1+3+3+3) Dr. Loh is making a Math Olympiad team of 3 boys and 3 girls from a group of students which has 6 girls and 8 boys. How many ways can he make a team this way
   (a) Without any restrictions? $\binom{6}{3}\binom{8}{3}$
   (b) If two of the girls refuse to be on the same team? WLOG lets say this is G1 and G2. $\binom{4}{3}$ ways to form a group without them. $\binom{4}{2}$ ways to put G1 in and leave G2 out. So $(4+2\times 6)\binom{8}{3}$. Another way of doing it would be to subtract all ways G1,G2 show up in the same group. This gives $(\binom{6}{3}-4)\binom{8}{3}$ which is the same.
   (c) If two of the boys refuse to be on the same team? Same logic. $\binom{6}{3}(\binom{8}{3}-6)$. Same as $\binom{6}{3}(\binom{6}{3}+2\binom{6}{2})=1000$.
   (d) If one girl and one boy refuse to be on the same team? WLOG lets this be B1 and G1. Now $\binom{5}{2}\binom{7}{2}$ ways to have both of them together. So the answer is $\binom{6}{3}\binom{8}{3}-\binom{5}{2}\binom{7}{2}=910$.

4. (1+3) Harry is dividing ten pieces of fizzing whizzbees[1] among his three kids. How many divisions are possible,
   (a) Without restrictions? This is textbook formula. $\binom{12}{2}$.

[1] Magical sweets from the wizarding world. Check pottermore.com for more details.

(b) If each child demands at least two? I want all integral solutions to $x_1 + x_2 + x_3 = 10$ such that $x_i > 2$. Convert this to the problem you know how to solve, for example the above problem. $y_i = x_i - 2$. Now we want number of solutions to $y_1 + y_2 + y_3 = 4$ such that $y_i \geq 0$. This is $\binom{4+3-1}{2} = \binom{6}{2}$.
Same thing can be done with $y_i = x_i - 1$. Then I have $y_1 + y_2 + y_3 = 7$ with $y_i > 0$. We know that number of solutions to this is $\binom{7-1}{3-1} = \binom{6}{2}$.