SDS321 Practice problems Solutions

1. A bag contains 8 pairs of shoes; each pair is a different style. You pick two random shoes from the bag.
   (a) What is the probability that the two shoes you picked out are a pair; i.e. left and right of the same style? 16*1/(16*15)=1/15
   (b) What is the probability you picked one left shoe and one right shoe? 16*8/(16*15)=8/15
2. How many solutions are there to the inequality $x_1 + x_2 + x_3 \leq 11$, where x_1, x_2, x_3 are non-negative integers. Note that in class we solved a version of this problem with an equality. Hint: Introduce an auxiliary variable x_4. Same as number of natural number solutions of $x_1 + x_2 + x_3 + x_4 = 11$. C(14,11).
3. Suppose a department has 10 men and 15 women. How many ways can we form a committee with six members, if it must have more women than men? C(15,6)+ C(15,5)C(10,1)+ C(15,4)C(10,2)
4. How many ways are there to choose a half dozen donuts from 10 varieties
   a. If there are no two donuts of the same variety. (means you need to select 6 varieties without replacement from 10) C(10,6)
   b. If there are at least two varieties. (not all same) |S| minus all same using the balls in boxes strategy to find |S|. To find all same choose one type from the 10. C(15,6)-C(10,1)
   c. If there must be at least one but no more than 4 glazed. All minus (none+at least 5). C(15,6)-(C(14,6)+C(10,1)) or at least one –at least five or exactly one +exactly 2 + exactly 3 + exactly 4.
5. Given a set A = {a,b,c,d,e},
   a. How many different sequences of type A of length n>0 exist that contain at most one a? ORDERED/ at most 1 is zero or 1 so 4^n+n4^{n-1}
   b. How many subsets of A are there? 2^5
   c. How many non-empty subsets of A are there? 2^5-1
   d. How many subsets of size 3 can you create from A? C(5,3)
   e. How many subsets of A are there that are entirely vowels or entirely consonants? (Assume empty is both). 2^2+2^3-1
   f. How many subsets of A are there that have at least one vowel and one consonant? $2^5-(2^2+2^3-1)$
   g. How many subsets of A of size 3 contain exactly one vowel. C(2,1)C(3,2)
   h. How many ways can you arrange the letters in A? 5!
   i. How many ways can the letters of A be arranged so that all of the vowels are together? 2!4!
   j. How many ways can you arrange the letters of A so that it is not the case that all of the vowels are together? 5!- 2!4!
   k. How many ways can you arrange the letters in A so that vowels and consonants alternate and the arrangement begins with a consonant. 3!2!

   For n>0, assume all strings of length *n* from the set *A* (allowing repetition) are equally likely.

   l. What is the probability that such a string has no *a*? $4^n/5^n$
   m. What is the probability that such a string has no *b* given that it has no *a*? $3^n/4^n$

6. How many distinct permutations are there of the letters in “perfect”? 7!/2
7. Billy takes two tests in his probability and statistics class. The probability that Billy would pass at least one test is 0.9. The probability that he passes both tests is 0.7. The tests are of equal difficulty (that is, the probability that Billy passes test 1 is the same as the probability that he passes test 2.) What is the conditional probability of Billy passing test 2 given the event that he passes test 1? 7/8
8. Let A and B be events such that $A \subseteq B$. Can A and B be independent? Wouldn't be interesting. $P(A \cap B)=P(A)$ so $P(B)=1$ or $P(A)=0$ if they are independent.
9. Three persons roll a fair 4-sided die once. Let B_{ij} be the event that person i and person j roll the same face. Show that the events B_{12}, B_{13}, and B_{23} are pairwise independent but are not independent.
   $P(B_{ij})=4*1/4*4=1/4$ for all j
   $P(B_{12}\cap B_{13})=P(\text{all three rolled same})=4/4*4*4=1/16= P(B_{12})P(B_{13})$
   $P(B_{12}\cap B_{23})=P(\text{all three rolled same})=4/4*4*4=1/16= P(B_{12})P(B_{23})$
   $P(B_{13}\cap B_{23})=P(\text{all three rolled same})=4/4*4*4=1/16= P(B_{13})P(B_{23})$
   But
   $P(B_{12}\cap B_{13}\cap B_{23})=P(\text{all three rolled same})=4/4*4*4=1/16\neq P(B_{12})P(B_{13})P(B_{23})$ so the events are pairwise independent but not independent.
10. Let $P(A)=.5$, $P(B)=.6$, and $P(A\cap B^c)=.2$. Are A and B independent? Yes since A and B^c are? What is $P(A\cap B|A\cup B)$? 3/8