Homework Assignment 4

Due via Canvas, Tuesday April 9th by midnight

SDS 384-11 Theoretical Statistics

1. Let $\mathcal{P}$ be the set of all distributions on the real line with finite first moment. Show that there does not exist a function $f(x)$ such that $Ef(X) = \mu^2$ for all $P \in \mathcal{P}$ where μ is the mean of P, and X is a random variable with distribution P.

2. Let g_1 and g_2 be estimable parameters within $\mathcal{P}$ with respective degrees m_1 and m_2.

   (a) Show $g_1 + g_2$ is an estimable parameter with degree $\leq \max(m_1, m_2)$.

   (b) Show $g_1 g_2$ is an estimable parameter with degree at most $m_1 + m_2$.

3. A continuous distribution with CDF $F(x)$, on the real line is symmetric about the origin if, and only if, $1 - F(x) = F(-x)$ for all real x. This suggests using the parameter,

$$\theta(F) = \int (1 - F(x) - F(-x))^2 dF(x) \tag{1}$$

$$= \int ((1 - F(-x))^2 dF(x) - 2\int (1 - F(-x))F(x)dF(x) + \int F(x)^2 dF(x) \tag{2}$$

   as a nonparametric measure of how asymmetric the distribution is. Find a kernel h, of degree 3, such that $E_F h(X_1, X_2, X_3) = \theta(F)$ for all continuous F. Find the corresponding U statistic.

4. Suppose the distribution of X is symmetric about the origin, with variance $\sigma^2 > 0$ and $EX^4 < \infty$. Consider the kernel, $h(x, y) = xy + (x^2 - \sigma^2)(y^2 - \sigma^2)$.

   (a) Show that the corresponding U statistic is degenerate of order 1, i.e. $\xi_1 = 0$, but $\xi_2 > 0$.

   (b) Find the asymptotic distribution of nU.

5. Look at the seminar paper "Probability Inequalities for Sums of Bounded Random Variables" by Wassily Hoeffding. It should be available via `lib.utexas.edu`. Read and reproduce the proof of equation 5.7 for large sample deviation of order r U statistics.