Homework Assignment 6

Due on Friday 10th by midnight via canvas

SDS 321 Intro to Probability and Statistics

1. (7pts) The CDF of a random variable is given by:

$$F(b) = \begin{cases} 0 & b < 0 \\ \frac{1}{2} & 0 \leq b < 1 \\ \frac{3}{5} & 1 \leq b < 2 \\ \frac{4}{5} & 2 \leq b < 3 \\ \frac{9}{10} & 3 \leq b < 3.5 \\ 1 & b \geq 3.5 \end{cases}$$

   (a) (3 pts) Calculate the PMF of X.

   (b) (2 pts) Calculate $E[X]$.

   (c) (2 pts) Calculate the variance of X.

2. (2+2 pts) Suppose that, in flight, airplane engines will fail with probability $1 - p$, independently from engine to engine. An airplane needs at least half of its engines operative to complete a successful flight.

   (a) If $p = 3/4$, which is preferable, a four-engine plane or a two-engine plane?

   (b) What about if $p = 1/2$?

3. (2+2 pts) The covariance between two random variables X and Y is defined as $\text{cov}(X, Y) := E[(X - E[X])(Y - E[Y])]$.

   (a) (2 pts) Show that if X and Y are independent, then $\text{cov}(X, Y) = 0$.

   (b) (2 pts) Consider $X \sim Binomial(n, p)$. Let Y denote $n - X$. Calculate $\text{cov}(X, Y)$.

4. (2+2+1 = 5pts) Let the number of cars in the UT campus roads on a given day be denoted by X. On a rainy day $X \sim Poisson(100)$, whereas on a sunny day $X \sim Poisson(60)$. Denote the event of rain by R. $P(R) = 0.1$.

   (a) Calculate $E[X]$.

   (b) Calculate $E[X^2]$.

   (c) Calculate $\text{var}[X]$.