

$$-\frac{d}{dx} \left(k(x) \frac{du(x)}{dx} \right) = 100x^2 \quad k(x) = \begin{cases} 1, & x \in (0, 1) \\ 2x, & x \in (1, 2) \end{cases}$$

$$u(2) = -20 \quad \Omega = (0, 2)$$

$$u'(0) + u(0) = 20 \Rightarrow u'(0) = 20 - u(0)$$

$$-(k(x) u'(x))' = 100x^2 \quad | \cdot v(x), v \in V, V = \{f \in H^1: f(2) = 0\}$$

$$-(k(x) u'(x))' v(x) = 100x^2 v(x) \quad | \int_{\Omega} dx$$

$$-\int_0^2 (k(x) u'(x))' v(x) dx = 100 \int_0^2 x^2 v(x) dx$$

$$-(k(x) u'(x) v(x)) \Big|_0^2 + \int_0^2 k(x) u'(x) v'(x) dx = 100 \int_0^2 x^2 v(x) dx$$

$$-k(2) u'(2) \underbrace{v(2)}_0 + \underbrace{k(0)}_1 u'(0) v(0) + \int_0^2 k(x) u'(x) v'(x) dx = 100 \int_0^2 x^2 v(x) dx$$

$$u'(0) v(0) = (20 - u(0)) v(0) = 20 v(0) - u(0) v(0)$$

$$20 v(0) - u(0) v(0) + \int_0^2 k(x) u'(x) v'(x) dx = 100 \int_0^2 x^2 v(x) dx$$

$$\underbrace{\int_0^2 k(x) u'(x) v'(x) dx - u(0) v(0)}_{B(u, v)} = \underbrace{100 \int_0^2 x^2 v(x) dx - 20 v(0)}_{L(v)}$$