# Dynamic Modeling III

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## Summary

$$\frac{\Theta(s)}{r} = \frac{\dfrac{k_d s^2 + k_p s + k_I}{I_{Tot}}}{s^3 + \dfrac{C_F + k_d H(s)}{I_{Tot}} s^2 + \dfrac{k_p H(s)}{I_{Tot}} s + \dfrac{k_I H(s)}{I_{Tot}}}$$

## Coefficients without Dynamic Coupling

$$k_d = \frac{-(2a+b) I_{Tot} - C_F}{H(s)} \qquad k_p = \frac{(2ab + a^2) I_{Tot}}{H(s)}$$

$$k_I = \frac{-a^2 b \, I_{Tot}}{H(s)}$$

## Equations of Motion with Dynamic Coupling

$$\ddot{\theta}_L I_{Tot,L} = \sum M_L + \cos(\theta_L) \cdot (L_i + \sin(\theta_L) L) \cdot L \cdot m \dot{\theta}_V^2$$

$$\ddot{\theta}_V I_{Tot,V} = \sum M_V - \dot{\theta}_V \frac{dI}{dt}$$

## Add Dynamic Coupling Terms to Plant TF - Vert Axis

### Solve for $\ddot{\theta}_V$

$$\ddot{\theta}_V I_{Tot,V} = \sum M_V - \dot{\theta}_V \frac{dI}{dt}$$

$$\ddot{\theta}_V = \frac{\sum M_V}{I_{Tot,V}} - \frac{\dot{\theta} \frac{dI_{Tot,V}}{dt}}{I_{Tot,V}} = \frac{\sum M_V}{I_{Tot,V}} - \frac{1}{I_{Tot,V}} \cdot \dot{\theta}_V \frac{dI_{Tot,V}}{dt}$$

$$\mathcal{L}\left\{\dot{\theta}_V \frac{dI_{Tot,V}}{dt}\right\} = s\,\Theta(s) \cdot \frac{dI_{Tot,V}}{dt}$$

$$\theta(s) = \frac{T_m(s)}{I_{Tot} s^2 + s C_F} - s\theta(s) \cdot \dot{I}_{Tot,v}$$

$$\theta(s) + s\theta(s) \cdot \dot{I}_{Tot,v} = \frac{T_m(s)}{I_{Tot,v} s^2 + s C_{F,v}}$$

$$\theta(s)\left(1 + s\dot{I}_{Tot,v}\right) = \frac{T_m(s)}{\left(I_{Tot,v} s^2 + C_{F,v} s\right)}$$

$$\theta(s) = \frac{T_m(s)}{\left(1 + s\dot{I}_{Tot,v}\right)\left(I_{Tot,v} s^2 + C_{F,v} s\right)}$$

$$G(s) = \frac{\theta(s)}{T_m(s)} = \frac{1}{\dot{I}_{Tot,v} \cdot I_{Tot,v} s^3 + I_{Tot,v} s^2 + \dot{I}_{Tot,v} \cdot C_{F,v} s^2 + C_{F,v} s}$$

$$= \frac{1}{\dot{I}_{Tot,v} \cdot I_{Tot,v} s^3 + \left(I_{Tot,v} + \dot{I}_{Tot,v} \cdot C_{F,v}\right) s^2 + C_{F,v} s}$$

$$G(s)_v C(s)_v = \frac{k_p + k_I \frac{1}{s} + k_d s}{\dot{I}_{Tot,v} \cdot I_{Tot,v} s^3 + \left(I_{Tot,v} + \dot{I}_{Tot,v} \cdot C_{F,v}\right) s^2 + C_{F,v} s}$$

$$\frac{1}{G(s)\, C(s)} = \frac{\dot{I}_{Tot,v} \cdot I_{Tot,v} s^3 + \left(I_{Tot,v} + \dot{I}_{Tot,v} \cdot C_{F,v}\right) s^2 + C_{F,v} s}{k_p + k_I \frac{1}{s} + k_d s}$$

$$\frac{\Theta(s)}{r} = \cfrac{1}{\cfrac{\dot{I}_{Tot,v} \cdot I_{Tot,v} s^3 + (I_{Tot,v} + \dot{I}_{Tot,v} \cdot C_{F,v}) s^2 + C_{F,v} s}{k_p + k_I \frac{1}{s} + k_d s} + H(s)}$$

$$= \frac{k_p + k_I \frac{1}{s} + k_d s}{\dot{I}_{Tot,v} \cdot I_{Tot,v} s^3 + (I_{Tot,v} + \dot{I}_{Tot,v} \cdot C_{F,v}) s^2 + C_{F,v} s + (k_p + k_I \frac{1}{s} + k_d s) H(s)}$$

$$= \frac{k_p + k_I \frac{1}{s} + k_d s}{\dot{I}_{Tot,v} \cdot I_{Tot,v} s^3 + (I_{Tot,v} + \dot{I}_{Tot,v} \cdot C_{F,v}) s^2 + C_{F,v} s + k_d H(s) s + k_I H(s) \frac{1}{s} + k_p H(s)}$$

$$= \frac{k_p + k_I \frac{1}{s} + k_d s}{\dot{I}_{Tot,v} \cdot I_{Tot,v} s^3 + (I_{Tot,v} + \dot{I}_{Tot,v} \cdot C_{F,v}) s^2 + (C_{F,v} + k_d H(s)) s + k_I H(s) \frac{1}{s} + k_p H(s)}$$

$$= \frac{k_d s^2 + k_p s + k_I}{\dot{I}_{Tot,v} \cdot I_{Tot,v} s^4 + (I_{Tot,v} + \dot{I}_{Tot,v} \cdot C_{F,v}) s^3 + (C_{F,v} + k_d H(s)) s^2 + k_p H(s) s + k_I H(s)}$$

$$= \cfrac{\cfrac{k_d s^2 + k_p s + k_I}{\dot{I}_{Tot,v} \cdot I_{Tot,v}}}{s^4 + \cfrac{(I_{Tot,v} + \dot{I}_{Tot,v} \cdot C_{F,v})}{\dot{I}_{Tot,v} \cdot I_{Tot,v}} s^3 + \cfrac{(C_{F,v} + k_d H(s))}{\dot{I}_{Tot,v} \cdot I_{Tot,v}} s^2 + \cfrac{k_p H(s)}{\dot{I}_{Tot,v} \cdot I_{Tot,v}} s + \cfrac{k_I H(s)}{\dot{I}_{Tot,v} \cdot I_{Tot,v}}}$$

What to do with the higher order terms?

$(s-a)^2(s-b)^2$ gives desired characteristic polynomial

$(s-a)^2(s-b)^2$ gives desired characteristic polynomial, with $a, b$ complex roots (no real roots)

$$a^2b^2 - 2a^2bs + a^2s^2 - 2ab^2s + 4abs^2 - 2as^3 + b^2s^2 - 2bs^3 + s^4$$

$$= s^4 - (2a+2b)s^3 + (a^2+4ab+b^2)s^2 - (2a^2b+2ab^2)s + a^2b^2$$

$k_d$: $$\frac{(C_{F,v} + k_d H(s))}{\dot{I}_{Tot,v} \cdot I_{Tot,v}} = a^2 + 4ab + b^2$$

$$C_{F,v} + k_d H(s) = (a^2+4ab+b^2)\,\dot{I}_{Tot,v} \cdot I_{Tot,v}$$

$$k_d = \frac{(a^2+4ab+b^2)\,\dot{I}_{Tot,v} \cdot I_{Tot,v} - C_{F,v}}{H(s)}$$

$k_I$: $$\frac{k_I H(s)}{\dot{I}_{Tot,v} \cdot I_{Tot,v}} = a^2 b^2$$

$$k_I H(s) = (a^2b^2)\,\dot{I}_{Tot,v} \cdot I_{Tot,v}$$

$$k_I = \frac{(a^2b^2)\,\dot{I}_{Tot,v} \cdot I_{Tot,v}}{H(s)}$$

$k_p$: $$\frac{k_p H(s)}{\dot{I}_{Tot,v} \cdot I_{Tot,v}} = -(2a^2b + 2ab^2)$$

$$k_p H(s) = -(2a^2b + 2ab^2)\,\dot{I}_{Tot,v} \cdot I_{Tot,v}$$

$$k_p H(s) = -(2a^2b + 2ab^2)\,\dot{I}_{Tot,V} \cdot I_{Tot,V}$$

$$k_p = -\frac{(2a^2b + 2ab^2)\,\dot{I}_{Tot,V} \cdot I_{Tot,V}}{H(s)}$$