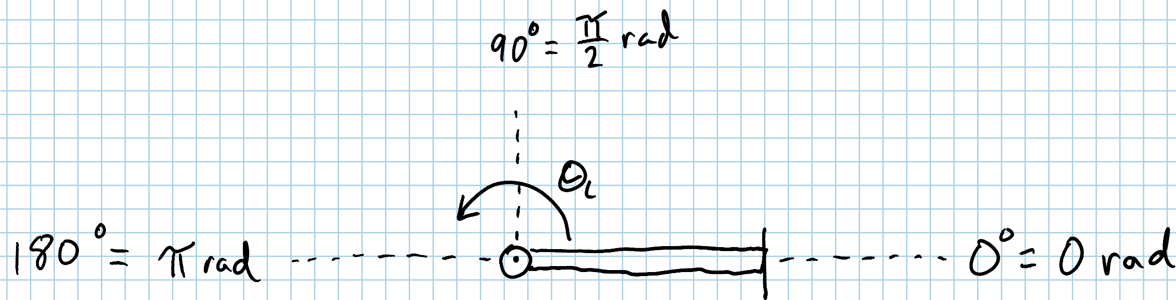
Dynamic Modeling

Tuesday, February 11, 2014
2:45 PM

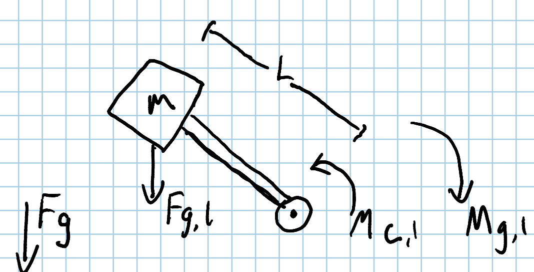
RTx - Control position of each axis
vertAxis
latAxis

latAxis - Coordinate system

Model assuming vertical axis is plumb, then scale by vertical axis position

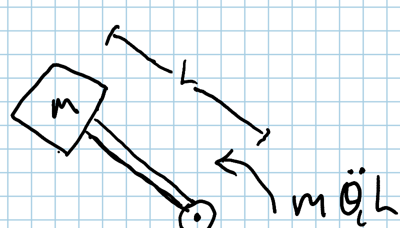


FBD



$M_{g,l}$: Rotational Force (moment) due to component of F_g

KD



$$\sum M: \quad m\ddot{\theta}L = M_{c,l} - M_{g,l}$$

Kinematic helpers:

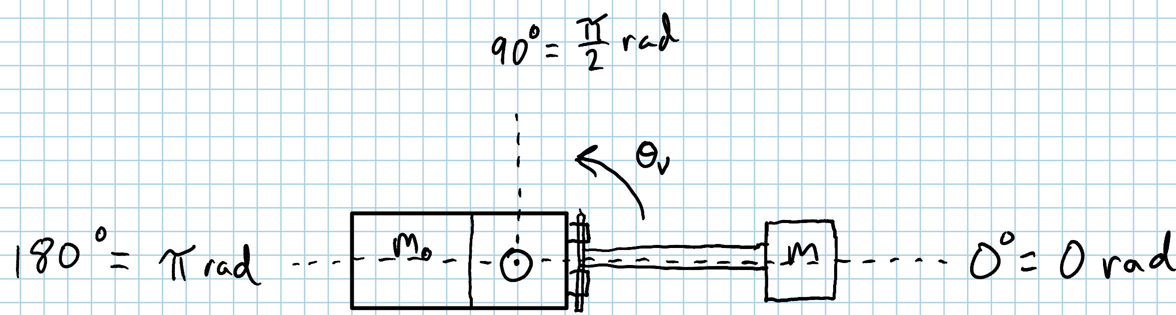
$M_{g,l}$: At $\theta_L = 0°$, $F_{g,l} = F_g$

At $\theta_L = 180°$, $F_{g,l} = -F_g$

At $\theta_L = 90°$, $F_{g,l} = 0$

$$M_{g,l} = \cos(\theta_L) \cdot mg \cdot L$$

Scale by position of vertical axis



Scaling of $M_{g,l}$: At $\theta_v = 0°$, $F'_{g,l} = 0$

At $\theta_v = 90°$, $F'_{g,l} = F_{g,l}$

At $\theta_v = 180°$, $F'_{g,l} = 0$

$$M'_{g,l} = \sin(\theta_v) \cdot \cos(\theta_L) \cdot mg \cdot L$$

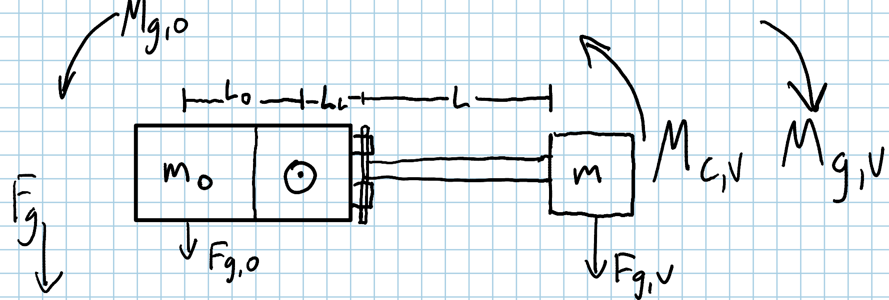
$$\Rightarrow L \cdot m \ddot{\theta}_L = M_{c,l} - M'_{g,l} = M_{c,l} - \sin(\theta_v) \cdot \cos(\theta_L) \cdot mg \cdot L$$

$$\Rightarrow \quad \ddot{\theta}_L = \frac{M_{c,l}}{mL} - \sin(\theta_v) \cdot \cos(\theta_L) g$$

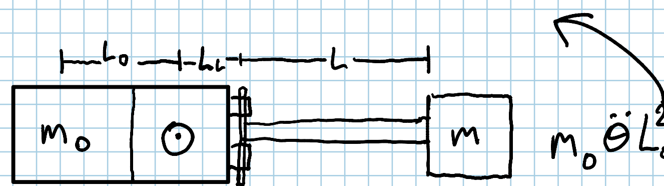
vertAxis - use established coordinate system.

model assuming lateral axis is straight, then scale by latAxis position.

FBD



KD



$$m_o \ddot{\theta} L_o^2 + m\ddot{\theta}(L_L + \cos(\theta_L)L)^2$$
$$= \ddot{\theta}\left(m_o L_o^2 + m(L_L + \cos(\theta_L)\cdot L\right)^2$$

kinematic helper:

$M_{g,v}$: At $\theta_v = 0°$, $M_{g,v} = (L_L + L)^2 mg - M_{g,o}$
At $\theta_v = 90°$, $M_{g,v} = 0$
At $\theta_v = 180°$, $M_{g,v} = M_{g,o} - (L_L + L)^2 mg$

$M_{g,o}$: At $\theta_v = 0°$, $M_{g,o} = m_o L_o^2 g$
At $\theta_v = 90°$, $M_{g,o} = 0$
At $\theta_v = 180°$, $M_{g,o} = -m_o L_o^2 g$

$$M_{g,o} = \cos(\theta_v)\cdot m_o L_o^2 g$$

$$M_{g,v} = \cos(\theta_v)\cdot(M_{g,o} - (L_L + L)^2 mg)$$

$$= \cos(\theta_v)\cdot(\cos(\theta_v)\cdot m_o L_o^2 g - (L_L + L)^2 mg)$$

$$= \cos(\theta_v) \cdot \left(\cos(\theta_v) \cdot m_o L_o^2 g - (L_L + L)^2 mg \right)$$

$$= \cos(\theta_v) \cdot g \left(\cos(\theta_v) \cdot m_o L_o^2 - (L_L + L)^2 m \right)$$

Scale by position of lateral axis

At $\theta_L = 0^\circ$, $M'_{g,v} = \cos(\theta_v) \cdot g\left(\cos(\theta_v) \cdot m_o L_o^2 - L_L^2 m\right)$
At $\theta_L = 90^\circ$, $M'_{g,v} = M_{g,v}$
At $\theta_L = 180^\circ$, $M'_{g,v} = \cos(\theta_v) \cdot g\left(\cos(\theta_v) \cdot m_o L_o^2 - L_L^2 m\right)$

$$\Sigma M : \quad \left(m_o L_o^2 + ((L_h + \sin(\theta_L) \cdot L)^2 m\right) \ddot{\theta} = M_{c,v} - M_{g,v}$$

$$M'_{g,v} = \cos(\theta_v) \cdot g \left(\cos(\theta_v) \cdot m_o L_o^2 - ((L_L + \cos(\theta_L) \cdot L)^2 m \right)$$

$$\Rightarrow g\left(m_o L_o^2 + ((L_h + \cos(\theta_L) \cdot L)^2 m\right) \ddot{\theta} = M_{c,v} - \cos(\theta_v) \cdot g\left(\cos(\theta_v) \cdot m_o L_o^2 - ((L_L + \cos(\theta_L) \cdot L)^2 m\right)$$

$$\Rightarrow \ddot{\theta} = \frac{M_{c,v}}{g\left(m_o L_o^2 + ((L_h + \cos(\theta_L) \cdot L)^2 m\right)} - \frac{\cos(\theta_v) \cdot \left(\cos(\theta_v) \cdot m_o L_o^2 - ((L_L + \cos(\theta_L) \cdot L)^2 m\right)}{g\left(m_o L_o^2 + ((L_h + \cos(\theta_L) \cdot L)^2 m\right)}$$