Dynamic Modeling

Sunday, February 16, 2014
4:47 PM

Friction - FBD

KD: $\ddot{\theta} I_{Tot}$

$$\ddot{\theta} I_{Tot} = T_M - T_F$$

kinematic helpers: $T_F = C_F \dot{\theta}$

$$\ddot{\theta} I_{Tot} = T_M - C_F \dot{\theta}$$

$$\ddot{\theta} = \frac{T_M}{I_{Tot}} - \frac{C_F}{I_{Tot}} \dot{\theta}$$

$\mathcal{L}\{x(t)\} = X(s)$ $\quad \mathcal{L}\{\dot{x}(t)\} = sX(s) - x(0)$ $\quad \mathcal{L}\{\ddot{x}(t)\} = s^2 X(s) - sx(0) - \dot{x}(0)$

$$s^2 \Theta(s) = \frac{T_M(s)}{I_{Tot}} - s\Theta(s) \cdot \frac{C_F}{I_{Tot}}$$

$$s^2 \Theta(s) + s\,\Theta(s) \cdot \frac{C_F}{I_{Tot}} = \frac{T_M(s)}{I_{Tot}}$$

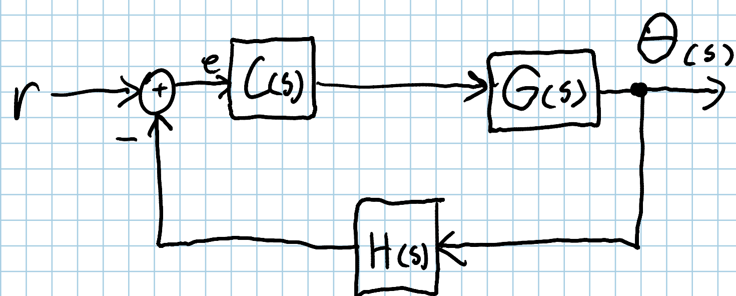
$$\Theta(s)\left(s^2 + s\frac{C_F}{I_{Tot}}\right) = \frac{T_M(s)}{I_{Tot}}$$

$$\Theta(s) = \frac{1}{s^2 + s\frac{C_F}{I_{Tot}}} \cdot \frac{T_M(s)}{I_{Tot}}$$

$$\Theta(s) = \frac{T_M(S)}{I_{Tot} s^2 + s C_F}$$

$$G(s) = \frac{\Theta(s)}{T_M(s)} = \frac{1}{I_{Tot}\, s^2 + s C_F}$$ Transfer Function: Plant

Control Structure



$$\Theta(s) = G(s)\,C(s) \cdot e$$

$$e = r - H(s) \cdot \Theta(s)$$

$$\Theta(s) = G(s)\,C(s)\,(r - H(s)\,\Theta(s))$$

$$\Theta(s) = G(s)\,C(s)\,r - G(s)\,C(s)\,H(s)\,\Theta(s)$$

$$\Theta(s) + G(s)\,C(s)\,H(s)\,\Theta(s) = G(s)\,C(s)\,r$$

$$\Theta(s)\,(1 + G(s)\,C(s)\,H(s)) = G(s)\,C(s)\,r$$

$$\Theta(s) = \frac{G(s)\,C(s)\,r}{1 + G(s)\,C(s)\,H(s)}$$

$$\frac{\Theta(s)}{r} = \frac{G(s)\,C(s)}{1 + G(s)\,C(s)\,H(s)}$$

$$\frac{\Theta(s)}{} = \frac{\frac{1}{G(s)C(s)} \cdot G(s)\,C(s)}{}$$

$$\frac{\Theta(s)}{r} = \frac{\frac{1}{G(s)C(s)} \cdot G(s)C(s)}{\frac{1}{G(s)C(s)}\left(1 + G(s)C(s)H(s)\right)}$$

$$\frac{\Theta(s)}{r} = \frac{1}{\frac{1}{G(s)C(s)} + \frac{G(s)C(s)H(s)}{G(s)C(s)}}$$

$$\frac{\Theta(s)}{r} = \frac{1}{\frac{1}{G(s)C(s)} + H(s)}$$ Transferfunction: System

Pole placement

roots: negative complex $\quad As^2 + Bs + C = 0$

solve for roots

Filters: $\frac{P}{s+R}$ 1st order

$\frac{P^2}{s^2 + Qs + R^2}$ 2nd order

$$G(s) = \frac{\Theta(s)}{T_M(s)} = \frac{1}{I_{Tot} s^2 + sC_F}$$

$C(s)$ - as a fn of I_{Tot}, $H(s)$, Coupling: Control structure

$$\oplus \longrightarrow \boxed{C(s)} \longrightarrow \boxed{G(s)} \longrightarrow \Theta(s)$$

Proportional-only : $C_{out} = e k_p = (r - H(s)G(s)) k_p$

Integral : $k_p + k_I \frac{1}{s}$

$$C_{out} = e\left(k_p + k_I \frac{1}{s}\right)$$

Derivative: $k_p + k_I \frac{1}{s} + k_d s = C(s)$

$$C_{out} = e\left(k_p + k_I \frac{1}{s} + k_d s\right)$$

$$C(s) = k_p + k_I \frac{1}{s} + k_d s$$

$$G(s)\,C(s) = \frac{1}{I_{Tot} s^2 + s C_F} \cdot k_p + k_I \frac{1}{s} + k_d s$$

$$= \frac{k_p + k_I \frac{1}{s} + k_d s}{I_{Tot} s^2 + s C_F}$$

$$\frac{1}{G(s)C(s)} = \frac{I_{Tot} s^2 + C_F s}{k_p + k_I \frac{1}{s} + k_d s}$$

$$\frac{\Theta(s)}{r} = \frac{1}{I_{Tot} s^2 + C_F s \quad \ldots + H(s)}$$

$$\frac{\Theta(s)}{r} = \frac{1}{\dfrac{I_{Tot} s^2 + C_F s}{k_p + k_I \frac{1}{s} + k_d s} + H(s)}$$

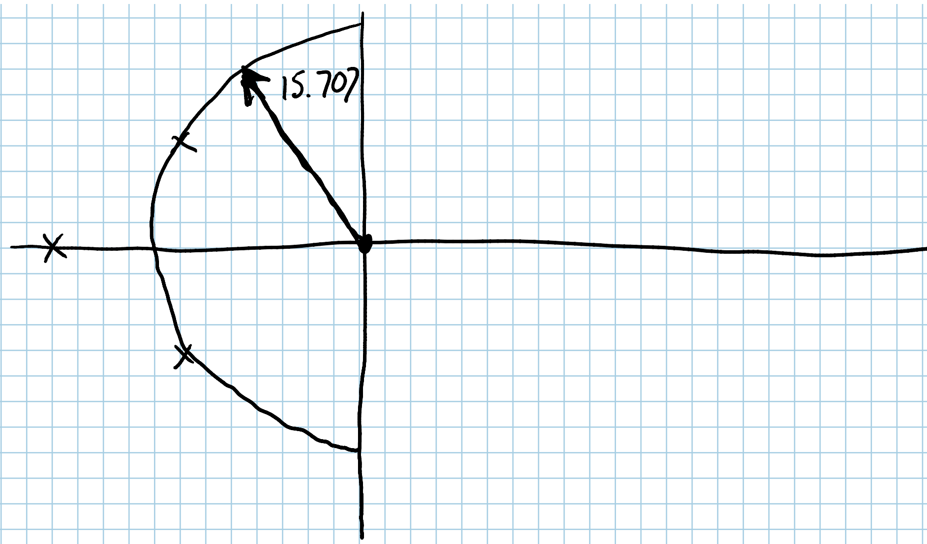
$$\frac{\Theta(s)}{r} = \frac{k_p + k_I \frac{1}{s} + k_d s}{I_{Tot} s^2 + C_F s + k_p H(s) + k_I \frac{1}{s} H(s) + k_d s H(s)}$$

$$\frac{\Theta(s)}{r} = \frac{k_p + k_I \frac{1}{s} + k_d s}{I_{Tot} s^2 + (C_F + k_d H(s)) s + k_p H(s) + k_I H(s) \frac{1}{s}} \cdot \frac{s}{s}$$

$$\frac{\Theta(s)}{r} = \frac{k_d s^2 + k_p s + k_I}{I_{Tot} s^3 + (C_F + k_d H(s)) s^2 + k_p H(s) s + k_I H(s)}$$

Choose desired poles, solve for k's

$$\omega_B = 2.5\,Hz = 2.5\,(2\pi)\,\frac{rad}{s} = 15.71\,\frac{rad}{s}$$



solve roots

$$\frac{\Theta(s)}{r} = \frac{k_d s^2 + k_p s + k_I}{I_{Tot} s^3 + (C_F + k_d H(s)) s^2 + k_p H(s) s + k_I H(s)}$$

$$\frac{\Theta(s)}{r} = \frac{\dfrac{k_d s^2 + k_p s + k_I}{I_{Tot}}}{s^3 + \dfrac{C_F + k_d H(s)}{I_{Tot}} s^2 + \dfrac{k_p H(s)}{I_{Tot}} s + \dfrac{k_I H(s)}{I_{Tot}}}$$

Place poles at $\omega_d = 15.707$, so

a: complex pole

b: real pole

$(s-a)^2(s-b)$

$$(s-a)^2(s-b)$$

$$s^3-(2a+b)s^2+(2ab+a^2)s-a^2b$$

k_d: $$\frac{C_F+H(s)k_d}{I_{Tot}}=-(2a+b)$$

$$C_F+H(s)k_d=-(2a+b)I_{Tot}$$

$$k_d=\frac{-(2a+b)I_{Tot}-C_F}{H(s)}$$

$$k_p=\frac{(2ab+a^2)I_{Tot}}{H(s)}$$

$$k_I=\frac{-a^2b\,I_{Tot}}{H(s)}$$

Dynamic Coupling:

$$\ddot{\theta}_v I_{Tot,v}=\sum M_v-\dot{\theta}_v\frac{dI}{dt}$$

$$\ddot{\theta}_L I_{Tot,L}=\sum M_L-\cos(\theta_L)\cdot\dot{\theta}_v\cdot mL$$?

$$\tau=\|r\|\cdot\|F\|\cos(\theta_L);\quad F_\perp=mr_{c,l}\omega^2;\ r=L$$

At $\theta_L=0^\circ$, $F_{c,l}=mL_L\dot{\theta}_v^2$

At $\theta_L=90^\circ$, $F_{c,l}=0$

At $\theta_l = 180°$, $F_{c,l} = -m L_l \dot{\theta}_v^2$

So, $F_{c,l} = \cos(\theta_l) \cdot m \cdot r \cdot \dot{\theta}_v^2$

$r_{c,l} = (L_l + \sin(\theta_l) L)$

$M_{c,l} = F_{c,l} \cdot L \cdot \cos(\theta_l) \cdot (L_l + \sin(\theta_l) L) m \dot{\theta}_v^2$

$\ddot{\theta}_l I_{Tot} = \sum M_l + \cos(\theta_l) \cdot (L_l + \sin(\theta_l) L) \cdot L \cdot m \dot{\theta}_v^2$