

Dynamic Modeling

Sunday, February 16, 2014
4:47 PM

Friction - FDD

$$KD: \ddot{\theta} I_{\text{Tot}}$$

$$\dot{\theta} I_{\text{Tot}} = T_m - T_f$$

$$\text{Kinematic helpers: } T_f = C_F \dot{\theta}$$

$$\dot{\theta} I_{\text{Tot}} = T_m - C_F \dot{\theta}$$

$$\ddot{\theta} = \frac{T_m}{I_{\text{Tot}}} - \frac{C_F}{I_{\text{Tot}}} \dot{\theta}$$

$$\mathcal{L}\{X(t)\} = X(s) \quad \mathcal{L}\{\dot{X}(t)\} = sX(s) - X(0) \quad \mathcal{L}\{\ddot{X}(t)\} = s^2 X(s) - sX(0) - \dot{X}(0)$$

$$s^2 \Theta(s) = \frac{T_m(s)}{I_{\text{Tot}}} - s\Theta(s) \cdot \frac{C_F}{I_{\text{Tot}}}$$

$$s^2 \Theta(s) + s\Theta(s) \cdot \frac{C_F}{I_{\text{Tot}}} = \frac{T_m(s)}{I_{\text{Tot}}}$$

$$\Theta(s) \left(s^2 + s \frac{C_F}{I_{\text{Tot}}} \right) = \frac{T_m(s)}{I_{\text{Tot}}}$$

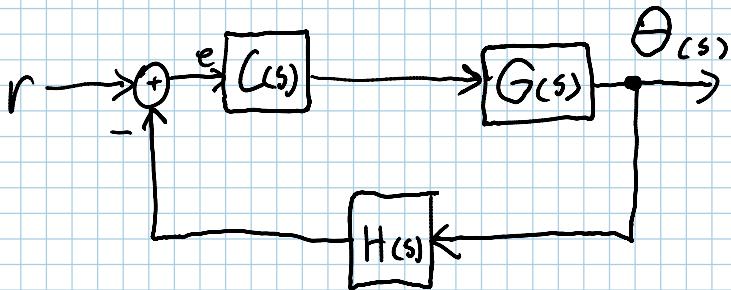
$$\Theta(s) = \frac{1}{s^2 + s \frac{C_F}{I_{\text{Tot}}}} \cdot \frac{T_m(s)}{I_{\text{Tot}}}$$

$$\Theta(s) = \frac{T_m(s)}{I_{\text{Tot}} s^2 + s C_F}$$

$$G(s) = \frac{\Theta(s)}{T_n(s)} = \frac{1}{I_{T_0} + s^2 + sC_F}$$

Transfer Function: Plant

Control Structure



$$\Theta(s) = G(s) C(s) \cdot e$$

$$e = r - H(s) \cdot \Theta(s)$$

$$\Theta(s) = G(s) C(s) (r - H(s) \Theta(s))$$

$$\Theta(s) = G(s) C(s) r - G(s) C(s) H(s) \Theta(s)$$

$$\Theta(s) + G(s) C(s) H(s) \Theta(s) = G(s) C(s) r$$

$$\Theta(s) (1 + G(s) C(s) H(s)) = G(s) C(s) r$$

$$\Theta(s) = \frac{G(s) C(s) r}{1 + G(s) C(s) H(s)}$$

$$\frac{\Theta(s)}{r} = \frac{G(s) C(s)}{1 + G(s) C(s) H(s)}$$

$$\underline{\Theta(s)} = \underline{\frac{1}{G(s) H(s)}} \cdot \underline{G(s) C(s)}$$

$$\frac{\Theta(s)}{r} = \frac{\frac{1}{G(s)C(s)} \cdot G(s)C(s)}{\frac{1}{G(s)C(s)}(1 + G(s)C(s)H(s))}$$

$$\frac{\Theta(s)}{r} = \frac{1}{\frac{1}{G(s)C(s)} + \frac{G(s)C(s)H(s)}{G(s)C(s)}}$$

$$\frac{\Theta(s)}{r} = \frac{1}{\frac{1}{G(s)C(s)} + H(s)}$$

Transfer function: System

Pole placement

roots: negative
complex $As^2 + Bs + C = 0$
 solve for roots

Filters: $\frac{P}{s + R}$ 1st order

$\frac{P^2}{s^2 + Qs + R^2}$ 2nd order

$$G(s) = \frac{\Theta(s)}{T_m(s)} = \frac{1}{I_{tot} + s^2 + sC_F}$$

$C(s)$ - as a fn of I_{tot} , $H(s)$, Coupling: Control structure



$$\text{Proportional-only : } C_{\text{out}} = e k_p = (r - H(s)G(s))k_p$$

$$\text{Integral : } R_p + R_I \frac{1}{s}$$

$$C_{\text{out}} = e \left(k_p + k_I \frac{1}{s} \right)$$

$$\text{Derivative: } k_p + k_I \frac{1}{s} + k_d s = C(s)$$

$$C_{\text{out}} = e \left(k_p + k_I \frac{1}{s} + k_d s \right)$$

$$C(s) = k_p + k_I \frac{1}{s} + R_d s$$

$$G(s)C(s) = \frac{1}{I_{\text{tot}} + s^2 + s C_F} \cdot k_p + k_I \frac{1}{s} + k_d s$$

$$= \frac{k_p + k_I \frac{1}{s} + R_d s}{I_{\text{tot}} s^2 + s C_F}$$

$$\frac{1}{G(s)C(s)} = \frac{I_{\text{tot}} + s^2 + C_F s}{k_p + k_I \frac{1}{s} + R_d s}$$

$$\frac{\Theta(s)}{r} = \frac{1}{I_{\text{tot}} + s^2 + C_F s}, \text{ L...}$$

$$\frac{\Theta(s)}{r} = \frac{\frac{I_{\text{tot}} + s^2 + C_F s}{k_p + k_I \frac{1}{s} + k_d s} + H(s)}$$

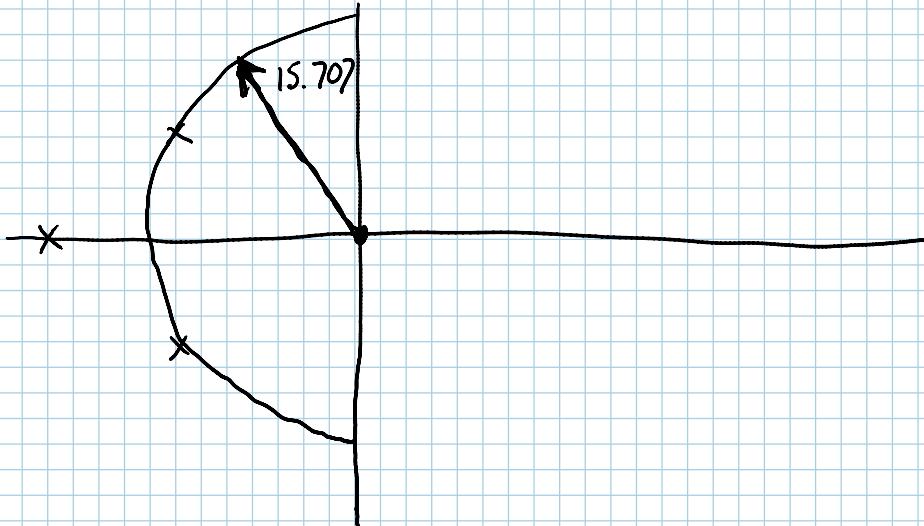
$$\frac{\Theta(s)}{r} = \frac{k_p + k_I \frac{1}{s} + k_d s}{I_{\text{tot}} + s^2 + C_F s + k_p H(s) + k_I \frac{1}{s} H(s) + k_d s H(s)}$$

$$\frac{\Theta(s)}{r} = \frac{k_p + k_I \frac{1}{s} + k_d s}{I_{\text{tot}} + s^2 + (C_F + k_d H(s)) s + k_p H(s) + k_I H(s) \frac{1}{s}} \cdot \frac{s}{s}$$

$$\frac{\Theta(s)}{r} = \frac{k_d s^2 + k_p s + k_I}{I_{\text{tot}} + s^3 + (C_F + k_d H(s)) s^2 + k_p H(s) s + k_I H(s)}$$

Choose desired poles, solve for k's

$$\omega_b = 2.5 \text{ Hz} = 2.5 (2\pi) \frac{\text{rad}}{\text{s}} = 15.71 \frac{\text{rad}}{\text{s}}$$



solve roots

$$\frac{\Theta(s)}{r} = \frac{k_d s^2 + k_p s + k_I}{I_{T_0+} s^3 + (C_F + k_d H(s)) s^2 + k_p H(s) s + k_I H(s)}$$

$$\frac{\Theta(s)}{r} = \frac{\frac{k_d s^2 + k_p s + k_I}{I_{T_0+}}}{s^3 + \frac{C_F + k_d H(s)}{I_{T_0+}} s^2 + \frac{k_p H(s)}{I_{T_0+}} s + \frac{k_I H(s)}{I_{T_0+}}}$$

Place poles at $\omega_d = 15.707$, so

a: complex pole

b: real pole

$$(s-a)^2(s-b)$$

$$(s-a)(s-b)$$

$$s^3 - (2a+b)s^2 + (2ab+a^2)s - a^2b$$

$$k_d : \frac{C_F + H(s)k_d}{I_{\text{tot}}} = -(2a+b)$$

$$C_F + H(s)k_d = -(2a+b)I_{\text{tot}}$$

$$k_d = \frac{-(2a+b)I_{\text{tot}} - C_F}{H(s)}$$

$$k_p = \frac{(2ab+a^2)I_{\text{tot}}}{H(s)}$$

$$k_I = \frac{-a^2b I_{\text{tot}}}{H(s)}$$

Dynamic Coupling:

$$\ddot{\Theta}_v I_{\text{tot},v} = \sum M_v - \dot{\Theta}_v \frac{dI}{dt}$$

$$\ddot{\Theta}_l I_{\text{tot},l} = \sum M_l - \cos(\Theta_l) \cdot \dot{\Theta}_v \cdot (mL) \quad ?$$

$$\tau = \|r\| \cdot \|F\| \cos(\theta_i); \quad F_i = m r_i \omega^2; \quad r = L$$

$$\text{At } \theta_i = 0^\circ, \quad F_{c,i} = mL_i \dot{\theta}_v^2$$

$$\text{At } \theta_i = 90^\circ, \quad F_{c,i} = 0$$

$$\text{At } \theta_i = 180^\circ, F_{c,i} = -m L_i \dot{\theta}_v^2$$

$$\text{So, } F_{c,i} = \cos(\theta_i) \cdot m \cdot r \cdot \dot{\theta}_v^2$$

$$r_i = (L_i + \sin(\theta_i) L)$$

$$M_{c,i} = F_{c,i} \cdot L \cdot \cos(\theta_i) \cdot (L_i + \sin(\theta_i) L) \cdot \dot{\theta}_v^2$$

$$\ddot{\theta}_i I_{T_{\text{tot}}} = \sum M_i + \cos(\theta_i) \cdot (L_i + \sin(\theta_i) L)^2 \cdot m \dot{\theta}^2$$