Dynamic Modeling V

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Coefficients - Vert and Lat Axis

$$k_d = \frac{-(2a+b)I_{Tot,v} - C_F}{H(s)} \qquad k_p = \frac{(2ab+a^2)I_{Tot,v}}{H(s)}$$

$$k_I = \frac{-a^2 b\, I_{Tot}}{H(s)}$$

$$\omega_{CB} = 2.5\ Hz = 2.5 \cdot 2\pi\ rad = 15.707\ rad/s$$

$$b = 15.707$$

$$a = \sin\left(\frac{\pi}{4}\right) \cdot 15.707(1+j) = 11.11 + 11.11j$$

$$H(s) = \frac{3789395}{s^3 + 321.3 s^2 + 48495 s + 3789395}$$

Let $p = 3789395$, $q = 321.3$, $u = 48495$

$$\frac{\Theta(s)}{r} = \frac{k_d s^2 + k_p s + k_I}{I_{Tot} s^3 + (C_F + k_d H(s)) s^2 + k_p H(s) s + k_I H(s)}$$

$$= k_d s^2 + k_p s + k_I \div \left[I_{Tot} s^3 + \left(C_F + k_d \cdot \frac{p}{s^3 + q s^2 + u s + p}\right) s^2 \right.$$

$$\left. + k_p \cdot \frac{p}{s^3 + q s^2 + u s + p} s + k_I \cdot \frac{p}{s^3 + q s^2 + u s + p} \right]$$

$$= k_d s^2 + k_p s + k_I \div \left[I_{Tot} s^3 + C_F s^2 + \frac{k_d \cdot p}{s^3 + q s^2 + u s + p} s^2 \right.$$

$$\left. + \frac{k_p \cdot p}{s^3 + q s^2 + u s + p} s + \frac{k_I \cdot p}{s^3 + q s^2 + u s + p} \right]$$

$$= k_d s^2 + k_p s + k_I \div \left[I_{Tot} \cdot \frac{s^3 + q s^2 + u s + p}{s^3 + q s^2 + u s + p} s^3 \right.$$

$$\left. + \frac{C_F (s^3 + q s^2 + u s + p) + k_d \cdot p}{s^3 + q s^2 + u s + p} s^2 + \frac{k_p \cdot p}{s^3 + q s^2 + u s + p} s + \frac{k_I \cdot p}{s^3 + q s^2 + u s + p} \right]$$

$$= k_d s^2 + k_p s + k_I \div \left[\frac{I_{Tot} (s^3 + q s^2 + u s + p)}{s^3 + q s^2 + u s + p} s^3 + \frac{C_F \cdot (s^3 + q s^2 + u s + p) + k_d \cdot p}{s^3 + q s^2 + u s + p} s^2 \right.$$

$$\left. + \frac{k_p \cdot p}{s^3 + q s^2 + u s + p} s + \frac{k_I \cdot p}{s^3 + q s^2 + u s + p} \right]$$

$$= \frac{(k_d s^2 + k_p s + k_I)(s^3 + q s^2 + u s + p)}{I_{Tot} (s^3 + q s^2 + u s + p) s^3 + C_F (s^3 + q s^2 + u s + p) s^2 + k_d \cdot p s^2 + k_p \cdot p s + k_I \cdot p}$$

$$= \frac{k_d s^5 + k_d \cdot q s^4 + k_d u s^3 + k_d p s^2 + k_p s^4 + k_p q s^3 + k_p u s^2 + k_p p s + k_I s^3 + k_I q s^2 + k_I u s + k_I p}{I_{Tot} (s^6 + q s^5 + u s^4 + p s^3) + C_F (s^5 + q s^4 + u s^3 + p s^2) + k_d \cdot p s^2 + k_p p s + k_I \cdot p}$$

$$= \frac{k_d s^5 + (k_d q + k_p) s^4 + (k_d u + k_p q + k_I) s^3 + (k_d p + k_p u + k_I q) s^2 + (k_p p + k_I u) s + k_I p}{}$$

$$I_{Tot}s^6+(I_{Tot}q+C_F)s^5+(I_{Tot}u+C_Fq)s^4+(I_{Tot}p+C_Fu)s^3+(C_Fp+k_dp)s^2+k_pps+k_Ip$$

Solving for coefficients - simultaneous equations?

- Nonlinear terms in I_{Tot}

Moment of Inertia and Gravity

Vertical Axis

$$I_{Tot,v}=\left(I_{CM,o}+m_oL_o^2\right)+\left(I_{cm}+m\cdot(\sin(\theta_l)L)^2\right)$$

Model payload moment of inertia as solid sphere for simplicity, with a radius of 30 cm (parameterize in firmware to allow real values to be entered)

L_m = length of motor

$$I_{cm}=\frac{2}{5}m\cdot(0.3\,m)^2=0.036m\,[kg\cdot m^2]$$

$$I_{cm,o}=\frac{1}{3}m_o\cdot L_m^2$$

$$M'_{g,v}=\cos(\theta_v)\cdot g\left(\cos(\theta_v)\cdot m_oL_o^2-\left((L_c+\cos(\theta_l)\cdot L\right)^2m\right)$$

Lateral Axis

$$I_{Tot,l}=I_{cm}+mL^2$$

$$M'_{g,l}=\sin(\theta_v)\cdot\cos(\theta_l)\cdot mg\cdot L$$