

UNIVERSIDAD
NACIONAL
DE COLOMBIA

CLASIFICACIÓN Y RECONOCIMIENTO DE PATRONES

REDUCCION DE LA DIMENSIONALIDAD

JOHN W. BRANCH

Profesor Titular

Departamento de Ciencias de la Computación y de la Decisión
Director del Grupo de I+D en Inteligencia Artificial – GIDIA

PEDRO ATENCIO ORTIZ

Profesor Invitado

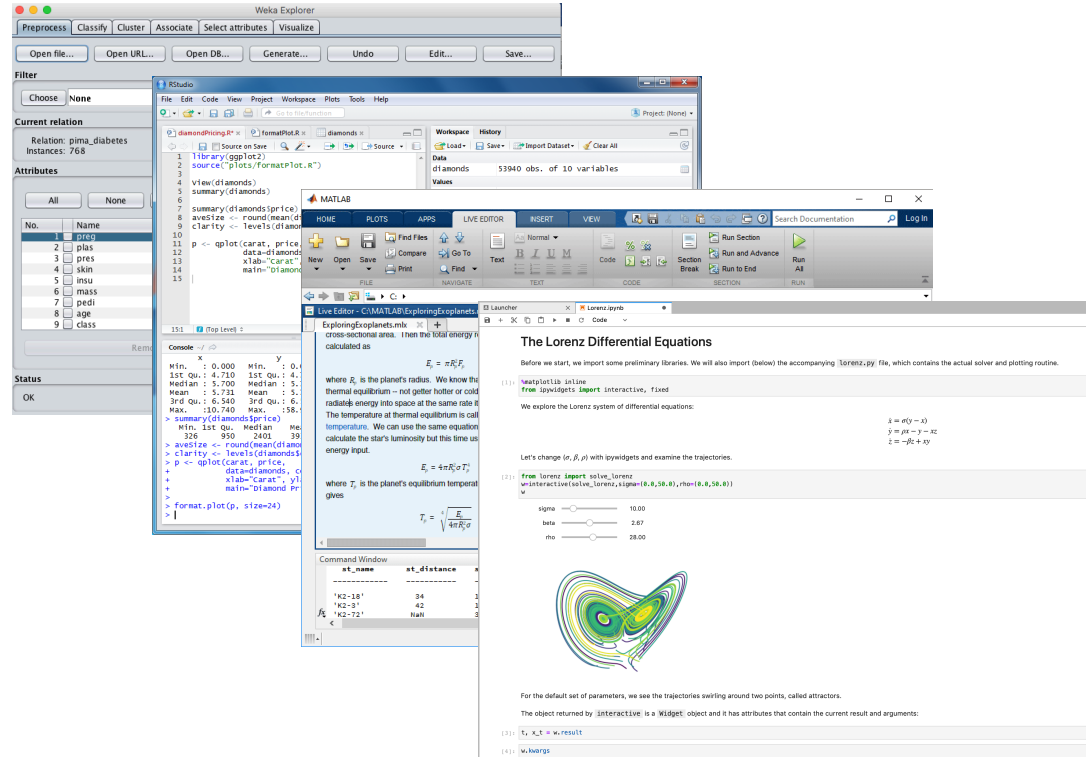
Facultad de Ingenierías
Instituto Tecnológico Metropolitano

Agenda

1. Data processing Tools.
2. Dimensionality reduction using Python and Scikit:
 1. Feature Selection.
 1. The Knapsack problem.
 2. Statistical approach (Filters).
 1. Variance criteria.
 2. Statistical tests.
 3. Wrappers and Embedded models.
 1. Wrappers and search.
 2. Recursive elimination.
 3. Select from model.
 2. Feature Extraction.
 1. Feature transformation.
 2. Principal Component Analysis (PCA).
 1. SVD vs SKLearn.
 2. Non-Linear PCA (Kernel).
3. Conclusions.
4. Classwork.

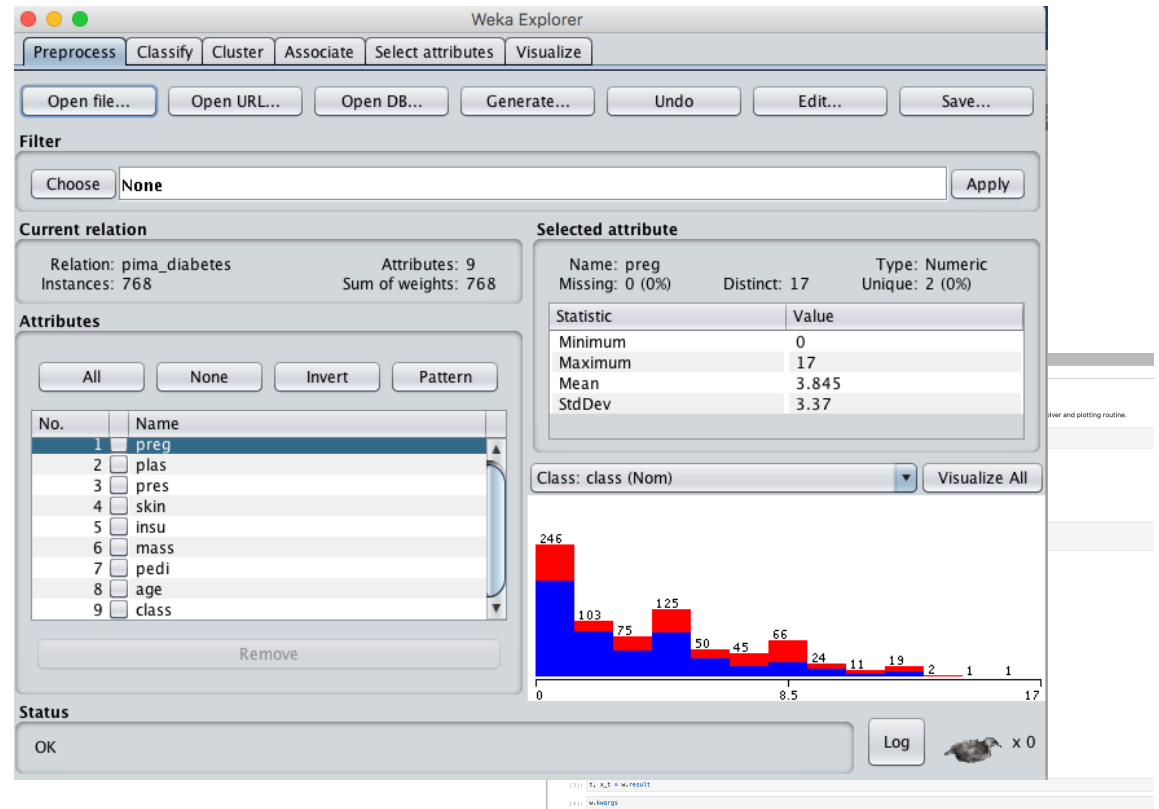
1. Data Processing Tools

- Weka
- R+RStudio
- MATLAB
- Python+Jupyter+Scikit



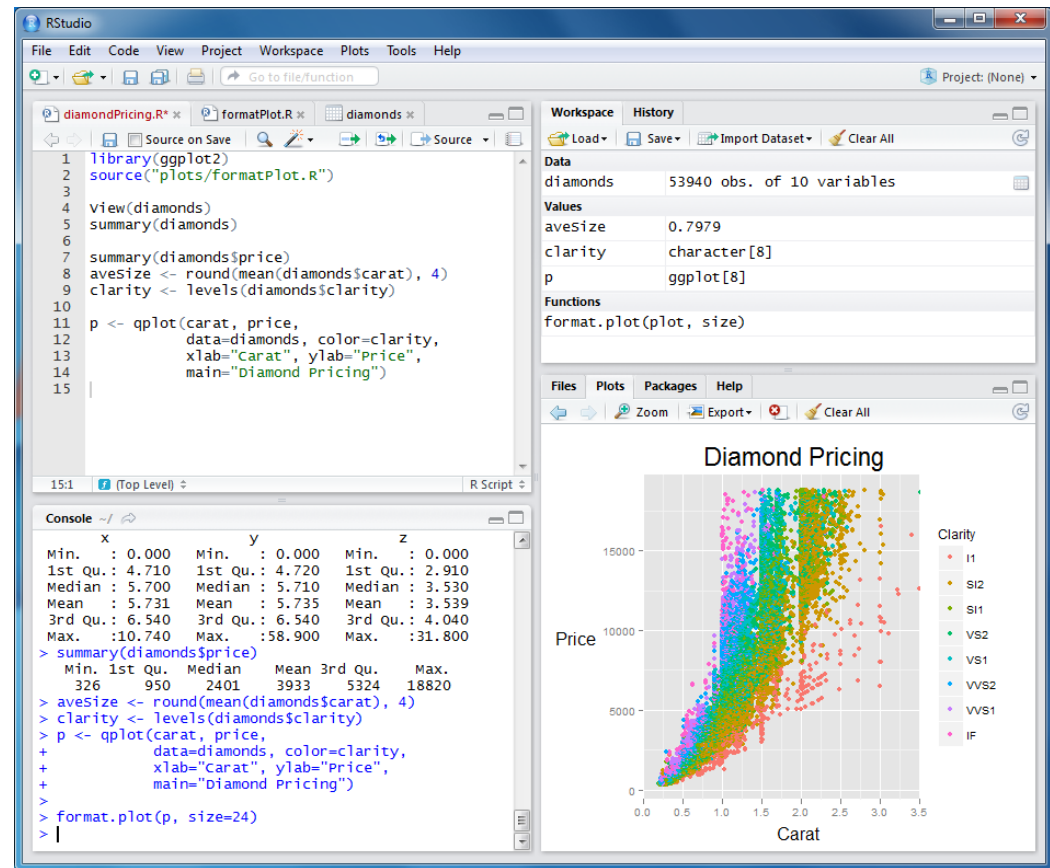
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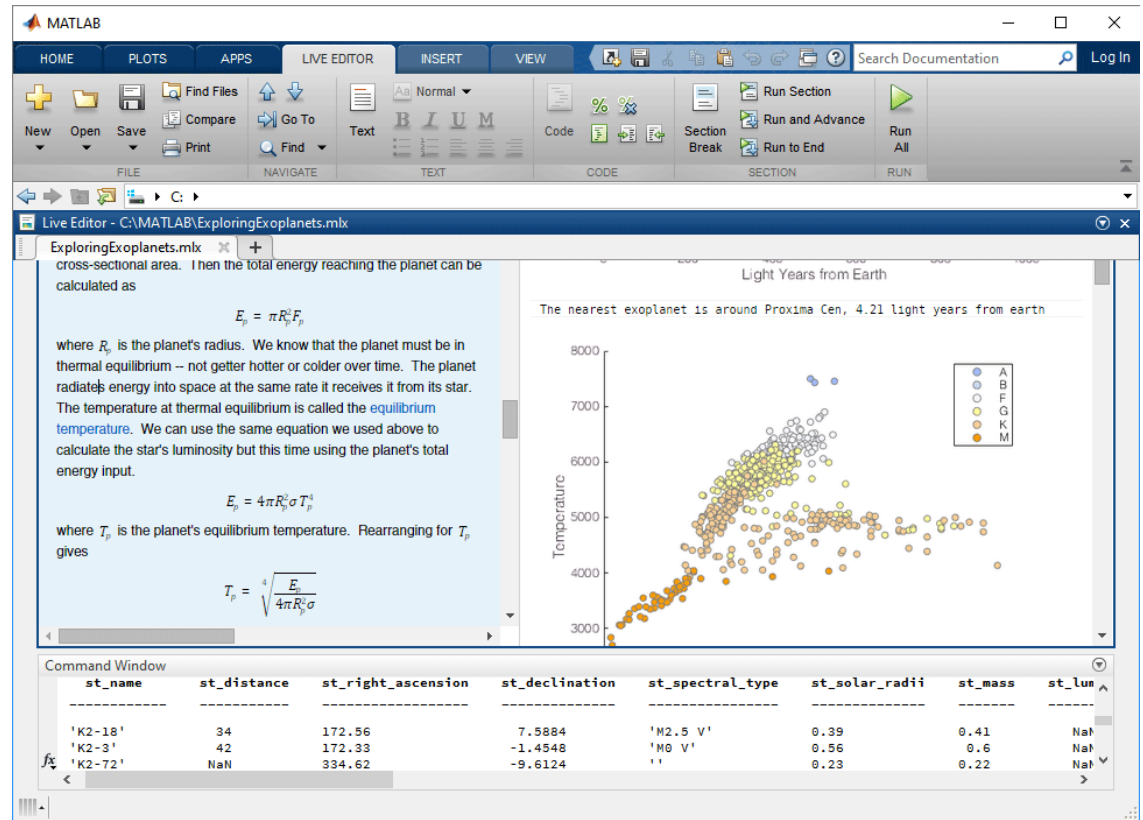
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Launcher X Lorenz.ipynb

The Lorenz Differential Equations

Before we start, we import some preliminary libraries. We will also import (below) the accompanying `lorenz.py` file, which contains the actual solver and plotting routine.

```
[1]: %matplotlib inline
from ipywidgets import interactive, fixed
```

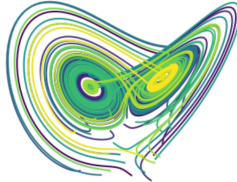
We explore the Lorenz system of differential equations:

$$\begin{aligned}\dot{x} &= \sigma(y - x) \\ \dot{y} &= \rho x - y - xz \\ \dot{z} &= -\beta z + xy\end{aligned}$$

Let's change (σ, β, ρ) with ipywidgets and examine the trajectories.

```
[2]: from lorenz import solve_lorenz
w=interactive(solve_lorenz, sigma=(0.0,50.0), rho=(0.0,50.0))
w
```

sigma 10.00
beta 2.67
rho 28.00



For the default set of parameters, we see the trajectories swirling around two points, called attractors.

The object returned by `interactive` is a `Widget` object and it has attributes that contain the current result and arguments:

```
[3]: t, X_t = w.result
```

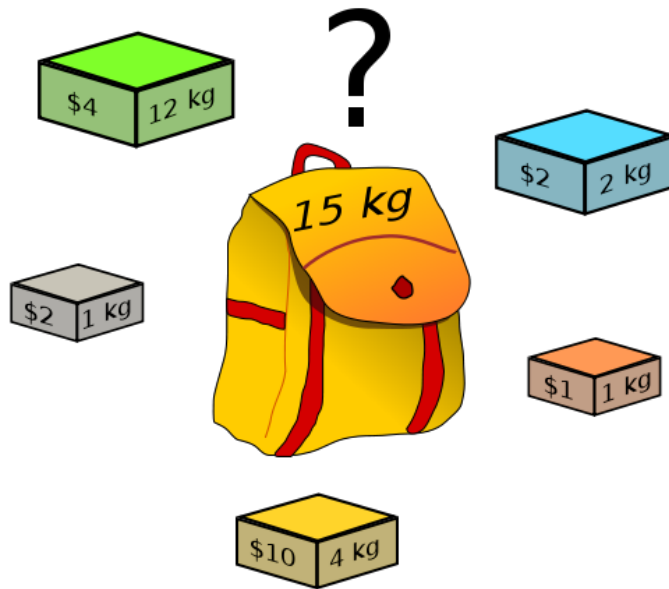
```
[4]: w.kwargs
```


2. Dimensionality Reduction Using Python and Scikit

2.1. Feature Selection

2.1.1 The Knapsack Problem

The feature selection problem can be understood as the Knapsack problem:



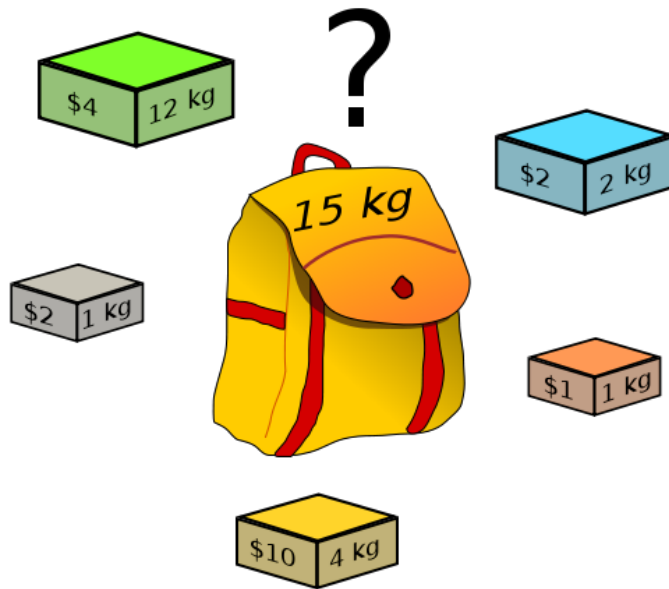
$$\text{maximize } \sum_{i=1}^n R(v_i x_i)$$

$$\text{s.t. } \sum_{i=1}^n |v_i|_0 \leq l$$

2.1. Feature Selection

2.1.1 The Knapsack Problem

The feature selection problem can be understood as the Knapsack problem:



$$\begin{aligned} & \text{maximize} \sum_{i=1}^n R(v_i; x_i) \\ & s.t. \sum_{i=1}^n |v_i|_0 \leq l \end{aligned}$$

Annotations for the equations:

- Problem performance (points to $R(v_i; x_i)$)
- features (points to x_i)
- Binary array (points to v_i)
- max features (points to l)
- num of selected features (points to $|v_i|_0$)



2.1. Feature Selection

2.1.2. Statistical Approach (Filters)

A score is assigned to each feature according to a statistical measure, for example:

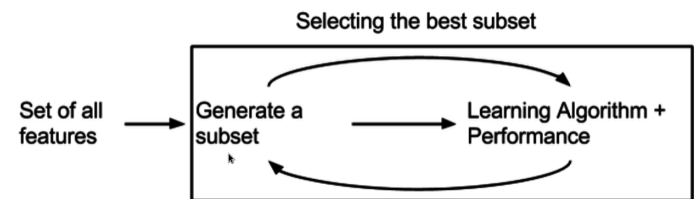
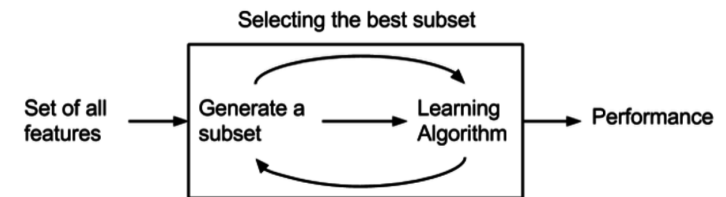
- Variance σ^2
- Chi - Squared χ^2
- Anova.
- Entropy.
- etc.

2.1. Feature Selection

2.1.3. Wrappers and Embedded Models

In simple words both approaches consists in:

- **Wrappers:** A predictive model is used to evaluate **different combinations of features using a search method**, for example, SFS, BFS, etc. Once, some combinations a tried, best subset of features is selected.
- **Embedded Models:** Use **two predictive models**, one to evaluate subset performance and one to LEARN which subset is best.



2.2. Feature Extraction

2.2.1. Feature Transformation

Feature transformation consists in obtaining set of features G from a lineal or non-linear mapping of an input set X .

$$G = f(X) = \alpha x_1 + \alpha x_2 + \alpha x_3 + \dots + \alpha x_n$$

$$s.t. |G| < |X|$$

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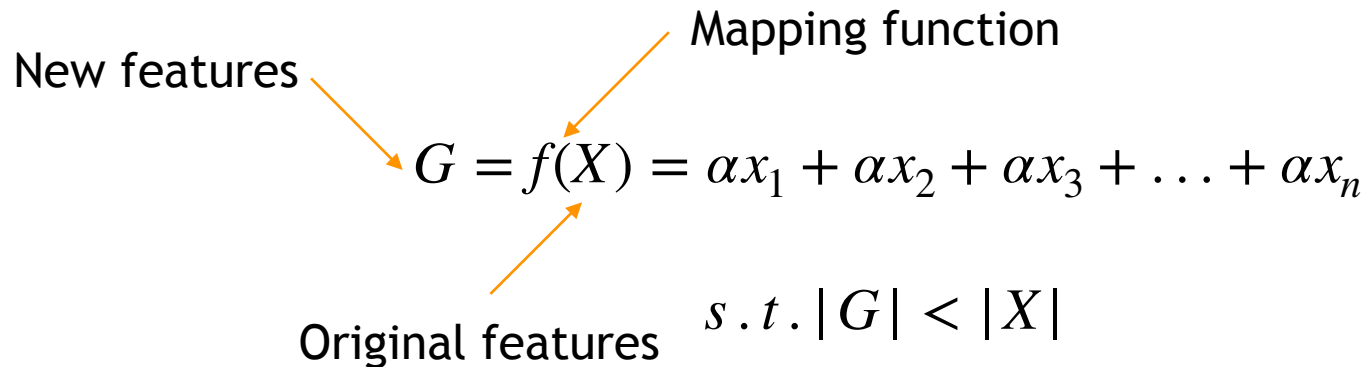
New features

Mapping function

$$G = f(X) = \alpha x_1 + \alpha x_2 + \alpha x_3 + \dots + \alpha x_n$$

Original features

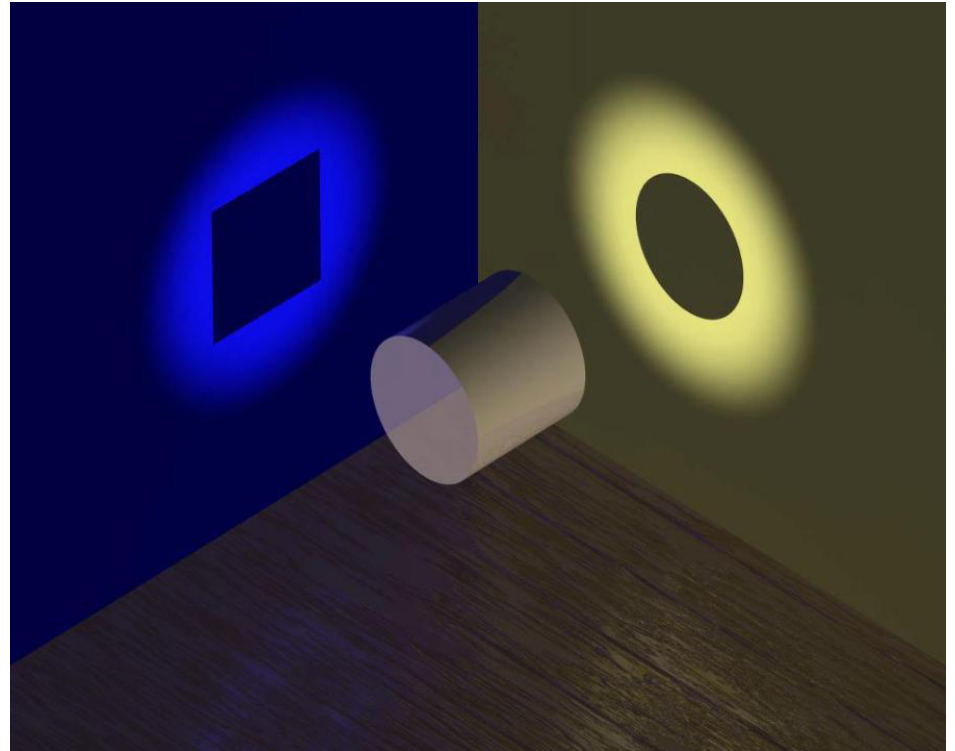
$s.t. |G| < |X|$



2.2. Feature Extraction

2.2.2. Principal Component Analysis

PCA consists in projecting a set of data over an hyperspace composed by principal dimensions of data.

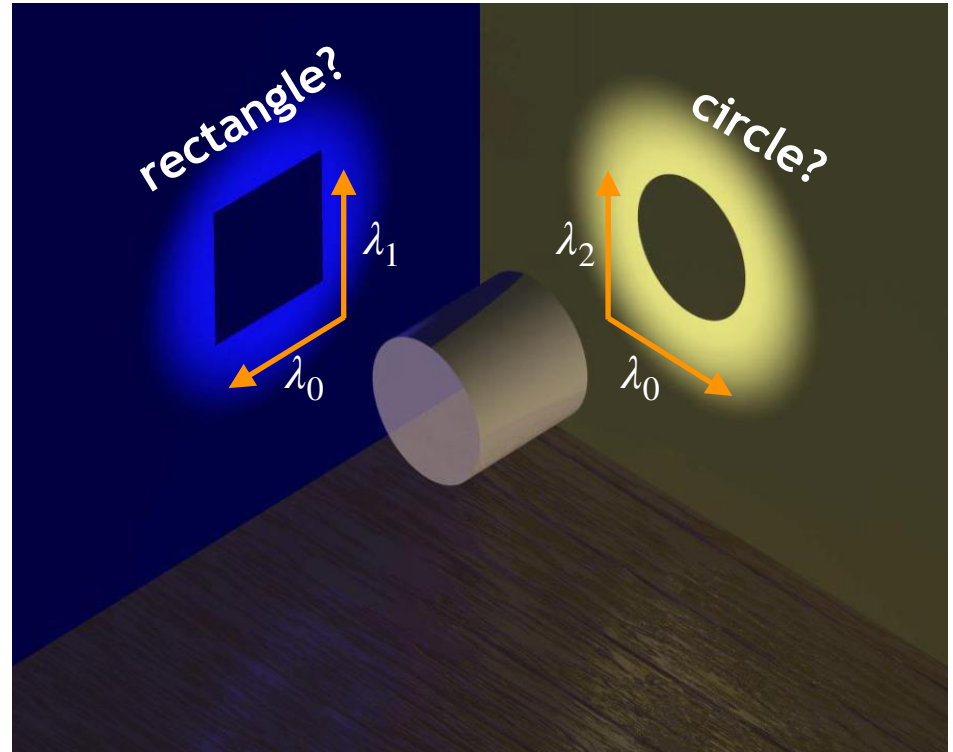


2.2. Feature Extraction

2.2.2. Principal Component Analysis

PCA consists in **projecting** a set of data over an hyperspace composed by principal dimensions of data.

Which $\lambda_i \in \Lambda$ are best?



2.2. Feature Extraction

2.2.2. Principal Component Analysis

Matrix factorization techniques can be used to extract Eigenvalues or principal components of a dataset. Single Value Decomposition is a matrix factorization technique:

$$X = U \cdot \Sigma \cdot V^T$$

Principal Components Matrix



2.2. Feature Extraction

2.2.2. Principal Component Analysis

Matrix factorization techniques can be used to extract Eigenvalues or principal components of a dataset. Single Value Decomposition is a matrix factorization technique:

$$X = U \cdot \Sigma \cdot V^T$$

Principal Components Matrix

Once V is extracted, we can project matrix X over one or more Eigenvalues or principal components and reduce original dimension of X :

$$X_{new} = X \cdot V'$$

Subset of V

3. Conclusions

There are many approaches that can be applied in order to reduce dimensionality which will be dependent of the problem we are trying to solve. Other some popular techniques are:

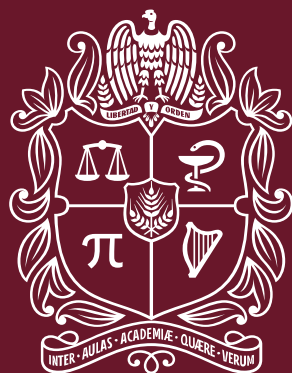
- Multidimensional Scaling (MDS) —> distance reduction.
- Isomap —> graph connections and geodesic preservation.
- t-SNE —> Embedding.
- LDA —> Classifier that learns discriminative axes.



4. Classwork

Work over next problems, using previous code as a base:

1. Load diabetes regression dataset using `load_diabetes()` from `sklearn.datasets`
 1. Tabulate data X and y .
 2. Apply a feature selection approach to obtain best 3 features.
 3. Plot using `plt.plot()` 3 plots: (x_0, y) , (x_1, y) , (x_2, y) .
2. Load breast_cancer classification dataset using `load_breast_cancer()` from `sklearn.datasets`:
 1. Tabulate data X and y .
 2. Apply a feature extraction approach to obtain 2 features.
 3. Plot using `plt.scatter()` to obtain a figure where can be observed elements and classes.



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