The highest temperature 6000 K represents nearly the surface temperature of th (5762 K). The other two temperatures, i.e., 1000 K represents the high temperature solar surface while 400 K depicts the low temperature solar heated surface.

Energy emitted by a blackbody at temperature T over the wavelengths is expressed

$$E_b = \int_0^\infty E_{\lambda b} d\lambda = \sigma T^4$$

where σ = 5.6697 × 10^{-8} W/m$^2 \cdot$K^4 and is called the Stefan–Boltzmann constant wavelength corresponding to maximum intensity of blackbody radiation at temperatu expressed by Wien's Displacement law as

$$\lambda_{\max} T = 2897.8\ \mu\text{m} \cdot \text{K}$$

It shows that an increase in temperature shifts the maximum blackbody radiation int towards the shorter wavelength.

The dotted line in Figure 4.3 indicates the displacement of wavelength for max intensity as given by Eq. (4.3). The radiation emitted by a real body is a fraction blackbody radiation, i.e.,

$$E = \varepsilon \sigma T^4$$

where ε represents the emissivity of a real body surface and is always less than 1.

4.5 RADIATION HEAT TRANSFER BETWEEN REAL BODIES

Radiation exchange between two surfaces takes place from a hot to a cold body. The r exchange of heat energy between two closely spaced parallel bodies, one at a temperature T with emissivity ε_1 and the other at a temperature T_2 and with emissivity ε_2, is given by

$$Q_{\text{rad}} = \frac{\sigma A}{[(1/\varepsilon_1) + (1/\varepsilon_2)] - 1} (T_1^4 - T_2^4)$$

To evaluate the performance of a solar collector, it is necessary to calculate the radi exchange between the collector and the sky. The net radiation to a body of surface area A emittance ε and temperature T from the sky is calculated from

$$Q = \varepsilon A \sigma (T_{\text{sky}}^4 - T^4)$$

where T_{sky} is called the sky temperature and it is the temperature of the equivalent black

To estimate T_{sky}, for clear skies, Swinbank (1963) proposed a relation of sky temperat local air ambient temperature T_{air} (kelvin) as given by the equation

$$T_{\text{sky}} = 0.0552\, T_{\text{air}}^{1.5}$$

Whiller (1967) proposed another simple relation,

$$T_{\text{sky}} = T_{\text{air}} - 12$$

or the relation

$$T_{\text{sky}} = T_{\text{air}} - 6$$

4.6 RADIATION OPTICS

Thermal radiation from a high temperature body to a lower temperature body causes transfer of heat through electromagnetic waves up to 0.1 μm–100 μm. The larger part of the terrestrial solar energy lies between 0.3 μm and 3 μm. Thermal radiation is in the infrared range and travels at the speed of light. When radiation strikes a body, a part is reflected, another is absorbed, and the remainder is transmitted through if the body is transparent. The law of conservation of energy dictates that the total sum of radiation components must be equal to incident radiation, i.e.,

$$I_\alpha + I_\rho + I_\tau = I \tag{4.10}$$

and

$$\alpha + \rho + \tau = 1 \tag{4.11}$$

where α, ρ and τ are absorptivity, reflectivity and transmissivity of the light-impinged body. I_α, I_ρ and I_τ are radiation components that are absorbed, reflected and transmitted respectively. The values of α, ρ and τ are always positive within the limits of 0 and 1.

For an opaque surface, $\tau = 0$, so

$$\alpha + \rho = 1$$

For a white surface which reflects all radiation, $\rho = \tau = 0$ and so $\alpha = 1$.

For a blackbody, α and τ are zero and $\rho = 1$ making it a body that absorbs all the energy incident on it.

4.7 TRANSMISSIVITY OF THE COVER SYSTEM

Transmissivity considering reflection only, when a light beam strikes a glass surface there are two losses—one is reflection loss from the top surface and the other is absorption loss as the beam passes through the glass material. First we find transmittance as if there is reflection loss only and then we find transmittance as if there is absorption loss only.

When a beam of light having intensity I_1 travelling in a transparent medium 1 strikes another transparent medium 2, a part of it is reflected and the major part is refracted (Figure 4.4).

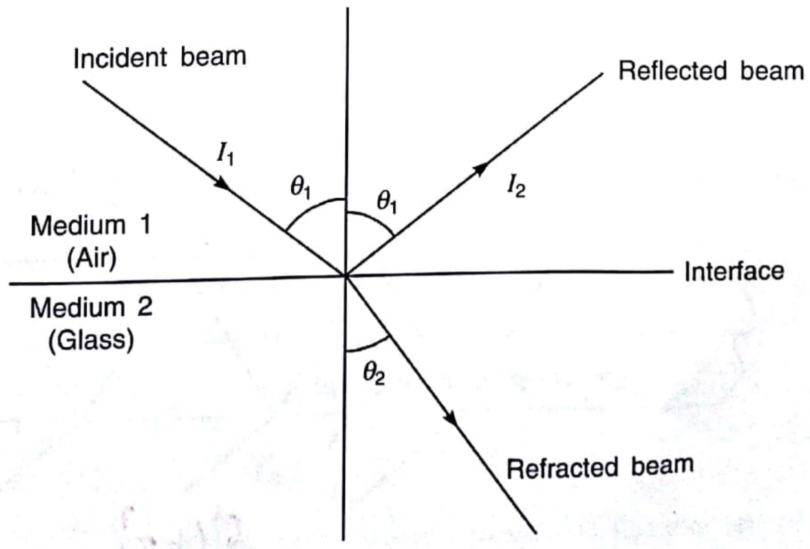

Figure 4.4 Reflection and refraction at the interface of two transparent media.

According to the Snell's law of refraction,

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1} \tag{4.12}$$

where θ_1 = angle of incidence, θ_2 = angle of refraction and n_1, n_2 = refractive indices of the two media.

Reflectivity is expressed by, $\rho = I_2/I_1$, where I_2 is the reflected beam intensity and I_1 is the incident beam radiation. Also,

$$\rho = \frac{1}{2}(\rho_1 + \rho_2) \tag{4.13}$$

where ρ_1 and ρ_2 are the reflectivities for the two components of polarization—one parallel to the plane of incidence and the other perpendicular to this plane—as given below:

$$\rho_1 = \frac{\sin^2(\theta_2 - \theta_1)}{\sin^2(\theta_2 + \theta_1)}$$

$$\rho_2 = \frac{\tan^2(\theta_2 - \theta_1)}{\tan^2(\theta_2 + \theta_1)}$$

For radiation at normal incidence, $\theta_1 = 0$ and for this case

$$\rho = \rho_1 = \rho_2 = \frac{(n_2 - n_1)^2}{(n_2 + n_1)^2} \tag{4.14}$$

Transmissivity τ can be expressed similar to that for ρ, i.e.,

$$\tau = \frac{1}{2}(\tau_1 + \tau_2) \tag{4.15}$$

where τ_1 and τ_2 are the polarization components of transmissivity.

The cover material used in solar appliances requires transmission of radiation through a slab or sheet having two interfaces per cover where reflection-refraction takes place. The cover interfaces with air on both sides. Multiple reflections and refractions will occur as shown in Figure 4.5.

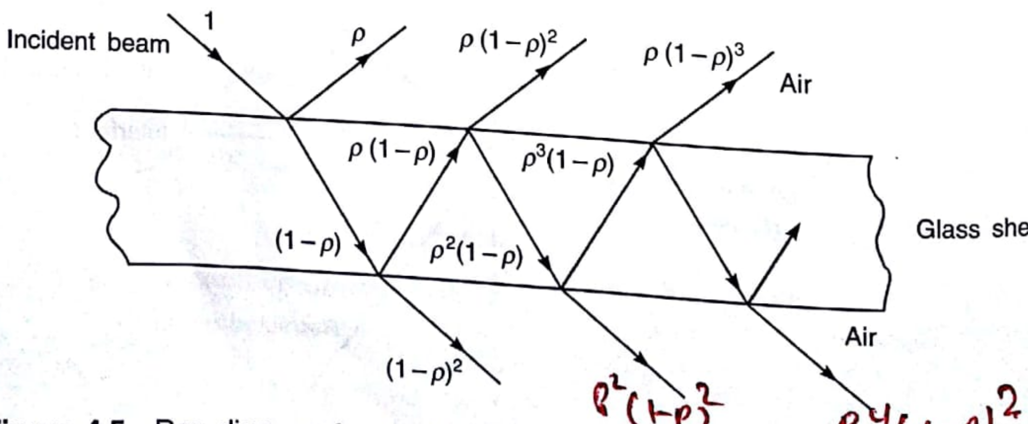

Figure 4.5 Ray diagram for single cover multiple reflections and refractions.

For each component of polarization, the incident beam depletes at the second surface. The amount of incidence beam reaching below the interface after reflection is only $(1 - \rho)$, i.e., in case of a unit incident beam after reflection, only $(1 - \rho)$ reaches the second interface. From this, $(1 - \rho)^2$ passes through the interface and $\rho(1 - \rho)$ is reflected back to the first interface, and the process is repeated. Summing up all the terms, the transmittance for a single cover is

$$\tau_1 = (1-\rho_1)^2 + (1-\rho_1)^2 \rho_1^2 + (1-\rho_1)^2 \rho_1^4 + \cdots$$

$$= (1-\rho_1)^2 (1 + \rho_1^2 + \rho_1^4 + \cdots)$$

$$= (1-\rho_1)^2 \frac{1}{1-\rho_1^2}$$

or
$$\tau_1 = \frac{1-\rho_1}{1+\rho_1}$$

Similarly
$$\tau_2 = \frac{1-\rho_2}{1+\rho_2}$$

For a system of N covers and of the same material, therefore, we can write

$$\tau_1 = \frac{1-\rho_1}{1+(2N-1)\rho_2}$$

$$\tau_2 = \frac{1-\rho_2}{1+(2N-1)\rho_1} \tag{4.16}$$

4.7.1 Transmittance Considering Absorption Only

Transmissivity, based on absorption, in a transparent material sheet, can be explained by the Bouger's law, i.e.,

$$dI = -KI\,dx$$

where dI is the decrease in radiation intensity, I is the initial value of intensity, K is a constant of proportionality known as 'extinction coefficient', x is the distance travelled by radiation. Assuming that K is a constant in the solar spectrum range, then integrating the expression for dI, we get

$$\int_{I_0}^{I_L} \frac{dI}{I} = -K\int_0^L dx$$

or
$$\log I\Big|_{I_0}^{I_L} = -Kx\Big|_0^L$$

or
$$\log I_L - \log I_0 = -KL$$

or
$$I_L = I_0 e^{-KL}$$

or
$$\tau_\alpha = \frac{I_L}{I_0} = e^{-KL} \tag{4.17}$$

where τ_α is the transmittance considering only absorption and L is distance travelled by radiation through the medium.

The extinction coefficient K is a physical property of the cover material. For clear white glass, the value of K is 0.04/cm, while for poor quality glass with greenish colour at its edges the value of K is 0.25/cm. A low value of K is preferred.

When the beam is incident at an angle θ_1, the path length through the cover would be ($L/\cos\,\theta_2$), where θ_2 is the angle of refraction. Thus, Eq. (4.17) is modified as

$$\tau_\alpha = e^{-KL/\cos\theta_2}$$

The transmissivity of the system allowing for both absorption and reflection is given by

$$\tau = \tau_\alpha \tau_\rho$$

EXAMPLE 4.1

Estimate τ_α, τ_ρ and τ for a glass cover system with the given data:

Angle of incidence = 10°
Number of covers = 4
Thickness of each cover = 3 mm
Refractive index of glass relative to air = 1.52
Extinction coefficient of glass = 15 m^{-1}

Solution

θ_1 = 10°, using Snell's law,

$$\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2} \qquad (\theta_1 = 10°)$$

n_2/n_1 = refractive index of glass relative to air = 1.52 (given)

So,
$$\theta_2 = \sin^{-1}\left(\frac{\sin 10°}{1.52}\right) = 6.55°$$

$$\rho_1 = \frac{\sin^2(6.55° - 10°)}{\sin^2(6.55° + 10°)} = 0.044$$

$$\rho_2 = \frac{\tan^2(6.55° - 10°)}{\tan^2(6.55° + 10°)} = 0.041$$

$$\tau_{\alpha_1} = \frac{1 - 0.044}{1 + 7 \times 0.041} = 0.733$$

$$\tau_{\alpha_2} = \frac{1 - 0.041}{1 + 7 \times 0.044} = 0.742$$

$$\tau_\alpha = \frac{1}{2}(0.733 + 0.742) = 0.737$$

$$\tau_\rho = e^{-KL/\cos\theta_2}$$

Given in the problem

$K = 15\ \text{m}^{-1}$

$L = 4 \times 3 \times 10^{-3}$

$\therefore \quad \tau_\rho = 0.836$

and $\quad \tau = \tau_\alpha \tau_\rho = 0.737 \times 0.836$

$= 0.616$

4.7.2 Transmissivity–Absorptivity Product

For solar collector analysis, it is required to calculate the transmissivity–absorptivity product ($\tau\alpha$). Here, τ is the transmissivity of glass cover and α is the absorptivity of absorber plate. It is defined as the ratio of solar flux absorbed by the absorber plate to the solar flux incident on the cover system.

Solar radiation after passing through the cover system falls on the absorber plate, where some radiation is reflected back to the cover system. Out of the reflected part, a portion is transmitted through the cover system and a part gets reflected back to the absorber plate. This activity of absorption and reflection is shown in Figure 4.6 which goes on indefinitely. However, the quantities involved in the process gradually get reduced.

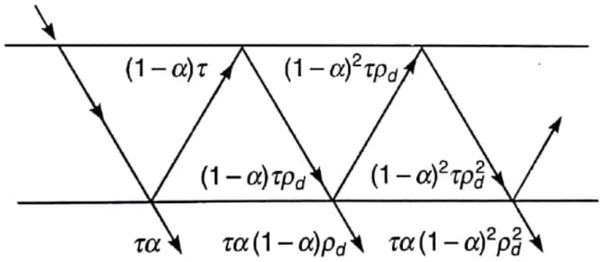

Figure 4.6 Absorption and reflection at an absorber plate.

Reflection from the absorber plate is more diffuse and let ρ_d be the reflectivity of glass cover for diffuse radiation. The fraction $(1-\alpha)\tau$ that reaches the cover plate is diffuse radiation, $(1-\alpha)\rho_d\tau$ is reflected back to the absorber plate.

Thus, the net radiation absorbed is the summation of

$$(\tau\alpha)_{\text{net}} = \tau\alpha + \tau\alpha(1-\alpha)\rho_d + \tau\alpha(1-\alpha)^2\rho_d^2 + \cdots$$

or

$$(\tau\alpha)_{\text{net}} = \frac{\tau\alpha}{1-(1-\alpha)\rho_d} \tag{4.18}$$

For an incident angle of 60°, the value of ρ_d is about 0.16, 0.24 and 0.29 for one, two and three glass covers respectively.

4.8 PERFORMANCE ANALYSIS OF A LIQUID FLAT-PLATE COLLECTOR

The performance of solar collector can be improved by enhancing the useful energy gain from incident solar radiation with minimum losses. Thermal losses have three components, namely the conductive loss, the convective loss and the radiative loss.

Conductive loss is reduced by providing insulation on the rear and sides of the absorber plate. Convective loss is minimized by keeping an air gap of about 2 cm between the cover and the plate. Radiative losses from the absorber plate are lowered by applying a spectrally selective absorber coating.

During normal steady-state operation, useful heat delivered by a solar collector is equal to the heat gained by the liquid flowing through the tubes welded on to the underside of the absorber plate minus the losses. The energy balance of the absorber can thus be represented by a mathematical equation, i.e.,

$$Q_u = A_p S - Q_L \tag{4.19}$$

where

Q_u = useful heat delivered by the collector (watts)

S = solar heat energy absorbed by the absorber plate (W/m^2)

A_p = area of the absorber plate (m^2)

Q_L = rate of heat loss by convection and reradiation from the top, by conduction and convection from the bottom and sides (watts).

From Eq. (3.23), the solar flux falling on an inclined surface is expressed by

$$I_T = I_b R_b + I_d R_d + (I_b + I_d) R_r$$

The flux absorbed is obtained if the above equation is multiplied by the transmissivity-absorptivity product $(\tau\alpha)$. Therefore,

$$S = I_b R_b (\tau\alpha)_b + [I_d R_d + (I_b + I_d) R_r] (\tau\alpha)_d \tag{4.20}$$

where $(\tau\alpha)_b$ is the transmissivity–absorptivity product for the beam radiation falling on the collector and $(\tau\alpha)_d$ is the transmissivity–absorptivity product for diffuse radiation impinging the collector.

Now, it is necessary to define two terms—*instantaneous collector efficiency* and *stagnation temperature* which are required to indicate the performances of the collector and also for comparing the designs of different collectors.

The instantaneous collector efficiency is defined as the ratio of *useful heat gain* to *radiation* falling on the collector. It is expressed by

$$\eta_i = \frac{Q_u}{A_p I_T} \tag{4.21}$$

Depending on the given data, the collector aperture area A_a or the collector gross area A_c is used in place of A_p in the above equation. The collector aperature area is the net opening in the top cover through which solar radiation passes into the collector. It is nearly 15% greater than the absorber plate area. The collector gross area is the top cover area including the frame and A_a is about 20% higher than A_p.

Thus, from Eq. (4.25),

$$U_s = \frac{(L_1 + L_2) L_3 K_i}{L_1 L_2 L_s} \tag{4.34}$$

4.10 SOLAR CONCENTRATING COLLECTORS

While dealing with flat-plate collectors with heat transport medium as water or air, the area of glass cover and that of absorber plate are practically the same. Thus, solar radiation intensity is uniformly distributed over the glass cover and the absorber, keeping the temperature rise of the solar device up to 100°C. If solar radiation falling over a large surface is concentrated to a smaller area of the absorber plate or receiver, the temperature can be enhanced up to 500°C. Concentration is achieved by an optical system either from the reflecting mirrors or from the refracting lenses. These concentrators are used in medium temperature or high temperature energy conversion cycles.

An optical system of mirrors or lenses projects the radiation on to an absorber of smaller area. This process compensates the reflection or absorption losses in mirrors or lenses and losses on account of geometrical imperfections in the optical system. A term called 'optical efficiency' takes care of all such losses. For higher collection efficiency, concentrating collectors are supported by a tracking arrangement that tracks the sun all the time, so that beam radiation is on to the absorber surface. As collectors provide a high degree of concentration, a continuous adjustment of collector orientation is required.

Some new terms that will be encountered in the text hereinafter are defined now for greater clarity. These are:

(i) 'Concentrator' is for the optical subsystem that projects solar radiation on to the absorber. The term 'receiver' shall be used to represent the sub-system that includes the absorber, its cover and accessories.

(ii) 'Aperture' (W) is the opening of the concentrator through which solar radiation passes.

(iii) 'Acceptance angle' ($2\theta_a$) is the angle across which beam radiation may deviate from the normal to the aperture plane and then reach the absorber.

(iv) 'Concentration ratio' (CR) is the ratio of the effective area of the aperture to the surface area of the absorber. The value of CR may change from unity (for flat-plate collectors) to a thousand (for parabolic dish collectors). The CR is used to classify collectors by their operating temperature range.

4.11 TYPES OF CONCENTRATING COLLECTORS

Plane receiver with plane collectors

It is a simple concentrating collector, having up to four adjustable reflectors all around, with a single collector as shown in Figure 4.9. The CR varies from 1 to 4 and the non-imaging operating temperature can go up to 140°C.

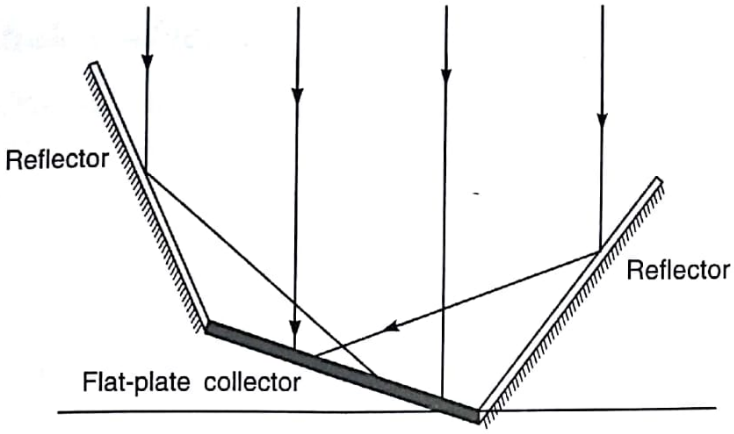

Figure 4.9 Plane receiver with plane reflectors.

Compound parabolic collector with plane receiver

Reflectors are curved segments that are parts of two parabolas (Figure 4.10). The CR varies from 3 to 10. For a CR of 10, the acceptance angle is 11.5° and tracking adjustment is required after a few days to ensure collection of 8 hours a day.

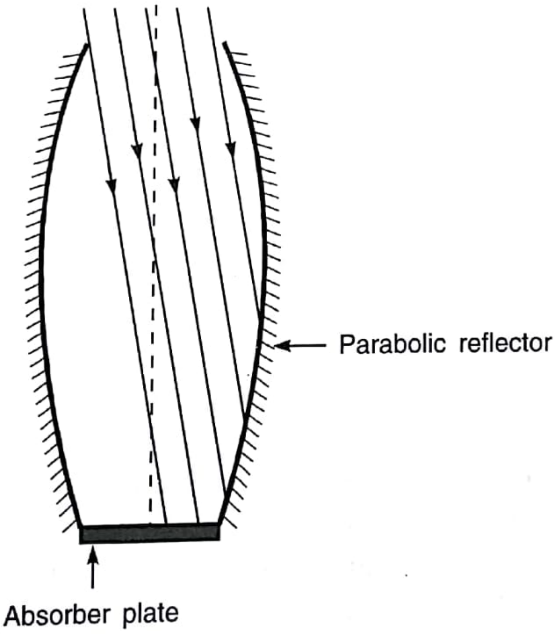

Figure 4.10 Compound parabolic collector with a plane receiver.

Cylindrical parabolic collector

The reflector is in the form of trough with a parabolic cross section in which the image is formed on the focus of the parabola along a line as shown in Figure 4.11. The basic parts are: (i) an absorber tube with a selective coating located at the focal axis through which the liquid to be heated flows, (ii) a parabolic concentrator, and (iii) a concentric transparent cover.

The aperture area ranges from 1 m^2 to 6 m^2, where the length is more than the aperture width. The CR range is from 10 to 30.

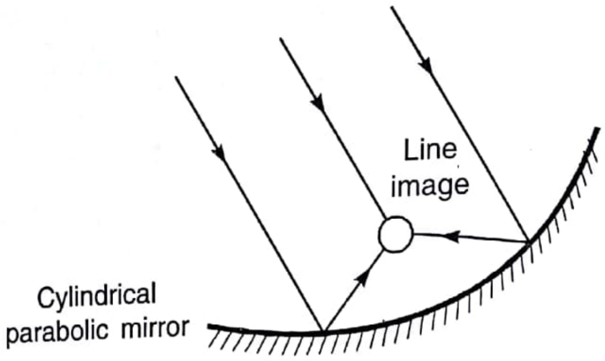

Figure 4.11 Cross section of a cylindrical parabolic collector.

Collector with a fixed circular concentrator and a moving receiver

The fixed circular concentrator consists of an array of long, narrow, flat mirror strips fixed over a cylindrical surface as shown in Figure 4.12. The mirror strips create a narrow line image that follows a circular path as the sun moves across the sky. The CR varies from 10 to 100.

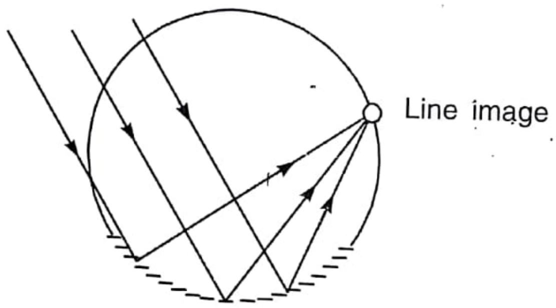

Figure 4.12 Cross section of a collector with a fixed circular concentrator and a moving receiver.

Fresnel lens collector

Fresnel lens refraction type focusing collector is made of an acrylic plastic sheet, flat on one side, with fine longitudinal grooves on the other as shown in Figure 4.13. The angles of grooves are designed to bring radiation to a line focus. The CR ranges between 10 and 80 with temperature varying between 150°C and 400°C.

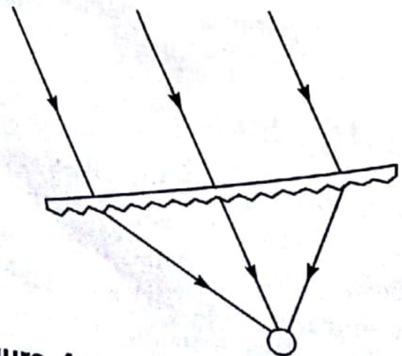

Figure 4.13 Fresnel lens collector.

Paraboloid dish collector

To achieve high CRs and temperature, it is required to build a point-focusing collector. A paraboloid dish collector is of point-focusing type as the receiver is placed at the focus of the paraboloid reflector (Figure 4.14).

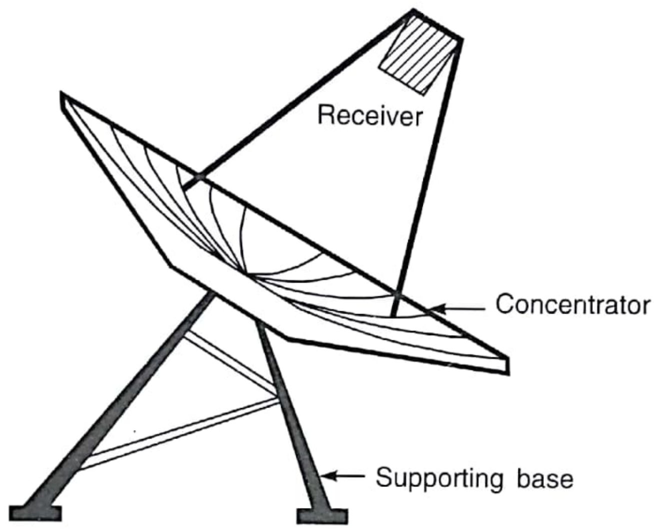

Figure 4.14 Paraboloid dish collector.

As a typical case, a dish of 6 m in diameter is constructed from 200 curved mirror segments forming a paraboloidal surface. The absorber has a cavity shape made of zirconium–copper alloy, with a selective coating of black chrome. The CR ranges from 100 to a few thousands with maximum temperature up to 2000°C. For this, two-axis tracking is required so that the sun may remain in line with the focus and vertex of the paraboloid.

Central receiver with heliostat

To collect large amounts of heat energy at one point, the 'Central Receiver Concept' is followed. Solar radiation is reflected from a field of heliostats (an array of mirrors) to a centrally located receiver on a tower (Figure 4.15).

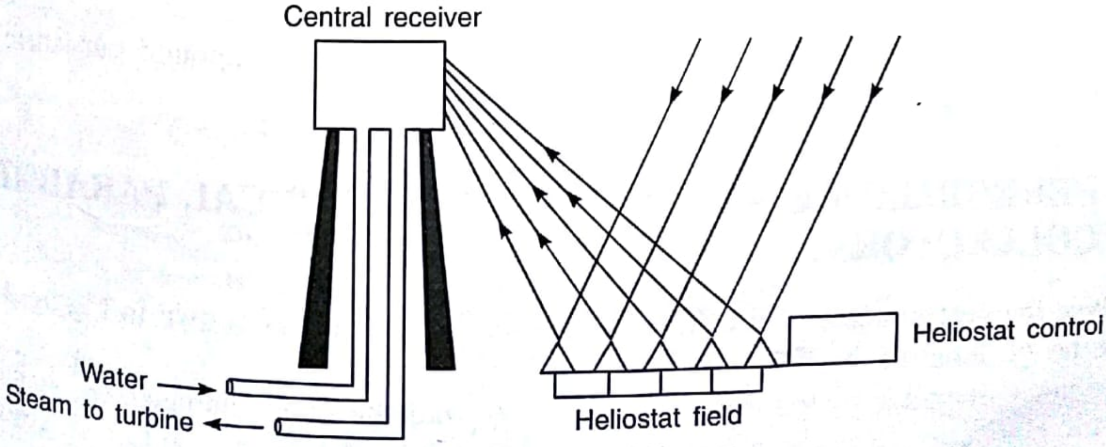

Figure 4.15 Central receiver tower with a field of heliostats.

Heliostats follow the sun to harness maximum solar heat. Water flowing through the receiver absorbs heat to produce steam which operates a Rankine cycle turbo generator to generate electrical energy.

With a central receiver optical system, a large number of small mirrors are installed, each steerable to have an image at the absorber on the central receiver. A curvature is provided to the mirrors so as to focus the sunlight in addition to directing it to the tower.

4.12 THERMODYNAMIC LIMITS TO CONCENTRATION

The function of a solar concentrator is to enhance the flux density of solar radiation. A solar concentrator is shown in Figure 4.16 where radiation is incident on an aperture area A_a, which is then concentrated on a smaller absorber plate area A_{ap}.

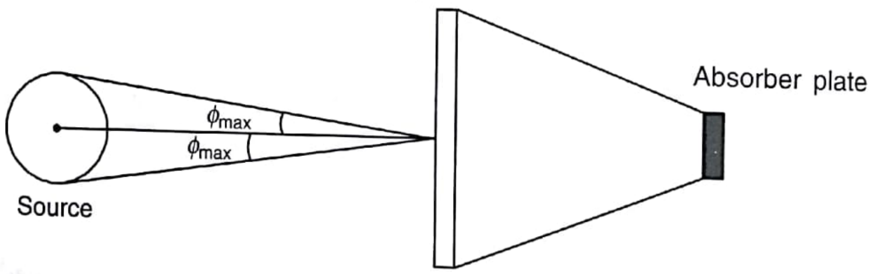

Figure 4.16 Schematic of source, aperture and absorber.

If the acceptance angle is $2\phi_{max}$, the concentration ratio (maximum) C_{max} is given by

$$C_{max,2D} = \frac{1}{\sin \phi_{max}} \tag{4.35}$$

For a linear 2D collector, the maximum value of C is 212.

For a dish concentrator (3D collector), the maximum value of C can be expressed as

$$C_{max,3D} = \frac{1}{\sin^2 \phi_{max}}$$

The maximum value of C for a 3D collector (dish having a compound curvature) is about 40,000, considering that the sun subtends an angle of ½°.

4.13 PERFORMANCE ANALYSIS OF CYLINDRICAL PARABOLIC COLLECTOR

To analyse the performance of a cylindrical parabolic collector as shown in Figure 4.17, let its aperture be W, length L and rim angle ϕ_{rim}.

The inner diameter of the absorber tube is D_i and the outer diameter D_o. It has either a concentric glass cover or a flat glass/plastic sheet covering the whole aperture area that protects the reflecting surface from weather effects.

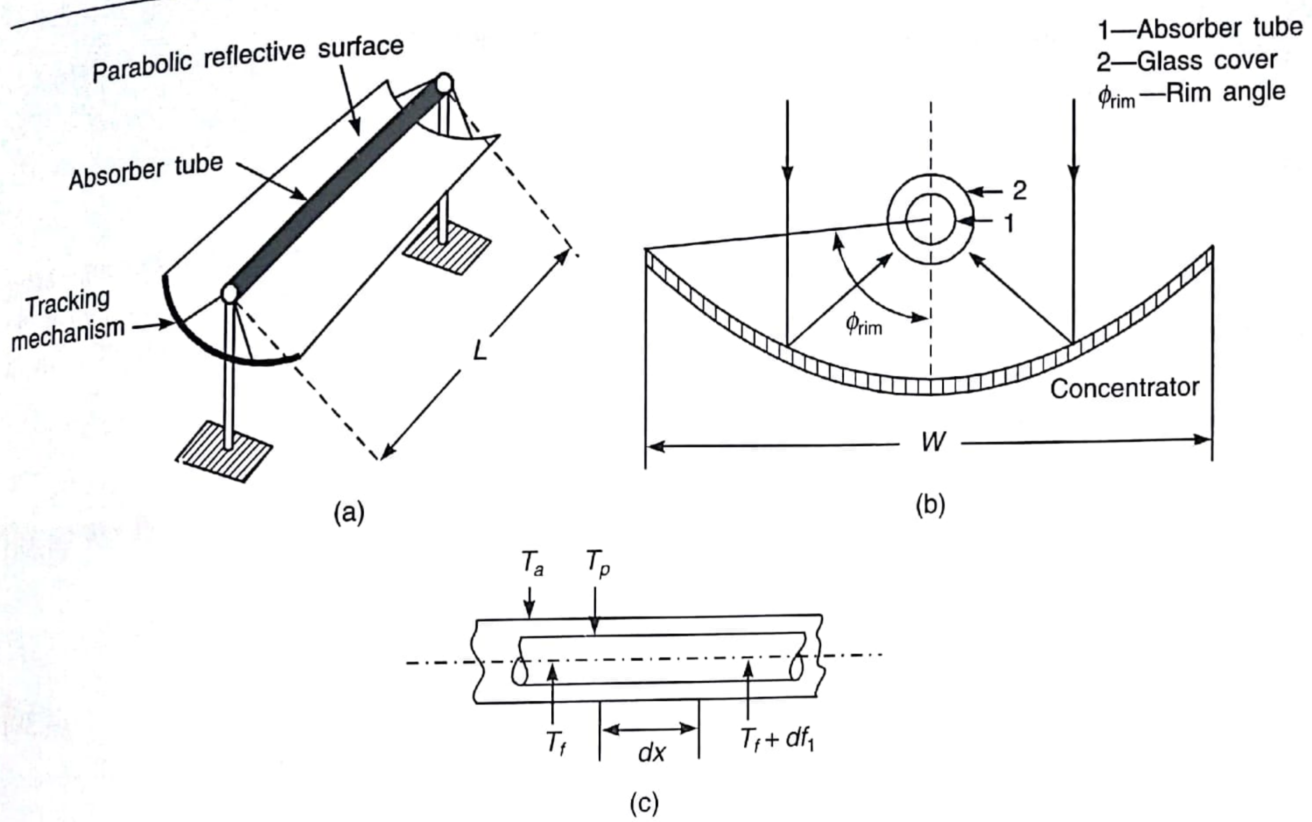

Figure 4.17 (a) Cylindrical parabolic collector, (b) cross section of the cylindrical parabolic collector, and (c) cross section of the absorber tube.

The following assumptions are made for analysis:

(a) Radiation flux is the same all along the length of the absorber tube.

(b) The temperature drop across the thickness of the absorber tube and that across the glass cover is negligible.

The 'concentration ratio' of the collector is expressed by

$$C = \frac{\text{effective aperture area}}{\text{absorber tube area}} = \frac{W - D_o}{\pi D_o} \tag{4.36}$$

For energy balance, if we consider an elemental length dx of the absorber tube at a distance x from the inlet, the following equation can then be written for the steady-state condition. Beam radiation normally incident on the aperture is $I_b R_b$.

$$dQ_u = [I_b R_b (W - D_o)\, \rho r (\tau\alpha)_b + I_b R_b D_o (\tau\alpha)_b - U_1 \pi D_o (T_p - T_a)]dx \tag{4.37}$$

where

dQ_u = useful heat gain rate for a length dx
I_b = beam radiation flux
R_b = beam radiation tilt factor
ρ = specular reflectivity of the concentrator surface
r = intercept factor, the fraction of the specularly reflected radiation intercepted by the absorber tube

$(\tau\alpha)_b$ = transmissivity–absorptivity product for beam radiation
U_l = overall heat loss coefficient
T_p = local temperature of absorber tube
T_a = ambient temperature.

The first term in the above equation represents the incident beam radiation on absorber tube after reflection. The second term indicates the absorbed incident beam radiation directly falling on the absorber tube. The second term can be ignored when the top of the tube is insulated. The third term is the loss by convection and re-radiation.

Now, the absorbed radiation flux S may be given as

$$S = I_b R_b \rho r (\tau\alpha)_b + I_b R_b (\tau\alpha)_b \left(\frac{D_o}{W - D_o}\right) \tag{4.38}$$

Equation (4.37) can be represented as

$$dQ_u = \left[S - \frac{U_1}{C}(T_p - T_a)\right](W - D_o)\,dx \tag{4.39}$$

The useful heat gain rate can also be represented as

$$dQ_u = h_f \pi D_i (T_p - T_f)\,dx \tag{4.40}$$

$$= \dot{m} c_p\, dT_f \tag{4.41}$$

where

D_i = inner diameter of the absorber tube
h_f = heat transfer coefficient on the inner surface of the tube
T_f = local fluid temperature
$\dot{m}$ = mass flow rate of the fluid being heated in the collector
T_{fi} = inlet temperature of the fluid
T_{fo} = outlet temperature of the fluid.

Combining Eqs. (4.40) and (4.41) with the elimination of T_p, the relation for the useful heat gain becomes

$$dQ_u = F'\left[S - \frac{U_1}{C}(T_f - T_a)\right](W - D_o)\,dx \tag{4.42}$$

Here, F' is the collector efficiency factor. Its value is

$$F' = \frac{1}{U_e[(1/V_e) + (D_o/D_i h_f)]} \tag{4.43}$$

Solving Eqs. (4.41) and (4.42), the following differential equation is obtained.

$$\frac{dT_f}{dx} = F'\pi \frac{D_o U_1}{\dot{m} c_p}\left[\frac{CS}{U_1} - (T_f - T_a)\right] \tag{4.44}$$

Integrating and applying the inlet condition at $x = 0$, $T_f = T_{fi}$, we get the temperature distribution as

$$\frac{[(CS/U_1)+T_a]-T_f}{[(CS/U_1)+T_a]-T_{fi}} = -\exp\left(-\frac{F'\pi D_o U_1 x}{\dot{m}c_p}\right) \tag{4.45}$$

The outlet temperature of the fluid can be had by putting $T_f = T_{fo}$ and $x = L$ in Eq. (4.45). With this substitution and then subtracting both sides from unity, we get

$$\frac{T_{fo}-T_{fi}}{[(CS/U_1)+T_a]-T_{fi}} = 1-\exp\left(-\frac{F'\pi D_o U_1 L}{\dot{m}c_p}\right) \tag{4.46}$$

The useful heat gain rate is

$$Q_u = \dot{m}c_p(T_{fo}-T_{fi})$$

or

$$Q_u = \dot{m}c_p\left(\frac{CS}{U_1}+T_a-T_{fi}\right)\left[1-\exp\left(-\frac{F'\pi D_o U_1 L}{\dot{m}c_p}\right)\right]$$

$$= F_R(W-D_o)L\left[S-\frac{U_1}{C}(T_{fi}-T_a)\right] \tag{4.47}$$

where F_R is the heat removal factor and it is given as

$$F_R = \frac{\dot{m}c_p}{\pi D_o L U_R}\left[1-\exp\left(-\frac{F'\pi D_o U_1 L}{\dot{m}c_p}\right)\right] \tag{4.48}$$

The *instantaneous collection efficiency* considering beam radiation only, η_{ib}, in percentage (neglecting ground reflected radiation) is given by

$$\eta_{ib} = \frac{Q_u}{(I_b R_b)WL}\times 100 \tag{4.49}$$

In general, the *instantaneous collection efficiency*, η_i, can be expressed as

$$\eta_i = \frac{Q_u}{(I_b R_b + I_d R_d)WL}\times 100 \tag{4.50}$$

The heat loss coefficient U_1 can be calculated from

$$U_1 = \left(\frac{1}{h_{\text{wind}}}+\frac{1}{h_r}\right)^{-1} \tag{4.51}$$

where

h_{wind} = film coefficient due to wind
= $5.7 + 3.8v$ W/m^2 °C
with v as the wind velocity in m/s.

h_r = radiation coefficient.

The radiation coefficient h_r can be calculated as

$$h_r(T_r - T_a) = \sigma\varepsilon_r(T_r^4 - T_a^4)$$

where

σ = Stefan–Boltzmann constant = 5.67×10^{-8} W/m$^2 \cdot$ °C

ε_r = emissivity of the surface

T_r = temperature of the radiant surface.

Hence,

$$h_r = \sigma\varepsilon_r(T_r + T_a)(T_r^2 + T_a^2)$$

Assuming that $T_r \approx T_a$ and $\overline{T} = (T_r + T_a)/2$, we have

$$h_r = 4\sigma\varepsilon_r\overline{T}^3 \tag{4.5}$$

EXAMPLE 4.2

For a parabolic collector of length 2 m, the angle of acceptance is 15°. Find the concentrati ratio of the collector.

Solution

$$\text{Concentration ratio, CR} = \frac{1}{\sin \phi_{\max}}$$

$$\phi_{\max} = \frac{\text{angle of acceptance}}{2} = 7.5°$$

$$\text{So, CR} = \frac{1}{\sin 7.5°} = 7.66$$

EXAMPLE 4.3

For a cylindrical parabolic concentrator of 2.5 m width and 9 m length, the outside diameter the absorber tube is 6.5 cm. Find the concentration ratio of the collector.

Solution

$$\text{Concentration ratio} = \frac{W - D_o}{\pi D_o}$$

$$= \frac{2.5 - 6.5 \times 10^{-2}}{\pi \times 6.5 \times 10^{-2}}$$

$$= 11.93$$

EXAMPLE 4.4

Calculate the heat removal factor, the useful heat gain, the exit fluid temperature and the collection efficiency for a cylindrical parabolic concentrator having 2.5 m width and 9 m length, the outside diameter of the absorber tube being 6.5 cm. The temperature of the fluid to be heated at the inlet is 16°C with a flow rate of 450 kg/h. The incident beam radiation is 700 W/m^2. The ambient temperature is 28°C. The optical properties are as given below:

$\rho = 0.85$, $(\tau\alpha)_b = 0.78$, $\tau = 0.93$
$c_p = 1.256$ kJ/kg · °C
Collector efficiency factor, $F' = 0.85$
Heat loss coefficient, $U_l = 7.0$ W/m^2 · °C

Solution

From the given data, $I_b R_b = 700$ W/m^2
Absorbed radiation flux,

$$S = I_b R_b \, \rho\tau (\tau\alpha)_b + I_b R_b \, (\tau\alpha)_b \frac{D_o}{W - D_o}$$

$$S = 700 \times 0.85 \times 0.93 \times 0.78 + 700 \times 0.78 \left(\frac{0.65}{2.5 - 0.65}\right)$$

$$= 431.61 + 0.02 = 431.63 \text{ W/m}^2$$

Heat removal factor,
$$F_R = \frac{\dot{m}c_p}{\pi D_o L U_1}\left[1 - \exp\left(-\frac{F'\pi D_o U_1 L}{\dot{m}c_p}\right)\right]$$

$$\dot{m} = \frac{450}{3600} = 0.125 \text{ kg/s}$$

$$\frac{\dot{m}c_p}{\pi D_o U_1 L} = \frac{0.125 \times 1.256 \times 10^3}{3.14 \times 0.015 \times 7.0 \times 9} = 12.21$$

$\therefore$
$$F_R = 12.21\left[1 - \exp\left(-\frac{0.85}{12.21}\right)\right]$$

$$= 12.21[1 - \exp(0.00696)]$$

$$= 12.21(1 - 0.9327)$$

$$= 0.821$$

Concentration ratio,
$$C = \frac{W - D}{\pi D} = \frac{2.5 - 0.065}{3.14 \times 0.065}$$

$$= 11.93$$

Useful heat gain Eq. (4.47) is

$$Q_u = F_R (W - D_o) L \left[S - \frac{U_1}{C}(T_{fi} - T_a)\right]$$

$$= 0.821\,(2.5 - 0.65)\,9\left[431.6 - \frac{7}{11.93}(150 - 28)\right]$$

$$= 0.821 \times 2.435 \times 9 \times 360.015$$
$$= 6477.5 \text{ W}$$

Also, $$Q_u = \dot{m}c_p(T_{fo} - T_{fi})$$

or $$T_{fo} = \frac{Q_u}{\dot{m}c_p} + T_{fi}$$

$$= \left(\frac{6477.5}{0.125 \times 1.256 \times 10^3}\right) + 150$$

$$= 191.2°\text{C}$$

$$\eta_{ib} = \frac{Q_u}{(I_b R_b)WL} \times 100 = \frac{6477.5}{700 \times 2.5 \times 9} \times 100\%$$

$$= 41\%$$

4.14 COMPOUND PARABOLIC CONCENTRATOR (CPC)

A two-dimensional CPC is shown in Figure 4.18. It has two segments BE and CF which are parts of parabolas 2 and 1 respectively. BC is the aperture of width W while EF is the absorber surface of width b. Both parabolas are positioned in such a way that the focus of parabola 1 lies at E while that of parabola 2 at F. Also, the tangents drawn at points B and C to the parabolas are parallel to the axis of CPC.

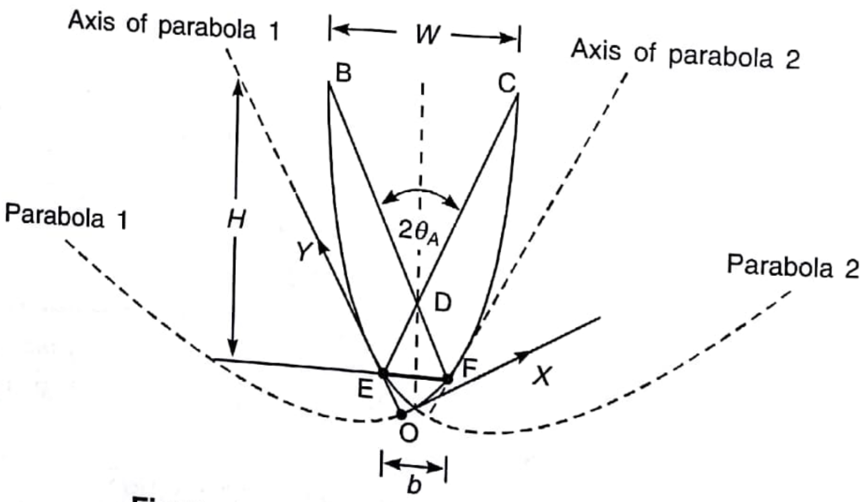

Figure 4.18 Geometry of a CPC collector.

Acceptance angle $\angle BDC = 2\theta_A$

Concentration ratio, $C = \dfrac{W}{b} = \dfrac{1}{\sin\theta_A}$

Equation of parabola 1 having vertex at O, i.e., the origin of X–Y co-ordinate is given by

$$y = \frac{x}{2b(1+\sin\theta_A)} \tag{4.53}$$

And focal length,

$$\text{OE} = \frac{b}{2(1+\sin\theta_A)}$$

The coordinates of F are

$$x = b \cos \theta_A$$

$$y = \frac{b}{2(1 - \sin \theta_A)}$$

The coordinates of C are

$$x = (b + W) \cos \theta_A$$

$$y = \frac{b}{2}(1 - \sin \theta_A)\left(1 + \frac{1}{\sin \theta_A}\right)^2$$

The ratio of height to aperture can be expressed as

$$\frac{H}{W} = \frac{1}{2}\left(1 + \frac{1}{\sin \theta_A}\right)\cos \theta_A$$

$$= \frac{1}{2}(1 + C)\left(1 - \frac{1}{C^2}\right)^{1/2} \quad (4.54)$$

The surface area of the concentrator can be calculated by integrating along the parabolic arc. However, for a concentration ratio of more than 3, a simple equation provides a nearly correct value as

$$\frac{\text{concentrator area}}{\text{aperture area}} = \frac{A_{conc}}{A_a} = 1 + C$$

EXAMPLE 4.5

A CPC, 1.5 m long has an acceptance angle of 20°. The surface of the absorber is flat with a width of 15 cm. Evaluate the concentration ratio, the aperture height and the surface area of the concentrator.

Solution

$$C = \frac{1}{\sin 10^\circ} = 5.76$$

Aperture, $W = 5.76 \times 15 = 86.4$ cm

$$\frac{H}{W} = \frac{1}{2}\left(1 + \frac{1}{\sin 10^\circ}\right)\cos 10^\circ = 3.328$$

$\therefore$

$$H = 3.328 \times 86.4 = 287.53$$

$$\frac{A_{conc}}{WL} = 1 + 5.76 = 6.76$$

$\therefore$

$$A_{conc} = 6.76 \times 0.867 \times 1.5$$

$$= 8.79 \text{ m}^2$$