(6 Question)

28/11/22
Monday

Unit - 1

1] Introduction to Mathematical Modeling.

⇒ There are many ways in which models can be described

- Words
- Drawings / sketches
- Physical models
- computer programs
- Mathematical Models

⇒ Mathematical Model — is a representation of the behavior of real objects and phenomena in the language of mathematics.

⇒ Mathematical Modeling — is a process that uses mathematics

- To represent
- Analyze
- Make Prediction
- Provide insight into real world phenomena.

Types of Mathematical Modeling

(1) Theoretical Models :- are models that are based on 1st principles & physical laws. can be extrapolated to a wide variety of situations.

Ex- $v = u + at$

(2) Empirical Models :- are models based on experimental data. These models are applicable in the areas with identical conditions as those in which the relationship was formulated.

imp

Process of Mathematical Modeling.

Understanding the problem & identifying the parameters

↓

Mathematical Modeling Assumptions

↓

Formulation or Governing Equations

Analytical method (or) Numerical method

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1$

Solutions

↓

Validation → (NO) → Mathematical Modeling Assumptions

↓ (yes)

Application

30/11/22 Wednesday

⇒ The 1^{st} step towards mathematical modeling is abt understanding the problem & also identifing the parameters.

(1)

[In this step we will analyze the problem & see which parameters have a major influence on the solution to the problem].

⇒ The next is to construct the basic framework of
(2) the model by making certain assumptions.

[In this step we state those parameters which are not essential & can be neglected].

⇒ If the assumptions are sufficiently precise, they
(3) may lead directly to the formulation or governing Equations.

"In some cases the formulation itself is the solution".
(So box 3 & 4 are same)

- If the formulation is not the solution then we apply analytical method to solve the Equation.

- When analytical methods are unproductive we can use numerical methods to obtain the solution.

⇒ After obtaining the solution we start testing the validity
(5) of the model by comparing the theoretical & practical results.

- If the model is valid then we move towards the application

- If not we recheck our assumptions and repeat the steps until we get a valid model.

Questions

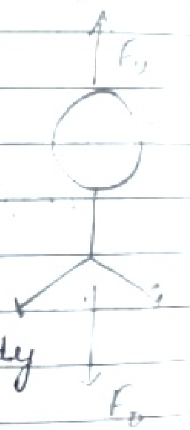
2] Understang the Problem :-

7 M Find the velocity prior to opening of parachute

Identifying parameters

Forces on the body & mass.

Mathematical modeling assumptions :
No horizontal force is acting on the body & mass of parachute is negligible.



Formulation or governing Equations :-

By Newton's Second law,

$$F = ma$$

where, F = Net force acting on body
m = mass (kg)
a = acceleration (m/s^2)

3 Since 2 forces acting on a body are downward force (F_D) & upward force (F_u)

$$\therefore \ F = F_u + F_D \quad \text{------} \quad (1)$$

F_D = force due to gravity = mg
F_u = force due to air resistance = $-cV$
[$-c$ = drag co-efficient in opp. dirn]

classmate
Date
Page

Linear differential eqⁿ

$$\frac{dy}{dx} + Py = Q$$

① $IF = e^{\int P dx}$

② Solⁿ

$y(IF) = \int Q(IF)\,dx + k$

Eqⁿ ① reduces to

$$F = -cv + mg$$

$$ma = -cv + mg$$

$$m\frac{dv}{dt} = -cv + mg$$

$$\frac{dv}{dt} = \frac{-cv}{m} + g$$

$$\boxed{\frac{dv}{dt} + \left(\frac{c}{m}\right)v = g}$$

Which is a linear differential Equation.

IF = integrating factor

Here, $P = \frac{c}{m}$, $Q = g$

$$IF = e^{\int \frac{c}{m} dt} = e^{\frac{c}{m}\int dt} = e^{\frac{c}{m}t}$$

$$\boxed{IF = e^{c/m\, t}}$$

Solution is, $\Rightarrow$ $v(IF) = \int g\,Q\,(IF)\,dx + k$

$$v\left(e^{\frac{c}{m}t}\right) = \int g\left(e^{c/m\, t}\right)dt + k$$

$$ve^{c/m\, t} = g\,\frac{e^{c/m\, t}}{\frac{c}{m}} + k$$

$$\boxed{\int e^{ax}dx = \frac{e^{ax}}{a}}$$

$$\boxed{v = \frac{mg}{c} + ke^{-\frac{c}{m}t}} \quad \text{——— ②}$$

When $t=0$, $v=0$ using in Eqⁿ ②

$$0 = \frac{mg}{c} + ke^{-\frac{c}{m}(0)}$$

$$0 = \frac{mg}{c} + k$$

put this in Eq (2)

$$\boxed{k = -\frac{mg}{c}}$$

Eq^n (2) reduces to

$$\boxed{V = \frac{mg}{c}\left[1 - e^{-\frac{c}{m}t}\right]}$$

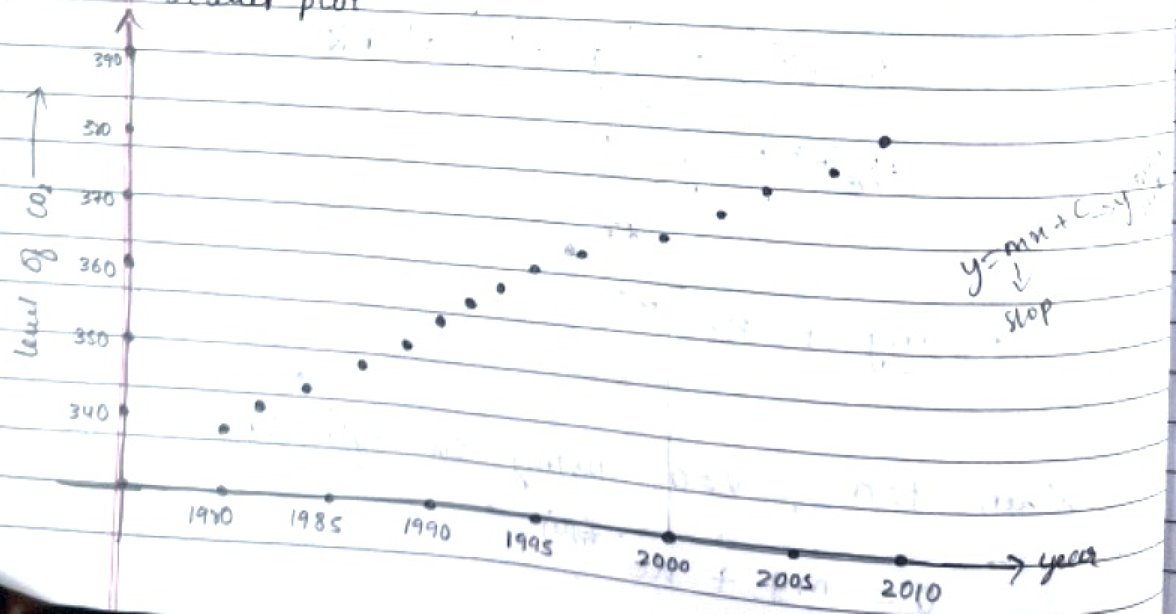
is the sol^n of the model.

3] <u>Understand the problem.</u>

From the given data we have to find the level of CO_2 in atmosphere.

<u>Identifying the Parameters</u> :

Relation b/w the variables CO_2 and year by Scatter plot



Mathematical Modeling Assumptions :-
No assumptions [Emperical model].

Formulation or Governing Equation :-
As the data coins appear to lie close to a st. line, its natural to choose a linear model in this case

$CO_2 = a\,[\text{years}] + b$ —— (1) $\quad \therefore\ y = \underbrace{m}_{\text{slope}}x + \underbrace{c}_{\text{y-intercept}}$

here, $a = \dfrac{(CO_2)_2 - (CO_2)_1}{(\text{year})_2 - (\text{year})_1}$ $\quad \left[m = \dfrac{y_2 - y_1}{x_2 - x_1} \right]$

$a = \dfrac{(385.6) - (338.7)}{2008 - 1980}$

$a = \dfrac{46.9}{28}$

slope, $\boxed{a = 1.6675}$

Eq^n (1) reduces to,

$CO_2 = 1.6675\,(\text{year}) + b \longrightarrow$ (2)

To get b = CO_2 intercept :-

put year = 1980
and CO_2 = 338.7 in Eq^n (2)

$338.7 = 1.6675 \times 1980 + b$
$339.7 = \cancel{3301.65}\ 3316.5 + b$
$b = -2977.8$

$\therefore$ Eq 2 reduces to

$\boxed{CO_2 = 1.675\,(\text{year}) - 2977.8}$

2] Functions & Graphs

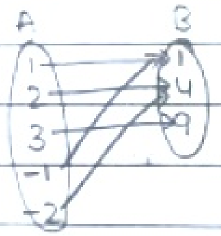
\# A function is a rule which takes certain no.s as inputs and assigns to each a unique output.

=> A function from a set A to B is denoted by
$f : A \longrightarrow B$ [A = domain & B - co-domain]

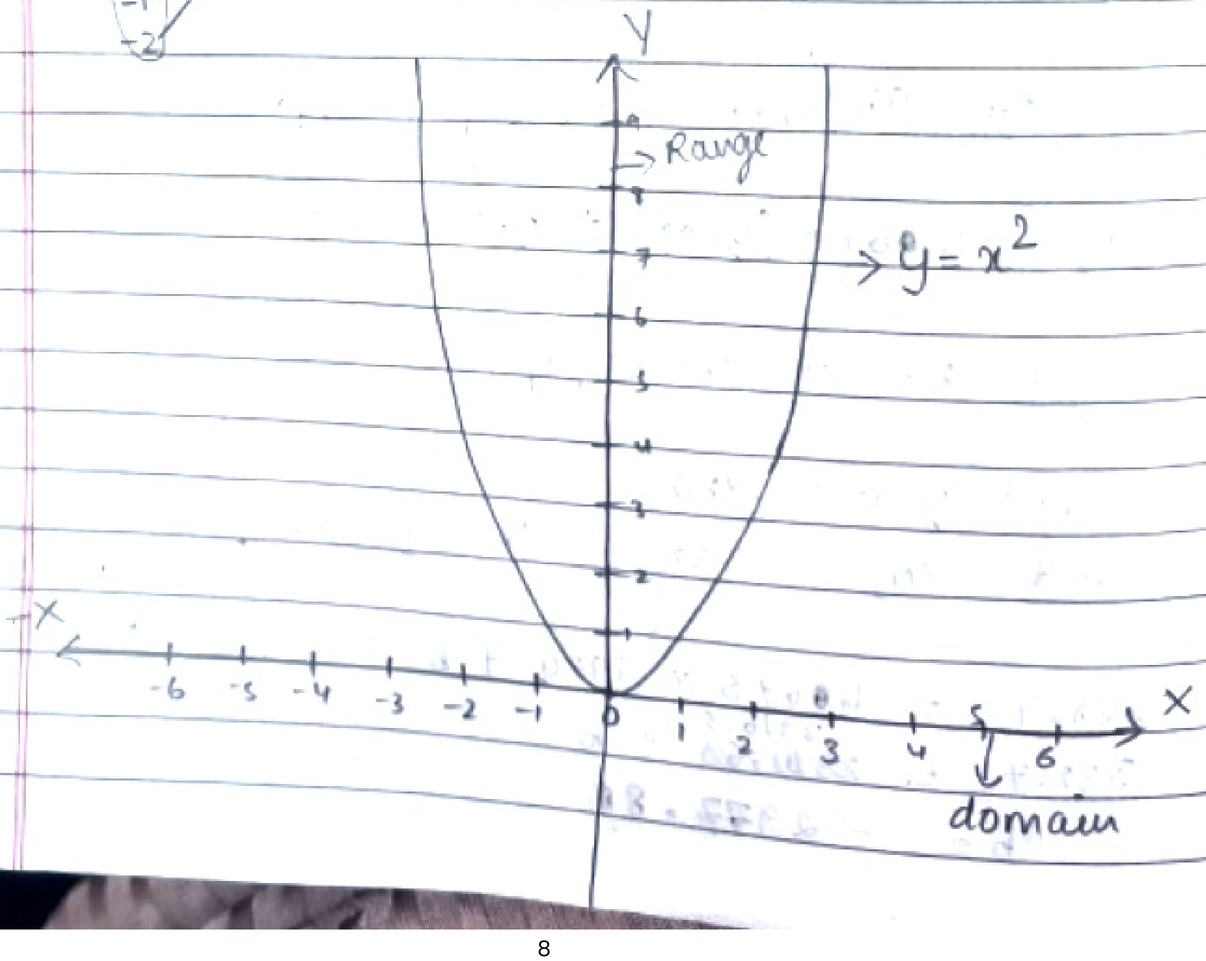
=> Domain of a function :- The set of all input no.s is called domain of a funⁿ.

=> Range of a function :- The set of all output no.s is called Range of a funⁿ.

$f(x) = x^2$ domain → set of all NO.

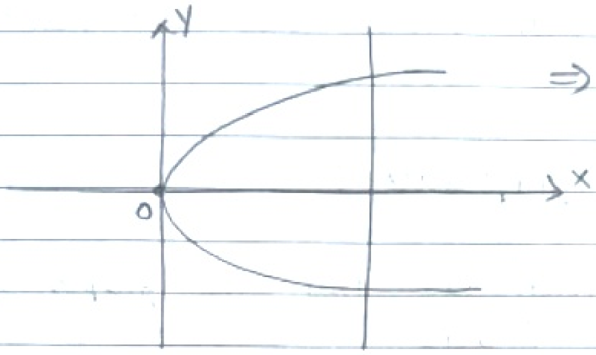


domain → on x-axis
Range → on y-axis.

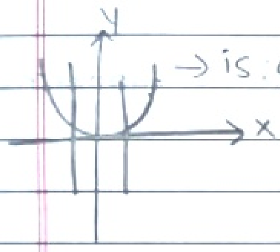


Vertical Line Test :-

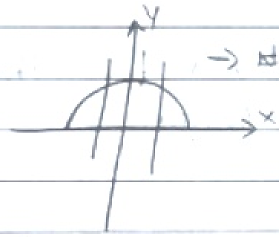
A curve in the XY - plane is the graph of a function of x if and only if no vertical line intersects the curve more than once.



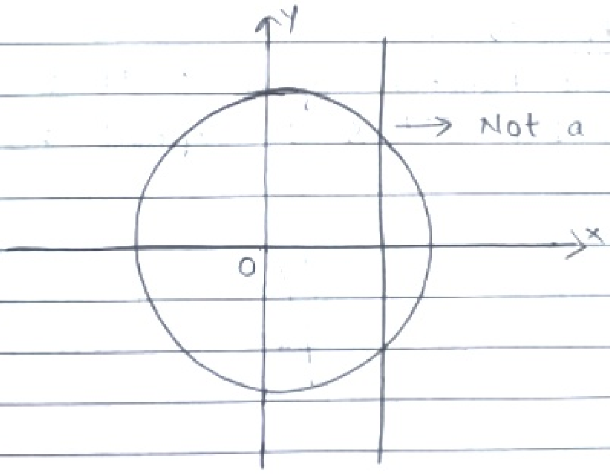
⇒ Not a function



→ is a function



→ ~~is~~ function



→ Not a function.

Even & Odd functions :-

$\forall$ - for every

For any function f,

f is even if, $f(-x) = f(x)$, for every x

f is odd if, $f(-x) = -f(x)$, for every x

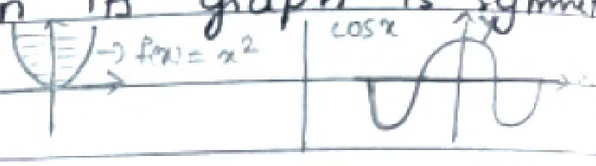
Example :-

① $f(x) = x^2$, $f(x) = \cos x$ are even fun^n

② $f(x) = x^3$, $f(x) = \sin x$ are odd fun^n

Geometrical Significance :-

→ If f is an even fun^n then its graph is symmetric wrt the Y-axes.

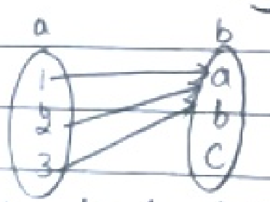


→ If f is an odd fun^n then its graph is symmetric wrt the Origin.

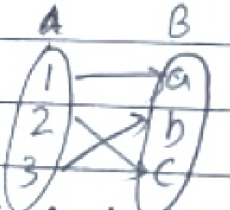
12/22 Friday

Inverse Function :-

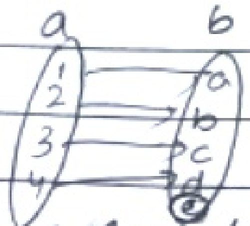
Inverse Function of a 1-to-1 fun^n $y = f(x)$ is denoted by f^{-1} & is defined as $f^{-1}(y) = x$



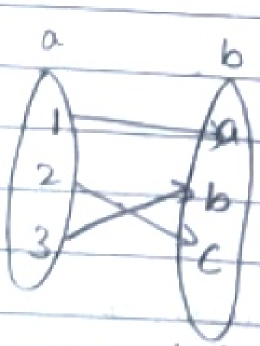
Not 1-1 fu^n nor onto fu^n.



is 1-1 fu^n & onto f^n.



is 1-1 fu^n not onto fu^n



onto fu^n

1-to-1 = bijective

	Domain	Range
~~sin~~ sin:	R	$[-1,1]$
$\sin^{-1}$:	$[-1,1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Note :-

1] Domain of f^{-1} = Range of f
2] Range of f^{-1} = Domain of f
3] Graph of f^{-1} is the reflection of graph of f about the line $y = x$.

Linear Function :-

Linear funⁿ is a fuⁿ whose graph is a st. line.

$$f(x) = mx + c$$

where m = slope of line &
c = y-intercept

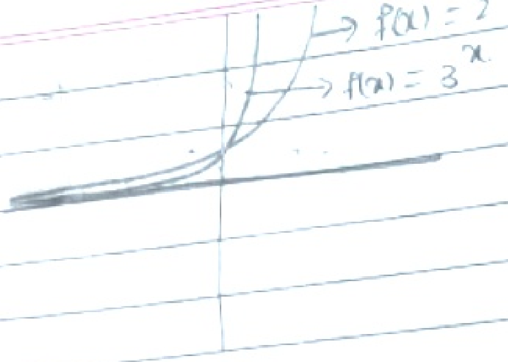
Exponential Functions :-

Funⁿ of the form $f(x) = a^x$, where base 'a' is a +ve constant

Note

1] If $a > 1$, we have Exponential growth.
graph will be ↑

2] If $0 < a < 1$, we have Exponential decay.
graph will be ↓



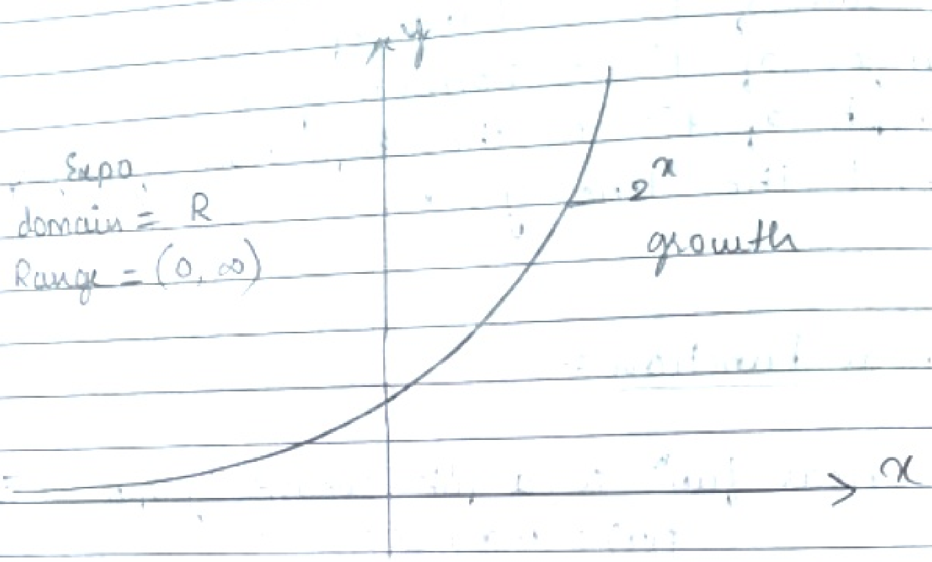
$2^2 = 4$, $3^2 = 9$

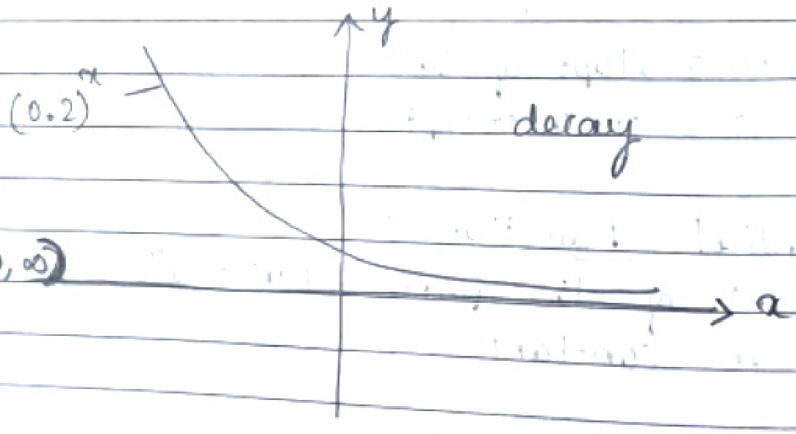
$2^{-2} = \frac{1}{4}$ $3^{-2} = \frac{1}{9}$

ln ← Expo

Range ← domain = R

domain ← Range = $(0, \infty)$





domain = R

range = $\boxed{(0, \infty)}$

General Exponential function:

$P = P_0 a^t$ $\boxed{P = P_0 a^t}$

where, P_0 = Initial quantity

a = +ve constant

$f(x) = a^x$

$f(t) = a^t$

$P = P_0 a^t$

Logarithmic functions :-

Funⁿ of the form,

$y = \log_a x \, , \quad a > 0$

$\log_e x = \ln x$

$\log_2 x = \ln_2 x = \dfrac{\ln x}{\ln 2}$

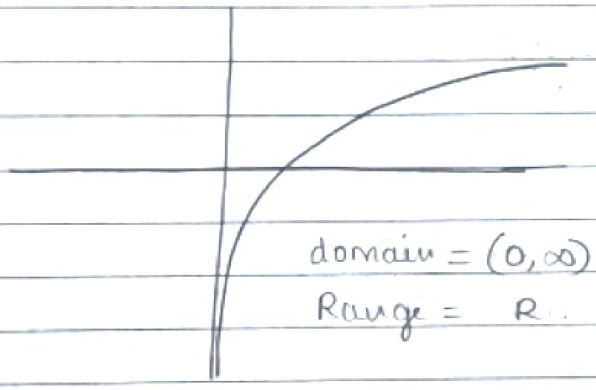
Note :

1) If $a > 1$, $\log_a x$ gives growth

2) If $0 < a < 1$, $\log_a x$ gives decay.

$\ln 1 = 0$

$\ln 0 = ND$



domain $= (0, \infty)$

Range $= R$

$y = 2^x \Rightarrow x = \ln_2 y$

Periodic Function :-

$f(t)$ is a periodic function if $f(t + T) = f(t)$ for all t & $T \in R$

Example

1) $\sin(x + 2\pi) = \sin x$

2) $\cos(x + 2\pi) = \cos x$

Amplitude :-

Amplitude = (Max. value − min value)/2

Amplitude & Period of Sinusoidal Functions :

If $f(t) = A \sin(Bt)$ & $g(t) = A\cos(Bt)$ then

- Amplitude = $|A|$
- Period = $\frac{2\pi}{|B|}$

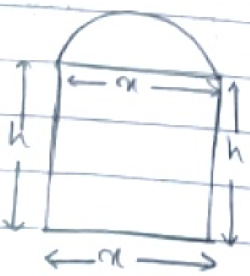
5/12/22
Monday

$f(x) = 4 \sin(3x)$
$g(x) = -6 \cos\left(\frac{x}{4}\right)$

Amplitude = $|-6| = 6$
Period = $\frac{2\pi}{1/4} = 8\pi$

Problem : pg − 7 of 29

1] i)



Given : width = x
perimeter = 30

To find : area of as a funⁿ of x 2 6M

radius of semicircle $= \frac{x}{2}$

perimeter $= h + x + h + \pi$ (radius)

$30 = 2h + x + \pi \frac{x}{2}$

$60 = 4h + 2x + \pi x$

Solving for h,

$4h = 60 - 2x - \pi x$

$\boxed{h = 15 - \frac{x}{2} - \frac{\pi x}{4}} \longrightarrow (1)$

Area of window = Area of Rect. + Area of semi ci

$A = hx + \frac{\pi r^2}{2}$

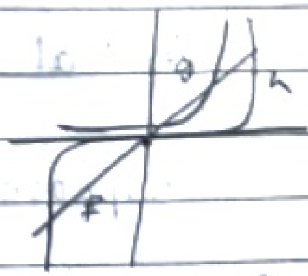
$A = \left[15 - \frac{x}{2} - \frac{\pi x}{4}\right] x + \frac{\pi}{2}\left(\frac{x}{2}\right)^2$

refer to eq (1) — for (h).

$A = 15x - \frac{x^2}{2} - \frac{\pi x^2}{4} + \frac{\pi x^2}{8}$

$\boxed{A = 15x - \frac{x^2}{2} - \frac{\pi x^2}{8}}$

2) (M)



a) $y = 5x \longrightarrow f$ (Linear fun^n)

b) $y = 5^x \longrightarrow g$ (Exponential, growth, x^0)

c) $y = x^5 \longrightarrow h$ (odd fun^n, symmetric wrt origin)

Equation		
$y = 5x$	f	$y = 5x$ is a linear fuⁿ of x. its graph is a st. line
$y = 5^x$	g	$y = 5^x$ is an exponential fuⁿ of x with base $(=5>1)$. so graph gives an exponential growth.
$y = x^5$	h	$y = 5x$ is an odd fuⁿ so its ~~grow~~ graph is symmetric abt origin.

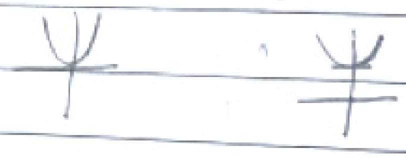
Transformations of Functions

(imp)

It is a method of obtaining a new funⁿ from old function.

Eg $y = f(x)$ $y = x^2$ $y = x^2 + 5$

$y = f(x) + 5$



I ⇒ Vertical & Horizontal Shifts :-

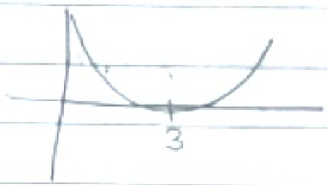
Suppose $c > 0$. To obtain the graph of

1) $y = f(x) + c$, Shift the graph of $y = f(x)$ a distance 'c' units upward.

2) $y = f(x) - c$, shift the graph of $y = f(x)$ a distance 'c' units downward.

$= f(x-c)$, shift the graph of $y = f(x)$ a distance 'c' units to the right.

$y = (x-3)^2$



$\Rightarrow$ horizontal shift

$f(x+c)$, shift the graph of $y = f(x)$ a distance units to the left.

$\Rightarrow$ horizontal shift.

tical & Horizontal Stretching and Reflecting :-

uppose $c > 1$. To obtain the graph of

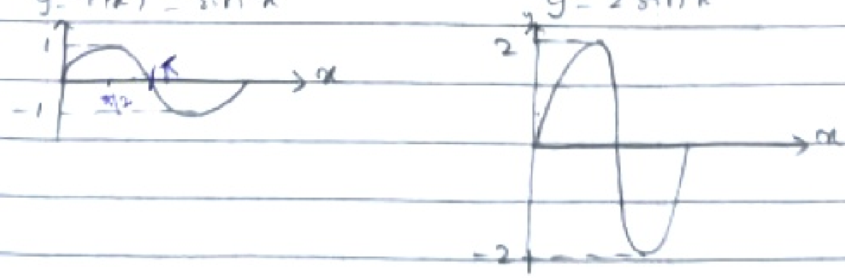
$= c\, f(x)$, stretch the graph of $y = f(x)$ vertically y a factor of c.

$(1/c)\, f(x)$, compress the graph of $y = f(x)$ rtically by a factor of c.

$y = f(cx)$, compress the graph of $y = f(x)$ izontally by a factor of c.

$: f(x)$ $\quad y = f(x) = \sin x$ $\quad\quad y = 2 \sin x$

$2 f(x)$



$f(x/c)$, stretch the graph of $y = f(x)$, horizontally y a factor of c.

5) $y = -f(x)$, reflect the graph of $y = f(x)$ abt x-axis.

6) $y = f(-x)$, reflect the graph of $y = f(x)$ abo… y-axis.

Q] 3] 7M imp

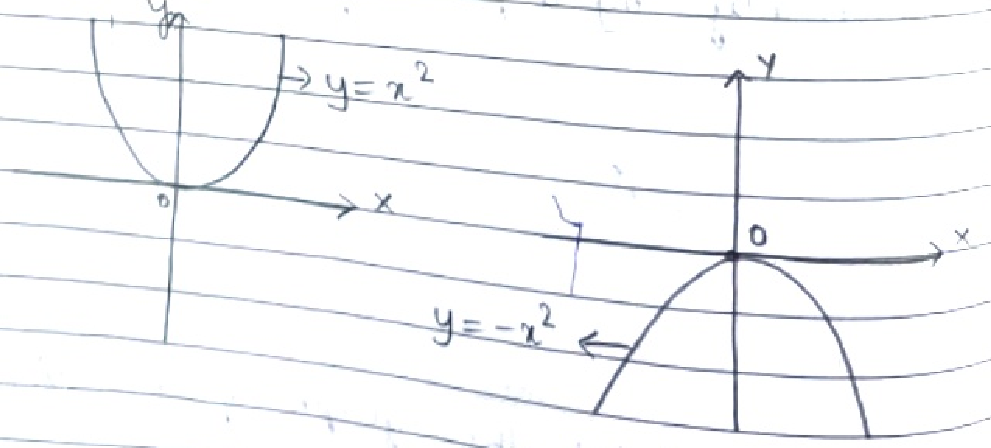
(ii) $y = 1 - x^2$

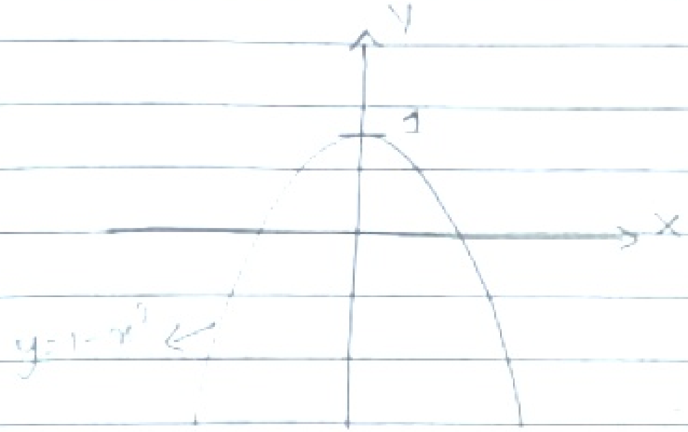
parent graph = $y = x^2$

steps = ① $y = x^2$
② $y = -x^2$ (x-axis) Reflect
③ $y = -x^2 + 1$ (Shift 1 unit upward)

To obtain the graph of $y = 1 - x^2$ we start with

① The graph of $y = x^2$
② Reflect the graph of $y = x^2$ abt x-axis to get the graph of $y = -x^2$
③ Shift the graph of $y = -x^2$, 1 unit upward to get $y = -x^2 + 1$





Domain = $\mathbb{R} = (-\infty, \infty)$

Range = $(-\infty, 1]$

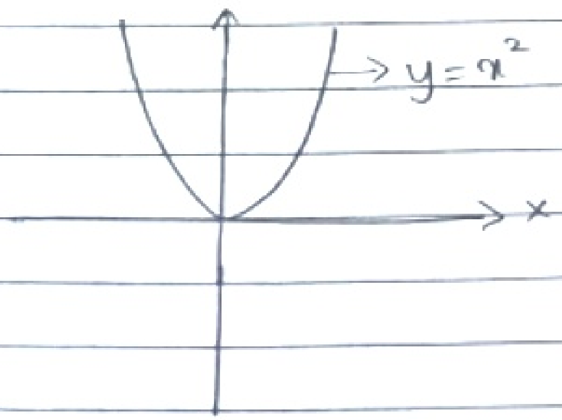
12/22
Tuesday

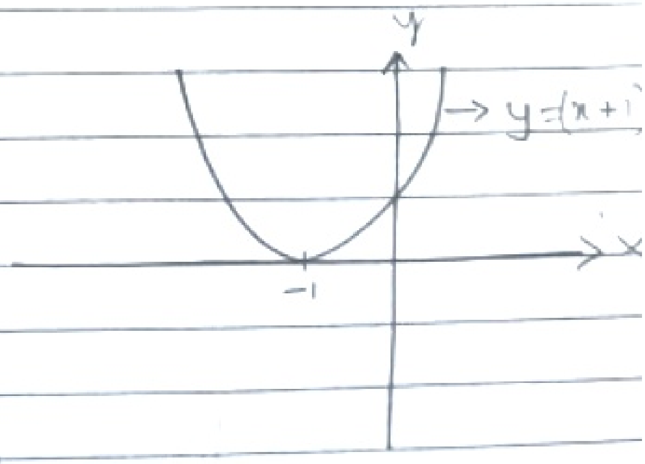
(iii) $y = (x+1)^2$

= To obtain the graph of $y = (x+1)^2$, we start with

① Graph of $y = x^2$

② Shift the graph of $y = x^2$, 1 unit to the left to g
$y = (x+1)^2$.





domain =

Range =

(iv) $y = x^2 - 4x + 3$

$\Rightarrow$ $y = x^2 - 4x + 3$

$y = x^2 - 4x + 4 - 4 + 3$

$y = (x-2)^2 - 1$

Basic Graph $\Rightarrow$ ① $y = x^2$

② Shift 2 units right

③ Shift 1 units downward.

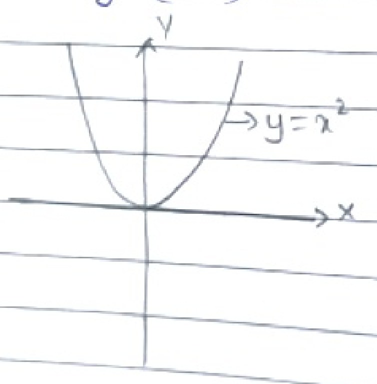
To obtain the graph of $y = x^2 - 4x + 3 = (x-2)^2 - 1$

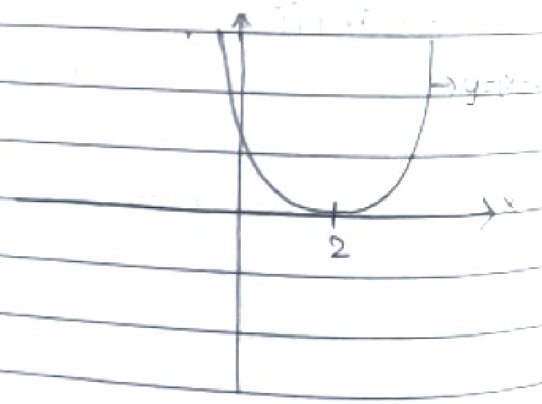
We start with.

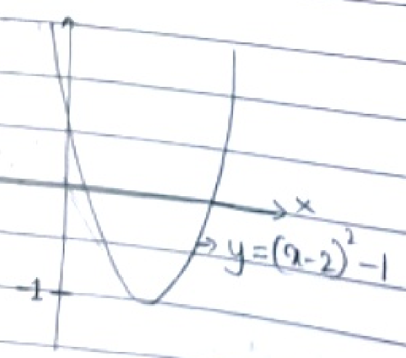
① Graph of $y = x^2$

② Shift graph of $y = x^2$, 2 units to the right to get $y = (x-2)^2$

③ Shift $y = (x-2)^2$, 1 unit downward to get $y = (x-2)^2 - 1$.







domain $= (-\infty, \infty) = R$

range $= [-1, \infty)$

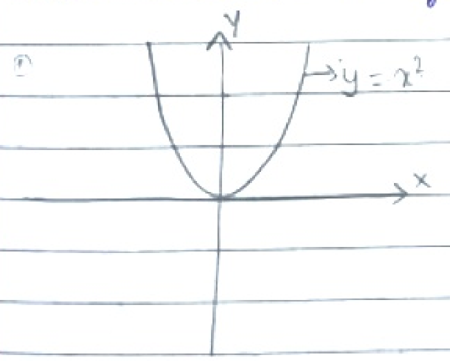
(xii) $y = |x^2 - 2x|$

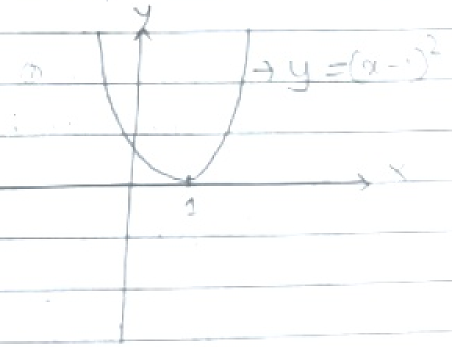
Consider , $x^2 - 2x$

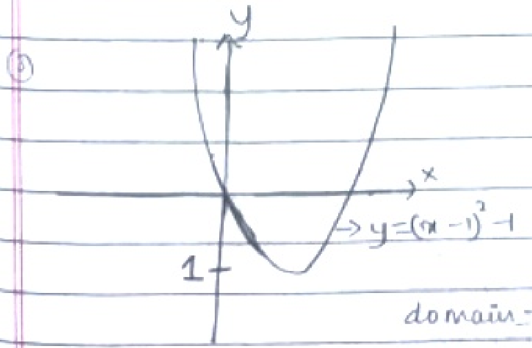
$$x^2 - 2x = x^2 - 2x + 1 - 1$$
$$x^2 - 2x = (x-1)^2 - 1$$
$$\therefore y = |x^2 - 2x| = |(x-1)^2 - 1|$$

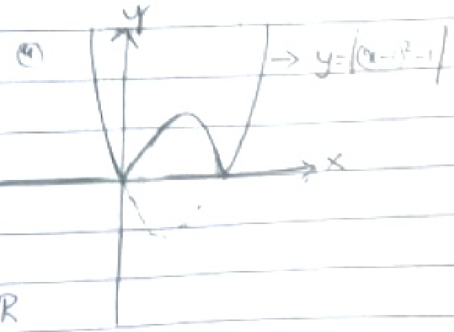
To obtain the graph of $y = |x^2 - 2x| = |(x-1)^2 - 1|$ we start with,

1. Start with graph of $y = x^2$
2. Shift graph $y = x^2$, 1 unit to the right to get $y = (x-1)^2$
3. Shift $y = (x-1)^2$, 1 unit downward to get. $y = (x-1)^2 - 1$
4. Reflect the part of graph $y = (x-1)^2 - 1$ below x-axis abt X-axis to get $y = |(x-1)^2 - 1|$









domain $= (-\infty, \infty) = R$

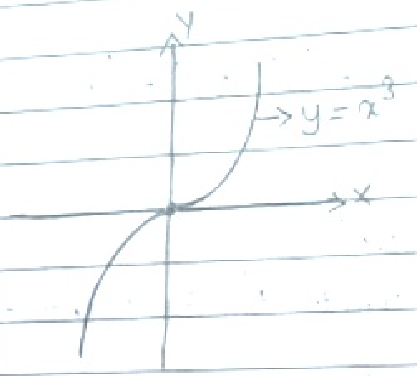
range $= [0, \infty)$

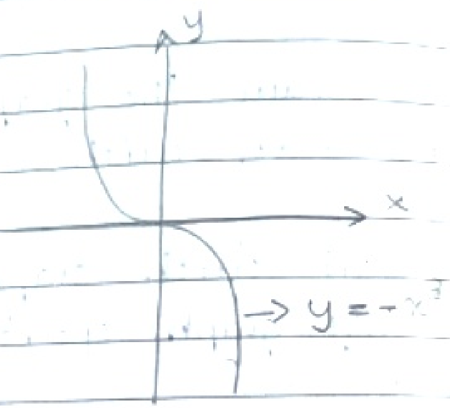
(i) $y = -x^3$

⇒ To obtain the graph of $y = x^3$, we start with,

① Graph of $y = x^3$

② Reflect $y = x^3$ about x-axis to get $y = -x^3$





domain = R

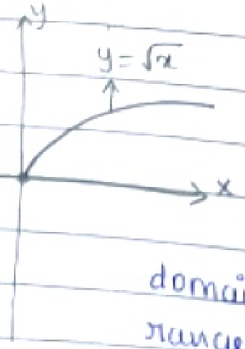
range = R

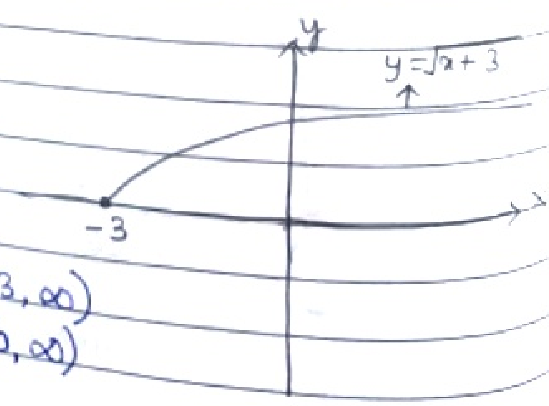
(vii) $y = \sqrt{x+3}$

To obtain the graph of $y = \sqrt{x+3}$, we start with.

① Graph of $y = \sqrt{x}$

② Shift $y = \sqrt{x}$, 3 units to left to get $y = \sqrt{x+3}$





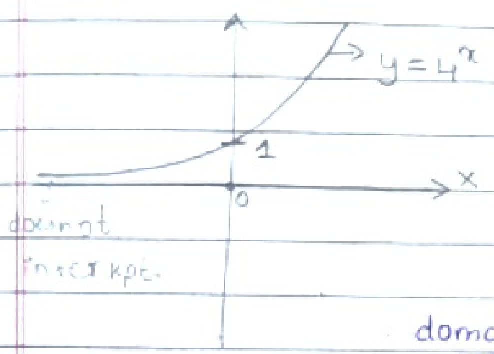
domain = $[-3, \infty)$

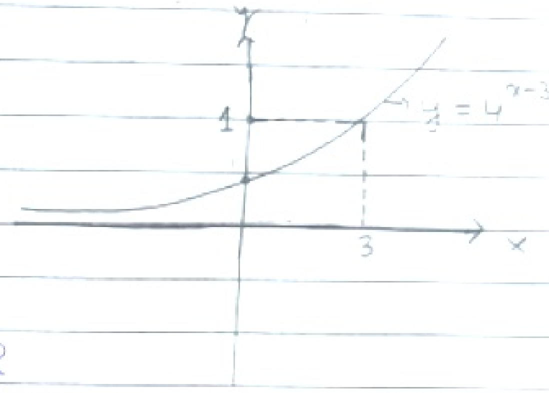
range = $[0, \infty)$

(ix) $y = 4^{x-3}$

To obtain the graph of $y = 4^{x-3}$, we start with

① Graph of $y = 4^x$
② Shift $y = 4^x$, 3 units to right to get $y = 4^{x-3}$.



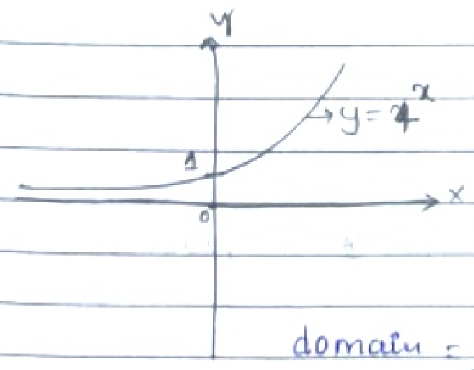


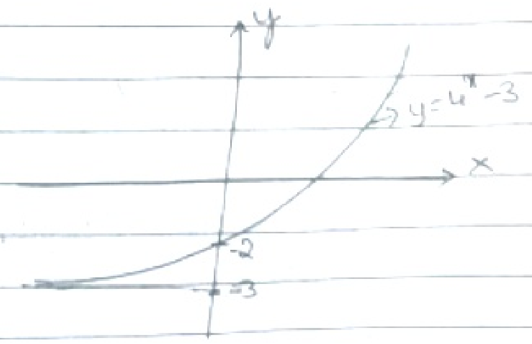
domain = R
range = $(0, \infty)$

(x) $y = 4^x - 3$

To obtain the graph of $y = 4^x - 3$, we start with

① Graph of $y = 4^x$
② Shift the $y = 4^x$, 3 units to downward to get $y = 4^x - 3$





domain = R
range = $(-3, \infty)$

(xi) $y = 3 - e^x$

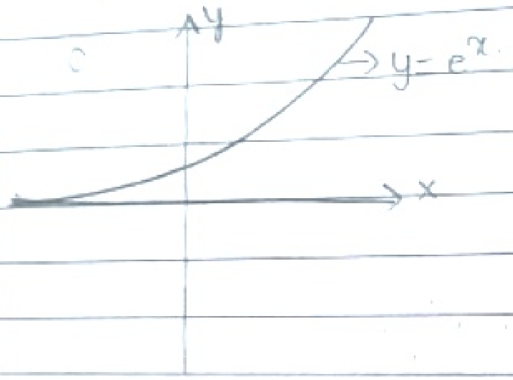
To obtain the graph of $y = 3 - e^x$, we start with

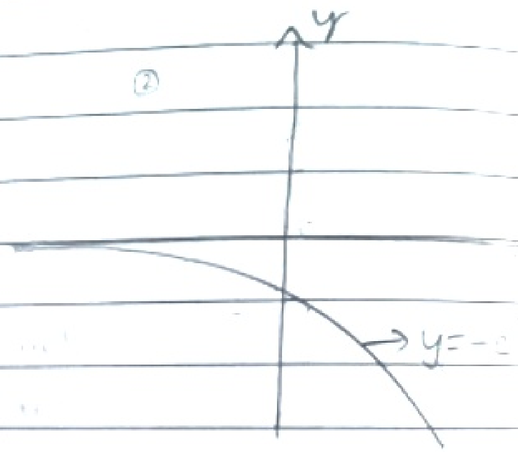
① Graph of $y = e^x$

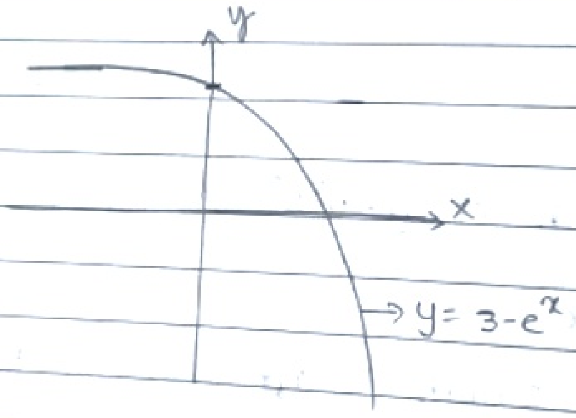
② ~~Shift~~ Reflect $y = e^x$ about x-axis to get $y = -e^x$

③ Shift $y = -e^x$, 3 units upwards to get .

$y = -e^x + 3 = 3 - e^x$.







domain = R

range = $(-\infty, 3]$

7/12/22

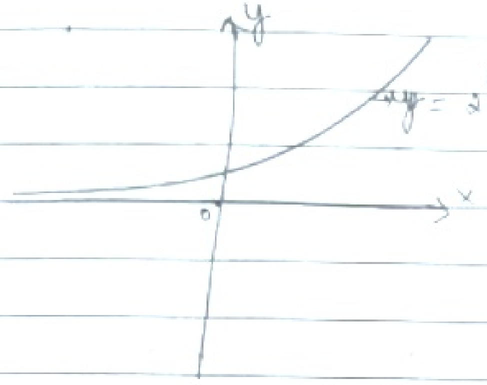
Wednesday

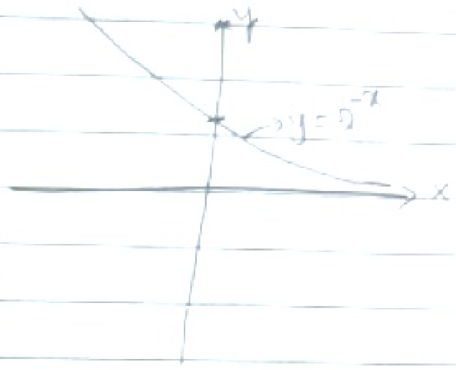
(viii) $y = -2^{-x}$

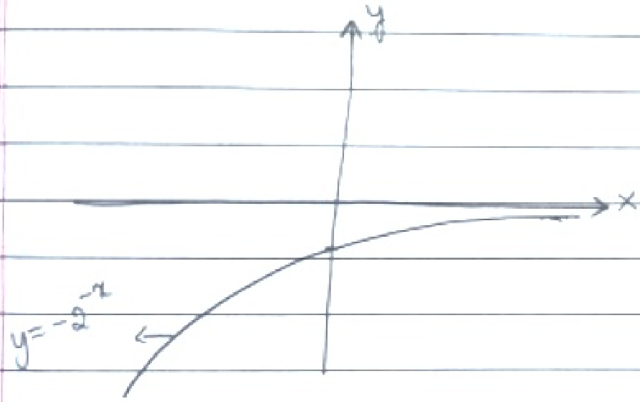
To obtain the graph of $y = -2^{-x}$, we start with

① Graph of $y = 2^x$

2) Reflect the $y = 2^x$ abt y-axis to get $y = 2^{-x}$
3) Reflect the $y = 2^{-x}$ abt x-axis to get $y = -2^{-x}$





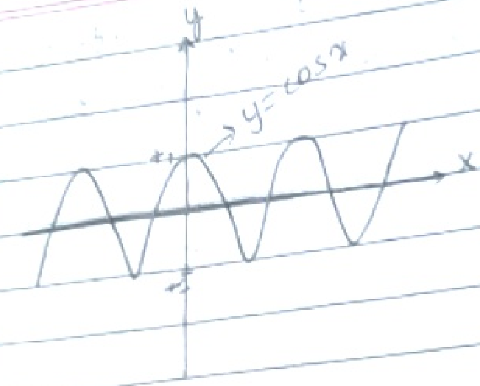


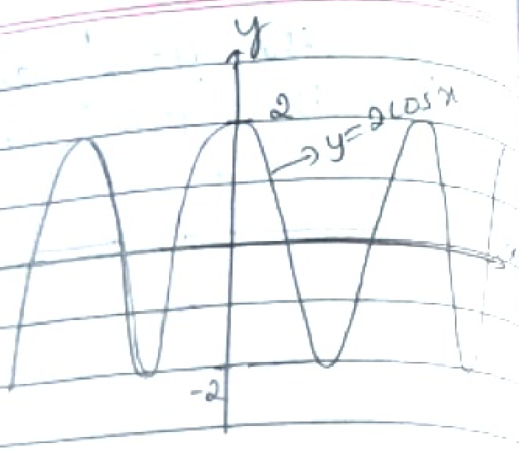
Domain = R = $(-\infty, \infty)$
Range = $(-\infty, 0)$

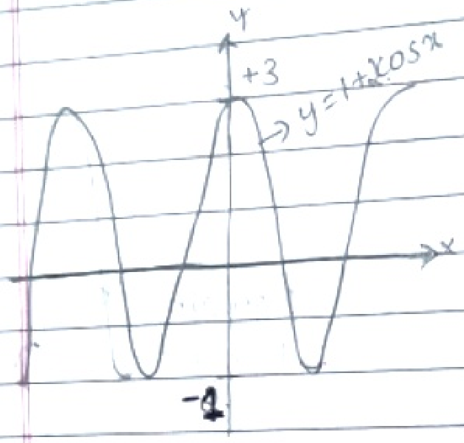
(v) $y = 1 + 2\cos x$

$\Rightarrow$ To obtain the graph of $y = 1 + 2\cos x$, we start with.

① Graph of $y = \cos x$
② Stretch $y = \cos x$ vertically by a factor of 2 to get $y = 2\cos x$
③ Shift $y = 2\cos x$, 1 unit upward to get $y = 1 + 2\cos x$
i.e. $y = 1 + 2\cos x$.







Domain = $(-\infty, \infty) = \mathbb{R}$

Range = $[-1, 3]$

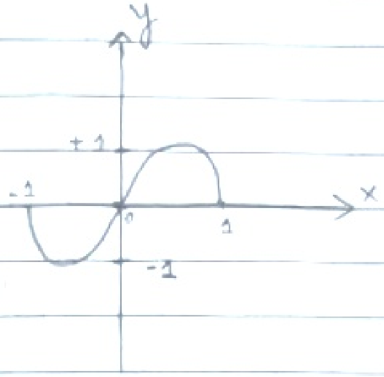
(vi) $y = \sin\left(\frac{x}{2}\right)$

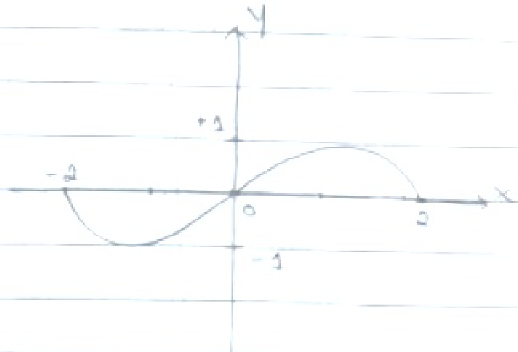
$\Rightarrow$ To obtain the graph of $y = \sin\left(\frac{x}{2}\right)$, we start with.

① Graph of $y = \sin x$.

② Stretch the graph $y = \sin x$ horizontally by a factor of 2 to get $y = \sin\left(\frac{x}{2}\right)$

[↑ width & height remain same]



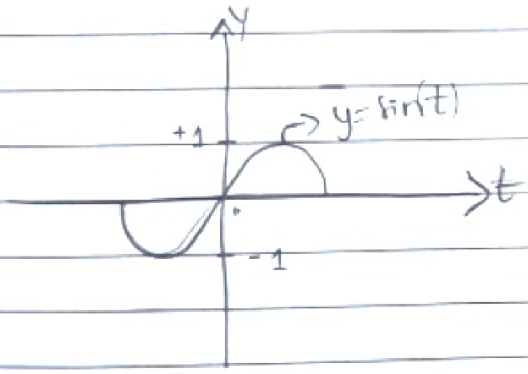


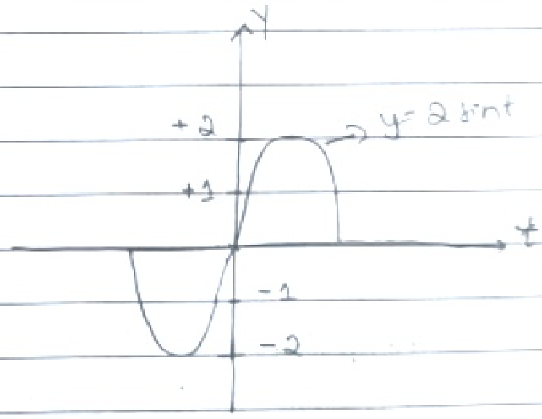
domain = $(-\infty, \infty) = R$

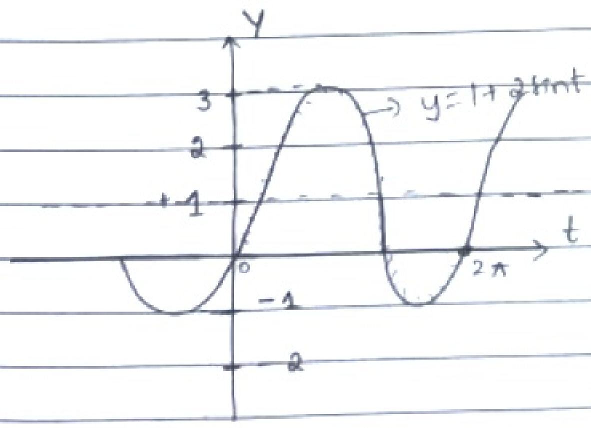
Range = $[-1, 1]$

6] Find & show on a graph the amplitude & period of the function.

(i) $y = 1 + 2\sin(t)$





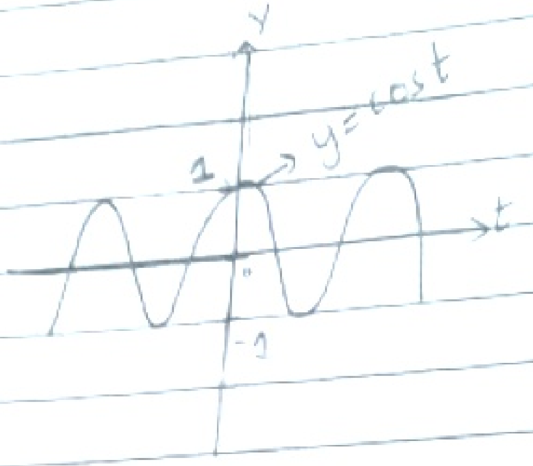


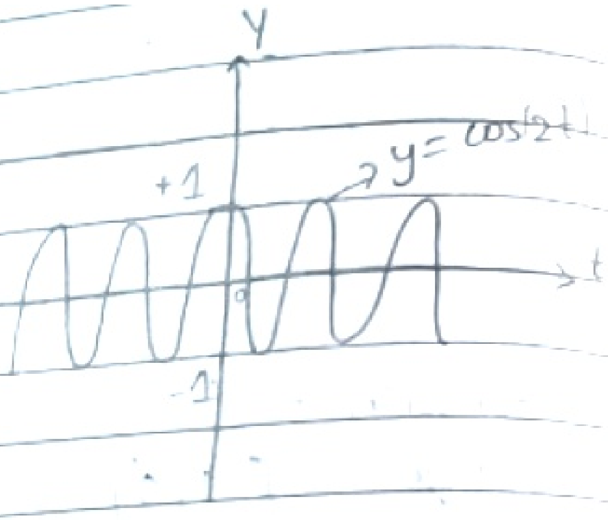
Amplitude = 2

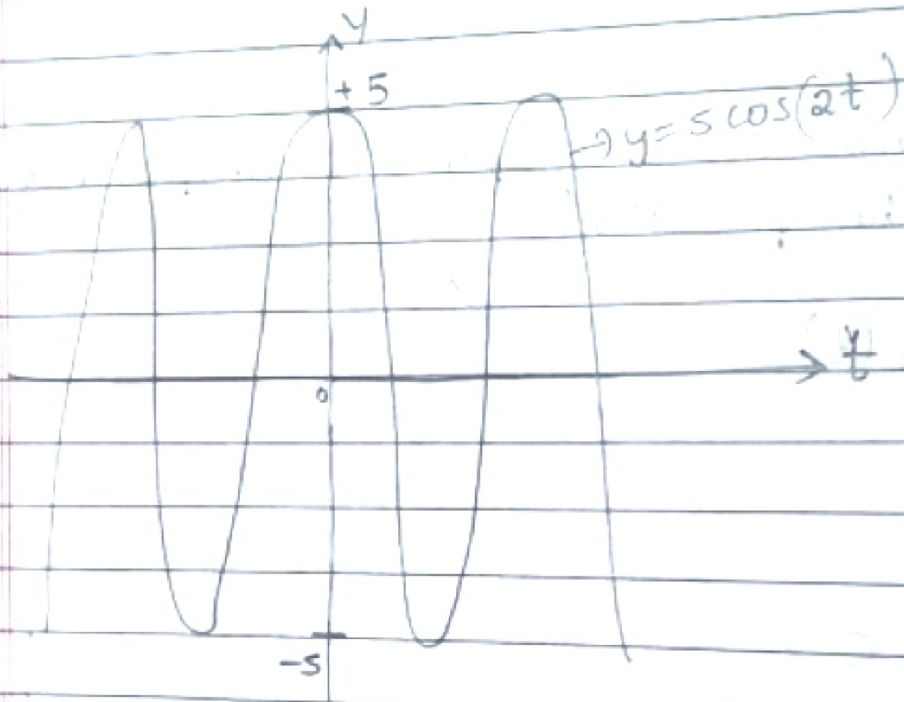
period = $\frac{2\pi}{1}$

$= 2\pi$

(ii) $y = 5\cos(2t)$



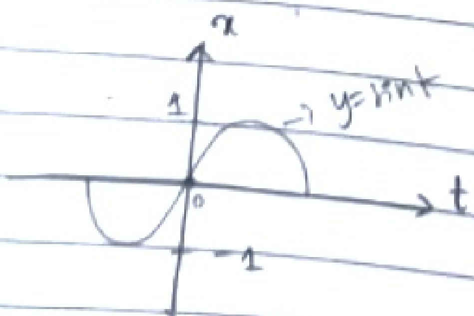




Amplitude = 5

Period $= \frac{2\pi}{2} = \pi$

(iii) $y = -5\sin\left(\frac{t}{2}\right)$



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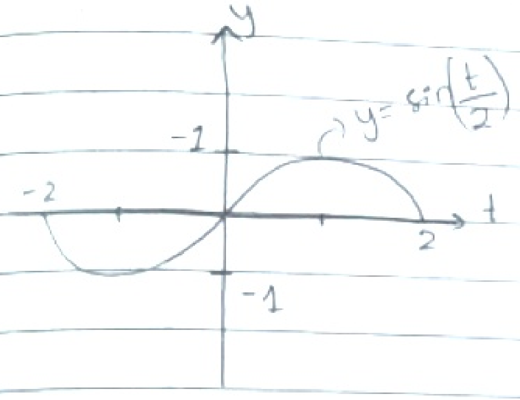
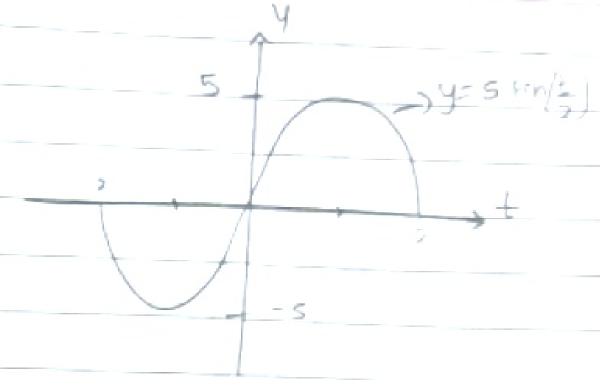
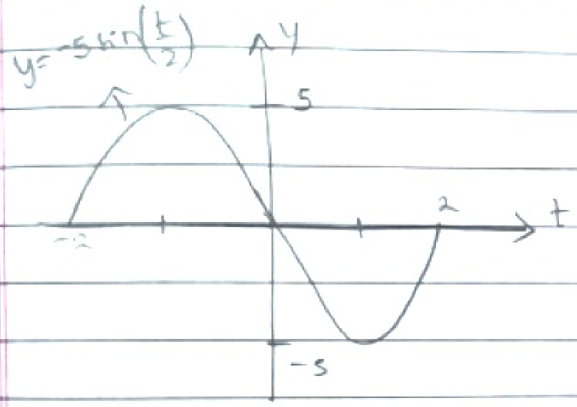






① $\sin t$

② $\sin\left(\frac{t}{2}\right)$ - Stretch hori

③ $5\sin\left(\frac{t}{2}\right)$ - Stretch verti

④ $-5\sin\left(\frac{t}{2}\right)$ = reflect x-axis

Amplitude $= |-5| = 5$

period $= \frac{2\pi}{1/2} = 4\pi$

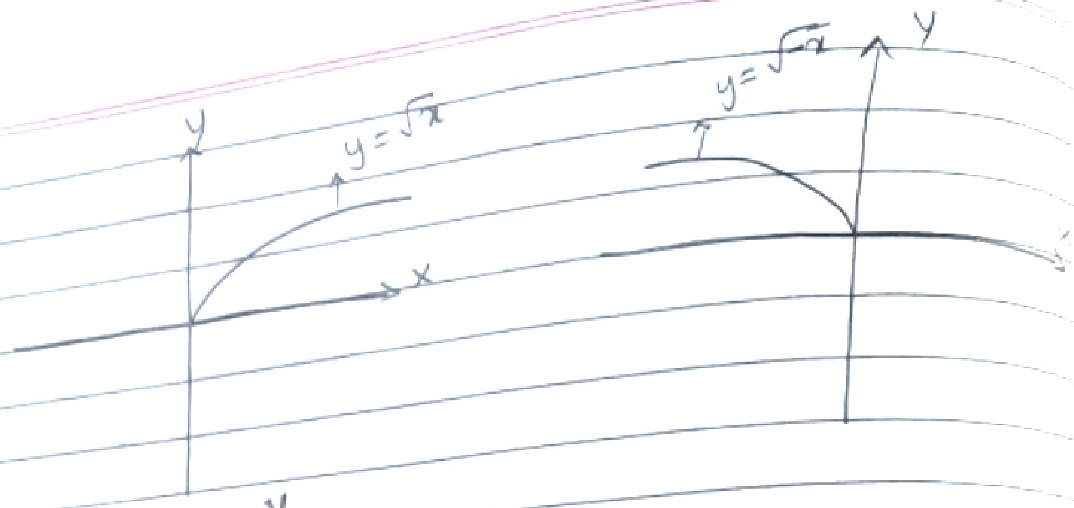
4] Sketch the graph $f(x) = \sqrt{-1-x}$ & its inverse on the same coordinate axes.

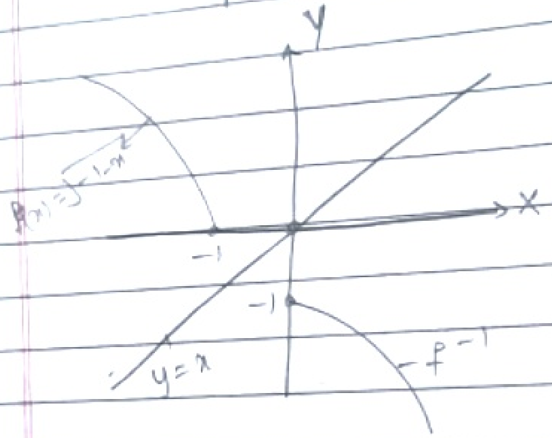
=) To obtain the graph of $fx = \sqrt{-1-x}$, we start with.

① Graph of $y = \sqrt{x}$

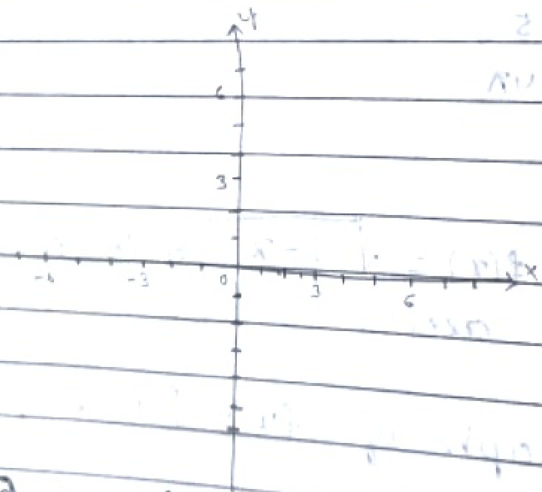
② reflect $y = \sqrt{x}$ about y-axis to get $y = \sqrt{-x}$

③ shift $y = \sqrt{-x}$, 1 unit downward to get,
$y = \sqrt{-x-1} = \sqrt{-1-x}$.





5)



ⓐ $y = f(x-4)$ — 3

ⓑ $y = f(x) + 3$ — 1

ⓒ $y = (1/3)\, f(x)$ — 4

ⓓ $y = -f(x+4)$ — 5

ⓔ $y = 2f(x+6)$ — 2

Eq.	Graph	Reason
(a) $y = f(x-4)$	3	$y = f(x-4)$ is graph of $y = f(x)$ shifted 4 unit to the right
(b) $y = f(x) + 3$	1	$y = f(x) + 3$ is the graph of $y = f(x)$ shifted 3 units upward
(c) $y = f(x)/3$	4	$y = f(x)/3$ is the graph of $y = f(x)$ compressed vertically by a factor of 3
(d) $y = -f(x+4)$	5	$y = -f(x+4)$ is graph obtained 1st by shifting the graph of $y = f(x)$ 4 units to the left & then by reflecting it abt the x-axis
(e) $y = 2f(x+6)$	2	$y = 2f(x+6)$ is the graph obtained 1st by shifting the graph of $y = f(x)$ 6 units to the left & then by stretching it vertically by a factor of 2.

(7) **Given:** $T_1 = 70$ $\quad N_1 = 113$

$T_2 = 80$ $\quad N_2 = 173$

(i) $T = f(N)$

given f is linear.

$\cancel{T \neq N}$ $\quad T - T_1 = \dfrac{T_2 - T_1}{N_2 - N_1} (N - N_1)$

$$T - 70 = \frac{10}{60}(\ldots$$

$$T - 70 = \frac{1}{6}(N - 113)$$

$$T = \frac{1}{6}N - \frac{113}{6} + 70$$

$$\boxed{T = \left(\frac{1}{6}\right)N + \frac{307}{6}} \longrightarrow (1)$$

(ii) Slope $= \frac{1}{6}$

Each ↑ of 6 chirps per minute correspond to an ↑ of 1^0 F ~~...~~

(iii) put $N = 150$ in Eq. 1

$$T = \frac{1}{6}(150) + \frac{307}{6}$$

$$= \frac{457}{6}$$

$$\boxed{T = 76.167^\circ F}$$

(8)

Given :- $t = 0$, $P_0 = 1000$

(i) Population ↑ by 50 people a year.

$t = 1$, $P_1 = 1050 = P_0 + 50 = 1000 + 50$

$t = 2$, $P_2 = P_1 + 50 = 1100$

$\Rightarrow (1000 + 50) \ldots$

$t=3$, ; $P_3 = P_2 + 50$

$= 1000 + 50(3)$

$t=t$; $\boxed{P_t = 1000 + 50t}$

(ii) Population ↑ by 5% in a year.

$t=1$, $P_1 = P_0 + 5\% \, P_0$

$P_1 = P_0 + 0.05 \, P_0$

$P_1 = (1 + 0.05) \, P_0$

$t=1$, $P_1 = P_0 (1.05)$

$t=2$ $P_2 = P_1 + 5\% \, P_1$

$= P_1 + 0.05 \, P_1$

$= P_1 (1 + 0.05)$

$= (1.05) P_0 (1.05)$

$t=2$, $P_2 = P_0 (1.05)^2$

$t=3$, $P_3 = P_0 (1.05)^3$

⋮

$t=t$, $P_t = P_0 (1.05)^t$

$\boxed{P_t = 1000 \, (1.05)^t}$

(9) $t=0$, $Q_0 = 2g$

$t_{1/2} = 15 \text{ hr}$, $Q = \frac{Q_0}{2} = 1g$

__To find :-__ (i) $t = 60$, $Q = ?$

(ii) $t = t$, $Q = ?$

(iii) $t = 4$ days, $Q = ?$

WKT, $Q = Q_0 a^t$ ——— (1)

$Q = 2a^t$

put $t = 15$ & $Q = 1$ in Eq (1)

$1 = 2a^{15}$

$\frac{1}{2} = a^{15}$

$a = \left(\frac{1}{2}\right)^{1/15}$

Equation (1) reduces to,

$$\boxed{Q = 2\left(\frac{1}{2}\right)^{t/15}} \longrightarrow (2)$$

(i) put $t = 60$ in Eq (2)

$Q = 2\left(\frac{1}{2}\right)^{60/15}$

$= 2\left(\frac{1}{2}\right)^4$

$= 2\left(\frac{1}{2^4}\right)$

$= \frac{1}{2^3} = \frac{1}{8} = 0.125\ g$

$\frac{7}{4}$

$Q = 0.125\ g$

(ii) put $t = 4$ days $= 4 \times 24 = 96$ hrs in Eq^n (2)

$Q = 2 \cdot \left(\frac{1}{2}\right)^{96/15}$

$Q = 0.024\ g$

8/12/22 Thursday

10. Let y be the water level in feet.
Let 't' be the time measured in hours.

<u>Given</u> - High tide = 9.9
low tide = 0.1

$\therefore$ amplitude = $\frac{\text{high tide} - \text{low tide}}{2}$

$$= \frac{9.9 - 0.1}{2}$$

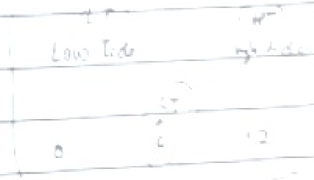
$$\boxed{A = 4.9}$$

Next high tide occurs at 12 noon
$\Rightarrow$ period = 12
$\therefore \frac{2\pi}{B} = 12$

$$\boxed{B = \frac{\pi}{6}}$$

Since the water level is <u>highest</u> at <u>midnight</u> the oscillations are best represented by <u>cosine</u> curve

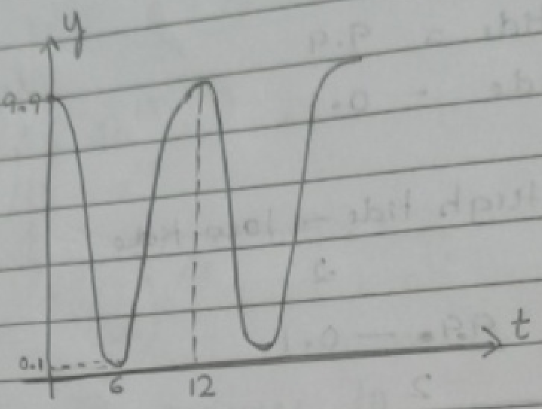
~~highest~~ Height about average $\Big\} = A\cos(Bt)$



$$= 4.9\cos\left(\frac{\pi}{6}t\right)$$

Average water level = $\frac{9.9 + 0.1}{2} = 5$

$$\therefore \boxed{y = 5 + 4.9\cos\left(\frac{\pi}{6}t\right)}$$



8/12/22
Thursday

Chapter (3)

3] Calculus of functions & Models :-

\# We write $\lim_{x \to a} f(x) = L$ and say "the limit of $f(x)$ as 'x' approaches a, equals L" if we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a (on either side of a) but not equal to a.

Note :

$\lim_{x \to a} f(x)$ exists iff. ~~LHL ≠ RHL~~

$$LHL = RHL$$

$$\Rightarrow \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

(1) $f(x) = \dfrac{x-1}{x^2-1} = \dfrac{1-1}{1-1} = \dfrac{0}{0}$ $\quad$ fun is does not exist but limit exist.

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{x-1}{(x-1)(x+1)}$$

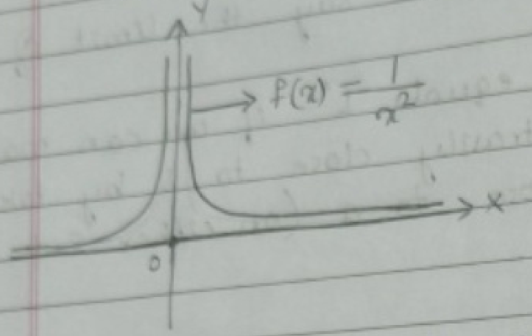
$$= 0.5$$

(2) $f(x) \begin{cases} \dfrac{x-1}{x^2-1}, & \forall x \neq 1 \\ 3, & \forall x = 1 \end{cases}$

$$\lim_{x \to 1} f(x) = 0.5$$

Q) (a) (i) $\lim_{x \to 0} \frac{1}{x^2}$

lim does not exist.



From the graph of $f(x) = \frac{1}{x^2}$, we observe

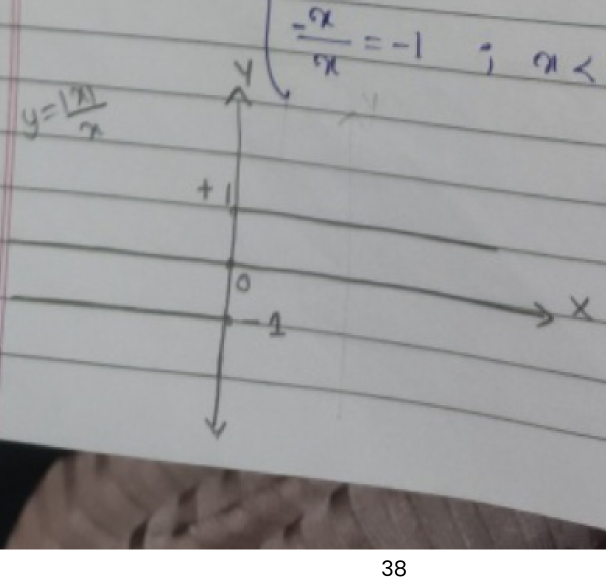
that, the values of $f(x)$ can be made, arbitrarily large by taking x close enough to zero. Thus $f(x)$ does not approach a real number.

$\therefore \lim_{x \to 0} \frac{1}{x^2}$ does not exists.

(ii) $\lim_{x \to 0} \frac{|x|}{x}$

$f(x) = |x| = \begin{cases} +x & \text{if } x > 0 \\ -x & \text{if } x < 0 \end{cases}$

$$\frac{|x|}{x} = \begin{cases} \frac{x}{x} = 1 & ; \quad x > 0 \\ \frac{-x}{x} = -1 & ; \quad x < 0 \end{cases}$$



from the graph,

$$LHL = \lim_{x \to 0^-} \frac{|x|}{x} = -1$$

$$RHL = \lim_{x \to 0^+} \frac{|x|}{x} = +1$$

$\therefore$ LHL $\neq$ RHL

$\Rightarrow \lim_{x \to 0} \frac{|x|}{x}$ DNE $\rightarrow$ does not exist.

$\rightarrow$ intersection at ∞

Horizontal Asymptote :-

Defn: The line $y = L$ is called a horizontal asymptote of the curve $y = f(x)$ if

$\boxed{\lim_{x \to \infty} f(x) = L}$ or $\boxed{\lim_{x \to -\infty} f(x) = L}$

Vertical Asymptote :-

Defn: The line $x = a$ is called a vertical asymptote of the curve $y = f(x)$ if

$$\boxed{\lim_{x \to a} f(x) = \infty}$$

* **Note** :- For vertical asymptote

① $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = \infty$

② $\lim_{x \to a} f(x) = \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = -\infty$

2] (i) $y = \frac{x^2}{x^2-1}$

7M

$\Rightarrow HA = \lim_{x \to \infty} y = \lim_{x \to \infty} \frac{x^2}{x^2-1}$

Take x^2 as common factor from denominator

$= \lim_{x \to \infty} \frac{\cancel{x^2}}{\cancel{x^2}\left(1 - \frac{1}{x^2}\right)}$

$= \frac{1}{1 - \frac{1}{\infty}}$

$= \frac{1}{1-0} = 1$

$\boxed{y = 1 \text{ as } HA}$

$\Rightarrow$ VA : $y \longrightarrow \infty$

$\Rightarrow \left(\frac{x^2}{x^2-1}\right) \longrightarrow \infty$

$\frac{a}{b} \rightarrow \infty$

$\boxed{b = 0}$

$\Rightarrow x^2 - 1 = 0$

$\Rightarrow \underbrace{x = -1, \; x = 1}_{\text{are VA's}}$

9/12/22 Friday

(ii) $y = \frac{x^3}{x^2+3x-10}$

$\Rightarrow$ HA : $\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{x^3}{x^2+3x-10}$

$= \lim_{x\to\infty} \left[\frac{x^{\cancel{3}}}{\cancel{x^2}\left(1+\frac{3}{x}-\frac{10}{x^2}\right)} \right]$

$= \lim_{x\to\infty} \left[\frac{x}{1+\frac{3}{x}-\frac{10}{x^2}} \right]$

$= \frac{\infty}{1+\frac{3}{\infty}-\frac{10}{\infty^2}}$

$= \frac{\infty}{1+0-0}$

$\lim_{x\to\infty} y = \frac{\infty}{1} = \infty$

NO HA.

VA :- $y \to \infty$

$= \frac{x^3}{x^2+3x-10} = \infty = 1/0$

$= x^2+3x-10 = 0$

$= (x+5)(x-2) = 0$

$= x=-5 \; ; \; x=2$ are the VA's.

Q) Ex: $y = \dfrac{2e^x}{e^x - 5}$

HA:-

$\lim_{x\to\infty} y = \lim_{x\to\infty} \dfrac{2e^x}{e^x - 5}$

$= \lim_{x\to\infty} \dfrac{2e^x}{e^x - 5}$

$= \lim_{x\to\infty} \dfrac{2\cancel{e^x}}{\cancel{e^x}\left(1 - \frac{5}{e^x}\right)}$

$= \lim_{x\to\infty} \dfrac{2}{[1 - 5e^{-x}]}$

$= \lim_{x\to\infty} \dfrac{2}{(1 - 5e^{-\infty})}$

$= \lim_{x\to\infty} \dfrac{2}{1 - 5(0)}$

$\boxed{y = 2}$

VA:- $y \longrightarrow \infty$

$\dfrac{2e^x}{e^x - 5} \longrightarrow \infty$

$= e^x - 5 = 0$

$e^x = 5$

$x = \ln 5$ is VA//

Q) $y = \dfrac{x^4 + 1}{x^2 - x^4}$

HA $= \lim_{x\to\infty} y = \lim_{x\to\infty} \dfrac{x^4 + 1}{x^2 - x^4}$

$$\lim_{x \to \infty} = \frac{x^4(1 + x^{-4})}{x^4(x^{-2} - 1)}$$

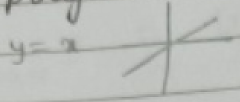
* **Continuity :-**

A function $f(x)$ is continuous at a point 'a'

if $\boxed{\lim_{x \to a} f(x) = f(a)}$ $\boxed{\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)}$

Examples:

① All polynomial functions are continuous.



② $\sin x$, $\cos x$, e^x, $\ln x$ are continuous in their respective domains.

③ $f(x) = \dfrac{1}{x-2}$ is continuous for all 'x' except at $x = 2$.

3] Define continuity. For wt value of constant 'c' is the funⁿ.

$$f(x) = \begin{cases} x^2 - c^2, & x < 4 \\ cx + 20, & x \geq 4 \end{cases}$$ continuous on $(-\infty, \infty)$.

⇒ Given, $f(x) = \begin{cases} x^2 - c^2, & x < 4 \\ cx + 20 & x \geq 4 \end{cases}$

f is continuous on $(-\infty, \infty)$

f is also continuous at $x = 4$

⇒ LHL = RHL at $x = 4$

$$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x)$$

$$\lim_{x \to 4^-} [x^2 - c^2] = \lim_{x \to 4^+} [cx + 20]$$

$16 - c^2 = 4c + 20$

$c^2 + 4c + 4 = 0$

$(c+2)^2 = 0 \Rightarrow \boxed{c = -2}$

Example:- $f(x) = \begin{cases} cx+1 , & x < 3 \\ cx^2 - 1 , & x \geq 3 \end{cases}$

f is continuous at $x = 3$

LHL = RHL at $x = 4$.

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$$

$$\lim_{x \to 3^-} (cx+1) = \lim_{x \to 3^+} (cx^2 - 1)$$

$$3c + 1 = 9c - 1$$

$$2 = 6c$$

$$c = \frac{1}{3}$$

4

$\left(\lim_{x \to b} f(x) \text{ exist}\right) + \left(f \text{ is not continuous at } b\right)$

b		Reason
$b = -1$	False	Limit does not exist [vertical asymptote]
$b = 0$	True	Limit exist & f is not continuous
$b = 1$	False	Limit exist but f = continuous
$b = 2$	False	Limit of f does not exist
$b = 3$	False	Limit exists but f is continuous

Intermediate Value Theorem :-

If f is continuous on a closed interval $[a,b]$ then for any k between $f(a)$ & $f(b)$, $\exists$ c between a & b such that $f(a) \neq f(b)$, $\boxed{f(c)=k}$
Lim exist

& output must be of opp. signs

12/12/22
Monday

Result:- If f is continuous on $[a,b]$ $f(a)$ & $f(b)$ are of opposite signs then $\exists$ $c \in (a,b)$ such that $f(c)=0$

Transcendental Equation :-

Equations containing polynomials, exponential functions, logarithmic fun^n, trignometric fun^n etc are called transcendental Equations.

Example: ① $x^2 = \sin x$
② $e^x + \log x = \tan x$.

(10 & 29)

Q) 5] (i) $x^3 - 6x + 1 = 0$ between 2 & 3

$\Rightarrow$ Let $f(x) = x^3 - 6x + 1$

Clearly $f(x)$ is continuous, as polynomial Eq are continuous

$\exists \Rightarrow$ there exist

$f(2) = 2^3 - 6(2) + 1 = 8 - 12 + 1 = -3 < 0$

$f(3) = 3^3 - 6(3) + 1 = 27 - 18 + 1 = 10 > 0$

By IVT $\exists$ $c \in (2,3)$ such that $f(c) = 0$

$\therefore$ Root exist b/w 2 & 3

(ii) $\ln(x) = e^{-x}$ in $(1, 2)$

$\Rightarrow$ $\ln(x) - e^{-x} = 0$

Let $f(x) = \ln(x) - e^{-x}$

clearly $f(x)$ is continuous.

$f(1) = \ln(1) - e^{-1} = 0 - e^{-1} = \frac{-1}{e} = -0.3679$

$f(2) = \ln(2) - e^{-2} = 0.6931 - e^{-2}$
$= 0.6931 - 0.1353$
$= +0.5578$

By IVT, $\exists\, c \in (1,2)$ such that $f(c) = 0$

$\therefore$ Root exist between 1 & 2

gives approximate Root but not exact Root.

Bisection Method :-

Statement :- If f is continuous in $[a, b]$ with $f(a)$ & $f(b)$ having opposite signs then an approx. root to the desired root is given by $\boxed{c = \frac{a+b}{2}}$, where c is the 1^{st} approximation.

6] (i) $x^3 - 2x - 5 = 0$
(7M)

$\Rightarrow$ Let, $f(x) = x^3 - 2x - 5$
Clearly $f(x)$ is continuous.

{ The graph can be b/w one +ve & -ve no. }

$f(0) = 0^3 - 2(0) - 5 = -5$
$f(1) = 1^3 - 2(1) - 5 = -6$
$f(2) = 2^3 - 2(2) - 5 = -1$
$f(3) = 3^3 - 2(3) - 5 = 16$

$f(2) = 2^3 - 2(2) - 5$
$= -16$
$f(3) = 3^3 - 2(3) - 5$
$= 16$

By IVT, $\exists$ a root b/w 2 & 3.
(a value is +ve but output is -ve)

Iterations	$\bar{a}$	$c = \frac{a+b}{2}$	$\overset{+}{b}$	$f(c)$
1	2	2.5	$\boxed{3}$	+5.6250
2	2	2.25	2.5	+1.8906
3	2	2.1250	2.25	+0.3457
4	$\boxed{2}$ replaced	2.0625	2.125	−0.3513
5	2.0625	$\boxed{2.0938}$	2.125	−0.0094

$\therefore$ an approximate root is $x = 2.0938$.

f(x)
f(x)
f(x)
f(x)
f(x)

(ii) $\cos(x) - 1.3x = 0$ $(0.6, 0.63)$

$\Rightarrow$ Let, $f(x) = \cos(x) - 1.3x$
Clearly $f(x)$ is continuous.

$f(0.6) = \cos(0.6) - 1.3(0.6)$

$f(0.63) = \cos(0.63) - 1.3(0.63)$
$= -0.0110$

By IVT, $\exists$ a root b/w 0.6 & 0.63
(a) (b)

Iterations	a (+)	$c = \frac{a+b}{2}$	b (−)	f(c)
1	0.6	0.6150	0.63	0.0173
2	0.6150	0.6225	0.63	0.0032
3	0.6225	0.6263	0.63	−0.0040
4	0.6225	0.6244	0.6263	−0.0004
5	0.6225	$\boxed{0.6235}$	0.6244	+0.0013

$\therefore$ approximate root is $x = 0.6235$

13/12/22
Tuesday

(iii) $x + \log_{10} x = 3.375$

$\Rightarrow x + \log_{10} x - 3.375 = 0$

Let $f(x) = x + \log_{10} x - 3.375$.

Clearly, $f(x)$ is continuous.

$f(2) = 2 + \log_{10} 2 - 3.375$
$= -1.0740$

$f(3) = 3 + \log_{10} 3 - 3.375$
$= +0.1021$

By IVT, ∃ a root b/w 2 & 3.

Iteration	a (−)	$c=\frac{a+b}{2}$	b (+)	f(c)
1	2	2.5	3	−0.4771
1	2	2.5	3	−0.1857
2	2.5	2.75	3	−0.0414
3	2.75	2.8750	3	+ 0.0305
4	2.875	2.9375	2.9375	−0.0054
5	2.875	2.9063		

12/12 Mon

∴ approximately root is $x = 2.9063$

For, $x + \log_3 x = 3.375$

$$x + \frac{\log_e x}{\log_e 3} - 3.375 = 0$$

$$x + \left(\frac{1}{\log 3}\right)\log_e x - 3.375 = 0$$

Eg: $x^3 - 4x + 9 = 0$

Let $x^3 - 4x + 9 = 0$

$f(x) = x^3 - 4x + 9$

Clearly $f(x)$ is continuous

$f(-2) = (-2)^3 - 4(-2) + 9$

$= -8 + 8 + 9$

$= (+)9$

$f(0) = 9$

$f(1) = 1 - 4 + 9 = 6$

$f(-2) = -8 + 8 + 9 = 9$

$f(-3) = -27 + 12 + 9 = -6$

$f(-3) = (-3)^3 - 4(-3) + 9$

$= -27 + 12 + 9 = (-)6$

By IVT root lies b/w $\overset{a}{-3}$ & $\overset{b}{-2}$

Iteration	a (−)	$c = \frac{a+b}{2}$	b (+)	f(c)
1	−3	−2.5	−2	+3.3750
2	−2.5	−2.250	−2	+6.6094
3	−2.250	−2.1250	−2	+7.9043
4	−2.125	−2.0625	−2	+8.4763
5				

1	−3	−2.5	−2	+3.3750
2	−3	−2.75	−2.5	−0.7969
3	−2.75	−2.625	−2.5	+1.4121
4	−2.75	−2.6875	−2.625	+0.3391
5	−2.75	−2.7188	−2.6875	−0.2218

approximate root is $x = -2.7188$

Newton - Raphson Method :-

N-R Formula :-

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

Where $x_n = n^{th}$ approximation

$f'(x_n) \neq 0$

Note :-

② N-R method fails if $f' = 0$ at any testing point (say x_n)

Question

7] (i) $x^3 - 2x - 5 = 0$

(7M)

$\Rightarrow \boxed{f(x) = x^3 - 2x - 5}$

Clearly $f(x)$ is continuous

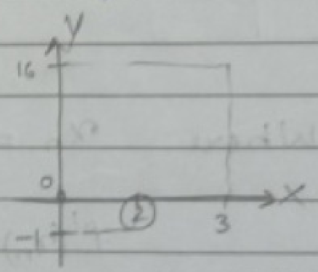
$\boxed{f'(x) = 3x^2 - 2}$

$$f(2) = 2^3 - 2(2) - 5$$
$$= 8 - 4 - 5$$
$$= -$$

$$f(3) = 3^3 - 2(3) - 5$$
$$= 27 - 6 - 5$$
$$= +16$$

By IVT, root lies b/w 2 & 3

NR formula :- $\boxed{x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}}$



as 2 is closer to zero

so $x_0 = 2$

Let $\boxed{x_0 = 2}$

1st iteration $(n=0)$,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 2 - \frac{f(2)}{f'(2)}$$

$$= 2 - \frac{(-1)}{(10)}$$

$$\boxed{x_1 = 2.1}$$

2nd iteration $(n=1)$:-

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.1 - \frac{f(2.1)}{f'(2.1)}$$

$$= 2.1 - \frac{(0.0610)}{(11.2300)}$$

$$\boxed{x_2 = 2.0946}$$

3rd iteration $(n=2)$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.0946 - \frac{f(2.0946)}{f'(2.0946)}$$

$$= 2.0946 - \frac{(0.0005)}{(11.1620)}$$

$$\boxed{x_3 = 2.0946}$$

$$x_2 = x_3$$

$\therefore$ an approximate root is $x = 2.0946$

15/10/22
Thursday

(ii) $x \sin x + \cos x = 0$ near $x = \pi$

$\Rightarrow$ $f(x) = x \sin x + \cos x$

$f'(x) = x[\cos x] + \cancel{[\sin x]} - \cancel{\sin x}$

$f'(x) = x \cos x$

Given, $x_0 = \pi$

N-R Formula :-

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

1st iteration $(n=0)$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = \pi - \frac{f(\pi)}{f'(\pi)}$$

$$x_1 = \pi - \frac{(-1)}{(-\pi)}$$

$$x_1 = \pi - \frac{1}{\pi}$$

$$\boxed{x_1 = 2.8233}$$

2nd iteration $(n=1)$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 2.8233 - \frac{f(2.8233)}{f'(2.8233)}$$

$$= 2.8233 - \left(\frac{+0.0662}{+2.6815}\right)$$

$$= 2.8233 - 0.0247$$

$$x_2 = 2.7986$$

3rd iteration $(n=2)$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.7986 - \frac{f(2.7986)}{f'(2.7986)}$$

$$= 2.7986 - \left(\frac{+0.0006}{+2.6356}\right)$$

$$= 2.7986 - 0.0002$$

$$x_3 = 2.7984$$

4th iteration $(n=3)$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 2.7984 - \frac{f(2.7984)}{f'(2.7984)}$$

$$= 2.7984 - 0$$

$$x_4 = 2.7984$$

$\therefore$ an approximate root is, $\boxed{x = 2.7984}$

$\boxed{x_3 = x_4}$

Eg: $x \log_{10} x = 1.2$

$\Rightarrow x \log_{10} x - 1.2 = 0$

$\frac{1}{\log_e 10}$ $\boxed{1 - \ln(10)}$

$f(x) = x \log_{10} x - 1.2$

$f(x) = \frac{x \log_e x}{\log_e 10} - 1.2$

$\boxed{f(x) = (0.4343)\, x \ln x - 1.2}$

$f'(x) = (0.4343)\left[x \cdot \frac{1}{x} + \ln x\,(1)\right]$

$\boxed{f'(x) = 0.4343\,[1 + \ln x]}$

$f(2) = (0.4343)\,(2)\ln(2) - 1.2$

$= -0.5979$

$f(3) = (0.4343)\,(3)\ln 3 - 1.2$

$= +0.2314$

By IVT, $\exists$ a root between 2 & 3.

N-R formula :- $\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$

Take $\boxed{x_0 = 3}$

1st iteration $(n=0)$:-

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 3 - \frac{f(3)}{f'(3)}$

$$x_1 = 3 - \frac{(0.2314)}{(0.9114)}$$

$$x_1 = 3 - 0.2539$$

$$x_1 = 2.7461$$

2nd iteration $(n=1)$:-

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.7461 - \frac{f(2.7461)}{f'(2.7461)}$$

$$= 2.7461 - \frac{(0.0048)}{(0.8730)}$$

$$= 2.7461 -$$

$$x_2 = 2.7406$$

3rd iteration, $(n=2)$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.7406 - \frac{f(2.7406)}{f'(2.7406)}$$

$$= 2.7406 -$$

$$x_3 = 2.7406$$

$\therefore$ an approximate root is ~~$x=$~~

$$\boxed{x = 2.7406}$$

Date
Page

8] $y = \cos(x) - xe^{x}$ crosses the x-axis.

$\Rightarrow y = 0$

$\cos(x) - xe^{x} = 0$

$xe^{x} - \cos x = 0$

$f(x) = xe^{x} - \cos x$

$f'(x) = + \sin x + [xe^{x} + e^{x}]$

$f(0) = 0e^{0} - \cos 0 = -1$

$f(1) = 1e^{1} - \cos 1$

$= +2.1780$

take ($x_0 = 0$)

N-R formula :- $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

1st iteration ($x = 0$) :

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 0 - \frac{f(0)}{f'(0)}$

$= 0 - \frac{-1}{1}$

$x_1 = 1$

2^{nd} iteration

$x_2 = 0.6531$

$x_3 = 0.5314$

$x_4 = 0.5179$

$x_5 = 0.5178$

$x_6 = 0.5178$

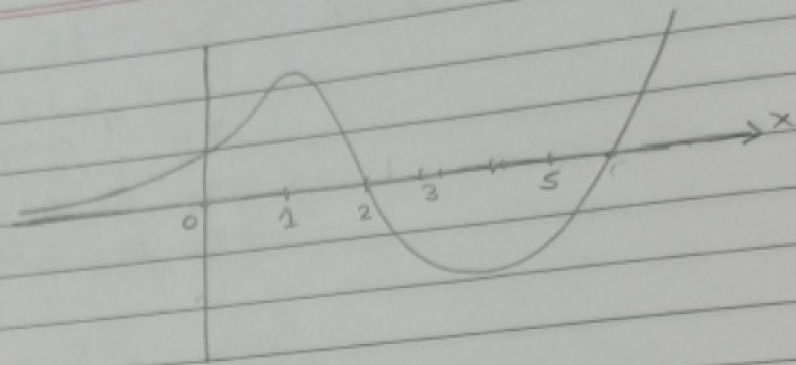
classmate

Date
Page

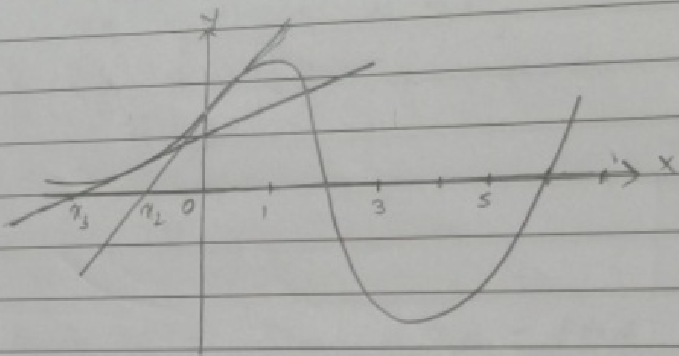
NR-method = method of tangent

16/12/22
Friday

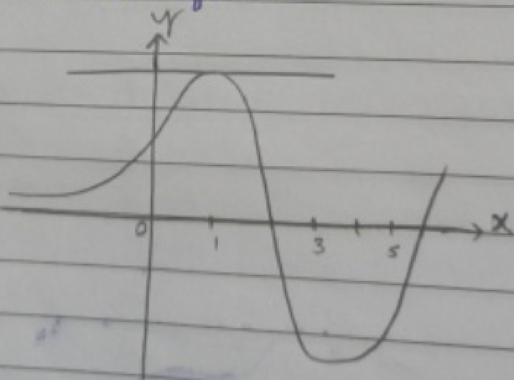
(9)



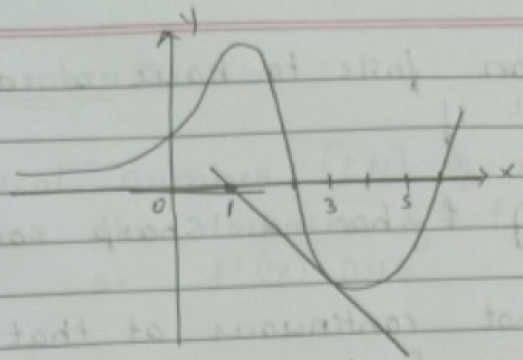
a) At $x_1 = 0$, x_2 is -ve and x_3 is more -ve, the sequence of approximation donot workout hence Newton's method fails.



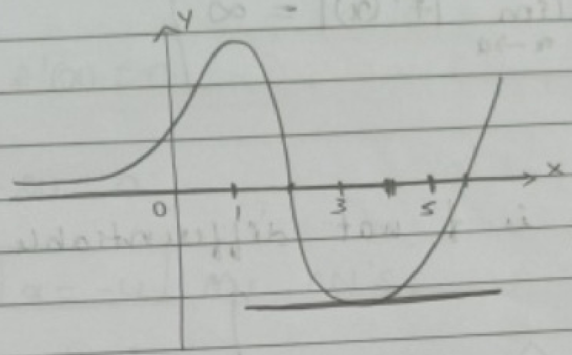
b) At $x_1 = 1$, the tangent line is horizontal & Newton's method fails.



c) At $x_1 = 3$, $x_2 = 1$ and we have the same situation as in part (b), Newton's method fails again.



d) At $x_1 = 4$, the tangent line is horizontal & Newton's method fails.



e) At $x_1 = 5$, Newton's method will lead us to the root.

Derivatives :-

Derivative of a function at a point 'a' is denoted by $f'(a)$ & is defined as.

(3) A function fails to have a derivative at a point 'a' if

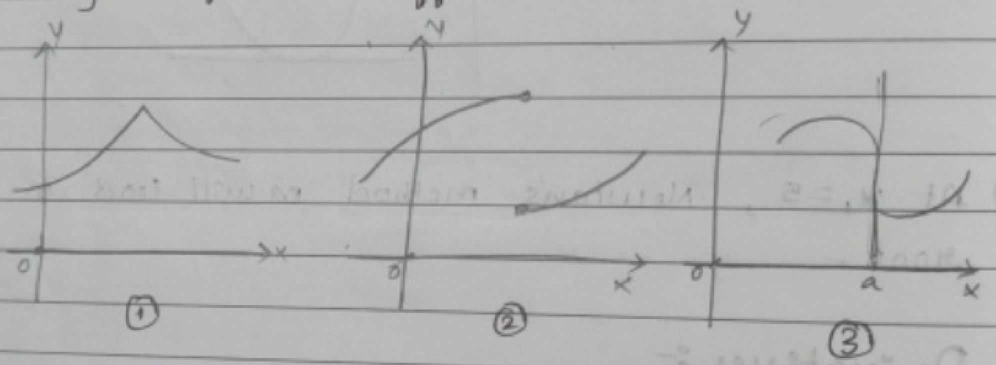
(i) Graph of f has a sharp corner at that point.

(ii) f is not continuous at that point.

(iii) Graph of f has a vertical tangent at that point.

$$\boxed{\lim_{x \to a} |f'(x)| = \infty}$$

10] Why is f not differentiable at $x = a$



In Fig (1), f is not differentiable
∵ there is a sharp corner

In Fig (2), f is not differentiable
∵ f is discontinuous

In Fig (3), f is not differentiable,
∵ f has a vertical tangent.

number [CN] of a functi
the points in the domain of f such
$f'(x) = 0$ or $f'(x)$ DNE

(11) i] $f(x) = x^3 + 3x^2 - 24x$

$\Rightarrow f'(x) = 3x^2 + 6x - 24$

for CN, $f'(x) = 0$

$3x^2 + 6x - 24 = 0$

$\boxed{x = 2}$, $\boxed{x = -4}$ are CN's.

ii] $f(x) = \dfrac{x+1}{x^2+x+1}$

$$\Rightarrow f'(x) = \frac{(x^2+x+1)\frac{d}{dx}(x+1) - (x+1)\frac{d}{dx}(x^2+x+}{(x^2+x+1)^2}$$

$$= \frac{(x^2+x+1)(1) - (x+1)(2x+1)}{x^2+x+1)}$$

$$= \frac{(x^2+x+1) - (2x^2+x+2x+1)}{(x^2+x+1)}$$

$$= \frac{(x^2+x+1) - (2x^2+3x+1)}{x^2+x+1}$$

$$= \frac{x^2+x+\cancel{1} - 2x^2 - 3x - \cancel{1}}{x^2+x+1}$$

$= -x^2 - 2x$

CN :- $f'(x) = 0$

$$\frac{-x^2 - 2x}{(x^2 + x + 1)^2} = 0$$

$\Rightarrow -x^2 - 2x = 0$

$\Rightarrow x = 0,\ x = -2$ are CN's ;

19/12/22 Monday

(iii) $f(\theta) = 2\cos\theta + \sin^2\theta$

$\Rightarrow f'(\theta) = -2\sin\theta + 2\sin\theta\cos\theta$

$f'(\theta) = -2\sin\theta\,(1 - \cos\theta)$

CN :- $f'(\theta) = 0$

$-2\sin\theta\,(1-\cos\theta) = 0$

$\Rightarrow \sin\theta = 0$; $1 - \cos\theta = 0$

$\Rightarrow \sin\theta = 0$; $\cos\theta = 1$

$\Rightarrow \boxed{\theta = n\pi}$; $\boxed{\theta = 2n\pi}$, $n \in \mathbb{Z}$

$\Rightarrow \theta = n\pi$ is CN

Absolute Maxima :-

A function $f(x)$ has an absolute maximum (global maximum) at a point $p \in [a,b]$ if $\boxed{f(p) \geq f(x),\ \forall x}$

Absolute Minimum :-

A function $f(x)$ has an absolute minimum (global minimum) at 'p' if $\boxed{f(p) \leq f(x),\ \forall x}$

Example :

① $f(x) = \sin x$ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ has absolute max $= 1$ & absolute min $= -1$.

Local Maximum :-

A function $f(x)$ has a local maximum at a point 'c' if $\boxed{f(c) \geq f(x),\ \forall\ x \text{ near } c}$

Local Minimum :-

A function $f(x)$ has a local minimum at a point 'c' if $\boxed{f(c) \leq f(x),\ \forall\ x \text{ near } c}$

eg :- 1^{st} Rank to college → (Absolute max)

3^{rd} Rank to University → Local.

Working Rule :-

To find absolute maximum and minimum of f in $[a, b]$

① Find CN's of f in $[a, b]$
② Evaluate f at CN's obtained in step 1
③ Find $f(a)$ & $f(b)$
④ Largest value from step ② & ③ is absolute max. & smallest from step ② & ③ is absolute min.

12] (i) $f(x) = 2x^3 - 3x^2 - 12x + 1$, $[-2, 3]$

$\Rightarrow f'(x) = 6x^2 - 6x - 12$

CN : $f'(x) = 0$

$6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$\Rightarrow x = -1$, $x = 2$ are CN's

$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1 = +8$

$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1 = -19$

$f(-2) = 2(-2)^3 - 3(-2)^2 - 12(-2) + 1 = -3$

$f(3) = 2(3)^3 - 3(3)^2 - 12(3) + 1 = -8$

Abs max occurs at $x = -1$

Abs min " " $x = 2$ //

(ii) $f(x) = \frac{\ln x}{x}$ in $[1, 3]$

$$\Rightarrow f'(x) = \frac{x \frac{d}{dx}[\ln x] - (\ln x)\frac{d}{dx}[x]}{x^2}$$

$$f'(x) = \frac{x\left(\frac{1}{x}\right) - (\ln x)(1)}{x^2}$$

$$\boxed{f'(x) = \frac{1-\ln x}{x^2}}$$

CN : $f'(x) = 0$

$$\frac{1-\ln x}{x^2} = 0$$

$\Rightarrow 1 - \ln x = 0$

$\ln x = 1$

$\boxed{x = e^1 = e}$ is CN

ab max

$f(e) = \frac{\ln e}{e} = \frac{1}{e} = e^{-1} = 0.3679$

$f(1) = \frac{\ln 1}{1} = 0$ ab min

$f(3) = \frac{\ln 3}{3} = 0.3662$

Absolute max occurs at ~~x~~ ~~or~~ $x = e$

" min $x = 1$

<u>Ex:</u> $f(x) = \sin x + \cos x$, $\left[0, \frac{\pi}{3}\right]$

$\Rightarrow f'(x) = \cos x - \sin x$.

<u>CN</u> :- $f'(x) = 0$

$\cos x - \sin x = 0$

$\cos x = \sin x$

$x = \frac{\pi}{4}$ is CN

$$f(\pi/4) = \sin\frac{\pi}{4} + \cos\frac{\pi}{4}$$

$$= \sqrt{2} = 1.414.$$

$$f(0) = \sin 0 + \cos 0 = 1$$

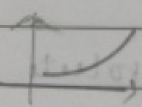
$$f(\pi/3) = \sin\frac{\pi}{3} + \cos\frac{\pi}{3}$$

$$= \frac{\sqrt{3}+1}{2} = 1.3660$$

Abs. max. occurs at $x = \frac{\pi}{4}$

Abs. min " " $x = 0$

Increasing / Decreasing Test :-

① If $f' > 0$ then f is increasing. 

② If $f' < 0$ then f is decreasing 

③ If $f' = 0$ then f has horizontal tangent.

First Derivative Test for Local Max or Min.

Suppose 'c' is a CN of f.

① If f' changes sign from +ve to −ve at 'c' then f' has a local maximum.

(2) If f' changes sign from $-ve$ to $+ve$ at 'c' then 'f' has a local minimum.

(3) If 'f' does not change sign at 'c' then it is neither local max. nor local min.

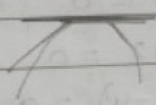
Concave Up :-

If graph of f lies above all of its tangents then f is CU.

Eg :- $f(x) = x^2$

Concave Down :-

If graph of f lies below all of its tangents then f is CD.



Eg :- $f(x) = -x^2$

Concavity Test :-

(1) 'f' is CU if $f'' > 0$

(2) 'f' is CD if $f'' < 0$

Point of Inflection :-

A point where function changes its concavity is pt. of inflection.

classmate
Date
Page

20/12/22
Tuesday

⇒ NOTE :-
Possible pts of inflection are the pt. when either $f''(p) = 0$ or f'' DNE.

I/D Test		FDT	CT	IP
				Possible
$f' > 0$	$f \uparrow$	f' + to −ve Lmax	$f'' > 0$ CU	$f'' = 0$
$f' < 0$	$f \downarrow$	f' − to +ve Lmin	$f'' < 0$ CD	f'' DNE
$f' = 0$	f HT (Horizontal tangent)	f' does not neither		

13 (i) $f(x) = x^3 - 12x + 1$

7M

$\Rightarrow f'(x) = 3x^2 - 12$
$f''(x) = 6x$

CN : $f'(x) = 0$
$\Rightarrow 3x^2 - 12 = 0$
$\Rightarrow 3(x^2 - 4) = 0$
$\Rightarrow x = -2, \ x = +2$ are CN's

(a)

Intervals	Sign of f'	Nature of f
$(-\infty, -2)$	+	Increasing
$(-2, 2)$	−	Decreasing
$(2, \infty)$	+	Increasing.

(b) f' changes sign from + to − at −2
∴ f has local max. at $x = -2$ (By FDT)
∴ $\boxed{f(-2) = +17}$

f' changes sign from $-ve$ to $+ve$ at $x=2$

$\therefore f$ has local min at $x=2$ (by FDT)

$$\boxed{f(2) = -15}$$

(c) $f''(x) = 0$

$\Rightarrow \quad 6x = 0$

$\Rightarrow \quad \boxed{x = 0}$

$-\infty$ ( )$_0$( ) ∞

Intervals	Sign of f''	concavity
$(-\infty, 0)$	$-ve$	concave down
$(0, \infty)$	$+ve$	concave up

$\therefore x = 0$ is an inflection point.

(ii) $f(x) = x^{3/5}(4-x)$

$$\Rightarrow f'(x) = x^{3/5}(-1) + (4-x)\frac{3}{5}x^{3/5-1}$$ [derivative rule]

$$f'(x) = -x^{3/5} + \frac{3}{5}(4-x)x^{-2/5}$$

$$= -x^{3/5} + \frac{3(4-x)}{5x^{2/5}}$$

$$f'(x) = \frac{-5x + 12 - 3x}{5x^{2/5}}$$

$$\boxed{f'(x) = \frac{12-8x}{5x^{2/5}}}$$

$5x^{2/5} \times x^{3/5}$

$\frac{2}{5} + \frac{3}{5} = \frac{5}{5}$

$$f''(x) = \frac{5x^{\frac{2}{5}} \frac{d}{dx}(12-8x) - (12-8x)\frac{d}{dx}\left(5x^{2/5}\right)}{\left(5x^{2/5}\right)^2}$$

$$f''(x) = \frac{5x^{2/5}(-8) - (12-8x)\frac{5\times 2}{5}x^{-3/5}}{25x^{4/5}}$$

$$f''(x) = \frac{-40x^{2/5} - (24-16x)\frac{1}{x^{3/5}}}{25x^{4/5}}$$

$$f''(x) = \frac{-40x - 24 + 16x}{25x^{7/5}}$$

$$\boxed{f''(x) = \frac{-24x-24}{25x^{7/5}} = \frac{-24(x+1)}{25x^{7/5}}.}$$

CN: $f' = 0 \Rightarrow \frac{12-8x}{5x^{2/5}} = 0$

$\Rightarrow 12 - 8x = 0$

$$\boxed{x = \frac{3}{2}}$$

$f' = \infty \Rightarrow \frac{12-8x}{5x^{2/5}} = \infty$

$5x^{2/5} = 0$

$\Rightarrow \boxed{x = 0}$ are the CN's

(a)

Intervals	Sign of f'	Nature of f
$(-\infty, 0)$	+ve	Increasing
$(0, 3/2)$	+ve	Increasing
$(3/2, \infty)$	-ve	Decreasing

(b) f' changes sign from +ve to -ve at $x = 3/2$
f has local max at $x = 3/2$ (By FDT)

$$\therefore f(3/2) = 3.1886.$$

(c) $f''(x) = 0$

$$\frac{-24(x+1)}{25x^{7/5}} = 0$$

$$\Rightarrow \boxed{x = -1}$$

$$f''(x) = \infty$$

$$\frac{-24(x+1)}{25x^{7/5}} = \infty$$

$$\Rightarrow \boxed{x = 0}$$

Intervals	Sign of f''	concavity
$(-\infty, -1)$	-ve	concave down
$(-1, 0)$	+ve	concave up
$(0, \infty)$	-ve	concave down

$\therefore x = 0$ & $x = -1$ are the IP's.

classmate
Date
Page

21/12/22
Wednesday

(iii) $f(x) = \cos^2 x - 2\sin x$; $0 \leq x \leq 2\pi$

$\Rightarrow f'(x) = -2\cos x \sin x - 2\cos x$

$f'(x) = -\sin 2x - 2\cos x$

$\boxed{f''(x) = -2\cos 2x + 2\sin x}$

CN : $f' = 0$

$-2\cos x \sin x - 2\cos x = 0$

$-2\cos x(\sin x + 1) = 0$

$\cos x = 0$; $\sin x + 1 = 0$

$\cos x = 0$; $\sin x = -1$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$; $x = \frac{3\pi}{2}$

$\therefore x = \frac{\pi}{2}, \frac{3\pi}{2}$ are CN's

(a)

Intervals	sign of f'	Nature of f
$(0, \pi/2)$	$-ve$	Decreasing
$(\pi/2, 3\pi/2)$	$+ve$	Increasing
$(3\pi/2, 2\pi)$	$-ve$	Decreasing

(b) f' changes sign from $-ve$ to $+ve$ at $x = \pi/2$

$\therefore f$ has local min at $x = \pi/2$ (FDT)

$\therefore \boxed{f(\pi/2) = -2}$

f' changes from $-$ to $+$ at $x = 3\pi/2$
f has local max. at $x = 3\pi/2$

$$\boxed{f(3\pi/2) = 2}$$

(c) $f'' = 0$

$$-2\cos(2x) + 2\sin x = 0$$
$$2\sin x = 2\cos(2x)$$
$$\sin x = \cos(2x)$$
$$\sin x = 1 - 2\sin^2 x$$
$$2\sin^2 x + \sin x - 1 = 0$$
$$2\sin^2 x + 2\sin x - \sin x - 1 = 0$$
2

$ax^2 + bx + c = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\boxed{\sin x = \frac{1}{2}}$, $\boxed{\sin x = 1}$

$\Rightarrow \boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$; $\boxed{x = \frac{3\pi}{2}}$

(d)

Intervals	sign of f''	concavity
$(0, \pi/6)$	$-$ve	CD
$(\pi/6, 5\pi/6)$ mid$(\pi/2)$	$+$ve	CU
$(5\pi/6, 3\pi/2)$ (π)	$-$ve	CD
$(3\pi/2, 2\pi)$ (2π)	$-$ve	CD

Second Derivative Test :-

Suppose f is continuous near c

(1) If $f'(c) = 0$ & $f''(c) < 0$
then f has a local maximum.

(2) If $f'(c) = 0$ and $f''(c) > 0$
then f has a local minimum.

(3) If $f'(c) = 0$ and $f''(c) = 0$
then the test is inconclusive.

(11 Df 29)

Q 14 (i) $f(x) = x^3 - 9x^2 - 48x + 52$

$\Rightarrow$ $f(x) = x^3 - 9x^2 - 48x + 52$
$f'(x) = 3x^2 - 18x - 48$
$f''(x) = 6x - 18$

CN : $f'(x) = 0$
$3x^2 - 18x - 48 = 0$
$\Rightarrow$
$x = 8$, $x = -2$ are CN's

$f''(x) = 6x - 18$
$f''(-2) = (-2)6 - 18$
$= -30 (< 0)$

$f''(8) = 6(8) - 18$
$= +30 \ (> 0)$

By SDT :-

f has local max at $x = -2$ & $f(-2) = 104$
f " " min at $x = 8$ & $f(8) = -396$

(ii) $f(x) = \frac{x}{x^2+4}$

$$\Rightarrow \frac{D\frac{d}{x}(N) - N\frac{d}{x}(D)}{D^2}$$

$$\Rightarrow f'(x) = \frac{(x^2+4) - x(2x)}{(x^2+4)^2}$$

$$= \frac{(x^2+4)\frac{d}{dx}(x) - x\frac{d}{dx}(x^2+4)}{(x^2+4)^2}$$

$$= \frac{x^2+4-2x^2}{(x^2+4)^2}$$

$$\boxed{f'(x) = \frac{4-x^2}{(x^2+4)^2}}$$

$$f''(x) = \frac{(x^2+4)^2\frac{d}{dx}(4-x^2) - (4-x^2)\frac{d}{dx}(x^2+4)^2}{(x^2+4)^4}$$

$$= \frac{(x^2+4)^{\cancel{2}}(-2x) - (4-x^2)\,2\cancel{(x^2+4)}(2x)}{(x^2+4)^{\cancel{4}\,3}}$$

$$= \frac{(x^2+4)(-2x) - (4-x^2)(4x)}{(x^2+4)^3}$$

$$= \frac{(-2x)\left[x^2+4+2(4-x^2)\right]}{(x^2+4)^3}$$

$$f''(x) = \frac{(-2x)\cdot\left[-x^2+12\right]}{(x^2+4)^3}$$

classmate

Date
Page

CN : $f' = 0$

$x = \pm 2$ are CN's

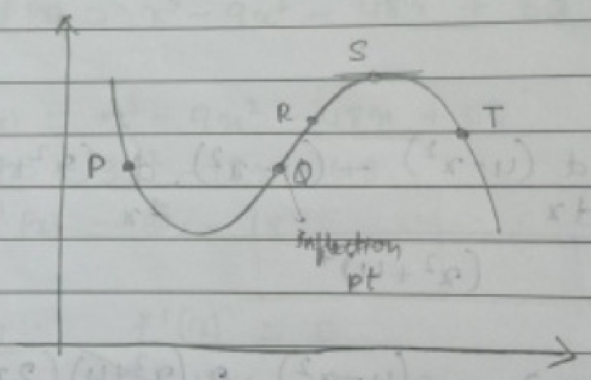
$f''(+2) = -1/16 = -0.0625$

$f''(-2) = +1/16 = 0.0625$

f has local max at $x = 2$
& $f(2) = 0.25$

f has local min at $x = -2$
& $f(-2) = -0.25$.

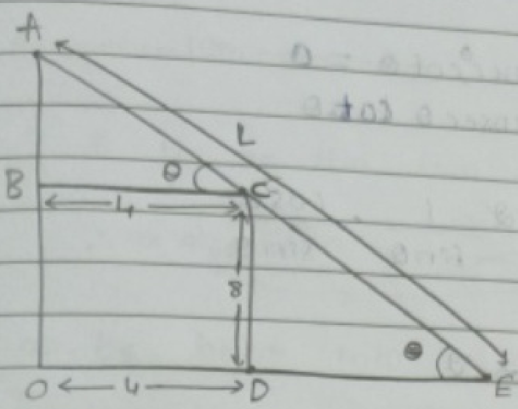
Q 15



Points	y'	y''
P	−	+
Q	+	0
R	+	−
S	0	−
T	−	−

Optimization Problems :-

Q17



OA = Building
CD = fence
AE = ladder

Let L = AE

$L = AC + CE \longrightarrow (1)$

From Δ^{le} ABC,

$$\cos\theta = \frac{BC}{AC} = \frac{4}{AC}$$

$$\boxed{AC = 4\sec\theta} \longrightarrow (2)$$

In Δ^{le} CDE,

$$\sin\theta = \frac{CD}{CE}$$

$$\sin\theta = \frac{8}{CE}$$

$$\boxed{CE = 8\operatorname{cosec}\theta} \longrightarrow (3)$$

from (2) & (3), Eq^n (1) reduces to,

$$\boxed{L = 4\sec\theta + 8\operatorname{cosec}\theta} \longrightarrow (4)$$

Differentiation (4) wrt 'θ'

$$\boxed{L' = 4\sec\theta\tan\theta - 8\operatorname{cosec}\theta\cot\theta}$$

CN: $L' = 0$

$4\sec\theta\tan\theta - 8\operatorname{cosec}\theta\cot\theta = 0$

$4\sec\theta\tan\theta = 8\operatorname{cosec}\theta\cot\theta$

$$\frac{1}{\cos\theta}\cdot\frac{\sin\theta}{\cos\theta} = 2\,\frac{1}{\sin\theta}\cdot\frac{\cos\theta}{\sin\theta}$$

$$\frac{\sin^3\theta}{\cos^3\theta} = 2$$

$$\tan^3\theta = 2$$

$$\therefore \tan\theta = 2^{1/3}$$

$\Rightarrow \theta = \tan^{-1}(2^{1/3})$

$\boxed{\theta = 0.8999}$ $(-\infty, 0.89999)$

Intervals	sign of L'
$\theta < 0.8999$ (0.5)	$-ve$
$\theta > 0.8999$	$+ve$

By FDT :- L' changes sign from $-ve$ to $+ve$

$\therefore L$ has minimum at
$\theta = 0.8999$

$\therefore$ Shortest length from (4)

$L = 4\sec(0.8999) + 8\operatorname{cosec}(0.8999)$

$L = \underline{16.647}$ ft.

Q 18]

10m

Let x & y be the two pieces of the wire

$$\therefore x + y = 10 \quad \text{——} \quad (1)$$

Let x be bent into a square of side $\frac{x}{4}$

Let y be bent into an Equilateral Δ^{le} of side $\frac{y}{3}$

$\therefore$ Total area $= \left(\text{Area of square}\right) + \left(\text{Area of Equilateral } \Delta^{le}\right)$

$$A = \frac{x^2}{16} + \frac{\sqrt{3}}{4}\left(\frac{y}{3}\right)^2$$

$$A = \frac{x^2}{16} + \frac{\sqrt{3}}{4} \cdot \frac{y^2}{9}$$

$$\boxed{A = \frac{x^2}{16} + \frac{\sqrt{3}}{36}(10-x)^2} \quad \text{refer eq}^n\ (1) \quad \text{——} \quad (2)$$

where, $0 \leq x \leq 10$

diff w.r.t x $\quad A' = \frac{2x}{16} + \frac{\sqrt{3}}{36}\, 2(10-x)(-1)$

$$A' = \frac{x}{8} - \frac{\sqrt{3}}{18}(10-x)$$

CN: $A' = 0$

$$\frac{x}{8} - \frac{\sqrt{3}}{18}(10-x) = 0$$

2	8	18
2	4	9
2	2	9
3	1	9
3		3
		1

$$\frac{x}{8} = \frac{\sqrt{3}}{18}(10-x)$$

$$\Rightarrow 18x = 8\sqrt{3}(10-x)$$

$$18x = 80\sqrt{3} - 8\sqrt{3}x$$

$$18x + 8\sqrt{3}x = 80\sqrt{3}$$

$$31.8564x = 80\sqrt{3}$$

$$x \doteq 4.3496 \text{ is CN}$$

from (2),

(3) $$A(4.3496) = \frac{(4.3496)^2}{16} + \frac{\sqrt{3}}{36}(10-4.3496)^2$$

$$= 1.1824 + 1.5361$$

$$= 2.7185$$

$$A(0) = \frac{0^2}{16} + \frac{\sqrt{3}}{36}(10-0)^2$$

$$= 4.81$$

$$A(10) = \frac{10^2}{16} + \frac{\sqrt{3}}{36}(10-10)^2$$

$$= 6.25$$

22/12/22
Thursday

16

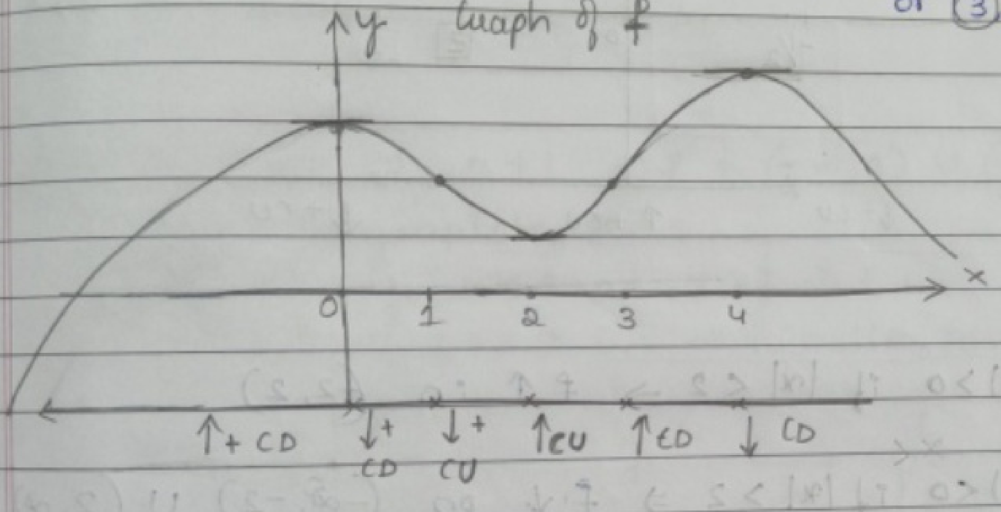
(I) $f'(0) = f'(2) = f'(4) = 0 \Rightarrow f$ has HT's at $x=0,\ 2,\ 4$

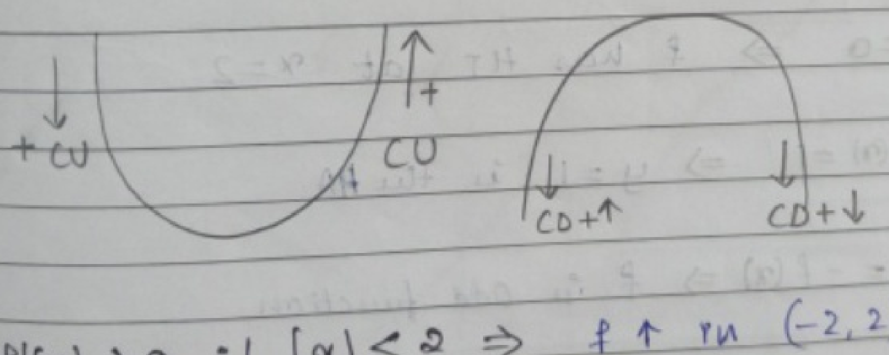
$f'(x) > 0$ if $x<0$ or $2<x<4 \Rightarrow f\uparrow$ on $(-\infty, 0)$ or $(2,4)$

$f'(x) < 0$ if $0<x<2$ or $x>4 \Rightarrow f\downarrow$ on $(0,2)$ or $(4,\infty)$

$f''(x) > 0$ if $1<x<3 \Rightarrow f$ is CU on $(1,3)$

$f''(x) < 0$ if $x<1$ or $x>3 \Rightarrow f$ is CD on $(-\infty,1)$ or $(3,\infty)$





(II) $f'(x) > 0$ if $|x| < 2 \Rightarrow f\uparrow$ on $(-2,2)$

$\lim_{x \to \infty} f(x) = L$ HA

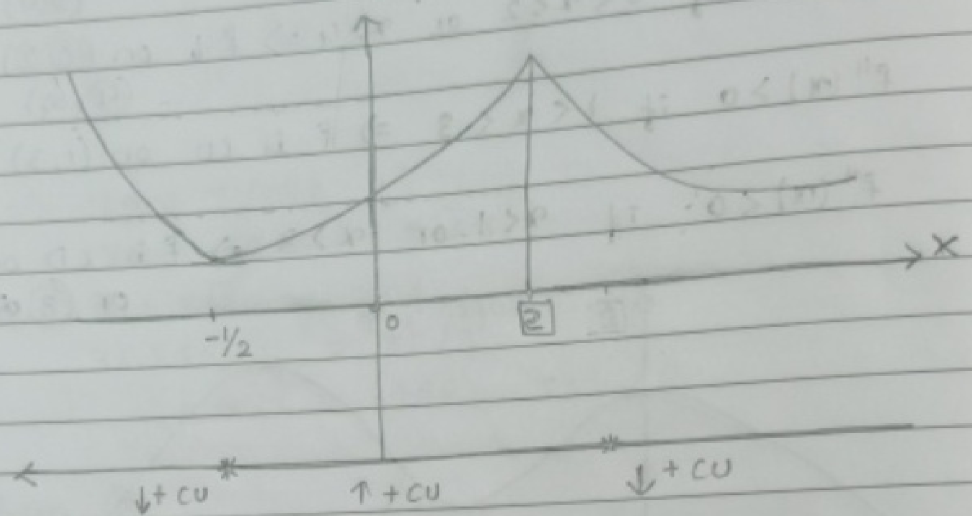
$\lim_{x \to a} f(x) = \infty$ VA

$\lim_{x \to a} |f'(x)| = \infty$ VT

$\lim_{x \to 2} |f'(x)| = \infty \Rightarrow f$ has VT at $x = 2$

$f''(x) > 0$ if $x \neq 2 \Rightarrow f$ is CU $(-\infty, 2) \cup (2, \infty)$

Graph of f



III

$f'(x) > 0$ if $|x| < 2 \Rightarrow f \uparrow$ in $(-2, 2)$

$f'(x) < 0$ if $|x| > 2 \Rightarrow f \downarrow$ on $(-\infty, -2) \cup (2, \infty)$

$f'(2) = 0 \Rightarrow f$ has HT at $x = 2$

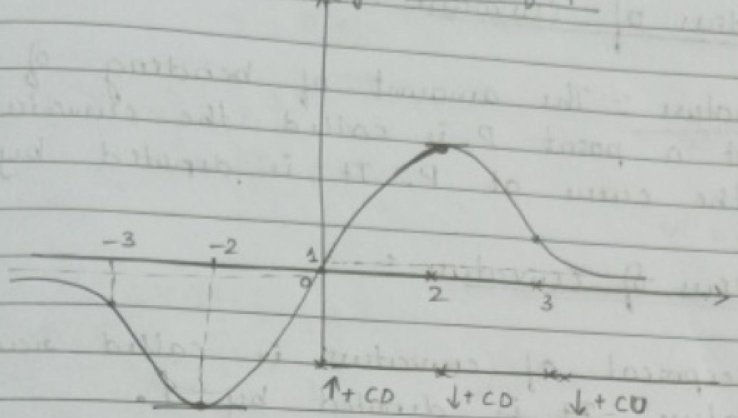
$\boxed{\lim_{x \to \infty} f(x) = 1} \Rightarrow y = 1$ is the HA

$f(-x) = -f(x) \Rightarrow f$ is Odd function

$f''(x) < 0$ if $0 < x < 3 \Rightarrow f$ is CD in $(0, 3)$

$f''(x) > 0$ if $x > 3 \Rightarrow f$ is CU in $(3, \infty)$

Graph of f



23/12/22
Friday

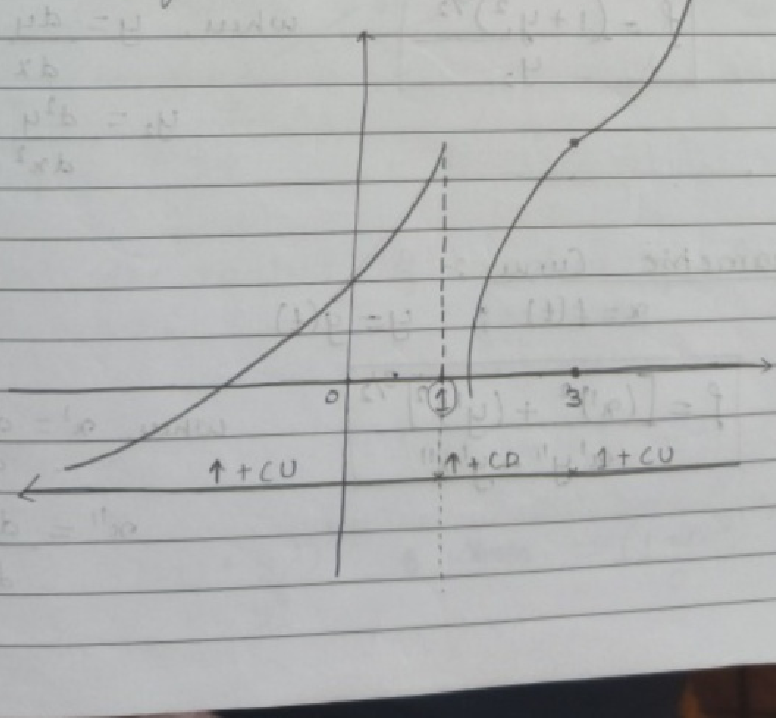
Ex: $f'(x) > 0 \ \forall \ x \neq 1 \Rightarrow f \uparrow (-\infty, 1) \cup (1, \infty)$

vertical asymptote $x = 1$

$f''(x) > 0$ if $x < 1$ or $x > 3 \Rightarrow f$ is CU $(-\infty,1) \cup (3,\infty)$

$f''(x) < 0$ if $1 < x < 3 \Rightarrow f$ is CD on $(1,3)$

Graph of f,



Radius of Curvature :-

Curvature :- The amount of bending of a curve at a point P is called the curvature of the curve at P. It is denoted by K.

Radius of Curvature :-

Reciprocal of curvature is called radius of curvature. It is denoted by ρ.

$$\boxed{\rho = \frac{1}{K}}$$

Formulae :-

1] For Cartesian Curve :-

$y = f(x)$ or $f(x, y) = 0$

$$\boxed{\rho = \frac{(1+y_1^2)^{3/2}}{y_2}}$$

where, $y = \frac{dy}{dx}$

$y_2 = \frac{d^2y}{dx^2}$

2] Parametric Curves :-

$x = f(t)$; $y = g(t)$

$$\boxed{\rho = \frac{[(x')^2 + (y')^2]^{3/2}}{x'y'' - y'x''}}$$

where, $x' = \frac{dx}{dt}$

$x'' = \frac{d^2x}{dt^2}$

3] Polar Curve :-

$$r = f(\theta)$$

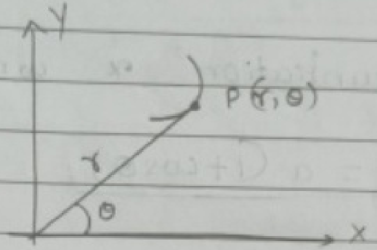
$$\boxed{\rho = \frac{[r^2 + r_1^2]^{3/2}}{r^2 + 2r_1^2 - r r_2}}$$

where, $r' = \frac{dr}{d\theta}$

$r'' = \frac{d^2 r}{d\theta^2}$

$r = f(\theta)$ → angle

radius vector

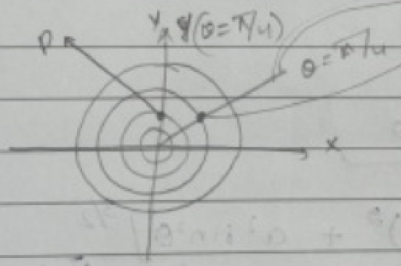


$P(x, y) \Rightarrow y = f(x)$

$P(r, \theta) \Rightarrow r = f(\theta)$

Eg: $P(2, \pi/2)$ $P(3, \pi/4)$

$\downarrow r$ $\downarrow \theta$



Ex:- Find the radius of curvature of the st. line $y = mx + c$

$\Rightarrow$ $y = mx + c$

$y_1 = m$

$y_2 = 0$

ROC :- $\rho = \frac{(1 + y_1^2)^{3/2}}{y_2} = \frac{(1 + m^2)^{3/2}}{0} = \infty$

Q19] $x = a(\theta + \sin\theta)$, $y = a(1-\cos\theta)$

7M Given:- $x = a(\theta + \sin\theta)$

$y = a(1-\cos\theta)$

ROC : $\rho = \dfrac{[(x')^2 + (y')^2]^{3/2}}{x'y'' - y'x''}$

Differentiation x wrt 'θ'

$\boxed{x' = a(1+\cos\theta)}$

Diff^n x' wrt θ'

$\boxed{x'' = a(-\sin\theta)}$

Diff^n y wrt 'θ',

$\boxed{y' = a(\sin\theta)}$

Diff^n $\boxed{y'' = a\cos\theta}$

$\therefore \rho = \dfrac{[(a^2 (1+\cos\theta)^2 + a^2\sin^2\theta]^{3/2}}{a(1+\cos\theta)\, a\cos\theta - (a\sin\theta)(-a\sin\theta)}$

$\rho = \dfrac{a^3 [(1+\cos\theta)^2 + \sin^2\theta]^{3/2}}{\cancel{a^2}[(1+\cos\theta)\cos\theta + \sin^2\theta]}$

$\rho = \dfrac{[a[(1+\cos\theta)^2 + (1-\cos^2\theta)]^{3/2}}{\cos\theta + \cos^2\theta + \sin^2\theta}$

$$\rho = \frac{a\left[(1+\cos\theta)^2 + (1-\cos\theta)(1+\cos\theta)\right]^{3/2}}{1+\cos\theta}$$

$$\rho = \frac{a\,(1+\cos\theta)^{3/2}\left[(1+\cancel{\cos\theta})+(1-\cancel{\cos\theta})\right]^{3/2}}{1+\cos\theta}$$

$$\rho = 2^{3/2}\, a\, (1+\cos\theta)^{1/2}$$

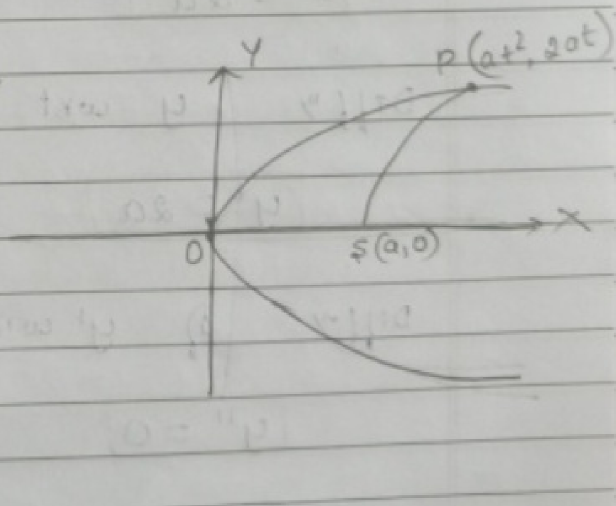
$$\rho = 2^{3/2}\, a \left(2\cos^2\frac{\theta}{2}\right)^{1/2}$$

$$\rho = 2^{3/2}\, a\; 2^{1/2}\cos\frac{\theta}{2}$$

$$\boxed{\rho = 4a\cos\frac{\theta}{2}}$$

Q 20 Given = $y^2 = 4ax$

7M Parametric Equations of Parabola $y^2 = 4ax$:-



$x = at^2$
$y = 2at$

$y^2 = 4a(at^2)$
$y^2 = 4a^2t^2$
$y = 2at$

$SP = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$$SP = \sqrt{a^2t^4 + a^2 - 2a^2t^2 + 4a^2t^2}$$

$$SP = \sqrt{a^2t^4 + a^2 + 2a^2t^2}$$

$$SP = \sqrt{(at^2+a)^2}$$

$$SP = at^2 + a$$

$$SP = a(1+t^2)$$

ROC :- $\rho = \dfrac{[(x')^2 + (y')^2]^{3/2}}{x'y'' - y'x''}$

Diff^y x wrt 't'

$$\boxed{x' = 2at}$$

Diff^y x' wrt 't'

$$\boxed{x'' = 2a}$$

Diff^y y wrt 't'

$$\boxed{y' = 2a}$$

Diff^y of y' wrt t

$$\boxed{y'' = 0}$$

$$\rho = \frac{[4a^2t^2 + 4a^2]^{3/2}}{0 - (4a^2)}$$

$$\rho = \frac{(4a^2)^{3/2}\,[t^2+1]^{3/2}}{-(4a^2)}$$

$$\rho = -(4a^2)^{1/2}\,(1+t^2)^{3/2}$$

$$\rho = -2a\,(1+t^2)^{3/2}$$

sq. both sides,

$$\rho^2 = 4a^2\,(1+t^2)^3$$

$$\rho^2 = 4a^2\left(\frac{SP}{a}\right)^3 \quad (\text{refer } 1)$$

$$\rho^2 = 4a^2\,\frac{(SP)^3}{a^3}$$

$$\rho^2 = \frac{4}{a}(SP)^3$$

$$\rho^2 \propto (SP)^3 \;//$$

26|12|22
Monday

Q 21] $r = a(1-\cos\theta)$ varies as $\sqrt{r}$

7M

$\Rightarrow$ Given:- $r = a(1-\cos\theta)$ ——— (1)

To prove:- $\rho \propto \sqrt{r}$

Formula :- $\rho = \dfrac{(r^2+r_1^2)^{3/2}}{r^2+2r_1^2-rr_2}$ ——— (2)

Differentiate (1) wrt 'θ'

$$\boxed{r_1 = a(\sin\theta)}$$

Differentiate 'r_1' wrt 'θ'

$$\boxed{r_2 = a\cos\theta}$$

Eqn (2) reduces to

$$\rho = \frac{\left[a^2(1-\cos\theta)^2 + a^2\sin^2\theta\right]^{3/2}}{a^2(1-\cos\theta)^2 + 2a^2\sin^2\theta - a^2(1-\cos\theta)\cos\theta}$$

$$\rho = \frac{a^3\left[(1-\cos\theta)^2 + \sin^2\theta\right]^{3/2}}{a^2\left[(1-\cos\theta)^2 + 2\sin^2\theta - (1-\cos\theta)\cos\theta\right]}$$

$$= \frac{a\left((1-\cos\theta)^2 + \sin^2\theta\right)^{3/2}}{\left[(1-\cos\theta)^2 + 2\sin^2\theta - (1-\cos\theta)\cos\theta\right]}$$

$$= \frac{a(1-2\cos\theta+\cos^2\theta+\sin^2\theta)^{3/2}}{(1-2\cos\theta+\cos^2\theta+2\sin^2\theta - \cos\theta + \cos^2\theta)}$$

$$= \frac{a(2-2\cos\theta)^{3/2}}{\left[1-3\cos\theta + 2\sin^2\theta + 2\cos^2\theta\right]}$$

$$= \frac{a\, 2^{3/2}(1-\cos\theta)^{3/2}}{1-3\cos\theta + 2(\sin^2\theta+\cos^2\theta)}$$

$$= \frac{2^{3/2}a\,(1-\cos\theta)^{3/2}}{(3-3\cos\theta)}$$

$$= \frac{2^{3/2}a\,(1-\cos\theta)^{3/2}}{3(1-\cos\theta)}$$

$1-\frac{3}{2}$ $\quad$ $\frac{2-3}{2}$

$$= \frac{2^{3/2} a (1-\cos\theta)^{1/2}}{3}$$

$$\rho = \frac{2^{3/2}}{3} a \sqrt{1-\cos\theta}$$

From (1), $\frac{r}{a} = 1-\cos\theta$

$$\therefore \rho = \frac{2^{3/2}}{3} a \sqrt{\frac{r}{a}}$$

$$\rho = \left(\frac{2^{3/2}}{3}\sqrt{a}\right)\sqrt{r}$$

$\therefore \rho \propto \sqrt{r}$ //

or

$$\rho = \frac{\cancel{a^3}\left[1-2\cos\theta + \cos^2\theta + \sin^2\theta\right]^{3/2}}{\cancel{a^2}\left[(1-\cos)^2 + 2(1+\cos^2\theta) - (1-\cos\theta)\cos\theta\right]}$$

$$\rho = \frac{a\left[2-2\cos\theta\right]^{3/2}}{(1-\cos\theta)^2 + 2(1-\cos\theta)(1+\cos\theta) - (1-\cos\theta)\cos\theta}$$

$$\rho = \frac{2^{3/2} a \left[1-\cos\theta\right]^{3/2}}{(1-\cos\theta)^1\left[1-\cos\theta + 2(1+\cos\theta) - \cos\theta\right]}$$

$$\rho = \frac{2^{3/2} a (1-\cos\theta)^{1/2}}{1-\cancel{\cos\theta} + 2 + 2\cancel{\cos\theta} - \cancel{\cos\theta}}$$

Indeterminate Form

The following are the indeterminate forms:

$\frac{0}{0}$, $\frac{\infty}{\infty}$, $0\times\infty$, $\infty-\infty$, 0^0, 1^∞, ∞^0

L-Hospital's Rule :-

If $f(x)$ & $g(x)$ are two functions such that

(1) $f(a) = g(a) = 0$

(2) $f'(x)$ & $g'(x)$ exists with $g'(x) \neq 0$ then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

2/22 ...day

22 (1) $\lim_{x\to 0} \frac{2\sin x - \sin 2x}{x^3}$

ans = 1

$\Rightarrow$ let $L = \lim_{x\to 0} \frac{2\sin x - \sin(2x)}{x^3}$ $\left(\frac{0}{0}\right)$

Applying L-Hospital's rule.

$$L = \lim_{x\to 0} \frac{2\cos x - 2\cos(2x)}{3x^2} \quad \left(\frac{0}{0}\right)$$

$$L = \lim_{x\to 0} \frac{-2\sin x + 4\sin(2x)}{6x} \quad \left(\frac{0}{0}\right)$$

$$L = \lim_{x \to 0} \frac{-2\cos x + 8\cos(2x)}{6}$$

$$L = \frac{-2+8}{6} = \frac{6}{6} = 1$$

$$\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$$

let $L = \lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right) \quad (\infty - \infty)$

$$L = \lim_{x \to 0} \left(\frac{x - \sin x}{x \sin x} \right) \quad \left(\frac{0}{0} \right)$$

Applying L-Hospital's Rule.

$$L = \lim_{x \to 0} \left(\frac{1 - \cos x}{x\cos x + \sin x} \right) \quad \left(\frac{0}{0} \right)$$

$$L = \lim_{x \to 0} \left(\frac{\sin x}{-x\sin x + \cos x + \cos x} \right)$$

$$L = \frac{0}{0+1+1} = \frac{0}{2} = 0$$

$$\lim_{x \to \infty} \left(\frac{\log x}{x} \right)$$

$$\boxed{\begin{array}{l} \log \infty = \infty \\ \log 0 = -\infty \end{array}}$$

$\Rightarrow$ let $L = \lim_{x \to \infty} \left(\frac{\log x}{x} \right) \quad \left(\frac{\infty}{\infty} \right)$

Applying L-Hospitals rule.

$$L = \lim_{x \to \infty} \left(\frac{1/x}{1} \right)$$

$$L = \frac{1}{\infty} = 0$$

Ex: $\lim_{x \to \infty} \left(\frac{e^x}{x^2} \right)$

$\Rightarrow$ Let $L = \lim_{x \to \infty} \left(\frac{e^x}{x^2} \right) \left(\frac{\infty}{\infty} \right)$

Applying L-Hospitals rule

$$L = \lim_{x \to \infty} \left(\frac{e^x}{2x} \right) \left(\frac{\infty}{\infty} \right)$$

$$L = \lim_{x \to \infty} \left(\frac{e^x}{2} \right)$$

$$L = \frac{e^\infty}{2}$$

$$L = \frac{\infty}{2}$$

$$\boxed{L = \infty}$$

(iii) $\lim_{x\to 0} (\tan x)^{\tan 2x}$

$\Rightarrow$ Let $L = \lim_{x\to 0} (\tan x)^{\tan 2x}$ $[0^0]$

Applying log on both sides.

$$\log_e L = \log \lim_{x\to 0} (\tan x)^{\tan 2x}$$

$$\log_e L = \lim_{x\to 0} \log_e (\tan x)^{\tan 2x}$$

$$\log_e L = \lim_{x\to 0} \tan(2x) \log(\tan x) \quad [0 \times \infty]$$

$$\log_e L = \lim_{x\to 0} \frac{\log(\tan x)}{\cot(2x)} \quad \left(\frac{\infty}{\infty}\right)$$

Applying L-Hospitali rule.

$$\log_e L = \lim_{x\to 0} \left(\frac{\frac{1}{\tan x} \cdot \sec^2 x}{-2\operatorname{cosec}^2(2x)} \right)$$

$$\log_e L = \lim_{x\to 0} \left(\frac{\frac{\cos x}{\sin x} \cdot \frac{1}{\cos^2 x}}{\frac{-2}{\sin^2(2x)}} \right)$$

$$\log_e L = \lim_{x\to 0} \frac{\sin^2(2x)}{-\sin(2x)}$$

$$\log_e L = \lim_{x\to 0} [-\sin(2x)]$$

$$\log_e L = 0$$

$$\Rightarrow \boxed{L = e^0 = 1}$$

(iv) $\lim_{x\to 0} (\cot x)^{\frac{1}{\log x}}$

$$\Rightarrow L = \lim_{x\to 0} (\cot x)^{\frac{1}{\log x}} \quad [\infty^0]$$

Applying log on both sides

$$\log_e L = \lim_{x\to 0} \log (\cot x)^{1/\log x}$$

$$= \lim_{x\to 0} \frac{1}{\log x} \log(\cot x) \quad \left[\frac{\infty}{\infty}\right]$$

Applying L-Hospital's Rule ;

$$= \lim_{x\to 0} \frac{\frac{-1}{\cot x} \cdot \operatorname{cosec}^2 x}{\left(\frac{1}{x}\right)}$$

$$= \lim_{x\to 0} \frac{\left(\frac{-\sin x}{\cos x} \cdot \frac{1}{\sin^2 x}\right)}{\left(\frac{1}{x}\right)}$$

$$= \lim_{x\to 0} \frac{\left(\frac{-1}{\cos x} \cdot \frac{1}{\sin x}\right)}{\left(\frac{1}{x}\right)}$$

$$\Rightarrow \lim_{x\to 0} \left(\frac{-1}{\cos x}\right) \cdot \left(\frac{x}{\sin x}\right)$$

$$\left(\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1\right)$$

$$\Rightarrow \quad = \frac{-1}{1} \times 1$$

$$\log_e L = -1$$

$$\boxed{L = e^{-1}}$$

(V) $\lim_{x\to 0} \left[\frac{\sin x}{x}\right]^{1/x^2}$

$\Rightarrow$ Let $L = \lim_{x\to 0} \left(\frac{\sin x}{x}\right)^{1/x^2}$ (1^{∞}) $\left(\lim_{\theta\to 0} \left(\frac{\sin\theta}{\theta}\right) = 1\right)$

$\Rightarrow$ Applying Log on both sides

$$\log_e L = \lim_{x\to 0} \log \left(\frac{\sin x}{x}\right)^{1/x^2}$$

$$\log_e L = \lim_{x\to 0} \frac{1}{x^2} \log \left(\frac{\sin x}{x}\right)$$

$$= \lim_{x\to 0} \frac{\cdots\left(\frac{\cdots}{x}\right)}{x^2} \quad \left[\frac{0}{0}\right]$$

Applying L-Hospital's rule, using Quotient rule

$$= \lim_{x\to 0} \frac{\left(\frac{x}{\sin x}\right)\left\{\frac{x\cos x - \sin x}{x^2}\right\}}{2x}$$

$$= \lim_{x\to 0} \frac{\left(\frac{x\cos x - \sin x}{x \cdot \sin x}\right)}{2x}$$

$$= \lim_{x\to 0} \frac{x\cos x - \sin x}{2x^2 \cdot \sin x}$$ (devide & multiply with x.)

$$= \lim_{x\to 0} \frac{x\,(x\cos x - \sin x)}{2x^3 \sin x}$$

$$= \lim_{x\to 0} \left(\frac{x}{\sin x}\right)\left(\frac{x\cos x - \sin x}{2x^3}\right)$$

$$= (1) \lim_{x\to 0} \left(\frac{x\cos x - \sin x}{2x^3}\right)$$

$$= \frac{1}{2} \lim_{x\to 0} \left(\frac{x\cos x - \sin x}{x^3}\right)$$

$$= \frac{1}{2} \lim_{x\to 0} \left(\frac{-x\sin x + \cancel{\cos x} - \cancel{\cos x}}{3x^2}\right)$$

$$= \frac{1}{2} \lim_{x\to 0} \left(\frac{-\cancel{x}\sin x}{\cdots}\right)$$

$$= \frac{-1}{6} \lim_{x \to 0} \frac{\sin x}{x}$$

$$= \frac{-1}{6}$$

$$\boxed{L = e^{-1/6}}$$

Ex: $\lim_{x \to 0} \left(\frac{1}{x}\right)^{\sin x}$

$\Rightarrow$ Let, $L = \lim_{x \to 0} \left(\frac{1}{x}\right)^{\sin x}$

30/12/22
Friday

Ex: Find the constants a & b if,

$$\lim_{x\to 0} \frac{x(1-a\cos x)+b\sin x}{x^3} = \frac{1}{3}$$

$\Rightarrow$ Let limit $= L = \frac{1}{3}$

$$\therefore \lim_{x\to 0} \frac{x(1-a\cos x)+b\sin x}{x^3} = L \quad \left[\frac{0}{0}\right]$$

Applying L-Hospital's rule,

$$\lim_{x\to 0} \frac{x(+a\sin x)+(1-a\cos x)+b\cos x}{3x^2} = L$$

$$\left[\frac{1-a+b}{0}\right]$$

In order to apply L-Hospital's rule we must have $1-a+b=0 \Rightarrow \boxed{a-b=1}$ ← (1)

$$\therefore \lim_{x\to 0} \frac{a\sin x + ax\cos x + a\sin x - b\sin x}{6x} = L$$

$$\lim_{x\to 0} \frac{ax\cos x + (2a-b)\sin x}{6x} = L \quad \left[\frac{0}{0}\right]$$

$$\lim_{x\to 0} \frac{-ax\sin x + a\cos x + (2a-b)\cos x}{6} = L$$

$$\Rightarrow \frac{a+(2a-b)}{6} = L$$

$$\Rightarrow \frac{3a-b}{\cancel{6}\,2} = \frac{1}{\ }$$

$\Rightarrow$ 3a

solving (1) & (2) we get $\boxed{a = \frac{1}{2}}$, $\boxed{b = \frac{-1}{2}}$

Eg:- Find the constant 'a' if,

$$\lim_{x \to 0} \frac{\sin 2x + a \sin x}{x^3} = -1$$

$\Rightarrow$ Let limit $L = -1$,

$$\therefore \lim_{x \to 0} \frac{\sin 2x + a \sin x}{x^3} = L \quad \left[\frac{0}{0}\right]$$

Applying L-Hospital's Rule,

$$\lim_{x \to 0} \frac{2\cos 2x + a\cos x}{3x^2} = -1 \quad \left[\frac{2+a}{0}\right]$$

In order to apply L-Hospital's rule we must have $2 + a = 0$, $\Rightarrow \boxed{a = -2}$

Eg:- $\lim_{x \to 0} \left(\frac{a^x + b^x + c^x}{3}\right)^{1/x}$

$\Rightarrow L = \lim_{x \to 0} \left(\frac{a^x + b^x + c^x}{3}\right)^{1/x} \quad \left[1^\infty\right]$

Applying $\log_e$

$$\log_e L = \lim_{x \to 0} \log\left(\frac{a^x + b^x + c^x}{3}\right)^{1/x}$$

$$\therefore \lim_{x\to 0} \frac{1}{x} \log\left(\frac{a^x+b^x+c^x}{3}\right)$$

$$\Rightarrow \lim_{x\to 0} \frac{\log\left(\frac{a^x+b^x+c^2}{3}\right)}{x}$$

$$\Rightarrow \lim_{x\to 0} \frac{\log(a^x+b^x+c^x)-\log 3}{x} \quad \left[\frac{0}{0}\right]$$

Applying L-Hospital's rule,

$$\Rightarrow \lim_{x\to 0} \frac{1}{a^x+b^x+c^x}\left[a^x\log a + b^x \log b + c^x \log c\right]$$

$$\log_e L \Rightarrow \frac{1}{3}(\log a + \log b + \log c)$$

$$\log_e L = \frac{1}{3}\log(abc)$$

$$\Rightarrow \log_e L = \log(abc)^{1/3}$$

$$\Rightarrow \boxed{L = (abc)^{1/3}}$$

classmate

Model Q.P (page 21 of 29)

Q2] b] Let r & h be the radius & height respectively of the cylindrical can.

Given :- Volume $= V = 1L$

$\Rightarrow V = 1000 \ cm^3$

$\Rightarrow \boxed{\pi r^2 h = 1000}$ — (1)

In order to minimize the cost we minimize the surface area of the cylinder.

Surface area :-

$A = 2\pi r^2 + 2\pi r h$

from (1), $\boxed{h = \frac{1000}{\pi r^2}}$

$\therefore A = 2\pi r^2 + 2\cancel{\pi}\cancel{r}\left(\frac{1000}{\cancel{\pi} r^{\cancel{2}}}\right)$

$\boxed{A = 2\pi r^2 + \frac{2000}{r}}$ — (2)

where $0 < r < \infty$

diff (2) wrt (r),

$\boxed{A' = 4\pi r - \frac{2000}{r^2}}$

CN :- $A' = 0$

$$4\pi r - \frac{2000}{r^2} = 0$$

$$4\pi r = \frac{2000}{r^2}$$

$$r^3 = \frac{2000}{4\pi}$$

$$r^3 = \frac{500}{\pi}$$

$$r = \left(\frac{500}{\pi}\right)^{1/3}$$

$$\boxed{r = 5.4190}$$

Intervals	Sign of A'
$r < 5.4190$	-ve
$r > 5.4190$	+ve

Since A' changes sign from -ve to the +ve at $r = 5.4190$

$\therefore$ Surface area $= A$ is minimum at r.

$r = 5.4190$ cm & $h = \frac{1000}{\pi r^2} = 10.8396$ cm

2/1/23 Monday

Bisection method

Q) $\sin x = \frac{1}{x}$.

$\Rightarrow \sin x = \frac{1}{x}$

$x \sin x = 1$

$x \sin x - 1 = 0$

$f(x) = x \sin x - 1$

$f(1) = (1)\sin(1) - 1 = -0.1585$

$f(2) = 2 \sin 2 - 1 = 0.8186$

$\ominus$ a	$c = a+b/2$	$\oplus$ b	f (c)
1	1.5	2	+0.4962
1	1.25	1.5	+0.1862
1	1.1250	1.25	+0.451
1	1.0625	1.125	−0.0718
1.0625	1.0938	1.125	−0.0283

∴ an aproximate root is $x = 1.0938$

NR-Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Q) $10^x + x - 4 = 0$

$\Rightarrow f(x) = 10^x + x - 4$

$f'(x) = 10^x \log_e 10 + 1$
$= (2.3026) 10^x + 1$

$f(0) = 10^0 + 0 - 4 = -3$

$f(1) = 10 + 1 - 4 = 7$

By IVT 0 & 1

$x_0 = 0$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0 - \frac{(-3)}{3.3026}$$

$x_1 = 0.9084$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.9084 - \frac{f(0.9084)}{f'(0.9084)}$$

$$= 0.9084 - \frac{(5.0068)}{19.6474}$$

$x_2 = 0.6536$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.6536 - \frac{f(0.6536)}{f'(0.6536)}$$

$= 0.6536 -$

$x_3 = 0.5518$

$x_4 =$

$x_4 = 0.5393$

$x_5 = 0.5392$

$x_6 = 0.5392$

app root is 0.5392

(13) (i) $f(x) = x^4 - 4x^3 + 10$

imp

CN :- $f' = 4x^3 - 12x^2$

$f' = 0 \qquad 4x^3 - 12x^2 = 0$

$4x^2(x-3) = 0$

$x = 0, \quad x = 3.$

intervals	sign of f'	Nature f
$(-\infty, 0)$ (-1)	$-$	↓
$(0, 3)$	$-$	↓
$(3, \infty)$	$+$	↑

(b) local min is at $x = 3$.

$f(3) = -17$

(c) $f'' = 12x^2 - 24x.$

$f'' = 0 \Rightarrow 0, 2$ are IP's

intervals	sign of f''	concavity
$(-\infty, 0)$	$+$	CU
$(0, 2)$	$-$	CD
$(2, \infty)$	$+$	CU

Ex: $f'(x) > 0 \quad \forall\ x \neq 1$

IMP VA: $x = 1$

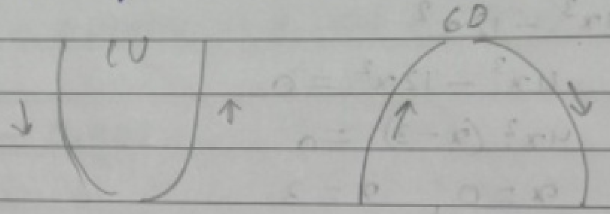
IMP $f''(x) > 0$ if $x < 1$ or $x > 3$

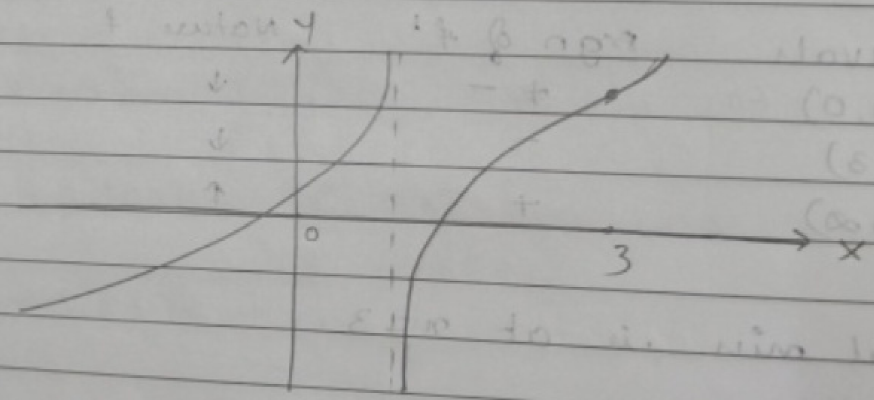
$f''(x) < 0$ if $1 < x < 3$.

$\Rightarrow f' > 0,\ \forall\ x \neq 1 \Rightarrow f$ is $\uparrow$ $(-\infty, 1) \cup (1, \infty)$

$f'' > 0$, if $x < 1$ or $x > 3 \Rightarrow f$ CU $(-\infty, 1) \cup (3, \infty)$

$f'' < 0$ if $1 < x < 3 \Rightarrow f$ is CD $(1, 3)$





Radius of Curvature:-