Unit - I Chapter : 1

MATHEMATICAL MODELING

Mathematical Model : There are many ways in which models can be described

- Words
- Drawing or sketches
- Physical Models
- Computer Programs
- Mathematical model.

Mathematical model is representation of the behaviour of real world objects and phenomena in the language of mathematics

Mathematical Modeling : is the process that uses mathematics

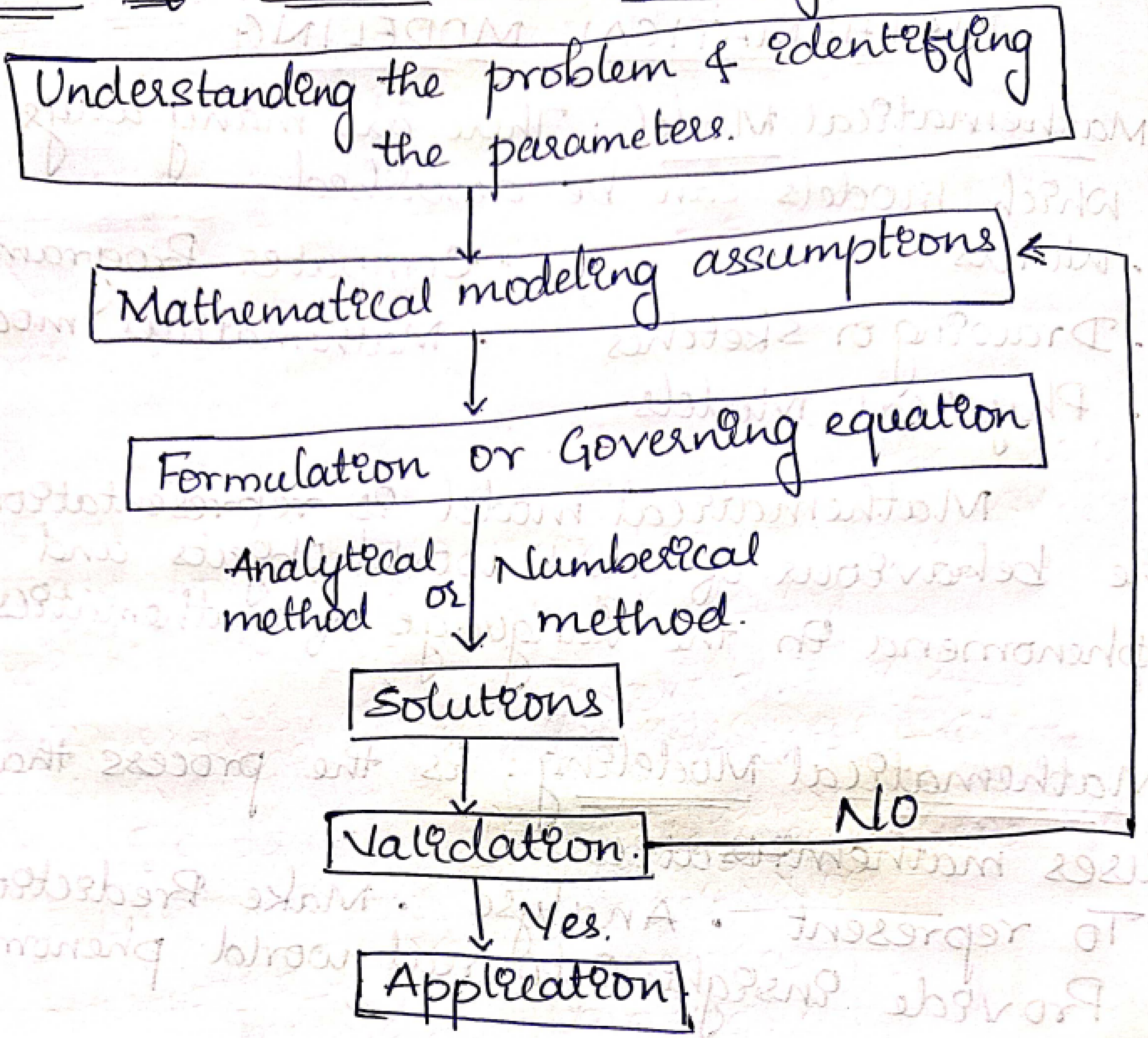
- To represent
- Analyse
- Make Prediction
- Provide insight into real world phenomen

Types of Mathematical Model

1. **Theoretical models** : Theoretical models are models that are based on first principles & physical laws. can be extrapolated to a wide variety of situations.
2. **Empirical models** : Empirical models are models based on experimental data. These models are applicable in the areas with identical conditions as those in which relationship was formulated.

Process of Mathematical Modeling (7m)

1).



→ The first step towards mathematical modeling is about understanding the problem & identify-ing the parameters. (In this step we analyse the problem and see which parameters have major influence on the solution to the problem

→ The next step is to construct the basic frame-work of the model by making certain assumptions (In this step we state those parameters which are not essential and can be neglected)

→ If the assumptions are sufficiently precise they may lead directly to the formulation or governing equations " In some cases the formulation itself is the solution "

If the formulation is not the solution then we apply analytical method to solve the equation when analytical methods are unproductable we can use numberical methods ~~are~~ to optain solution.

→ After obtaining the solution we start testin the validity of the model by comparing the theoretical and practical result the If the model is valid then we move towards applicati If not we recheck our assumptions and repeat the steps until we get a valid model.

2> Build a mathematical model for velocity prior to opening the parachutist, when a parachutist of mass m kg jumps out & a stationary hot air balloon where drags coefficient is c kg/s

Solⁿ : Understanding the Problem : Find the velocity prior to opening of the parachutes

Identifying parameters : Forces acting on the body and mass.

Assumptions: No Horizontal force is acting on the body and mass of the parachute is negligible.

Formulation or governing equation: By Newton's Second law, $F = ma$

where, F = net force acting on the body
m = mass of the object (kg)
a = its acceleration (m/s^2)

Since two forces are acting on the body, upward force (F_u) and downward force (F_D)

$\therefore F = F_u + F_D$ — ①

F_D : is due to gravity $= mg$

F_u : is due to air resistance $= -cv$ in opposite direction

c drag co-efficient

Eqn ① reduces to.

$$F = -cv + mg$$

$$\therefore ma = -cv + mg$$

$$\Rightarrow m\frac{dv}{dt} = mg - cv \quad \because a = \frac{dv}{dt}$$

$$\therefore \frac{dv}{dt} = g - \left(\frac{c}{m}\right)v$$

$$\boxed{\frac{dv}{dt} + \left(\frac{c}{m}\right)v = g}$$

Note:

$$\frac{dy}{dx} + Py = Q$$

$$I.F = e^{\int P dx}$$

Soln of DE

$$y(I.F) = \int Q(I.F)dx + K$$

$I.F = e^{\int \frac{c}{m} dt} = e^{(c/m)t}$

sol^n by Analytical method is.

$$v(IF) = \int g(IF)\,dt + K$$

$$v\,e^{(c/m)t} = \int g\,e^{(c/m)t}\,dt + K.$$

$$v\,e^{(c/m)t} = g\int e^{(c/m)t}\,dt + K$$

$$v\,e^{(c/m)t} = g\,\frac{e^{(c/m)t}}{c/m} + K \quad \Big| \because \int e^{ax}dx = e^{ax}/a + c$$

$$v\,e^{(c/m)t} = \left(\frac{mg}{c}\right)e^{(c/m)t} + K \quad -② $$

we have $t=0$, $v=0$

$$0 = \left(\frac{mg}{c}\right) + K \Rightarrow \boxed{K = -\frac{mg}{c}}$$

Eq^n ② reduces to

$$v\,e^{(c/m)t} = \left(\frac{mg}{c}\right)e^{(c/m)t} - \frac{mg}{c}$$

$$v\,e^{(c/m)t} = \frac{mg}{c}\left[e^{(c/m)t} - 1\right]$$

$$\boxed{v = \frac{mg}{c}\left[1 - e^{-(c/m)t}\right]}$$

end

<u>Validation</u> : $m = 68.1\,kg$, $c = 12.5\,kg/s$, $g = 9.8\,m/s^2$; $v(t) = \frac{gm}{c}(1 - e^{-ct/m})$

$\therefore v(t) = 53.39(1 - e^{-0.18355t})$

3. Construct a mathematical model for the CO_2 level using the data given in the table where table lists the average CO_2 level in atomsphere, measured in ppm (parts per million) at Mauna Lao observatory from 1980 - 2008.

Year	CO_2 level (in ppm)	Year	CO_2 level (in ppm)
1980	338.7	1996	362.4
1982	341.2	1998	366.5
1984	344.4	2000	369.4
1986	347.2	2002	373.2
1988	351.5	2004	377.5
1990	354.2	2006	381.9
1992	356.3	2008	385.6
1994	358.6		

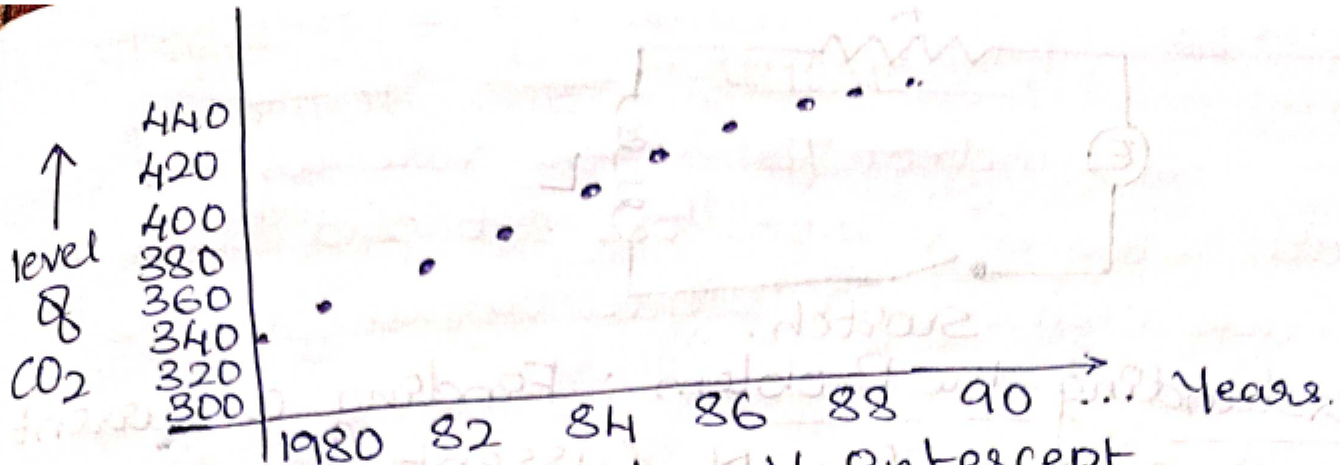
Solⁿ : Understanding the Problems : We have to find level of CO_2 from the given data

Parameters : Years & level of CO_2

Assumptions : No assumptions in case of empirical model.

Formulation or Governing eqⁿ : As the data points appears to lie close to a straight line, we choose a linear model.

$CO_2 = a(\text{year}) + b$ — ① [we have $y = mx + c$]



where a = slope, b = y-intercept

From the table, $a = \dfrac{385.6 - 338.7}{2008 - 1980}$

$$\boxed{a = 1.675}$$

Eq^n ① reduces to

$CO_2 = 1.675\,(\text{year}) + b$ — ②

Put year $= 1980$ and $CO_2 = 338.7$ in eq^n ②

$338.7 = 1.675\,(1980) + b$

$$\boxed{b = -2977.8}$$

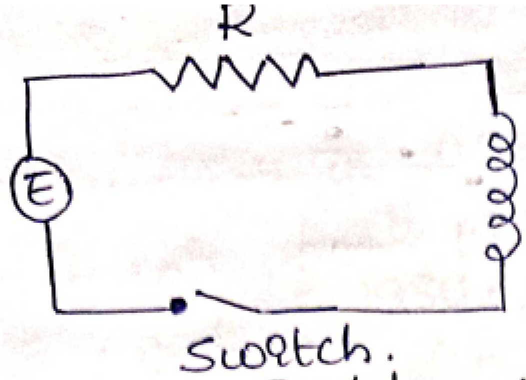
Eq^n ② reduces to

$$\boxed{CO_2 = 1.675\,(\text{year}) - 2977.8}$$

is model for level of CO_2

4. Build a mathematical model for the current I in a circuit where the resistance is R ohm the inductance is L. Henry, a battery gives a constant voltage of E voltags, and the switch is turned on when $t=0$, what is the limiting value of the current ?

Solⁿ



Understanding the Problem : Finding a current and limiting value of current.

Parameters : Resistance (R), Inductance (L)

Assumptions : R and L are connected in series

Formulation or Governing eqⁿ :

Ohm's ~~eqⁿ~~ Law : $V_R = RI$, $V_L = L\frac{dI}{dt}$

Kirchhoff's Law : $V_R + V_L = E$

$$RI + L\frac{dI}{dt} = E$$

$$L\frac{dI}{dt} + RI = E \quad (1)$$

$$\frac{dI}{dt} + \left(\frac{R}{L}\right) I = \frac{E}{L} \quad (\because \text{Dividing by } L)$$

$$I.F = e^{\int \frac{R}{L} dt} = e^{(R/L)t}$$

Solⁿ by Analytical method is

$$I(IF) = \int \frac{E}{L}(IF)\,dt + K \quad \Big| \; y(IF) = \int Q(IF)\,dx + k$$

$$I e^{(R/L)t} = \int \frac{E}{L}\left(e^{(R/L)t}\right) dt + K \quad (2)$$

$$I\, e^{(R/L)t} = \frac{E}{L}\int e^{(R/L)t}\, dt + K$$

$$I\, e^{(R/L)t} = \frac{E}{\cancel{L}} \cdot \frac{e^{(R/L)t}}{R/\cancel{L}} + K$$

$$I\, e^{(R/L)t} = \frac{E\, e^{(R/L)t}}{R} + K$$

$$\boxed{I = \frac{E}{R} + K e^{-(R/L)t}} \quad - (3)$$

we have, $t = 0$, $I = 0$.

$$\Rightarrow 0 = \frac{E}{R} + K \Rightarrow \boxed{K = -\frac{E}{R}}$$

Eq^n (2) reduce to

$$I\, e^{(R/L)t} = \left(\frac{E}{R}\right) e^{(R/L)t} - \frac{E}{R}.$$

$$I\, e^{(R/L)t} = \frac{E}{R}\left[e^{(R/L)t} - 1\right]$$

$$I = \frac{E}{R}\left[1 - e^{-(R/L)t}\right] - (4)$$

In (4), apply limit as $t \to \infty$

$$\lim_{t\to\infty} I = \lim_{t\to\infty} \frac{E}{R}\left[1 - e^{-(R/L)t}\right]$$

$$\boxed{I = \frac{E}{R}} \qquad \because\ e^{-\infty} = e^{1/\infty} = \frac{1}{\infty} = 0$$

//