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# ENGINEERING MECHANICS

UNIT-II Chapter – 4

COPLANAR NON-CONCURRENT FORCE SYSTEM(EQUILIBRIUM)

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## **Chapter Content:**

- Conditions of Equilibrium, Types of supports and reactions for 2D structures, statically determinate beam, loads acting on the beam.
- Determination of support reactions and Numerical problems on equilibrium of coplanar non-concurrent force system for unknown forces.

### ● **Conditions of equilibrium:**

When several coplanar non-current forces acts on a body, the body may attain different states. They are as follows:

1. The body may move in any direction.
2. Body may move in any direction + rotate also.
3. Body may rotate about itself without motion.
4. Body may be at rest.

According to the 1<sup>st</sup> state, to achieve equilibrium the resultant R should be equal to zero. So the resultant will be zero when,

$$\sum F_x = 0, \sum F_y = 0$$

According to the 2<sup>nd</sup> state, to achieve equilibrium the resultant R should be equal to zero and moment should also be equal to zero.

$$\bullet \quad \sum F_x = 0, \sum F_y = 0 \text{ and } \sum M = 0$$

According to the 3<sup>rd</sup> state, to achieve equilibrium moment should be equal to zero.

$$\sum M = 0$$

According to the 4<sup>th</sup> state, the body is already in equilibrium because, the resultant R is equal to zero and moment also is equal to zero. Therefore the following conditions are satisfied.

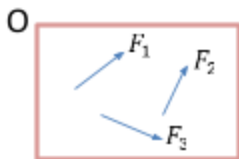
$$\sum F_x = 0, \sum F_y = 0 \text{ and } \sum M = 0$$

To summarise all the 4 states we can say that, to attain the equilibrium of the body:

$$\sum F_x = 0, \sum F_y = 0 \text{ and } \sum M = 0$$

## Law of moments:

If several coplanar forces act on a system which is in equilibrium, then according to the law of moments “the sum of clockwise moments is equal to the sum of anticlockwise moments” or “the algebraic sum of moments of all the forces about any point on the plane of the forces should be zero.”



Sum of clockwise moments = Sum of anticlockwise moments

or

$$\sum M_o = F_1 \times d_1 + F_2 \times d_2 + F_3 \times d_3 = 0$$

## Types of supports and their reactions:

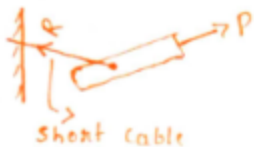
### 1. Reactions equivalent to a force with known line of action.

- Roller.
- Rocker.
- Smooth surface.



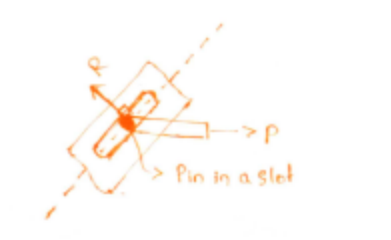
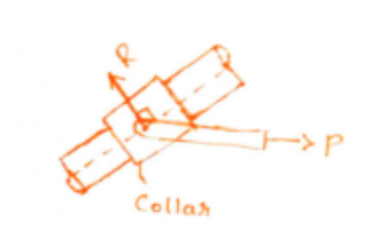
In all the above supports the movement of the body is restrained in only vertical direction therefore there will be only one reaction.

- Short cable.
- Short link.



In all the above supports the movement of the body is restrained in only one direction therefore there will be only one reaction along the cable or link.

- Collar on frictionless a rod.
- Frictionless pin in a slot.

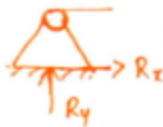
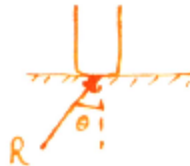


In all the above supports the movement of the body is restrained in only one direction therefore there will be only one reaction normal to the axis of the rod..



## 2. Reactions equivalent to a force of unknown direction.

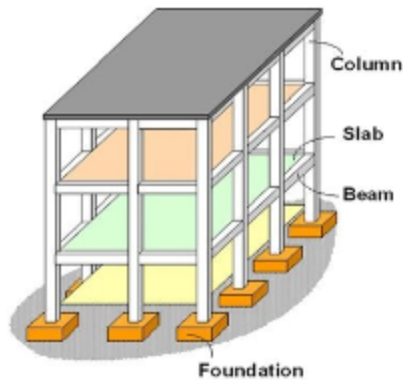
- Rough surface.
- Hinges.
- Frictionless pins in a fitted hole.



In all the above supports the movement of the body is restrained in two directions therefore there will be two reactions.

### 3. Reactions equivalent to a force of unknown direction and a couple.

- Fixed supports.



In the above supports the movement of the body is restrained in three directions therefore there will be three reactions.

#### Sign conventions:

All the upward reactions are positive and downward reactions are negative.

All the right-side going reactions are positive and left side going reactions are negative.

All the clockwise moments are positive and anticlockwise moments are negative.

**Statically determinate beam:**

It is the beam which can be solved for the unknown reactions completely using only three equilibrium equations.

$$\sum F_x = 0, \sum F_y = 0 \text{ and } \sum M = 0$$

Ex: simply supported beam.

Note: Statically indeterminate beams require additional equations along with three equilibrium equations to get all the unknown reactions.

## Types of beams:

Based on the support conditions we have following types of beams.

### 1. Simply supported beam:

The beam which has simple supports at either end is known as simply supported beam.

Ex:



## 2. Roller supported beam:

The beam which has roller supports at either end is known as roller supported beam.

Ex:



## 3. Hinged/pinned supported beam:

The beam which has hinged or pinned supports at either end is known as hinged/pinned supported beam.

Ex:



#### 4. Overhanging beam

The beam in which certain length is extended beyond the supports is called as overhanging beam.

Ex:

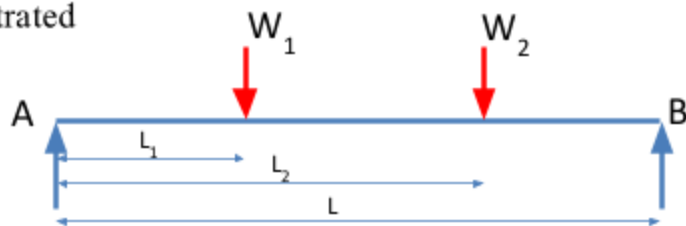


## Types of loads:

### 1. Point load/concentrated load:

If the area occupied by the load is very less compared to the total area on which the load is applied is called as point load/concentrated

Ex:



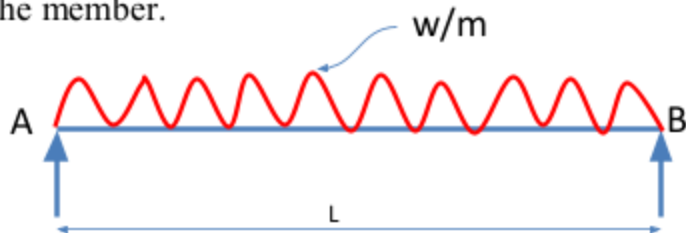
Here  $W_1$  and  $W_2$  are two point loads acting on the beam of length ' $L$ '. The unit of point load may be N or kN.

Sometimes point load can be inclined also. Finding moment of point loads about any point.

Considering  $A$  as reference point,  $M_A = W_1 \times L_1 + W_2 \times L_2$

## 2. Uniformly distributed load (UDL):

The load which is spread over a member in such a way that, the intensity of this load remains constant over each unit length of the member.



Consider the above figure in which a simply supported beam is loaded with UDL over its entire length 'L'.

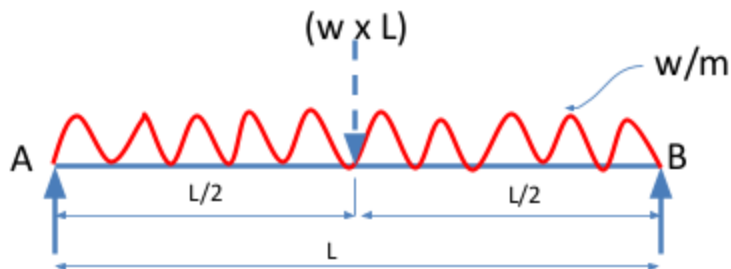
For UDL two things are very important:

1. Intensity of load = 'w' per unit length either in N/m or kN/m (depending on the unit of force)
2. Span of the load = l in 'm'

For the above figure span of the UDL is equal to span of the beam

$$\text{i.e. } l = L$$





To find out moment of UDL about any point its very important to find its equivalent concentrated load (ECL).

Equivalent concentrated load of UDL can be found by multiplying intensity of load with **span of the load**.

Equivalent concentrated load (ECL) =  $w \times L$  (here  $l = L$ )

This ECL acts at the midpoint of **span of the UDL ( $L/2$ )**.

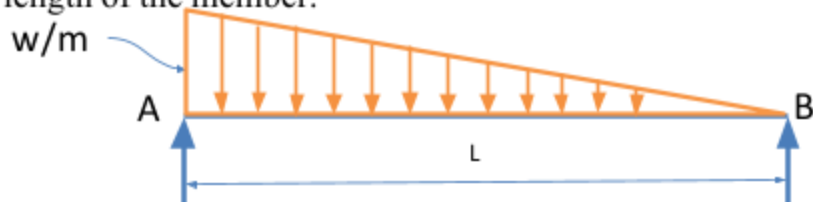
**To calculate moment of UDL about any point:**

$$M_A = (wXL)XL/2$$

$$M_B = -(wXL)XL/2$$

### 3. Uniformly varying load (UVL): Triangular variation UVL

The load which is spread over a member in such a way that, the intensity of this load varies uniformly over each unit length of the member.



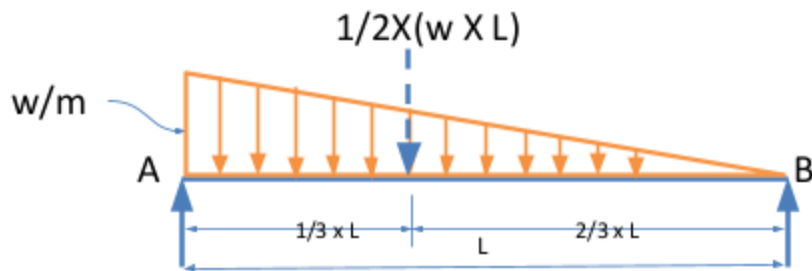
Consider the above figure in which a simply supported beam is loaded with UVL over its entire length 'L'.

For UVL two things are very important:

1. Intensity of load = 'w' per unit length either in N/m or kN/m (depending on the unit of force)
2. Span of the load =  $l$

For the above figure span of the UVL is equal to span of the beam

$$\text{i.e. } l = L$$



To find out moment of UVL about any point its very important to find its equivalent concentrated load(ECL).

ECL of UVL can be found by applying area of triangle,  $(1/2 \times \text{intensity of UVL} \times \text{span of the UVL})$ .

Equivalent concentrated load(ECL) =  $\frac{1}{2} \times w \times L$  (here  $l = L$ )

This equivalent point load acts at the centroid of the area of the triangle.

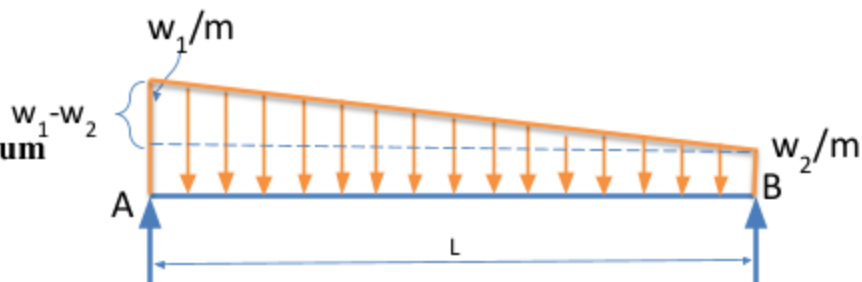
Here the area of UVL resembles the shape of the triangle. Hence its centroidal distances are  $\frac{1}{3} \times L$  and  $\frac{2}{3} \times L$  from base and apex respectively.

**To calculate moment of UVL about any point:**

$$M_A = \frac{1}{2} (w \times L) \times \frac{1}{3} \times L$$

$$M_B = -\frac{1}{2} (w \times L) \times \frac{2}{3} \times L$$

● UVL can also be in the shape of trapezium



Consider the above figure in which a simply supported beam is loaded with UVL over its entire length 'L'.

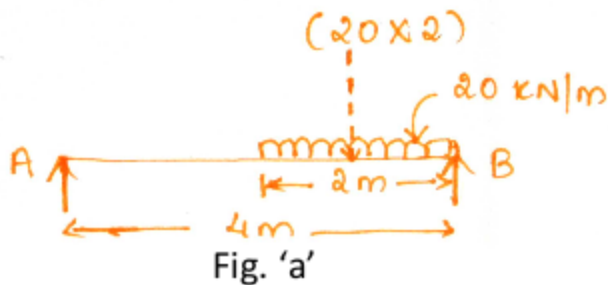
We can solve the above problem conveniently by splitting the UVL into triangular load of intensity  $(w_1 - w_2)/m$  and UDL of intensity  $w_2/m$ .

**To calculate moment of UVL about any point:**

$$M_A = \frac{1}{2}(w_1 - w_2)XL \times \frac{1}{3}XL + (w_2XL) \times \frac{L}{2}$$

$$M_B = -\frac{1}{2}(w_1 - w_2)XL \times \frac{2}{3}XL - (w_2XL) \times \frac{L}{2}$$

Some examples on various loadings:



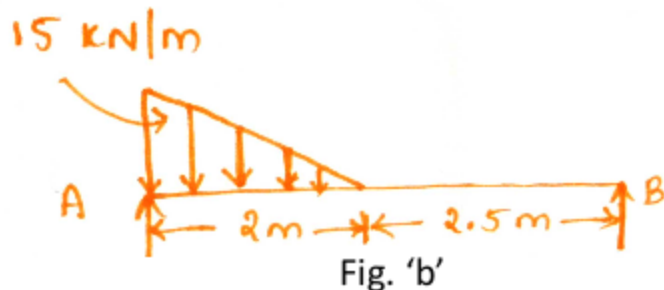
To find  $M_A$  and  $M_B$ :

For the beam given in Fig. 'a'  $M_A = (20 \times 2) \times \left(2 + \frac{2}{2}\right) = 120 \text{ kNm}$

$$M_B = -(20 \times 2) \times \left(\frac{2}{2}\right) = -40 \text{ kNm}$$

For the beam given in Fig. 'b'  $M_A = \left(\frac{1}{2} \times 15 \times 2\right) \times \left(\frac{1}{3} \times 2\right) = 10 \text{ kNm}$

$$M_B = -\left(\frac{1}{2} \times 15 \times 2\right) \times \left(\frac{2}{3} \times 2 + 2.5\right) = -57.5 \text{ kNm}$$



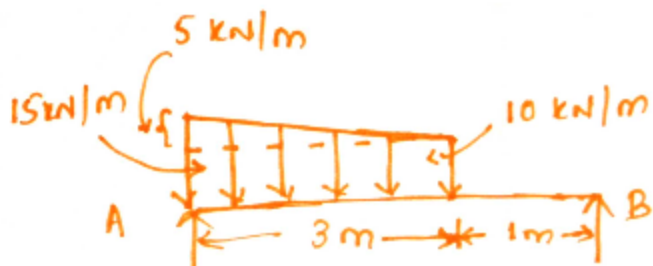


Fig. 'c'

**To find  $M_A$  and  $M_B$ :**

For the beam given in Fig. 'c'  $M_A = (10 \times 3) \times \left(\frac{3}{2}\right) + \left(\frac{1}{2} \times 5 \times 3\right) \times \left(\frac{1}{3} \times 3\right) = 52.5 \text{ kNm}$

$M_B = -(10 \times 3) \times \left(\frac{3}{2} + 1\right) - \left(\frac{1}{2} \times 5 \times 3\right) \times \left(\frac{2}{3} \times 3 + 1\right) = -97.5 \text{ kNm}$

**Problem 1:** A fixed crane has a mass of 1000 kg and is used to lift a crate of mass  $m = 2400$  kg. It is held in place by a pin at A and rocker at B. The CG of the crane is located at G. determine the components of the reactions at A & B. Take  $AB = 1.5$  m,  $x_1 = 2$  m,  $x_2 = 4$  m.

**Solution:** weight of crane:  $1000 \times 9.81$   
 $= 9.81$  kN

Weight of crate  $= 2400 \times 9.81$   
 $= 23.54$  kN

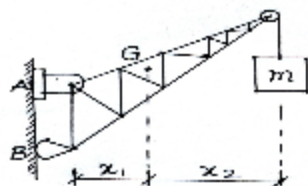
Let the reactions at 'A' & 'B' are as shown in FBD.

Using  $\sum M = 0$  & taking moment about 'A',

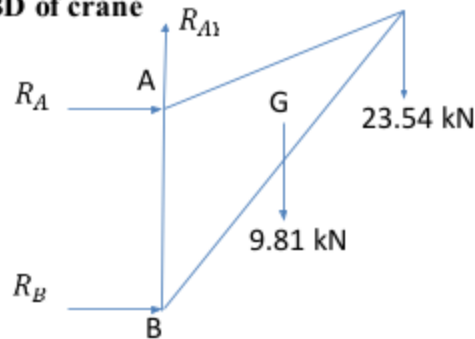
$$\sum M_A = 23.54 \times 6 + 9.81 \times 2 - R_{BX} \times 1.5 = 0$$

$$R_{BX} = \frac{160.884}{1.5} = 107.26 \text{ kN}$$

Using  $\sum F_x = 0$



**FBD of crane**





●

1

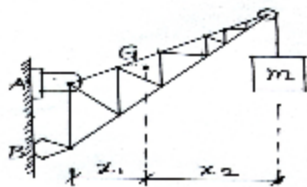
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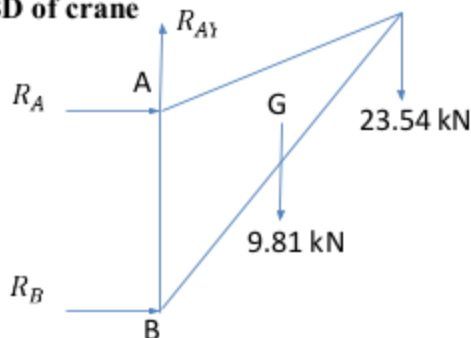
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### FBD of crane





**Problem 2:** Two links AB & DE are connected by a bell crank as shown in figure. Knowing that the tension in the link AB is 180 N, determine (i) tension in the link DE and (ii) the reaction at C.  $\theta_1 = 90^\circ$ , and  $x_1 = 40$  cm,  $x_2 = 60$  cm,  $y_1 = 30$  cm, &  $y_2 = 45$  cm

**Solution:** The forces and the reaction at different points are as shown in FBD.

Consider  $\sum M_c = 0$

From triangle BCN, using Pythagoras theorem,

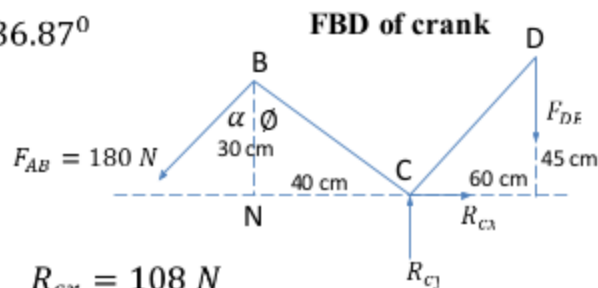
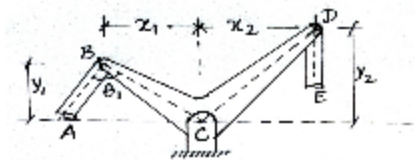
$$BC = 50 \text{ cm.} \quad \cos \phi = \frac{30}{50} \text{ or } \phi = 53.13^\circ \text{ \& } \alpha = 36.87^\circ$$

$$\sum M_c = -180 \times 50 + F_{DE} \times 60 = 0$$

$$F_{DE} = \frac{180 \times 50}{60} = 150 \text{ N}$$

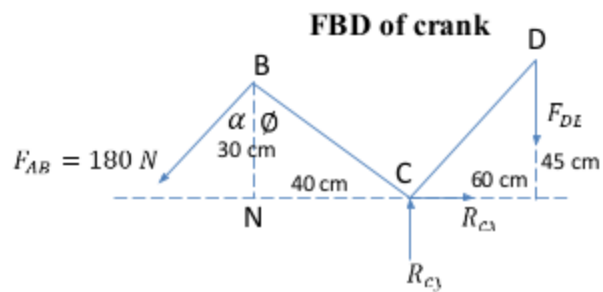
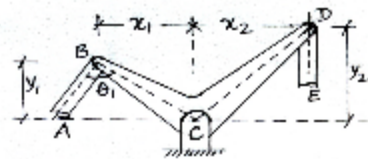
$$\text{Applying } \sum F_x = 0, R_{cx} - 180 \sin 36.87^\circ = 0 \quad R_{cx} = 108 \text{ N}$$

$$\text{Applying } \sum F_y = 0, R_{cy} - 180 \cos 36.87^\circ - F_{DE} = 0 \quad R_{cy} = 294 \text{ N}$$



$$P_{\bullet} = \sqrt{(108^2 + 294^2)} = 313.21 \text{ N}$$

$$\text{Direction } \tan \theta = \frac{294}{108} \quad \theta = 69.82^\circ$$



**Problem 3:** Find the distance 'x' measured along AB at which a horizontal force  $F_1 = 60 \text{ N}$  should be applied to hold the uniform bar AB in the position shown in figure. Bar AB is 3.0 m long and weighs 140 N. the incline and the floor are smooth.  $\theta_1 = 36.87^\circ$ ,  $\theta_2 = 56.3^\circ$ .

**Solution:** Let the reactions and forces acting on the bar are as shown in FBD.

Applying  $\sum F_x = 0$ ,  $60 - R_B \sin 56.3^\circ = 0$

$$R_B = 72.12 \text{ N}$$

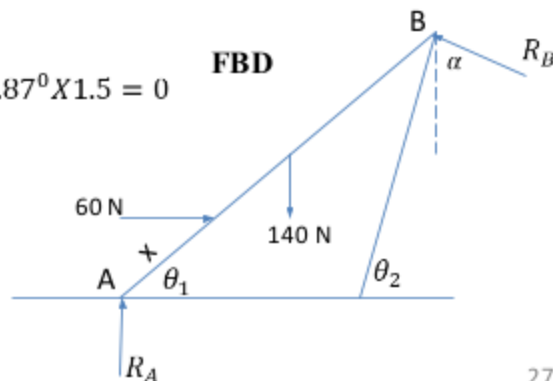
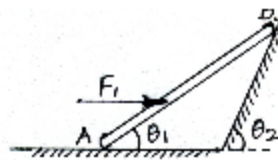
Applying  $\sum F_y = 0$ ,  $R_A - 140 + R_B \cos 56.3^\circ = 100 \text{ N}$

Applying  $\sum M = 0$  & taking moment about B

$$\sum M_B = 100 \times 3 \cos 36.87^\circ - 60 \times (3 - x) \sin 36.87^\circ - 140 \cos 36.87^\circ \times 1.5 = 0$$

$$= 240 - 108 + 36x - 168 = 0$$

$$x = 1 \text{ m.}$$



**Problem 4:** A loading car is at rest on a track forming an angle of  $25^\circ$  with the vertical. The gross weight of the car and its load is 25 kN, and it is applied at a point 0.75 m from the track, halfway between the two axles. The car is held by a cable attached 0.6 m from the track. Determine the tension in the cable and the reaction at each pair of wheels.

**Solution:** : Let the tension in the rope, reactions and forces acting on the loading car are as shown in FBD

Applying  $\sum F_x = 0$ ,  $-T + 25\cos 25 = 0$

$T = 22.657 \text{ kN}$ .

Applying  $\sum F_y = 0$ ,  $R_A + R_B - 25\sin 25 = 0$

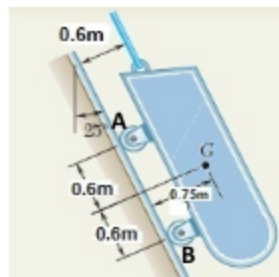
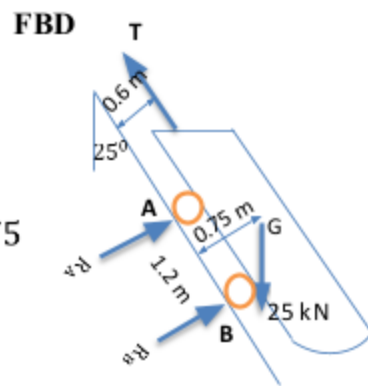
$R_A + R_B = 10.565 \text{ kN}$  -----(i)

Applying  $\sum M = 0$  & taking moment about A

$$\sum M_A = -22.657 \times 0.6 - R_B \times 1.2 + 25\cos 25 \times 0.75 + 25\sin 25 \times 0.6 = 0$$

$R_B = 8.115 \text{ kN}$ , from (i)

$R_A = 2.45 \text{ kN}$



**Problem 5:** Find the support reactions at A & B for the beam carrying loads as shown in the figure.

**Solution :** Let the reaction at A & B are as shown in FBD.

Applying  $\sum M_A = 0$ ,

$$60 \times 3 + 10 \times 4 \times 5 + 80 \sin 30^\circ \times 7 - R_{By} \times 9 - 20 \sin 30^\circ \times 2 = 0$$

$$R_{By} = 71.11 \text{ kN}$$

Applying  $\sum F_y = 0$ ,

$$R_{Ay} + R_{By} = 20 \sin 30^\circ + 60 + 10 \times 4 + 80 \sin 30^\circ$$

$$R_{Ay} = 78.9 \text{ kN}$$

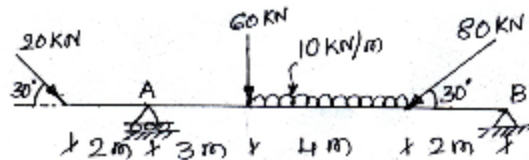
Applying  $\sum F_x = 0$ ,  $20 \cos 30^\circ - 80 \cos 30^\circ - R_{Bx} = 0$

$$R_{Bx} = -51.96 \text{ kN (i.e. sense of } R_{Bx} \text{ is to be changed)}$$

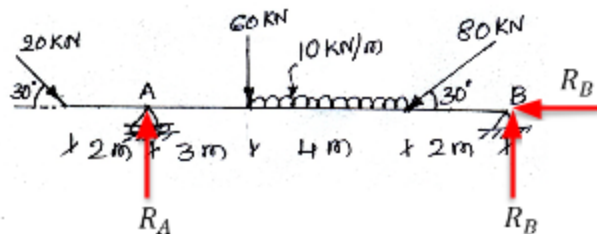
Resultant reaction at B,  $R_B = \sqrt{(71.11^2 + 51.96^2)} = 88.07 \text{ kN}$

$$\text{Direction, } \tan \theta = \frac{71.11}{51.96} = 1.368$$

$$\theta = 53.83^\circ$$



**FBD**



**Problem 6:** A simply supported beam AB of 6 m span is subjected to loading as shown. Find the support reactions at A and B.

**Solution:** : Let the reaction at A & B are as shown in FBD.

Applying  $\sum M_A = 0$ ,

$$-R_{By} \times 6 + \left( \frac{1}{2} \times 2 \times 3 \right) \times \left( 3 + \frac{2}{3} \times 3 \right) + 4 \times 2 + 4 \times 1 +$$

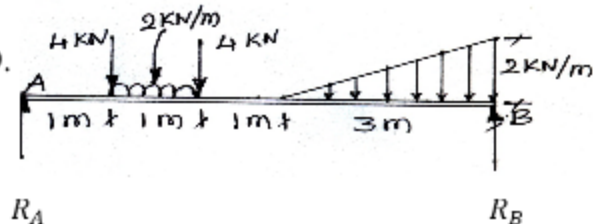
$$(2 \times 1) \times (1 + 0.5) = 0$$

$$R_{By} = 5 \text{ kN}$$

Applying  $\sum F_y = 0$ ,

$$R_{Ay} + R_{By} - 4 - (2 \times 1) - 4 - \left( \frac{1}{2} \times 2 \times 3 \right) = 0$$

$$R_{Ay} = 8 \text{ kN}$$



**Problem 7:** A beam AB 8.5 m long is hinged at A and supported on rollers over a smooth surface inclined at  $45^\circ$  to the vertical at B. the beam is loaded as shown. Determine the reactions at A and B.

**Solution:** :Let the reaction at A & B are as shown in FBD.

Applying  $\sum M_A = 0$ ,

$$-R_B \cos 45^\circ \times 8.5 + 5 \times 7 + 4 \sin 45^\circ \times 4 + 5 \times 2 = 0$$

$$R_B = 9.369 \text{ kN}$$

Applying  $\sum F_x = 0$ ,  $R_{Ax} + 4 \cos 45^\circ - 9.369 \sin 45^\circ = 0$

$$R_{Ax} = 3.796 \text{ kN}$$

Applying  $\sum F_y = 0$ ,

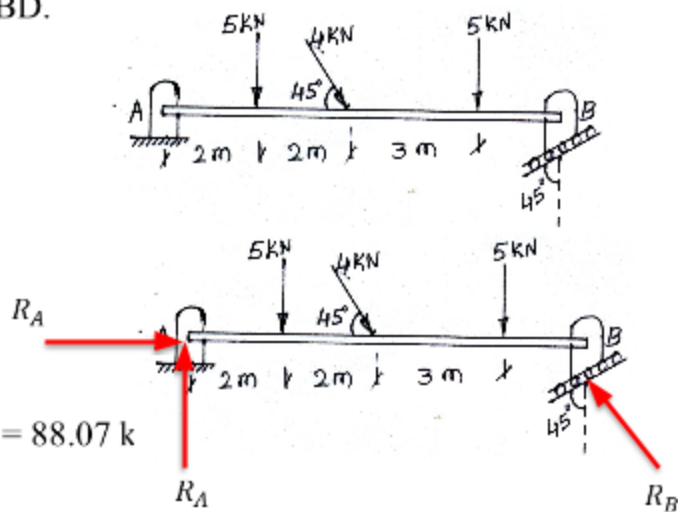
$$R_{Ay} + R_B \cos 45^\circ - 5 - 4 \sin 45^\circ - 5 = 0$$

$$R_{Ay} = 6.203 \text{ kN}$$

Resultant reaction at A,  $R_A = \sqrt{(3.796^2 + 6.203^2)} = 88.07 \text{ k}$

$$\text{Direction, } \tan \theta = \frac{6.203}{3.796} = 1.634$$

$$\theta = 53.53^\circ$$



**Problem 8:** Determine the support reactions at A, B & C for the arrangement of forces as shown in figure .

**Solution:** The above problem can be solved conveniently by considering span AD and BC separately  
FBD of beam AD is as shown

Applying  $\sum F_x = 0$ ,  $R_{Ax} = 0$

Applying  $\sum F_y = 0$ ,

$$R_{Ay} + R_{Dy} - (20 \times 4) - 30 = 0$$

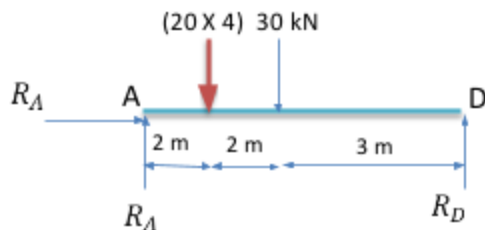
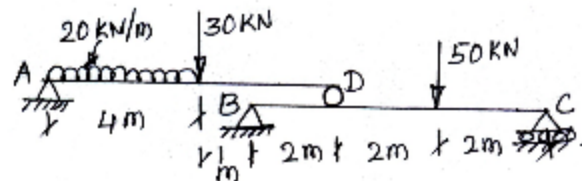
$$R_{Ay} + R_{Dy} = 120 \text{ -----(i)}$$

Applying  $\sum M_A = 0$ ,

$$-R_{Dy} \times 7 + 80 \times 2 + 30 \times 4 = 0$$

$$R_{Dy} = 40 \text{ kN}$$

From (i)  $R_{Ay} = 80 \text{ kN}$





The reaction  $R_{Dy}$  will act as a point load on span BC

FBD of beam BC is as shown

Applying  $\sum F_x = 0$ ,  $R_{Bx} = 0$

Applying  $\sum F_y = 0$ ,

$$R_{By} + R_{Cy} - 40 - 50 = 0$$

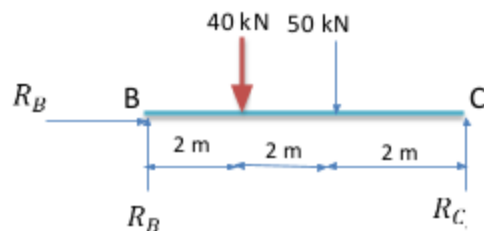
$$R_{By} + R_{Cy} = 90 \text{ -----(ii)}$$

Applying  $\sum M_B = 0$ ,

$$-R_{Cy} \times 6 + 40 \times 2 + 50 \times 4 = 0$$

$$R_{Cy} = 46.66 \text{ kN}$$

From (ii)  $R_{By} = 43.33 \text{ kN}$



**Problem 9:** Find the reaction at the supports A and B of the beam carrying the loads as shown in figure.

**Solution:** The FBD diagram of the beam is as shown.

Applying  $\sum M_A = 0$ ,

$$-R_B \sin 50^\circ \times 11.5 - 8 + 5 \sin 35^\circ \times 6.5 + (0.5 \times 5 \times 2.5) \times 3.83 + 6 \times 0.5 = 0$$

$$R_B = 4.265 \text{ kN}$$

Applying  $\sum F_x = 0$ ,

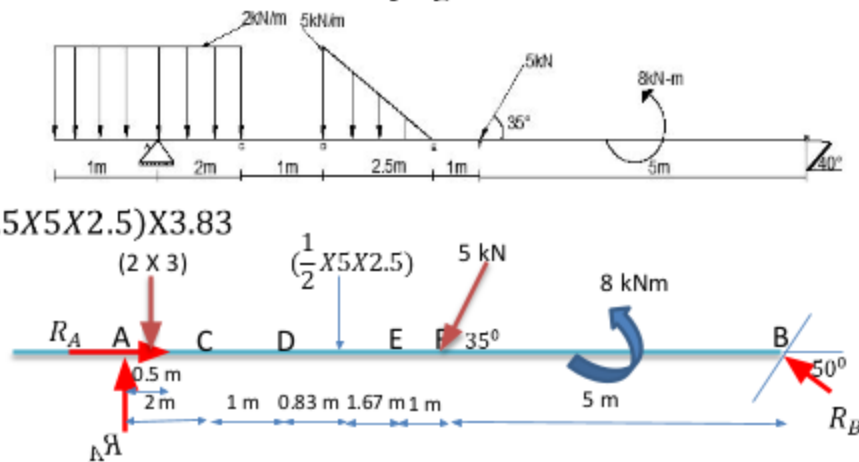
$$R_{Ax} - 5 \cos 35^\circ - 4.265 \cos 50^\circ = 0$$

$$R_{Ax} = 6.83 \text{ kN}$$

Applying  $\sum F_y = 0$ ,

$$R_{Ay} + 4.265 \sin 50^\circ - 6 - (0.5 \times 5 \times 2.5) - 5 = 0$$

$$R_{Ay} = 13.982 \text{ kN}$$



**Problem 10:** Determine the reactions at the supports of the truss shown in fig below.

**Solution:** The FBD diagram of the truss is as shown

Applying  $\sum M_A = 0$ ,

$$-R_B \times 16 + 4 \sin 60^\circ \times 16 + 8 \sin 60^\circ \times 12 + 6 \sin 60^\circ \times 8 - 8 \cos 60^\circ \times 2.309 - 6 \cos 60^\circ \times 4.61 = 0$$

$$R_B = 9.816 \text{ kN}$$

Applying  $\sum F_x = 0$ ,

$$R_{Ax} - 4 \cos 60^\circ - 8 \cos 60^\circ - 6 \cos 60^\circ = 0$$

$$R_{Ax} = 9 \text{ kN}$$

Applying  $\sum F_y = 0$ ,

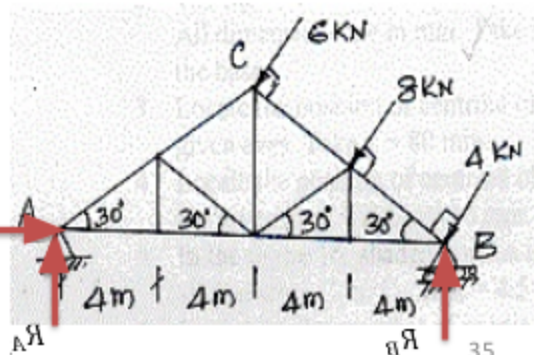
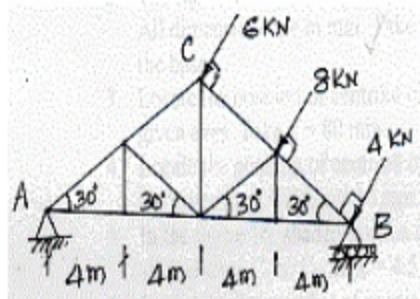
$$R_{Ay} - 4 \sin 60^\circ - 8 \sin 60^\circ - 6 \sin 60^\circ + 9.816 = 0$$

$$R_{Ay} = 5.77 \text{ kN}$$

Resultant reaction at A,  $R_A = \sqrt{(9^2 + 5.77^2)} = 10.69 \text{ kN}$

$$\text{Direction, } \tan \theta = \frac{5.77}{9} = 0.6411$$

$$\theta = 32.66^\circ$$



**Thank You**