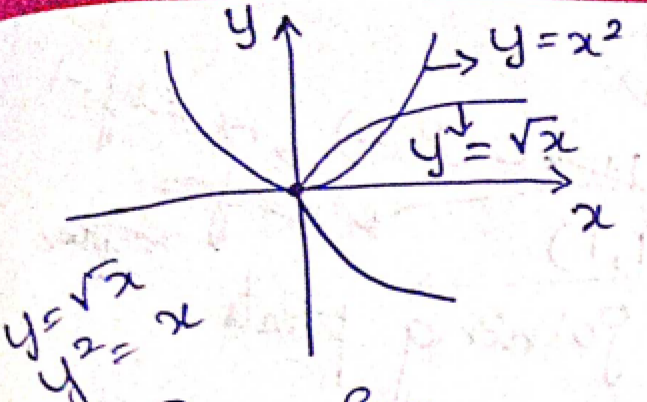
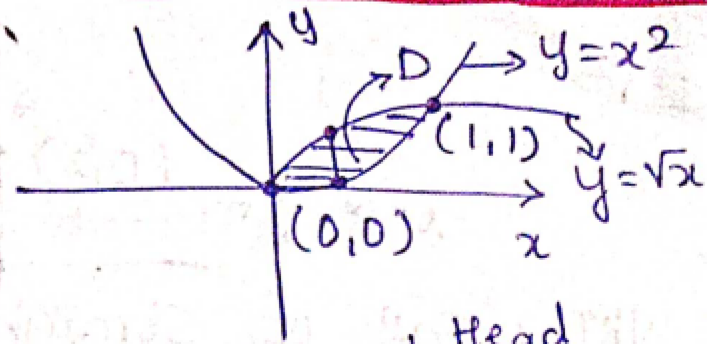




From fig,

$$\iint_D (x+y)\,dA = \int_0^1\int_{x^2}^{\sqrt{x}} (x+y)\,dy\,dx$$

Head → upper limit
foot → lower limit

$$I = \int_0^1 \left[xy + \frac{y^2}{2}\right]_{x^2}^{\sqrt{x}} dx = \int_0^1 \left[\left(x\sqrt{x} + \frac{(\sqrt{x})^2}{2}\right) - \left(x^3 + \frac{x^4}{2}\right)\right] dx$$

$$I = \int_0^1 \left(x^{3/2} + \frac{x}{2} - x^3 - \frac{x^4}{2}\right) dx$$

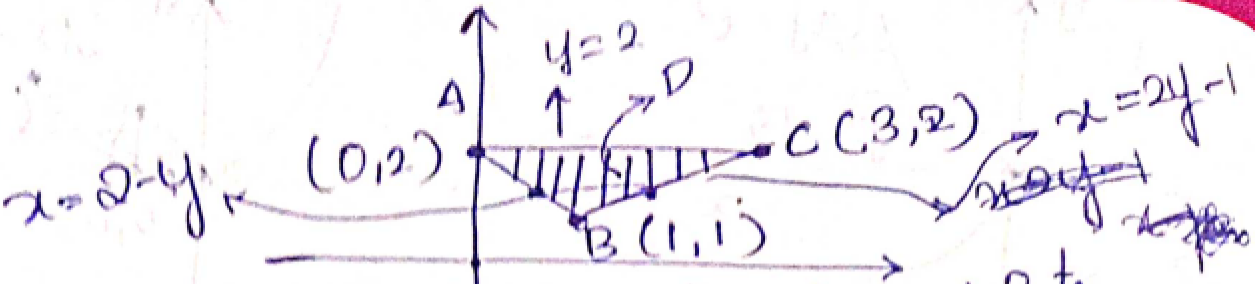
$$I = \left[\frac{2}{5}x^{5/2} + \frac{x^2}{4} - \frac{x^4}{4} - \frac{x^5}{10}\right]_0^1$$

$$I = \left[\frac{2}{5} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{4}} - \frac{1}{10}\right] = \frac{2}{5} - \frac{1}{10}$$

$$\boxed{I = \frac{3}{10}}$$

[M] $\iint_D y^3\,dA$, D is triangular region with vertices (0, 2), (1, 1) & (3, 2)

Sol^n:



WKT : Eq^n of straight line joining points (x_1, y_1) & (x_2, y_2) is

$$\boxed{y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)}$$

Eq^n: of AB : $y - 2 = \left(\frac{1-2}{1-0}\right)(x - 0)$

$$y - 2 = -x \quad \therefore \boxed{x = 2 - y}$$

Eq^n of BC : $y - 1 = \left(\frac{2-1}{3-1}\right)(x - 1)$

$$y - 1 = \frac{1}{2}(x - 1) \Rightarrow 2y - 2 = x - 1$$

$$\boxed{x = 2y - 1}$$

from fig, $\iint_D y^3 \, dA = \int_1^2 \int_{2-y}^{2y-1} y^3 \, dx \, dy$

$$I = \int_1^2 \left[y^3 x \right]_{2-y}^{2y-1} dy$$

$$I = \int_1^2 \left[y^3(2y-1) - y^3(2-y) \right] dy$$

$$I = \int_1^2 \left[2y^4 - y^3 - 2y^3 + y^4 \right] dy$$

$$I = \left[2\frac{y^5}{5} - \frac{y^4}{4} - 2\left(\frac{3y^4}{4}\right) + \frac{y^5}{5} \right]_1^2$$

$$I =$$

$$I = \int_1^2 y^3 \left[(2y-1) - (2-y) \right] dy$$

$$I = \int_1^2 y^3 [2y - 1 - 2 + y] \, dy$$

$$I = \int_1^2 y^3 [3y - 3] \, dy.$$

$$I = 3\int_1^2 y^3 [y-1] \, dy.$$

$$I = 3\int_1^2 [y^4 - y^3] \, dy$$

$$I = 3\left[\frac{y^5}{5} - \frac{y^4}{4} \right]_1^2$$

$$I = 3\left[\left(\frac{32}{5} - \frac{16}{4} \right) - \left(\frac{1}{5} - \frac{1}{4} \right) \right]$$

$$I = 3\left[\frac{32}{5} - \frac{16}{4} - \frac{1}{5} + \frac{1}{4} \right] = 3\left[\frac{31}{5} - \frac{15}{4} \right]$$

$$I = \frac{147}{20} = 7.35$$

Application of Double Integral

I. **Mass**: If $f(x,y)$ = density of an object (plate, people, charge etc)

then $$\boxed{Mass = \iint_D f(x,y)\, dA}$$

3.5 The population density of certain city is described by the function $f(x,y) = 10{,}000e^{-0.2x-0.1y}$, where the origin $(0,0)$ gives the location of the city hall, what is a population inside the rectangular area described by $R = \{(x,y) / -10 \le x \le 10; -5 \le y \le 5\}$

Sol: Given: $f(x,y)$ = density $= 10{,}000e^{-0.2x-0.1y}$

To find: Mass

$$Mass = \iint_R f(x,y)\, dA$$

$$Mass = \int_{-5}^{5}\int_{-10}^{10} 10{,}000e^{-0.2x-0.1y}\, dx\, dy$$

$$Mass = 10{,}000 \int_{-5}^{5}\left[\int_{-10}^{10} e^{-0.2x} \cdot e^{-0.1y}\, dx\right] dy$$