ENGINEERING MECHANICS

UNIT-I Chapter – 3

COPLANAR NON CONCURRENT FORCE SYSTEM(Resultant)

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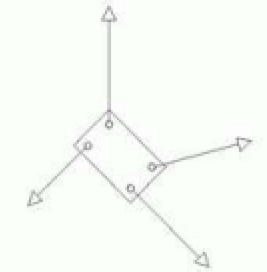


Chapter Content:

- Moment of a force, couple, moment of a couple, characteristics of a couple, equivalent force-couple system.

- Numerical problems - on moment of forces and couples, on equivalent force-couple system

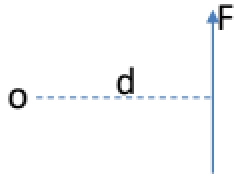
- Varignon's principle of moments, Composition of coplanar-non-concurrent force system

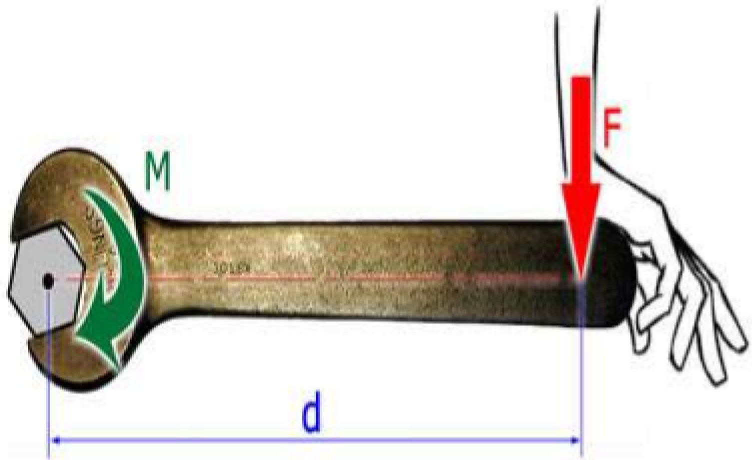
- Numerical problems on composition of coplanar non-concurrent force system

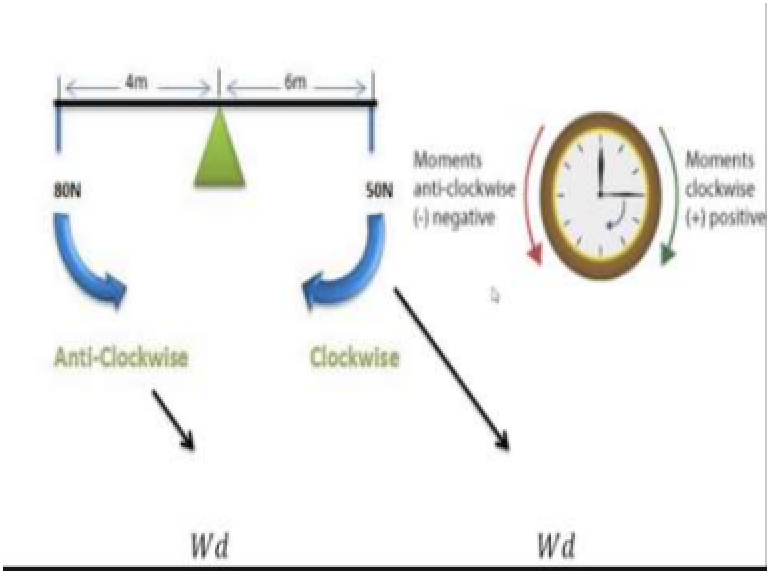
Moment of a force

- When a force is applied to a body, it has a tendency to turn or rotate the body about some point.
- This turning tendency of a force about point is called Moment of a force about that point
- The moment of a force is equal to the product of the force and the perpendicular distance of the point about which the moment is required from the line of action of force

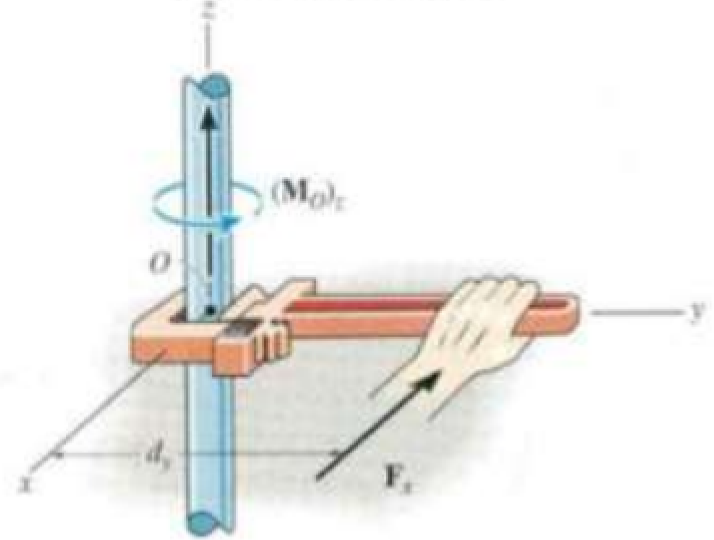
  - Mathematically , M=Fxd where
  - F= force acting on a body
  - d=Moment arm or perpendicular distance of the point from the line of action of the force

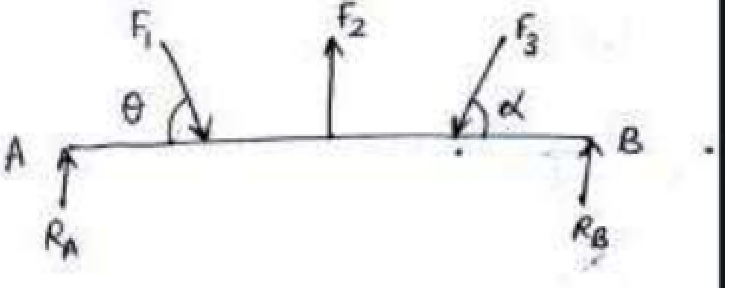



M
F
d


4m
6m
80N
50N
Moments
anti-clockwise
(-) negative
Moments
clockwise
(+) positive
Anti-Clockwise
Clockwise
Wd
Wd

point or axis.





Moment of a force

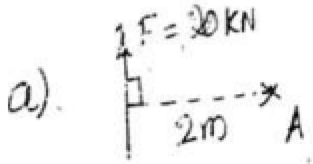
Unit of moment depends upon the unit of force & the distance Ex- N-m, N-mm, kN-m etc.

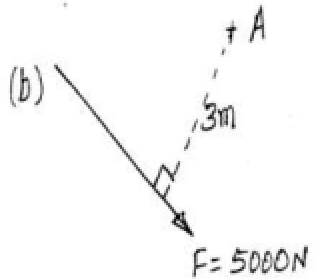
If the effect of a force is to turn the body in the same direction in which the hands of a clock move, it is said to be a clockwise moment

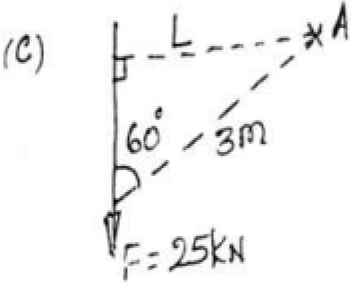
If the effect of a force is to rotate the body in the opposite direction in which the hands of a clock move, it is said to be an anticlockwise moment

Note: generally, the clockwise moments are taken as **+ve**   and anticlockwise e moments are taken as **-ve**

1. Find the moment of the force F about 'A' in the following cases.







Moment of a force about any point is given by M= Fxd there fore Moment about 'A' is

(a) $M_A = F \times d$

$= 20 \times 2$

$= 40$ **kNm**

(b) $M_A = -5000 \times 3$

$= -15000$ **N-m**

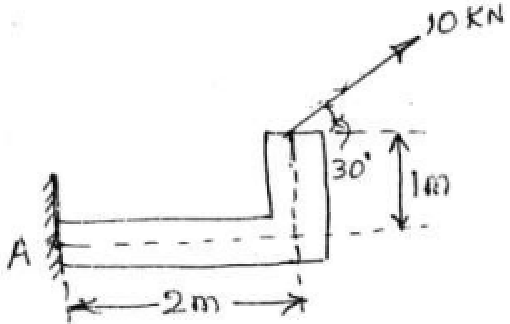
(C) $L = 3 \sin 60^0$

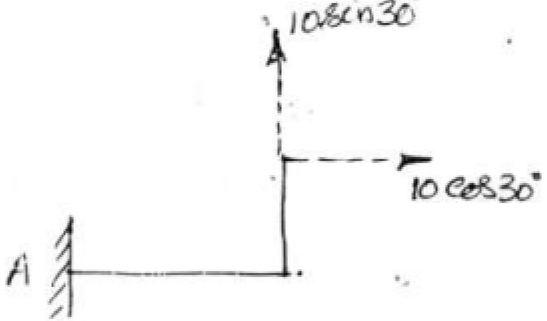
$= 2.598$ **m**

$M_A = -25 \times 2.598$

$= -64.95$ **kNm**

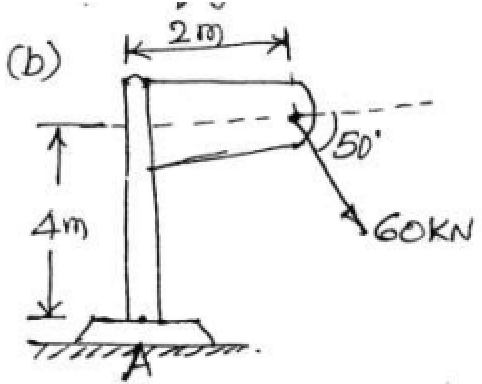
2.Find the moment of the force F about 'A'





$$M_A = 10 \cos 30^0 \times 1 - 10 \sin 30^0 x2$$
$$= 10x0.866 - 5x2$$
$$=8.66-10$$
$$=-1.34 \text{ kN-m}$$

3.Find the moment of the force F about 'A'

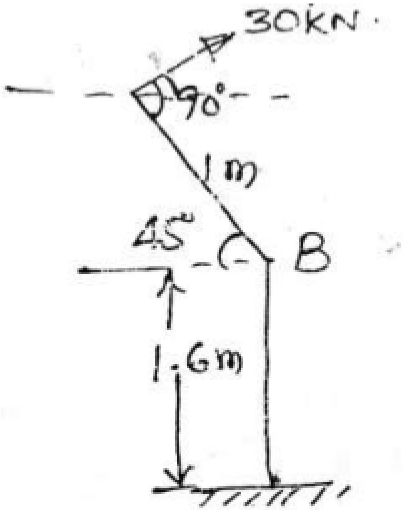


Resolving the given force into it's Components in x and y directions & taking Moment about A

$\sum M_A = 2(60 \sin 50^0) + (60 \cos 50^0 x\ 4)$
$=+246.19$ kN-m

$M_A = 246.19$ KN-m

4.Find the moment of the force F about 'A' and 'B'



$\sum M_B =(30\cos45^0 * 1\sin45^0)+(30\sin45^0 * 1\cos45^0)$

= 30 kN-m

$\sum M_A=(30\cos45^0)*(1\sin45^0+1.60)+30\sin45^0 * 1\cos45^0$

=63.93 kN-m

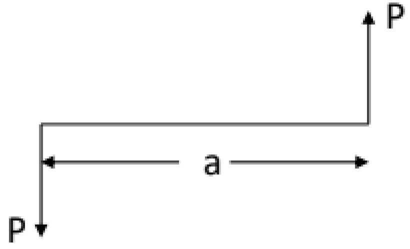
- **Couple:** Two equal and unlike parallel forces whose lines of action are not the same, are said to constitute a couple.

- The perpendicular distance ‘a’ between the lines of action of two equal and opposite forces is known as “arm of the couple”

- The moment of a couple is the product of the force P and the arm of the couple
  - Mathematically, moment of a couple is

$$M=P \times a$$

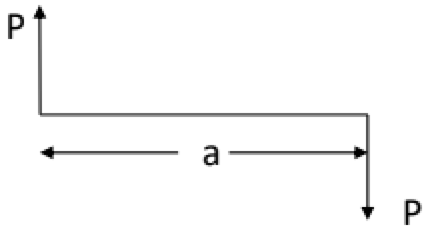
  - Unit of couple N-m, N-mm, kN-m etc.

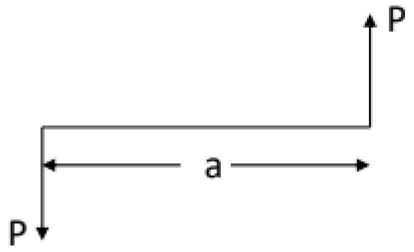


- Couple may be clockwise or anticlockwise

- If the effect of a couple is to rotate the body in the same direction in which the hands of a clock move, it is said to be a clockwise couple.

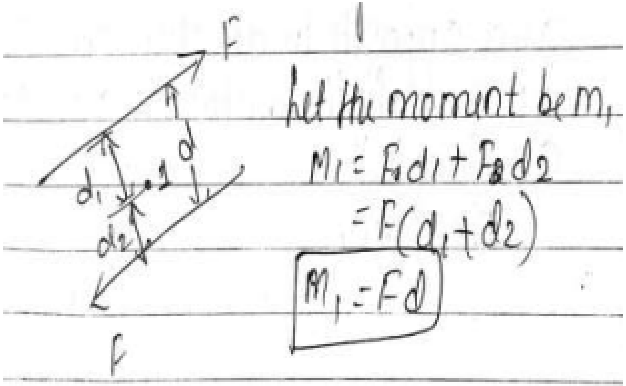
- If the effect of a couple is to rotate the body in the opposite direction in which the hands of a clock move, it is said to be an anticlockwise couple.

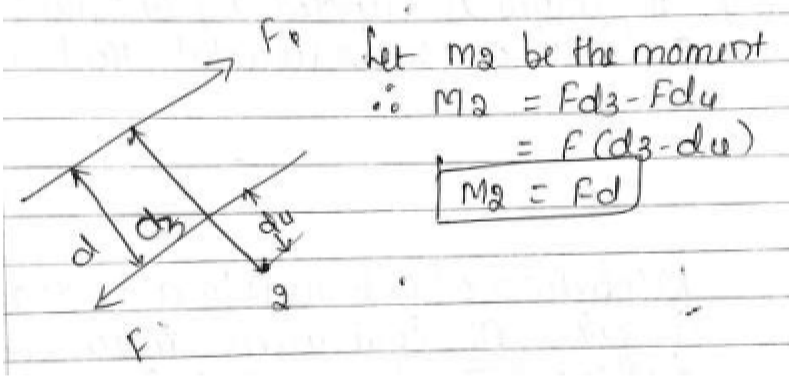




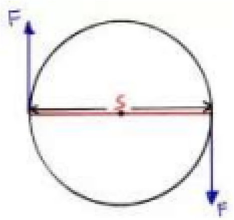
Characteristics of a couple:

- The algebraic sum of the forces constituting the couple is zero
- The algebraic sum of the moments of the forces constituting the couple about any point is same.

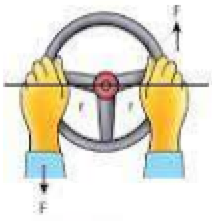


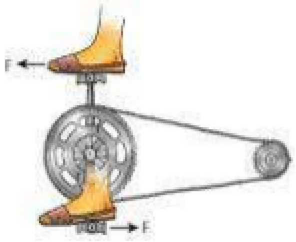


- A couple cannot be balanced by a single force, but can be balanced only by a couple of opposite sense.



- The translatory effect of a couple on the body is zero.





Equivalent force-couple system:

- In many engineering mechanics problems, it will be advantageous to resolve a force acting at a point on a body into a force acting at some other suitable point on the body and a couple.

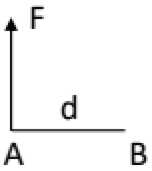


Fig.1

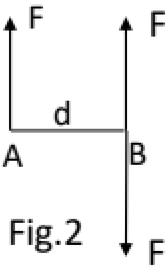


Fig.2

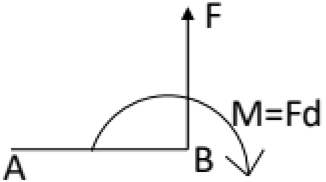


Fig.3

- Figure 1 shows a force F acting on a body at A. By applying equal and opposite forces F parallel to original force at B on the body as shown in Fig 2, the system of forces is not disturbed.
- Now the original force F at A & the opposite force at B, form a couple. Its effect may be replaced by the couple moment Fd acting at B. thus, the given force F at A is replaced by a force Fat B and a moment Fd as shown in figure 3.

5.Replace the horizontal 800 N force acting on the lever by an equivalent system consisting of a force & a couple at "o".

Ans: Apply a two force system in equilibrium each of 800N at ‘O’.

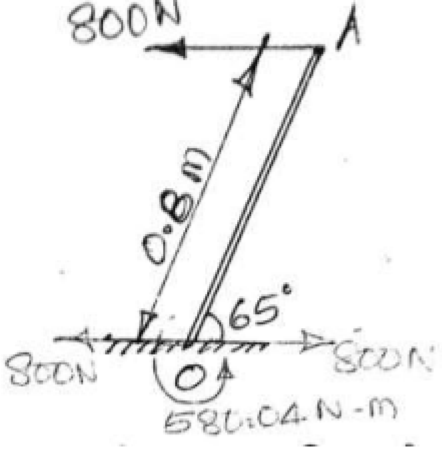
Consider the couple given by the given force & opposite force at 'o', We get an anticlockwise couple at 'o'.

i.e.$-800 \times 0.8 \sin 65 = -580.04$ N-m

Equivalent force at ‘o' is 800 N .

Thus, the original force of 800 N at A

is equivalent to 800N force & 580.04 N·m couple moment at ‘o'



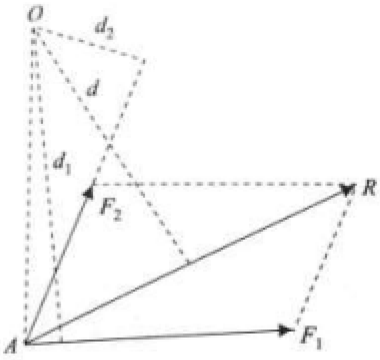
Varignon's theorem:

"If a number of coplanar forces are acting simultaneously on a particle the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point"

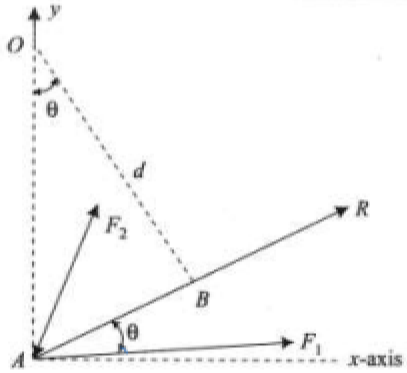
$$Rd = F_1d_1 + F_2d_2$$

$$Rd = \Sigma M_o$$

$$d = \Sigma M_o / R$$



(a)



(b)

- Referring to above Figure let R be the resultant of forces F_1 and F_2, and 'O' be the moment center.
- Let d, d_1, and d_2, be the moment arms of the forces R, F_1 , and F_2 respectively. Then in this case we have to prove

$Rd = F_1\ d_1 + F_2\ d_2$ --------(1)

- Join OA and consider it as y-axis. Draw x-axis to it with A as origin [Ref. Fig.(b)].Let resultant make an angle θ with x-axis. Noting that angle AOB is also θ, we can write

$Rd = R \times AO\ \cos\theta$

$= AO \times (R \cos\theta)$

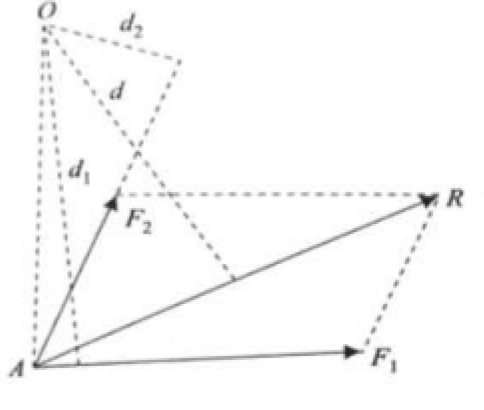
$= AO \times (R_x)$ ------------------(i)

- Where R_x denotes the component of R in x-direction. Similarly, if F_{1x} and F_{2x} are the components of F_1 and F_2 in x-direction,

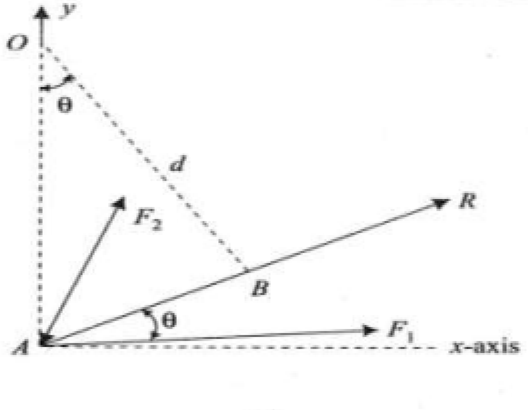
then,

$F_1 d_1 = AO \times F_{1x}$ -------------------------(ii)

$F_2 d_2 = AO \times F_{2x}$ -------------------------(iii)



(a)



(b)

and

From eqns. (ii) and (iii), we get

$$F_1d_1 + F_2d_2 = AO\,(F_{1x} + F_{2x})$$

$$= AO \times R_x \text{ ----------------------(iv)}$$

From eqns. (i) and (iv), we observe

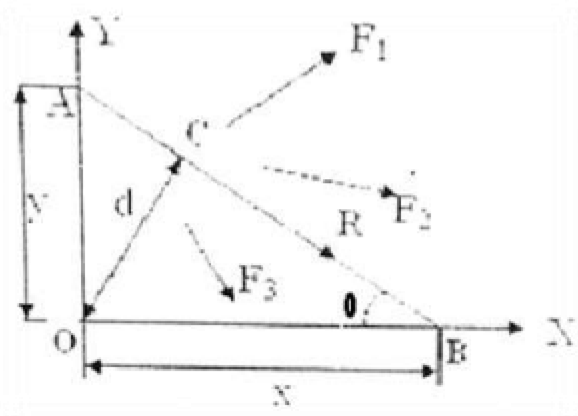
$$F_1d_1 + F_2d_2 = Rd$$

Thus we find sum of the moment of forces about a moment centre is same as moment of their resultant about the same centre.

X and Y intercepts of resultant:

Let R be the resultant of a system of forces shown, 'd' be the perpendicular distance of the point o from line of action of R and 'a be the angle made by R with x-axis. Then, intercepts are given by

In triangle BOC, X= $\frac{d}{sin\theta}$ and



In triangle AOC , Y= $\frac{d}{\sin(90-\theta)}$ or

$$Y= \frac{d}{cos\theta}$$

Resultant of Coplanar Non-concurrent forces:

The resultant force of a given system of forces may be found out by the method of resolution.

- Resolve all the forces along X-axis and find algebraic sum of all the X- components ($\sum F_x$).

- Resolve all the forces along Y-axis and find algebraic sum of all the Y -components ($\sum F_y$).

- Assign proper sign to each of the forces. For X-components, consider forces acting towards right as +ve and towards left as -ve.

- For Y components, consider forces acting upward as +ve and downward as-ve.

- The resultant R of the given forces is obtained by $\mathbf{R}=\sqrt{(\mathbf{F}_1^2+\mathbf{F}_2^2)}$.

- The inclination of the resultant with horizontal is given by $\mathbf{Tan\theta} = [\frac{\sum F_y}{\sum F_x}]$

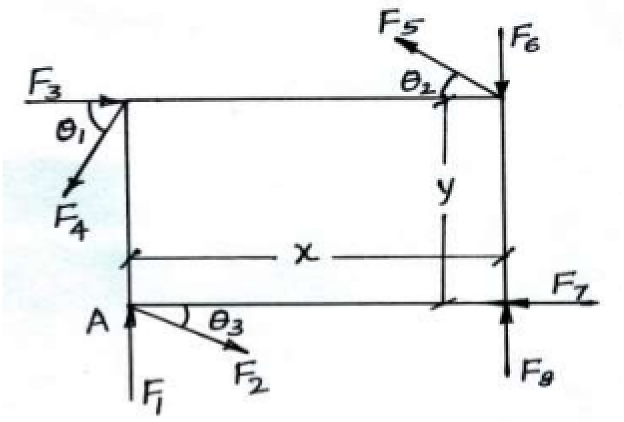
- Apply Varignon's theorem to locate the position of resultant with respect to given point by d=∑M/R

Problems:

1. Find the resultant for the system of forces acting on a rectangle ABCD as shown in figure, with respect to corner A, if x = 5 m and y = 2.50 m, F_1 = 2 kN, F_2 = 3 kN, F_3 = 8 kN, F_4 = 9 kN, F_5 = 7 kN, F_6 = 6 kN, F_7 = 5 kN, F_8 = 4kN, $\theta_1 = 60°$ $\theta_2 = 40°$ & $\theta_3 = 20°$.

Solution: Resolving all the forces horizontally we get

$$\sum F_x = F_2\cos\theta_3 + F_3 - F_4\cos\theta_1 - F_5\cos\theta_2 - F_7$$
$$= 3\cos20^0 + 8 - 9\cos60^0 - 7\cos40^0 - 5$$
$$= -4.04 \text{ kN}$$



Resolving all the forces vertically we get

$$\sum F_y = F_1 - F_2\sin\theta_3 - F_4\sin\theta_1 + F_5\sin\theta_2 - F_6 + F_8$$
$$= 2 - 3\sin20^0 - 9\sin60^0 + 7\sin40^0 - 6 + 4$$
$$= -4.32 \text{ kN}$$

Magnitude of the resultant:

$$R = \sqrt{\sum F_x^{\,2} + \sum F_y^{\,2}}$$

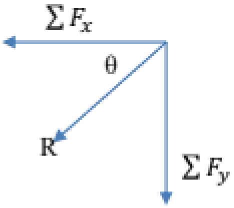
$$R = \sqrt{(-4.04)^2 + (-4.32)^2}$$

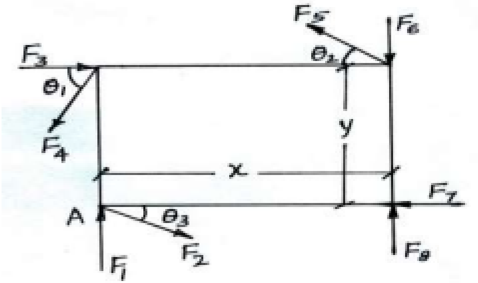
Magnitude R = 5.91 N

Direction of the resultant $\theta = \tan^{-1}\left[\frac{\sum F_y}{\sum F_x}\right]$

$$\theta = \tan^{-1}\left[\frac{4.32}{4.04}\right]$$

$$\theta = 46.91^0$$





Position of Resultant force :

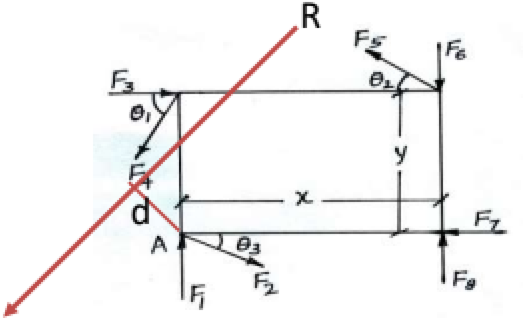
Let 'd' be the perpendicular distance of the corner 'A' from the line of action'R'. Using Varignon's theorem and taking moment about 'A'

$R \times d = \sum M_A$

$\sum M_A = (F_3 * y) - (F_4 \cos\theta_1 * y) - (F_5 \cos\theta_2 * y) - (F_5 \sin\theta_2 * x) + (F_6 * \mathbf{x}) - (F_8 * \mathbf{x})$

$= (8 \times 2.5) - (9\cos 60^0 \times 2.5) - (7\cos 40^0 \times 2.5) - (7\sin 40^0 \times 5)) + (6 \times 5) - (4 \times 5)$

$\sum M_A = -17.15$ kN-m



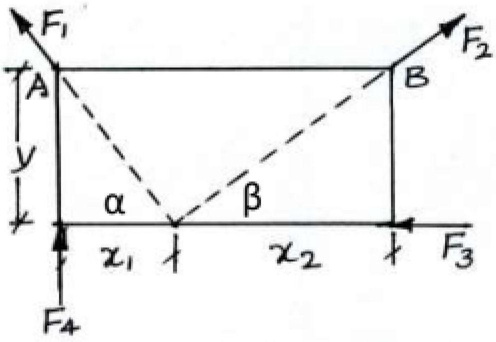
$-R \times d = \sum M_A$

$-5.91 \times d = -17.15$

$d = 2.90$m

2) Four forces act on a 700 mm X 350 mm plate. (i) Find the resultant of these forces. (ii) Locate the point where the line of action of the resultant intersects the edge AB of the plate shown in figure if F_1 = 350 N, F_2 = 500 N, F_3 = 750 N, F_4 = 600 N and x_1 = 200 mm, x_2 = 500 mm, y = 350 mm.

Solution: Let α and β be the angles made by F_1 and F_2 w.r.t to X-axis



$\tan\beta=[\frac{350}{500}]$ $\beta= \tan^{-1}[\frac{350}{500}] =35^0$

$\tan\alpha = [\frac{350}{200}]$ $\alpha= \tan^{-1}[\frac{350}{200}] =60.25^0$

Resolving all the forces horizontally we get

$\sum F_x = (-F_1\cos \alpha)+(F_2 \cos \beta)-F_3$

$=-350\cos 60.25^0 +500 \cos 35^0 - 750$

$= -514.01N$

Resolving all the forces vertically we get

$\sum F_y = (F_1\sin\alpha) +(F_2 \sin\beta) + F_4$

$=350\sin60.25^0+500\sin35^0+600$

$= 1190.65N$

Magnitude of the resultant:

$R = \sqrt{\sum F_x^{\,2} + \sum F_y^{\,2}}$

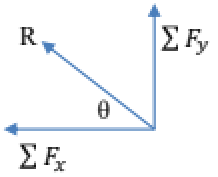
$R = \sqrt{(514.01)^2+(1190.65)^2}$

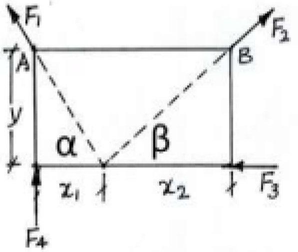
Magnitude R = 1296.89 N

Direction of the resultant $\theta = \tan^{-1}\left[\frac{\sum F_y}{\sum F_x}\right]$

$$\theta = \tan^{-1}\left[\frac{1190.65}{514.01}\right]$$

$$\theta = 66.65^0$$





Let 'x' be the distance from A at which resultant intersects line AB.
Taking moment about A and apply Varignon's theorem

$R x d = \sum M_A$

$\sum M_A = (-F_2 \sin\beta * (x_1 + x_2)) + (F3 * y)$

$= (-500 \sin 35^0 x 700) + (750 x 350)$

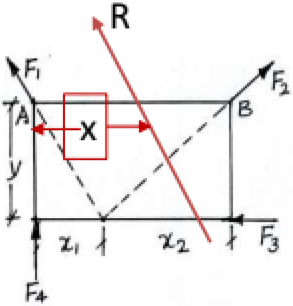
$\sum M_A = 61{,}748.24$ N-mm

$d = \sum M_A / R = 61748.24 / 1296.89 = 47.61$mm

$$x = \left[\frac{d}{\sin\theta}\right]$$

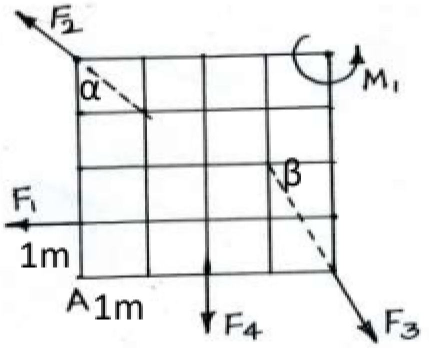
$$= \left[\frac{47.61}{\sin 66.65}\right]$$

$$x = 51.86$$



3) Four forces and a couple are acting on a lamina as shown in figure. Each block is a square of side 1 m. Find the position of resultant w.r.t. point A. $F_1 = 5$ kN, $F_2 = 10$ kN, $F_3 = 50$ kN, $F_4 = 30$ kN, $M_1 = 5$ kN-m

Solution: Let α and β be the angles made by F_2 and F_3 w.r.t to X-axis



$\tan\alpha = [\frac{1}{1}]$ $\alpha = \tan^{-1}[\frac{1}{1}] = 45^0$

$\tan\beta = [\frac{2}{1}]$ $\beta = \tan^{-1}[\frac{2}{1}] = 63.43^0$

Resolving all the forces horizontally we get

$\sum F_x = -F_1 - F_2\cos\alpha + F_3\cos\beta$

$= -5 - 10\cos 45^0 + 50\cos 63.43^0$

$= 10.293$ kN

Resolving all the forces vertically we get

$\sum F_y = F_2\sin\alpha - F_3\sin\beta - F_4$

$= 10\sin45^0 - 50\sin63.43^0 - 30$

$= -67.648$ kN

Magnitude of the resultant:

$$R = \sqrt{\sum F_x{}^2 + \sum F_y{}^2}$$

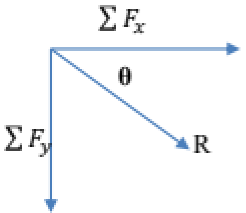
$$R = \sqrt{(10.293)^2 + (67.648)^2}$$

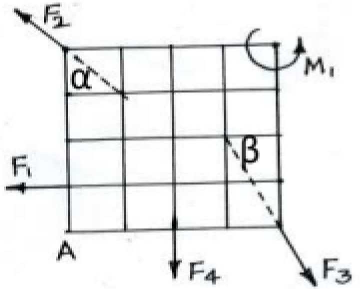
Magnitude $R = 68.426$ kN

Direction of the resultant $\theta = \tan^{-1}\left[\frac{\sum F_y}{\sum F_x}\right]$

$$\theta = \tan^{-1}\left[\frac{67.648}{10.293}\right]$$

$$\theta = 81.35^0$$





Position of Resultant force :

Let 'd' be the perpendicular distance of point 'A' from the line of action 'R' using varignon s theorem and taking moment about 'A'

$Rxd = \sum M_A$

$Rxd = (-F_1 * x) - (F_2 \cos\alpha * 3x) - (F_2 \sin\alpha * x) - M_1 + (F_3 \sin\beta * 3\ x) + (F_3 \cos\beta * 2x) + (F_4 * 2x)$

$Rxd = (-5x1) - (10\cos 45^0 x3) - (10\sin 45^0 x1) - 5 + (50\sin 63.43^0 x3) + (50 \cos$

$Rxd = 200.60$

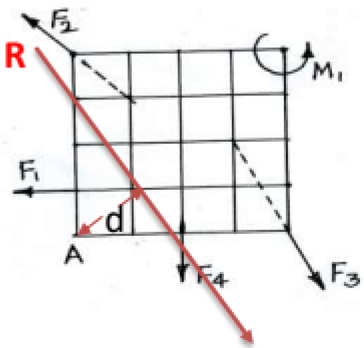
$d = 2.931m$

'x' intercept on horizontal line

$x = \left[\frac{d}{\sin\theta}\right]$

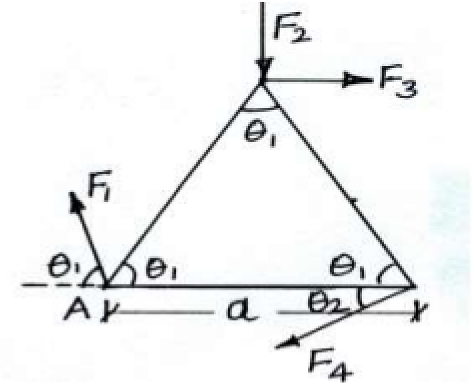
$= \left[\frac{2.931}{\sin 81.35^0}\right]$

$x = 2.965m$



4) Find the resultant of the force system shown in figure acting on a lamina of equilateral triangular shape of side a = 200 mm. Locate its X-intercept from point 'A'. $F_1 = 100$ N, $F_2 = 80$ N, $F_3 = 60$ N, $F_4 = 120$ N, $\theta_1 = 60°$ $\theta_2 = 30°$.

Solution:



Resolving all the forces horizontally we get

$\sum F_x = -F_1 \cos\theta_1 + F_3 - F_4 \cos\theta_2$

$= -100\cos 60^0 + 60 - 120\cos 30^0$

$= -93.92$ N

Resolving all the forces vertically we get

$\sum F_y = F_1 \sin\theta_1 - F_2 - F_4 \sin\theta_2$

$= 100\sin 60^0 - 80 - 120\sin 30^0$

$= -53.39$N

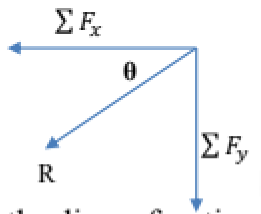
Magnitude of the resultant:

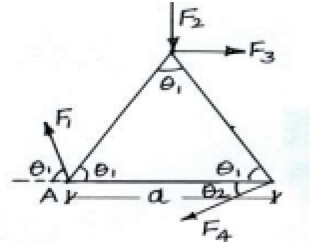
$$R = \sqrt{\sum F_x^{\,2} + \sum F_y^{\,2}}$$

$$R = \sqrt{(93.92)^2 + (53.39)^2}$$

Magnitude R = 108 N

Direction of the resultant $\theta = \tan^{-1}\left[\frac{\sum F_y}{\sum F_x}\right]$





$$\theta = \tan^{-1}\left[\frac{53.39}{93.92}\right]$$

$$\theta = 29.61^0$$

Position of Resultant force :

Let 'd' be the perpendicular distance of point 'A' from the line of action 'R' using Varignon's theorem and taking moment about 'A'

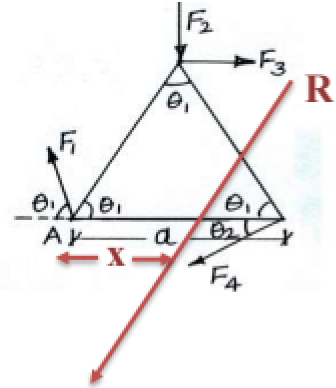
$Rxd = \sum M_A$

$\sum M_A = (F_2\cos\theta_1 * a) + (F_3\cos\theta_1 * a) + (F_4\sin\theta_1 * a)$

$= (80x200\cos 60^0) + (60x200\sin 60^0) + (120x200\sin 30^0)$

$\sum M_A = 30392.30$

'x' intercept on horizontal line



$$x = \left[\frac{\sum MA}{\sum F_y}\right]$$

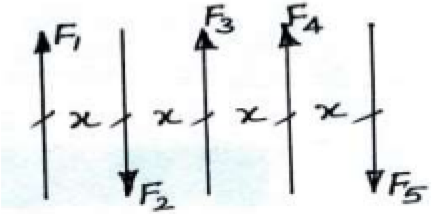
$$= \left[\frac{30392.30}{53.39}\right]$$

$$x = 569.54mm$$

5) Find the resultant for the system of parallel forces shown in figure. $F_1 = 10$ kN, $F_2 = 2$ kN, $F_3 = 3$ kN, $F_4 = 1$ kN, $F_5 = 5$ kN, $x = 1.0$ m.

Solution:

The magnitude of the resultant of a number of parallel forces is given by the algebric summation of all the forces



$R=F_1-F_2+F_3+F_4-F_5$

$=10-2+3+1-5$

$R = 7kN\ (\uparrow)$

Location:

Let 'd' be the distance of the resultant from point 'x' taken on the force 5kN acting downward. Using Varignon's theorem and taking moment about 'x'

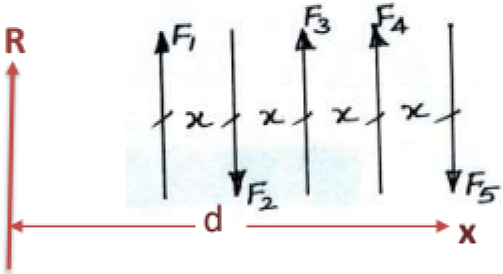
$R x d = \sum M_A$

$Rxd=(F1*4x)-(F2*3x)+(F3*2x)+(F4*x)$

$=(10x4)-(2x3)-(3x2)+(1x1)$

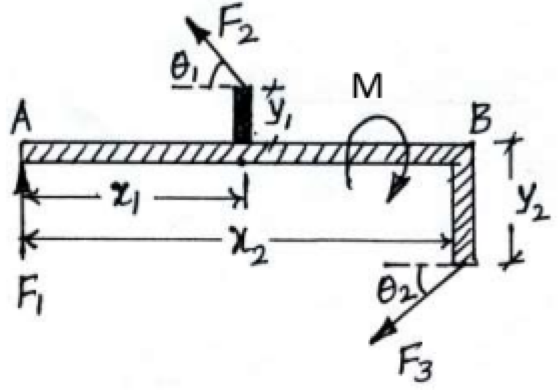
$7xd = 41$

$d=5.857m$



6) A bracket is subjected to 3 forces and a clockwise couple of M=50 N-m as shown in figure. Take F_1 = 200 N, F_2 = 400 N, F_3 = 150 N, θ_1 =45°, θ_2 =30° and x_1 = 3 m, x_2 = 6 m, y_1 = 0.6 m, & y_2 = 1 m. Check the safety of bracket for the given system of loading, if the bracket is safe for resultant value is less than 600N.

Solution:



Resolving all the forces horizontally we get

$\sum F_x = -F_2 \cos\theta_1 - F_3 \cos\theta_2$

$= -400\cos 45^0 - 150\cos 30^0$

$= -412.74$ N

Resolving all the forces vertically we get

$\sum F_y = F_1 + F_2\sin\theta_1 - F_3\sin\theta_2$

$= 200 + 400\sin 45^0 - 150\sin 30^0$

$= 407.84$ N

Magnitude of the resultant:

$R = \sqrt{\sum F_x{}^2 + \sum F_y{}^2}$

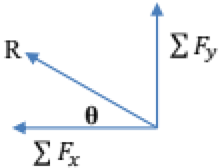
$R = \sqrt{(412.74)^2 + (407.84)^2}$

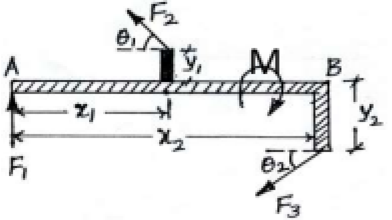
Magnitude R = 580.24 N

Direction of the resultant $\theta = \tan^{-1}\left[\frac{\sum F_y}{\sum F_x}\right]$

$\theta = \tan^{-1}\left[\frac{407.84}{412.74}\right]$

$\theta = 44.65^0$





Let the resultant intersect line AB at a distance of ‘x’ from point ‘A’. Using Varignon’s theorem taking moment about ‘A’

$Rxd = \sum M_A$

$\sum M_A = (-F_2\cos\theta_1 * y_1) - (F_2\sin\theta_1 * x_1) + M + (F_3\cos\theta_2 * y_2) + (F_3\sin\theta_2 * x_2)$

$= (-400x0.6\cos 45^0) - (400x3\sin 45^0) + 50 + (150x1\cos 30^0) + (150\sin 30^0 x6)$

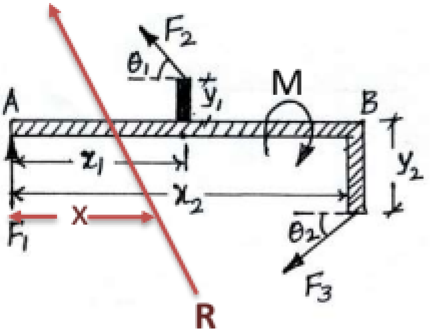
$\sum M_A = -388.32$ N-m

‘x’ intercept on horizontal line

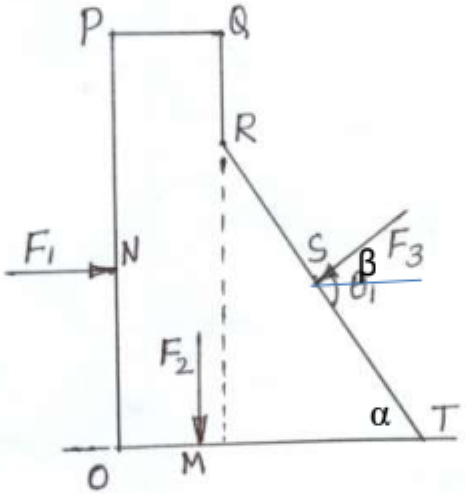
$x = \left[\frac{\sum MA}{\sum F_y}\right]$

$x = \left[\frac{388.32}{407.84}\right]$

$x = 0.952$ m



7) Find the magnitude, direction of the resultant of the force system acting on the dam section as shown in figure. Take $F_1 = 100$ kN, $F_2 = 150$ kN, $F_3 = 120$ kN, $\theta 1 = 90°$, PQ = 3 m, QR = 3 m, OM = 2.7 m and MT = 4.3 m, ON = 4.0 m, PN = 6.0 m. S is the midpoint of RT and check whether the dam is safe?



Solution: Let α be the inclination of RT w.r.t. horizontal

$\tan\alpha = 7/4$, $\alpha = 60.25^0$

Also $\cos 60.25^0 = 4.0/RT$

RT=8.06m and TS=4.03m.

$\beta = 29.75^0$

Resolving all the forces horizontally we get

$\sum F_x = F_1 - F_3\cos\beta$

$= 100 - 120\cos 29.75^0$

$= -4.1838$ kN

Resolving all the forces vertically we get

$\sum F_y = -F_2 - F_3 \sin\beta$

$= -150 - 120\sin 29.75^0$

$= -209.54$ kN

Magnitude of the resultant:

$$R = \sqrt{\sum F_x^2 + \sum F_y^2}$$

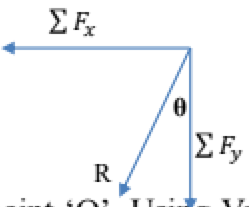
$$R = \sqrt{(4.18)^2 + (209.54)^2}$$

Magnitude R = 209.586 kN

Direction of the resultant $\theta = \tan^{-1}\left[\frac{\sum F_y}{\sum F_x}\right]$

$\theta = \tan^{-1}\left[\frac{209.586}{4.184}\right]$

$\theta = 88.85^0$



Let the resultant intersect line OT at a distance of 'x' from point 'O'. Using Varignon's theorem taking moment about 'O'

$R x d = \sum M_A$

$R_y * x = \sum M_o$

$x = \sum M_o / R_y$

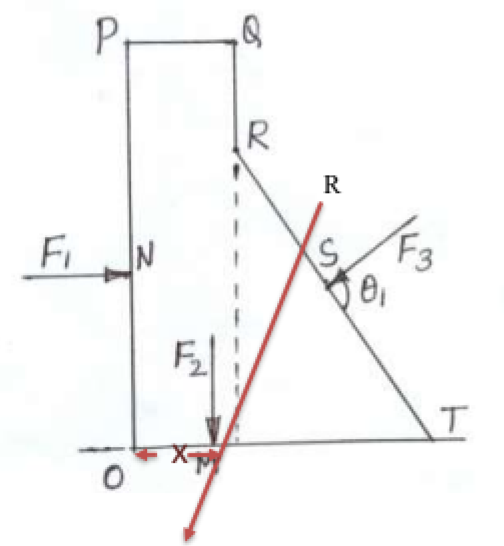
$=(F1*ON)+(F2*OM)-(F3\cos\beta*TS\sin\alpha)$

$+F3\sin\beta*(OT-TS\cos\alpha)/R_y$

$=(100*4)+(150*2.7)-(120\cos 29.75*4.03\sin 60.25)$

$+(120\sin 29.75(7-4.03\cos 60.25)) / 209.545$

x=3.522 m from 'O'



Thank You