$2\sqrt{\tfrac{1}{2}+\tfrac{4}{2}}$ $\quad 6 \quad 1\tfrac{1}{2} \quad \tfrac{3}{2}\cdot\tfrac{1}{2}\,\Gamma(\tfrac{1}{2})$

$$= \frac{4}{3}\,\frac{1!}{(3/2)(1/2)} = \frac{4}{3}\cdot\frac{2\cdot 2}{3\cdot 1} = \frac{16}{9}\ //$$

$$= \underline{\underline{1.7778}}$$

$$\boxed{\begin{array}{l}\Gamma(n+1) = n! \\ \Gamma(n+1) = n\,\Gamma(n)\end{array}}$$

Q. Write a double integral $\iint f(x,y)\,dA$ which gives volume of top half of R a solid ball of radius 5. Hence evaluate.

Sol^n: Eq^n. of solid ball of radius 5

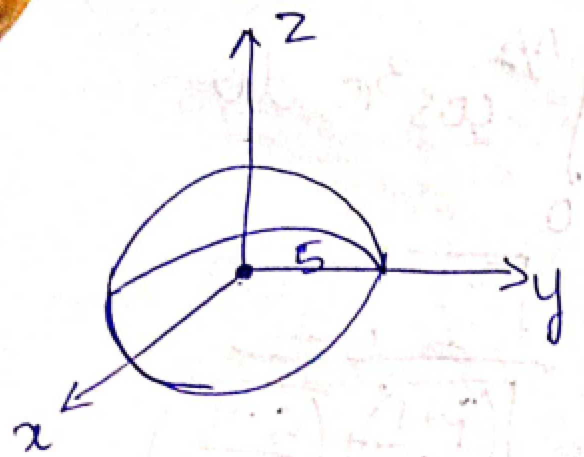
i.e, $x^2+y^2+z^2 = 5^2$

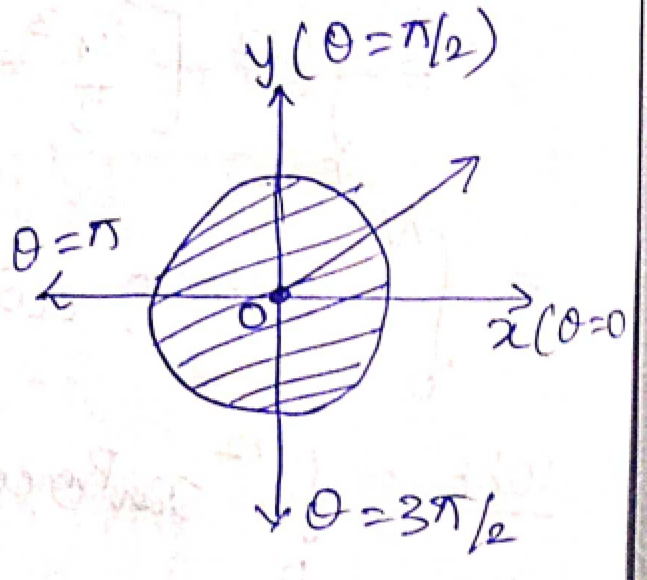
$\Rightarrow x^2+y^2+z^2 = 25 \Rightarrow z^2 = 25 - x^2 - y^2$

$z = +\sqrt{25-x^2-y^2}$; (top half +ve)

$\therefore$ volume $= \iint_R f(x,y)\,dA = \iint_R z\,dA$

$= \iint_R \sqrt{25-x^2-y^2}\,dA$





In polar coordinates, $x = r\cos\theta$, $y = r\sin\theta$ } $x^2+y^2=r^2$

$$\text{volume} = \iint \sqrt{25-(x^2+y^2)}\, dA$$

$$\text{volume} = \int_0^{2\pi}\int_0^5 \sqrt{25-r^2}\; r\,dr\,d\theta .$$

$$\text{volume} = \int_0^{2\pi} d\theta \int_0^5 (25-r^2)^{1/2} \frac{(-2r)}{-2}\, dr$$

$$V = [\theta]_0^{2\pi} \left[\frac{(25-r^2)^{1/2+1}}{1/2+1}\right]_0^5$$

$$\because \int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1}$$

$$V = [2\pi - 0]\left(\frac{1}{\cancel{-2}}\right)\left[(25-r^2)^{3/2}\right]_0^5 \left(\frac{\cancel{2}}{3}\right)$$

$$V = \frac{[2\pi-0]}{-3}\left[(25-5^2)^{3/2} - (25-0)^{3/2}\right]$$

$$V = -\frac{2\pi}{3}\left[0 - 5^3\right] = \frac{2\pi}{3}(125)$$

$$\boxed{V = \frac{250\pi}{3} \text{ cubic units}}$$ //

II <u>Area</u> :

1. If $f(x,y)=1$ then area is

$$\boxed{A = \iint_D 1\, dA}$$

$$A = 4\int_{\pi/6}^{\pi/2}[2(2\sin^2\theta)-1]\,d\theta$$

$$A = 4\int_{\pi/6}^{\pi/2}[2(1-\cos 2\theta)-1]\,d\theta \quad \{\because 2\sin^2\theta = 1-\cos 2\theta\}$$

$$A = 4\int_{\pi/6}^{\pi/2}[2-2\cos 2\theta-1]\,d\theta$$

$$A = 4\int_{\pi/6}^{\pi/2}[1-2\cos 2\theta]\,d\theta = 4\left[\theta-\cancel{2}\,\frac{\sin(2\theta)}{\cancel{2}}\right]_{\pi/6}^{\pi/2}$$

$$A = 4\left[\left(\pi/2-0\right)-\left(\frac{\pi}{6}-\sin\left(\frac{\pi}{3}\right)\right)\right]$$

$$A = 4\left[\frac{\pi}{2}-\left(\frac{\pi}{6}-\frac{\sqrt{3}}{2}\right)\right] = 4\left[\frac{\pi}{2}-\frac{\pi}{6}+\frac{\sqrt{3}}{2}\right]$$

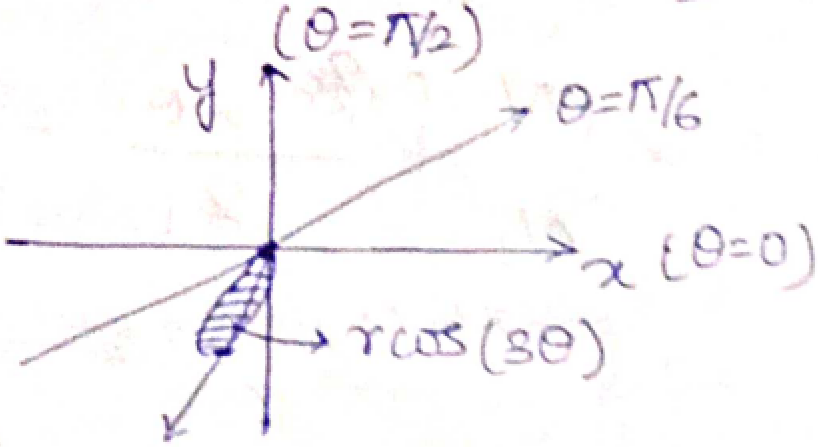
$A = 7.6529$ sq. units

Q5 One loop of the rose $r=\cos(3\theta)$

Sol: For one loop

$r=0 \Rightarrow \cos(3\theta)=0 \Rightarrow 3\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\Rightarrow \theta = \frac{\pi}{6}, \pi/2$



$$\therefore\ A = \iint_R r\,dr\,d\theta$$

$$A = \int_{\pi/6}^{\pi/2}\int_0^{\cos 3\theta} r\,dr\,d\theta = \int_{\pi/6}^{\pi/2}\left[\frac{r^2}{2}\right]_0^{\cos(3\theta)} d\theta$$

$$= \int_{\pi/6}^{\pi/2} \frac{(\cos(2\theta))^2}{2} d\theta = \frac{1}{2} \int_{\pi/6}^{\pi/2} \left[\cos^2(2\theta)\right] d\theta$$

WKT : $1 + \cos(2\theta) = 2\cos^2\theta$

$$\Rightarrow \frac{1 + \cos(2\theta)}{2} = \cos^2\theta$$

$$\Rightarrow \boxed{\frac{1 + \cos(6\theta)}{2} = \cos^2(3\theta)}$$

$$A = \frac{1}{2} \int_{\pi/6}^{\pi/2} \left[\frac{1 + \cos(6\theta)}{2}\right] d\theta$$

$$A = \frac{1}{4}\left[\theta + \frac{\sin(6\theta)}{6}\right]_{\pi/6}^{\pi/2}$$

$$= \frac{1}{4}\left\{\left(\frac{\pi}{2} + \frac{\sin(\cancel{6}^{3}(\pi/\cancel{2}))}{6}\right) - \left(\frac{\pi}{6} + \frac{\sin(\cancel{6}(\pi/\cancel{6}))}{6}\right)\right\}$$

$$= \frac{1}{4}\left\{(1.5708 + 0.0273) - (0.5236 + 0.0091)\right\}$$

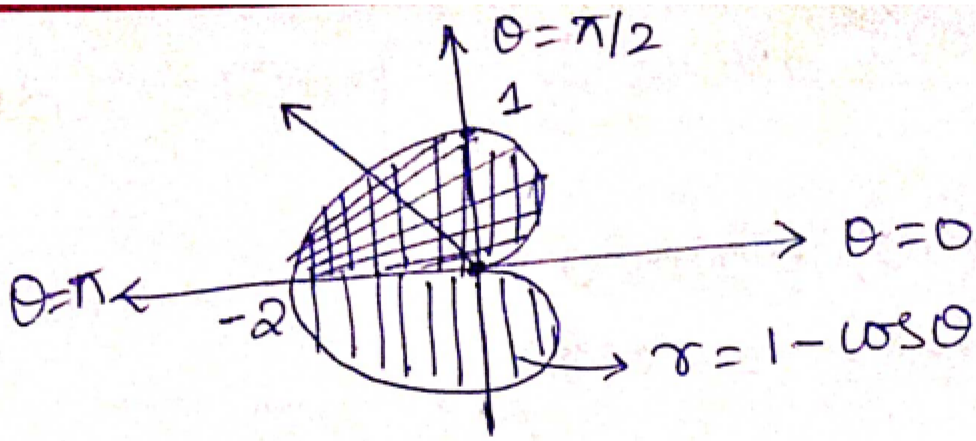
$$= \cancel{\tfrac{1}{4}}\ 0.2618 \text{ sq. units}$$

OR

$$A = \frac{1}{4}\left[\left(\frac{\pi}{2} + 0\right) - \pi/6\right] = \frac{1}{4}\left[\pi/2 - \pi/6\right]$$

$$A = \frac{1}{4}\left[\pi/3\right] = \pi/12 \text{ square units}$$

$$A = 0.2618 \text{ sq. units}$$



Given: $r = 1(1-\cos\theta)$

Required area = 2(area above x-axis)

$$A = 2\int_0^{\pi}\int_0^{1-\cos\theta} r\,dr\,d\theta = 2\int_0^{\pi}\left[\frac{r^2}{2}\right]_0^{1-\cos\theta} d\theta$$

$$A = \frac{2}{2}\int_0^{\pi}\left[(1-\cos\theta)^2\right] d\theta$$

WKT: $1-\cos 2\theta = 2\sin^2\theta$

$\Rightarrow 1-\cos\theta = 2\sin^2 \theta/2$

$$A = \int_0^{\pi}\left[2\sin^2\theta/2\right]^2 d\theta = 4\int_0^{\pi}\sin^4(\theta/2)\,d\theta$$

put $\theta/2 = t \Rightarrow d\theta/2 = dt \Rightarrow d\theta = 2dt$

$\theta = 0,\ \theta = \pi$; $t = 0$; $t = \pi/2$

$$A = 4\int_0^{\pi/2}\sin^4 t\, 2dt = 8\int_0^{\pi/2}\sin^4 t\,dt$$

Here, $p=4$, $q=0$

$$A = \frac{8\,\Gamma\frac{4+1}{2}\,\Gamma\frac{0+1}{2}}{2\,\Gamma\left(5/2+1/2\right)} = 4\,\frac{\Gamma 5/2\,\Gamma 1/2}{\Gamma 6/2} = 4\,\frac{\Gamma 5/2\,\Gamma 1/2}{\Gamma 3}$$

$$A = 4\,\frac{\frac{3}{2}\cdot\frac{1}{2}\,\Gamma 1/2\,\Gamma 1/2}{2!} = \frac{\cancel{4}\cdot\frac{3}{\cancel{2}}\cdot\frac{1}{\cancel{2}}\sqrt{\pi}\sqrt{\pi}}{\cancel{2}}$$

$A = \frac{3\pi}{2}$ sq. units