Fourier Series

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→ • A signal can be defined as a function that conveys information.

• Mathematically signals are represented as a function of one or more independent variable.

$$x(t) = x_1(t) + x_2(t) + x_3(t) + \dots$$

• The best example of signal is heart beat, music.

→ **Classification of signals:**

① Continuous time and discrete time signals:

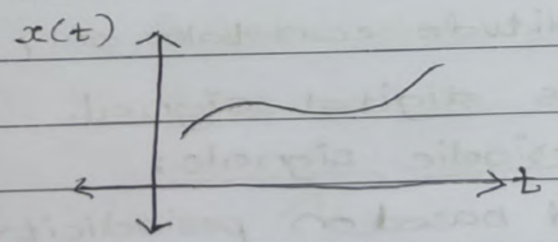
• signal can be given as continuous time signal or discrete time signal.

• continuous time signal:

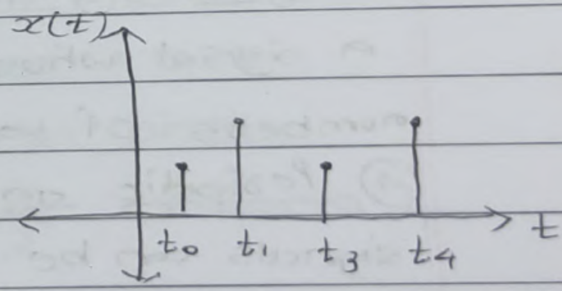
• A signal $x(t)$ is said to be a continuous time signal if it has value of amplitude for all time 't'.

• Discrete time signal:

• A discrete time signal is defined only at discrete instant of time.



(continuous)



(discrete)

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② Even and Odd signals:

- Signal can be expressed in whether they are symmetrical about y-axis or symmetric about origin.

Even signal:

A signal is said to be even signal if it satisfies the following condition:

$$x(-t) = x(t) \quad \forall t$$

Odd signal:

A signal is said to be odd signal if it satisfies the following condition:

$$x(-t) = -x(t) \quad \forall t$$

③ Analog and digital signals:

→ signals can be divided based on their amplitude

ⓐ Analog signal:

A signal whose amplitude can take any value from $-\infty$ to ∞ is analog signal.

ⓑ Digital signal:

A signal whose amplitude can take only finite numbers of values is digital signal.

④ Periodic and Aperiodic signals:

signals can be divided based on periodicity of the signal

ⓐ Periodic signal:

A signal which repeats itself after finite time 'T' is called as periodic signal.

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$$x(t) = x(t+T)$$

where 'T' is fundamental period.

ⓑ Aperiodic signal:

A signal which is not periodic is called as aperiodic signal.

P1 :) i) $x(t) = \sin\left(\frac{2\pi}{3}\right)t$

$y = A\sin\omega t$

Amplitude $= |A|$

period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi/3} \Rightarrow \underline{T=3}$

$x(t)$ is periodic signal with $T=3$.

ii) $x(t) = \cos 2t + \sin 3t$

for $\cos 2t$, $T = \frac{2\pi}{\omega} \Rightarrow T = \frac{2\pi}{2} \Rightarrow T_1 = \pi$

for $\sin 3t$, $T = \frac{2\pi}{\omega} \Rightarrow T = \frac{2\pi}{3} \Rightarrow T_2 = \frac{2\pi}{3}$

T_1 is fundamental period of $\cos 2t$

T_2 is fundamental period of $\sin 3t$.

$\frac{T_1}{T_2} = \frac{\pi}{2\pi/3} \Rightarrow \frac{T_1}{T_2} = \frac{3}{2}$ (rational)

Since $\left(\frac{T_1}{T_2}\right)$ is rational,

$\therefore x(t)$ is periodic.

Fundamental period of $x(t)$, T

function is a periodic function.

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$\frac{T_1}{T_2} = \frac{3}{2} \Rightarrow 2T_1 = 3T_2 = T$

$T = 2T_1 = 2(\pi) \Rightarrow T = 2\pi$

$T = 3T_2 = 3\left(\frac{2\pi}{3}\right) \Rightarrow T = 2\pi$

LP2 iii) $x(t) = \cos 2t + \sin \pi t$

T_1 is fundamental period of $\cos 2t$.

$T_1 = \frac{2\pi}{\omega} \Rightarrow T_1 = \frac{2\pi}{2} \Rightarrow T_1 = \pi$

T_2 is fundamental period of $\sin \pi t$.

$T_2 = \frac{2\pi}{\omega} \Rightarrow T_2 = \frac{2\pi}{\pi} \Rightarrow T_2 = 2$

$\frac{T_1}{T_2} = \frac{\pi}{2}$ (irrational)

$\therefore$ the given signal is aperiodic.

Q) $x(t) = \left(\sin\left(t - \frac{\pi}{6}\right)\right)^2$ $\qquad \cos 2\theta = 1 - 2\sin^2\theta$

$x(t) = \frac{1 - \cos\left(2t - \frac{\pi}{3}\right)}{2} = \frac{1}{2} - \frac{\cos\left(2t - \frac{\pi}{3}\right)}{2}$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$x(t) = \frac{1}{2} - \frac{1}{2}\left(\cos 2t \cdot \cos\frac{\pi}{3} + \sin 2t \cdot \sin\frac{\pi}{3}\right)$

$x(t) = \frac{1}{2} - \frac{1}{2}\left(\cos 2t \left(\frac{1}{2}\right) + \sin 2t \left(\frac{\sqrt{3}}{2}\right)\right)$

constant function is a periodic function

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$x(t) = \frac{1}{2} - \frac{1}{4}\left(\cos 2t + \sqrt{3}\sin 2t\right)$

T_1 is fundamental period of $\cos 2t$.

$T_1 = \frac{2\pi}{\omega} \Rightarrow T_1 = \frac{2\pi}{2} \Rightarrow T_1 = \pi$

T_2 is fundamental period of $\sin 2t$.

$T_2 = \frac{2\pi}{\omega} \Rightarrow T_2 = \frac{2\pi}{2} \Rightarrow T_2 = \pi$

$\frac{T_1}{T_2} = 1$ (Rational)

$\therefore$ $x(t)$ is periodic.

T is fundamental period of $x(t)$

$\frac{T_1}{T_2} = 1 \Rightarrow T_1 = T_2 = T$

$\therefore \underline{T = \pi}$.

LP1 iii) $x(t) = e^{j(\pi t - 1)}$ $\qquad (e^{j\theta} = \cos\theta + j\sin\theta)$

~~$x(t) = \cos(\pi t - 1) + j\sin(\pi t - 1)$~~

$x(t) = e^{j\pi t} \cdot e^{-j}$ $\qquad (e^{j\omega t}$

T is fundamental period of $x(t)$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} \Rightarrow \underline{T = 2}$

$\rightarrow e^{j\theta} = \cos\theta + j\sin\theta$

$e^{-j\theta} = \cos\theta - j\sin\theta$

$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$

$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j} \times \frac{j}{j} \Rightarrow \sin\theta = \frac{(e^{-j\theta} - e^{j\theta})j}{2}$

$i \times i = i^2 = \underline{-1}$

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$\sin\theta = -\frac{e^{j\theta}}{2} \cdot j + \frac{e^{-j\theta}}{2} \cdot j$

$\rightarrow e^{\pm ik\pi} = (-1)^k \quad (\cos k\pi + i\sin k\pi)$

$(\cos k\pi + 0)$

$((-1)^k)$

$\Rightarrow e^{\pm i2k\pi} = 1$

Fourier series can be defined as:

$x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos k\omega t + b_k \sin k\omega t$

$x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k\left(\frac{e^{jk\omega t} + e^{-jk\omega t}}{2}\right) + b_k\left(-\frac{e^{jk\omega t}}{2} j + \frac{e^{-jk\omega t}}{2} j\right)$

$x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} e^{jk\omega t}\left(\frac{a_k}{2} - \frac{jb_k}{2}\right) + e^{-jk\omega t}\left(\frac{a_k}{2} + \frac{jb_k}{2}\right)$

let $X(k) = \frac{a_k}{2} - \frac{jb_k}{2}$

$\Rightarrow X(-k) = \frac{a_k}{2} + \frac{jb_k}{2}$

$x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} e^{jk\omega t}\left(\frac{a_k}{2} - \frac{b_k j}{2}\right) + \sum_{k=1}^{\infty} e^{-jk\omega t}\left(\frac{a_k}{2} + \frac{b_k j}{2}\right)$

$x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} e^{jk\omega t} X(k) + \sum_{k=1}^{\infty} e^{-jk\omega t} X(-k)$

$x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} e^{jk\omega t} X(k) + \sum_{k=-\infty}^{-1} e^{jk\omega t} X(k)$

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$x(t) = \sum_{k=-\infty}^{\infty} e^{jk\omega t} X(k)$

$$X(k) = \begin{cases} \frac{a_k + b_k j}{2} & k \leq -1 \\ \frac{a_0}{2} & k = 0 \\ \frac{a_k - b_k j}{2} & k \geq 1 \end{cases}$$

while applying Fourier series a periodic and single valued signal $x(t)$ should satisfy following conditions:

① $x(t)$ is bounded.

$\int_0^T |x(t)|\,dt < \infty$ (Finite value)

② $x(t)$ has at most finite number of maxima and minima in one period.

③ $x(t)$ has at most a finite number of discontinuties in one period.

$\rightarrow$ A Fourier Series representation of Periodic continuous time signal $x(t)$ is given by

$x(t) = \sum_{k=-\infty}^{\infty} e^{jk\omega t} X(k) \rightarrow ①$

<u>Note</u>: $\tan^{-1}(\infty) = \frac{\pi}{2}$

$\tan^{-1}(-\infty) = -\frac{\pi}{2}$

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where $X(k)$ is called coefficient of $x(t)$ ~~fourier series~~ which is defined as

$$X(k) = \frac{1}{T} \int_{\langle T \rangle} x(t) e^{-jk\omega t} \, dt \rightarrow (2)$$

where 'T' is fundamental period of $x(t)$
and 'w' is fundamental angular frequency

$$\omega = \frac{2\pi}{T}$$

(1) known as synthesis eqn of Fourier series.
(2) known as analysis eqn of Fourier series.

$$x(t) \underset{}{\overset{F.S}{\rightleftarrows}} X(k)$$

(time domain) (frequency domain)

→ Coefficient of $x(t)$ is always an complex number.
i.e $X(k) = A(k) + jB(k)$

From this coefficient, we can find magnitude spectra and face spectra.

<u>Magnitude Spectra:</u>

Magnitude spectra of coefficient of $x(t)$ is defined as follows

$$|X(k)| = \sqrt{(A(k))^2 + (B(k))^2}$$

<u>Phase spectra:</u>

Face spectra of coefficient of $x(t)$ is defined as follows:

$$\phi(k) = \tan^{-1}\left(\frac{B(k)}{A(k)}\right)$$

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i) $x(t) = \cos(\omega_0 t)$

$x(t)$ is periodic signal with

Fundamental period, $T = \frac{2\pi}{\omega_0}$

$$\cos(\omega_0 t) = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

$$\cos(\omega_0 t) = \frac{1}{2} e^{j\omega_0 t} + \frac{1}{2} e^{-j\omega_0 t} \rightarrow ①$$

we know that F.S representation given as:

$$x(t) = \sum_{k=-\infty}^{\infty} X(k) e^{jk\omega_0 t} \rightarrow ②$$

$$x(t) = \ldots\ldots + X(-1) e^{-j\omega_0 t} + X(0) + X(1) e^{j\omega_0 t} + \ldots \rightarrow ③$$

$$x(t) = \frac{1}{2} e^{j\omega_0 t} + \frac{1}{2} e^{-j\omega_0 t}$$

$X(1) = \frac{1}{2}$ $\quad X(-1) = \frac{1}{2}$ $\quad X(0) = 0$ (By comparing ① & ③)

$X(k) = 0$ if $(k \neq \pm 1)$

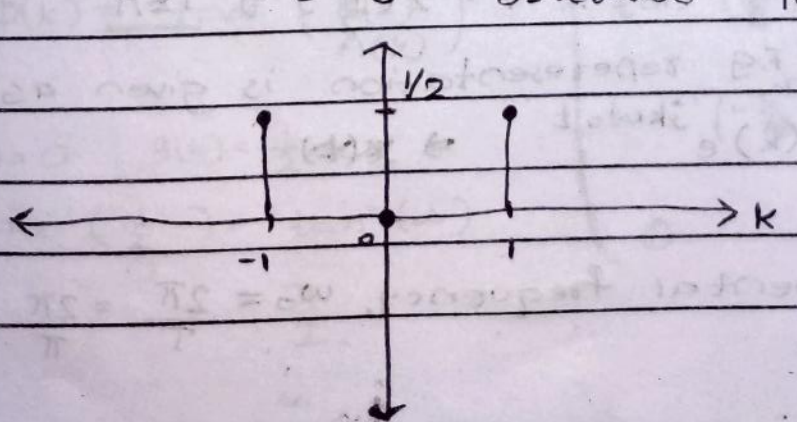
$$\therefore x(t) = \sum_{k=-\infty}^{\infty} X(k) e^{jk\omega_0 t}$$

where $X(k) = \begin{cases} 1/2 & \text{if } k = \pm 1 \\ 0 & \text{otherwise} \end{cases}$

magnitude spectra:

$|X(k)| = \begin{cases} 1/2 & k = \pm 1 \\ 0 & \text{otherwise} \end{cases}$ $\qquad X(k) = A(k) + jB(k)$

$\qquad |X(k)| = \sqrt{(A(k))^2 + (B(k))^2}$



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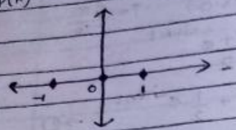
phase spectra: $\phi(k) = \tan^{-1}\left(\frac{B(k)}{A(k)}\right)$

$\phi(k) = 0 \quad \forall k$



LP2) $x(t) = \cos 4t + \sin 6t$

$$x(t) = \frac{e^{j4t}}{2} + \frac{e^{-j4t}}{2} - \frac{je^{j6t}}{2} + \frac{je^{-j6t}}{2} \rightarrow (1)$$

T_1 be fundamental period of $\cos 4t$

T_2 be fundamental period of $\sin 6t$

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{6} = \frac{\pi}{3}$$

$$\frac{T_1}{T_2} = \frac{\pi/2}{\pi/3} = \frac{3}{2} \text{ (rational)} \Rightarrow \text{periodic.}$$

'T' be fundamental period of $x(t)$

$$\frac{T_1}{T_2} = \frac{3}{2} \Rightarrow 2T_1 = 3T_2 = T$$

$$\therefore T = 2\left(\frac{\pi}{2}\right) \Rightarrow \underline{T = \pi}$$

W.R.T F.S representation is given as

$$x(t) = \sum_{k=-\infty}^{\infty} X(k) e^{jk\omega_0 t} \Rightarrow \cancel{x(t)}$$

Fundamental frequency, $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{\pi} \Rightarrow \omega_0 = 2$

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$$x(t) = \sum_{k=-\infty}^{\infty} X(k) e^{jk2t}$$

$$x(t) = \ldots X(-3)e^{-6jt} + X(-2)e^{-4jt} + \ldots + X(0) + X(3)e^{6jt} + X(2)e^{4jt} + \ldots \rightarrow (3)$$

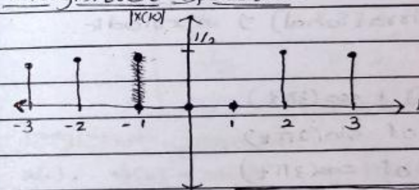
comparing (1) and (3)

$$X(-3) = \frac{j}{2} \quad X(-2) = \frac{1}{2} \quad X(2) = \frac{1}{2} \quad X(3) = \frac{-j}{2}$$

$X(0) = 0$

$$\therefore X(k) = \begin{cases} 1/2 & k = \pm 2 \\ j/2 & k = -3 \\ -j/2 & k = 3 \\ 0 & \text{otherwise} \end{cases}$$

magnitude spectra:



$$|X(k)| = \begin{cases} 1/2 & k = \pm 2 \\ 1/2 & k = \pm 3 \\ 0 & \text{otherwis} \end{cases}$$

$$|X(k)| = \sqrt{(A(k))^2 + (B(k))^2} \quad (X(k) = A(k) + jB(k))$$

phase spectra:

$$\phi(k) = \tan^{-1}\left(\frac{B(k)}{A(k)}\right) = \begin{cases} \tan^{-1}(\infty) = \frac{\pi}{2} & k = -3 \\ \tan^{-1}(-\infty) = \frac{-\pi}{2} & k = +3 \\ 0 & \cancel{\text{otherwise}} \end{cases}$$

$\frac{j}{2}$ $A(k) = 0 \quad B(k) = \frac{1}{2}$

$$\tan^{-1}\left(\frac{1/2}{0}\right) = \tan^{-1}(\infty) = \frac{\pi}{2}.$$

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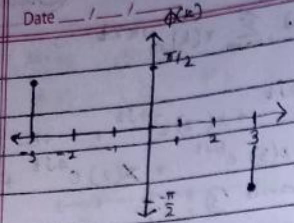

2b2 iii) $x(t) = \cos 2t + \sin \pi t$

T_1 be fundamental period of $\cos 2t$

T_2 be fundamental period of $\sin \pi t$

$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{2} = \pi$

$T_2 = \frac{2\pi}{\pi} = 2$

$\therefore \frac{T_1}{T_2} = \frac{\pi}{2}$ (irrational) $\Rightarrow$ aperiodic.

d) $x(t) = \sin(2\pi t) + \cos(3\pi t)$,

T_1 be F.P of $\sin(2\pi t)$

T_2 be F.P of $\cos(3\pi t)$

$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{2\pi} = 1$ $\quad T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{3\pi} = \frac{2}{3}$

$\frac{T_1}{T_2} = \frac{1}{2/3} = \frac{3}{2}$; $\quad \frac{T_1}{T_2} = \frac{3}{2} \Rightarrow 2T_1 = 3T_2 = T$

$\therefore T = 2T_1 \Rightarrow T = 2(1) \Rightarrow T = 2$.

Angular frequency, $\omega = \frac{2\pi}{T} = \frac{2\pi}{2}$

$\Rightarrow \underline{\omega = \pi}$.

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$x(t) = \sin(2\pi t) + \cos(3\pi t)$

$x(t) = -\frac{j}{2} e^{j2\pi t} + \frac{j}{2} e^{-j2\pi t} + \frac{1}{2} e^{j3\pi t} + \frac{1}{2} e^{-j3\pi t} \rightarrow ①$

Fourier series representation is given as:

$x(t) = \sum_{k=-\infty}^{\infty} X(k) e^{jk\omega t} = \sum_{k=-\infty}^{\infty} X(k) e^{jk\pi t}$

$x(t) = \ldots\ldots X(-3) e^{-j3\pi t} + X(-2) e^{-j2\pi t} + \ldots X(0) + \ldots$
$X(2) e^{j2\pi t} + X(3) e^{j3\pi t} + \ldots \rightarrow ②$

comparing ① and ②,

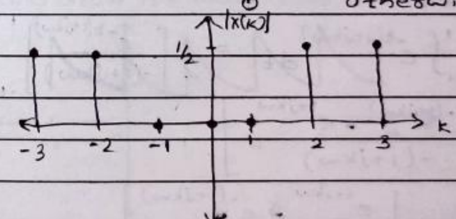
$X(-3) = \frac{1}{2} \quad X(-2) = \frac{j}{2} \quad X(2) = \frac{-j}{2} \quad X(3) = \frac{1}{2}$

$$X(k) = \begin{cases} \frac{1}{2} & k = \pm 3 \\ j/2 & k = -2 \\ -j/2 & k = 2 \\ 0 & \text{otherwise} \end{cases}$$

<u>magnitude spectra</u>:

$X(k) = A(k) + jB(k) \Rightarrow |X(k)| = \sqrt{(A(k))^2 + (B(k))^2}$

$$|X(k)| = \begin{cases} 1/2 & k = \pm 3, \pm 2 \\ 0 & \text{otherwise} \end{cases}$$



$$\int e^{at}\,dt = \frac{e^{at}}{a}$$

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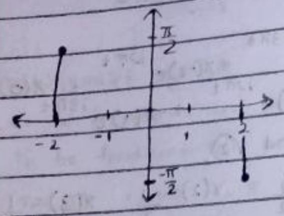
Phase spectra:

$$\phi(k) = \tan^{-1}\left(\frac{B(k)}{A(k)}\right) \Rightarrow \phi(k) = \begin{cases} \tan^{-1}(\infty) = \frac{\pi}{2} & k=-2 \\ \tan^{-1}(-\infty) = -\frac{\pi}{2} & k=2 \\ 0 & k=\pm 3 \end{cases}$$



LP3) Determine F.S representation for periodic signal $x(t)$ with period 2, $x(t) = e^{-t}$ $-1 < t < 1$

Soln: $x(t) = \sum_{k=-\infty}^{\infty} X(k)\, e^{jk\omega t}$

$$X(k) = \frac{1}{T}\int_{\langle T\rangle} x(t)\, e^{-jk\omega t}\, dt$$

fundamental Period, $T = 2$, let $-1 \le t \le 1$

$$X(k) = \frac{1}{2}\int_{-1}^{1} e^{-t} e^{-jk\omega t}\, dt = \frac{1}{2}\int_{-1}^{1} e^{-t-jk\omega t}\, dt$$

$$= \frac{1}{2}\int_{-1}^{1} e^{-t(1+jk\omega)}\, dt = \frac{1}{2}\left[\frac{e^{-t(1+jk\omega)}}{-(1+jk\omega)}\right]_{-1}^{1}$$

$$= \frac{1}{2}\left[\frac{e^{-(1+jk\omega)} - e^{1+jk\omega}}{-(1+jk\omega)}\right]$$

$$= \frac{1}{2(1+jk\omega)}\left[e^{1+jk\omega} - e^{-(1+jk\omega)}\right]$$

$\omega = \frac{2\pi}{T}$

$\omega = \frac{2\pi}{2}$

$\omega = \pi$

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let $0 \le t \le 2$

$$\frac{1}{2}\left[\frac{e^{-t(1+jk\omega)}}{-(1+jk\omega)}\right]_0^2 = \frac{1}{-2(+1+jk\omega)}\left[e^{-2(1+jk\omega)} - 1\right]$$

$$= \frac{1}{-2(+1+jk\omega)}\left[e^{-2}\cdot e^{-2jk\omega} - 1\right] = \frac{1}{-2(+1+\pi kj)}\left[e^{-2}\cdot e^{-2\pi kj}\right]$$

$$= \frac{1}{-2(+1+\pi kj)}\left[e^{-2}\left(\cos(-2\pi k) + j(\sin(-2\pi k))\right) - 1\right]$$

$$= \frac{1}{-2(+1+\pi kj)}\left(e^{-2}(1) - 1\right) = \frac{1}{-2(+1+\pi kj)}\left(e^{-2}-1\right)$$

$$= \left(\frac{e^{-2}-1}{-2}\right)\left(\frac{1}{1+jk\pi}\right)\times\left(\frac{1-jk\pi}{1-jk\pi}\right)$$

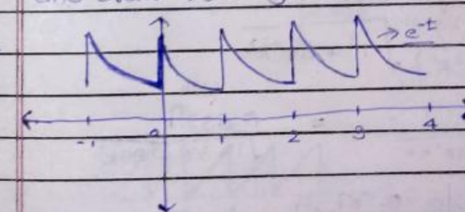
$$= \left(\frac{e^{-2}-1}{-2}\right)\left(\frac{1-jk\pi}{1-j^2k^2\pi^2}\right) = \left(\frac{e^{-2}-1}{-2}\right)\left(\frac{1-jk\pi}{1+k^2\pi^2}\right)$$

$$\frac{e^{-2}-1}{-2(1+k^2\pi^2)}\left(1-jk\pi\right) = \frac{1-e^{-2}}{2(1+k^2\pi^2)}\left(1-jk\pi\right)$$

$$= \frac{(1-e^{-2}) - j\left(k\pi(1-e^{-2})\right)}{2(1+k^2\pi^2)}$$

Q) For signal $x(t)$ shown below, find F.S representation and draw it's magnitude and face spectra.

Soln:



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from above figure, $x(t) = e^{-t}$

fundamental period, $T = 1$ $\quad (0 \leq t \leq 1)$

$$X(k) = \frac{1}{T}\int_{\langle T\rangle} x(t) e^{-tjk\omega}\, dt \qquad \omega = \frac{2\pi}{T}$$

$$X(k) = \frac{1}{1}\int_0^1 e^{-t} e^{-jtk\omega}\, dt = \int_0^1 e^{-t-tjk\omega}\, dt$$

$$= \int_0^1 e^{-t(1+jk\omega)}\, dt = \left[\frac{e^{-t(1+jk\omega)}}{-(1+jk\omega)}\right]_0^1$$

$$= \frac{e^{-(1+jk\omega)} - 1}{-(1+jk\omega)} = \frac{1}{-(1+jk\omega)}\left(e^{-1} e^{-jk\omega} - 1\right)$$

$$= \frac{1}{-(1+jk\omega)}\left(e^{-1}\left(\cos(-k\omega) + j\sin(-k\omega)\right) - 1\right)$$

$$= \frac{1}{-(1+2\pi jk)}\left(e^{-1}\left(\cos(-2\pi k) + j(\sin(-2\pi k)\right) - 1\right)$$

$$= \frac{1}{-(1+2\pi kj)}\left(e^{-1}(1) - 1\right) = \frac{1-e^{-1}}{1+2\pi kj} \times \frac{(1-2\pi kj)}{(1-2\pi kj)}$$

$$= \frac{1-e^{-1}\,(1-2\pi kj)}{1+4\pi^2k^2} = \frac{(1-e^{-1}) - j(2\pi k)(1-e^{-1})}{1+4\pi^2k^2}$$

magnitude spectra:

$$|X(k)| = \sqrt{(A(k))^2 + (B(k))^2}$$

$$|X(k)| = \frac{(1-e^{-1})}{(1+4\pi^2k^2)}\sqrt{1^2 + 4\pi^2k^2}$$

$$= \frac{(1-e^{-1})}{\sqrt{1+4\pi^2k^2}} = \frac{0.6321}{\sqrt{1+39.4784k^2}}$$

$k=1$, $|X(k)| = 0.9935$

$k=-1$ $\quad |X(k)| = 0.9935$

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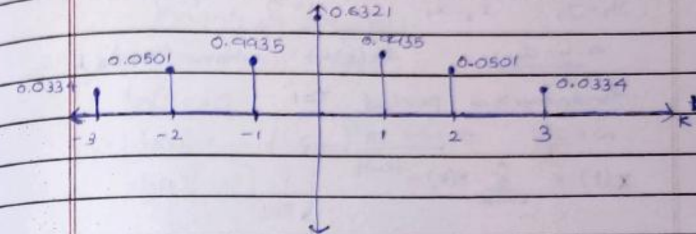
$k=2,-2$ $\quad |X(k)| = 0.0501$

$k=3,-3$ $\quad |X(k)| = 0.0334$

$k=0$ $\quad |X(k)| = 0.6321$



Phase spectra:

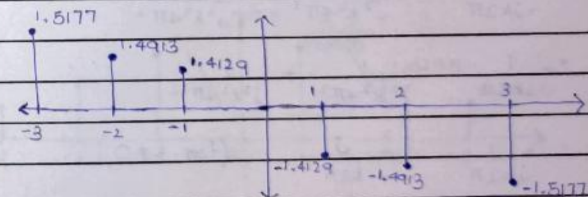
$$\tan^{-1}\left(\frac{B(k)}{A(k)}\right) = \tan^{-1}\left(\frac{-2\pi k}{1}\right)$$

$k=1$, $\tan^{-1}(-2\pi) = -1.4129$

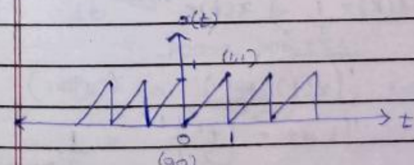
$k=-1$ $\quad \tan^{-1}(+2\pi) = +1.4129$

$k=2$, $\tan^{-1}(-4\pi) = -1.4913$, $k=-2$ $\quad \tan^{-1}(+4\pi) = +1.4913$

$k=3$, $\tan^{-1}(-6\pi) = -1.5177$, $k=-3$ $\quad \tan^{-1}(+6\pi) = +1.5177$



Q)



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from figure, $x(t) = t$?

straight line b/w $\overset{x_1, y_1}{(0,0)}$ and $\overset{x_2, y_2}{(1,1)}$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1} \Rightarrow \frac{y-0}{1-0} = \frac{x-0}{1-0}$$

$\Rightarrow \underline{y=x}$ $\therefore$ $x(t)=t$ where $0 \leq t \leq 1$

fundamental period $T=1$

$\omega = \frac{2\pi}{T} \Rightarrow \underline{\omega = 2\pi}$.

$$x(t) = \sum_{k=-\infty}^{\infty} X(k) e^{jk\omega t}$$

$$X(k) = \frac{1}{T}\int_{\langle T \rangle} x(t) e^{-jk\omega t}\, dt = \frac{1}{1}\int_0^1 t\, e^{-jk(2\pi)t}\, dt$$

$$= \int_0^1 \left[t\left(\frac{e^{-jk2\pi t}}{-jk2\pi}\right) - (1)\left(\frac{e^{-jk2\pi t}}{(-jk2\pi)^2}\right)\right]_0^1$$

$$= \left[\frac{e^{-jk2\pi}}{-jk2\pi} - \frac{e^{-jk2\pi}}{j^2k^2 4\pi^2}\right] - \left[0 - \frac{1}{j^2k^2 4\pi^2}\right]$$

$$= \frac{1}{-jk2\pi} - \frac{e^{-jk2\pi}}{j^2k^2 4\pi^2} + \frac{1}{j^2k^2 4\pi^2}$$

$$= -\frac{1}{jk2\pi} - \cancel{\frac{1}{j^2k^2 4\pi^2}} + \cancel{\frac{1}{j^2k^2 4\pi^2}}$$

$$= -\frac{1}{jk2\pi} = \frac{j}{k2\pi} \quad (\text{for } k \neq 0)$$

for $k=0$, $X(k) = \frac{1}{T}\int_0^1 x(t) e^{-jk\omega t}\, dt$

$$= \int_0^1 x(t)\, dt \quad (e^0 = 1)$$

$$= \int_0^1 t\, dt = \left[\frac{t^2}{2}\right]_0^1 = \underline{\frac{1}{2}}$$

$\int uv = uv_1 - u'v_2 + u''v_3 - \ldots\ldots$

$u' \rightarrow$ differentiation

$v_1 \rightarrow$ integration

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$$X(k) = \begin{cases} \dfrac{j}{2k\pi} & \text{if } k \neq 0 \\ \dfrac{1}{2} & \text{if } k = 0. \end{cases}$$

magnitude spectra:

$$|X(k)| = \sqrt{(A(k))^2 + (B(k))^2}$$

for $k \neq 0$,

$$|X(k)| = \sqrt{\left(\tfrac{1}{2k\pi}\right)^2}.$$

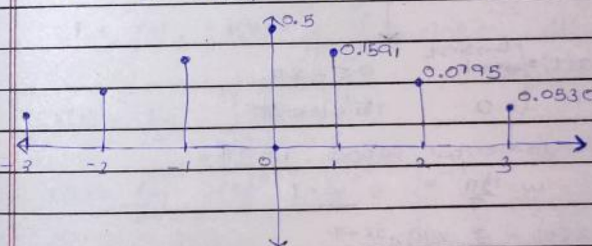
$$|X(k)| = \left|\frac{1}{2k\pi}\right|$$

$k=1$, $|X(k)| = \frac{1}{2\pi}$ | $k=-1$, $|X(k)| = +\frac{1}{2\pi}$ (0.1591)

$k=2$ $|X(k)| = \frac{1}{4\pi}$ | $k=-2$ $|X(k)| = +\frac{1}{4\pi}$ (0.0795)

$k=3$ $|X(k)| = \frac{1}{6\pi}$ | $k=-3$ $|X(k)| = +\frac{1}{6\pi}$ (0.0530)

if $k=0$, $|X(k)| = \sqrt{\left(\frac{1}{2}\right)^2} = \frac{1}{2}$.



phase spectra:

$$\Phi(k) = \tan^{-1}\left(\frac{B(k)}{A(k)}\right)$$

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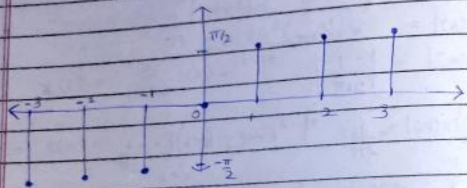
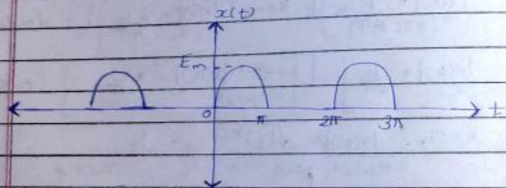
for $k=0$, $B(k)=0$ $\therefore \tan^{-1}(0)=0$.

for $k\neq 0$,

$$\Phi(k) = \tan^{-1}\left(\frac{\frac{1}{2k\pi}}{0}\right) = \begin{cases} \tan^{-1}(\infty) = \frac{\pi}{2} & k \geq 1 \\ \tan^{-1}(-\infty) = -\frac{\pi}{2} & k \leq -1 \end{cases}$$



Q)



$$x(t) = \begin{cases} E_m \sin t & 0 \leq t \leq \pi \\ 0 & \pi < t \leq 2\pi \end{cases}$$

fundamental period $T = 2\pi$

$\omega = \frac{2\pi}{T}$ $\Rightarrow$ $\omega = 1$

$$x(t) = \sum_{k=-\infty}^{\infty} X(k)\, e^{jk\omega t}$$

$$X(k) = \frac{1}{T}\int_{\langle T\rangle} x(t)\, e^{-jk\omega t}\, dt$$

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$$X(k) = \frac{1}{2\pi}\left[\int_0^{\pi} E_m \sin t\, e^{-jkt}\, dt + \int_{\pi}^{2\pi} 0\, e^{-jkt}\, dt\right]$$

$$X(k) = \frac{E_m}{2\pi}\int_0^{\pi} \sin t\, e^{-jkt}\, dt.$$

$$\int e^{ax}\sin bx\, dx = \frac{e^{ax}}{(a^2+b^2)}\left[a\sin bx - b\cos bx\right]$$

$$X(k) = \frac{E_m}{2\pi}\left[\frac{e^{-jkt}}{(1^2+(-jk)^2)}\left[(-jk)\sin x - (1)\cos x\right]\right]_0^{\pi}$$

$$= \frac{E_m}{2\pi(1+(jk)^2)}\left(e^{-\pi jk}\left((-jk)(0) - (-1)\right) - 1(0-1)\right)$$

$$= \frac{E_m}{2\pi(1+j^2k^2)}\left(e^{-jk\pi}(1) + 1\right)$$

$$= \frac{E_m}{2\pi(1-k^2)}\left[e^{-jk\pi}+1\right] \qquad k \neq \pm 1$$

$$= \frac{E_m}{2\pi(1-k^2)}\left((-1)^k + 1\right)$$

let $k=1$, $X(k) = \frac{1}{T}\int_0^{\pi} E_m \sin t\, e^{-jt}\, dt$

$$X(k) = \frac{E_m}{2\pi}\int_0^{\pi} \sin t\, e^{-jt}\, dt$$

$$X(k) = \frac{E_m}{2\pi}\int_0^{\pi}\left(\frac{-j}{2}e^{jt} + \frac{j}{2}e^{-jt}\right)e^{-jt}\, dt$$

$$X(k) = \frac{E_m}{2\pi}\int_0^{\pi}\left(\frac{-j}{2} + \frac{j}{2}e^{-2jt}\right)dt.$$

$$X(k) = \frac{E_m}{2\pi}\left[\frac{-jt}{2} + \frac{j}{2}\frac{e^{-2jt}}{(-2j)}\right]_0^{\pi}$$

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$$X(k) = \frac{E_m j}{4\pi}\left[-t + \frac{e^{-2jt}}{(-2j)}\right]_0^{\pi}$$

$$X(k) = \frac{E_m j}{4\pi}\left[\left(-\pi + \frac{e^{-2\pi j}}{-2j}\right) - \left(0 + \frac{1}{-2j}\right)\right]$$

$$X(k) = \frac{E_m j}{4\pi}\left[-\pi + \cancel{\frac{1}{(-2j)}} + \cancel{\frac{1}{2j}}\right]$$

$$X(k) = -\frac{E_m j}{4}$$

let $k=-1$, $X(k) = \frac{1}{T}\int_0^{\pi} E_m \sin t \, e^{jt}\, dt$

$$X(k) = \frac{E_m}{2\pi}\int_0^{\pi}\left(\frac{-je^{jt}}{2} + \frac{je^{-jt}}{2}\right)e^{jt}\,dt$$

$$X(k) = \frac{E_m j}{4\pi}\int_0^{\pi}\left(-e^{2jt} + 1\right)dt$$

$$X(k) = \frac{E_m j}{4\pi}\left[-\frac{(e^{2jt})}{2j} + t\right]_0^{\pi}$$

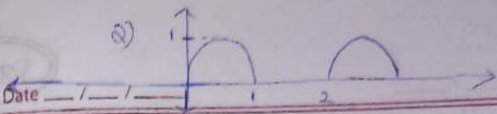
$$= \frac{E_m j}{4\pi}\left[\left(-\frac{e^{2\pi j}}{2j} + \pi\right) - \left(\frac{-1}{2j} + 0\right)\right]$$

$$= \frac{E_m j}{4\pi}\left[-\cancel{\frac{1}{2j}} + \pi + \cancel{\frac{1}{2j}}\right]$$

$$= \frac{E_m j \pi}{4\pi} = \frac{E_m j}{4}$$

$$X(k) = \begin{cases} \frac{E_m}{2\pi(1-k^2)}\left((-1)^k + 1\right) & k \neq \pm 1 \\ \frac{E_m j}{4} & k = -1 \\ -\frac{E_m j}{4} & k = 1 \end{cases}$$

2)



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if k is even $(-1)^k = 1$ and if k is odd $(-1)^k = -1$

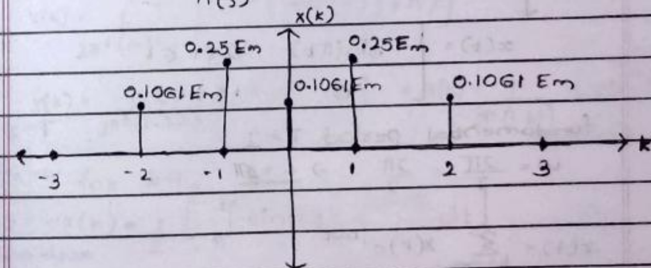
$$\therefore X(k) = \begin{cases} 0 & \text{if } k \text{ is odd} \quad k \neq \pm 1 \\ \frac{E_m}{\pi(1-k^2)} & k \text{ is even} \end{cases}$$

magnitude spectra:

$$|X(k)| = \begin{cases} \frac{\pi E_m}{4} & k = \pm 1 \\ 0 & k \text{ is odd and } k \neq \pm 1 \\ \left|\frac{E_m}{\pi(1-k^2)}\right| & k \text{ is even} \end{cases}$$

when $k = \pm 2$ $\quad |X(k)| = \left|\frac{E_m}{\pi(1-k^2)}\right| = \left|\frac{E_m}{\pi(1-4)}\right| = \frac{E_m}{\pi}\left|\frac{1}{-3}\right|$

$$= \frac{E_m}{\pi(3)} = 0.1061 E_m$$



phase spectra:

$$\phi(k) = \tan^{-1}\left(\frac{B(k)}{A(k)}\right)$$

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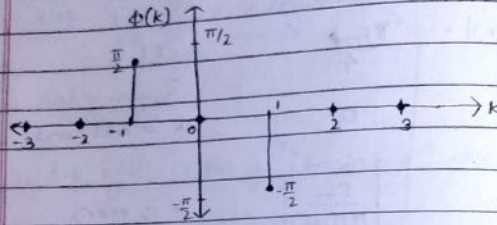
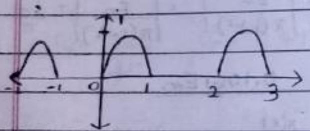
$$\Phi(k) = \begin{cases} \tan^{-1}(-\infty) = -\frac{\pi}{2} & k=1 \\ \tan^{-1}(\infty) = \frac{\pi}{2} & k=-1 \\ \tan^{-1}(0) = 0 & k \neq \pm 1 \end{cases}$$



3)



$$x(t) = \begin{cases} \sin(\pi t) & 0 \leq t \leq 1 \\ 0 & 1 \leq t \leq 2 \end{cases}$$

fundamental period, $T = 2$.

$\omega = \frac{2\pi}{T} = \frac{2\pi}{2} \Rightarrow \underline{\omega = \pi}$

$T = 2$

$\omega = \frac{2\pi}{2}$

$\omega = \pi$

$x(t) = A\sin\omega t$

$x(t) = \sin\pi t$

$$x(t) = \sum_{k=-\infty}^{\infty} X(k) e^{jk\omega t}$$

$$X(k) = \frac{1}{T} \int_{\langle T \rangle} x(t) e^{-jk\omega t} dt$$

$$X(k) = \frac{1}{2} \left[\int_0^1 \sin(\pi t) e^{-jk\omega t} dt + \int_1^2 0 . e^{-jk\omega t} dt \right]$$

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$$X(k) = \frac{1}{2} \left[\int_0^1 \sin(\pi t) e^{-jk\omega t} dt \right]$$

$$\cancel{X(k) = \frac{1}{2} \int_0^1 \left(\frac{-je^{j\pi t}}{2} + \frac{je^{-j\pi t}}{2} \right) e^{-jk\pi t}}$$

$$\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2} \left[a\sin bx - b\cos bx \right]$$

$$X(k) = \frac{1}{2} \left[\frac{e^{-jk\omega t}}{(-jk\pi)^2 + (\pi)^2} \left((-jk\pi)\sin\pi t - \pi\cos\pi t \right) \right]_0^1$$

$$X(k) = \frac{1}{2} \left[\frac{e^{-jk\pi t}}{(1+j^2k^2)(\pi^2)} \left(-jk\pi \sin\pi t - \pi\cos\pi t \right) \right]_0^1$$

$$\cancel{X(k) = \frac{1}{2\pi^2(1+j^2k^2)} (-jk\pi \sin\pi}$$

$$X(k) = \frac{1}{\pi^2 2(1-k^2)} \left[e^{-jk\pi} \left(-jk\pi \sin(\pi) - \pi\cos\pi \right) - 1 \left(0 - \pi(1) \right) \right]$$

$$X(k) = \frac{1}{2\pi^2(1-k^2)} \left[(-1)^k \left(-\pi(-1) \right) + \pi \right]$$

$$X(k) = \frac{1}{2\pi^2(1-k^2)} \left[\pi(-1)^k + \pi \right] \Rightarrow X(k) = \frac{1}{2\pi(1-k^2)} \left((-1)^k + 1 \right) (k \neq \pm 1)$$

for $k=1$,

$$X(k) = \frac{1}{2} \left[\int_0^1 \sin(\pi t) . e^{-j\omega t} dt \right]$$

$$X(k) = \frac{1}{2} \left[\int_0^1 \left(\frac{-j}{2} e^{j\pi t} + \frac{j}{2} e^{-j\pi t} \right) e^{-j\pi t} dt \right]$$

$$X(k) = \frac{1}{2} \left[\int_0^1 \left(\frac{-j}{2} + \frac{j}{2} e^{-2j\pi t} \right) dt \right]$$

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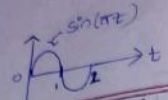

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$$x(k) = \frac{j}{4}\int_0^1 \left(-1 + e^{-2j\pi t}\right) dt$$

$$x(k) = \frac{j}{4}\left[-t - \frac{e^{-2j\pi t}}{2j\pi}\right]_0^1$$

$$x(k) = \frac{j}{4}\left[\left(-1 - \frac{e^{-2j\pi}}{2j\pi}\right) - \left(0 - \frac{1}{2j\pi}\right)\right]$$

$$= \frac{j}{4}\left[-1 - \cancel{\frac{1}{2j\pi}} + \cancel{\frac{1}{2j\pi}}\right]$$

$$x(k) = \frac{-j}{4}$$

for $k=1$

$$x(k) = \frac{1}{2}\left[\int_0^1 \sin(\pi t)\, e^{j\pi t}\, dt\right]$$

$$x(k) = \frac{1}{2}\left[\int_0^1 \left(\frac{-j}{2}e^{j\pi t} + \frac{j}{2}e^{-j\pi t}\right) e^{j\pi t}\, dt\right]$$

$$x(k) = \frac{1}{2}\left[\int_0^1 \left(\frac{-j}{2}e^{2j\pi t} + \frac{j}{2}\right) dt\right]$$

$$x(k) = \frac{j}{4}\left[\int_0^1 \left(-e^{2j\pi t} + 1\right) dt\right]$$

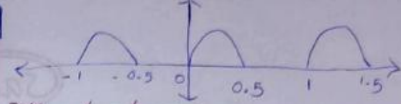
$$x(k) = \frac{j}{4}\left[\frac{-e^{2j\pi t}}{2j\pi} + t\right]_0^1$$

$$= \frac{j}{4}\left[\left(\frac{-e^{2j\pi}}{2j\pi} + 1\right) - \left(-\frac{1}{2j\pi} + 0\right)\right]$$

$$= \frac{j}{4}\left[-\cancel{\frac{1}{2j\pi}} + 1 + \cancel{\frac{1}{2j\pi}}\right]$$

$$= \frac{j}{4}$$

Q1]



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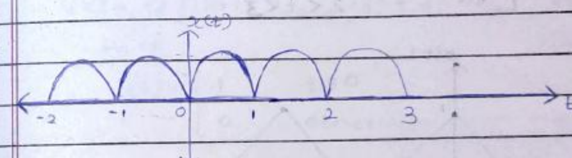
for $k=\text{even}$, $x(k) = \frac{1}{2\pi(1-k^2)}\left((+1)+1\right) = \frac{1}{\pi(1-k^2)}$

for $k=\text{odd}$, $k \neq \pm 1$, $\frac{1}{2\pi(1-k^2)}(-1+1) = 0$

$$x(k) = \begin{cases} \frac{j}{4} & k=-1 \\ \frac{-j}{4} & k=1 \\ 0 & k=\text{odd and } k\neq \pm 1 \\ \frac{1}{\pi(1-k^2)} & k \text{ is even} \end{cases}$$

Q2]



$x(t) = |\sin \pi t|$ $\quad T=1$

$$x(t) = \sum_{k=-\infty}^{\infty} x(k)\, e^{jk\omega t}$$

$$X(k) = \frac{1}{T}\int_{\langle T\rangle} x(t)\, e^{-jk\omega t}\, dt$$

$$= \frac{1}{T}\int_0^1 \sin \pi t\, e^{-jk\omega t}\, dt$$

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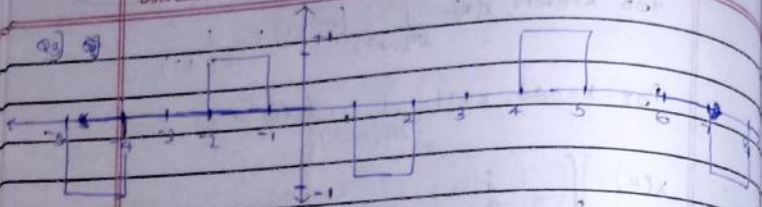
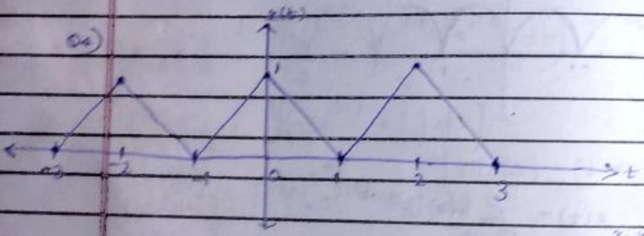
Q3)



$T = 6 \quad , \omega = \frac{2\pi}{6} \quad \omega = \frac{\pi}{3}$

$$x(t) = \begin{cases} 0 & -3 < t < -2 \\ 1 & -2 < t < -1 \\ 0 & -1 < t < 1 \\ -1 & 1 < t < 2 \\ 0 & 2 < t < 3 \end{cases}$$

Q4)



$T = 2 \quad \omega = \frac{2\pi}{2} \Rightarrow \omega = \pi$

$$x(t) = \begin{cases} t+1 & -1 \le t \le 0 \\ -t+1 & 0 \le t \le 1 \end{cases}$$

$x_1\ y_1 \quad x_2\ y_2$
$(-1,0) \quad (0,1)$

$\frac{y-0}{x+1} = \frac{1}{1}$

$y = x+1$

$x_1\ y_1 \quad x_2\ y_2$
$(0,1) \quad (1,0)$

$\frac{y-0}{x-1} = \frac{1}{-1}$

$y = -x+1$

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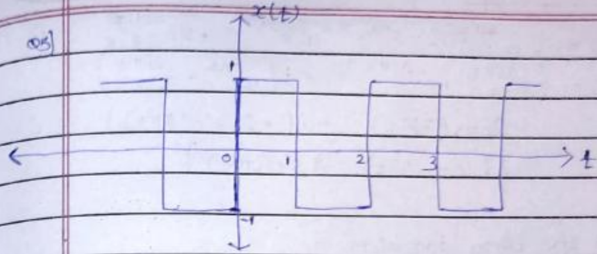
Q5)



$T = 2 \qquad \omega = \frac{2\pi}{T} \Rightarrow \omega = \pi$

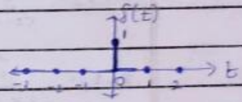
$$x(t) = \begin{cases} -1 & -1 < t < 0 \\ +1 & 0 < t < 1 \end{cases}$$

Find time domain signal whose F.S coefficient is

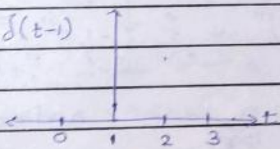
$x(k) = -j\,\delta(k+1) + \delta(k-3) + \delta(k+3) + j\,\delta(k-1)$

$\omega_0 = \pi$

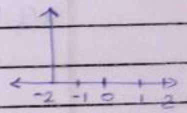
$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}$$



$\delta(t-1)$



$\delta(t+2)$



$x(t) = \sum_{k=-\infty}^{\infty} x(k)\, e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} x(k)\, e^{jk\pi t}$

$x(t) = \sum_{k=-\infty}^{\infty} \left(j\delta(k-1) - j\delta(k+1) + \delta(k-3) + \delta(k+3) \right) e^{jk\pi t}$

$x(t) = \sum_{k=-\infty}^{\infty} \left(j\delta(k-1)\, e^{jk\pi t} - j\delta(k+1)\, e^{jk\pi t} + \delta(k-3)\, e^{jk\pi t} + \delta(k+3)\, e^{jk\pi t} \right.$

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$\frac{e^{i\theta}+e^{-i\theta}}{2} = \cos\theta$

$\frac{e^{i\theta}-e^{-i\theta}}{2i} = \sin\theta$

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$x(t) = j e^{j\pi t} - j e^{-j\pi t} + e^{3j\pi t} + e^{-3j\pi t}$

$x(t) = \left(e^{3j\pi t} + e^{-3j\pi t}\right) + j\left(e^{j\pi t} - e^{-j\pi t}\right)$

$2\cos(3\pi t) + j(+2j\sin(\pi t))$

$2(\cos(3\pi t) - \sin(\pi t))$

Q) Find the time domain

$X(k) = (-1/2)^{|k|} \qquad \omega_0 = 1$

$x(t) = \sum_{k=-\infty}^{\infty} X(k)\, e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} X(k)\, e^{jkt}$

$x(t) = \sum_{k=-\infty}^{\infty} (-1/2)^{|k|} e^{jkt}$

$x(t) = \sum_{k=-\infty}^{-1} (-1/2)^{-k} e^{jkt} + \sum_{k=1}^{\infty} (-1/2)^{k} e^{jkt} + (-1/2)^0 e^0$

$x(t) = \sum_{k=-\infty}^{-1} \left(-\frac{1}{2}\right)^{-k} e^{jkt} + \sum_{k=0}^{\infty} (-1/2)^{k} e^{jkt}$

$x(t) = \sum_{k=1}^{\infty} \left(-\frac{1}{2}\right)^{k} e^{-jkt} + \sum_{k=0}^{\infty} (-1/2)^{k} e^{jkt}$

$x(t) = \sum_{k=1}^{\infty} \left(\frac{-1}{2} e^{-jt}\right)^k + \sum_{k=0}^{\infty} \left(\frac{-1}{2} e^{jt}\right)^k$

$\sum_{n=0}^{\infty} a r^{n-1} = \frac{a}{1-r}$; a→first term

$x(t) = \frac{\left(\frac{-1}{2} e^{-jt}\right)}{1-\left(\frac{-1}{2} e^{-jt}\right)} + \frac{1}{1-\left(-\frac{1}{2} e^{jt}\right)}$

$x(t) = \frac{\left(-\frac{1}{2} e^{-jt}\right)}{1+\frac{1}{2}e^{-jt}} + \frac{1}{1+\frac{1}{2}e^{jt}}$

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$X(k) = \begin{cases} \left(\frac{-1}{2}\right)^{-k} & k<0 \\ \left(\frac{-1}{2}\right)^{k} & k \geq 0 \end{cases}$

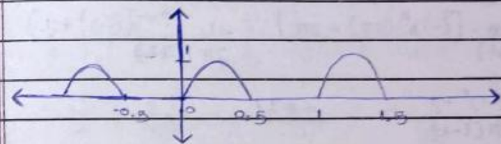
$x(t) = \frac{\left(\frac{-1}{2} e^{-jt}\right)\left(1+\frac{1}{2}e^{jt}\right) + 1 + \frac{1}{2}e^{-jt}}{\left(1+\frac{e^{-jt}}{2}\right)\left(1+\frac{1}{2}e^{jt}\right)}$

$= \frac{\left(-\frac{1}{2}e^{-jt} - \frac{1}{4}e^{0}\right) + 1 + \frac{1}{2}e^{-jt}}{\left(1+\frac{e^{-jt}}{2} + \frac{1}{2}e^{jt} + \frac{1}{4}e^{0}\right)}$

$= \frac{1-\frac{1}{4}}{1+\frac{1}{4}+\frac{1}{2}\left(e^{-jt}+e^{jt}\right)} = \frac{(3/4)}{\frac{5}{4} + \frac{1}{2}(2\cos t)}$

$x(t) = \frac{3}{5+4\cos t}$

Q]



$T=1, \quad \omega = \frac{2\pi}{T} \Rightarrow \omega = 2\pi$

$x(t) = A\sin\omega t \Rightarrow x(t) = \sin(2\pi t)$

$\therefore x(t) = \begin{cases} \sin(2\pi t) & 0 \leq t \leq 0.5 \\ 0 & 0.5 \leq t \leq 1 \end{cases}$

$x(t) = \sum_{k=-\infty}^{\infty} X(k)\, e^{jk\omega t}$

$X(k) = \frac{1}{T}\int_{\langle T\rangle} x(t)\, e^{-jk\omega t}\, dt$

$X(k) = \frac{1}{1}\left[\int_0^{0.5} \sin(2\pi t)\, e^{-jk\omega t}\, dt + \int_{0.5}^{1} (0)\, e^{-jk\omega t}\, dt\right]$

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$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{(a^2+b^2)}\left[a\sin bx - b\cos bx\right]$$

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$$x(k) = \int_0^{0.5} \sin(2\pi t)\, e^{-jk\omega t}\, dt = \int_0^{0.5} \sin(2\pi t)\, e^{-jk2\pi t}\, dt$$

$$x(k) = \left[\frac{e^{-jk2\pi t}}{(-jk2\pi)^2+(2\pi)^2}\left((-jk2\pi)\sin(2\pi t) - (2\pi)\cos(2\pi t)\right)\right]_0^{0.5}$$

$(-j)^2$

$(-1)^2(j)^2$

$= -1$

$$x(k) = \frac{1}{4\pi^2 - k^2(4\pi^2)}\left[e^{-jk2\pi t}\left((-jk2\pi)\sin(2\pi t) - 2\pi\cos(2\pi t)\right)\right]$$

$$x(k) = \frac{1}{4\pi^2(1-k^2)}\left[e^{-jk\pi}\left((-jk2\pi)\sin(\pi) - 2\pi\cos(\pi)\right) - 1\left((-jk2\pi)0 - 2\pi\cos(0)\right)\right]$$

$$= \frac{1}{4\pi^2(1-k^2)}\left[e^{-jk\pi}(0+2\pi) - 1(-2\pi)\right]$$

$$= \frac{1}{4\pi^2(1-k^2)}\left[\cos(k\pi)(2\pi) + 2\pi\right]$$

$$= \frac{1}{4\pi^2(1-k^2)}\left[(-1)^k(2\pi)+2\pi\right] = \frac{1}{2\pi(1-k^2)}\left((-1)^k+1\right)$$

$$x(k) = \frac{(-1)^k+1}{2\pi(1-k^2)} \qquad k\neq \pm 1$$

for $k=1$,

$$x(k)=x(1)=(1)\int_0^{0.5}\sin(2\pi t)e^{-jk\omega t}dt = \int_0^{0.5}\sin(2\pi t)e^{-j2\pi t}dt$$

$$= \int_0^{0.5}\left(\frac{e^{j2\pi t}-e^{-j2\pi t}}{2j}\right)e^{-j2\pi t}dt$$

$$\int_0^{0.5}\frac{e^{0} - e^{-4j\pi t}}{2j}dt$$

$$= \frac{1}{2j}\int_0^{0.5}\left(1-e^{-4j\pi t}\right)dt$$

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$$= \frac{-j}{2}\left[t + \frac{e^{-4j\pi t}}{4j\pi}\right]_0^{0.5}$$

$$= \frac{-j}{2}\left[\left(0.5 + \frac{e^{-2\pi j}}{4j\pi}\right) - \left(0 + \frac{1}{4j\pi}\right)\right]$$

$$= \frac{-j}{2}\left[\frac{1}{2} + \frac{1}{4j\pi} - \frac{1}{4j\pi}\right] = \frac{-j}{4}$$

for $k=-1$

$$x(k)=x(-1) = \int_0^{0.5}\sin(2\pi t)e^{-jk\omega t}dt = \int_0^{0.5}\sin(2\pi t)e^{j2\pi t}dt$$

$$= \int_0^{0.5}\left[\frac{e^{j2\pi t}-e^{-j2\pi t}}{2j}\right]\left(e^{j2\pi t}\right)dt$$

$$= \frac{1}{2j}\left[\int_0^{0.5}\left(e^{4j\pi t}-1\right)dt\right] = \frac{-j}{2}\left(\int_0^{0.5}\left(e^{4j\pi t}-1\right)dt\right)$$

$$= \frac{-j}{2}\left(\left[\frac{e^{4j\pi t}}{4j\pi} - t\right]_0^{0.5}\right) = \frac{-j}{2}\left(\left(\frac{e^{2j\pi}}{4j\pi}-0.5\right)-\left(\frac{1}{4j\pi}-0\right)\right)$$

$$= \frac{-j}{2}\left(\frac{1}{4j\pi}-\frac{1}{2}-\frac{1}{4j\pi}\right) = \frac{j}{4}$$

$$x(k) = \frac{(-1)^k+1}{2\pi(1-k^2)}$$

for $k=\text{even}$, $x(k) = \frac{1}{\pi(1-k^2)}$, for $k=\text{odd}$, $x(k) = 0$

$$x(k) = \begin{cases} \frac{1}{\pi(1-k^2)} & k=\text{even} \\ 0 & k=\text{odd} \quad k\neq\pm1 \\ \frac{j}{4} & k=-1 \\ \frac{-j}{4} & k=1 \end{cases}$$

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if $1 \le t \le 2$ $x(k) = \frac{1}{1}\int_1^2 -\sin\pi t \, e^{-jk\omega t}\, dt$

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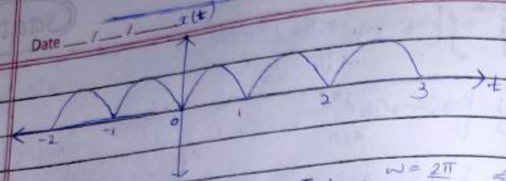
Q3)



$T = 1$ $\quad \omega = \frac{2\pi}{1} \Rightarrow \omega = 2\pi$

$x(t) = |\sin \pi t|$

$x(t) = \sum X(k) e^{jk\omega t}$

$X(k) = \frac{1}{T}\int_{\langle T\rangle} x(t) e^{-jk\omega t}\, dt$

$X(k) = \frac{1}{1}\left[\int_0^1 \sin\pi t\, e^{-jk\omega t}\, dt\right]$

$X(k) = \left[\frac{e^{-jk\omega t}}{(-jk\omega)^2 + (\pi)^2}\left((-jk\omega)\sin\pi t - \pi\cos(\pi t)\right)\right]_0^1$

$X(k) = \frac{1}{(-jk\omega)^2+\pi^2}\left[e^{-jk2\pi t}\left(-jk2\pi\sin\pi t - \pi\cos\pi t\right)\right]_0^1$

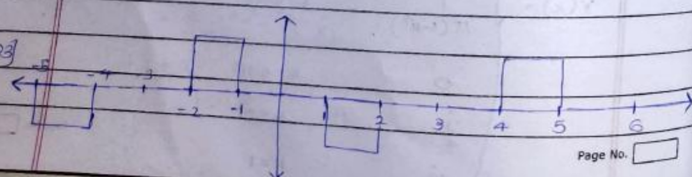
$X(k) = \frac{1}{\pi^2 + (-j2\pi k)^2}\left[\left(e^{-2\pi jk}\left(-jk2\pi\sin\pi - \pi\cos\pi\right) - 1(0 - \pi\cos 0)\right)\right]$

$X(k) = \frac{1}{\pi^2(1-4k^2)}\left[1(0+\pi) - 1(-\pi)\right]$

$X(k) = \frac{1}{\pi^2(1-4k^2)}(2\pi) = \frac{2}{\pi(1-4k^2)}$

$X(k) = \frac{2}{\pi(1-4k^2)}$

Q3)



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$T = 6$, $\quad \omega = \frac{2\pi}{T} \Rightarrow \omega = \frac{2\pi}{6} \Rightarrow \omega = \frac{\pi}{3}$

$$x(t) = \begin{cases} 0 & -3 < t < -2 \\ 1 & -2 < t < -1 \\ 0 & -1 < t < 1 \\ -1 & 1 < t < 2 \\ 0 & 2 < t < 3 \end{cases}$$

$X(k) = \frac{1}{T}\int_{\langle T\rangle} x(t) e^{-jk\omega t}\, dt$

$X(k) = \frac{1}{6}\left[\int_{-2}^{-1} 1 \cdot e^{-jk\omega t}\, dt + \int_1^2 (-1)\cdot e^{-jk\omega t}\, dt\right]$

$X(k) = \frac{1}{6}\left[\left[\frac{e^{-jk\omega t}}{-jk\omega}\right]_{-2}^{-1} - 1\left[\frac{e^{-jk\omega t}}{-jk\omega}\right]_1^2\right]$

$X(k) = \frac{1}{6(-jk\omega)}\left[\left[e^{-jk\omega t}\right]_{-2}^{-1} - \left[e^{-jk\omega t}\right]_1^2\right]$

$X(k) = \frac{1}{6(-jk\omega)}\left[\left(e^{jk\omega} - e^{jk2\omega}\right) - \left(e^{-2k\omega j} - e^{-k\omega j}\right)\right]$

$X(k) = \frac{1}{6\left(-jk\frac{\pi}{3}\right)}\left[e^{jk\pi/3} - e^{jk\frac{2\pi}{3}} - e^{-jk\frac{2\pi}{3}} + e^{-jk\frac{\pi}{3}}\right]$

$X(k) = \frac{1}{-2jk}\left[\left(e^{jk\pi/3} + e^{-jk\pi/3}\right) - \left(e^{jk2\pi/3} + e^{-jk2\pi/3}\right)\right]$

$X(k) = \frac{j}{2k}\left[2\cos\left(\frac{k\pi}{3}\right) - 2\cos\left(\frac{2k\pi}{3}\right)\right]$

$X(k) = \frac{j}{k}\left[\cos\left(\frac{k\pi}{3}\right) - \cos\left(2\frac{k\pi}{3}\right)\right]$ for $k \neq 0$

$= \frac{j}{k}\{$ 2sin

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for $k=0$,

$$x(k) = \frac{1}{6}\left[\int_{-2}^{-1} 1.e^{0}dt + \int_{1}^{2}(-1)\,e^{0}dt\right]$$

$$x(k) = \frac{1}{6}\left[[t]_{-2}^{-1} - [t]_{1}^{2}\right]$$

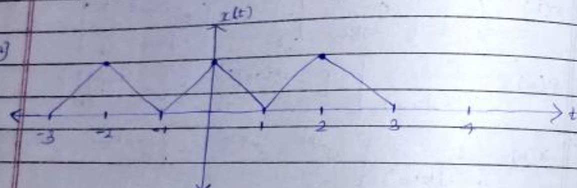
$$= \frac{1}{6}\left[[-1-(-2)] - [2-1]\right]$$

$$= \frac{1}{6}\left[-1+2-1\right]$$

$$= \underline{\underline{0}}$$

$$x(k) = \begin{cases} 0 & k=0 \\ \frac{j}{k}\left(\cos\frac{k\pi}{3} - \cos\frac{2k\pi}{3}\right) & \text{otherwise} \end{cases}$$

Q4]



$T=2,\quad \omega = \frac{2\pi}{T} = \frac{2\pi}{2} \Rightarrow \omega = \underline{\pi}$

$$x(t) = \begin{cases} t+1 & -1 \le t \le 0 \\ -t+1 & 0 \le t \le 1 \end{cases}$$

$$x(t) = \sum X(k)e^{jk\omega t}$$

$$X(k) = \frac{1}{T}\int_{\langle T\rangle} x(t)e^{-jk\omega t}\,dt$$

$$X(k) = \frac{1}{2}\left[\int_{-1}^{0}(t+1)e^{-jk\omega t}dt + \int_{0}^{1}(-t+1)e^{-jk\omega t}dt\right]$$

$$= \frac{1}{2}\left[\int_{-1}^{0}\left(t e^{-jk\omega t} + e^{-jk\omega t}\right)dt + \int_{0}^{1}\left(-t e^{-jk\omega t} + e^{-jk\omega t}\right)dt\right.$$

$$\int uv = uv_1 - u'v_2 + u''v_3 -$$

$$\int t e^{-at}dt = \left(t\frac{e^{-at}}{-a}\right) - (1)\left(\frac{e^{-at}}{a^2}\right) + (0) \dots$$

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$$= \frac{1}{2}\left[\frac{t e^{-jk\omega t}}{(-jk\omega)} - \frac{e^{-jk\omega t}}{(-jk\omega)^2} - \frac{e^{-jk\omega t}}{(+jk\omega)}\right]_{-1}^{0}$$

$$+ \frac{1}{2}\left[-\left(\frac{t e^{-jk\omega t}}{(-jk\omega)} - \frac{e^{-jk\omega t}}{(jk\omega)^2}\right) - \frac{e^{-jk\omega t}}{jk\omega}\right]_{0}^{1}$$

$$= \frac{1}{2}\left[\left(0 - \frac{1}{(jk\pi)^2} - \frac{1}{jk\pi}\right) - \left(\frac{(-1)e^{jk\pi}}{(-jk\pi)} - \frac{e^{jk\pi}}{(jk\pi)^2} - \frac{e^{jk\pi}}{jk\pi}\right)\right]$$

$$+ \frac{1}{2}\left[\left(-\left(\frac{e^{-jk\pi}}{(-jk\pi)} - \frac{e^{-jk\pi}}{(jk\pi)^2}\right) - \frac{e^{-jk\pi}}{jk\pi}\right) - \left(-\left(0 - \frac{1}{(jk\pi)^2}\right) - \frac{1}{jk\pi}\right)\right]$$

$$= \frac{1}{2}\left[-\frac{1}{(jk\pi)^2} - \cancel{\frac{1}{jk\pi}} - \cancel{\frac{(-1)^k}{(jk\pi)}} + \frac{(-1)^k}{(jk\pi)^2} + \cancel{\frac{(-1)^k}{(jk\pi)}}\right.$$

$$\left. + \cancel{\frac{(-1)^k}{jk\pi}} + \frac{(-1)^k}{(jk\pi)^2} - \cancel{\frac{(-1)^k}{jk\pi}} - \frac{1}{(jk\pi)^2} + \cancel{\frac{1}{jk\pi}}\right]$$

$$= \frac{2}{2}\left[\frac{-1}{j^2k^2\pi^2} + \frac{(-1)^k}{j^2k^2\pi^2}\right] = \left(\frac{1}{k^2\pi^2} - \frac{(-1)^k}{k^2\pi^2}\right)$$

$$X(k) = \frac{1}{k^2\pi^2}\left((-1)^k + 1\right) \qquad k \ne 0$$

for $k=0$,

$$X(k) = \frac{1}{2}\left[\int_{-1}^{0}(t+1)e^{0}dt + \int_{0}^{1}(-t+1)e^{0}dt\right]$$

$$= \frac{1}{2}\left[\left[\frac{t^2}{2}+t\right]_{-1}^{0} + \left[-\frac{t^2}{2}+t\right]_{0}^{1}\right]$$

$$= \frac{1}{2}\left[0 - \left(\frac{1}{2} - 1\right) + \left(-\frac{1}{2} + 1 - 0\right)\right]$$

$$= \frac{1}{2}\left[0 - \frac{1}{2} + 1 - \frac{1}{2} + 1\right] = \frac{1}{2}\left(2\left(1 - \frac{1}{2}\right)\right)$$

$$= \underline{\underline{\frac{1}{2}}}$$

~~X(k) = 0~~

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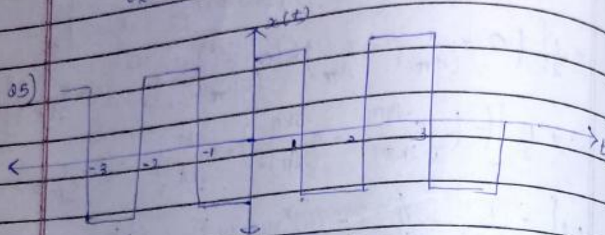
$$X(k) = \begin{cases} \frac{1}{2} & k=0 \\ \frac{1}{k^2\pi^2}\left((-1)^k+1\right) & \text{otherwise} \end{cases}$$

Q5)



$T=2 \qquad \omega = \frac{2\pi}{T} \Rightarrow \underline{\omega = \pi}$

$$x(t) = \begin{cases} -1 & -1<t\leq 0 \\ 1 & 0<t<1 \end{cases}$$

$$x(t) = \sum_{k=-\infty}^{\infty} X(k) e^{jk\omega t}$$

$$X(k) = \frac{1}{T}\int_{\langle T\rangle} x(t) e^{-jk\omega t}\, dt$$

$$X(k) = \frac{1}{2}\left[\int_{-1}^{0} -1 \cdot e^{-jk\omega t}\, dt + \int_{0}^{1} 1\, e^{-jk\omega t}\, dt\right]$$

$$= \frac{1}{2}\left[\left[\frac{e^{-jk\omega t}}{jk\omega}\right]_{-1}^{0} - \left[\frac{e^{-jk\omega t}}{jk\omega}\right]_{0}^{1}\right]$$

$$= \frac{1}{2}\left[\left(\frac{1}{jk\omega} - \frac{e^{jk\omega}}{jk\omega}\right) - \left(\frac{e^{-jk\omega}}{jk\omega} - 1\right)\right]$$

$$= \frac{1}{2jk\pi}\left(1 - e^{jk\pi} - e^{-jk\pi} + 1\right)$$

$$= \frac{1}{2jk\pi}\left(2 - \left(e^{jk\pi} + e^{-jk\pi}\right)\right)$$

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$$= \frac{1}{2jk\pi}\left(2 - 2\cos(k\pi)\right) = \frac{1-\cos(k\pi)}{jk\pi}$$

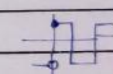
$$= \frac{(\cos(k\pi)-1)\, j}{k\pi} \qquad \text{for } k \neq 0.$$

for $\underline{k=0}$, $X(k)$ cannot be found, $x(t)$ does not have definite value for $\underline{k=1}$.

$$X(k) = \begin{cases} \cancel{\tfrac{1}{2}} & \cancel{k=0} \\ \frac{(\cos(k\pi)-1)j}{k\pi} & \text{otherwise} \end{cases}$$

$$\hookrightarrow \frac{((-1)^k-1)j}{k\pi} \rightarrow \begin{cases} 0 & \underline{k=\text{even}} \\ \frac{-2j}{k\pi} & k=\text{odd}. \end{cases}$$

if for $k=0$, $x(t)=1$



$$X(k) = \frac{1}{T}\int_{\langle T\rangle} x(t) e^{-jk\omega t}\, dt \Rightarrow X(k) = \frac{1}{T}\int_{\langle T\rangle} x(t) e^{0}\, dt$$

$$= \frac{1}{T}\int_{\langle T\rangle} x(t)\, dt = \frac{1}{2}\left[\int_0^1 (+1)\, dt\right]$$

$$= \frac{1}{2}\left[t\right]_0^1 = \frac{1}{2}(1-0) = \underline{\frac{1}{2}}.$$