2. Double Integrals

If a region R is bounded by the curves $x=a$, $x=b$, $y=c$ & $y=d$ then $\iint_R f(x,y)\,dA = \int_c^d \left[\int_a^b f(x,y)\,dx\right] dy$

1. Evaluate the double integrals

Ex: $\int_1^3 \int_0^1 (1+4xy)\,dx\,dy$

Sol^n: $I = \int_1^3 \left[\int_0^1 (1+4xy)\,dx\right] dy$

$$I = \int_1^3 \left[x + \cancel{4}y\frac{x^2}{\cancel{2}}\right]_0^1 dy$$

$$I = \int_1^3 [x + 2yx^2]_0^1\, dy = \int_1^3 [(1+2y) - 0]\, dy$$

$$I = \left[y + \cancel{2}\frac{y^2}{\cancel{2}}\right]_1^3 = [(3+9) - (1+1)]$$

$$\boxed{I = 10}$$

iii) $\iint_R \frac{xy^2}{x^2+1}\,dA$, $R = \{(x,y) \mid 0 \le x \le 1; -3 \le y \le 3\}$

Sol^n $I = \iint_R \frac{xy^2}{x^2+1}\,dA = \int_{-3}^3 \left[\int_0^1 \frac{xy^2}{x^2+1}\,dx\right] dy$

$$I = \int_{-3}^3 y^2 \left[\int_0^1 \left(\frac{x}{x+1}\right) dx\right] dy$$

$$\frac{d}{dx}\left[\frac{\ln(x^2+1)}{2}\right] = \frac{x}{x^2+1}$$

$$I = \int_{-3}^{3} y^2 \left[\frac{\ln(x^2+1)}{2}\right]_0^1 dy = \frac{1}{2}\int_{-3}^{3} y^2 [\ln 2 - \ln 1]\, dy$$

$$I = \frac{\ln 2}{2}\int_{-3}^{3} y^2\, dy = \frac{\ln 2}{2}\left[\frac{y^3}{3}\right]_{-3}^{3}$$

$$I = \frac{\ln 2}{6}\left[3^3 - (-3)^3\right] = \frac{\ln 2}{6}[27+27]$$

$$I = \frac{\ln 2}{\cancel{6}_3}\left[\cancel{2}(\cancel{27}^{9})\right] = \underline{\underline{9\ln 2}}$$

Q4 $\int_0^1 \int_x^{2-x} (x^2-y)\, dx\, dy$

Sol: $I = \int_0^1 \left[\int_x^{2-x} (x^2-y)\, dy\right] dx$

$$I = \int_0^1 \left[x^2 y - \frac{y^2}{2}\right]_x^{2-x} dx$$

$$I = \int_0^1 \left[\left\{x^2(2-x) - \frac{(2-x)^2}{2}\right\} - \left\{x^3 - \frac{x^2}{2}\right\}\right] dx$$

$$I = \int_0^1 \left[2x^2 - x^3 - \frac{(2-x)^2}{2} - x^3 + \frac{x^2}{2}\right] dx$$

$$I = \int_0^1 \left[\frac{5x^2}{2} - 2x^3 - \frac{(2-x)^2}{2}\right] dx$$

Note: $\int (a-x)^2 dx = \frac{(a-x)^3}{-3}$

$$I = \left[\frac{5}{2}\frac{x^3}{3} - 2\frac{x^4}{4} - \frac{1}{2}\frac{(2-x)^3}{-3}\right]_0^1$$

$$I = \left[\frac{5}{6}x^3 - \frac{x^4}{2} + \frac{1}{6}(2-x)^3\right]_0^1$$

$$I = \left[\left(\frac{5}{6} - \frac{1}{2} + \frac{1}{6}\right) - \left(0 - 0 + \frac{8}{6}\right)\right] = \left[\frac{-2}{6} - \frac{1}{2}\right]$$

$I = \cancel{1} - \frac{1}{3} - \frac{1}{2} = \frac{-5}{6}$ //

iii) $\int_0^{\pi}\int_0^{2} r\sin\theta \, dr\, d\theta$

Sol: $I = \int_0^{\pi}\left[\int_0^{2} r\sin\theta \, dr\right] d\theta$

$I = \int_0^{\pi} \sin\theta \left[\int_0^{2} r\, dr\right] d\theta = \int_0^{\pi} \sin\theta \left[\frac{r^2}{2}\right]_0^2 d\theta$

$I = \int_0^{\pi} \sin\theta \left[\cancel{4}^{2}/\cancel{2}_{1}\right] d\theta = \int_0^{\pi} 2\sin\theta \, d\theta$

$I = -2[\cos\theta]_0^{\pi} = -2\left[\cos\pi - \cos 0\right] = -2[-1-1]$

$\boxed{I = 4}$

2) Evaluate the double integrals

i) $\iint_D (x+y)\,dA$, D is bounded by $y = \sqrt{x}$ & $y = x^2$

Sol: Given: $y = \sqrt{x}$ — ① $y = x^2$ — ②

point of intersection (pt. of x^n :)

From ① & ② $\sqrt{x} = x^2$

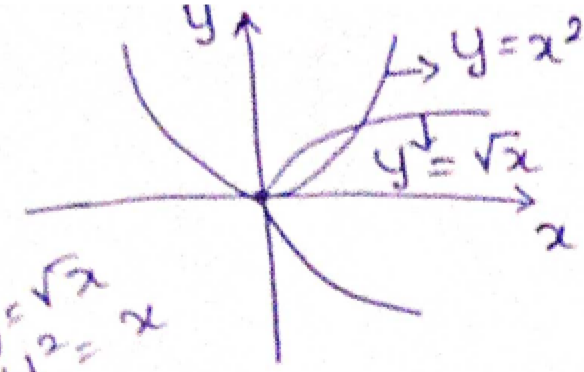
$\Rightarrow x = x^4 \Rightarrow x^4 - x = 0$

$\Rightarrow x(x^3 - 1) = 0$

$\Rightarrow \boxed{x = 0}$; $\boxed{x = 1}$

put $x = 0$ & $x = 1$ in ①

$y = 0$ & $y = 1$



$y = \sqrt{x}$
$y^2 = x$

$\Rightarrow$