CS 230 : Discrete Computational Structures

Fall Semester, 2023
Homework Assignment #11 [**Extra Credit**]
Due Date: Friday, December 8 at 5 pm

Suggested Reading: Rosen Chapter 6.5, CLRS Chapter on Graphs

These are the problems that you need to turn in. Always explain your answers and show your reasoning. **You will be graded based on whether you explain your answers. Just correct numerical answers will not be enough! For the computational problems, leaving answers in the form of $P(n, r)$ or $C(n, r)$, as a factorial, product or sum, are all acceptable. You do not need to compute the numerical solution!**

1. **[6 Pts]** A cookie shop sells 5 different kinds of cookies. How many different ways are there to choose 12 cookies if (a) you have no restrictions? (b) you pick at least two of each? (c) you pick at least 4 oatmeal cookies and at most 3 sugar cookies?

   (a) $\binom{n+k-1}{k-1} = \binom{12+5-1}{5-1} = \binom{16}{4} = 1820$
   There are 1820 ways to pick 12 cookies at random from the five types.

   (b) $12 - 10 = 2$ Open Spots, Still 5 types $\rightarrow \binom{n+k-1}{k-1} = \binom{2+5-1}{5-1} = \binom{6}{4} = 15$
   There are 15 ways to pick 12 cookies while picking at least 2 of each type.

   (c) This one will be the sum of four cases:
   4 Oatmeal, 0 Sugar, 8 any other kind + 4 Oatmeal, 1 Sugar, 7 any other kind + 4 Oatmeal, 2 Sugar, 6 any other kind + 4 Oatmeal, 3 Sugar, 5 any other kind
   Written in binomial, that is:
   $= \binom{8+4-1}{4-1} + \binom{7+4-1}{4-1} + \binom{6+4-1}{4-1} + \binom{5+4-1}{4-1}$
   $= \binom{11}{3} + \binom{10}{3} + \binom{9}{3} + \binom{8}{3}$
   $= 165 + 120 + 84 + 56$
   $= 425$
   There are 425 ways to chose 12 cookies while picking at least 4 oatmeal and at most 3 sugar cookies.

2. **[4 Pts]** If I have 4 bananas, 3 oranges, and 5 apples, how many ways can I distribute these to 12 friends, if each friend gets one fruit? Describe the kind of combinatorial problem you are solving.

   This is an r - permutation because if you order the friends, you can order the fruit and count the different orders of the fruits.
   An r - permutation is calculated: P(n, k) $= \frac{n!}{(n-k)! \cdot k!}$.
   This problem will have P(12, 12) combinations.

3. **[6 Pts]** How many ways are there to pack 24 different books into 8 boxes with 3 books each if (a) all 8 boxes are sent to different addresses? (b) all 8 boxes are sent to the same address? (c) 4 of the boxes are shipped to four different addresses while 4 are left to be addressed later?

(a) We can apply the product rule here for each box, removing three from n as we go to account for filled boxes.

$\frac{24!}{3!(21!)} \cdot \frac{21!}{3!(18!)} \cdot \frac{18!}{3!(15!)} \cdot \frac{15!}{3!(12!)} \cdot \frac{12!}{3!(9!)} \cdot \frac{9!}{3!(6!)} \cdot \frac{6!}{3!(3!)} \cdot \frac{3!}{3!(0!)}$

I'm not going to figure out what that number is, but it is calculated above. That's how many combinations there are.

(b) We can calculate this by taking C(24, 3) divided by 8 * 3!. That goes out to 2024 / 48, or 42 combinations.

(c) This will mimic the first part of the problem, only now with combinations instead of binomials.
There are [C(24, 3) * C(21, 3) * C(18, 3) * C(15, 3) * C(12, 3) * C(9, 3) * C(6, 3) * C(3, 3)] combinations.

4. **[6 Pts]** (a) How many ways are there to pack 5 identical books into 5 identical boxes with no restrictions placed on how many can go in a box (some boxes can be empty)? (b) What if the books are different?

(a) Part a is a simple P(n, r) problem. Since P(n, r) $= \binom{n+k-1}{k-1}$, we can sub in P(5, 5) to get :

$\binom{5+5-1}{5-1} = \binom{9}{4} = 126$ combinations.

(b) If they are different books, we take n^k to find our value.
$5^5 = 3125$ combinations.

5. [**6 Pts**] How many ways can we place 8 books on a bookcase with 5 shelves if the books are (a) indistinguishable copies (b) all distinct? Note that the position of the books on the shelves matter. Note that all books may be placed on the same shelf.

(a) For this part, we'll take the combinations of n + k - 1 and k - 1, which is $C(8+5-1, 5-1) = C(12, 4) = 495$ combinations.

(b) Like the last problem, for the distinct books we can take $5^8 = 390,625$ combinations.

6. [**4 Pts**] Consider a simple, undirected graph G that has 7 vertices with degrees of 5, 4, 3, 3, 2, 1, 1. Can G exist? Explain.

7. [**6 Pts**] If G is a simple, undirected graph with n vertices and $n - 1$ edges, (a) is G connected? (b) is G acyclic? For each question, if *yes*, give a short justification. If *no*, give a counterexample.

8. [**12 Pts**] Prove that a simple, undirected graph is a tree if and only if it is acyclic but adding any edge will create a cycle.

For more practice, work on problems in Rosen and LLM.