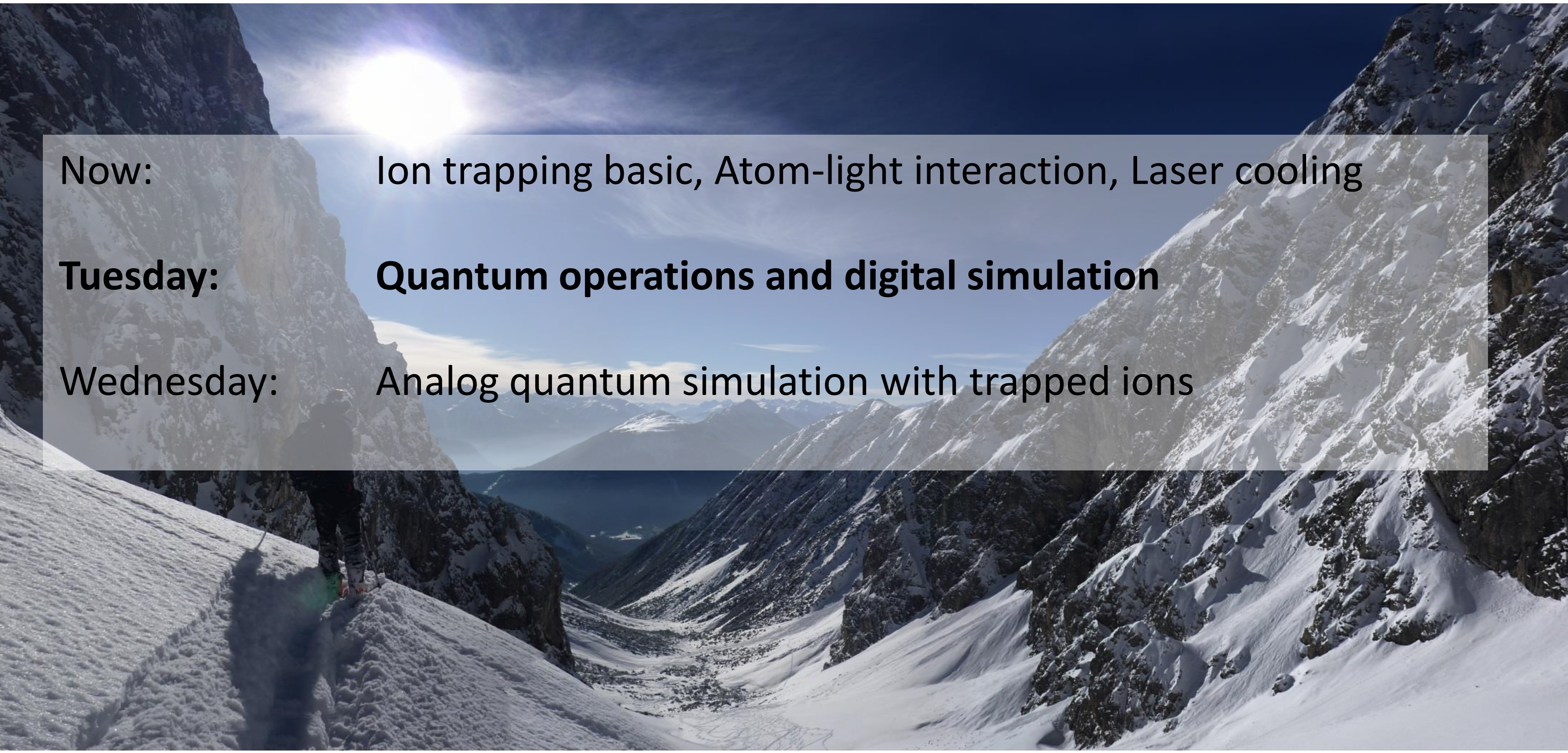
Quantum science with trapped ions (Pt2)

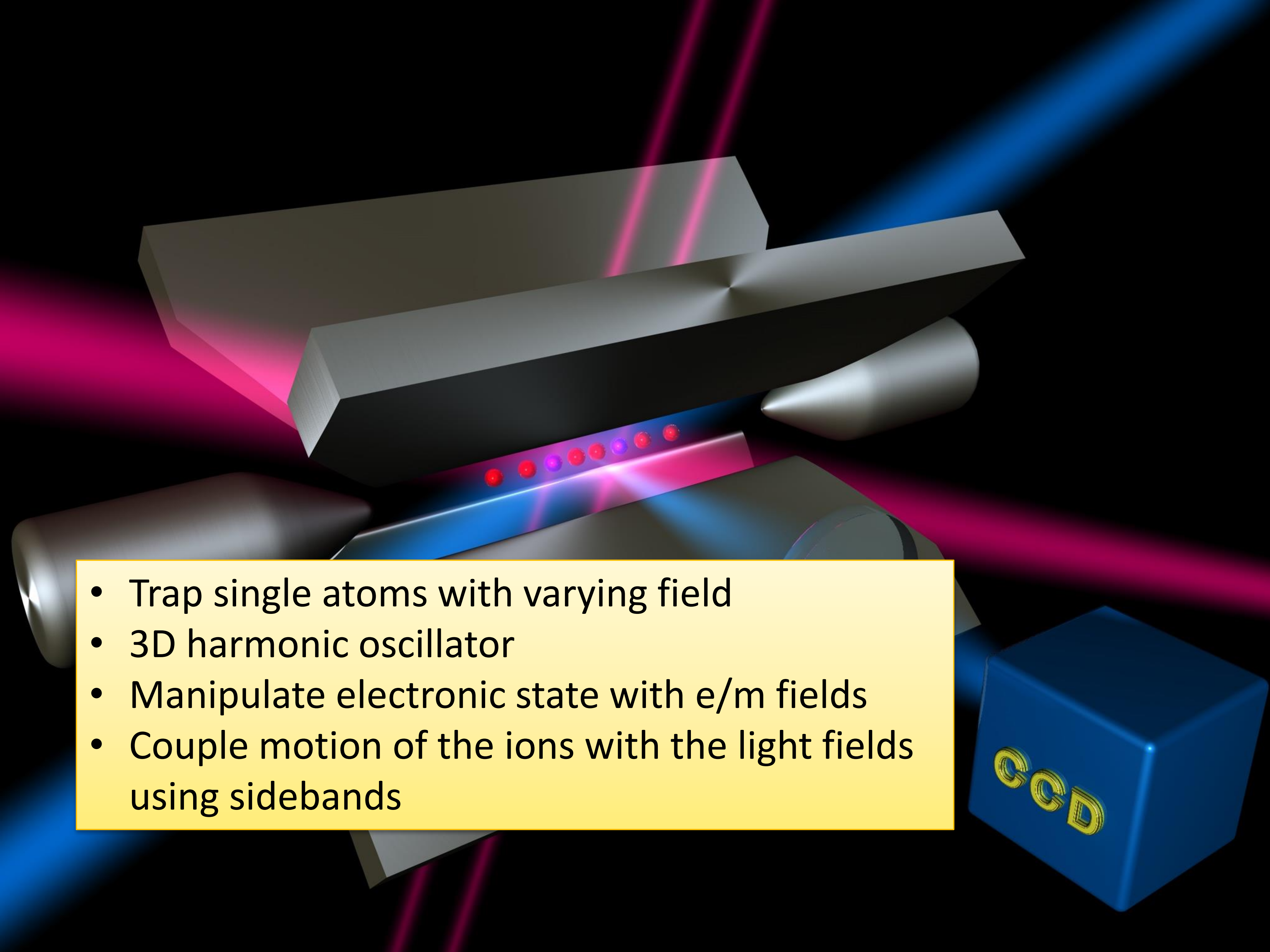


Now: Ion trapping basic, Atom-light interaction, Laser cooling

Tuesday: Quantum operations and digital simulation

Wednesday: Analog quantum simulation with trapped ions

Philipp Schindler
http://github.com/pschindler/natal



- Trap single atoms with varying field
- 3D harmonic oscillator
- Manipulate electronic state with e/m fields
- Couple motion of the ions with the light fields using sidebands

Repetition from this morning

Lamb-Dicke regime:

Extension of the ion‘s wave function Ψ much smaller than optical wavelength

$$\eta\sqrt{\langle\Psi|(a+a^\dagger)^2|\Psi\rangle} \ll 1$$

For $\Psi = |n\rangle \ : \ \eta\sqrt{2n+1} \ll 1$

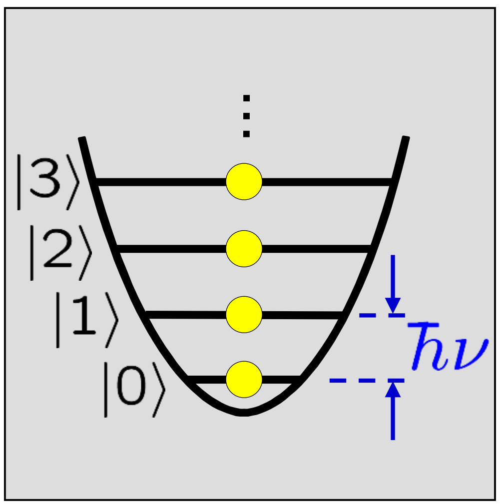
Taylor expansion of the exponential up to first order:

$$H_{int} = \frac{\hbar\Omega}{2}\sigma_+\{\mathbb{1}+i\eta(e^{-i\nu t}a+e^{i\nu t}a^\dagger)\}e^{-i\delta t+i\phi} + h.c.$$

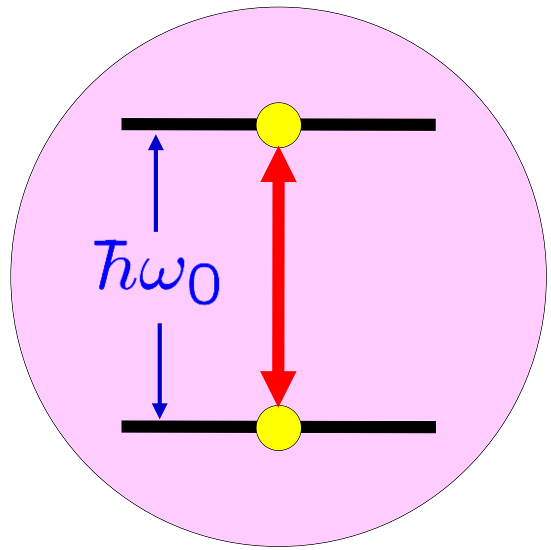
Laser cooling

Harmonic oscillator

Quantum bit



$\otimes$



Including spontaneous decay

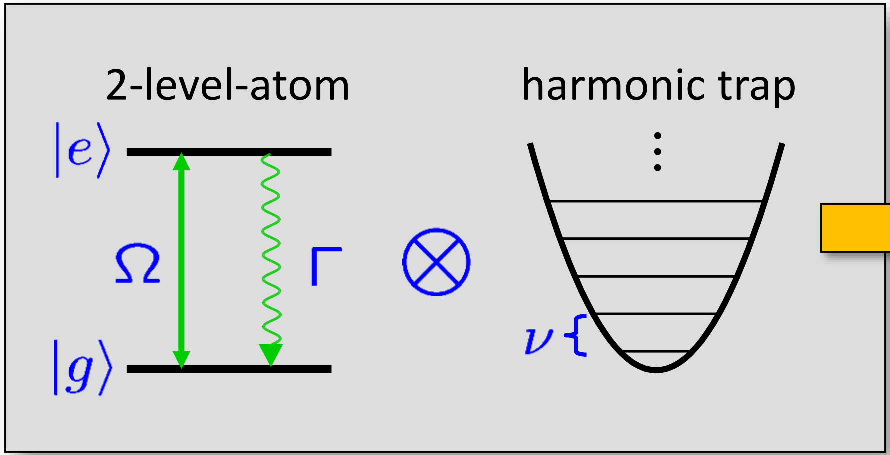
motional states

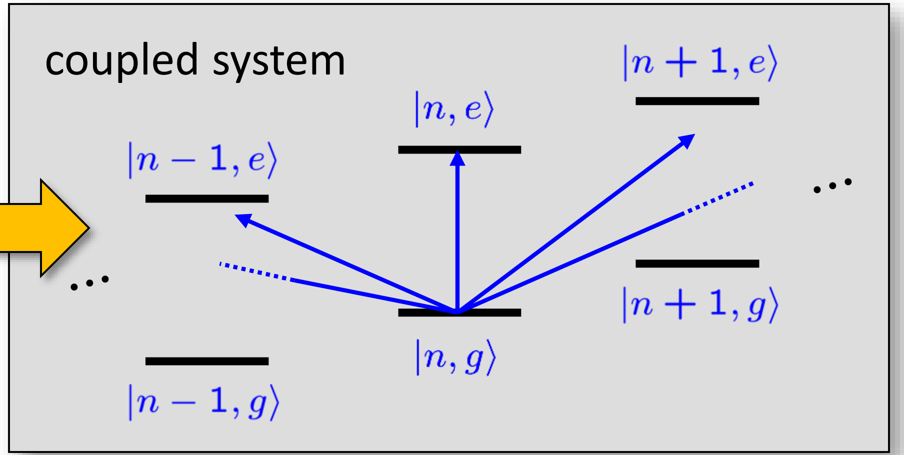
$|0\rangle, |1\rangle, |2\rangle, |3\rangle, \ldots$

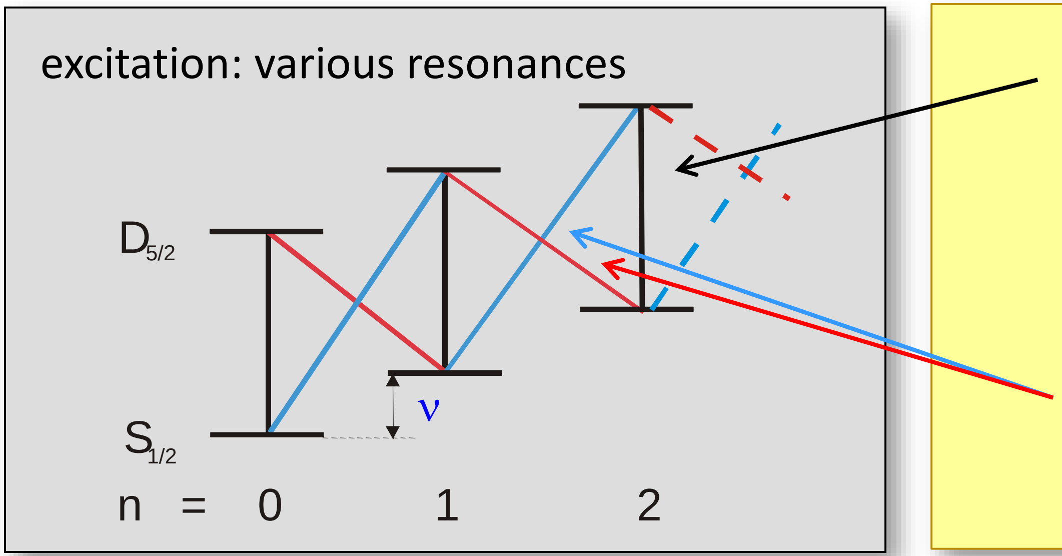
internal states

$|\uparrow\rangle, |\downarrow\rangle$

Qubit manipulation







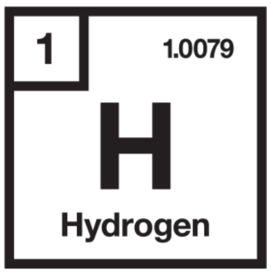
Carrier:
manipulate qubit
→ internal superpositions

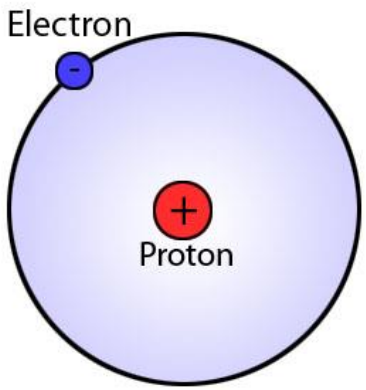
Sidebands:
manipulate motion and qubit
→ create entanglement

Outline

- Laser cooling and ion species
- Local operations
- Entangling operations
- Digital quantum simulation

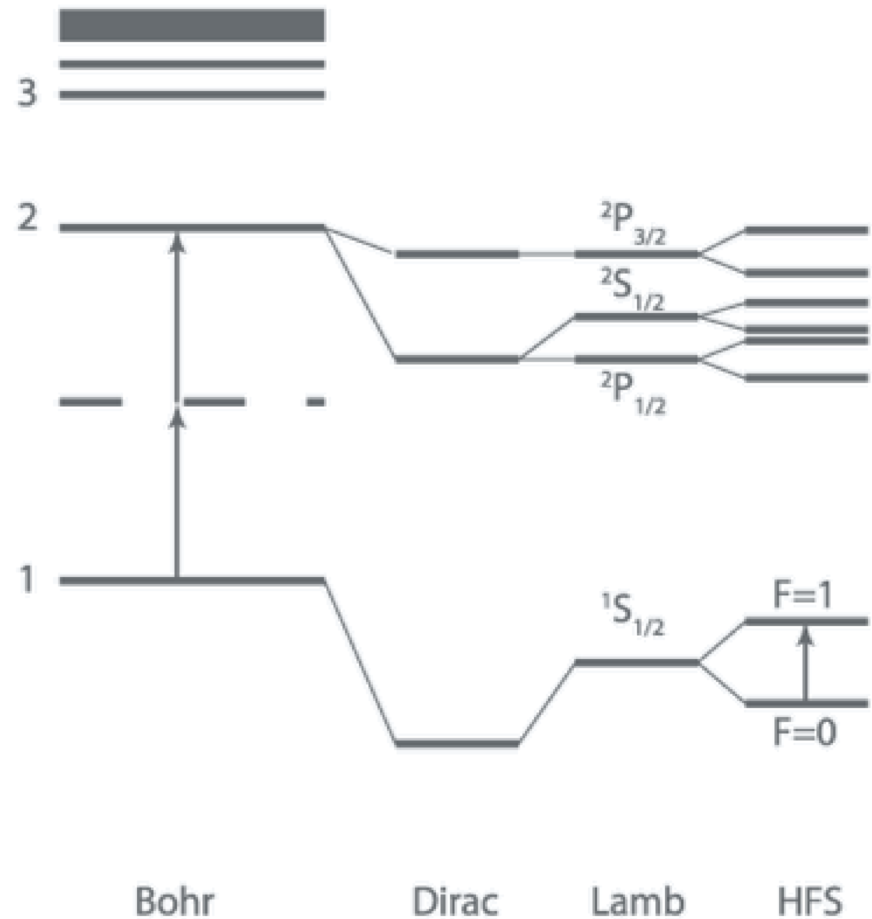
Physicists like it simple



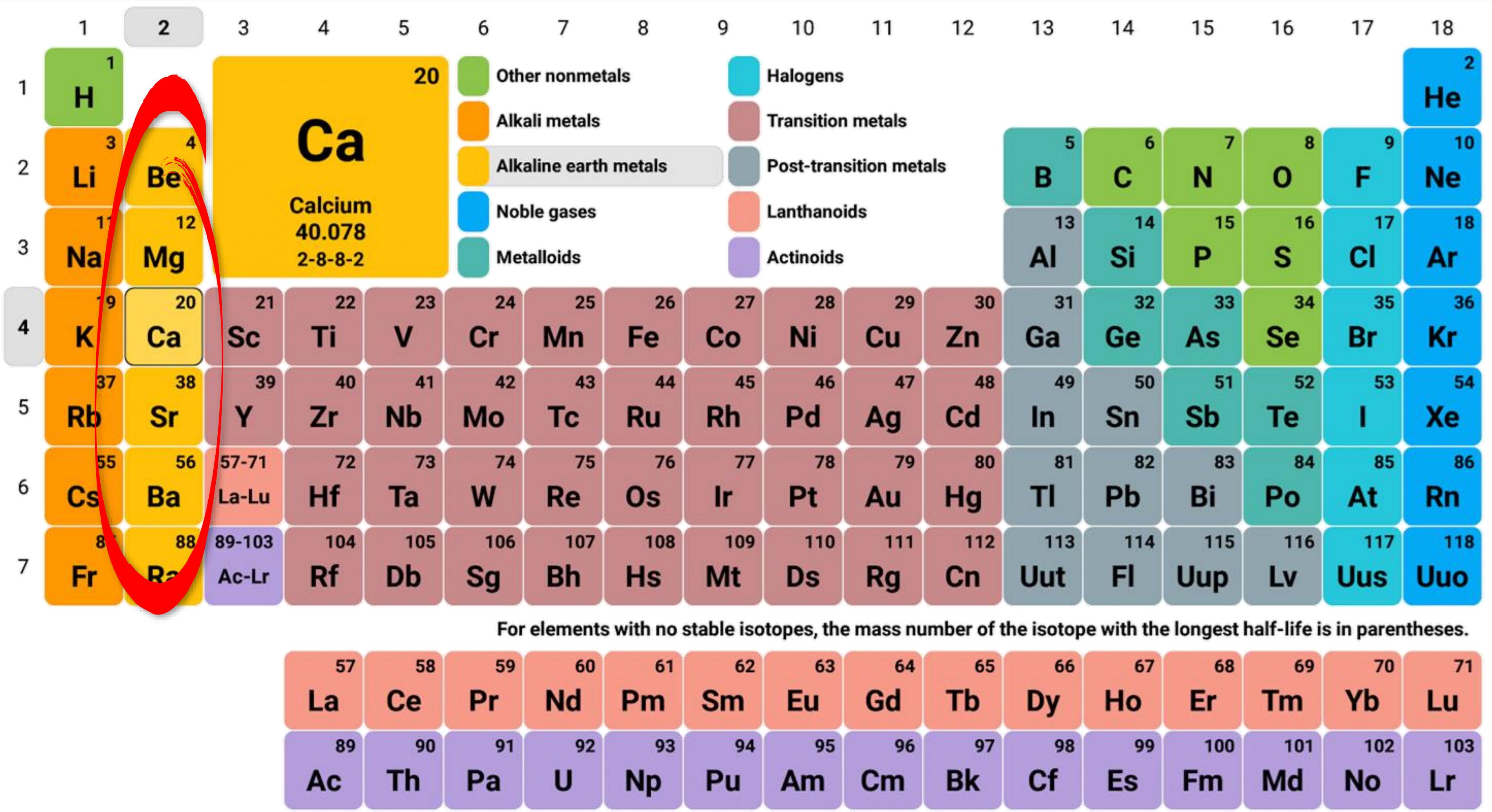


As simple as it gets:
- Only one electron
- No rotations
- No vibrations

Energy levels



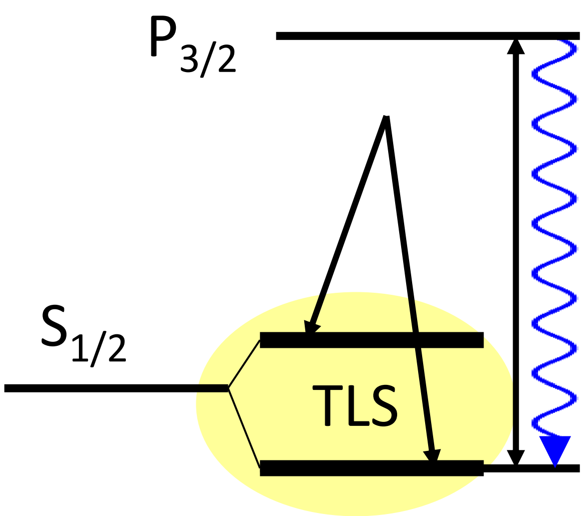
Ion trappers favorites



Possible qubits

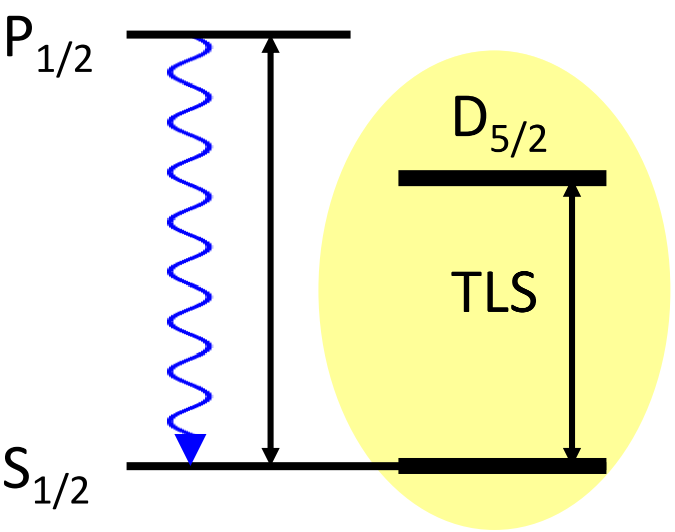
Storing and keeping quantum information requires long-lived atomic states:

- microwave transitions
  (hyperfine transitions,
  Zeeman transitions)
  alkaline earths:
  $^{9}Be^{+}$, $^{25}Mg^{+}$, $^{43}Ca^{+}$, $^{87}Sr^{+}$,
  $^{137}Ba^{+}$, $^{111}Cd^{+}$, $^{171}Yb^{+}$



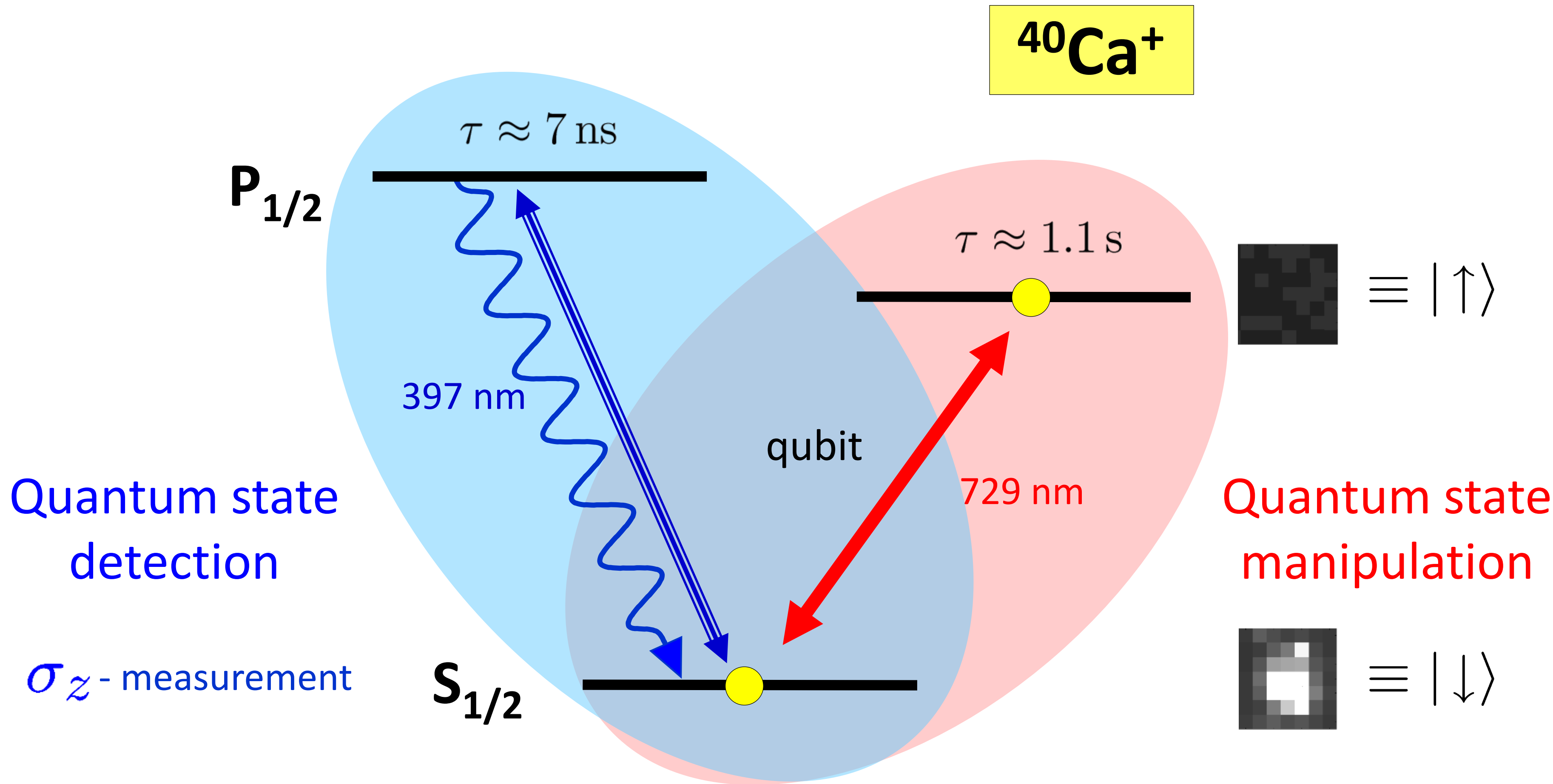
Boulder $^{9}Be^{+}$; Michigan $^{111}Cd^{+}$;
Innsbruck $^{43}Ca^{+}$, Oxford $^{43}Ca^{+}$;
Maryland $^{171}Yb^{+}$;

- optical transition frequencies
  (forbidden transitions,
  intercombination lines)
  S – D transitions in alkaline earths:
  Ca^{+}, Sr^{+}, Ba^{+}, Ra^{+}, (Yb^{+}, Hg^{+}) etc.



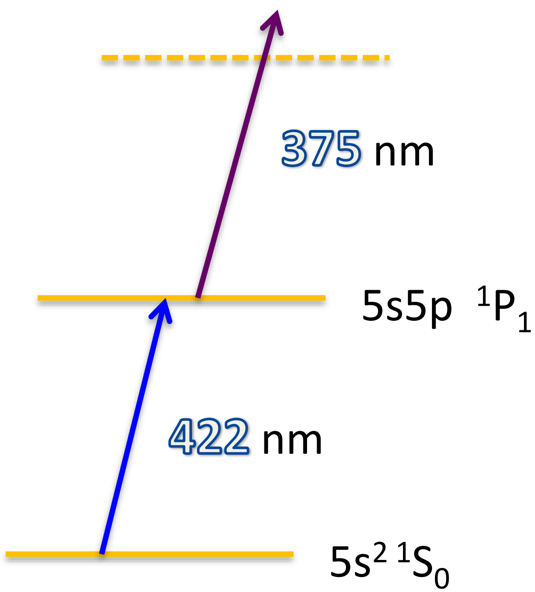
Innsbruck $^{40}Ca^{+}$

Calcium ions

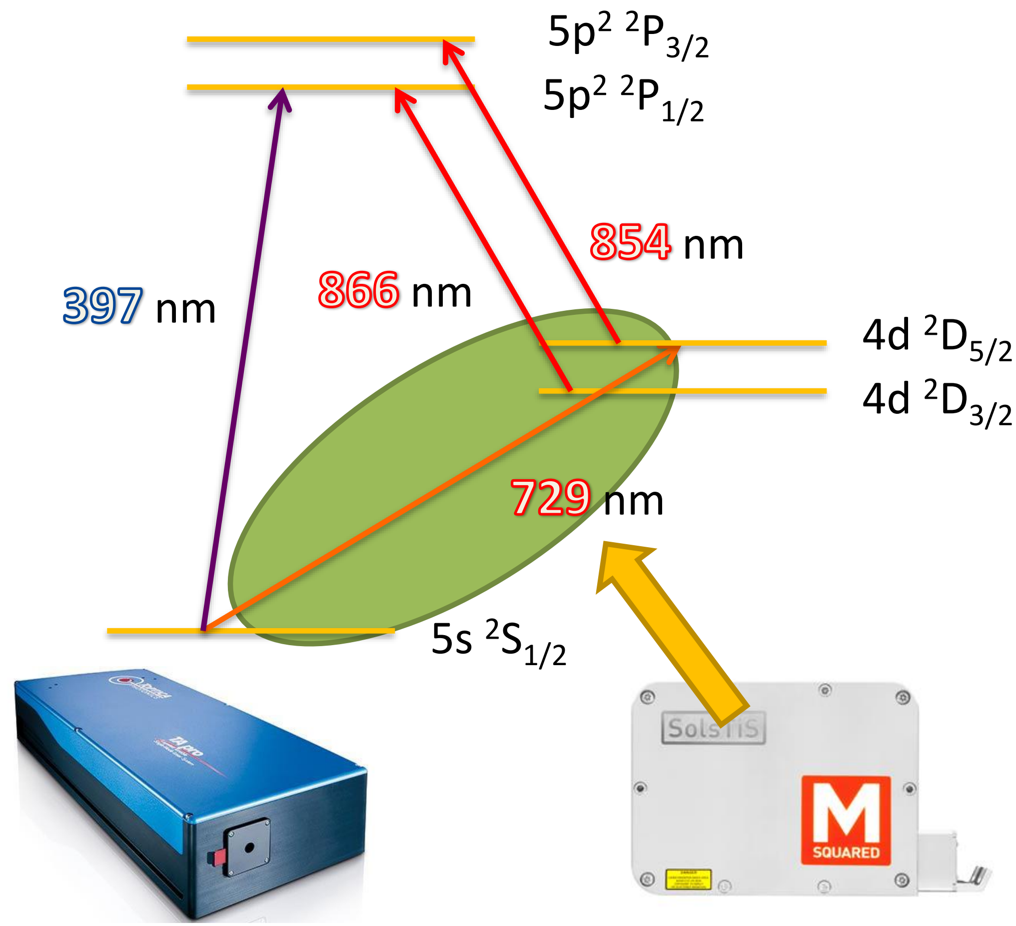


Required lasers

Ca energy levels

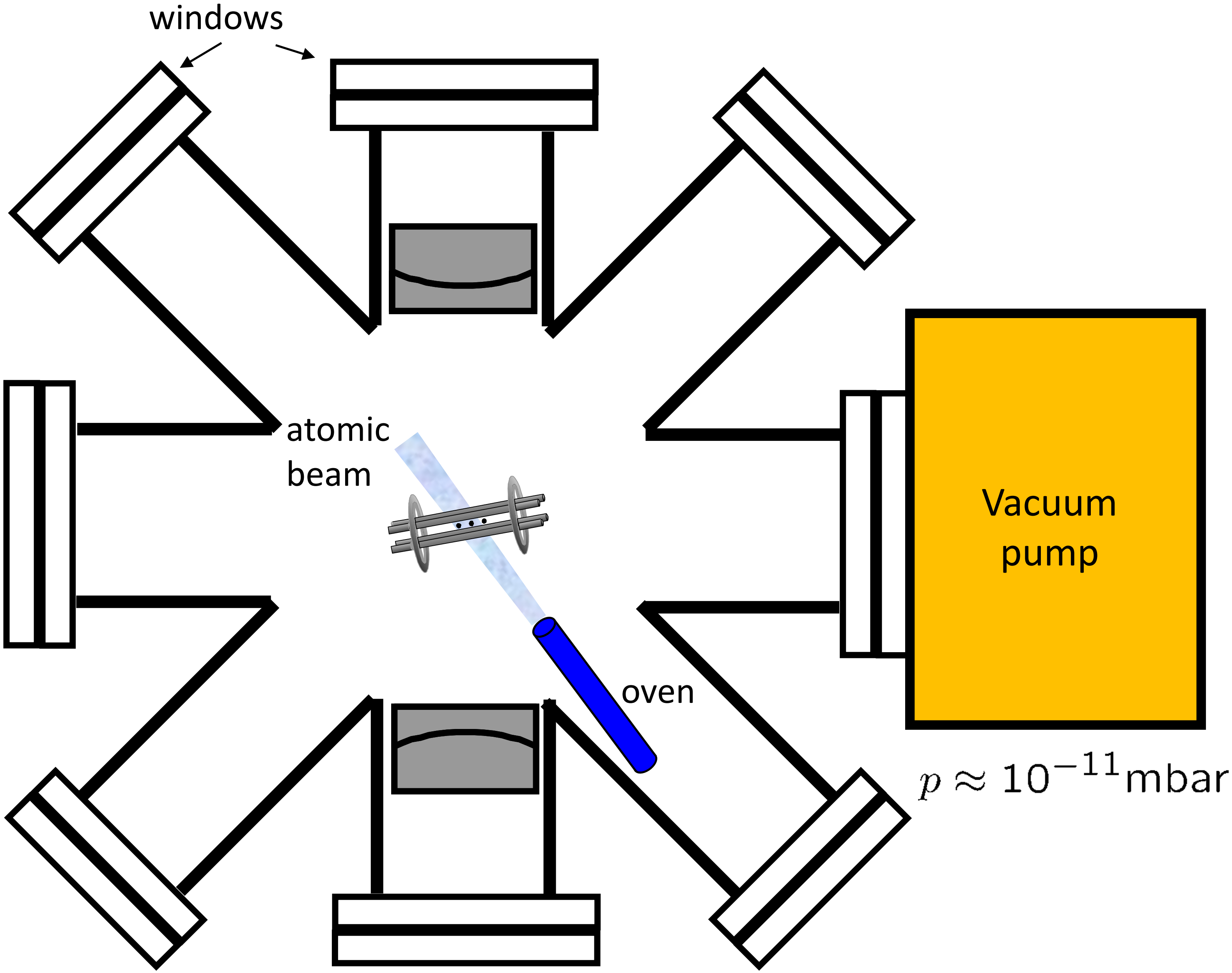


Ca$^+$ energy levels



6 laser systems required

Ion loading



Ion loading

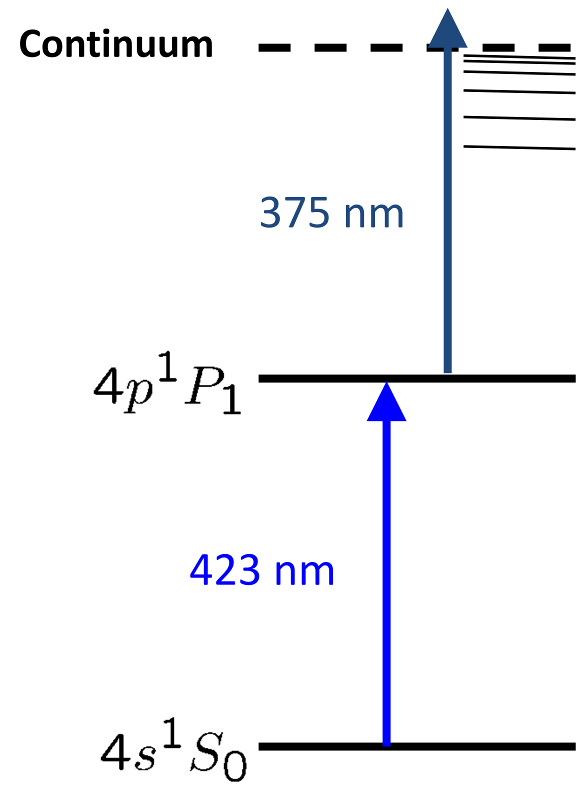
An oven produces a weak atomic beam of neutral calcium crossing the trap

Loading of ions into the trap by

- electron bombardement
- photoionization

(experimentally demonstrated for Mg^+, Ca^+, Cd^+)

2-step photoionization of neutral calcium

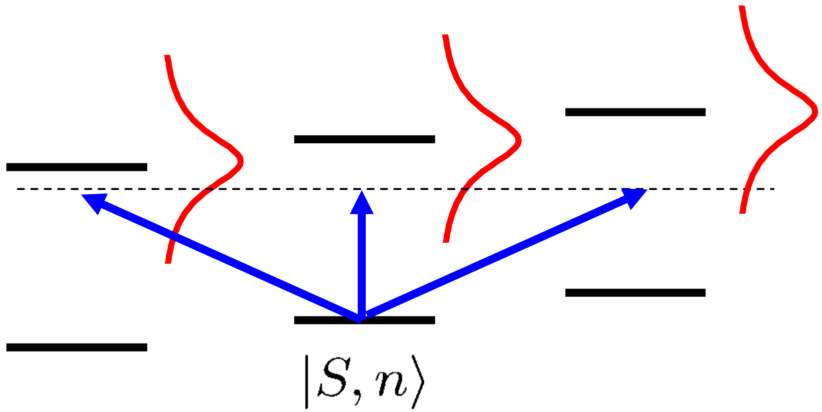


Advantages of photoionization:

- higher cross-section
- isotope-selective loading

Laser cooling regimes

Doppler cooling

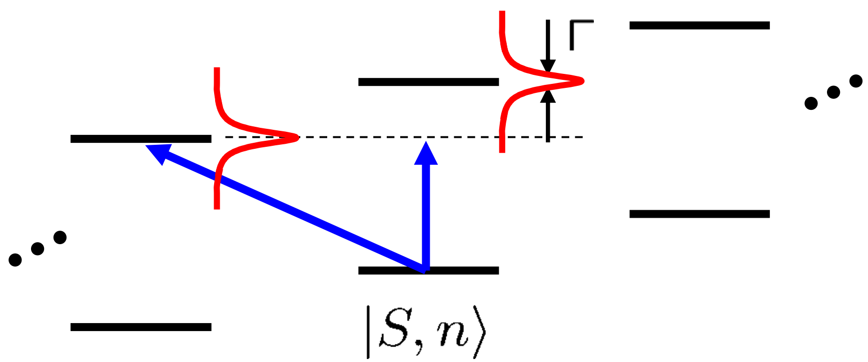


$\nu \ll \Gamma$ **weak** confinement, Doppler cooling

$$\langle n \rangle = \frac{\Gamma}{2\nu} > 1$$

if laser detuned by $\Delta = -\Gamma/2$

Sideband cooling



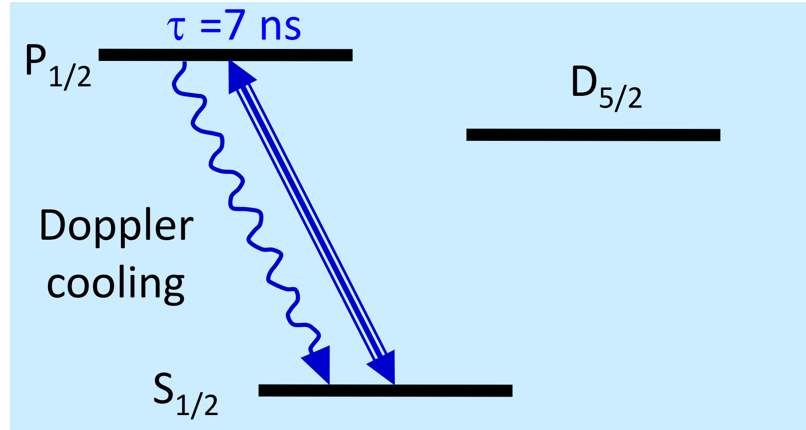
$\nu \gg \Gamma$ **strong** confinement, sideband cooling

$$\langle n \rangle = \frac{\Gamma^2}{4\nu^2} \ll 1$$

if laser detuned by $\Delta = -\nu$

Laser cooling in Ca

1. Doppler cooling on $S_{1/2}$-$P_{1/2}$ transition:

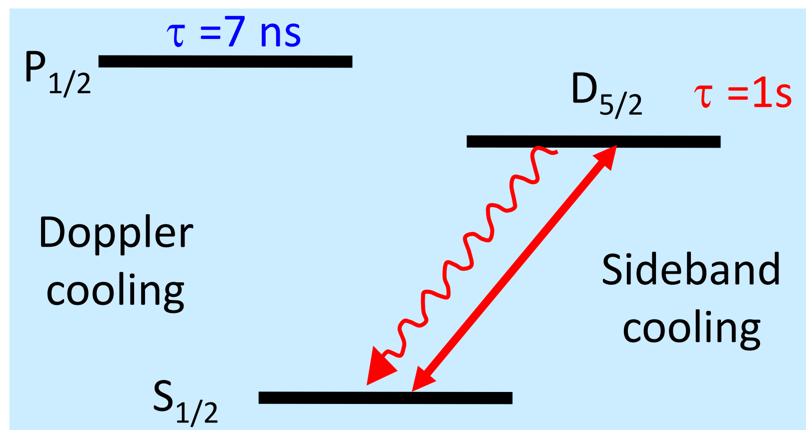


Fast cooling rate

$$\langle n \rangle = \frac{\Gamma}{2\nu} \approx 2 - 10$$

(2 ms Doppler cooling)

2. Sideband cooling on $S_{1/2}$-$D_{5/2}$ transition:



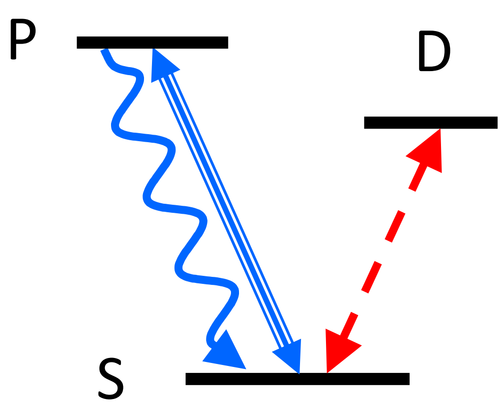
Slower cooling rate, but better cooling results

$$\langle n \rangle_{exp} \approx 0.001 - 0.01$$

(5-10 ms sideband cooling)

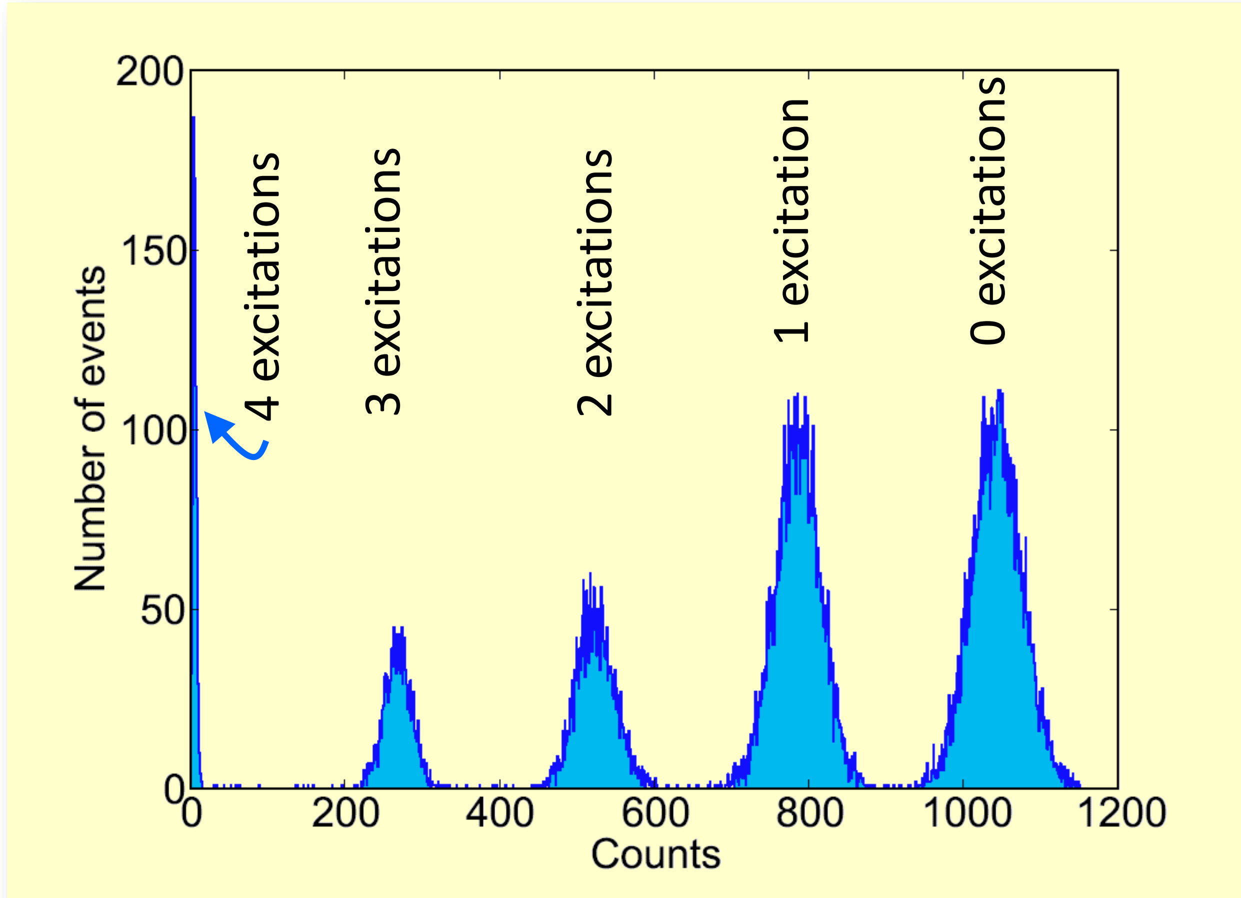
(For sideband cooling, the lifetime of the D5/2 state is artificially shortened by coupling it to the P3/2 state -> higher cooling rate)

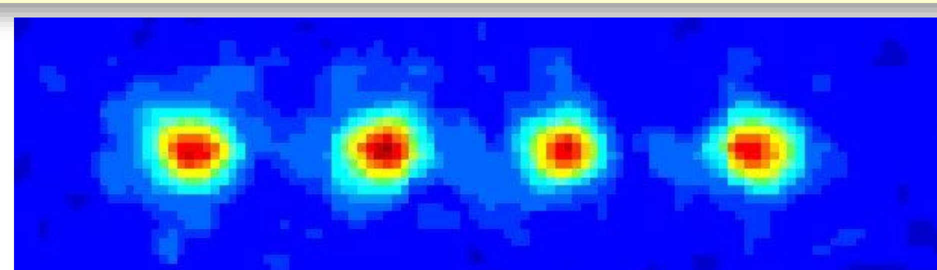
Qubit measurement

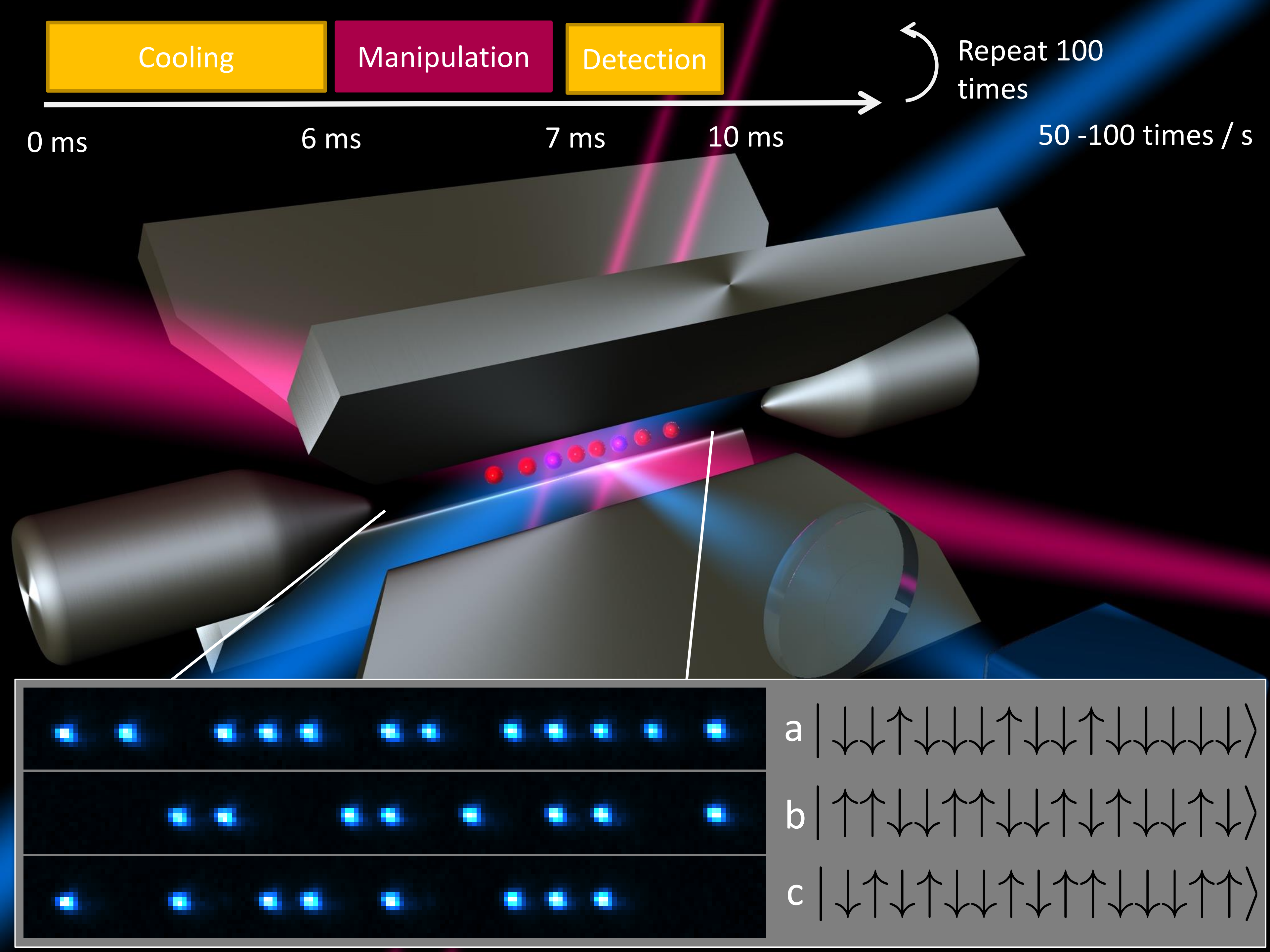


Detection:
Quantum Jumps

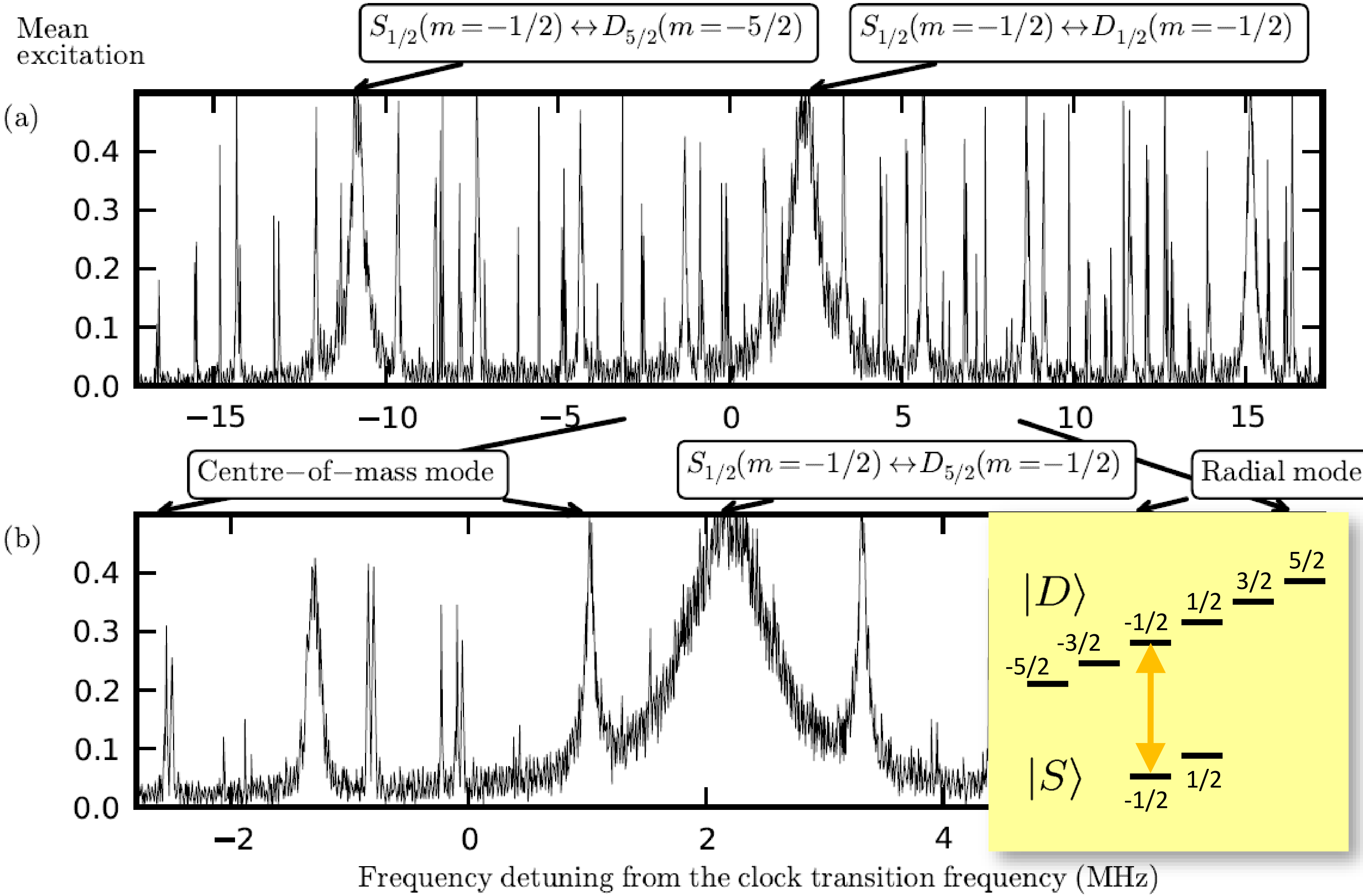
- Projection of ions to either S or D states,






Cooling
Manipulation
Detection
Repeat 100 times
0 ms
6 ms
7 ms
10 ms
50 -100 times / s
a |↓↓↑↓↓↓↑↓↓↑↓↓↓↓↓⟩
b |↑↑↓↓↑↑↓↓↑↓↑↓↓↑↓⟩
c |↓↑↓↑↓↓↑↓↑↑↓↓↓↑↑⟩

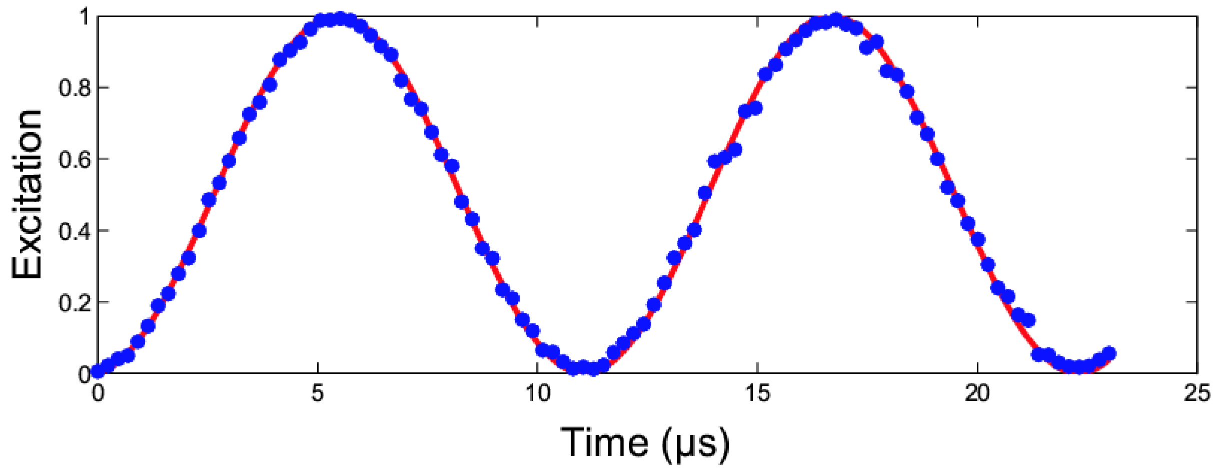
It’s a two-level system?

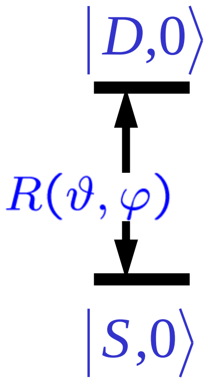


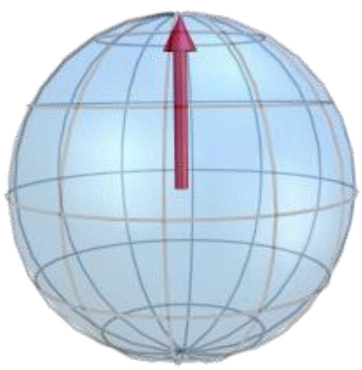
Outline

- Laser cooling and ion species
- Local operations
- Entangling operations
- Digital quantum simulation

Rabi flopping – Single qubit operations



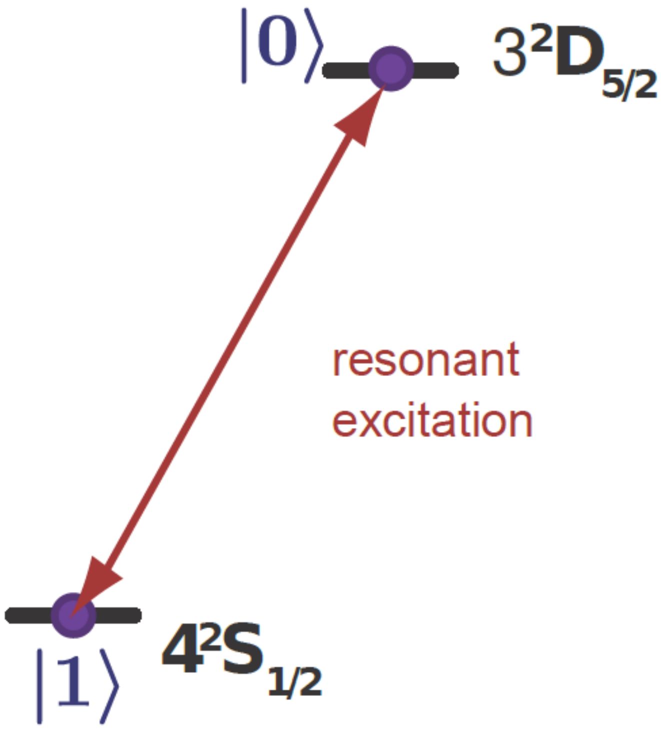


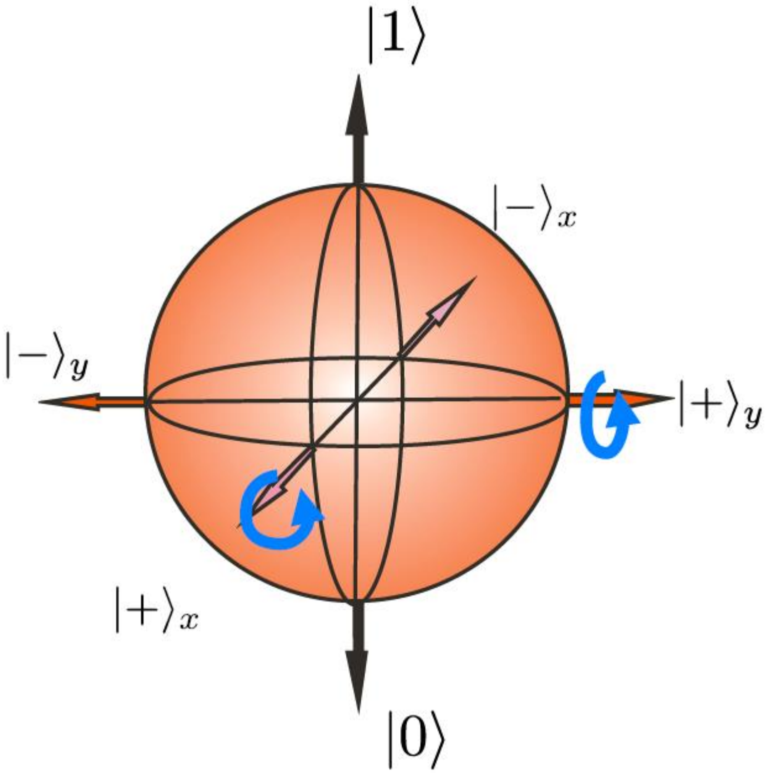


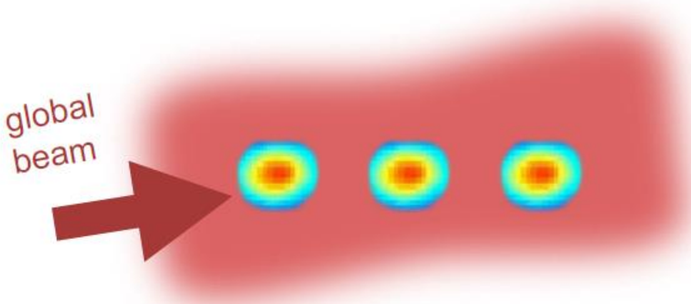
Carrier Rabi oscillations with Rabi frequencies

Rotations on the Bloch sphere

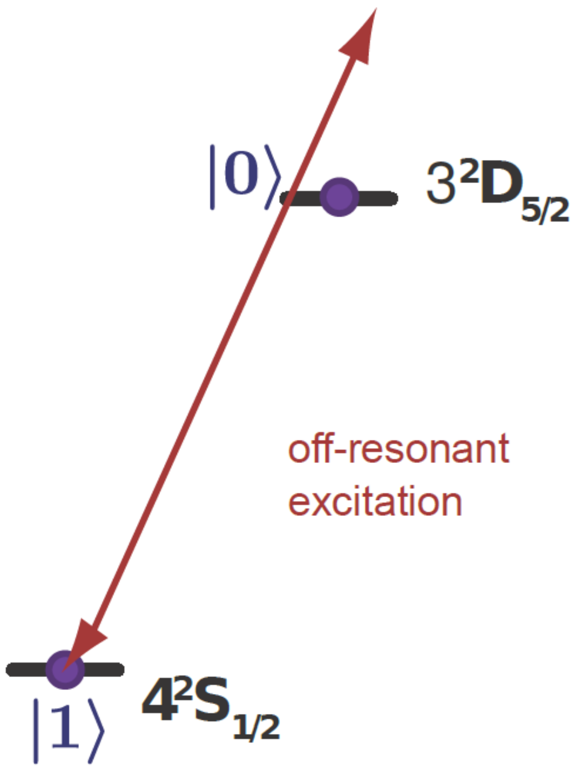
Collective local operations

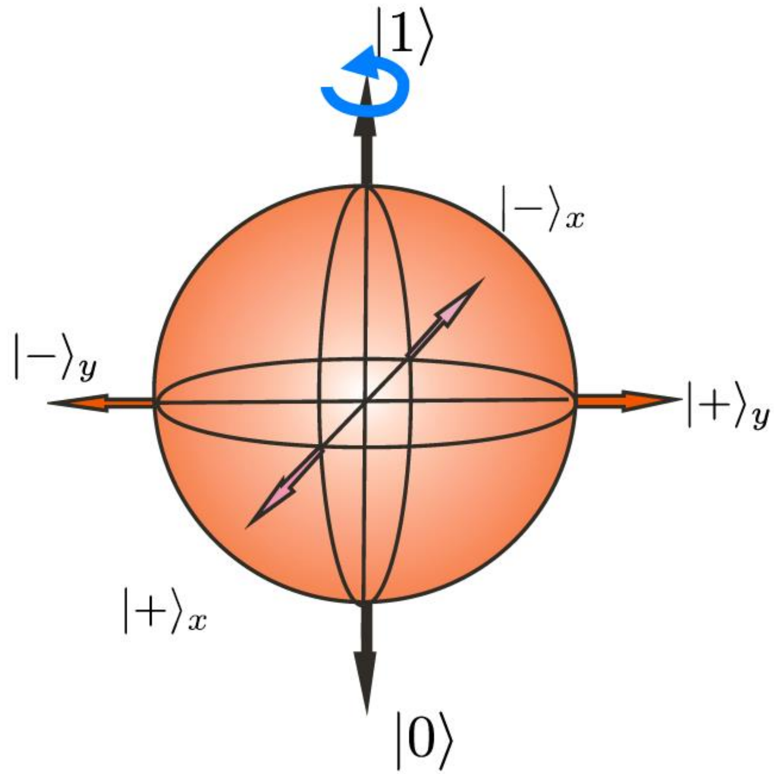


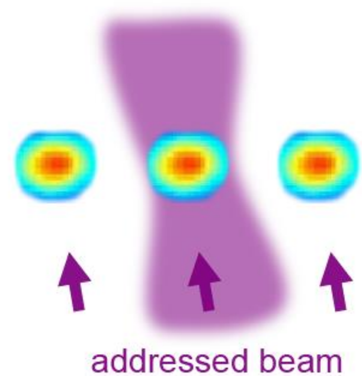




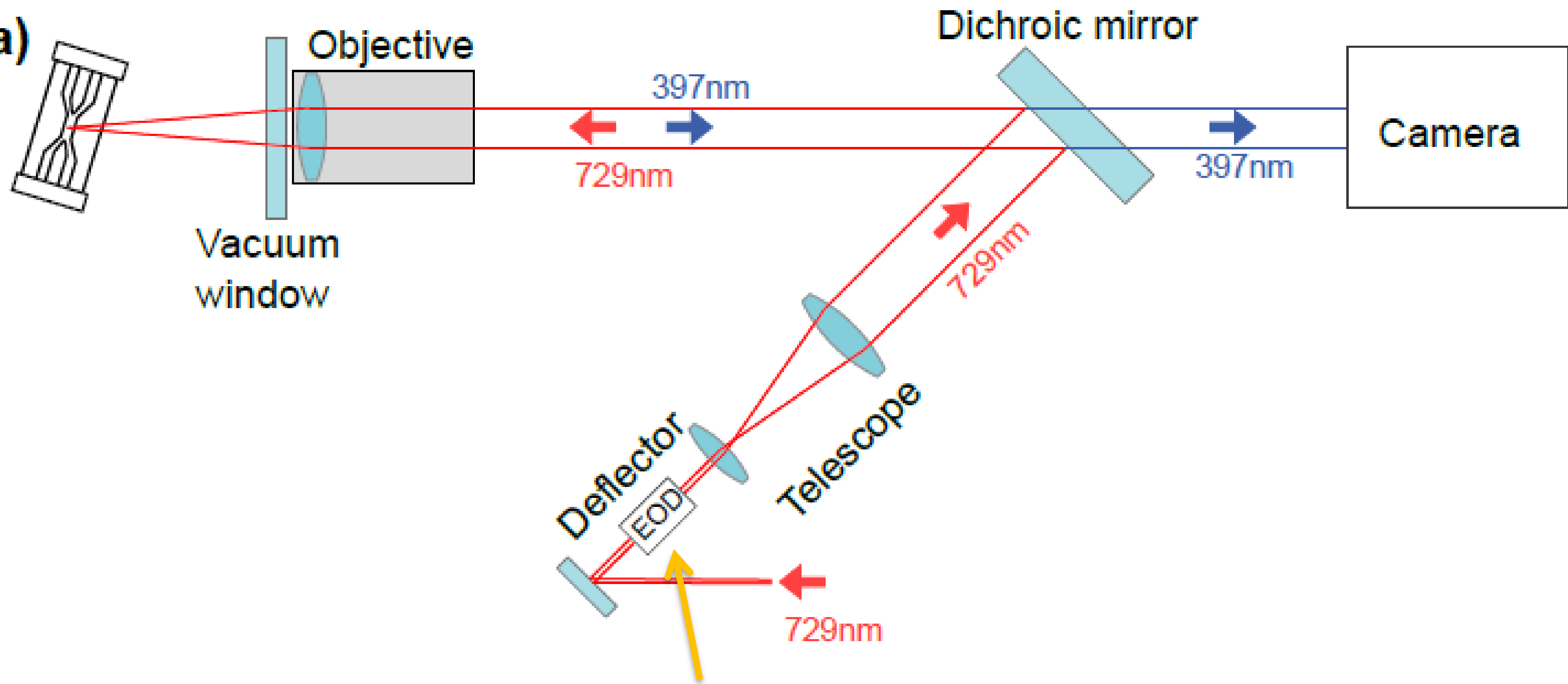
Single qubit operations







Single ion addressing



Electro optical deflector

Outline

- Laser cooling and ion species
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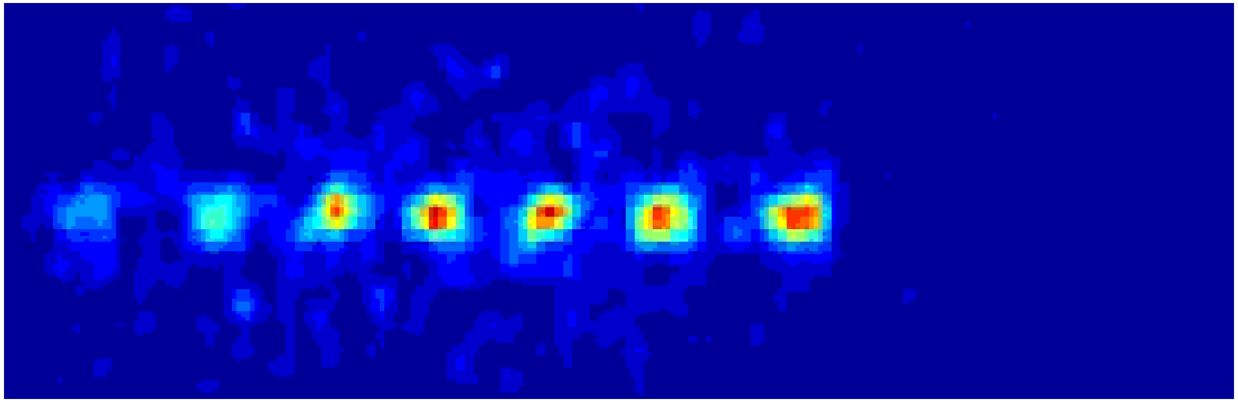
More ions



Normal modes of ion crystals

At low temperatures, ions oscillate around their equilibrium positions

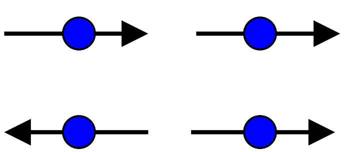
Coulomb interaction: coupling of ion motion

$\longrightarrow$ small excitations : collective normal modes

For the calculation of the normal mode frequencies:

Taylor expansion of Coulomb force and trapping force around the equil. positions

2 ions:



center of mass mode $\nu_1 = \nu_z$

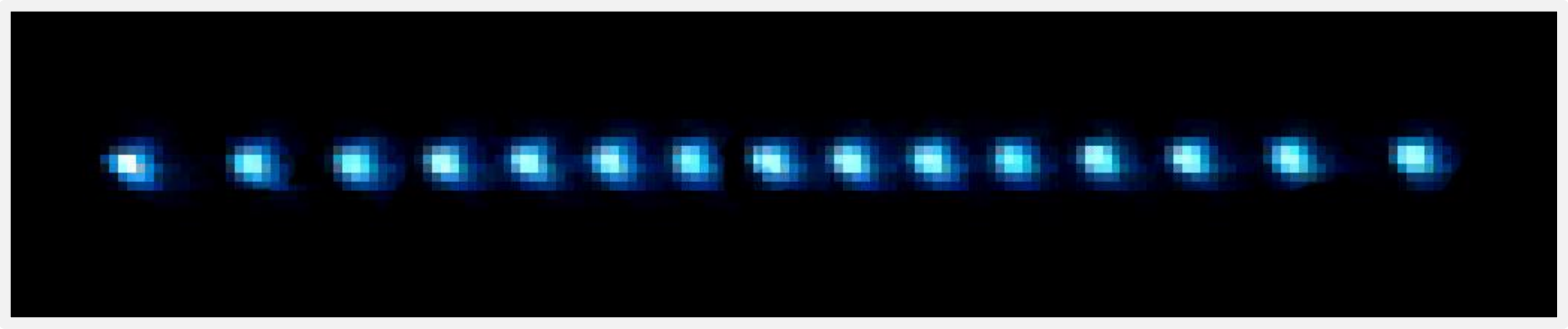
breathing mode $\nu_2 = \sqrt{3}\nu_z$

Ion crystals

Equilibrium positions:

Minimize potential energy of ions in a linear chain:

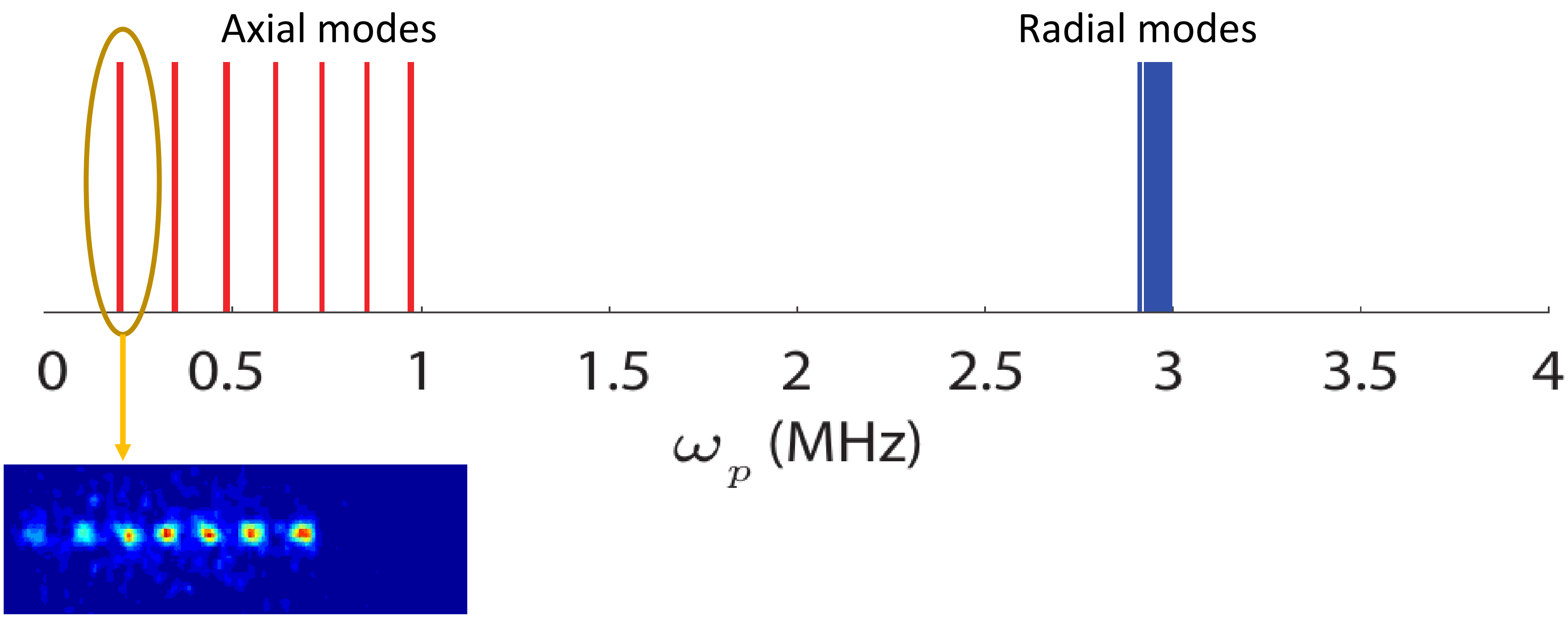
$$V = \frac{m\omega_z^2}{2}\sum_{i=1}^{N} z_i(t)^2 + \frac{(Ze)^2}{8\pi\varepsilon_0}\sum_{\substack{j,i=1\\ n\neq i}}^{N} \frac{1}{|z_j(t) - z_i(t)|}$$



Normal modes

Perform Taylor expansion around equilibrium positions to find normal modes.

Analogous to 3D classical coupled harmonic oscillator: 3N modes.



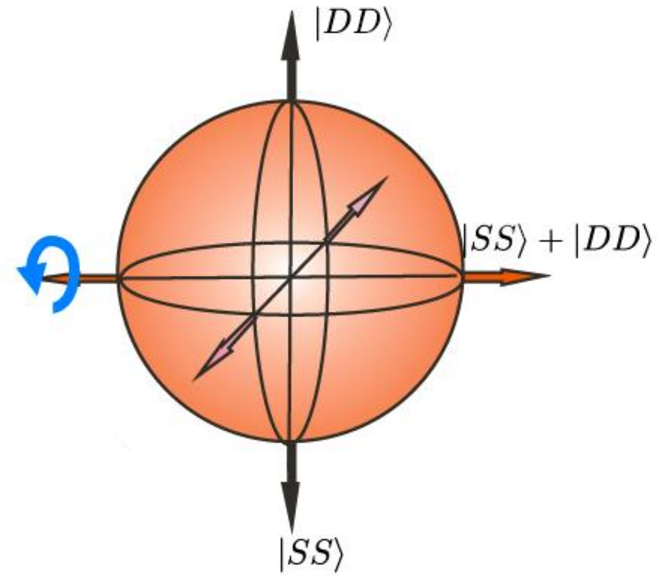
C. Marquet, F. Schmidt-Kaler, and D. F. V. James, Applied Physics B 76, 199 (2003)

Mølmer-Sørensen entangling operation

Based on state-dependent light forces.

Works for any number of qubits

Effective infinite range 2-body interaction.



T. Monz et al., *PRL.* **106**, 130506 (2011).

K. Mølmer and A. Sørensen, PRL 82, 1835 (1999).

Mølmer-Sørensen entangling operation

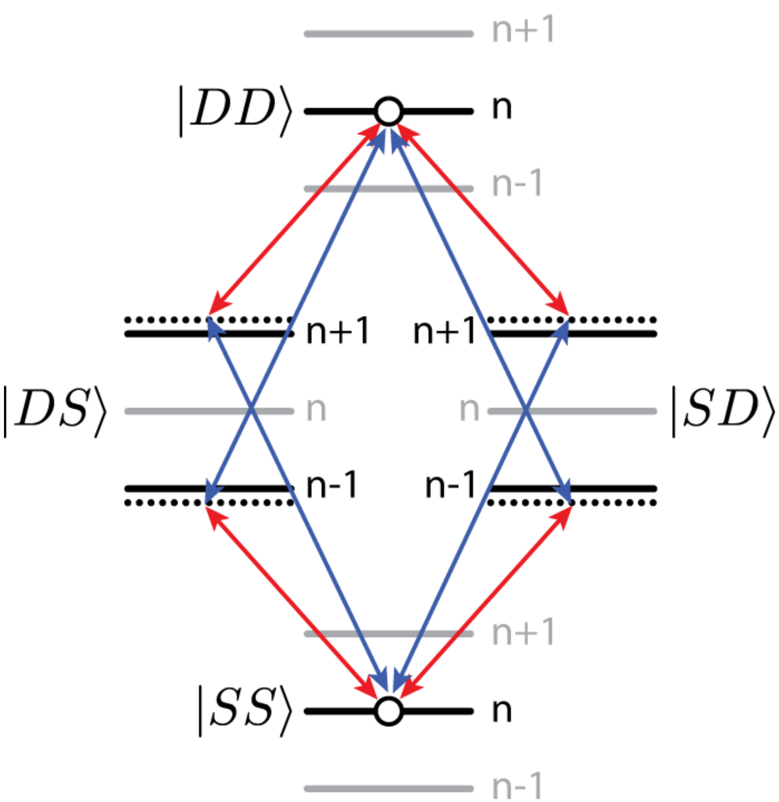
Lamb Dicke approximation interaction Hamiltonian for a single light field:

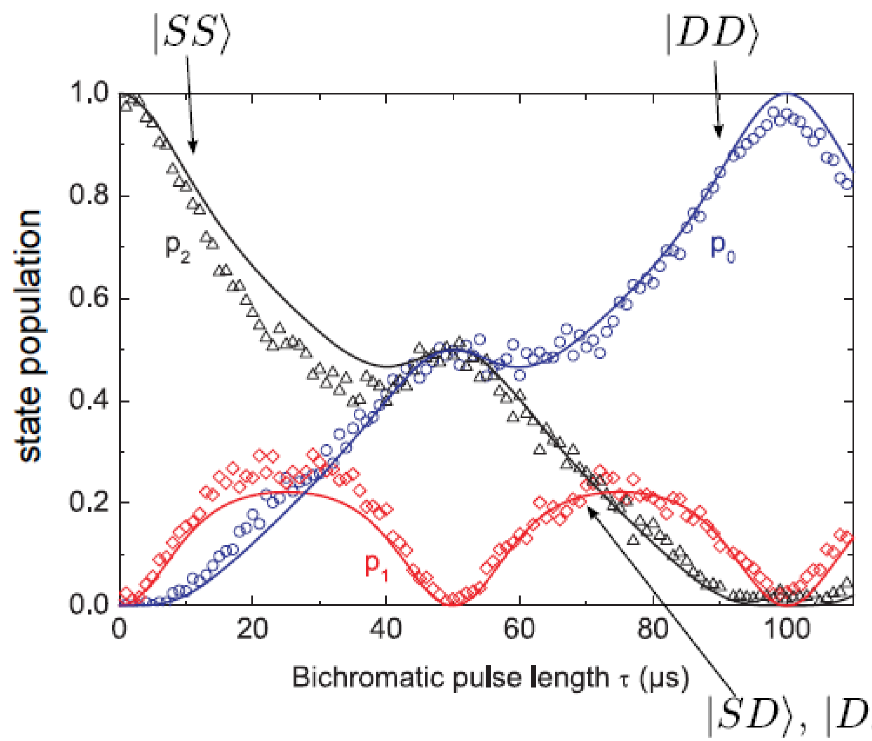
$$\hbar\frac{\Omega}{2}\left\{(e^{-i(\Delta t-\phi_L)}\sigma_+\left[1+i\eta\left(ae^{-i\omega_t t}+a^\dagger e^{i\omega_t t}\right)\right]+h.c.\right\}.$$

Further derivation on the blackboard.

C. F. Roos, New Journal of Physics 10, 013002 (2008)

Mølmer-Sørensen entangling operation

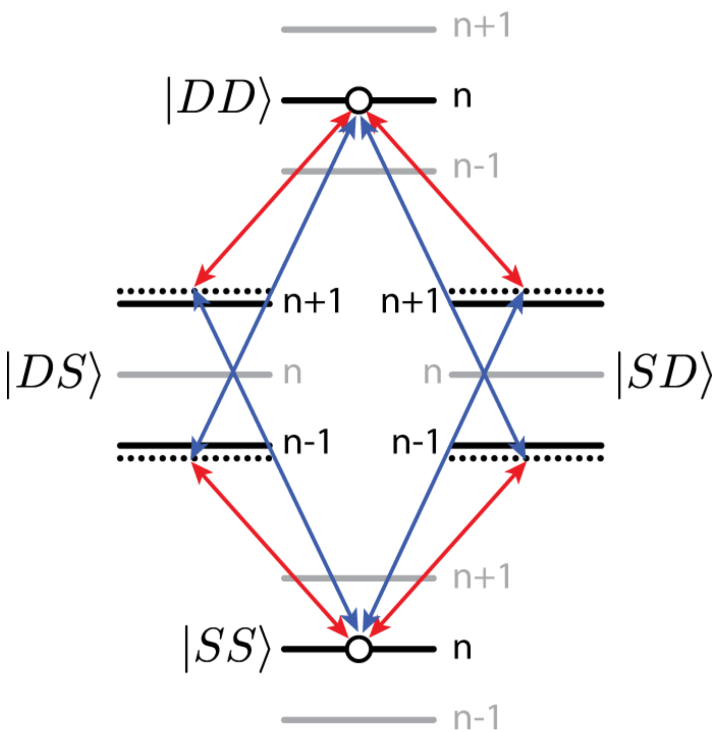


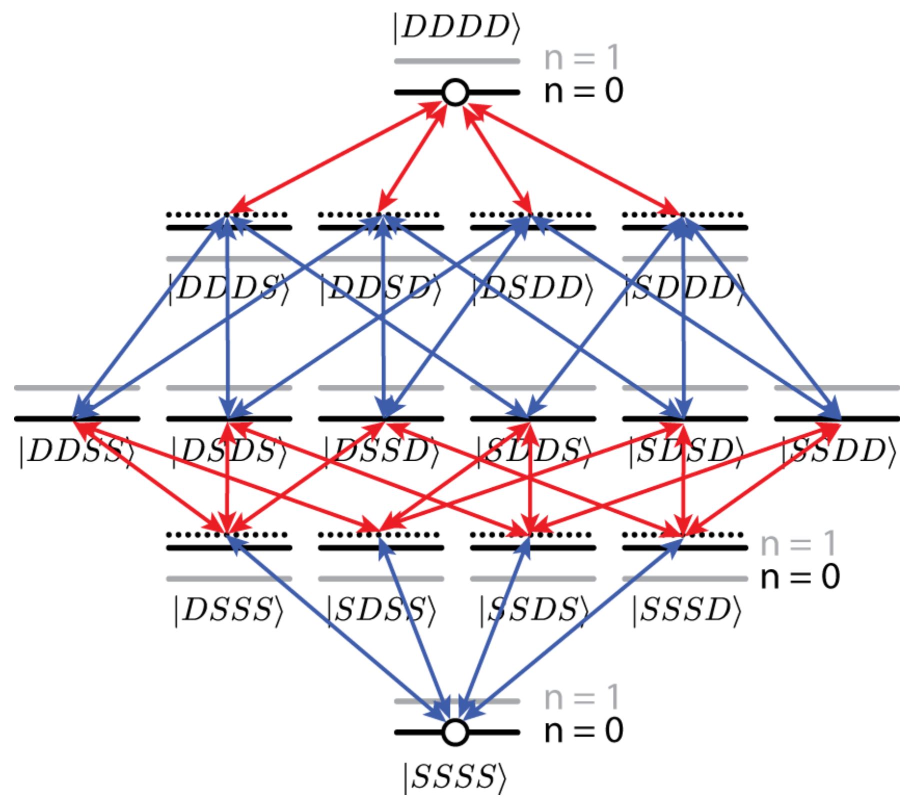


Off-resonant coupling to the sidebands
Unwanted populations interfere destructively

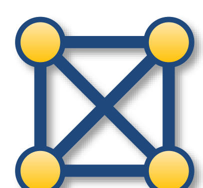
G. Kirchmair, et. al. New. J. Phys. 11, 023002 (2009)
K. Mølmer and A. Sørensen, PRL 82, 1835 (1999).

Multi path interferometer



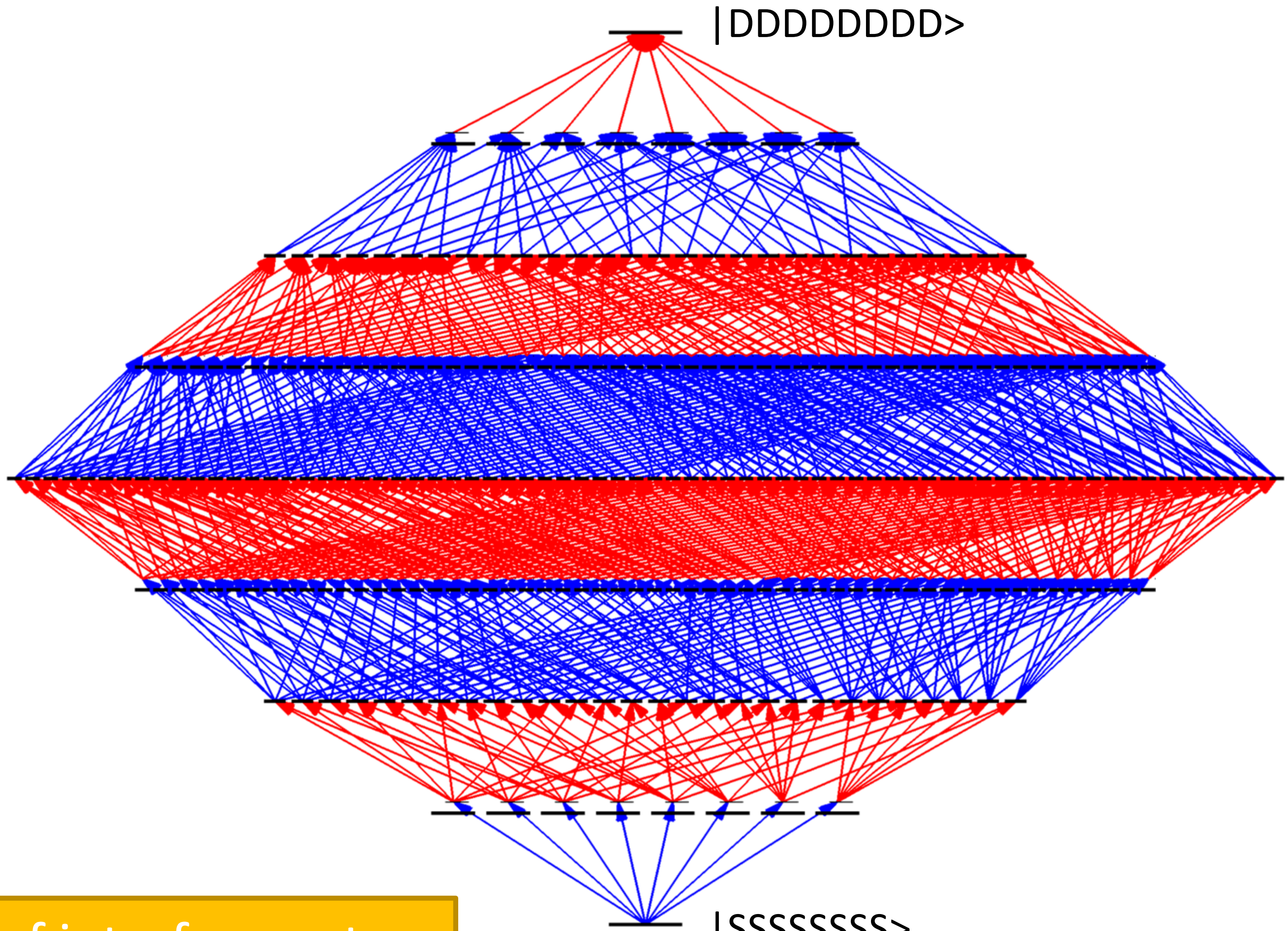


2 qubits



4 qubits

Multi path interferometer – 8 ions



Number of interferometer paths grows exponentially.

Outline

- Laser cooling and ion species
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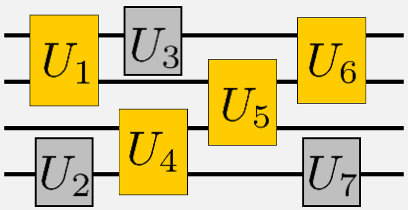
Digital (Discrete) Quantum Simulator

Goal

Simulate the physics of a quantum system of interest by another system that is easier to control and to measure.

Approach

Use a quantum computer as a quantum simulator



Decompose dynamics induced by system Hamiltonian into sequence of quantum gates

$$U_{\text{sim}} = \prod_{j=1}^{N} U_j \qquad U_{\text{sim}} \propto U_{\text{sys}} \qquad U_{\text{sys}} = e^{-\frac{i}{\hbar} H_{\text{sys}}\, \tau}$$

Example:

$$H = H_1 + H_2 + \ldots + H_k$$

$$e^{-\frac{i}{\hbar} H\, t} = \left(e^{-\frac{i}{\hbar} H_1\, t/n} e^{-\frac{i}{\hbar} H_2\, t/n} \ldots e^{-\frac{i}{\hbar} H_k\, t/n} \right)^n$$

R. Blatt, C. Roos, Nature Physics **8**, 277 (2012)

Universal Quantum Simulator

$$H = \sum_k h_k$$

← model of some local system to be simulated for a time t

0. have a universal set on ‘encoding’ degrees of freedom

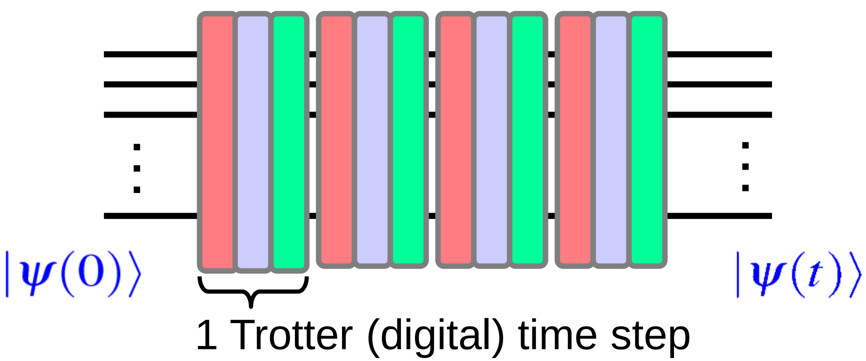
1. build each local evolution operator separately, for small time steps, using operation set

$$u_k = e^{-ih_k t/n}$$

2. approximate global evolution operator using the Trotter approximation

$$Et/h = \theta$$

$$U = e^{-iHt} \approx \left(e^{-ih_1 \frac{t}{n}} e^{-ih_2 \frac{t}{n}} e^{-ih_3 \frac{t}{n}} \ldots e^{-ih_k \frac{t}{n}} \right)^n$$



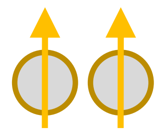
“Efficient for local quantum systems“

S. Lloyd,
Science **273**, 1073 (1996)

Trotterization of non-commuting terms

2-spin Ising system

$$H = \boxed{J\sigma_x^1\sigma_x^2} + \boxed{B(\sigma_z^1+\sigma_z^2)}$$



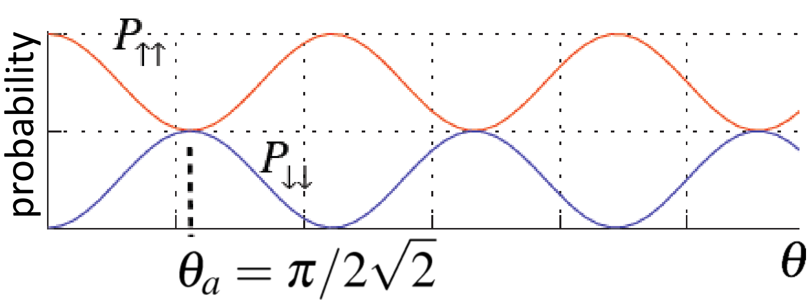
$$\tilde{H} = \sigma_x^1\sigma_x^2 + R(\sigma_z^1+\sigma_z^2) \qquad R = B/J$$

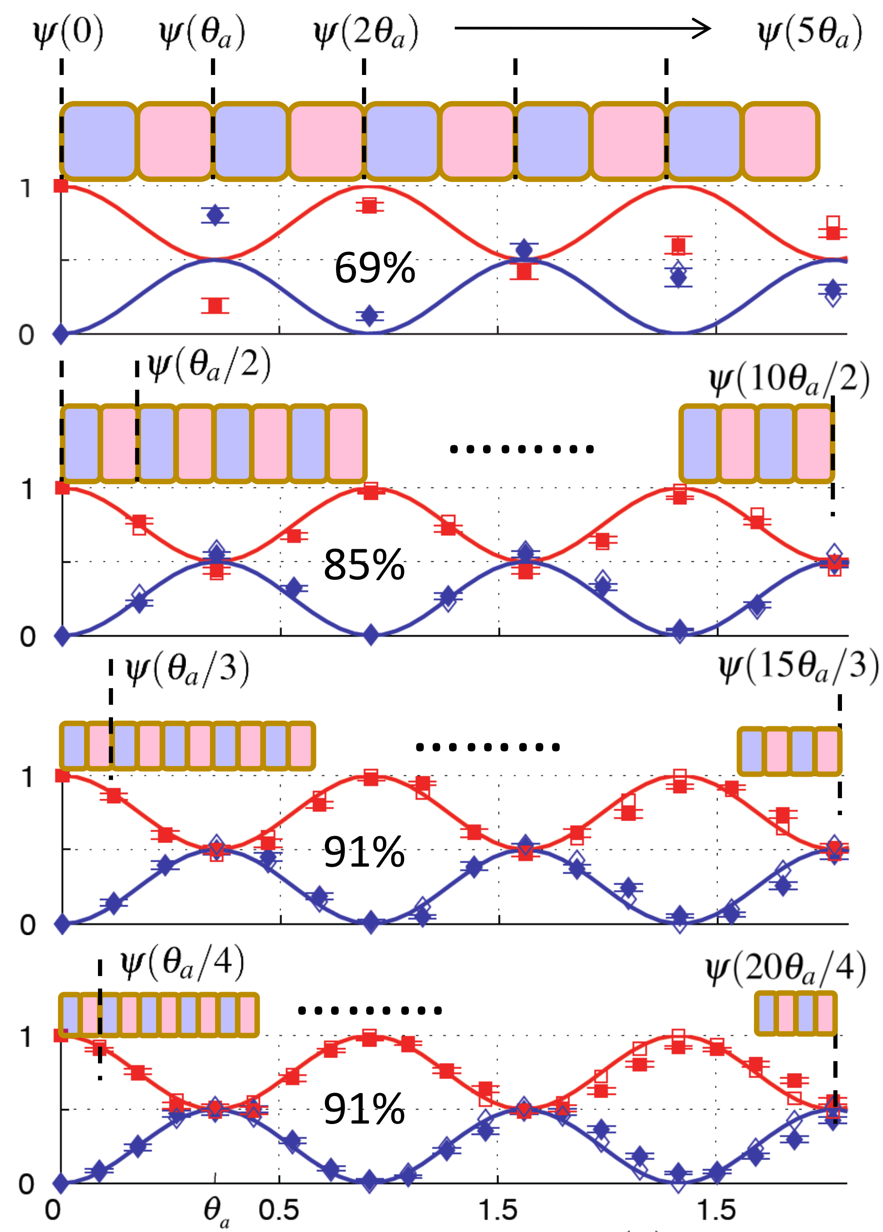
$$U(\theta) \simeq \left(\boxed{e^{-i\sigma_x^j\sigma_x^k\frac{\theta}{n}}}\boxed{e^{-i\sum_j\sigma_z^j R\frac{\theta}{n}}}\right)^n$$

Mølmer-Sørensen gate

AC-Stark gate

dynamics to simulate: $e^{-iH\theta}|\uparrow\uparrow\rangle,\ R = 0.5$





Simulating spins in small systems

2-spin simulations

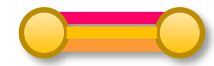
Ising

$$J\sigma_x^1\sigma_x^2 + B\sum_{i=1}^{n}\sigma_z^i$$

XY

$$\cdots + J_y\sigma_y^1\sigma_y^2$$

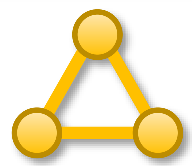
XYZ



$$\cdots + J_z\sigma_z^1\sigma_z^2$$

3-spin simulations

Ising type 1

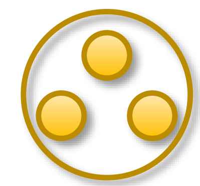


$$J\sum_{i\neq j}\sigma_x^i\sigma_x^j + B\sum_{i=1}^{n}\sigma_z^i$$

Ising type 2

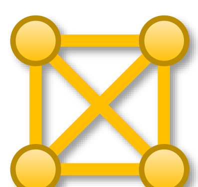
$$J_{12} = J_{23},\ J_{13} = 0$$

n-body

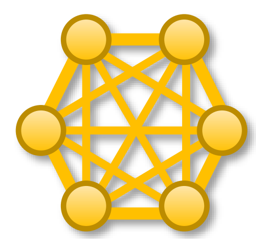


$$H = \sigma_x^{\otimes n}$$

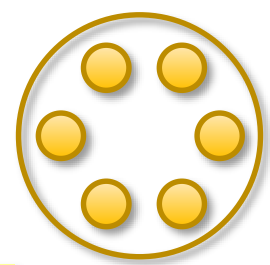
>3-spin simulations



4 spins



6 spins

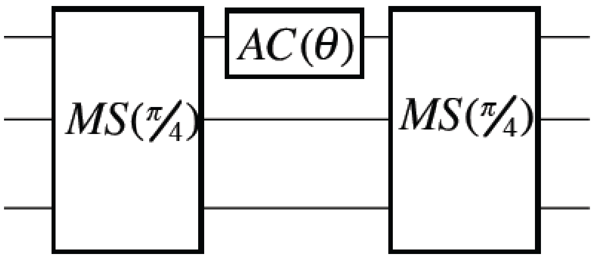


B. Lanyon et al.,
Science **334**, 6052 (2011)

Many-body interactions

Many-body interactions $H = \sigma_x^{\otimes n}$

B. Lanyon et al., Science **334**, 6052 (2011)



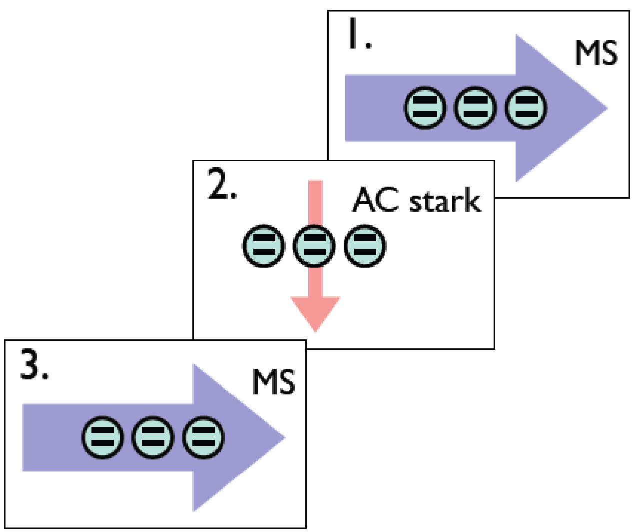
$= e^{i\,\sigma_z^1 \sigma_x^2 \sigma_x^3 \theta}$

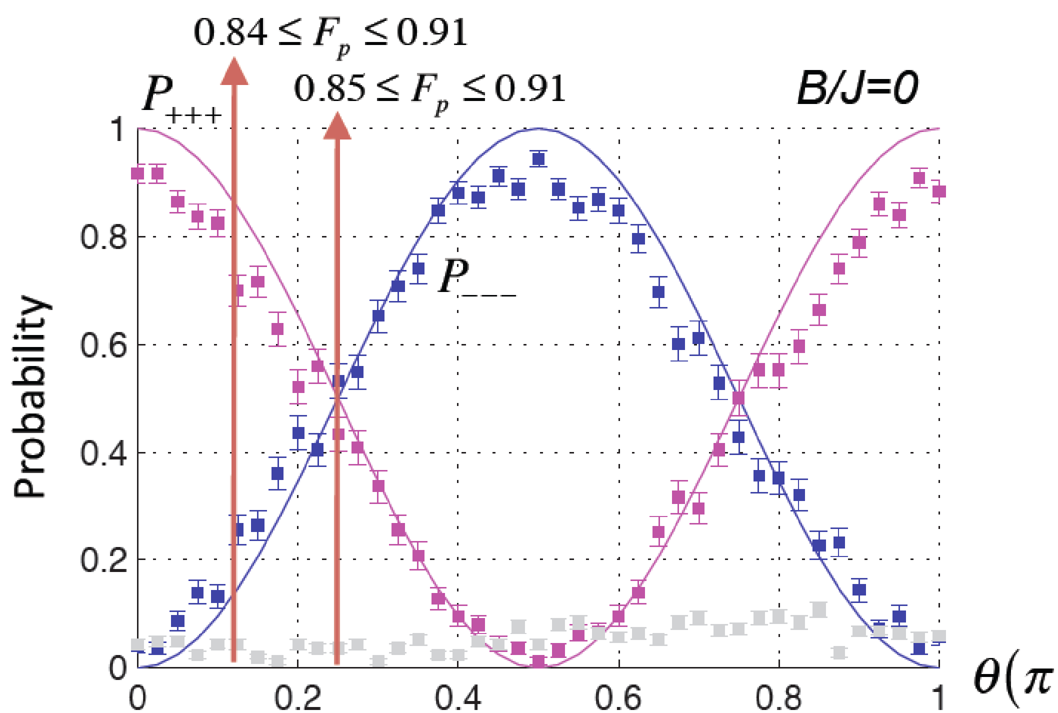
Effective 3-body Hamiltonian

Some interesting dynamics

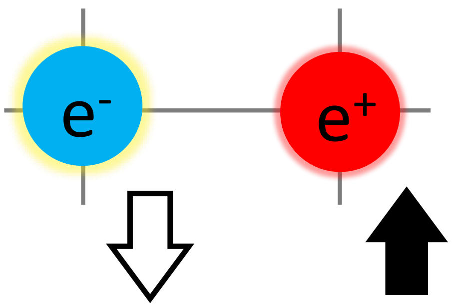
$|0\rangle + i|1\rangle$

$|+,+,+\rangle \Leftrightarrow |-,-,-\rangle$

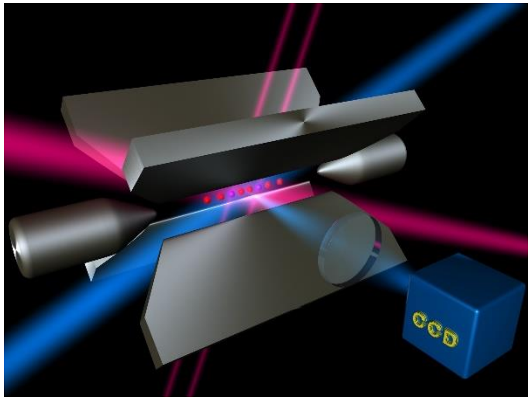




The model: lattice gauge theories



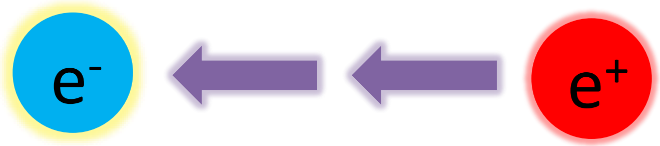
Trapped-ion quantum information processing



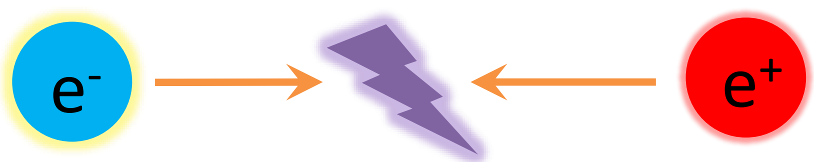
Quantum simulation of lattice gauge theories

Quantum electrodynamics

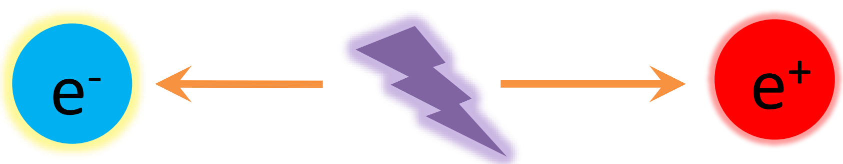
- Charged particles (electrons, e^-) and antiparticles (positrons, e^+) interact via electromagnetic force fields.



  - □ Particles and antiparticles can mutually annihilate.

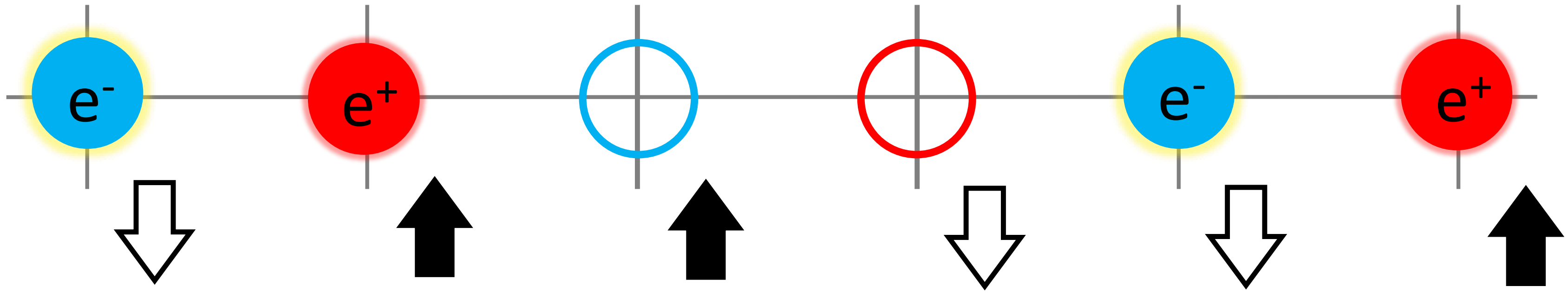


  - □ Prediction: spontaneous creation of particle-antiparticle pairs in strong static fields (Schwinger mechanism).

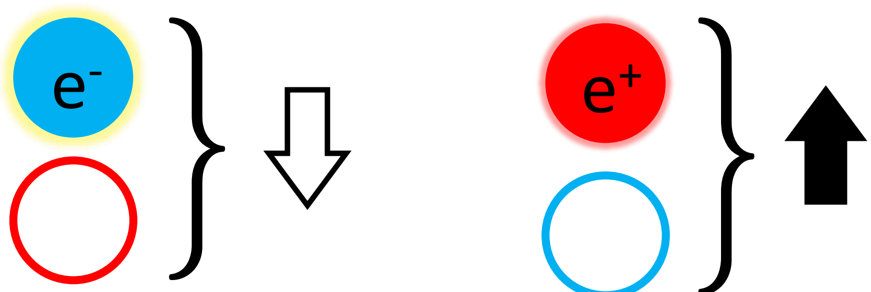


Lattice gauge theories

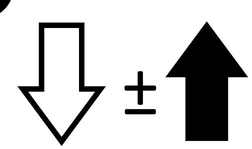
How can we simulate gauge theories?

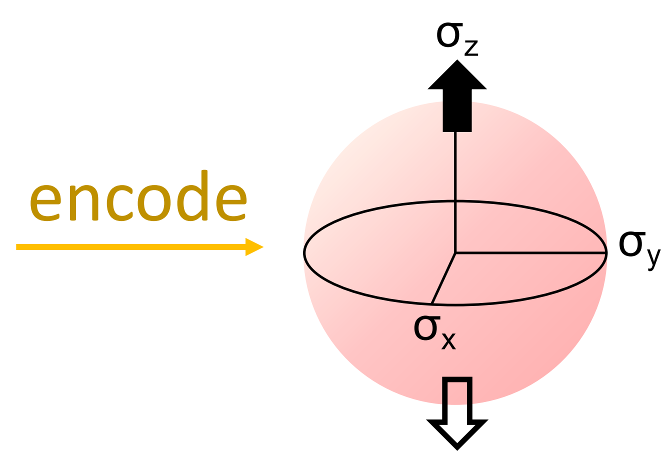


- Discretize space to a lattice (1D for simplicity).
- Blue sites hold particles, red sites hold antiparticles.
- Each site has two possible states (full/empty), encoding:



and also superpositions:





Time evolution

- The Hamiltonian of our spin system is:

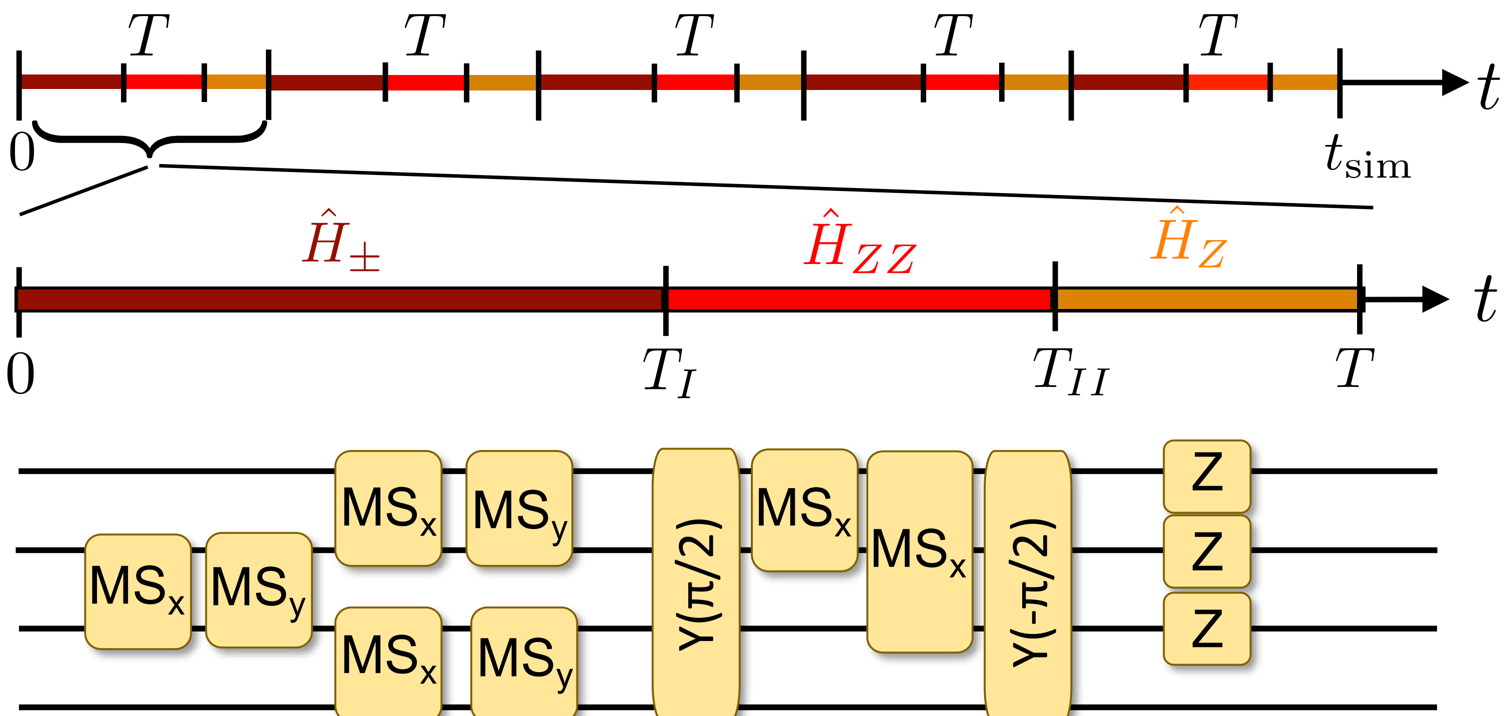
$$H = w\sum_{i=1}^{N-1} \sigma_i^+\sigma_{i+1}^- + \sigma_i^-\sigma_{i+1}^+ + J\sum_{i,j=1}^{N} c_{ij}\sigma_i^z\sigma_j^z + m\sum_{i=1}^{N} c_i\sigma_i^z + J\sum_{i=1}^{N} \tilde{c}_i\sigma_i^z$$

particle – antiparticle creation/annihilation

long-range interactions

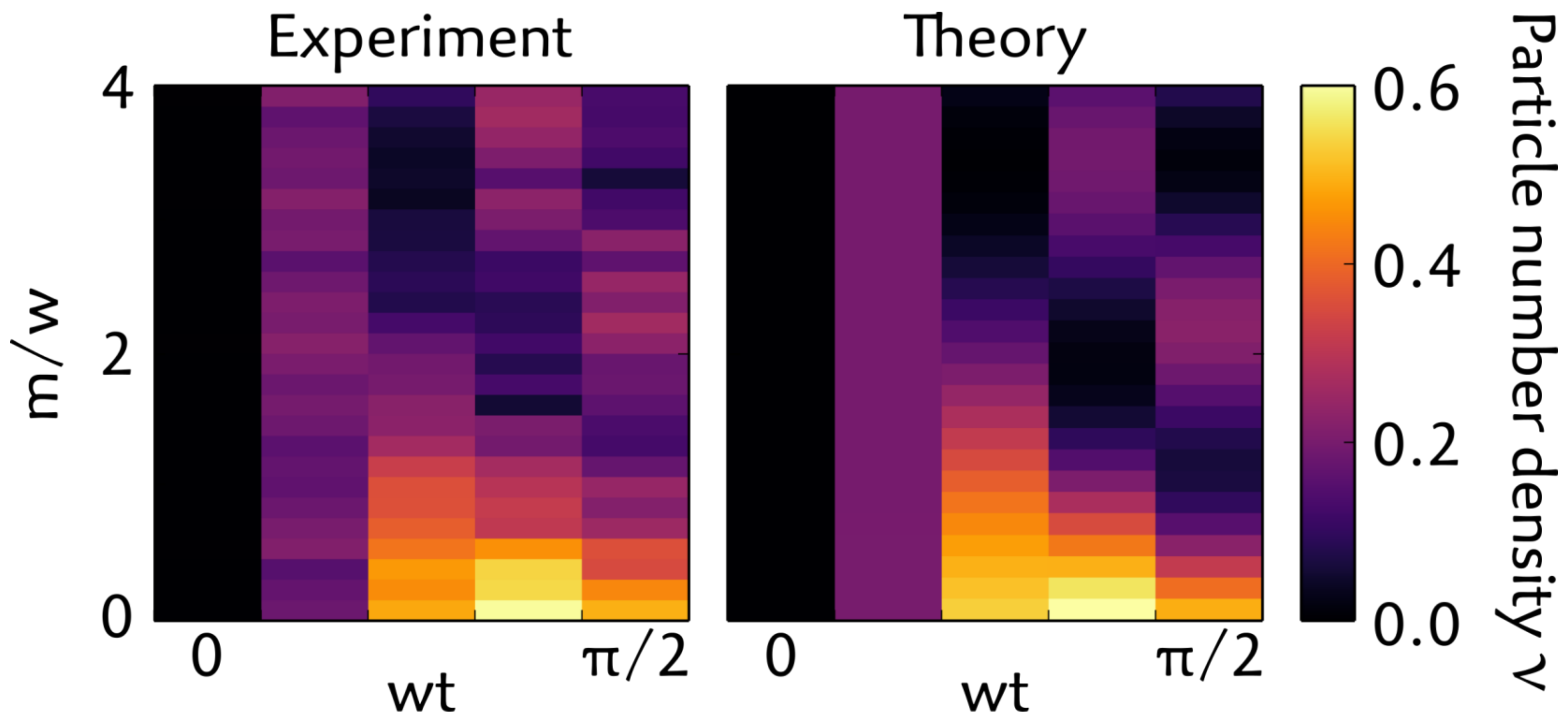
effective particle masses

- We slice the evolution in small time steps (Trotterization):



Dynamics of e^+e^- pair creation

Time evolutions for different particle masses *m:*

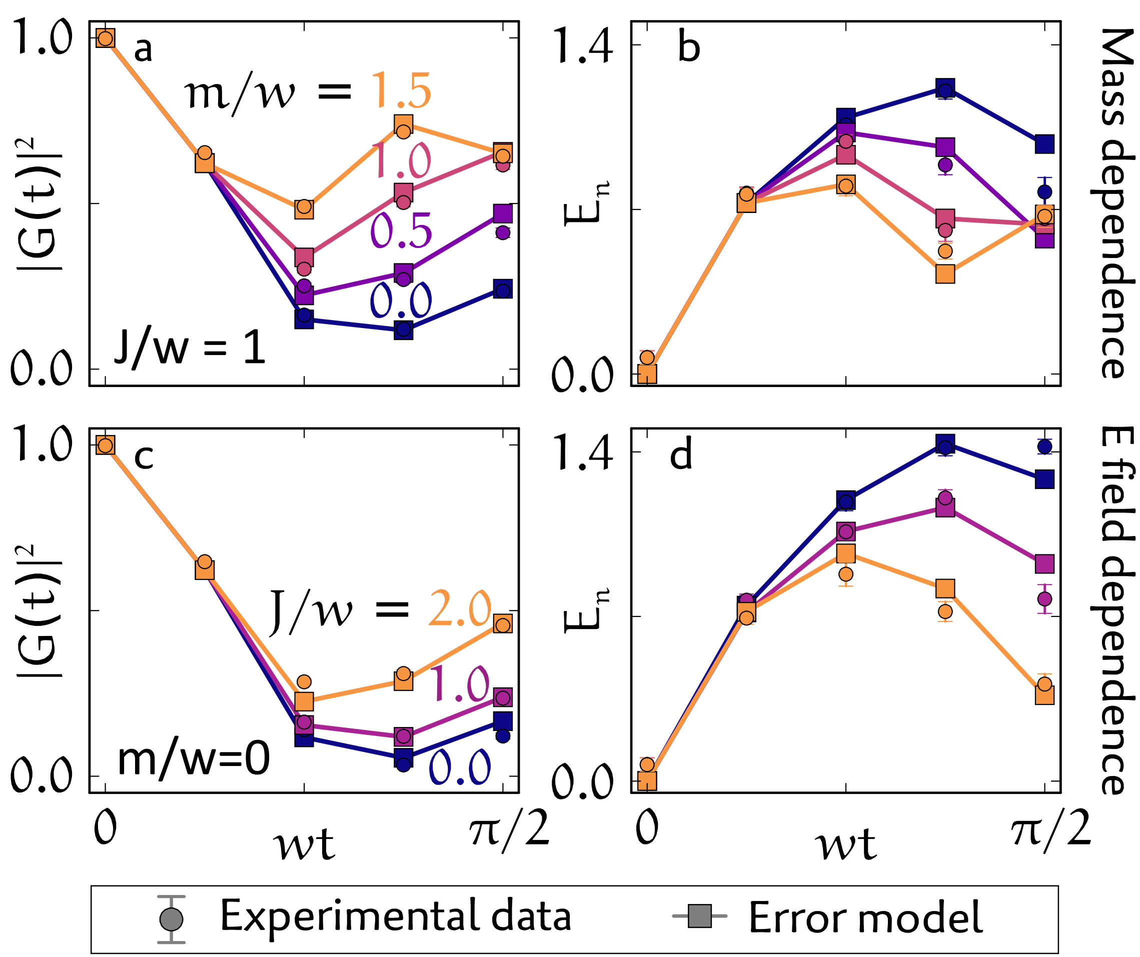


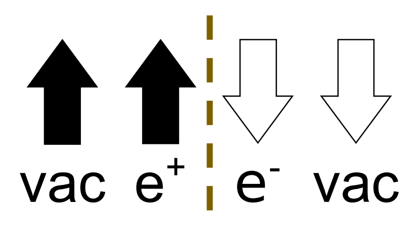
Higher mass → faster oscillations, less amplitude

What you cannot see at CERN

We have full experimental access to the system wavefunction:

$G(t) = \langle \text{vacuum} | \Psi(t) \rangle$ Entanglement

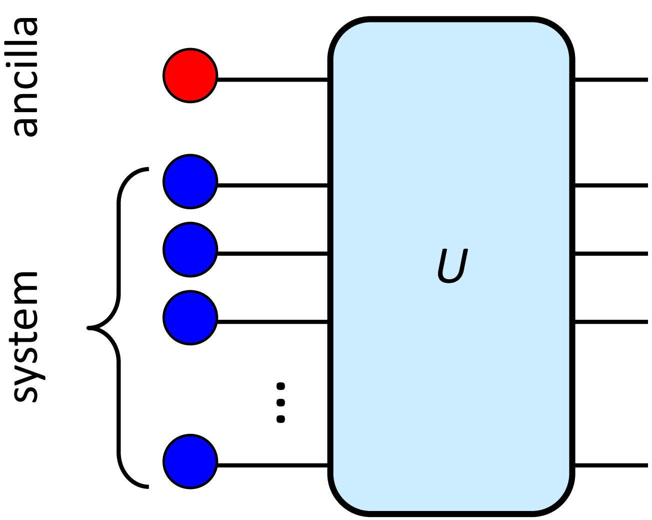


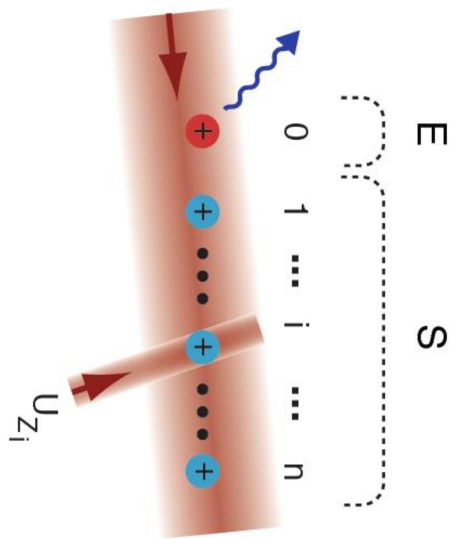


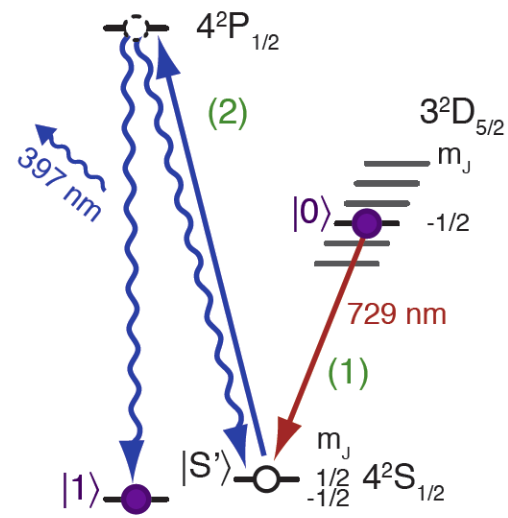
(E_n = log. negativity across this bipartition)

Simulating open systems

J. Barreiro, M. Müller et al., Nature **470**, 486 (2011)



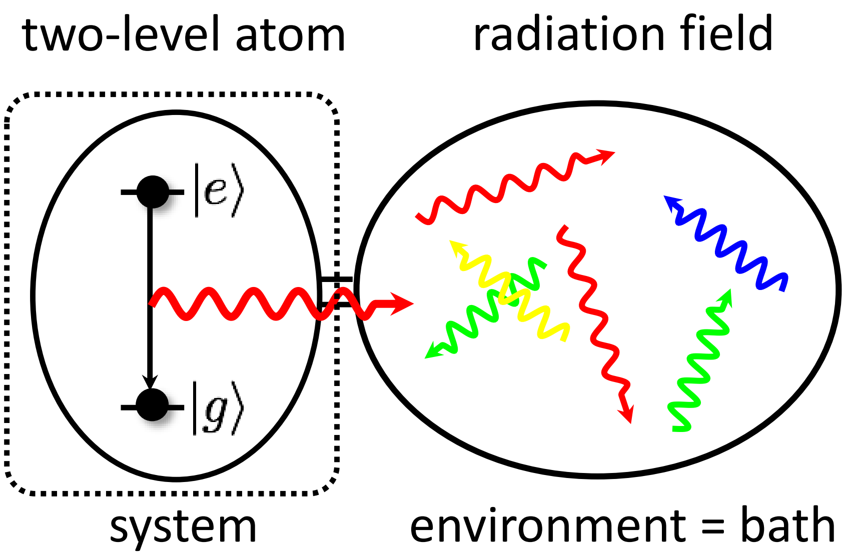


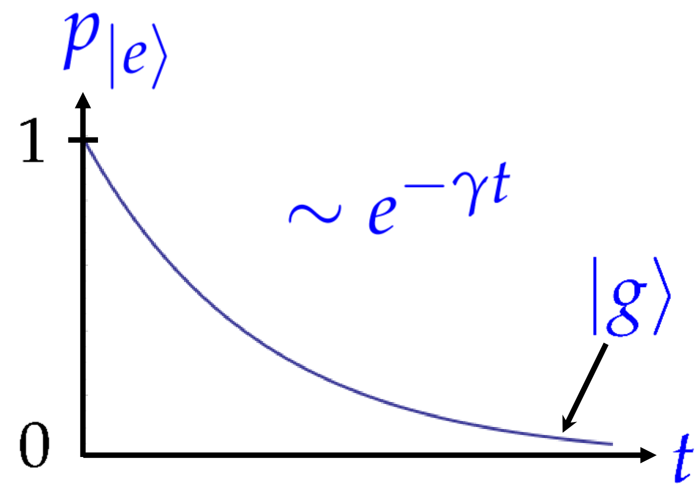


engineering many-body operations
engineering the environment
stabilizer pumping (Bell, GHZ)
QND measurements of many-body operators
dissipative state preparation

open systems
quantum simulations

Dissipative quantum systems





are weekly coupled

formal description: **reduced density matrix** of the system:

master equation: $$\frac{d}{dt}\rho = -i[H,\rho] + \mathcal{L}(\rho)$$

Liouvillian

$$\mathcal{L}(\rho) = \gamma\left(c\rho c^{\dagger} - \frac{1}{2}c^{\dagger}c\rho - \frac{1}{2}\rho c^{\dagger}c\right)$$

Hamiltonian dynamics

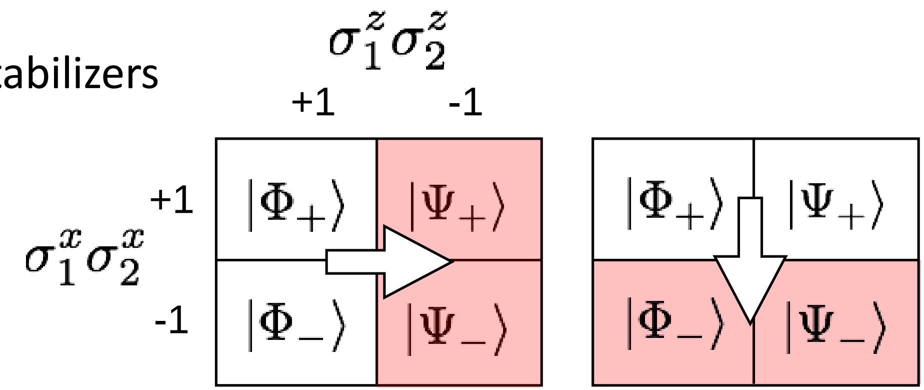
quantum jump operator $c = |g\rangle\langle e|$

Bell state pumping

Engineer dissipative dynamics that pumps a many-body system into an entangled state

M. Müller, P. Zoller (2010-11)

stabilizers



one pumping cycle

two qubits 1 2

$$|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$$

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

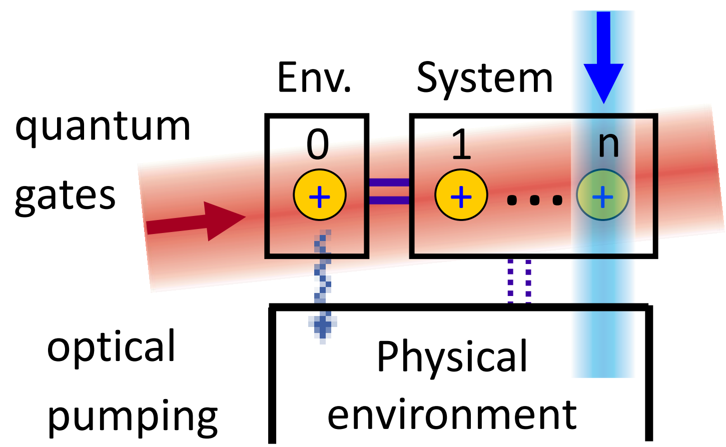
uniquely fixed by their eigenvalues with respect to $\sigma_1^z \sigma_2^z$ and $\sigma_1^x \sigma_2^x$

master equation for $\sigma_1^z \sigma_2^z$ - pumping:

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho = \gamma(c\rho c^\dagger - \frac{1}{2}c^\dagger c\rho - \rho\frac{1}{2}c^\dagger c)$$

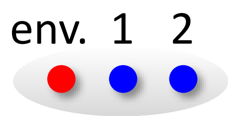
with $\quad c = \sigma_x^{(1)} \frac{1}{2}(1 + \sigma_z^{(1)}\sigma_z^{(2)})$

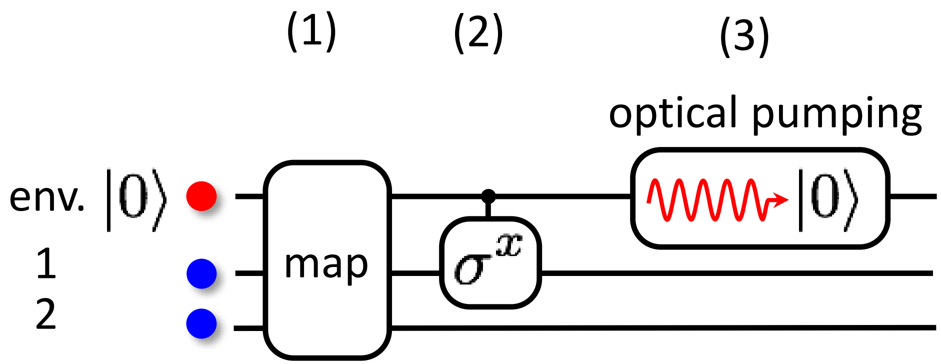
two-body quantum jump operator

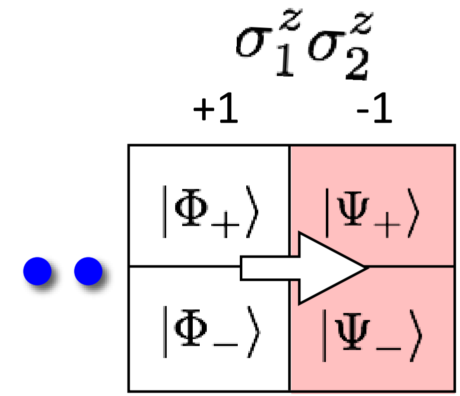


Implementing Pumping

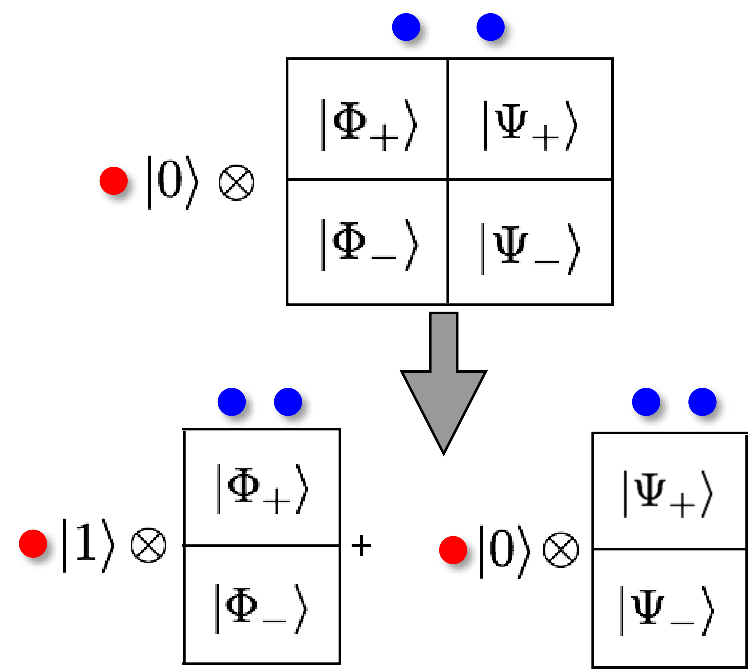
Implementation with 1+2 ions



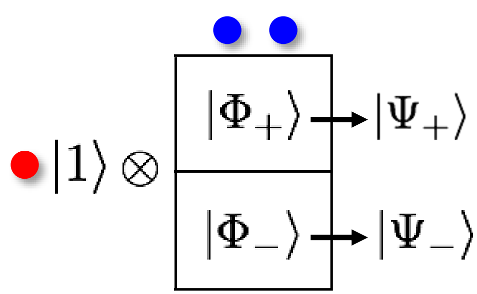




(1) coherent mapping



(2) two-qubit gate $C = |0\rangle\langle 0| \otimes 1 + |1\rangle\langle 1| \otimes \sigma_1^x$

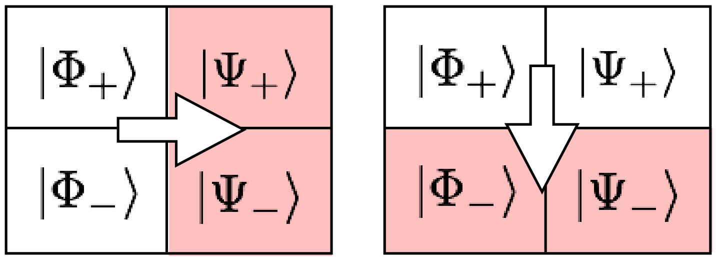


(3) optical pumping env. ion: $|1\rangle \rightsquigarrow |0\rangle$

... as the dissipative ingredient

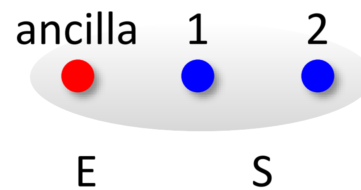
Experimental Bell state pumping

one pumping cycle

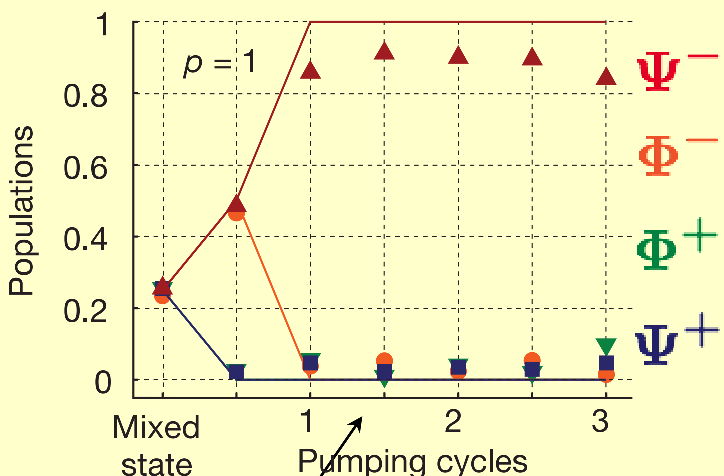


with pumping probability p

system ions

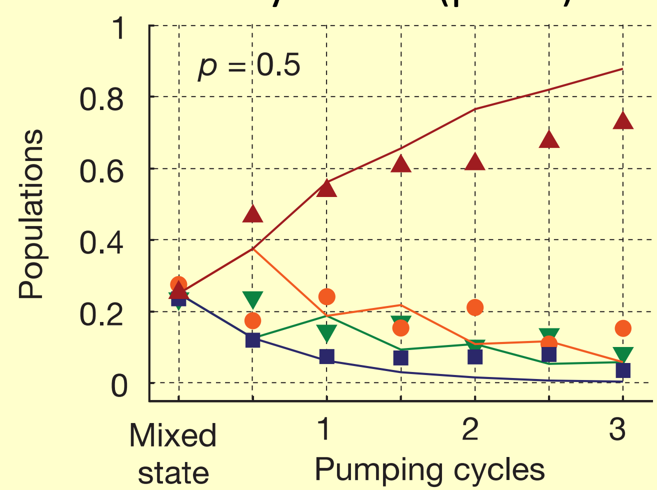


deterministic pumping (p=1)



$F_{process}$ = 87(1)%

towards master equation dynamics (p=0.5)



J. Barreiro, M. Müller et al., Nature **470**, 486 (2011)