Deakin University

Data Structures and Algorithms

OnTrack Submission

AVL-Trees

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Outcome	**Weight**
Complexity	♦◇◇◇◇
Implement Solutions	♦♦♦◇◇
Document solutions	♦◇◇◇◇

The task involves understanding the requirements for a valid AVL Tree and the rotations involved in balancing the tree after insertions and deletions. This relates to ULO2 by creating and using a range of data structures and algorithms.

September 3, 2020

Task 7.1

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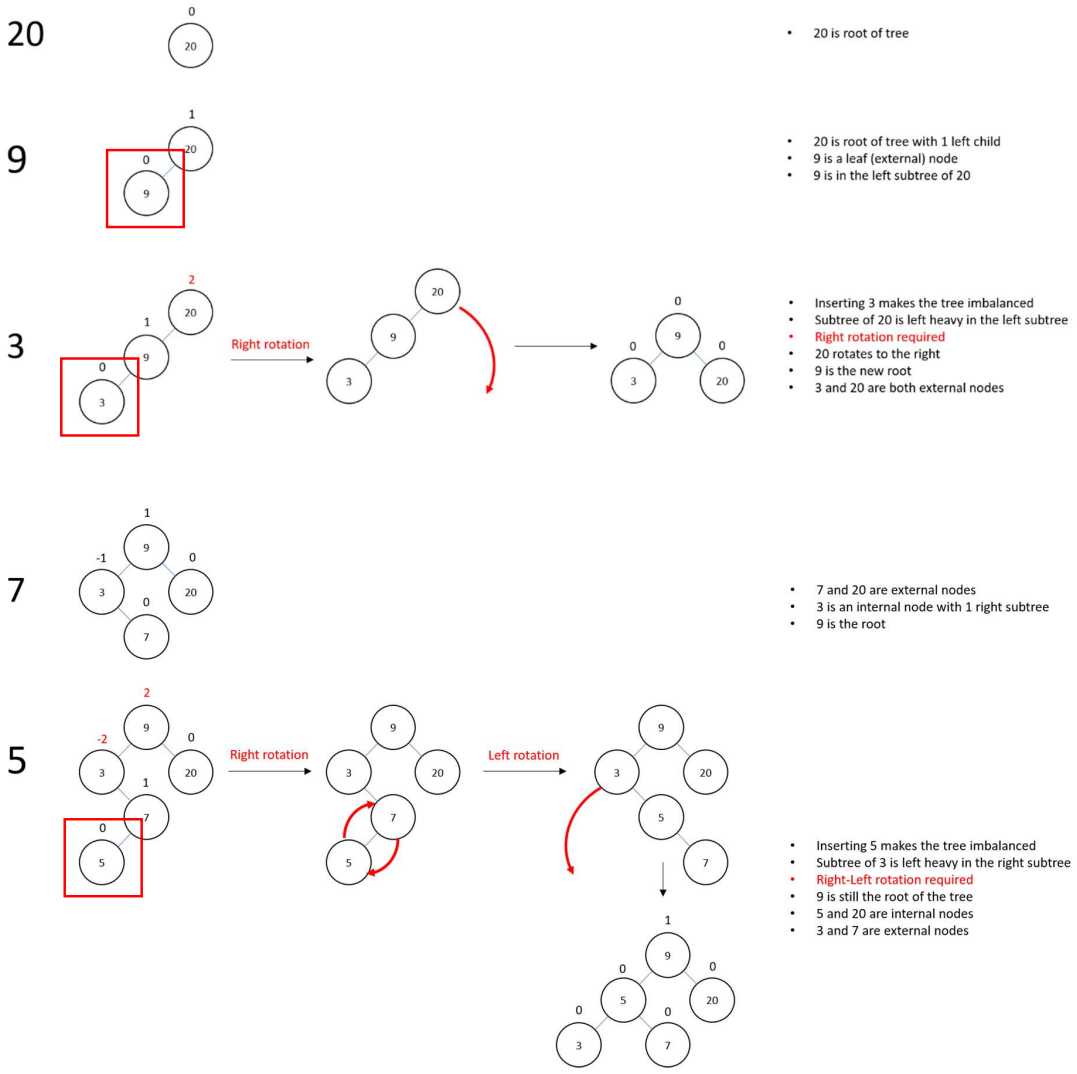
Question 1

Draw a series of figures demonstrating the insertion of the values:

20, 9, 3, 7, 5, 8, 25, 30, 15, 6, 17



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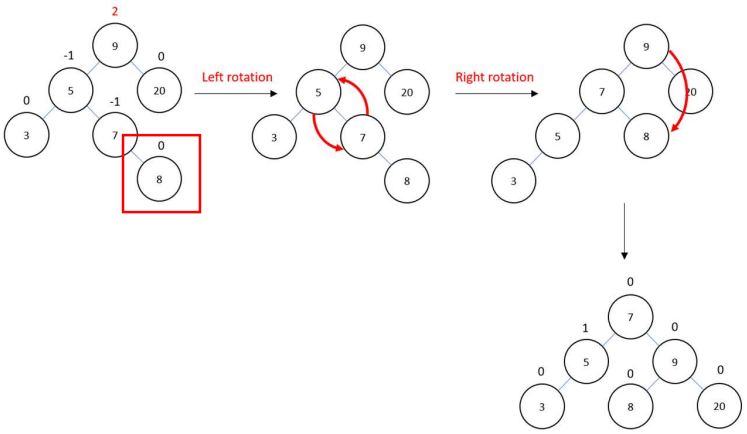
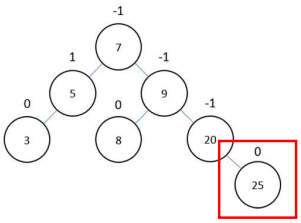


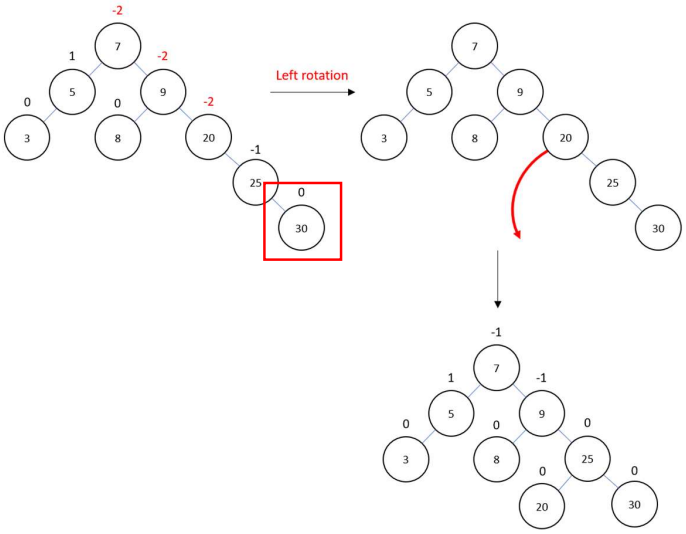
- Inserting 8 makes the tree imbalanced
- Subtree of 9 is right heavy in the left subtree
- Left-Right rotation required
- 7 becomes the new root of the tree
- 5 and 9 are internal nodes
- 3, 8 and 20 are external nodes

25



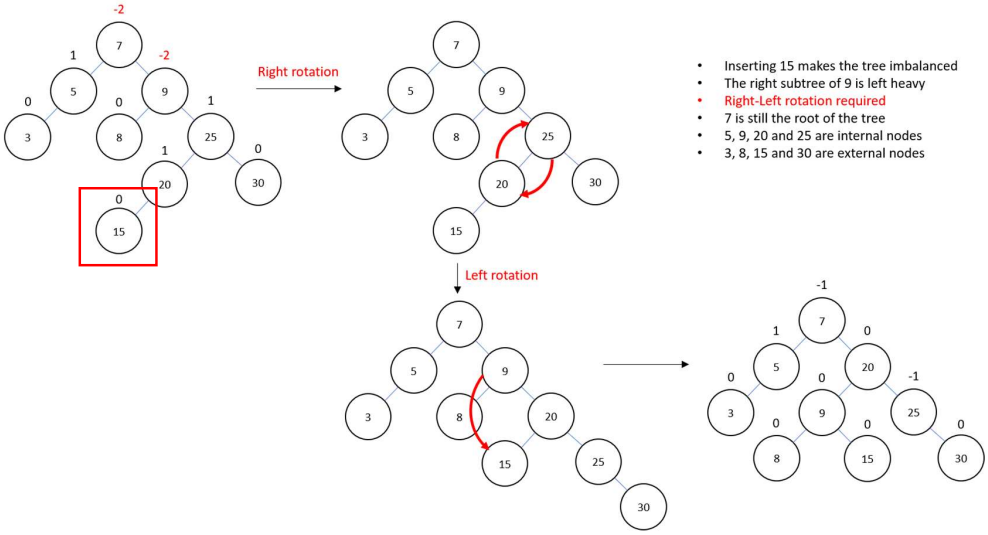
- No rotation required after inserting 25
- 20 is now an internal node
- 25 is the new external node

30



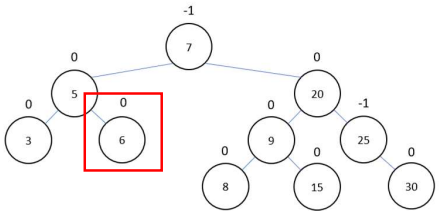
- Inserting 30 makes the tree imbalanced
- The right subtree of 9 is right heavy
- Left rotation required
- 7 is still the root of the tree
- 5, 9 and 25 are internal nodes
- 3, 8, 20 and 30 are external nodes

15



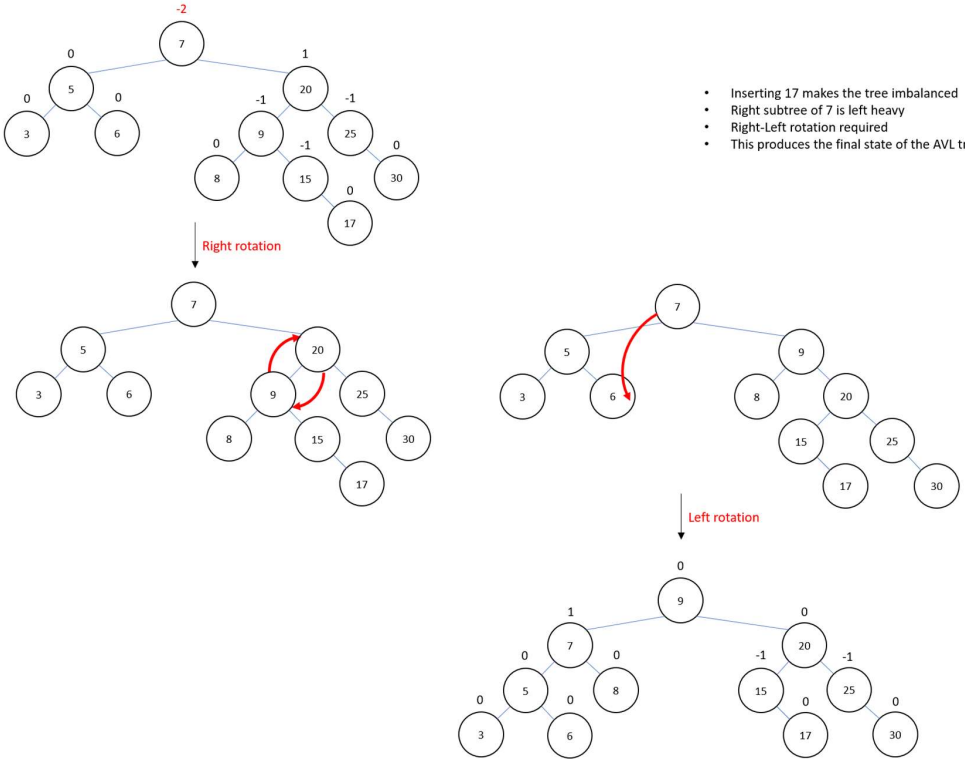
- Inserting 15 makes the tree imbalanced
- The right subtree of 9 is left heavy
- Right-Left rotation required
- 7 is still the root of the tree
- 5, 9, 20 and 25 are internal nodes
- 3, 8, 15 and 30 are external nodes

6



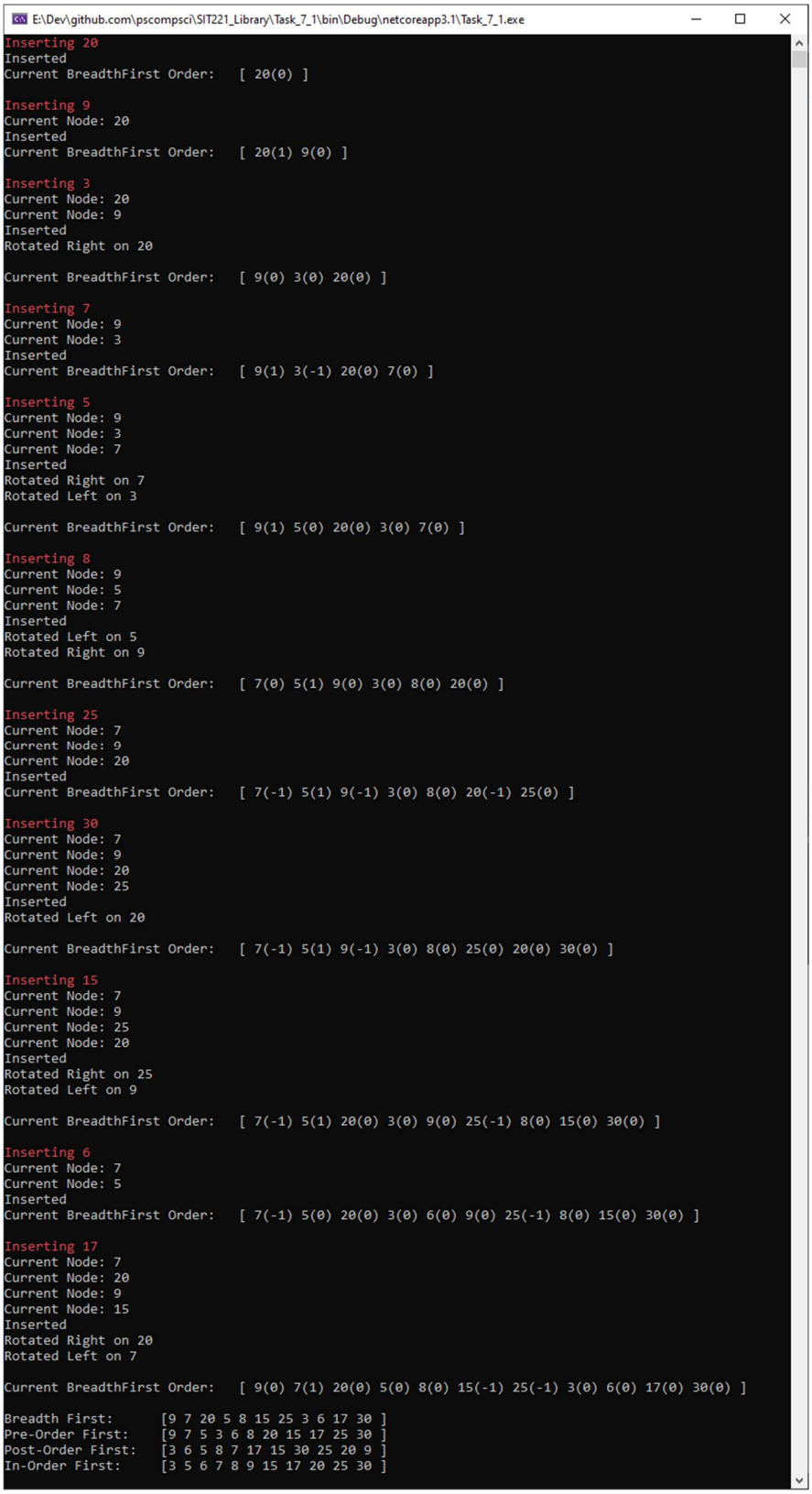
- No rotation required after inserting 6
- 7 remains the root
- 5, 20, 9 and 25 are internal nodes
- 3, 6, 8, 15 and 30 are external nodes

17



- Inserting 17 makes the tree imbalanced
- Right subtree of 7 is left heavy
- Right-Left rotation required
- This produces the final state of the AVL tree

To confirm my results, I wrote an AVL Tree class in C# and tester to print the operations:

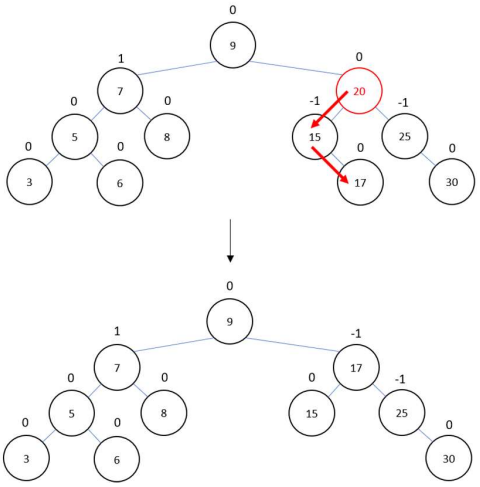


Question 2

Now, draw a series of figures showing the deletion of the values:

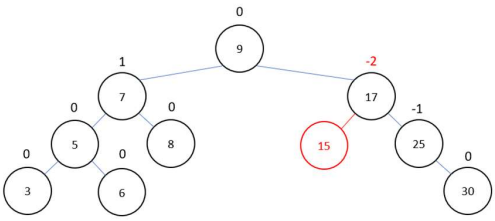
20, 15, 8, 25, 30, 9, 17, 5, 6, 3, 7

20



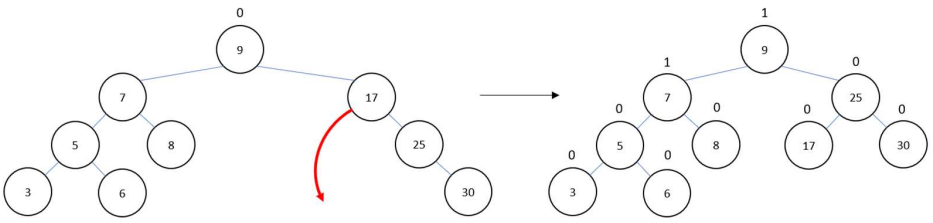
- Find the right most value in the left subtree
- This is the largest element smaller than 20
- Replace 20 with 17
- Delete 17
- No rebalance required

15



- 15 is an external node
- Delete 15
- Imbalance in the right subtree of 17
- Left rotation required

Left rotation



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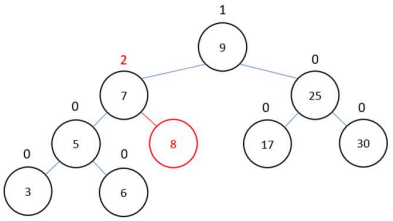
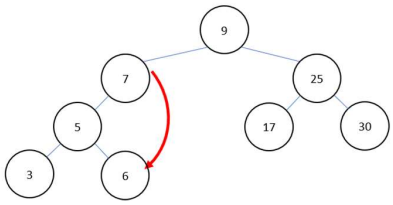
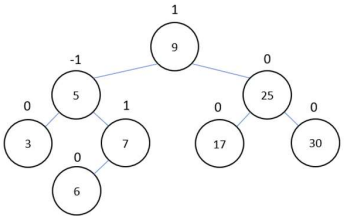


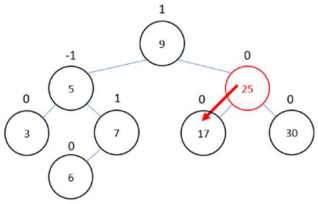
- 8 is an external node
- Delete 8
- Imbalance in the left subtree of 7
- Right rotation required

Right rotation

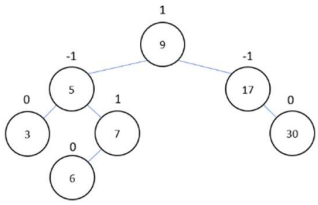




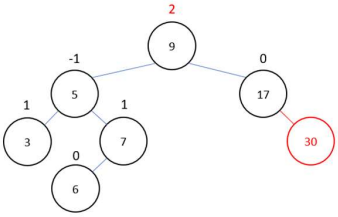
25



- 25 is an internal node with left and right subtree
- Find right-most value in left subtree
- Replace 25 with 17
- Delete 17
- No rotation required

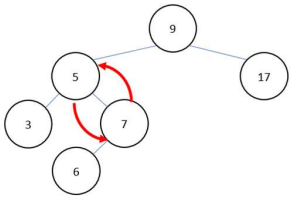


30

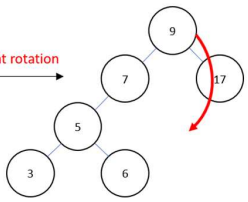


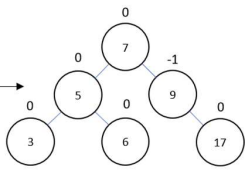
- 30 is an external note
- Delete 30
- Tree is now imbalanced
- Left subtree of 9 is right heavy
- Left-Right rotation required

Left rotation

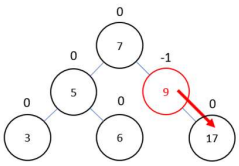


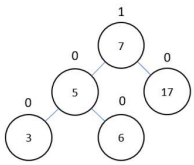
Right rotation





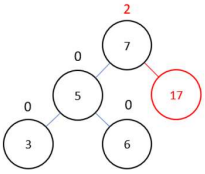
9



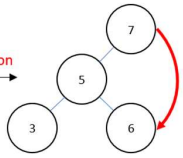


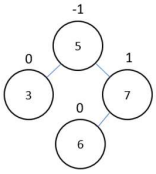
- 9 is an internal node with a right subtree
- Replace 9 with 17
- Delete 17
- No rebalance required

17



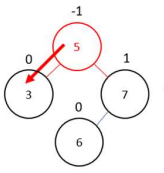
Right rotation

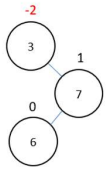




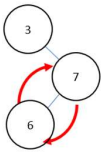
- 17 is an external node
- Delete 17
- Tree is now imbalanced
- Left subtree of 7 is left heavy
- Right rotation required

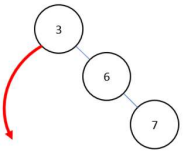
5



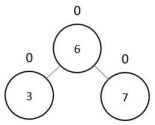


Right rotation



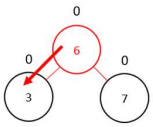


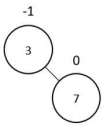
Left rotation



- 5 is the root
- Right-most key in left subtree is 3
- Replace 5 with 3
- Right subtree is left heavy
- Right-Left rotation required

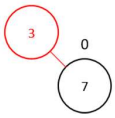
6





- 6 is the root
- Right-most key in left subtree is 3
- Replace 6 with 3
- No rebalance required

3



- 3 is the root
- Delete 3
- Make 7 the root

7

- Delete 7

Question 3

What order should we insert the elements {1, 2, … ,7} into an empty AVL tree so that we do not have to perform any rotations on it?

Any of the following orders will not require any rotation:

4, 2, 6, 1, 7, 3, 5

4, 6, 2, 1, 7, 3, 5

4, 2, 6, 7, 1, 3, 5

4, 6, 2, 7, 1, 3, 5

4, 2, 6, 1, 7, 5, 3

4, 6, 2, 1, 7, 5, 3

4, 2, 6, 7, 1, 5, 3

4, 6, 2, 7, 1, 5, 3

4, 2, 6, 7, 3, 5, 1

6, 4, 2, 7, 3, 5, 1

4, 2, 6, 7, 3, 1, 5

4, 2, 6, 7, 3, 5, 1

4, 2, 6, 5, 3, 7, 1

4, 2, 6, 5, 3, 1, 7

4, 6, 2, 5, 3, 7, 1

4, 6, 2, 5, 3, 1, 7

3, 2, 5, 1, 6, 4, 7

3, 5, 2, 1, 6, 4, 7

3, 2, 5, 6, 1, 4, 7

3, 5, 2, 6, 1, 4, 7

3, 2, 5, 1, 4, 6, 7

3, 5, 2, 1, 4, 6, 7

3, 2, 5, 4, 1, 6, 7

3, 5, 2, 4, 1, 6, 7

5, 3, 6, 2, 7, 4, 1

5, 6, 3, 2, 7, 4, 1

5, 3, 6, 7, 2, 4, 1

5, 3, 6, 7, 4, 2, 1

5, 3, 6, 4, 7, 2, 1

5, 6, 3, 4, 7, 2, 1

5, 3, 6, 2, 7, 1, 4

5, 6, 3, 2, 7, 1, 4

Attachment 1

AVLTree.cs

https://github.com/pscompsci/SIT221_Library/blob/master/Task_7_1/AVLTree.cs

```
using System;
using System.Collections.Generic;
using System.Diagnostics.Contracts;

#pragma warning disable 693

namespace Task_7_1
{
    public enum DisplayMethod
    {
        InOrder,
        PreOrder,
        PostOrder,
        BreadthFirst
    }

    public class AVLTree<T> where T : IComparable<T>
    {
        public class Node<T> : INode<T> where T : IComparable<T>
        {
            public T Key { get; set; }
            public Node<T> Left { get; set; }
            public Node<T> Right { get; set; }
            public Node(T key)
            {
               Key = key;
            }
        }

        public Node<T> Root { get; set; }
        public int Count { get; set; }
#if DEBUG
        private List<KeyValuePair<string, T>> _rotations;
#endif
        public AVLTree()
        {
#if DEBUG
```

```
            _rotations = new List<KeyValuePair<string, T>>();
#endif
        }

        public void Insert(T value)
        {
#if DEBUG
            _rotations.Clear();
            Console.ForegroundColor = ConsoleColor.Red;
            Console.WriteLine("Inserting {0}", value);
            Console.ResetColor();
#endif
            Node<T> node = new Node<T>(value);
            if (Root is null)
            {
                Root = node;
            }
            else
            {
                Root = RecursiveInsert(Root, node);
            }
#if DEBUG
            Console.WriteLine("Inserted");
            if (_rotations.Count > 0)
            {
                foreach(var pair in _rotations)
                {
                    Console.WriteLine("Rotated {0} on {1}", pair.Key,
pair.Value);
                }
                Console.WriteLine();
            }
            Console.Write("{0,-30}", "Current BreadthFirst Order:");
            Display(DisplayMethod.BreadthFirst);
#endif
            Count++;
        }

        private Node<T> RecursiveInsert(Node<T> current, Node<T> node)
        {
            if (current is null)
            {
                current = node;
                return current;
            }
#if DEBUG
```

```
            Console.WriteLine("Current Node: {0}", current.Key);
#endif
            if (node.Key.CompareTo(current.Key) < 0)
            {
                current.Left = RecursiveInsert(current.Left, node);
                current = BalanceTree(current);
            }
            else if (node.Key.CompareTo(current.Key) > 0)
            {
                current.Right = RecursiveInsert(current.Right, node);
                current = BalanceTree(current);
            }
            return current;
        }
        private Node<T> BalanceTree(Node<T> node)
        {
            int balanceFactor = BalanceFactor(node);
            if (balanceFactor > 1)
            {
                if (BalanceFactor(node.Left) > 0)
                {
                    node = RotateLeft(node);
                }
                else
                {
                    node = RotateLeftRight(node);
                }
            }
            else if (balanceFactor < -1)
            {
                if (BalanceFactor(node.Right) > 0)
                {
                    node = RotateRightLeft(node);
                }
                else
                {
                    node = RotateRight(node);
                }
            }
            return node;
        }
        public void Remove(T value)
        {
#if DEBUG
            Console.ForegroundColor = ConsoleColor.Green;
            Console.WriteLine("Removing {0}", value);
```

```
            Console.ResetColor();
#endif
            Root = Remove(Root, value);
#if DEBUG
            Console.WriteLine("Removed");
            Console.Write("{0,-30}", "Current BreadthFirst Order:");
            Display(DisplayMethod.BreadthFirst);
#endif
        }

        private Node<T> Remove(Node<T> node, T value)
        {
            Node<T> parent;
            if (node is null) return null;
            else
            {
                if (value.CompareTo(node.Key) < 0)
                {
                    node.Left = Remove(node.Left, value);
                    if (BalanceFactor(node) == -2)
                    {
                        if (BalanceFactor(node.Right) <= 0)
                        {
                            node = RotateRight(node);
                        }
                        else
                        {
                            node = RotateRightLeft(node);
                        }
                    }
                }
                else if (value.CompareTo(node.Key) > 0)
                {
                    node.Right = Remove(node.Right, value);
                    if (BalanceFactor(node) == 2)
                    {
                        if (BalanceFactor(node.Left) >= 0)
                        {
                            node = RotateLeft(node);
                        }
                        else
                        {
                            node = RotateLeftRight(node);
                        }
                    }
                }
```

```
            else
            {
                if (node.Right != null)
                {
                    parent = node.Right;
                    while (parent.Left != null)
                    {
                        parent = parent.Left;
                    }
                    node.Key = parent.Key;
                    node.Right = Remove(node.Right, parent.Key);
                    if (BalanceFactor(node) == 2)
                    {
                        if (BalanceFactor(node.Left) >= 0)
                        {
                            node = RotateLeft(node);
                        }
                        else
                        {
                            node = RotateLeftRight(node);
                        }
                    }
                }
                else
                {
                    return node.Left;
                }
            }
        }
        return node;
    }

    public Node<T> Find(T value)
    {
        return Find(Root, value);
    }

    private Node<T> Find(Node<T> node, T value)
    {

        if (value.CompareTo(node.Key) < 0)
        {
            if (value.Equals(node.Key))
            {
                return node;
            }
```

```
                else
                    return Find(node.Left, value);
            }
            else
            {
                if (value.Equals(node.Key))
                {
                    return node;
                }
                else
                    return Find(node.Right, value);
            }
        }

        private int Max(int l, int r)
        {
            return l > r ? l : r;
        }

        private int GetHeight(Node<T> node)
        {
            int height = 0;
            if (node != null)
            {
                int l = GetHeight(node.Left);
                int r = GetHeight(node.Right);
                int m = Max(l, r);
                height = m + 1;
            }
            return height;
        }
        private int BalanceFactor(Node<T> node)
        {
            int l = GetHeight(node.Left);
            int r = GetHeight(node.Right);
            return l - r;
        }

        private Node<T> RotateRight(Node<T> parent)
        {
#if DEBUG
            _rotations.Add(new KeyValuePair<string, T>("Left", parent.Key));
#endif
            Node<T> pivot = parent.Right;
            parent.Right = pivot.Left;
            pivot.Left = parent;
```

```
            return pivot;
        }

        private Node<T> RotateLeft(Node<T> parent)
        {
#if DEBUG
            _rotations.Add(new KeyValuePair<string, T>("Right", parent.Key));
#endif
            Node<T> pivot = parent.Left;
            parent.Left = pivot.Right;
            pivot.Right = parent;
            return pivot;
        }

        private Node<T> RotateLeftRight(Node<T> parent)
        {
            Node<T> pivot = parent.Left;
            parent.Left = RotateRight(pivot);
            return RotateLeft(parent);
        }

        private Node<T> RotateRightLeft(Node<T> parent)
        {
            Node<T> pivot = parent.Right;
            parent.Right = RotateLeft(pivot);
            return RotateRight(parent);
        }

        public List<T> InOrder()
        {
            List<T> result = new List<T>();
            InOrder(Root);

            return result;

            void InOrder(Node<T> node)
            {
                if (node.Left != null)
                {
                    InOrder(node.Left);
                }
                result.Add(node.Key);
                if (node.Right != null)
                {
                    InOrder(node.Right);
                }
```

```
        }
    }

    public List<T> PostOrder()
    {
        List<T> result = new List<T>();
        PostOrder(Root);

        return result;

        void PostOrder(Node<T> node)
        {
            if (node.Left != null)
            {
                PostOrder(node.Left);
            }
            if (node.Right != null)
            {
                PostOrder(node.Right);
            }
            result.Add(node.Key);
        }
    }

    public List<T> PreOrder()
    {
        List<T> result = new List<T>();
        PreOrder(Root);

        return result;

        void PreOrder(Node<T> node)
        {
            result.Add(node.Key);
            if (node.Left != null)
            {
                PreOrder(node.Left);
            }
            if (node.Right != null)
            {
                PreOrder(node.Right);
            }
        }
    }

    public List<T> BreadthFirst()
```

```
{
    if (Root is null) return null;
    List<T> result = new List<T>();
    Queue<Node<T>> nodes = new Queue<Node<T>>();
    BreadthFirst(Root);
    return result;

    void BreadthFirst(Node<T> node)
    {
        result.Add(node.Key);
        if (node.Left != null)
        {
            nodes.Enqueue(node.Left);
        }
        if (node.Right != null)
        {
            nodes.Enqueue(node.Right);
        }
        if (nodes.Count > 0)
        {
            BreadthFirst(nodes.Dequeue());
        }
    }
}

public void Display(DisplayMethod method)
{
    List<T> result;
    switch (method)
    {
        case DisplayMethod.InOrder:
            result = InOrder();
            break;
        case DisplayMethod.PreOrder:
            result = PreOrder();
            break;
        case DisplayMethod.PostOrder:
            result = PostOrder();
            break;
        case DisplayMethod.BreadthFirst:
        default:
            result = BreadthFirst();
            break;

    }
    if (result is null)
```

```
            {
                Console.WriteLine("[ ]");
                return;
            }
            Console.Write("[ ");
            foreach (var value in result)
            {
                Node<T> node = Find(value);
                int balanceFactor = BalanceFactor(node);
                Console.Write(value + "(" + balanceFactor + ") ");
            }
            Console.WriteLine("]\n");
        }
    }
}
```

Attachment 2

Tester.cs

https://github.com/pscompsci/SIT221_Library/blob/master/Task_7_1/Tester.cs

```
Using System;
using System.Collections.Generic;
using System.Globalization;

namespace Task_7_1
{
    class Tester
    {

        static void Display(List<int> list)
        {
            Console.Write("[");
            foreach(int i in list)
            {
                Console.Write(i + " ");
            }
            Console.WriteLine("]");
        }

        static void Main(string[] args)
        {
            AVLTree<int> tree = new AVLTree<int>();

            tree.Insert(20);
            tree.Insert(9);
            tree.Insert(3);
            tree.Insert(7);
            tree.Insert(5);
            tree.Insert(8);
            tree.Insert(25);
            tree.Insert(30);
            tree.Insert(15);
            tree.Insert(6);
            tree.Insert(17);

            List<int> breadth = tree.BreadthFirst();
            Console.Write("{0,-20}", "Breadth First:");
            Display(breadth);
```

```
            List<int> preorder = tree.PreOrder();
            Console.Write("{0,-20}", "Pre-Order First:");
            Display(preorder);

            List<int> postorder = tree.PostOrder();
            Console.Write("{0,-20}", "Post-Order First:");
            Display(postorder);

            List<int> inorder = tree.InOrder();
            Console.Write("{0,-20}", "In-Order First:");
            Display(inorder);

            Console.WriteLine("\n\n");

            tree.Remove(20);
            tree.Remove(15);
            tree.Remove(8);
            tree.Remove(25);
            tree.Remove(30);
            tree.Remove(9);
            tree.Remove(17);
            tree.Remove(5);
            tree.Remove(6);
            tree.Remove(3);
            tree.Remove(7);

            Console.ReadKey();
        }
    }
}
```