

# DEAKIN UNIVERSITY

## DATA STRUCTURES AND ALGORITHMS

### ONTRACK SUBMISSION

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## Algorithm complexity (pass)

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Outcome	Weight
Complexity	◆◆◆◆◆
Implement Solutions	◆◆◆◆◆
Document solutions	◆◆◆◆◆

Learning outcome 1 involves evaluating the computational complexity of different algorithms and being able to understand performance in terms of asymptotic notation. This task aligns very well with the learning outcome, only missing consideration of the memory use of the algorithms. It does not relate to creating data structures (ULO2) or documenting designs and constraints (ULO3)

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# Task 2.1

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## Question 1

1 i.

a)

Case	Assumptions	Operations
<b>Best</b>	<ul style="list-style-type: none"><li>rand() always returns a result very close to 1.0</li></ul>	11
<b>Average</b>	<ul style="list-style-type: none"><li>rand() return averages to 0.5</li></ul>	13
<b>Worst</b>	<ul style="list-style-type: none"><li>rand() always returns a result very close to 0.0</li></ul>	14

b)

Case	Big- $\Theta$ Notation
<b>Best</b>	$\Theta(1)$
<b>Average</b>	$\Theta(1)$
<b>Worst</b>	$\Theta(1)$

c) Overall Performance in Big-O:  $O(1)$

d) Overall Performance in Big- $\Omega$ :  $\Omega(1)$

e) Overall Performance in Big- $\Theta$ :  $\Theta(1)$

f)  $f(n) \sim g(1), O(1), \Omega(1), \Theta(1), o(\log n)$

1 ii.

a)

Case	Assumptions	Operations
<b>Best</b>	<ul style="list-style-type: none"><li><math>N &gt; 0</math></li><li>rand() always returns a value <math>\geq 0.5</math></li></ul>	$4N + 3$
<b>Average</b>	<ul style="list-style-type: none"><li><math>N &gt; 0</math></li><li>rand() results average to 0.5</li></ul>	$4.5N + 3$
<b>Worst</b>	<ul style="list-style-type: none"><li><math>N &gt; 0</math></li><li>rand() always returns a value <math>&lt; 0.5</math></li></ul>	$5N + 3$

b)

Case	Big- $\Theta$ Notation
<b>Best</b>	$\Theta(n)$
<b>Average</b>	$\Theta(n)$
<b>Worst</b>	$\Theta(n)$

- c) Overall Performance in Big-O:  $O(n)$
- d) Overall Performance in Big- $\Omega$ :  $\Omega(n)$
- e) Overall Performance in Big- $\Theta$ :  $\Theta(n)$
- f)  $f(n) \sim \Theta(n), O(n), \Omega(n), o(n \log n), \omega(\log n)$

1 iii.

a)

Case	Assumptions	Operations
<b>Best</b>	<ul style="list-style-type: none"> <li><math>N &gt; 0</math></li> <li>Unlucky is always false</li> </ul>	$3N + 3$
<b>Average</b>	<ul style="list-style-type: none"> <li><math>N &gt; 0</math></li> <li>unlucky is true half of the time</li> </ul>	$0.75N^2 + 4.5N + 3$
<b>Worst</b>	<ul style="list-style-type: none"> <li><math>N &gt; 0</math></li> <li>unlucky is always true</li> </ul>	$1.5N^2 + 6.5N + 3$

b)

Case	Big- $\Theta$ Notation
<b>Best</b>	$\Theta(n)$
<b>Average</b>	$\Theta(n^2)$
<b>Worst</b>	$\Theta(n^2)$

- c) Overall Performance in Big-O:  $O(n^2)$
- d) Overall Performance in Big- $\Omega$ :  $\Omega(n)$
- e) Overall Performance in Big- $\Theta$ : Cannot be described
- f)  $f(n) = O(n^2), \Omega(n), o(n^3), \omega(\log n)$

1 iv.

a)

Case	Assumptions	Operations
<b>Best</b>	<ul style="list-style-type: none"> <li>unlucky is false</li> </ul>	3
<b>Average</b>	<ul style="list-style-type: none"> <li><math>N &gt; 0</math></li> <li>unlucky is true half the time</li> </ul>	$1.5 \log_2(N) + 5$
<b>Worst</b>	<ul style="list-style-type: none"> <li><math>N &gt; 0</math></li> <li>unlucky is always true</li> </ul>	$3 \log_2(N) + 7$

b)

Case	Big- $\Theta$ Notation
<b>Best</b>	$\Theta(1)$
<b>Average</b>	$\Theta(\log n)$
<b>Worst</b>	$\Theta(\log n)$

- c) Overall Performance in Big-O:  $O(\log n)$
- d) Overall Performance in Big- $\Omega$ :  $\Omega(1)$
- e) Overall Performance in Big- $\Theta$ : Cannot be described
- f)  $f(n) = O(\log n), \Omega(1)$

1 v.

a)

Case	Assumptions	Operations
<b>Best</b>	<ul style="list-style-type: none"> <li><math>N &gt; 0</math></li> <li><math>\text{rand}()</math> always return <math>&lt; 0.5</math></li> </ul>	$4N + 6$
<b>Average</b>	<ul style="list-style-type: none"> <li><math>N &gt; 0</math></li> <li>Half the time, <math>\text{rand}()</math> returns a value <math>&lt; 0.5</math></li> </ul>	$6N + 6$
<b>Worst</b>	<ul style="list-style-type: none"> <li><math>N &gt; 0</math></li> <li><math>\text{rand}()</math> always returns a value <math>&lt; 0.5</math></li> </ul>	$8N + 6$

b)

Case	Big- $\Theta$ Notation
<b>Best</b>	$\Theta(n)$
<b>Average</b>	$\Theta(n)$
<b>Worst</b>	$\Theta(n)$

- c) Overall Performance in Big-O:  $O(n)$
- d) Overall Performance in Big- $\Omega$ :  $\Omega(n)$
- e) Overall Performance in Big- $\Theta$ :  $\Theta(n)$
- f)  $f(n) \sim \Theta(n), O(n), \Omega(n), o(n \log n), \omega(\log n)$

1 vi.

a)

Case	Assumptions	Operations
<b>Best</b>	<ul style="list-style-type: none"> <li><math>N &gt; 1</math></li> <li><math>a[j] &lt; a[j + 1]</math> always (i.e. already sorted ascending)</li> </ul>	$1.5N^2 + 2.5N - 2$
<b>Average</b>	<ul style="list-style-type: none"> <li><math>N &gt; 1</math></li> <li><math>a[j] &gt; a[j + 1]</math> half of the time</li> </ul>	$1.75N^2 + 2.25N - 2$
<b>Worst</b>	<ul style="list-style-type: none"> <li><math>N &gt; 1</math></li> <li><math>a[j] &gt; a[j + 1]</math> always (i.e. maximum unsorted)</li> </ul>	$2N^2 + 2N - 2$

b)

Case	Big- $\Theta$ Notation
<b>Best</b>	$\Theta(n^2)$
<b>Average</b>	$\Theta(n^2)$
<b>Worst</b>	$\Theta(n^2)$

- c) Overall Performance in Big-O:  $O(n^2)$
- d) Overall Performance in Big- $\Omega$ :  $\Omega(n^2)$
- e) Overall Performance in Big- $\Theta$ :  $\Theta(n^2)$
- f)  $f(n) = \Theta(n^2), O(n^2), \Omega(n^2), o(n^3), \omega(n \log n)$

## Question 2

Big-O notation bounds a function on the upper end, providing an expression for which the expression grows no faster than even under the worst conditions.

Since we are often interested in the impact on performance for large data sets and hence large set of inputs, understanding how the function will perform at its worst, generally provides more useful information when designing and architecting software, than other notations.

## Question 3

No.

While an algorithm that is bounded by  $\Theta(n^3)$  will always eventually be slower than an algorithm that is bounded by  $\Theta(\log n)$ , there may be some constant  $c_1, c_2 > 0$  that affects the performance of one or both algorithms up to some value of  $n$ , for which the  $\Theta(n^3)$  algorithm performs faster.

In figure 1, a constant  $c_1$  is affecting the  $\Theta(\log n)$  algorithm (green line) so that at low values of  $n$ , it's performance is slower.

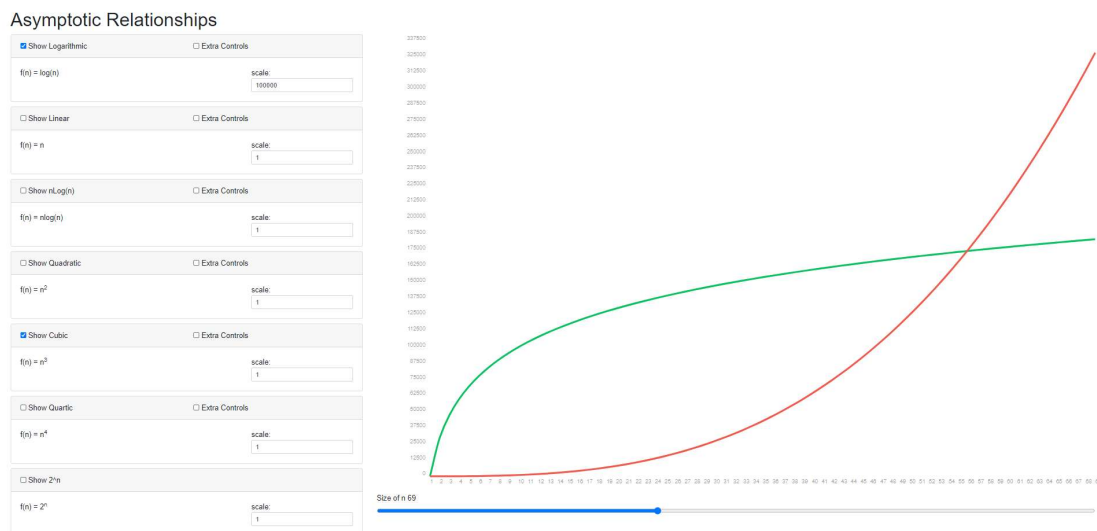


Figure 1:  $n^3$  v  $\log n$  performance at low  $n$ , with constant  $c_1$  affecting the  $(\log n)$  performance

Source: <https://asymptoticnot.z13.web.core.windows.net/> (created by Peter Stacey as part of studying for SIT221)

## Question 4

	Statement	Right/Wrong
1	$2n^2 + 6^{13}n = O(n^2)$	Right
2	$n \log n = O(n)$	Wrong
3	$n^3 + n^2 + 10^{16}n = \Theta(n^4)$	Wrong
4	$n \log n = \Omega(n)$	Right

