
Problem solving: Search and Stack

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Outcome	Weight
Complexity	◆◆◆◆
Implement Solutions	◆◆◆◆
Document solutions	◆◆◆◆

This task aligns with all three learning outcomes as it involves evaluating the complexity of different design possibilities for algorithms, the space complexity of different data structures (ULO1). It involves not only evaluating from design, but also implementing the designs to confirm the assumptions and assessment (not a mandatory component, but the best way to check) which aligns with ULO2; and it involves documenting the solution and constraints and trade-offs (ULO3).

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Task 6.1

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Question 1

Design a $\theta(n \log n)$ time algorithm that, given a set S of n integer numbers and another integer x , determines whether or not there exist two elements in S whose sum is exactly x .

Approach

Given a set S of n integers:

1. Sort S from smallest to largest (largest to smallest is also ok)
2. Create two pointer variables (eg. i and j) pointing at the start of sorted S (variable i) and the end of sorted S (variable j)
3. While the value at $S[j] + S[i] > \text{sum}$, $j--$
4. While the value at $S[j] + S[i] < \text{sum}$, $i++$
5. If $S[j] + S[i]$ equals sum , return true
6. Return false if no matching sum is found

Assumptions

- Sort is conducted no worse than $O(n \log n)$

C# Code

```
1  using System;
2
3  namespace Task_6_1
4  {
5      2 references
6      public class SumExists
7      {
8          2 references
9          public static bool Check(int[] numbers, int sum)
10         {
11             if (numbers is null) throw new ArgumentNullException(); // 1 operation
12             Array.Sort(numbers); // 0(n log n)
13             int i = 0; // 1 operation
14             int j = numbers.Length - 1; // 1 operation
15             while (i < j) // n in worst case
16             {
17                 if (numbers[i] + numbers[j] == sum) return true; // n - 1 in worst case
18                 else if (numbers[i] + numbers[j] < sum) i++; // 2(n - 1) in worst case
19                 else j--; // n - 1 in worst case
20             }
21             return false; // 1 operation
22         }
23     }
```

Tests

```
int[] numbers = {10, 5, 6, 8, 2, 19, 9, 11, 15, 1, -8, 50};

[Theory]
[InlineData(-2)] // 6 + -8
[InlineData(0)] // 8 + -8
[InlineData(1)] // 9 + -8
[InlineData(15)] // 10 + 5
[InlineData(26)] // 15 + 11
[InlineData(42)] // 50 + -8
[InlineData(69)] // 50 + 18
0 references
public void SumDoesExist(int sum)
{
    Assert.True(SumExists.Check(numbers, sum));
}

[Theory]
[InlineData(-100)]
[InlineData(50)]
[InlineData(100)]
0 references
public void SumDoesNotExist(int sum)
{
    Assert.False(SumExists.Check(numbers, sum));
}
```

Test Results

The Test Explorer window shows the following test results:

Test	Duration	Traits
Task_6_1Tests (14)	6 ms	
Task_6_1Tests (14)	6 ms	
UnitTest1 (14)	6 ms	
IntStackPopMin(numbers: [10, 9, 8, 7, 6, ...])	2 ms	
IntStackPushMin(numbers: [10, 9, 8, 7, 6, ...])	< 1 ms	
StackPopMin(numbers: [10, 9, 8, 7, 6, ...])	1 ms	
StackPushMin(numbers: [10, 9, 8, 7, 6, ...])	< 1 ms	
SumDoesExist (7)	3 ms	
SumDoesNotExist (3)	< 1 ms	

Group Summary

Task_6_1Tests

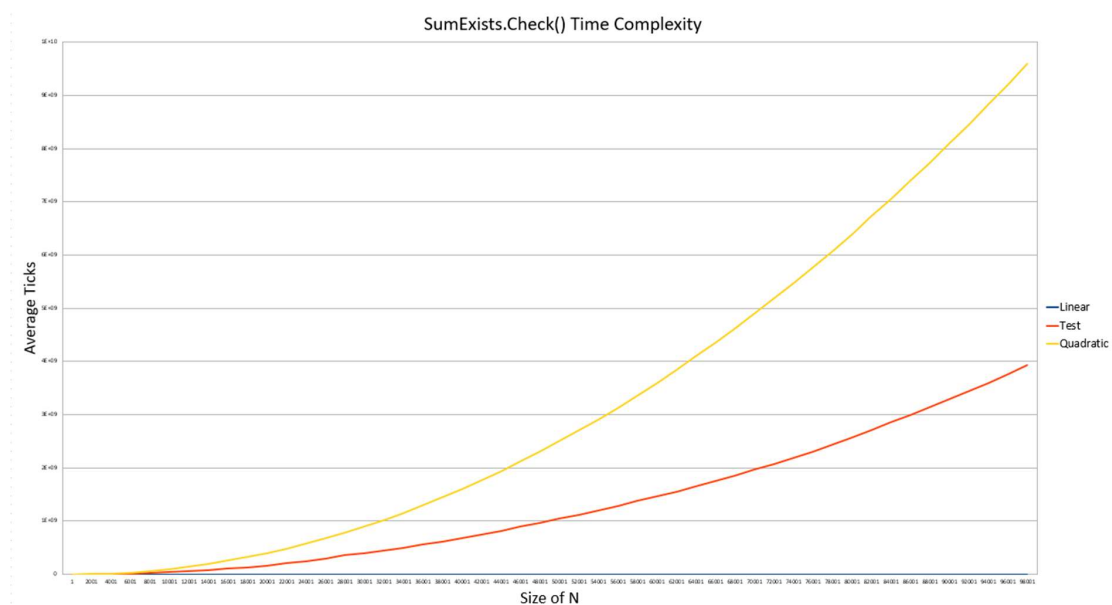
Tests in group: 14

Total Duration: 6 ms

Outcomes

14 Passed

Performance



Question 2

A Stack data structure provides Push and Pop, the two operations to write and read data, respectively. Using the Stack as a starting point design a data structure that, in addition to these two operations, also provides the Min operation to return the smallest element of the stack. Remember that the new data structure must operate in a constant $\mathcal{O}(1)$ time for all three operations.

Approach

There are a couple of different approaches immediately obvious, to ensure $\mathcal{O}(1)$ Push, Pop and Min methods.

These involve:

1. A stack for a pre-defined data type
2. A stack for a Generic data type

In developing a Stack for a pre-defined data type (eg. int), only one additional variable is required, to hold the current minimum value in the stack.

In developing a Stack for a generic data type, the generic type needs to implement `Comparable<T>` in order for values in the Stack to be compared, and a separate collection created to hold the set of minimum values.

Assumptions

- For a pre-defined data type stack, + and – operations can be applied to the data type
- For a generic data type stack, the generic type implements `Comparable<T>`
- For a generic data type stack, the use of $\mathcal{O}(n)$ additional space is acceptable

Option 1: Stack of integers (could also be applied to other numeric types)

```
1 using System;
2 using System.Collections.Generic;
3
4 namespace Task_6_1
5 {
6     public class IntStack
7     {
8         const int DEFAULT_MAX_SIZE = 100;
9         private int[] _stack;
10        private int _top;
11        private int _min;
12
13        public IntStack()
14        {
15            _stack = new int[DEFAULT_MAX_SIZE];
16            _top = -1;
17        }
18
19        public IntStack(int size)
20        {
21            if (size < 0) throw new ArgumentOutOfRangeException();
22            _stack = new int[size];
23            _top = -1;
24        }
25
26        private bool IsEmpty() => _top is -1 ? true : false;
27        private bool IsFull() => _top == _stack.Length - 1 ? true : false;
28
29        public void Push(int value)
30        {
31            if (IsFull()) throw new StackOverflowException();
32            if (IsEmpty())
33            {
34                _stack[++_top] = value;
35                _min = value;
36                return;
37            }
38
39            if (value < _min)
40            {
41                _stack[++_top] = (2 * value - _min);
42                _min = value;
43            }
44            else
45            {
46                _stack[++_top] = value;
47            }
48        }
49
50        public int Pop()
51        {
52            if (IsEmpty()) throw new InvalidOperationException();
53            int result = _stack[_top];
54
55            if (result < _min)
56            {
57                int temp = (2 * _min) - result;
58                result = _min;
59                _min = temp;
60            }
61
62            _top--;
63            return result;
64        }
65
66        public int Min()
67        {
68            if (IsEmpty()) throw new InvalidOperationException();
69            return _min;
70        }
71    }
72 }
```

Tests

```
[Theory]
[InlineData(10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0)]
0 references
public void IntStackPushMin(params int[] numbers)
{
    IntStack stack = new IntStack();
    for(int i = 0; i < numbers.Length; i++)
    {
        stack.Push(numbers[i]);
        Assert.Equal((numbers.Length - 1) - i, stack.Min());
    }
}

[Theory]
[InlineData(10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0)]
0 references
public void IntStackPopMin(params int[] numbers)
{
    IntStack stack = new IntStack();
    for(int i = 0; i < numbers.Length; i++)
    {
        stack.Push(numbers[i]);
    }

    for(int i = 0; i < numbers.Length - 1; i++)
    {
        stack.Pop();
        Assert.Equal(i + 1, stack.Min());
    }
}
```

Performance



Option 2: Generic Stack

```
1  using System;
2  using System.Collections.Generic;
3
4  namespace Task_6_1
5  {
6      6 references
7      public class Stack<T> where T : IComparable<T>
8      {
9          private const int MAX_STACK_SIZE = 100;
10         private T[] _stack;
11         private int _count;
12         private T[] _min;
13         private int _minCount;
14
15         2 references | 2/2 passing
16         public Stack()
17         {
18             _stack = new T[MAX_STACK_SIZE];
19             _count = 0;
20             _min = new T[MAX_STACK_SIZE];
21             _minCount = 0;
22         }
23
24         0 references
25         public Stack(int size)
26         {
27             if (size < 0) throw new ArgumentOutOfRangeException();
28             _stack = new T[size];
29             _count = 0;
30             _min = new T[size];
31             _minCount = 0;
32         }
33
34         3 references
35         private bool IsEmpty() => _count is 0 ? true : false;
36         1 reference
37         private bool IsFull() => _count >= _stack.Length - 1 ? true : false;
38
39         1 reference | 1/1 passing
40         public T Pop()
41         {
42             if (IsEmpty()) throw new InvalidOperationException();
43             T value = _stack[_count];
44             _count--;
45             if (_min[_minCount - 1].Equals(value))
46             {
47                 _min[_minCount - 1] = default(T);
48                 _minCount--;
49             }
50             return value;
51         }
52
53         2 references | 2/2 passing
54         public void Push(T value)
55         {
56             if (IsFull()) throw new StackOverflowException();
57             if (IsEmpty()) _min[_minCount++] = value;
58
59             if (value.CompareTo(_min[_minCount - 1]) < 0)
60             {
61                 _min[_minCount++] = value;
62             }
63             _stack[++_count] = value;
64         }
65
66         2 references | 2/2 passing
67         public T Min()
68         {
69             if (IsEmpty()) throw new InvalidOperationException();
70             return _min[_minCount - 1];
71         }
72     }
73 }
```


Tests

```
[Theory]
[InlineData(10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0)]
0 references
public void StackPushMin(params int[] numbers)
{
    Stack<int> stack = new Stack<int>();
    for(int i = 0; i < numbers.Length; i++)
    {
        stack.Push(numbers[i]);
        Assert.Equal((numbers.Length - 1) - i, stack.Min());
    }
}

[Theory]
[InlineData(10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0)]
0 references
public void StackPopMin(params int[] numbers)
{
    Stack<int> stack = new Stack<int>();
    for(int i = 0; i < numbers.Length; i++)
    {
        stack.Push(numbers[i]);
    }

    for(int i = 0; i < numbers.Length - 1; i++)
    {
        stack.Pop();
        Assert.Equal(i + 1, stack.Min());
    }
}
```

Performance

