IIT Bombay

Computer Programming

Dr. Deepak B Phatak
Dr. Supratik Chakraborty
Department of Computer Science and Engineering
IIT Bombay

Session: Solving Simultaneous Equations

Quick Recap

IIT Bombay

- An Array handles a set of values of same type
  - is declared with a type and size
  - elements of an array are accessed by using index expressions
  - value of index must be between 0 and size-1

Multidimensional Array (Matrix)

IIT Bombay

- We can declare and use arrays with more than one dimension

```
int A[50][40];
```

  - Declares a two dimensional array with 50 rows and 40 columns
- Each element is accessed by a reference requiring two index expressions, e.g.,

```
A[i][j] = 3782;
```

  - Row index ‘i‘, can have a value from 0 to 49,
  - Column index ‘j’ can have a value from 0 to 39
- All rules for index expression, apply to index for each dimension

Simultaneous Equations

IIT Bombay

- Matrices are used to represent a system of simultaneous equations in multiple variables
- Consider the following equations in two variables

$$2x + 4y = 8 \qquad \text{eq.1}$$

$$4x + 3y = 1 \qquad \text{eq.2}$$

These equations can be represented as

$$\Rightarrow \begin{pmatrix} 2 & 4 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 1 \end{pmatrix}$$

Gaussian Elimination Method

This method uses two properties of such systems to solve these equations :

1. ***The system of equations is not affected if an equation is multiplied by a constant***

$$2x + 4y = 8 \qquad \text{eq. 1}$$

Suppose we multiply eq.1 by 0.5, to get

$$1x + 2y = 4 \qquad \text{eq. 1'}$$

we now have the same system of equations, but in the following form:

$$1x + 2y = 4 \qquad \text{eq.1'}$$

$$4x + 3y = 1 \qquad \text{eq.2}$$

Simultaneous Equations …

IIT Bombay

- Representing these equations in the form of matrices:

$$\begin{matrix} 1\,x + 2\,y = & 4 \\ 4\,x + 3\,y = & 1 \end{matrix} \quad \Rightarrow \quad \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \quad \begin{matrix} \text{eq.1'} \\ \text{eq.2} \end{matrix}$$

Simultaneous Equations …

IIT Bombay

2. ***If an equation is replaced by a linear combination of itself and any other row, the system of equations remains same***

Multiply eq.1’ by 4 to get $4x + 8y = 16$

Subtract it from eq. 2, to get new eq.2’

$$
\begin{array}{rl}
4x + 3y = \quad 1 & \text{eq.2} \\
-\ (4x + 8y = \ 16) & \\
\mathbf{0x - 5y = -15} & \text{eq.2’}
\end{array}
$$

The system is now equivalent to

$$1x + 2y = 4$$
$$0x - 5y = -15$$

$$\Rightarrow \quad \begin{pmatrix} 1 & 2 \\ 0 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -15 \end{pmatrix} \quad \begin{array}{l} \text{eq.1’} \\ \text{eq.2’} \end{array}$$

Simultaneous Equations …

IIT Bombay

If we multiply eq.2’ by -0.2, (or equivalently, divide by -5), we will get

$$1\,x + 2\,y = 4$$
$$0\,x + 1\,y = 3$$

$$\Rightarrow \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \quad \begin{matrix} \text{eq.1'} \\ \text{eq.2''} \end{matrix}$$

Starting from the last equation, and using back-substitution, we get the solution for all variables.

$0 * x + 1 * y = 3$ eq. 2’’ directly gives us $y = 3$

$1 * x + 2 * 3 = 4$ we back-substitute this value of y in eq.1’

This gives us $x + 6 = 4$, which in turn gives us $x = -2$

Simultaneous Equations …

- **The essence of the method is to reduce the coefficient-matrix to an upper triangular matrix (all elements on the diagonal are 1), and then use back-substitution**
- The process is susceptible to round off errors
- There are other variations, such as:
  - Gauss Jordan elimination, Pivoting
  - L U decomposition
- A useful reference:

  "Numerical recipes in C++",   [also in C, Fortran]

  - by William H Press, Saul A Teukolsky,

  William T Vetterling, and Brian P Flannery

Simultaneous Equations …

IIT Bombay

- In general, a system of linear equations in n variables can be represented by the following matrices

$$
\begin{pmatrix}
a_{00} & a_{01} & a_{02} & \cdots & a_{0n-1} \\
a_{10} & a_{11} & a_{12} & \cdots & a_{1n-1} \\
a_{20} & a_{21} & a_{22} & \cdots & a_{2n-1} \\
. & & & & \\
. & & & & \\
a_{n0} & a_{n1} & a_{n2} & \cdots & a_{n-1n-1}
\end{pmatrix}
\begin{pmatrix}
x_0 \\ x_1 \\ x_2 \\ . \\ . \\ x_{n-1}
\end{pmatrix}
=
\begin{pmatrix}
b_0 \\ b_1 \\ b_2 \\ . \\ . \\ b_{n-1}
\end{pmatrix}
$$

Simultaneous Equations …

IIT Bombay

- The Gaussian elimination technique essentially reduces the coefficient matrix to an upper triangular form:

$$\begin{pmatrix} 1 & a_{01} & a_{02} & \dots & a_{0n-1} \\ 0 & 1 & a_{12} & \dots & a_{1n-1} \\ 0 & 0 & 1 & & . \\ & & & & \\ & & & & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ . \\ \\ x_{n-1} \end{pmatrix} = \begin{pmatrix} b_0 \\ b_1 \\ . \\ \\ b_{n-1} \end{pmatrix}$$

Summary

IIT Bombay

- In this session,
  - We examined the properties of a system of simultaneous equations in many variables
  - Understood how matrices can be used to represent such systems
  - Studied Gauss elimination technique to solve the system
- In the next session, we will write a program to implement this algorithm