Computer Programming

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Session: More on Two's Complement Representation

Quick Recap of Relevant Topics

- Representation of integers in a computer

    Unsigned integers

    Signed integers

Overview of This Lecture

- A closer look at two's complement representation
- Magnitude of negative integers in two's complement representation

Representing Signed Integers

- Treat MSB as sign bit: negative if MSB is 1, positive if MSB is 0
  - **Sign-magnitude representation**
  - Consider integers represented using 3 bits

+0
000

+1
001

-3
111

+2
010

-2
110

011
+3

101

100
-0

-1

**Wasteful:
Two representations
of zero**
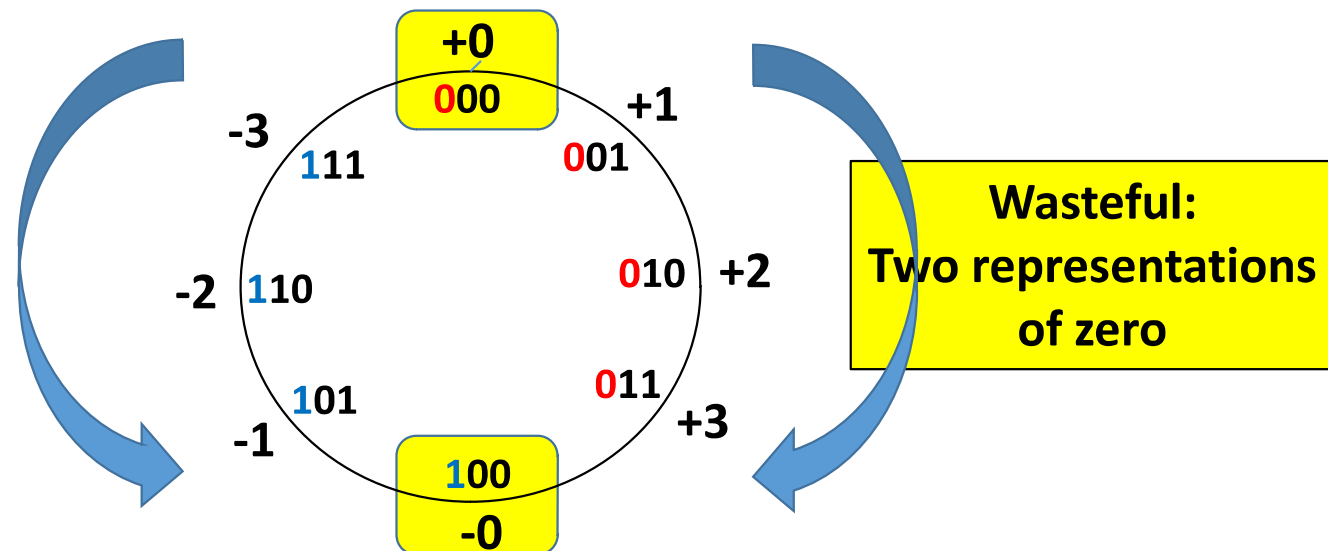
How Else Could We Represent?

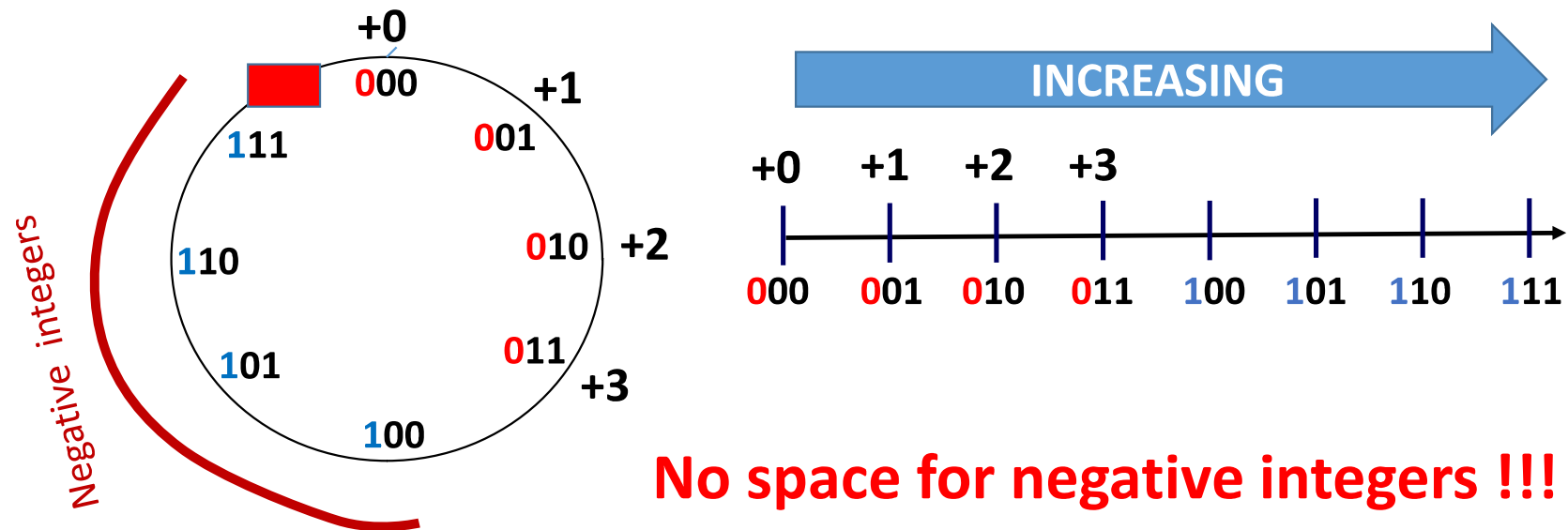- Using MSB to represent sign is convenient
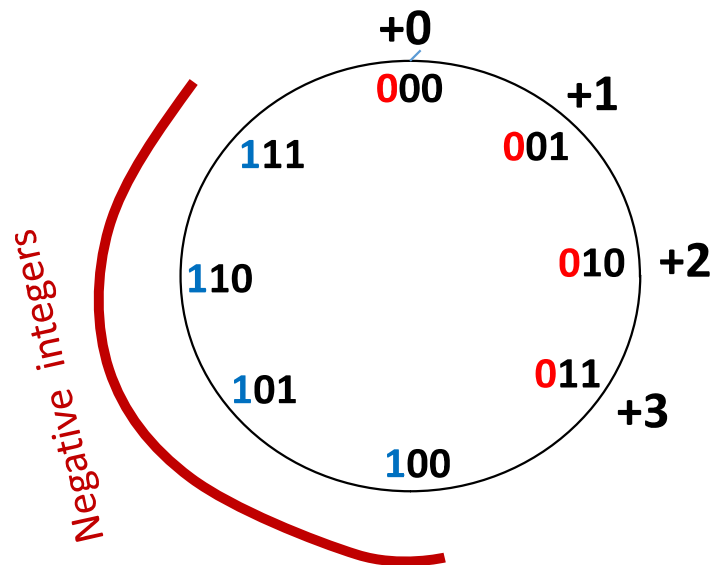
What negative number should 100 represent?

How about 101, 111, …?
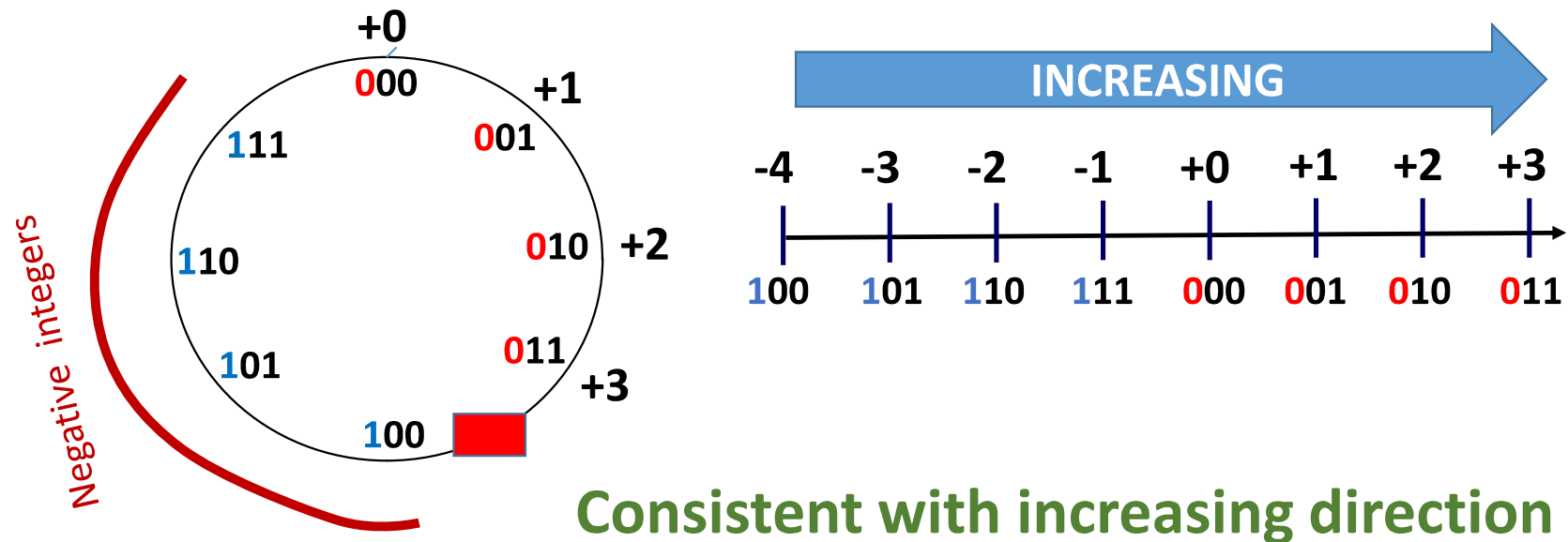
Can we think of the circle as a wrapped-around number line?

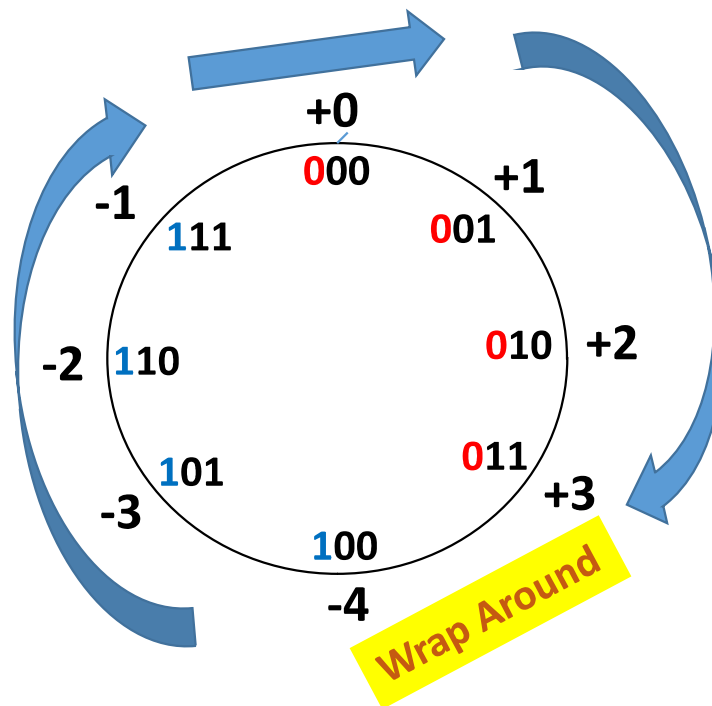# How Else Could We Represent?

- Where do we break the circle?

INCREASING

+0   +1   +2   +3

000   001   010   011   100   101   110   111

No space for negative integers !!!

How Else Could We Represent?

- Where do we break the circle?

Negative integers

+0
000

+1
001

+2
010

+3
011

100

101
101

110
110

111
111

INCREASING

-4	-3	-2	-1	+0	+1	+2	+3
100	101	110	111	000	001	010	011

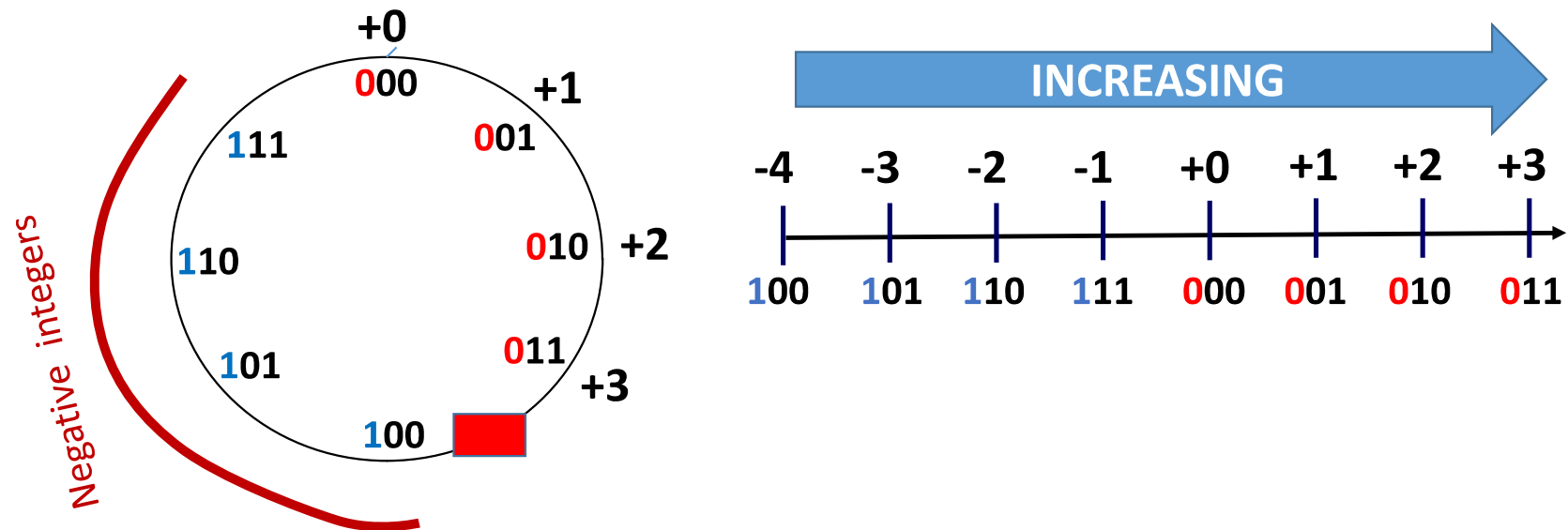**Consistent with increasing direction**

Two's Complement Representation

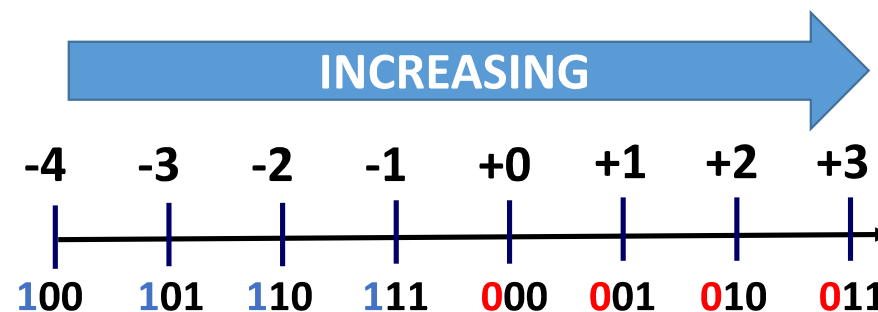**8 numbers represented:**
-4 through +3

Only one representation of 0

Magnitudes of Negative Integers

Need a way to map binary representation to magnitudes of negative integers

Magnitudes of Negative Integers

INCREASING

-4	-3	-2	-1	+0	+1	+2	+3
100	101	110	111	000	001	010	011

Desired map: 11 ⟶ 1, 10 ⟶ 2, 01 ⟶ 3, 00 ⟶ 4

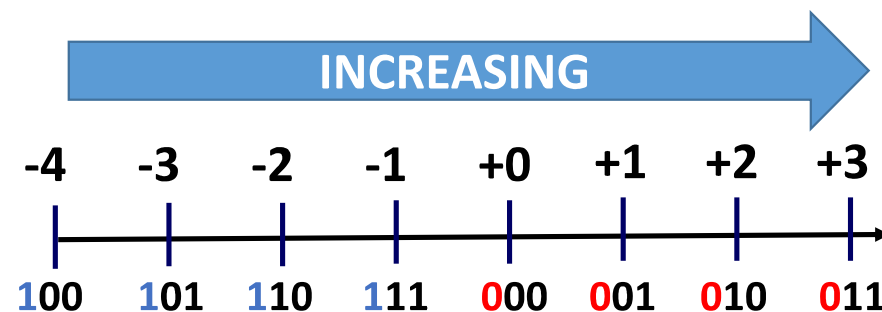Observation: 11 (represents unsigned 3) ⟶ $2^2 - 3 = 1$

10 (represents unsigned 2) ⟶ $2^2 - 2 = 2$

01 (represents unsigned 1) ⟶ $2^2 - 1 = 3$

00 (represents unsigned 0) ⟶ $2^2 - 0 = 4$

Magnitudes of Negative Integers

INCREASING →

-4	-3	-2	-1	+0	+1	+2	+3
100	**1**01	**1**10	**1**11	**0**00	**0**01	**0**10	**0**11

Desired map: 11 → 1, 10 → 2, 01 → 3, 00 → 4

Observation: 11 (represents unsigned 3) → 00 (= 0) + 1 = 1

10 (represents unsigned 2) → 01 (= 1) + 1 = 2

01 (represents unsigned 1) → 10 (= 2) + 1 = 3

00 (represents unsigned 0) → 11 (= 3) + 1 = 4

Magnitude of Negative Integers

- Is there an easy way to figure out the magnitude of what 10111 represents in 2's complement?
  - 10111 has MSB 1: Negative integer
  - To get absolute value of 10111
    - Ignore MSB: 10111
    - Flip every bit in 0111: 1000 (decimal 8)
    - Add 1: decimal 9
    - Absolute value is 9
  - Answer: -9

Two's Complement Representation

- Is there an easy way to figure out the magnitude of what 10111 represents in 2's complement?
  - 10111 has MSB 1: Negative integer
  - To get absolute value of 10111
    - Ignore MSB: 10111 (decimal 7)
    - Magnitude: $2^4 - 7 = 16 - 7 = 9$
  - Answer: -9

No. of bits in magnitude = No. of bits - 1

Summary

- Rationale behind two's complement representation
- Simple ways of getting magnitude of negative integers from two's complement representation