WAVE PROPAGATION NEAR-SHORE IN THE PRESENCE OF BREAKWATER

A TERM PAPER REPORT

Submitted by

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Abstract

Numerical modelling of near-shore wave propagation in the presence of breakwater based on linear wave theory and taking mild slope equation (MSE) as governing equation is used to solve wave elevation in different direction. Numerical resolution of waves into ideal domains and different directional iterations through a break water shows that numerical model developed here is effectively representing wave diffraction, reflection and radiations of wave due to the presence of break waters.

Keywords: Break water, Mild slope equation, Linear wave theory, Diffraction, Radiation, Refraction, Reflection.

INTRODUCTION

The finite element method is a widely used method in ocean engineering for numerically solving differential equation arising in engineering and mathematical modelling. Near-shore wave propagation in the presence of breakwater is a complex process combining diffraction, refraction, radiation and reflection. Achieving an efficient and accurate model for the analysis of break-water is complex process in coastal engineering problems. Mild slope equation provides a steady-state solution for linear regular waves over varying depths and is proven to be effective available mathematical models to describe combined wave effects.

Treatment of boundary is considered very important in the formulation of numerical models. The different direction of waves approaching near shore and interacting with the breakwater is the focus of mathematical modelling in this term paper. The numerical problem is solved using MATLAB, and generation of mesh is done using GAMBIT. Finally output of the program is a plot showing the wave profile and wave propagation.

Procedure

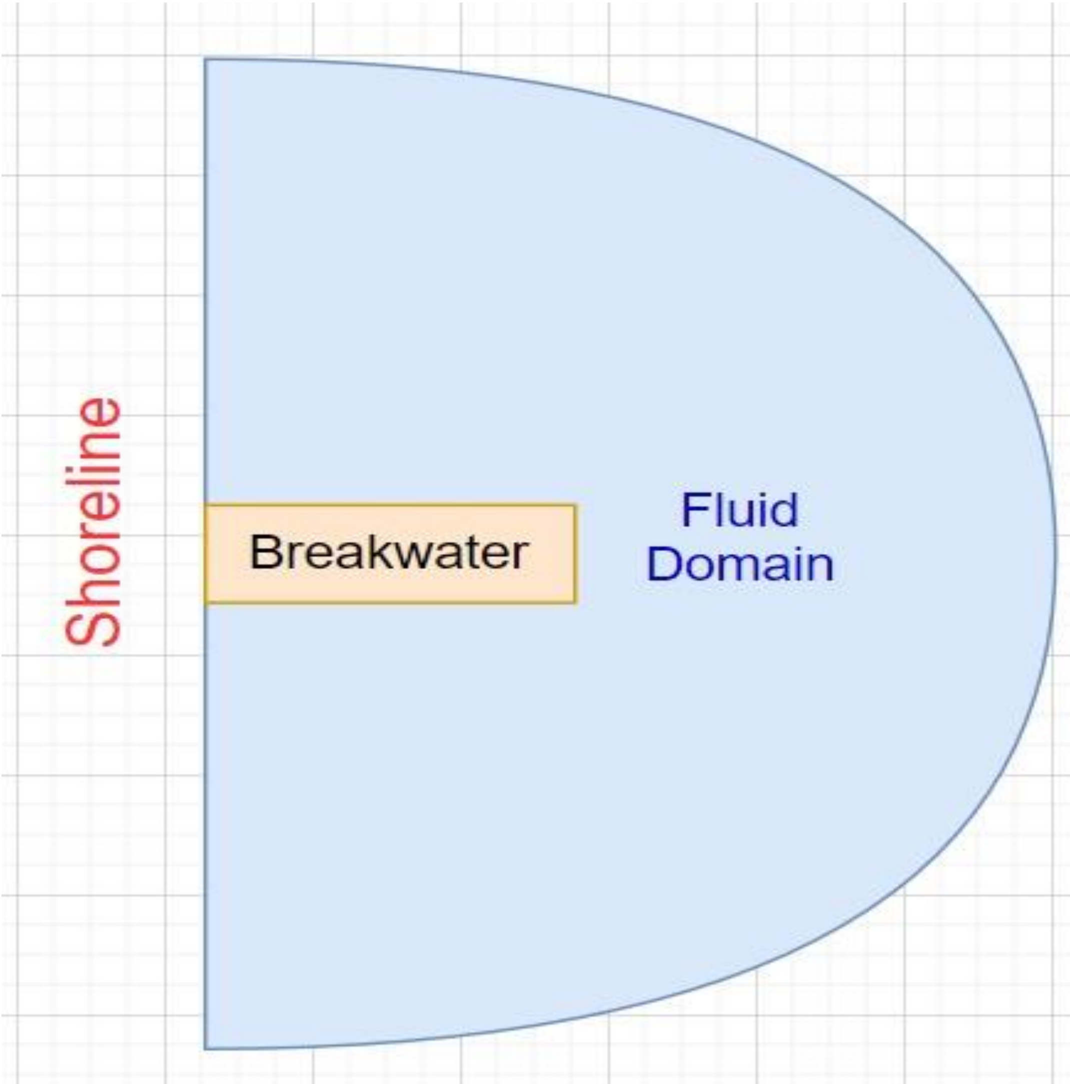
1. Fixing the domain and finalizing the boundary condition
2. Finite element formulation of stiffness matrix.
3. Discretization of the domain.
4. Numerical modelling in MATLAB.

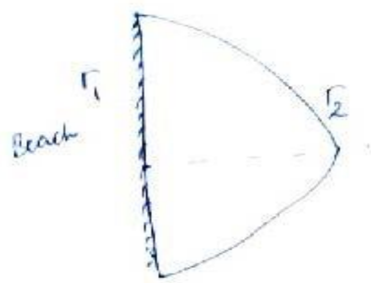
Fixing the domain and finalizing the boundary condition

- To construct a finite element numerical model the first step is to generate an appropriate domain.
- The dimensions of breakwater taken is as 0.7m length, 0.2 width and circular domain of radius 3m.
- The assigned boundary for break water is (τ_3). Breakwater is considered as rigid.
- For the beach side partial absorption of incoming waves with a domain of (τ_1)
- For the circular domain assigned with a symbol of (τ_2), Sommerfield radiation condition is taken.
- Origin of the whole domain is at the centre of the break water on the beach side.
- Water depth changes exactly as the x coordinate of consideration.

Finite element formulation of stiffness matrix.

- The governing equation used is Mild slope equation.
- The Galerkian formulation followed by Green-Gauss theorem is used to create the finite element formulation for the stiffness matrix.
- The descriptive part of the formulation is shown below





Governing equation
Mild slope equation

$$\nabla(CC_g \nabla\phi) + \omega^2 \frac{C_g}{C}\phi = 0$$

Boundary condition

at $\Gamma_1 \Rightarrow \frac{\partial\phi}{\partial n} + ik\alpha\phi = 0$

$\alpha = \frac{R-1}{R+1}\cos\delta$
R: reflection coefficient
δ = angle

$\Gamma_2 \Rightarrow \frac{\partial\phi}{\partial n} + ik\phi = 0$

and $\frac{\partial\phi}{\partial n} = \frac{\partial\phi_I}{\partial n} + \frac{\partial\phi_s}{\partial n}$

$$\phi_I = +a_0 e^{-ikn}$$

$$\frac{\partial\phi_I}{\partial n} = \mp a_0\, \partial ik e^{-ikn}$$

$$= -ik\phi_I$$

$$\boxed{\frac{\partial\phi_I}{\partial n} = -ik\phi_I}$$

$$\frac{\partial\phi_s}{\partial n} \mp ik\phi_s = 0$$

$$\boxed{\frac{\partial\phi_s}{\partial n} = ik\phi_s = ik(\phi - \phi_I)}$$

$$\therefore \frac{\partial\phi}{\partial n} = -ik\phi_I + ik\phi - ik\phi_I = \underline{\underline{ik\phi - 2ik\phi_I}}$$

FEM formulation

Galerkin method

$$\iint_{\Omega^e} N_j \left(\nabla(CC_g \nabla\phi) + \omega^2 \frac{C_g}{C} \phi \right) d\Omega = 0$$

$$\iint_{\Omega^e} N_j \nabla(CC_g \nabla\phi)\, d\Omega + \iint_{\Omega^e} \omega^2 \frac{C_g}{C} \phi \, d\Omega = 0$$

$\Downarrow$ GGT

$$-\iint_{\Omega^e} \nabla N_j \cdot CC_g \nabla\phi \, d\Omega + \int_{\Gamma} N_j CC_g \nabla\phi \, d\Gamma + \iint_{\Omega^e} \omega^2 \frac{C_g}{C} \phi \, d\Omega = 0$$

$$-\underbrace{\iint_{\Omega^e} \left[CC_g \nabla N_j \nabla\phi \, \cancel{d\Omega} - N_j \omega^2 \frac{C_g}{C} \phi \, \cancel{d\Omega} \right] d\Omega}_{I^{st}\ \text{term}} + \underbrace{\int_{\Gamma = \Gamma_1 + \Gamma_2} N_j CC_g \nabla\phi \, d\Gamma}_{II^{nd}\ \text{term}}$$

$$\phi = \Sigma N_i \phi_i \qquad \nabla\phi = \Sigma \nabla N_i \{\phi_i\}$$

1st term

$$-\iint_{\Omega^e} \left[CC_g \nabla N_j \nabla N_i - \omega^2 \frac{C_g}{C} N_j N_i \right] d\Omega \, \{\phi_i\} \qquad \frac{\omega^2}{C^2} = k^2$$

$$-\iint_{\Omega^e} CC_g \left[\nabla N_j \nabla N_i - k^2 N_j N_i \right] d\Omega \, \{\phi_i\}$$

2nd term

$$\int_{\Gamma_1} N_j CC_g \frac{\partial\phi}{\partial n} d\Gamma + \int_{\Gamma_2} N_j CC_g \frac{\partial\phi}{\partial n} d\Gamma$$

$$-i\alpha \int_{\Gamma_1} kCC_g N_j N_i \, d\Gamma \, \{\phi_i\} + i \int_{\Gamma_2} CC_g N_j (k\phi - 2k\phi_I) \, d\Gamma$$

$$-i\alpha \int_{\Gamma_1} k C C_g N_j N_i \, d\Gamma \, \{\phi_i\} + i \int_{\Gamma_2} C C_g k N_j \cancel{\phi} N_i \, d\Gamma \, \{\phi_i\}$$

$$- 2i \int_{\Gamma_2} C C_g \, k N_j N_i d\Gamma \, \{\phi_{I_i}\}$$

$\therefore \; \mathrm{I} + \mathrm{II} = 0$

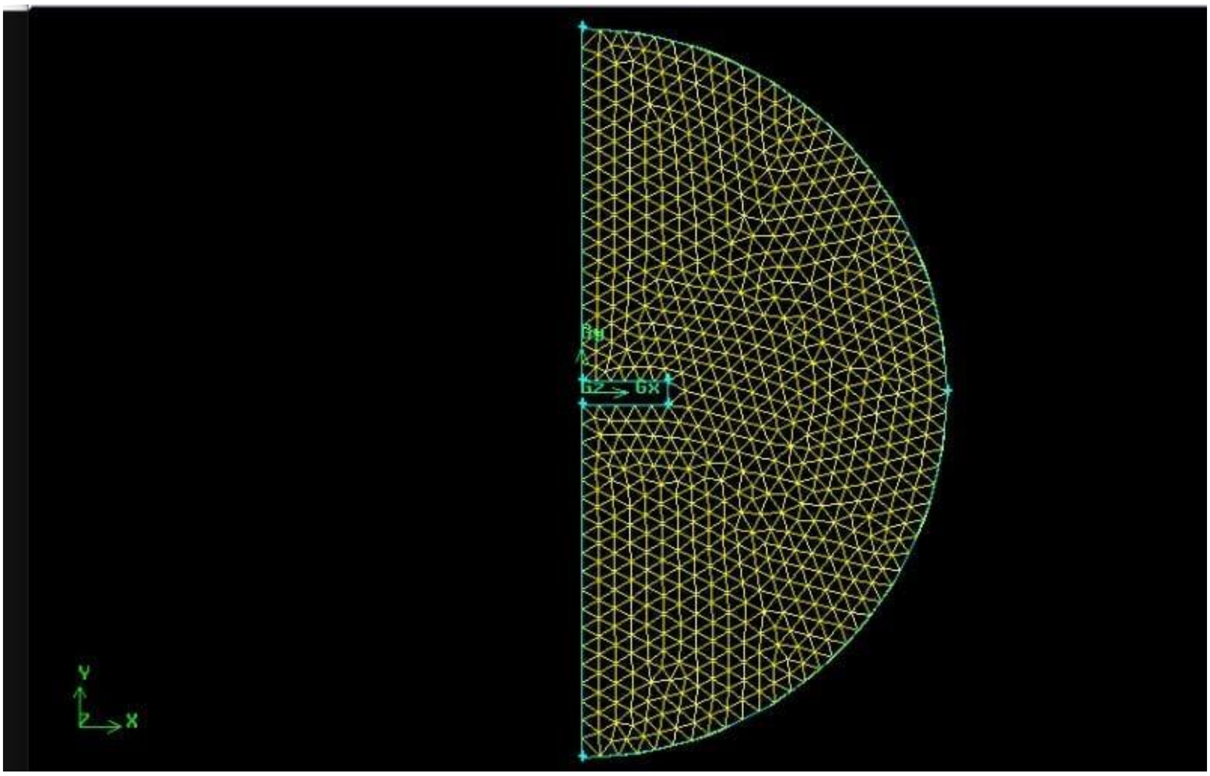
$$-\iint_{\Omega} C C_g \left[\nabla N_j \nabla N_i - k^2 N_j N_i \right] d\Omega \, \{\phi_i\}$$

$$- i \int_{\Gamma_1} k\alpha \, C C_g N_j N_i \, d\Gamma \, \{\phi_i\} + i \int_{\Gamma_2} C C_g k N_j N_i \, d\Gamma \, \{\phi_i\}$$

$$= 2i \int_{\Gamma_2} C C_g \, k N_j N_i \, d\Gamma \, \{\phi_{I_i}\}$$

Discretization of the domain.

- The triangular element is considered for discretising the domain.
- In order to create uniform triangular meshes with a spacing of 0.1m GAMBIT software is used.
- The nodal coordinates with connectivity of edges and triangles in the mesh from GAMBIT are taken as input values in MATLAB program.



Numerical modelling in MATLAB.

Formation of Local K and F Matrix

- There was a need to solve six integrals out of which two were area integral and remaining four were line integrals.
- The computation time for the area integral consisting of $\nabla N^t \nabla N$ increased drastically, which lead us to compute it manually and then feed it to the program which resulted in 3x3 matrices.
- The other area integral consisting of $N^t N$ was not able to compute manually so we used "int" function. This function gives us the result as definite and indefinite integrals. To compute this area integral, we considered scalene triangle and found out the limits in X and Y direction and computed this integral obtaining 3x3 matrices.
- One of the line integral along the shore line turn out to be zero as the celerity and group celerity at the shoreline will be zero. Also, other line integral along the boundary of the break water turned out to be zero as we have considered break water as rigid.
- The integral along the curved boundary was performed by converting cartesian coordinate system to the polar coordinate system having θ as variable. So here we got 2x2 local K matrices.
- The force was computed along the curved boundary as we have assumed that the incident waves approach the shore. We have incorporated the direction of incident wave while calculating the incident potential. So here we got 2x1 local F matrix.

Formation of Global K and F matrix

- Assembled two local 3x3 K matrices and one 2x2 K matrices for each node to get final global K matrix.
- One local F of 2x1 to get one global F.

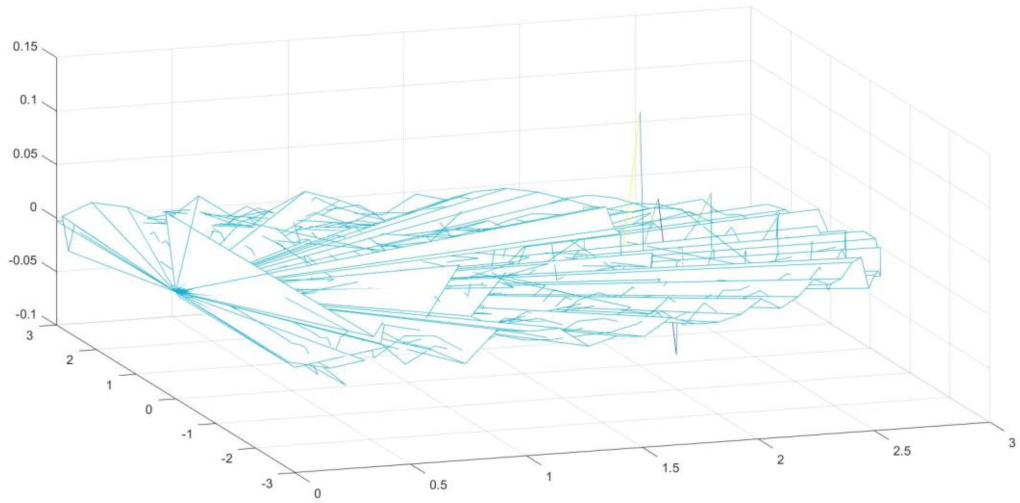
Applying boundary conditions and solving

- The Φ value along the shore-line is taken as constant (say 0) as no wave propagation and wave elevation across the boundary is present.
- Removed the rows and columns in the global K and F matrix of those nodes where Φ was set to zero.
- Solved $K\Phi = F$ equation to get the Φ values at all nodes.

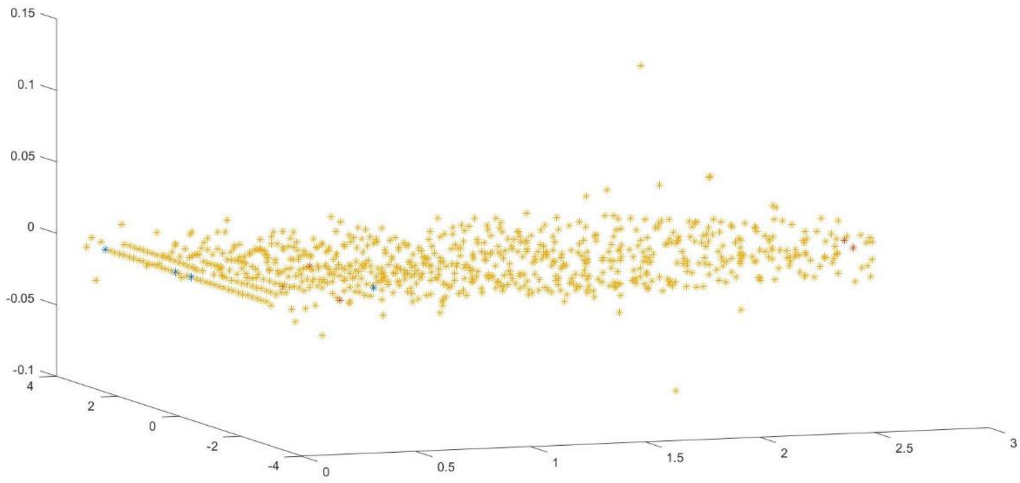
Plotting

- Differentiated this Φ value with respect to time and multiplied it with -1/g to obtain wave elevation η.
- Wave amplitude is obtained by taking complex magnitude and phase angle is obtained by angle command.
- Plotted wave amplitude and Phase angle with respect to X and Y.

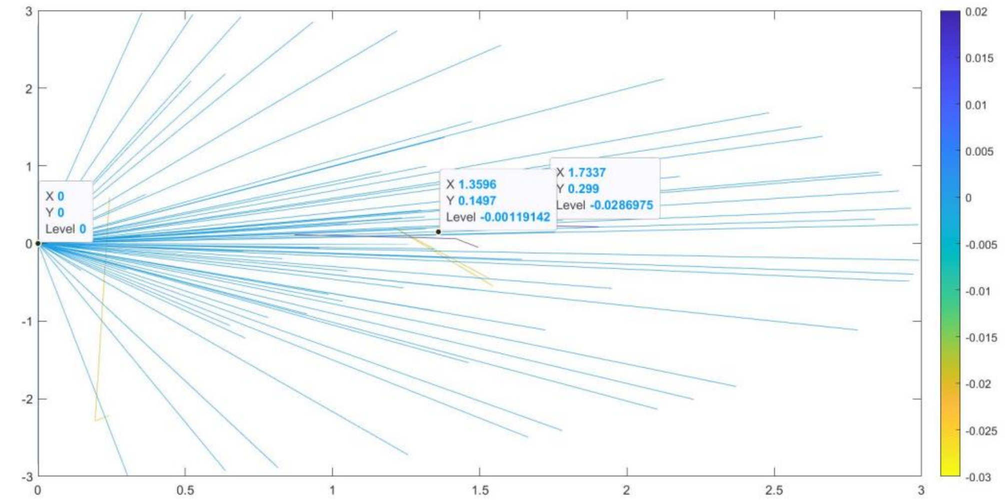
PLOTS



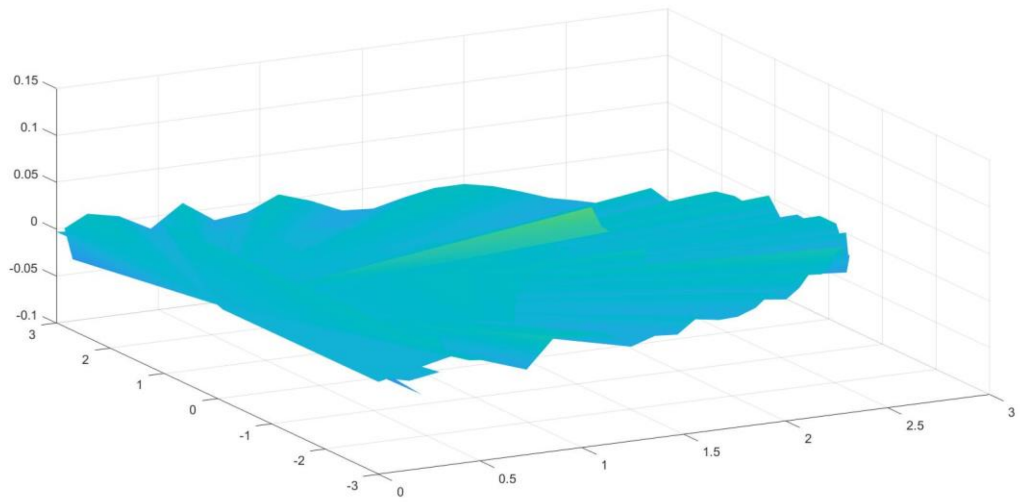
WAVE PROFILE IN X, Y DOMAIN USING MESH COMMAND



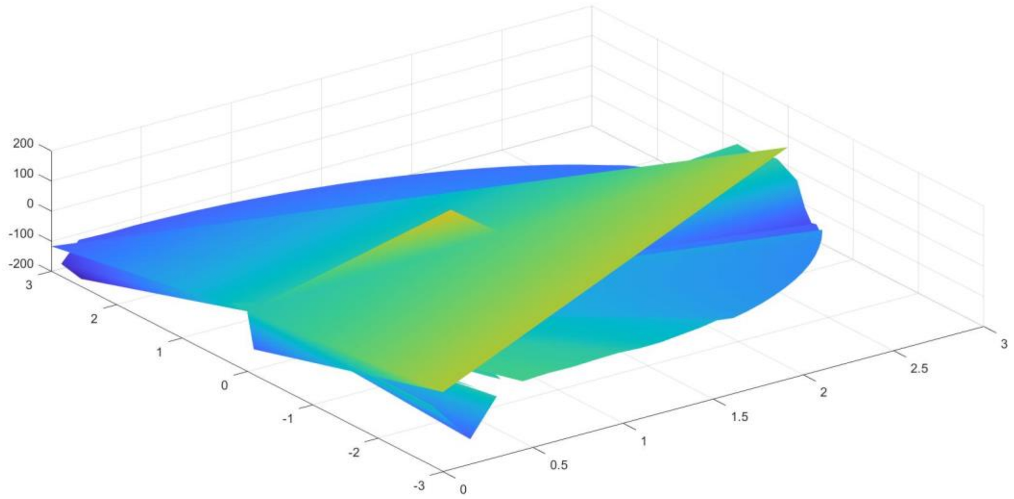
WAVE PROFILE IN X, Y DOMAIN USING PLOT3 COMMAND



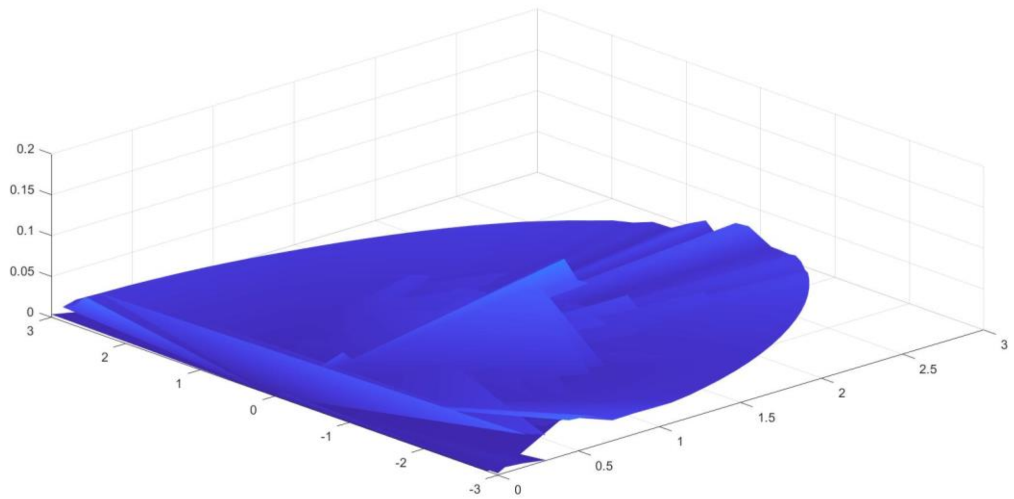
WAVE PROFILE IN X, Y DOMAIN USING CONTOUR PLOT



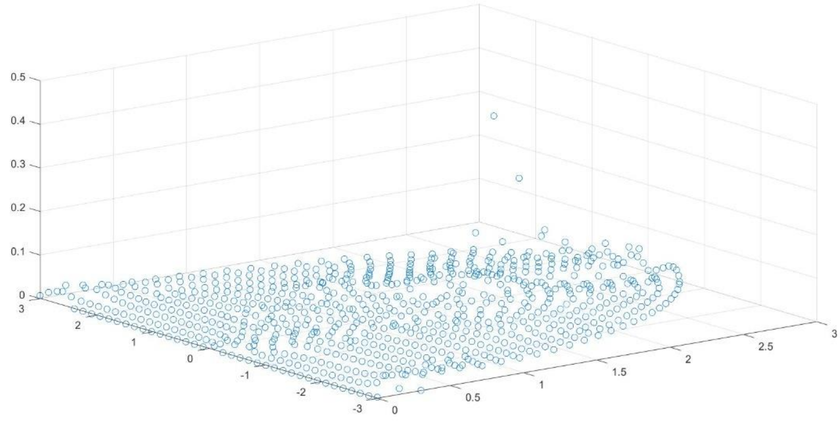
WAVE PROFILE IN X, Y DOMAIN USING SURF COMMAND



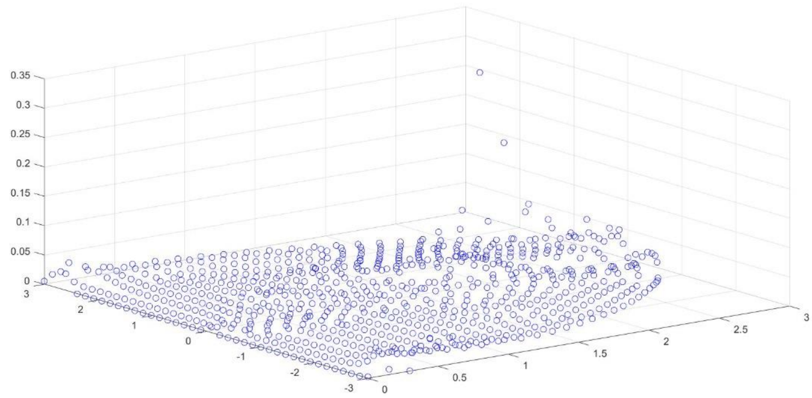
PHASE ANGLE WITH RESPECT TO X AND Y



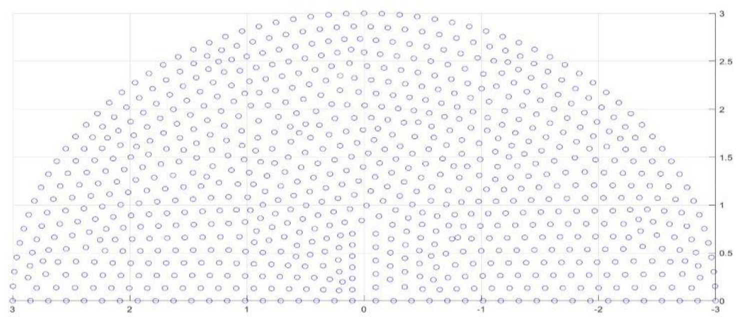
WAVE AMPLITUDE WITH RESPECT TO X AND Y



WAVE AMPLITUDE AT 0^0 AND t =0



WAVE AMPLITUDE AT 45^0 AND t = 0.4



WAVE AMPLITUDE AT 45^0 AND t =0

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