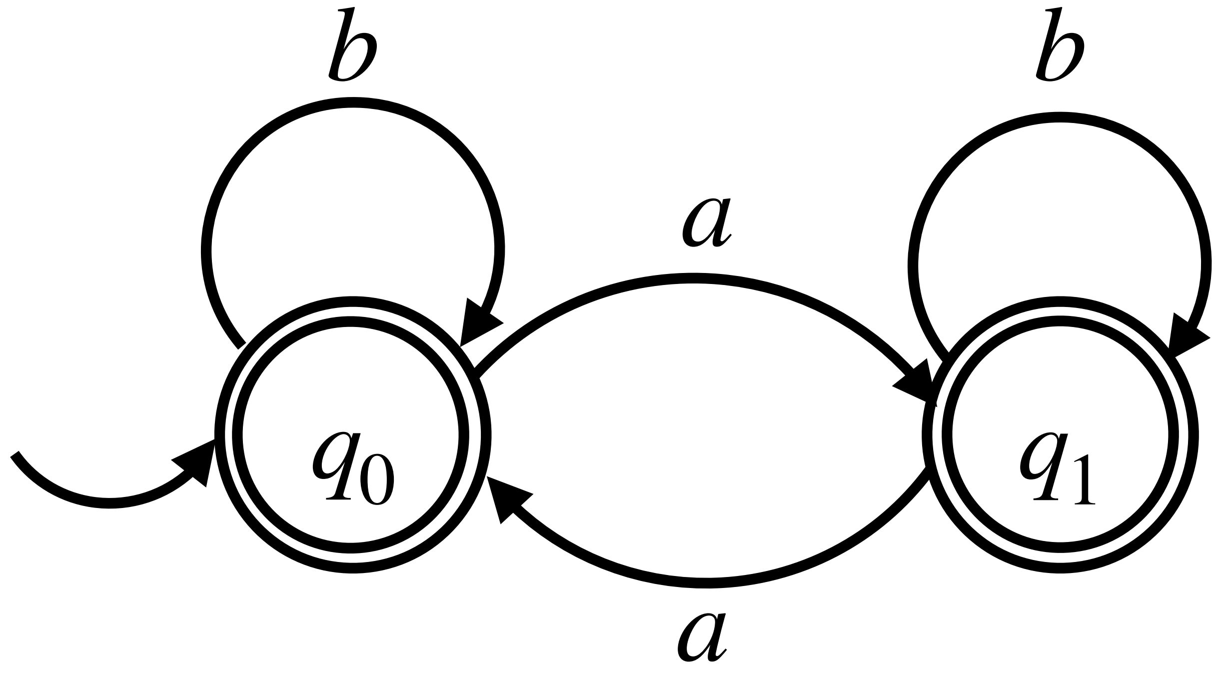
DFA :
Equivalence &
Minimal State Automaton

CS301 Theory of Computation

Sumesh Divakaran@CSE, College of Engineering Trivandrum

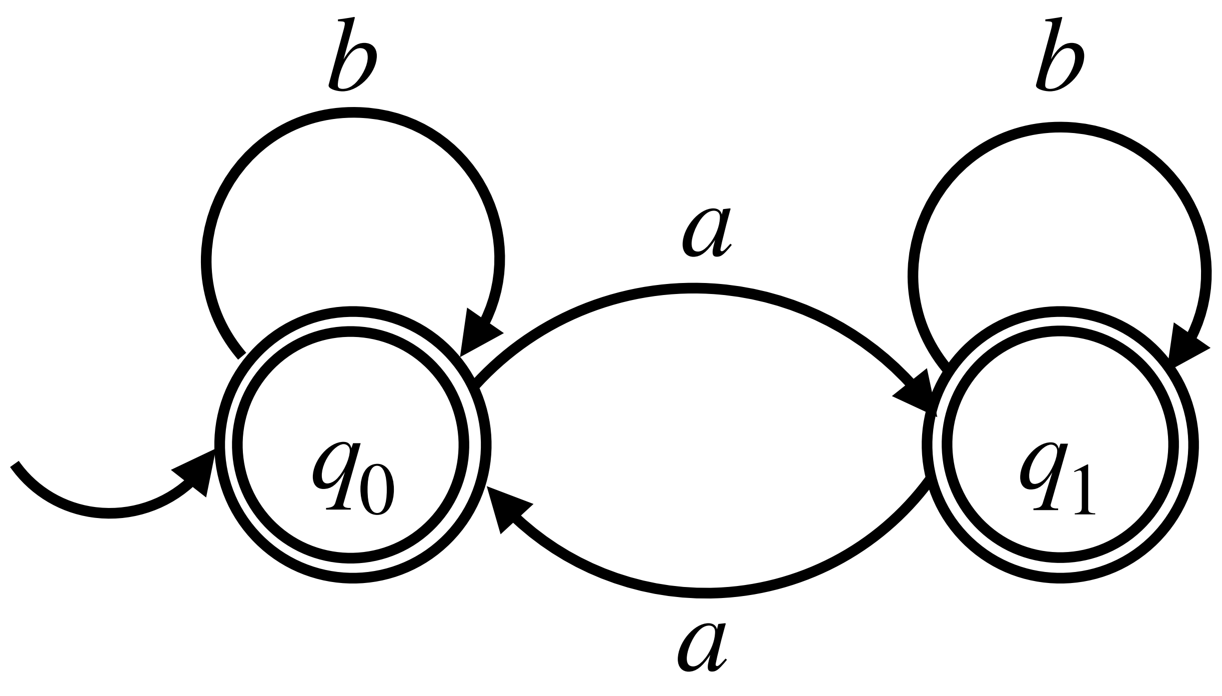
Equivalent DFAs

Equivalent DFAs



$\mathscr{A}$

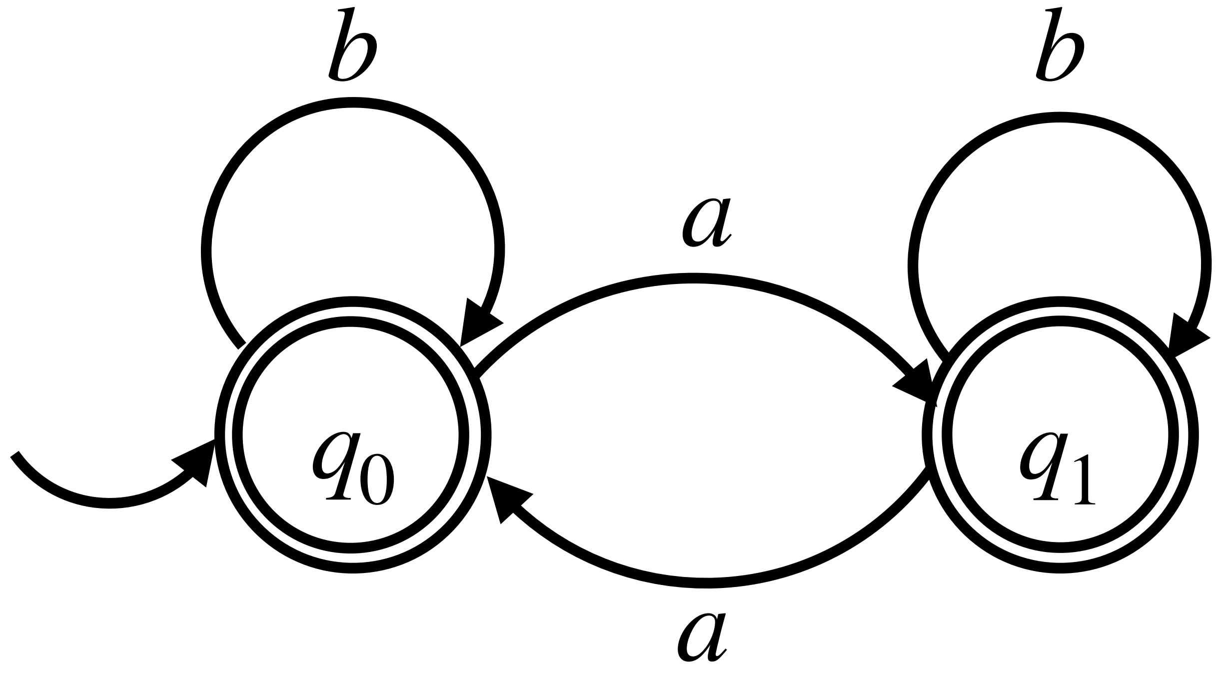
Equivalent DFAs



$\mathscr{A}$

$L(\mathscr{A}) = \ ?$

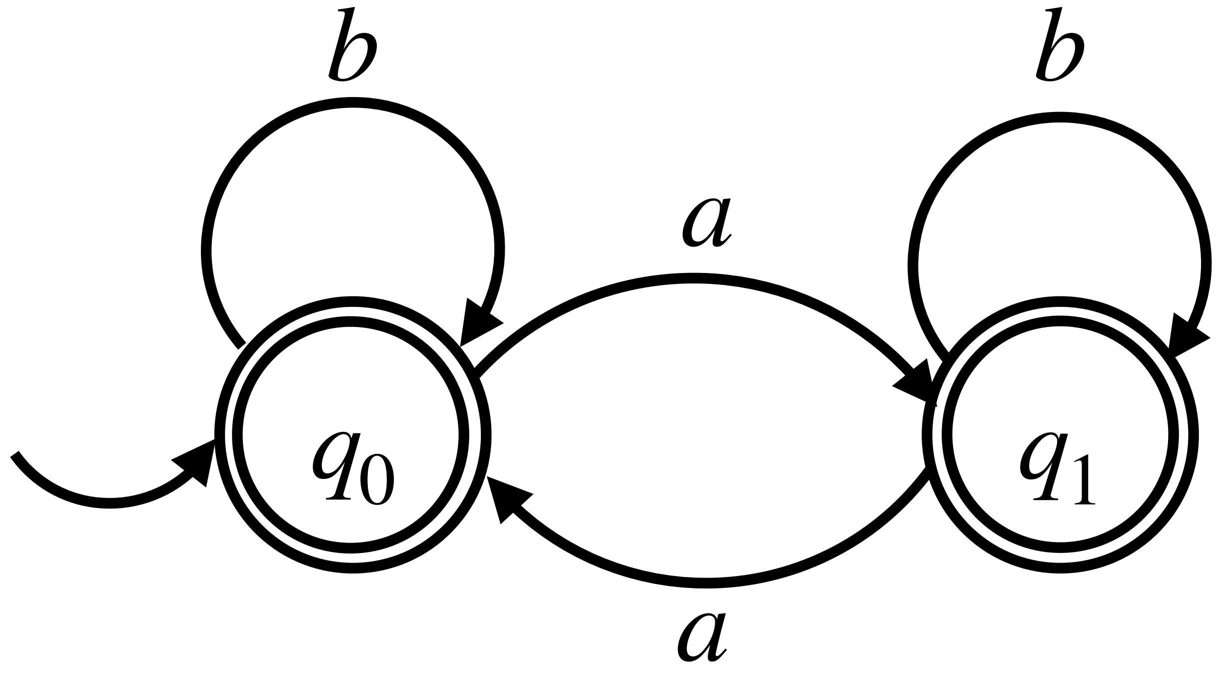
Equivalent DFAs



$\mathscr{A}$

$L(\mathscr{A}) = \{a, b\}^*$

Equivalent DFAs



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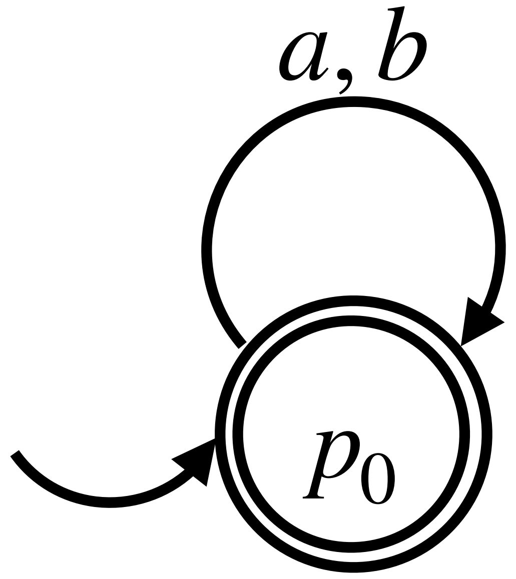
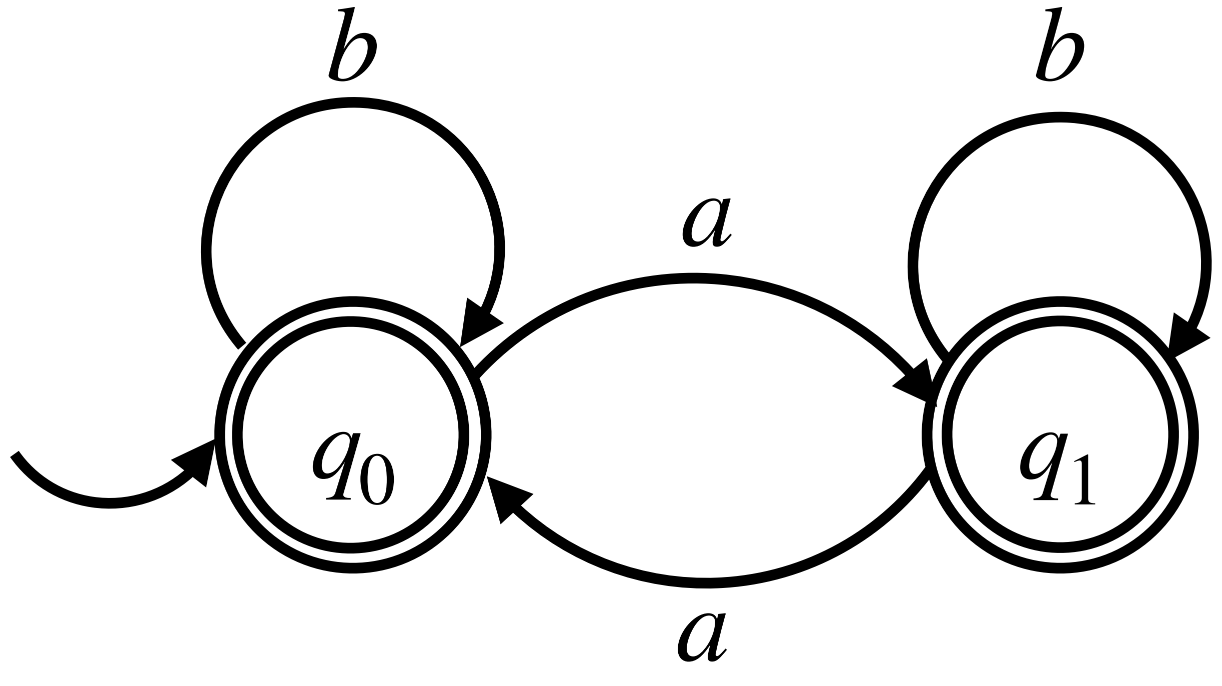


$\mathscr{B}$

$$L(\mathscr{A}) = \{a, b\}^*$$

Equivalent DFAs



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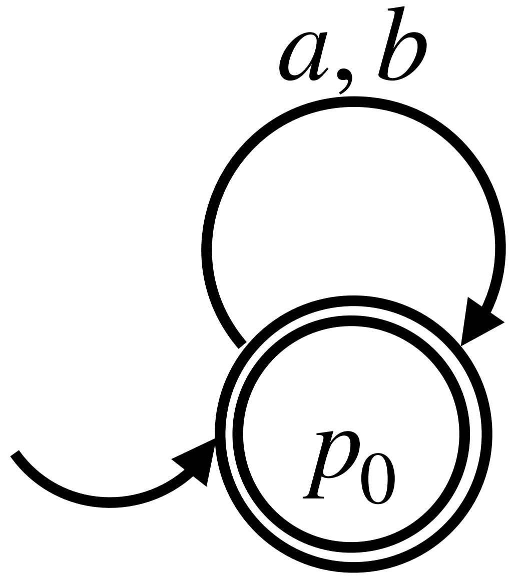
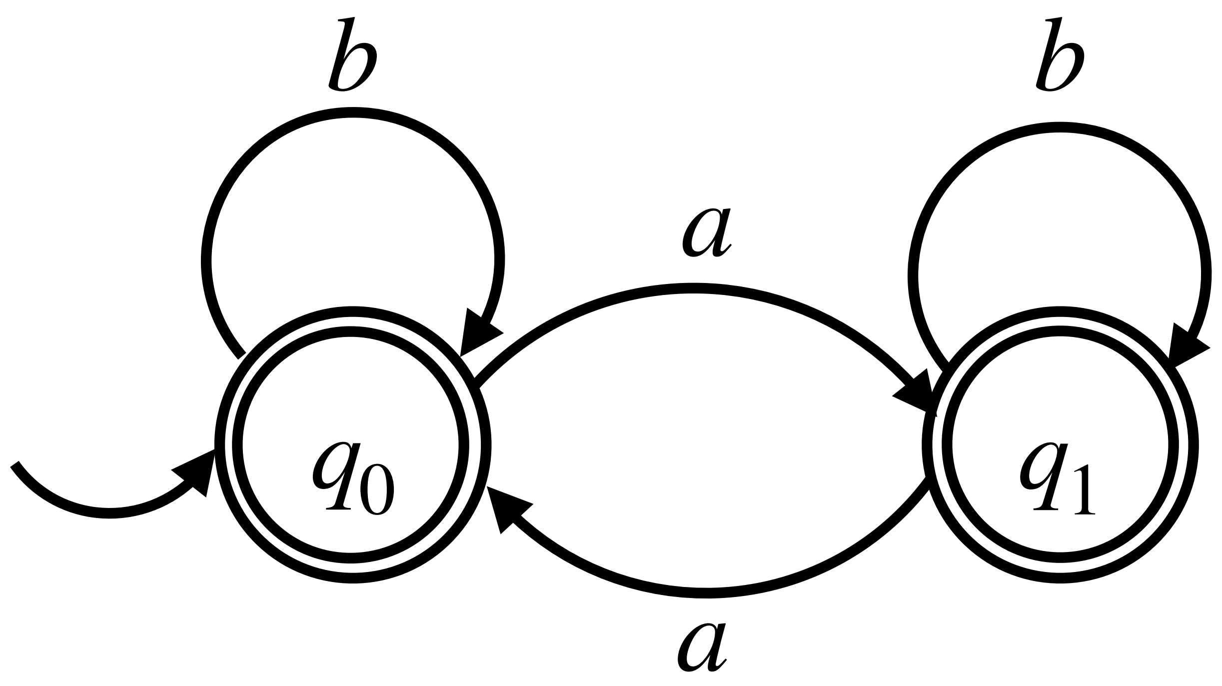


$\mathscr{B}$

$$L(\mathscr{A}) = \{a, b\}^* = L(\mathscr{B})$$

Equivalent DFAs



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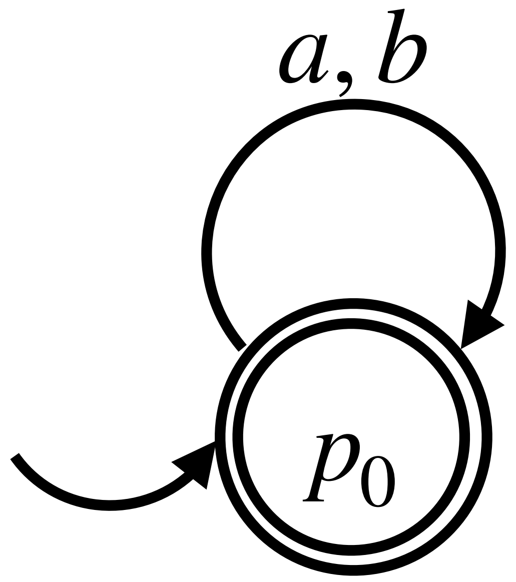
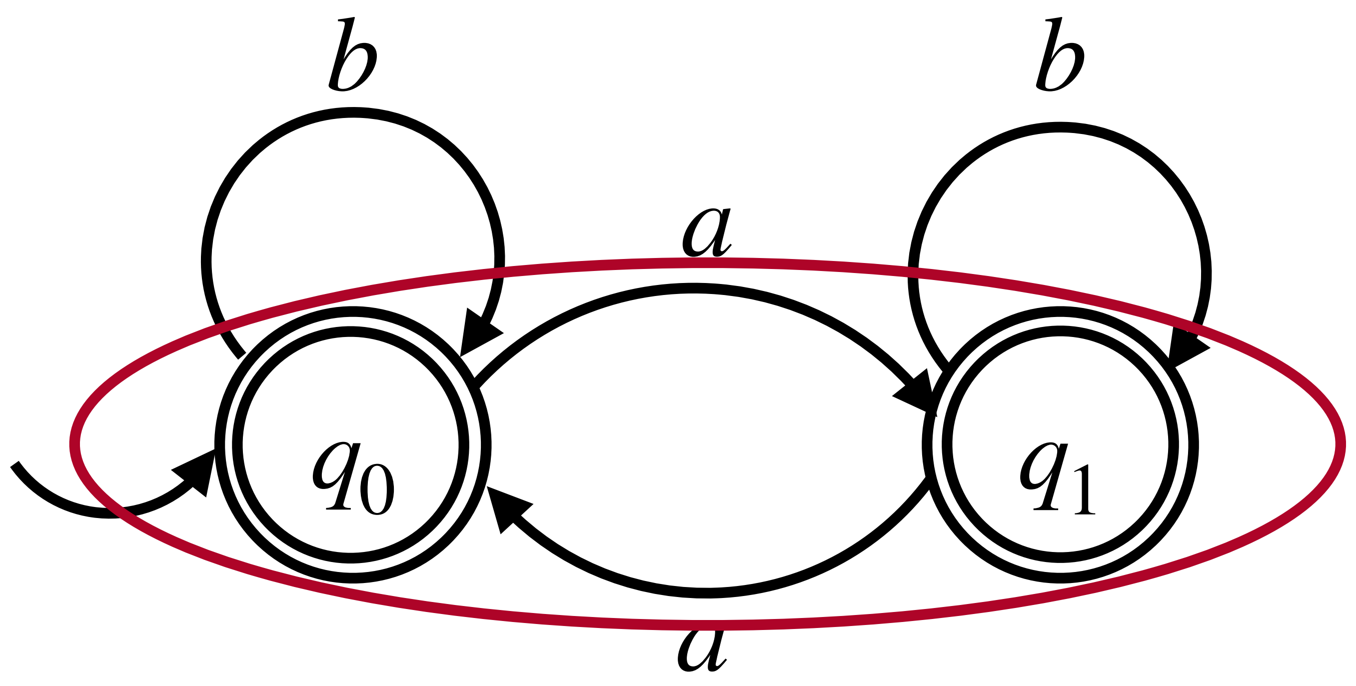


$\mathscr{B}$

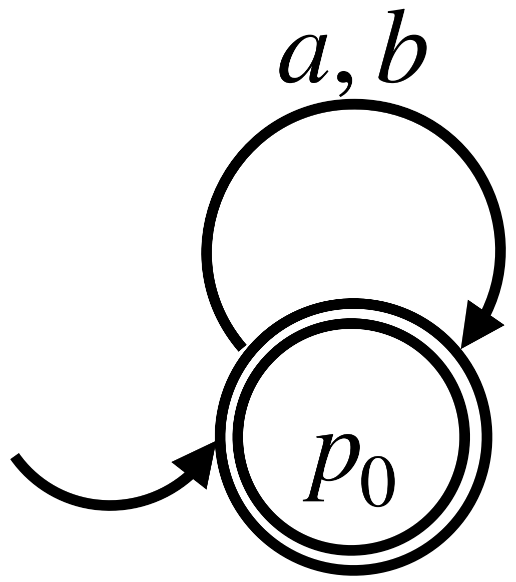
$$L(\mathscr{A}) = \{a, b\}^* = L(\mathscr{B})$$

- Hence, $\mathscr{A}$ and $\mathscr{B}$ are equivalent DFAs

Equivalent DFAs



$\mathscr{A}$



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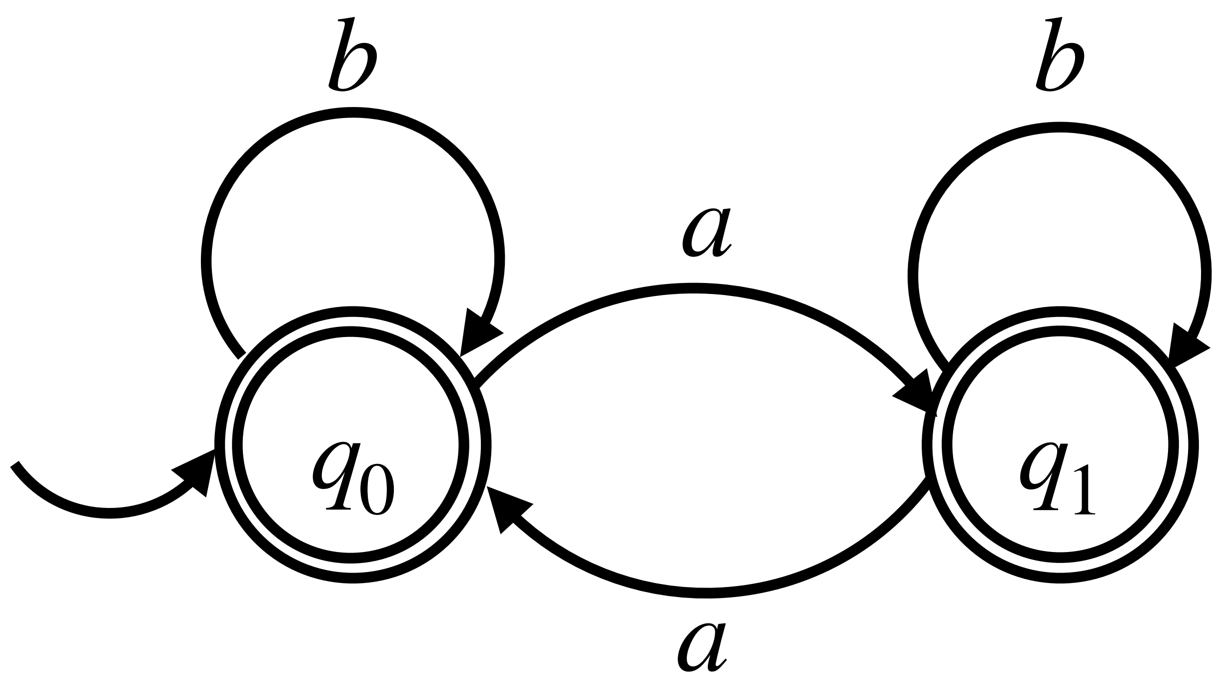
$$L(\mathscr{A}) = \{a, b\}^* = L(\mathscr{B})$$

- Hence, $\mathscr{A}$ and $\mathscr{B}$ are equivalent DFAs

Equivalent states can be collapsed to get a DFA with fewer number of states

Equivalent DFAs



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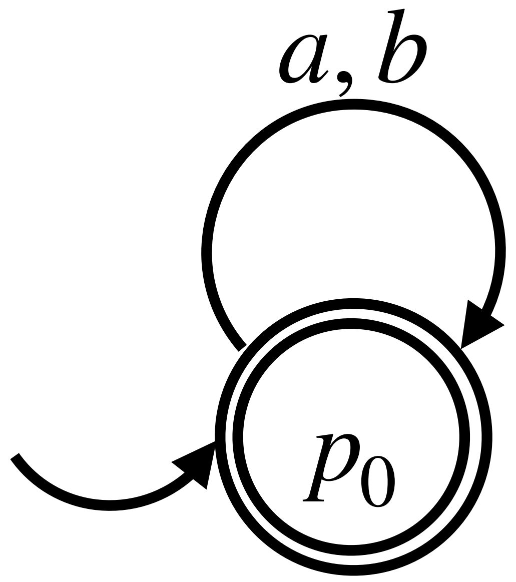

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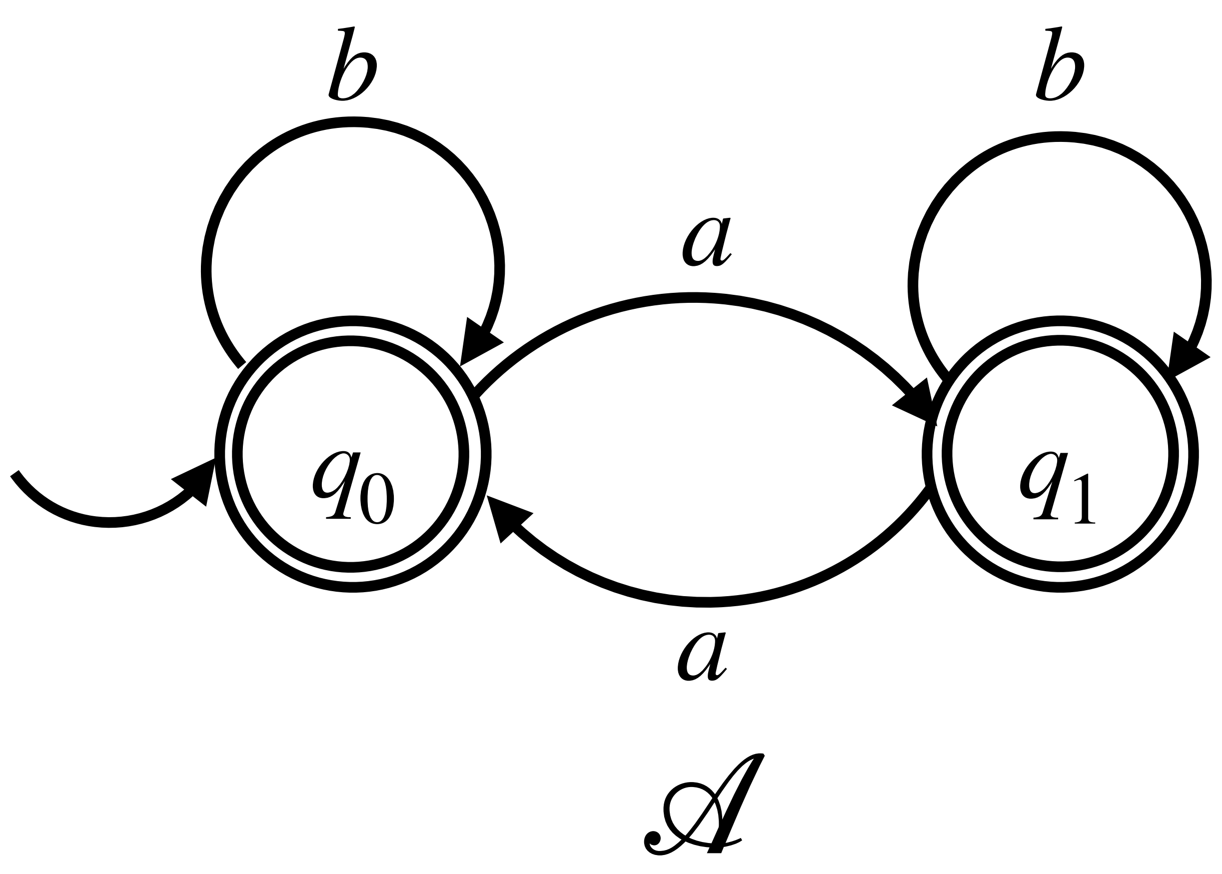
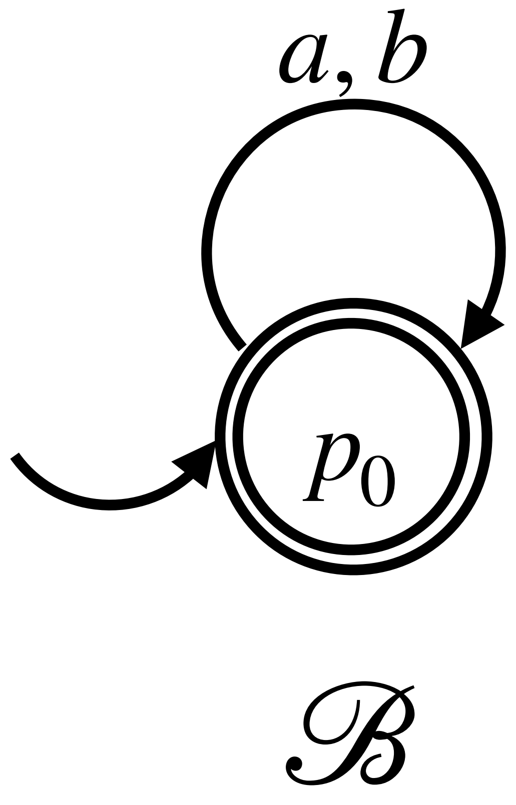
> Two DFAs $\mathscr{A}$ and $\mathscr{B}$ are said to be *equivalent* iff $L(\mathscr{A}) = L(\mathscr{B})$.

Equivalent DFAs





$$L(\mathscr{A}) = \{a, b\}^* = L(\mathscr{B})$$

- Hence, $\mathscr{A}$ and $\mathscr{B}$ are equivalent DFAs

> Two DFAs $\mathscr{A}$ and $\mathscr{B}$ are said to be *equivalent* iff $L(\mathscr{A}) = L(\mathscr{B})$.

- $\mathscr{B}$ is a DFA with fewer number of states

Equivalent DFAs

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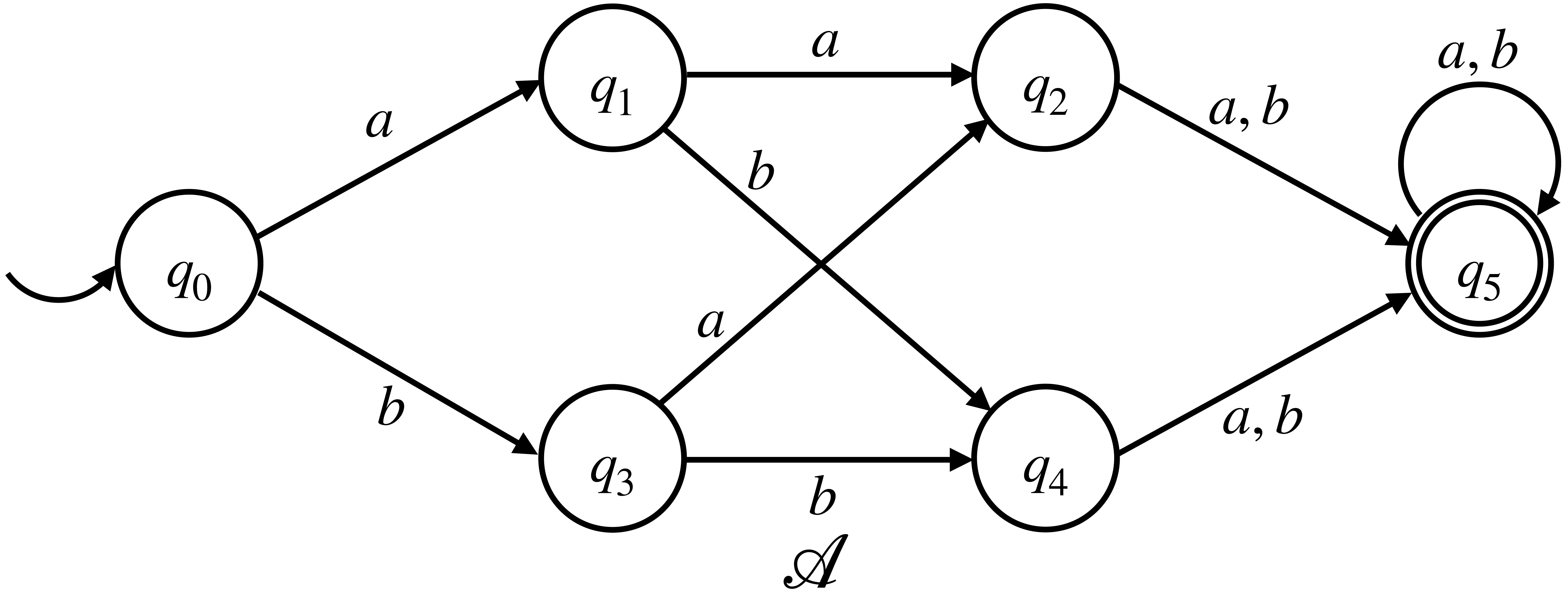

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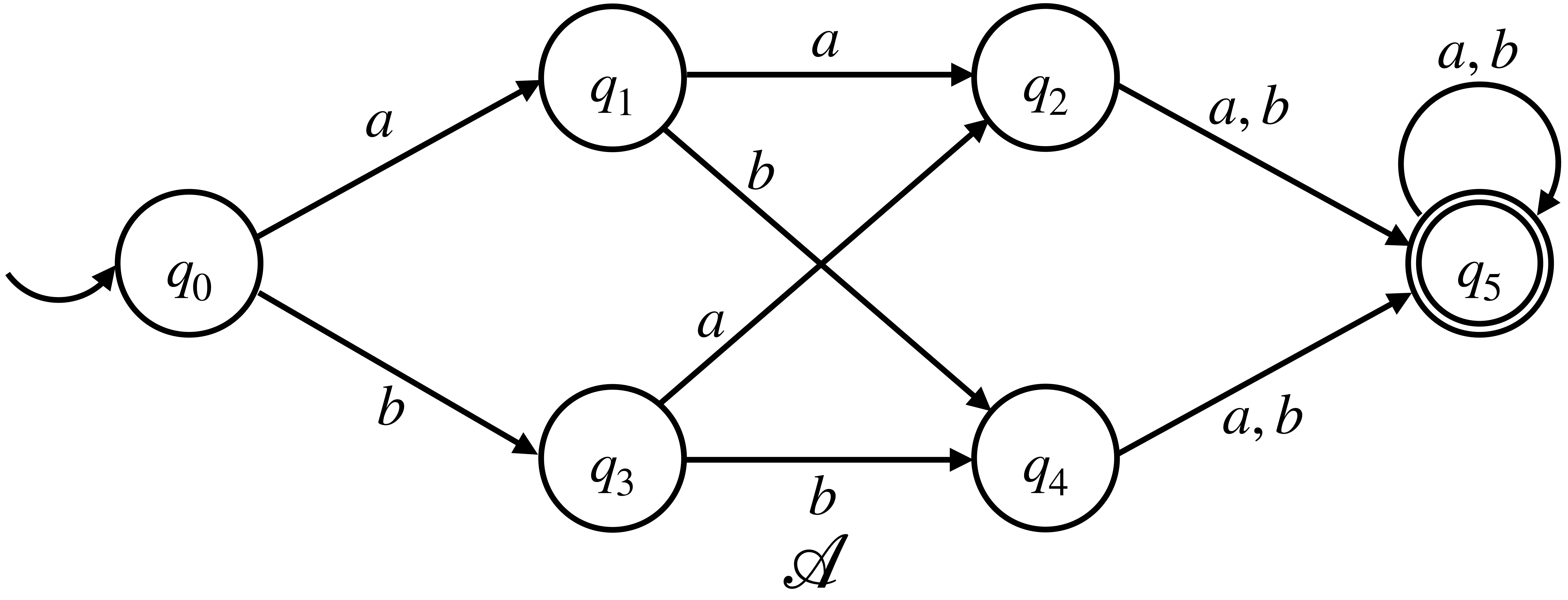
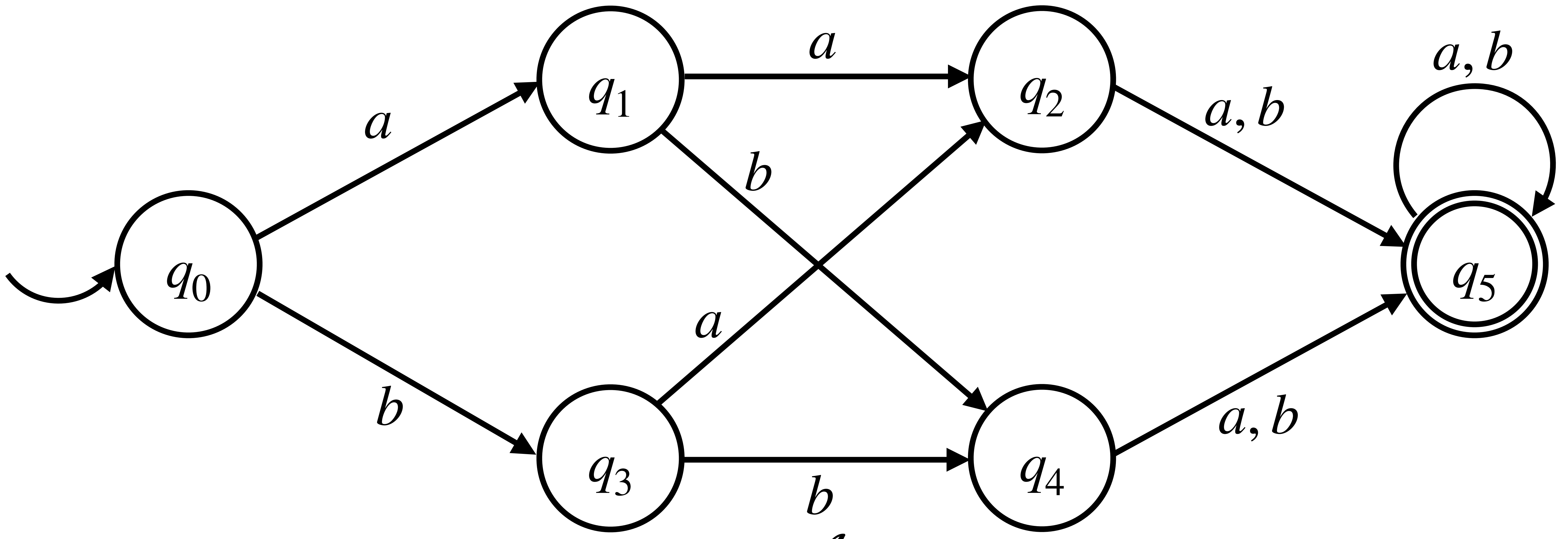


$L(\mathscr{A}) = \ ?$

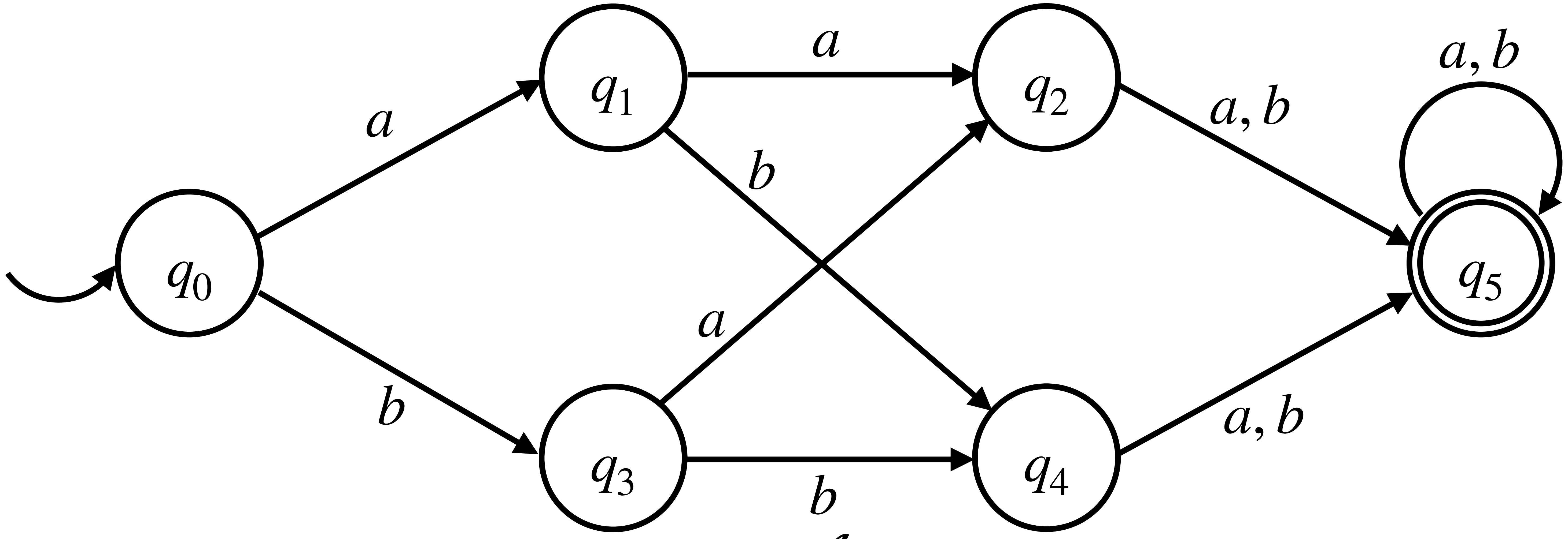
Equivalent DFAs



$\mathscr{A}$

$$L(\mathscr{A}) = \{x \in \{a, b\}^* \mid |x| \geq 3\}$$

Equivalent DFAs



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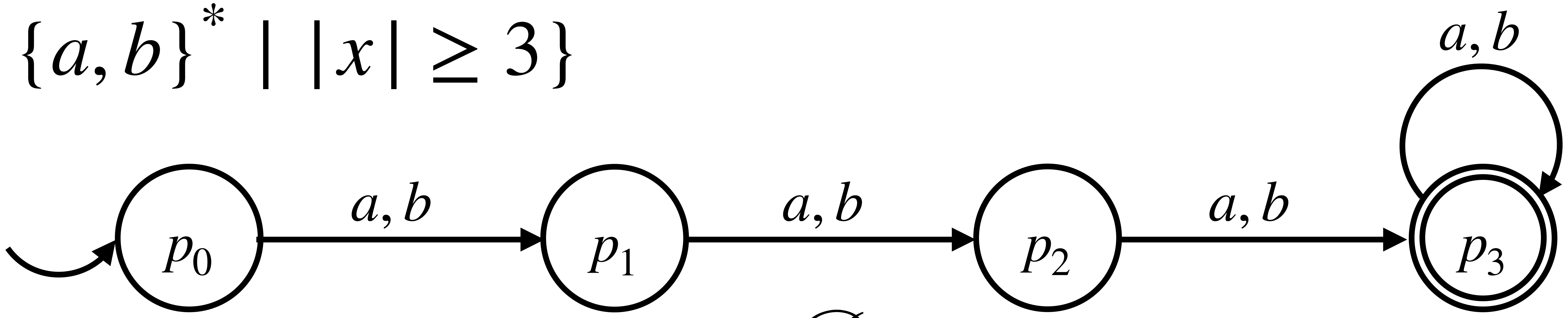
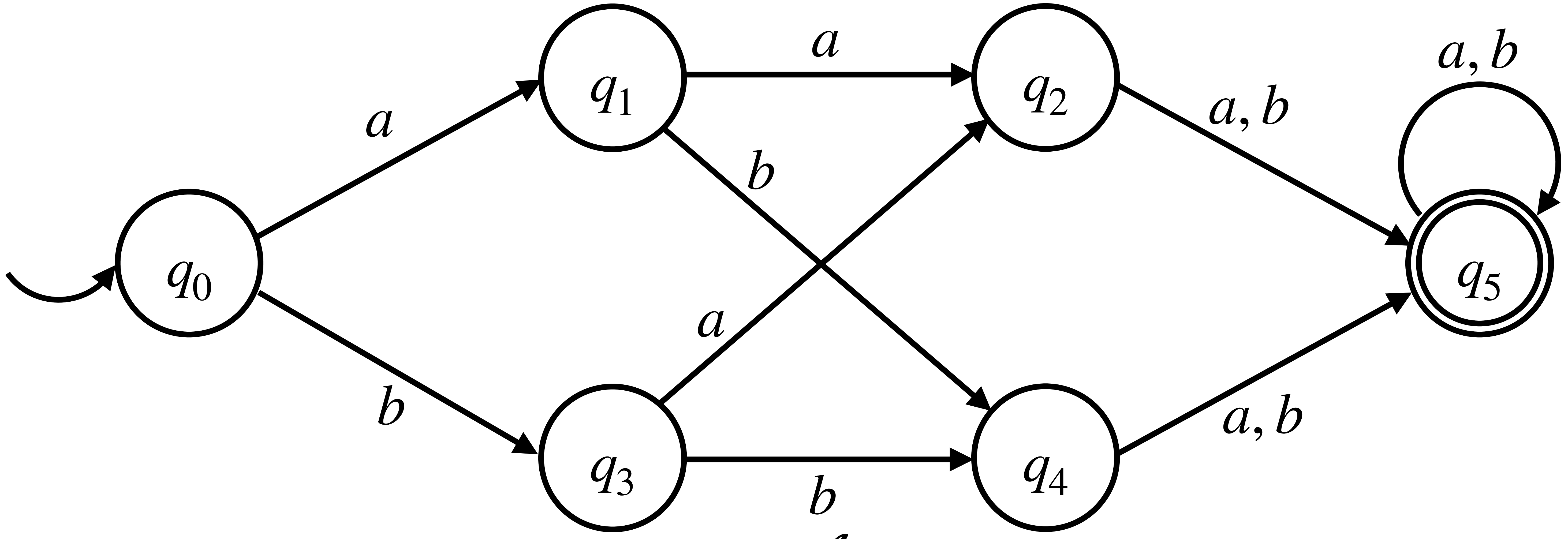


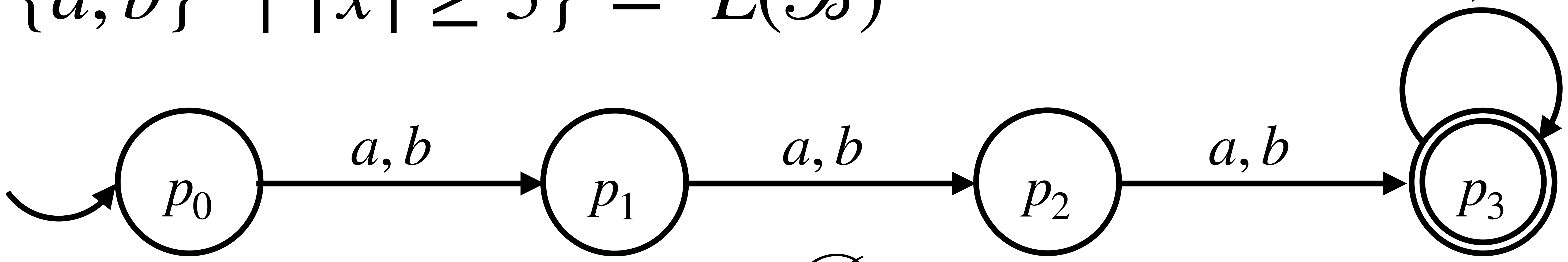
$\mathscr{B}$

Equivalent DFAs



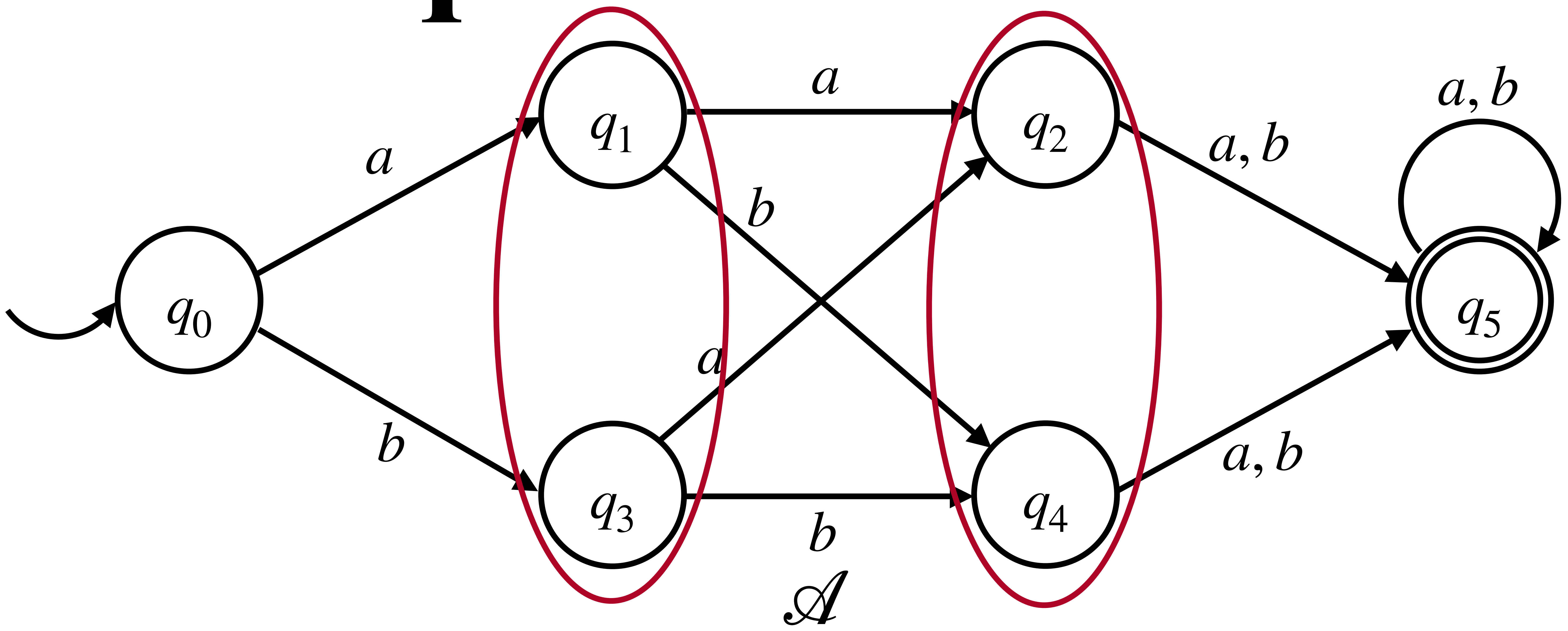
$\mathscr{A}$

$$L(\mathscr{A}) = \{x \in \{a, b\}^* \mid |x| \geq 3\} = L(\mathscr{B})$$



$\mathscr{B}$

Equivalent DFAs



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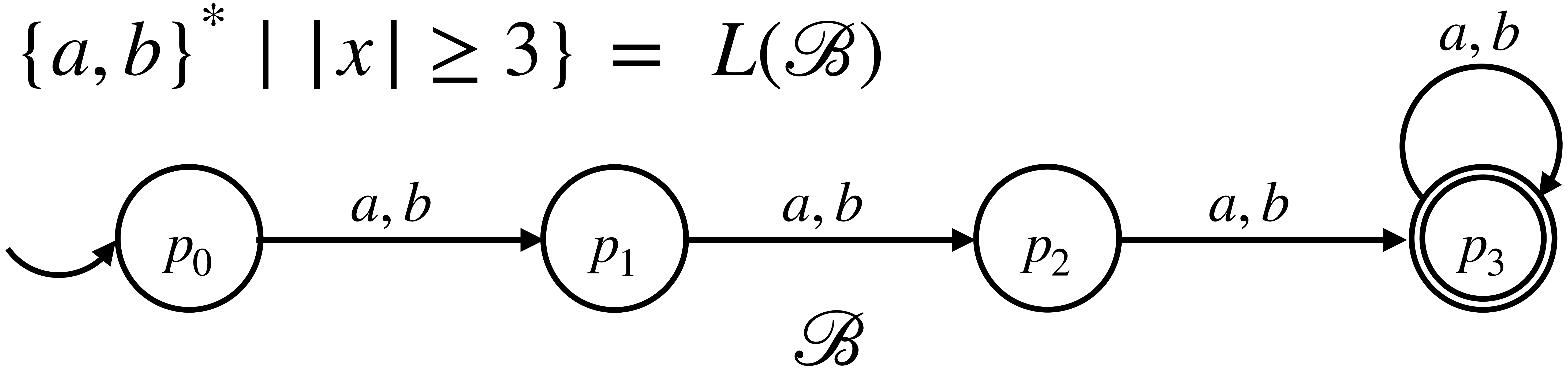
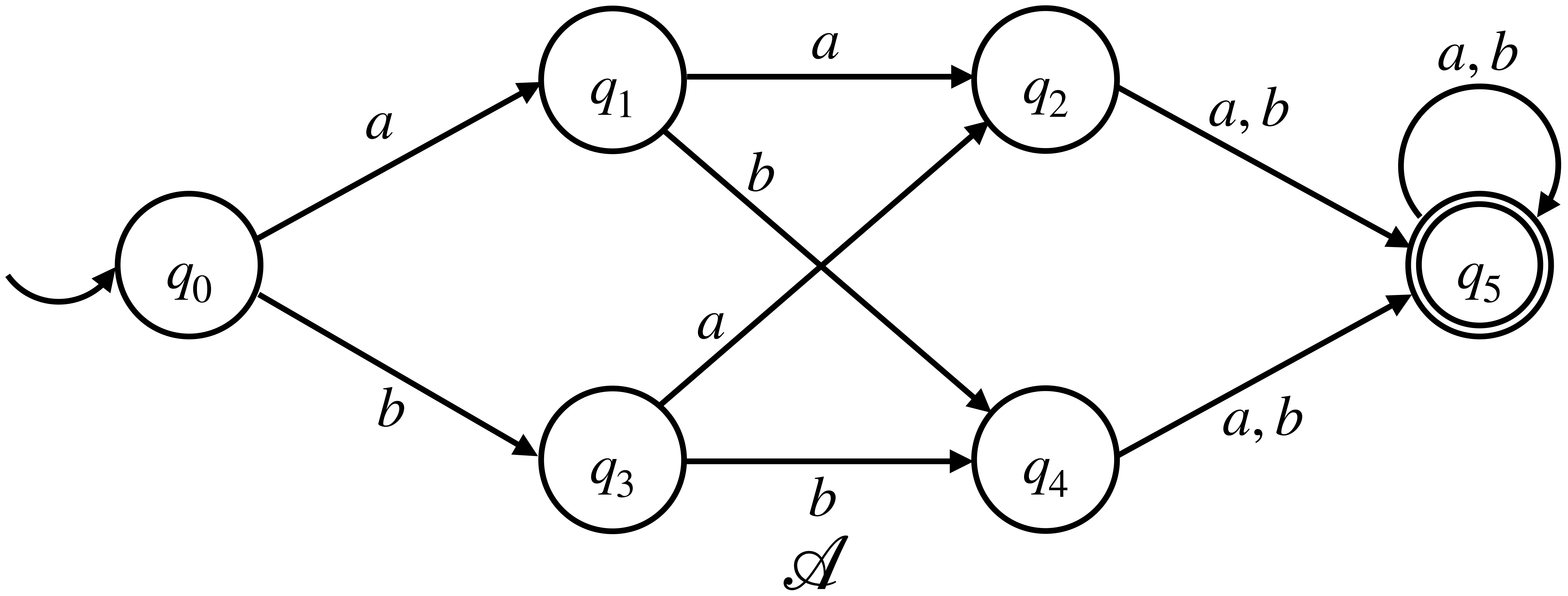


Equivalent states can be collapsed to get a DFA with fewer number of states

Equivalent DFAs



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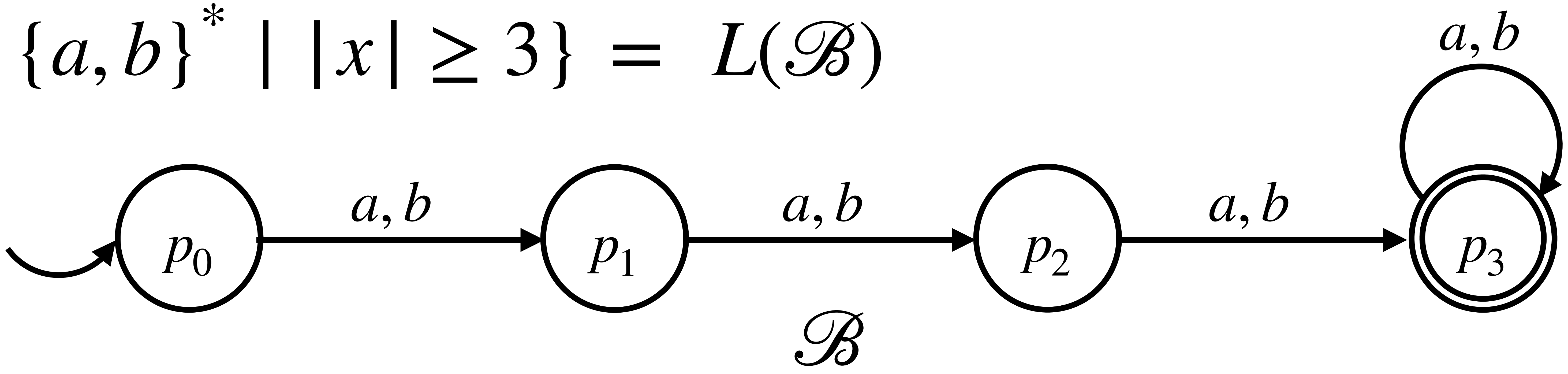

- $\mathscr{B}$ is a DFA with fewer number of states

Equivalent DFAs

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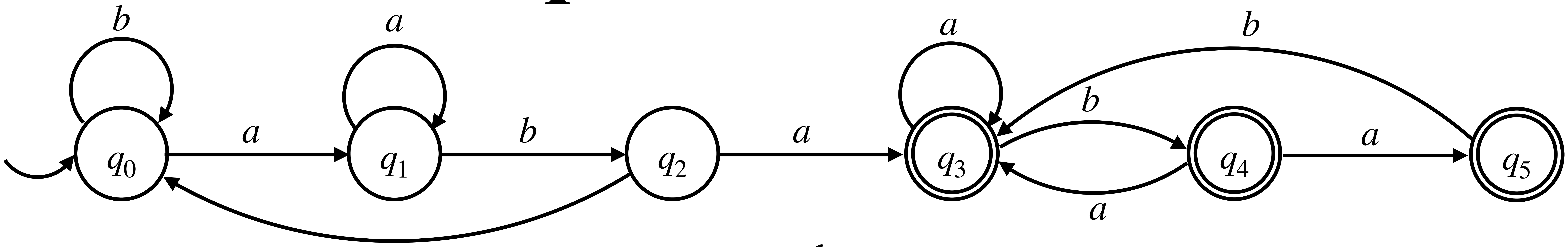
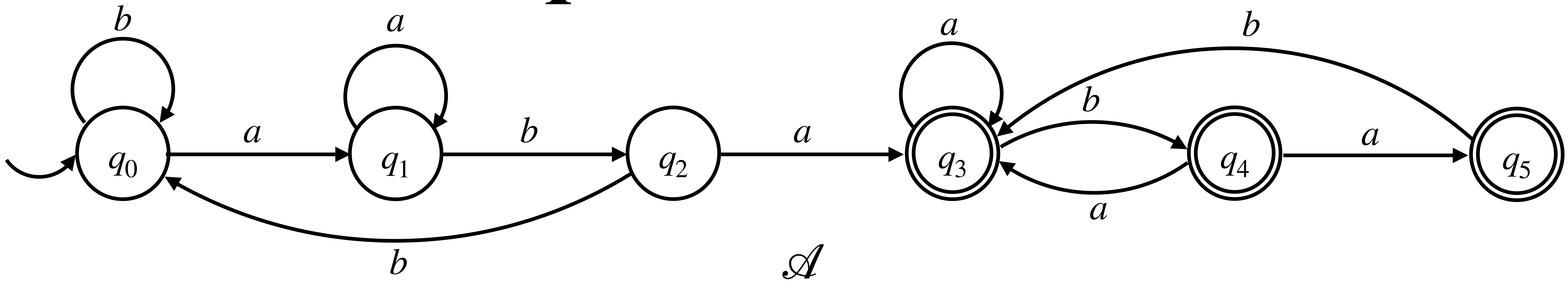


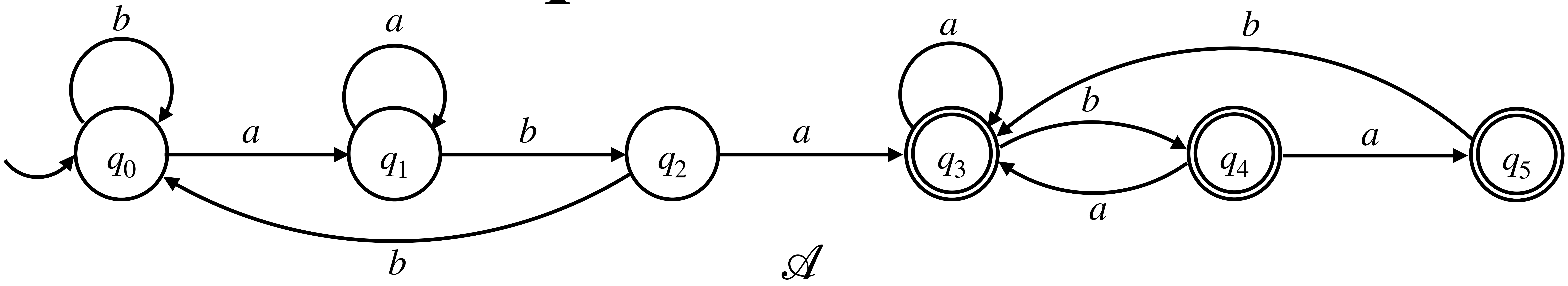
$\mathscr{A}$

Equivalent DFAs



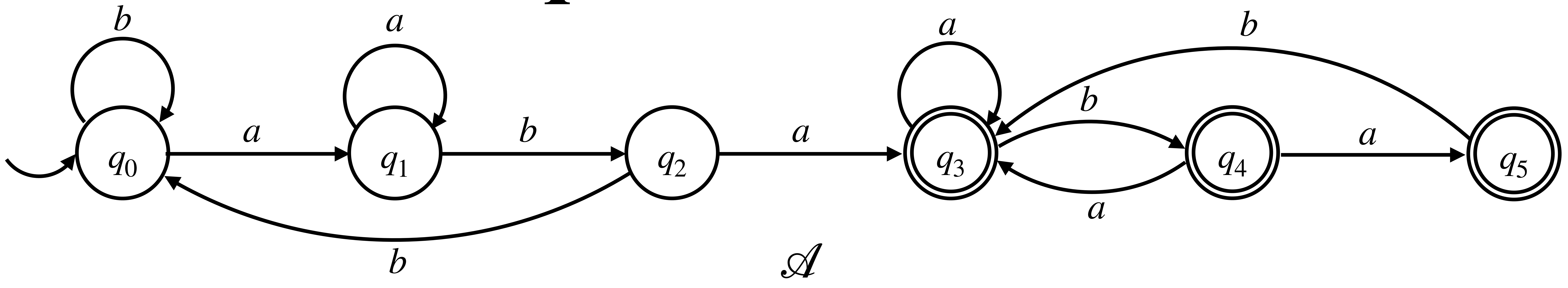
$L(\mathscr{A}) = \ ?$

Equivalent DFAs



$$L(\mathscr{A}) = \{x \in \{a, b\}^* \mid aba \text{ is a substring in } x\}$$

Equivalent DFAs



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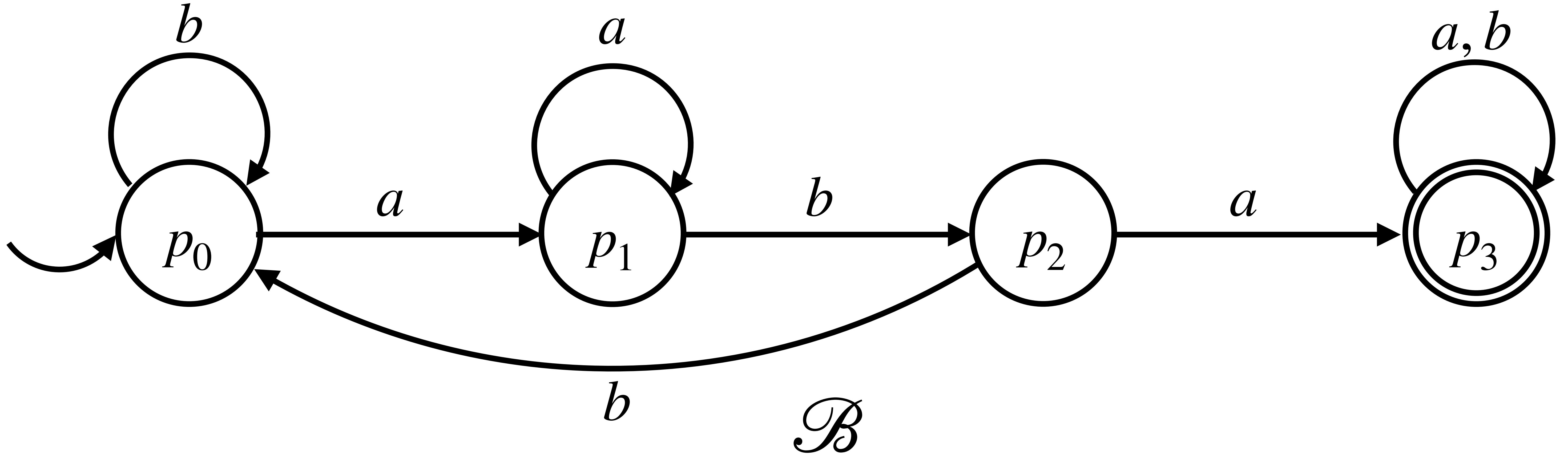

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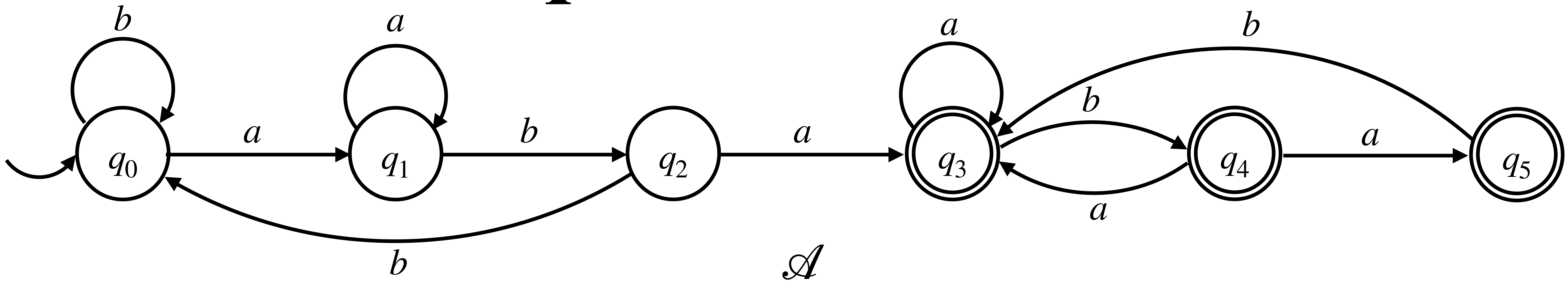

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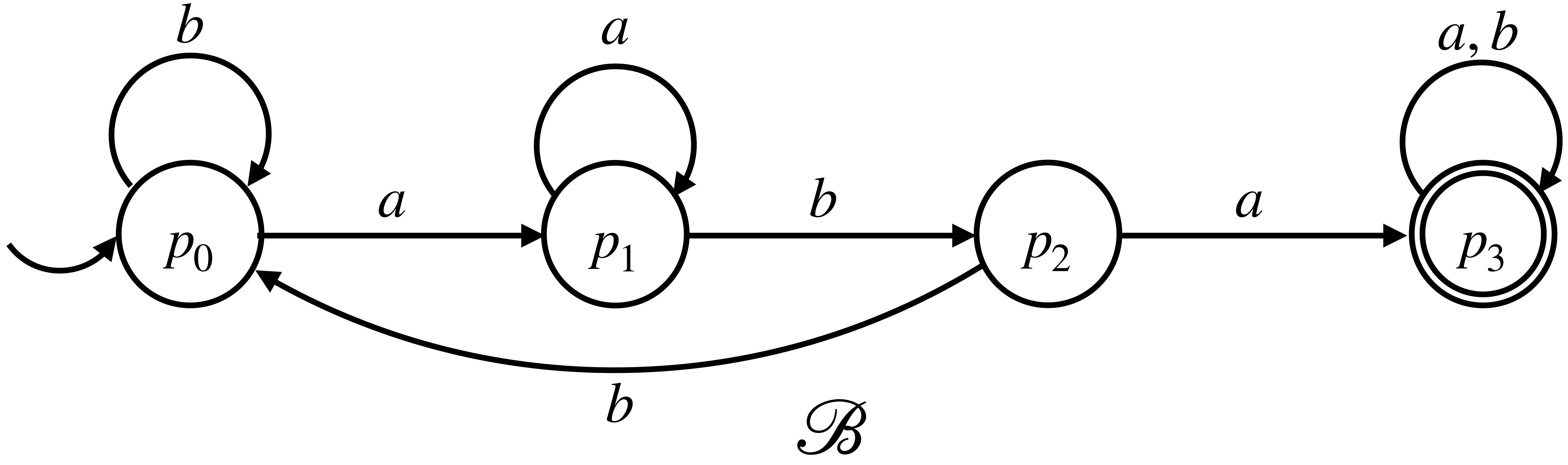

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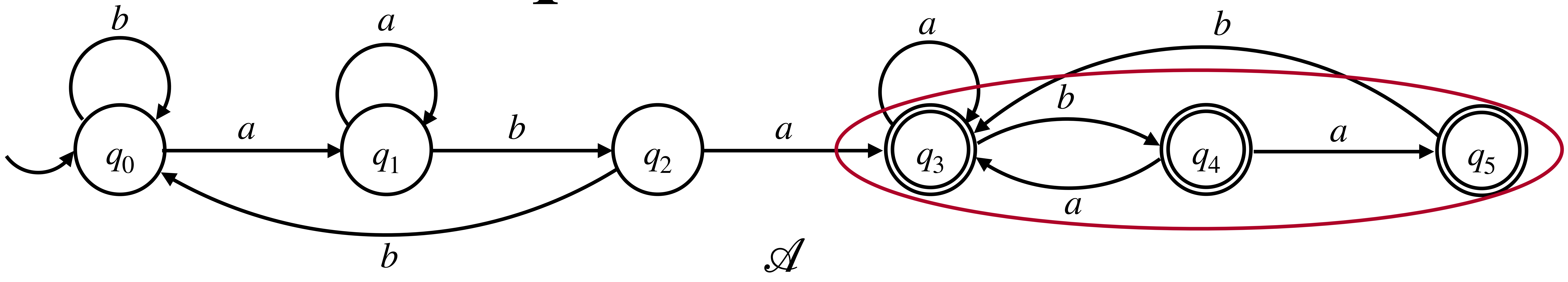

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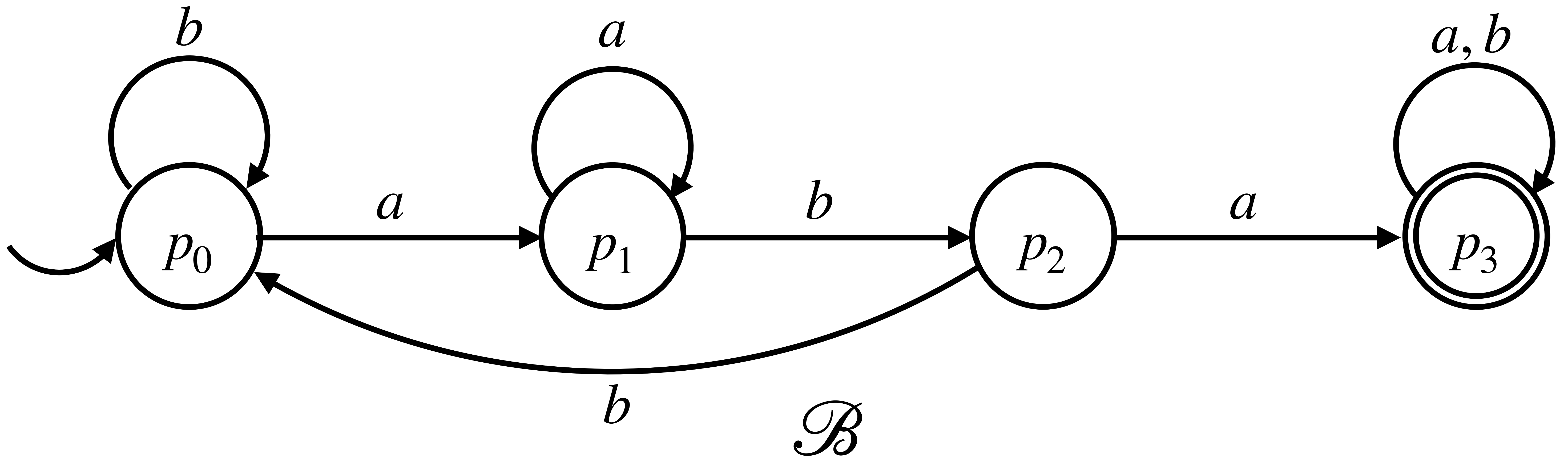
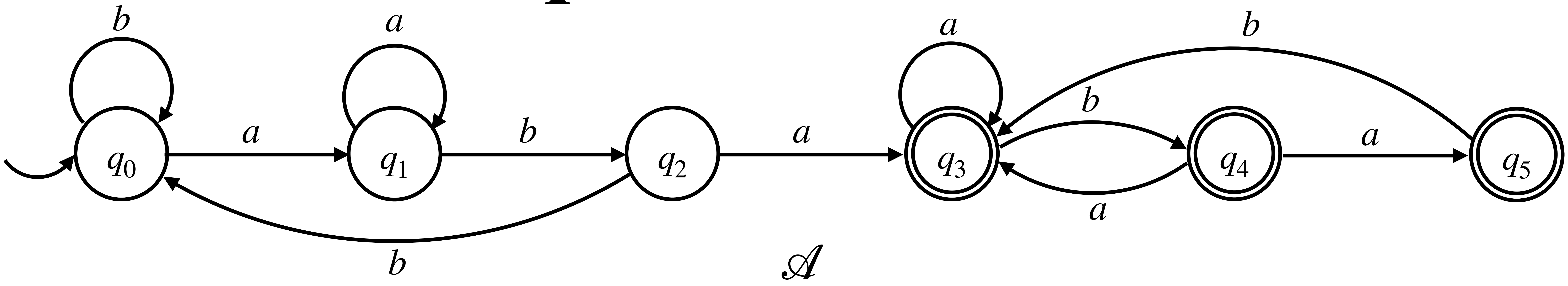


Equivalent states can be collapsed to get a DFA with fewer number of states

Equivalent DFAs



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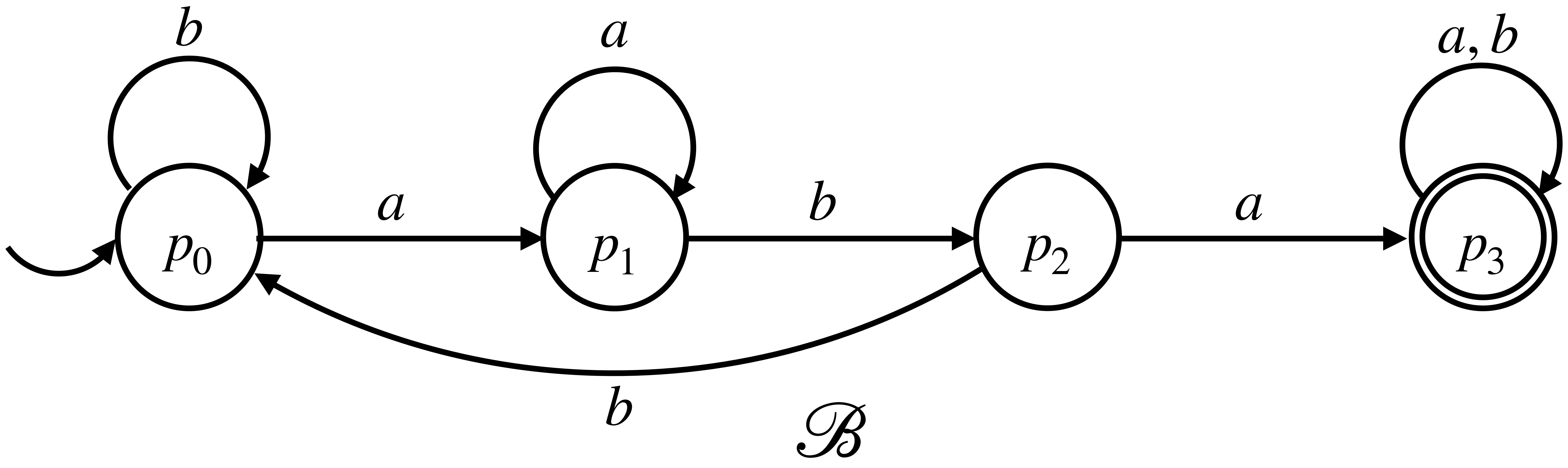
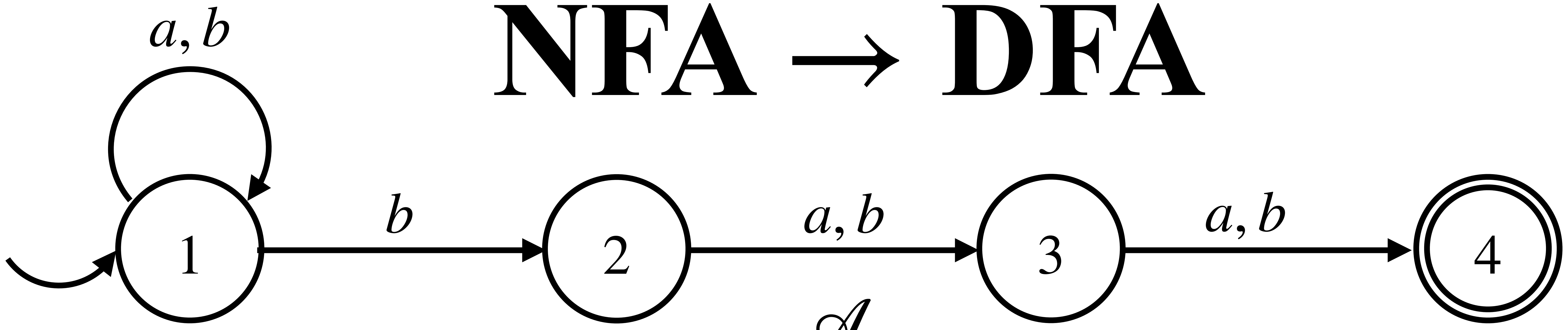


- $\mathscr{B}$ is a DFA with fewer number of states

NFA $\rightarrow$ DFA

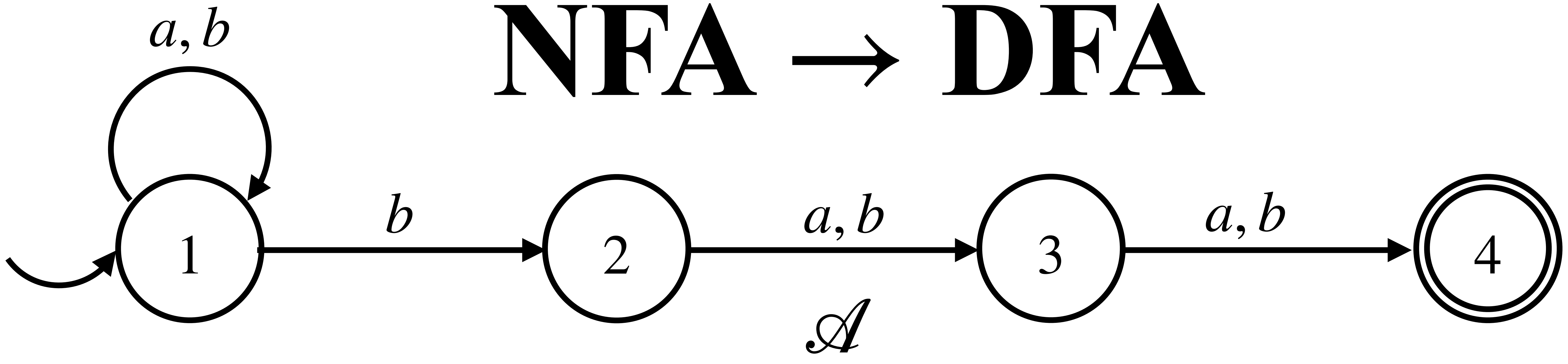
NFA → DFA



$\mathscr{A}$

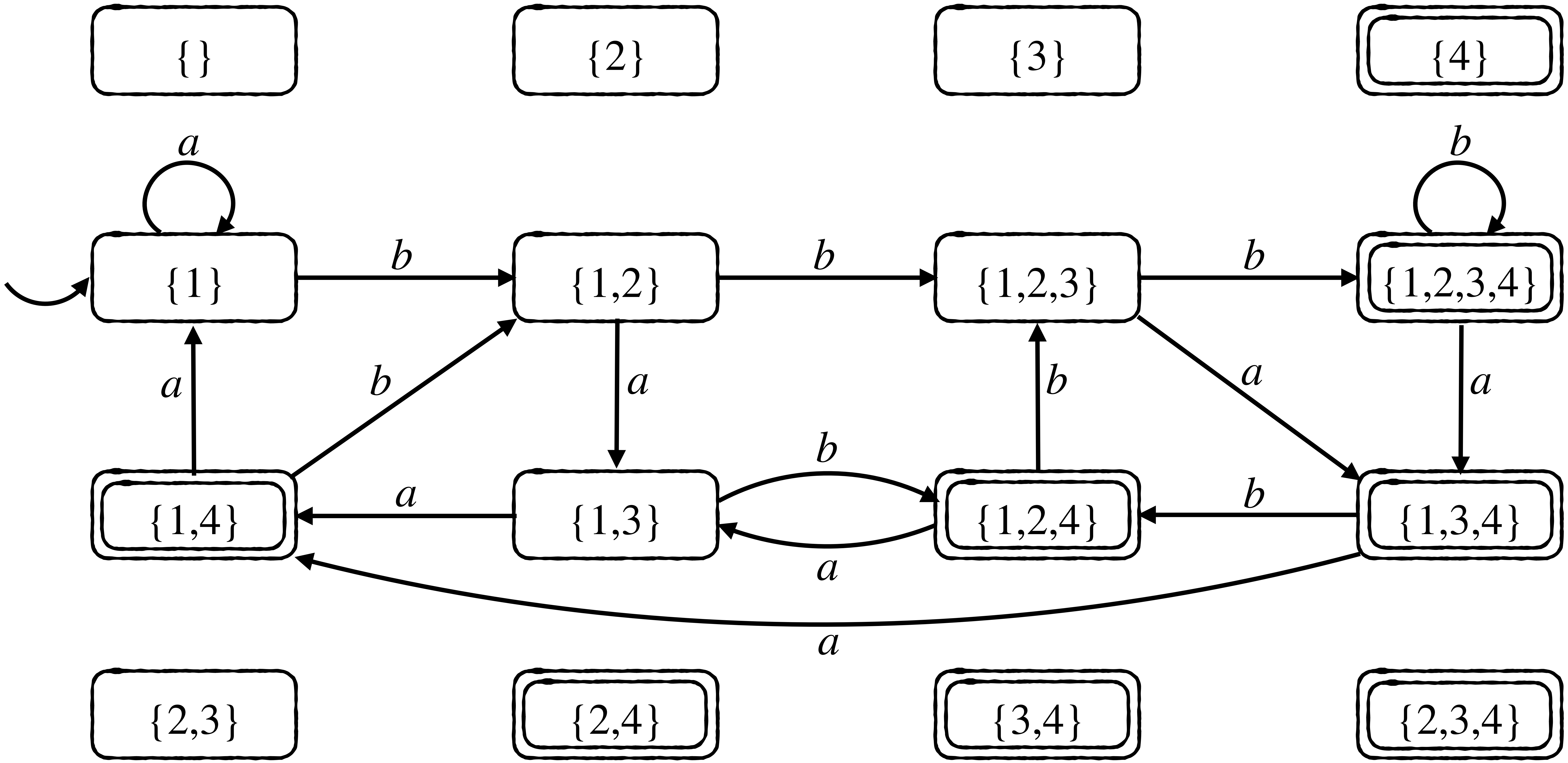
$$L(\mathscr{A}) = \{x \in \{a, b\}^* \mid \text{third last symbol in } x \text{ is } b\}$$

NFA → DFA

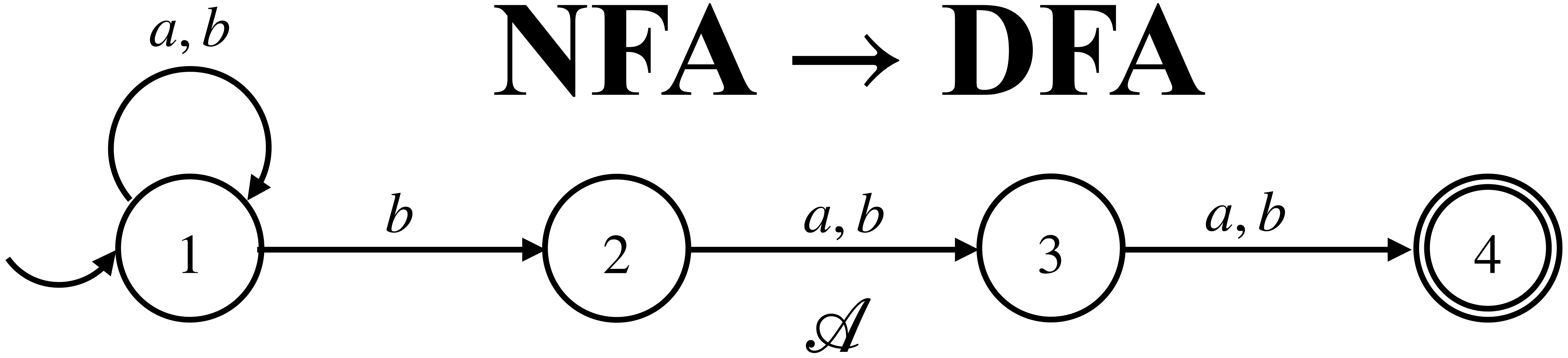


$L(\mathscr{A}) = \{x \in \{a, b\}^* \mid \text{third last symbol in } x \text{ is } b\}$

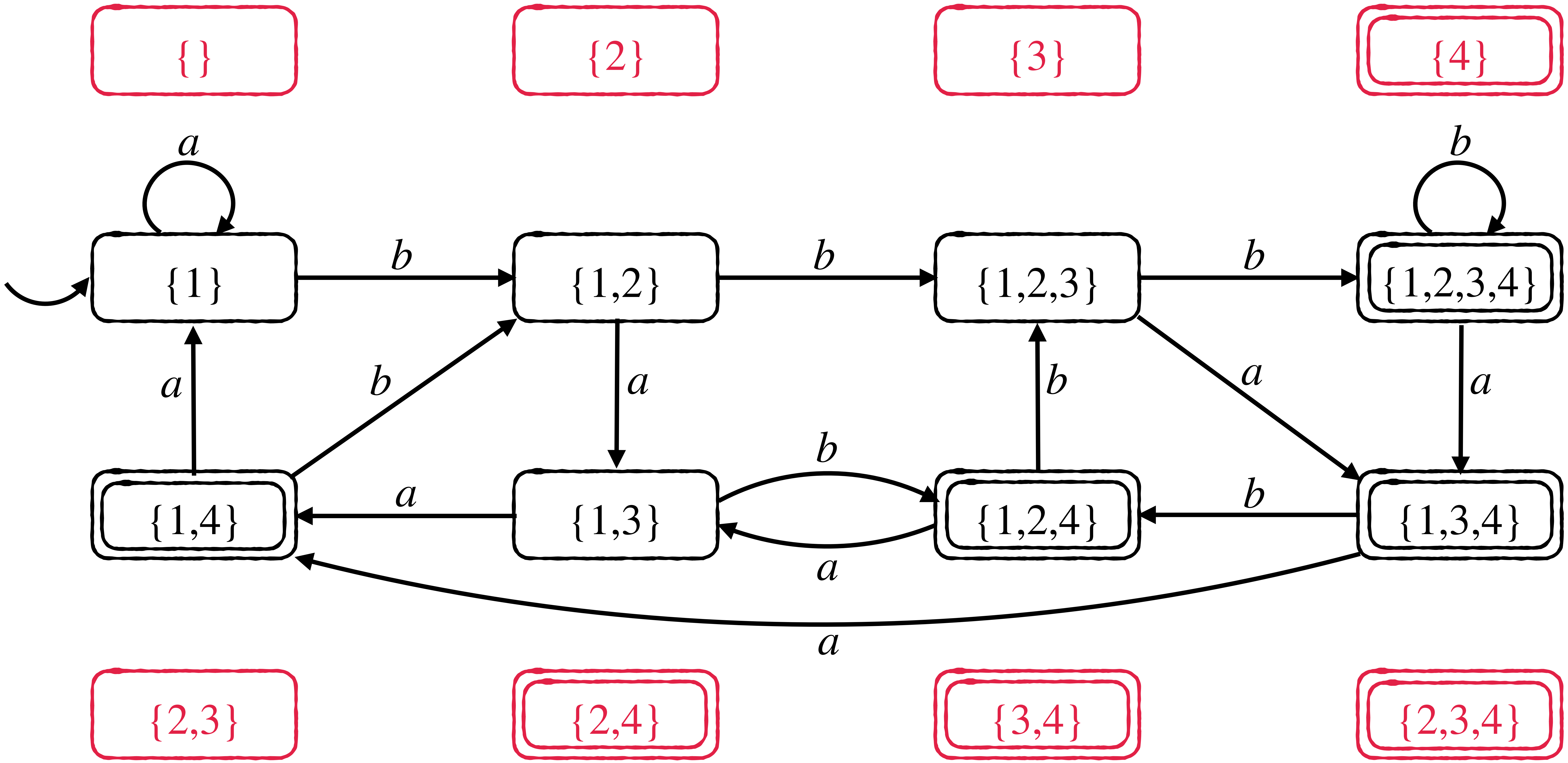
Subset construction on $\mathscr{A}$ gives the DFA:



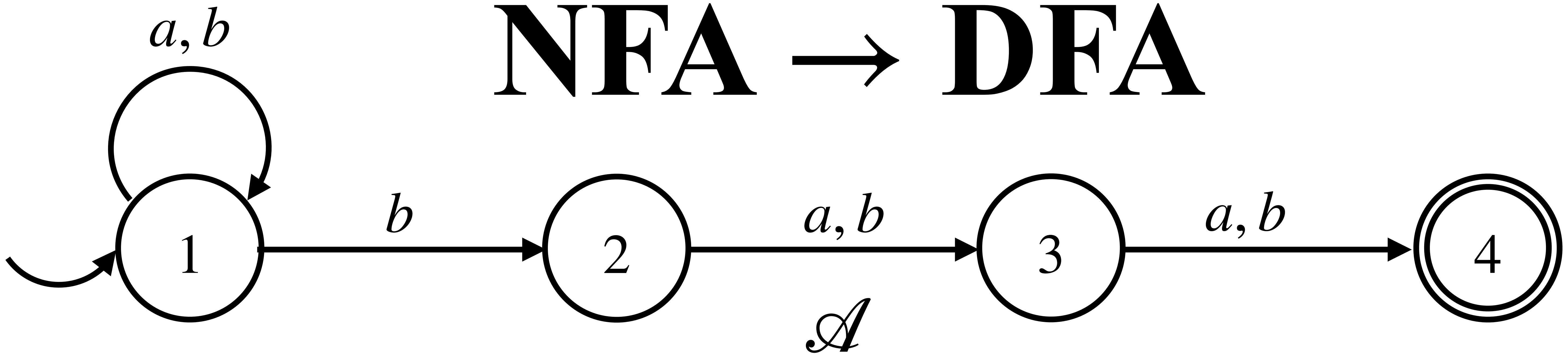
NFA → DFA



$L(\mathscr{A}) = \{x \in \{a,b\}^* \mid \text{third last symbol in } x \text{ is } b\}$ These are non reachable states in the DFA:

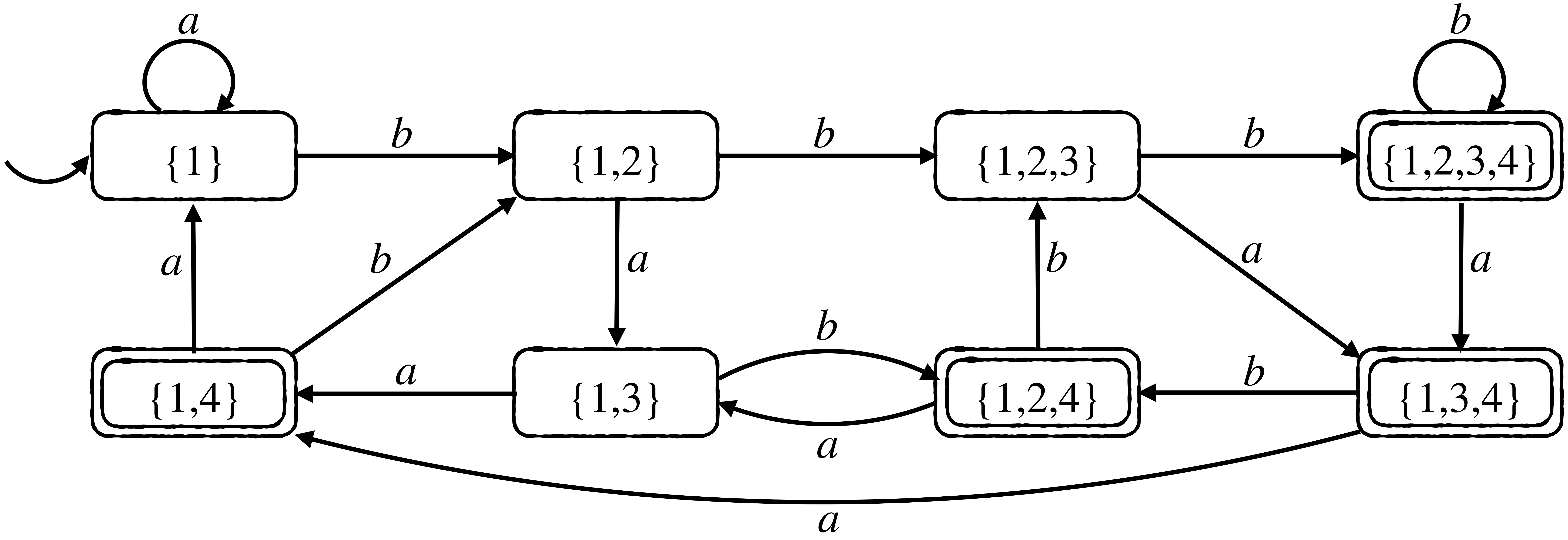


NFA → DFA

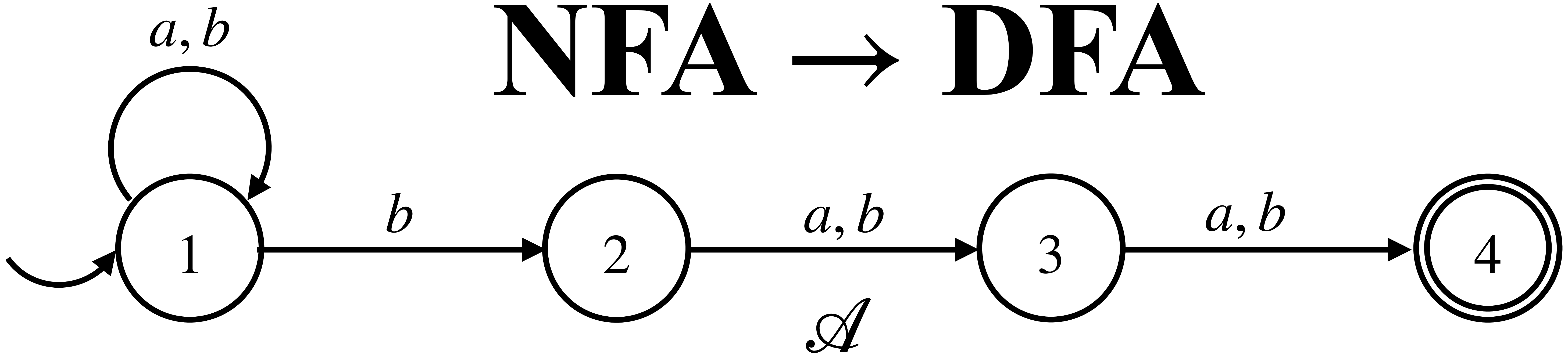


$$L(\mathscr{A}) = \{x \in \{a, b\}^* \mid \text{third last symbol in } x \text{ is } b\}$$

Removing non reachable states gives the DFA:



NFA → DFA



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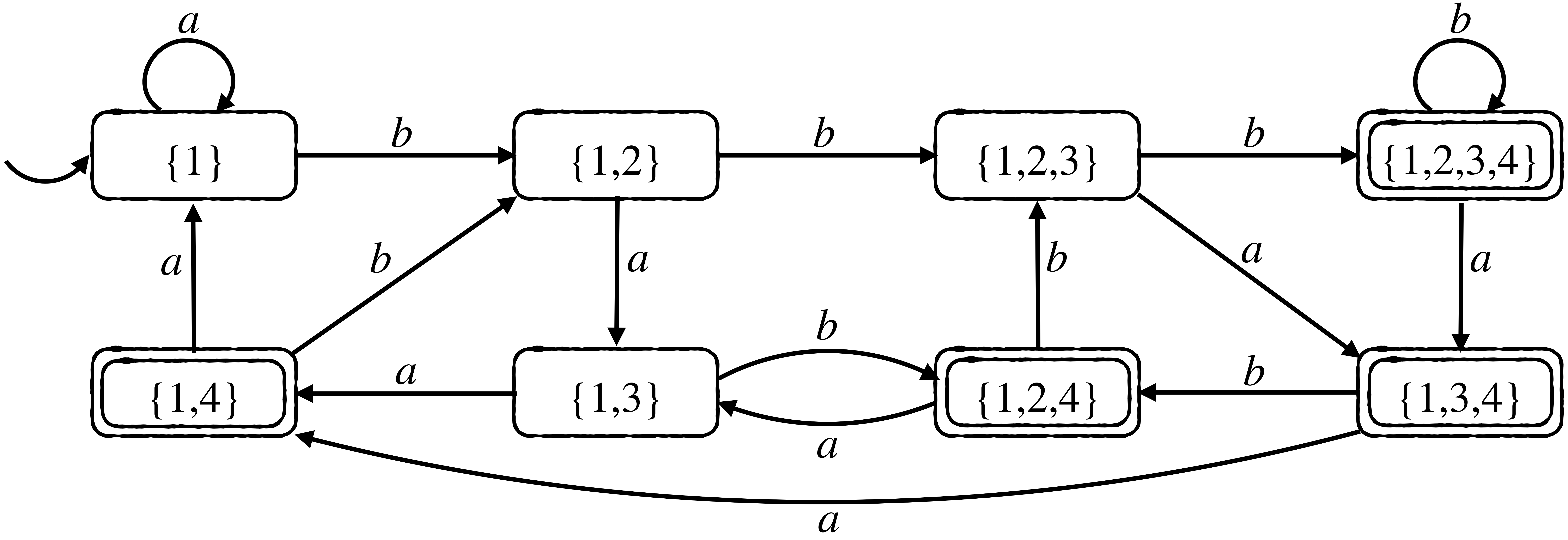

Non-reachable states can be removed to get an equivalent DFA with fewer number of states

Minimal-state DFA

- Given a DFA $\mathscr{A}$, can we automatically find the minimal-state DFA equivalent to $\mathscr{A}$?

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- Yes. $\exists$ algorithm to find the equivalent minimal-state DFA from a given DFA.
- Steps in the algorithm:
    - Remove ***non-reachable*** states if any.
    - Collapse ***equivalent*** states if any.