Equivalence Relation

CS301 Theory of Computation

Sumesh Divakaran@CSE, College of Engineering Trivandrum

Relation

Relation

S = { Romeo, Bill Gates, William, Melinda, Harry, Juliet, Ginny, Kate Middleton, Antony,
Bill Clinton,Cleopatra, Hillary}

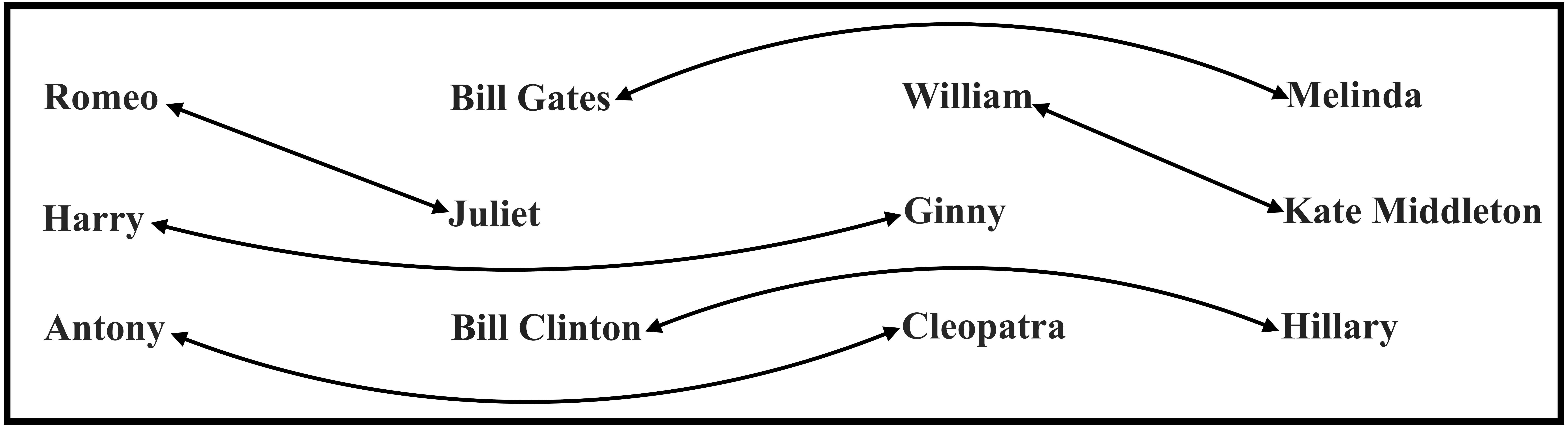
Relation

S = { Romeo, Bill Gates, William, Melinda, Harry, Juliet, Ginny, Kate Middleton, Antony, Bill Clinton,Cleopatra, Hillary}

Romeo	Bill Gates	William	Melinda
Harry	Juliet	Ginny	Kate Middleton
Antony	Bill Clinton	Cleopatra	Hillary

Relation

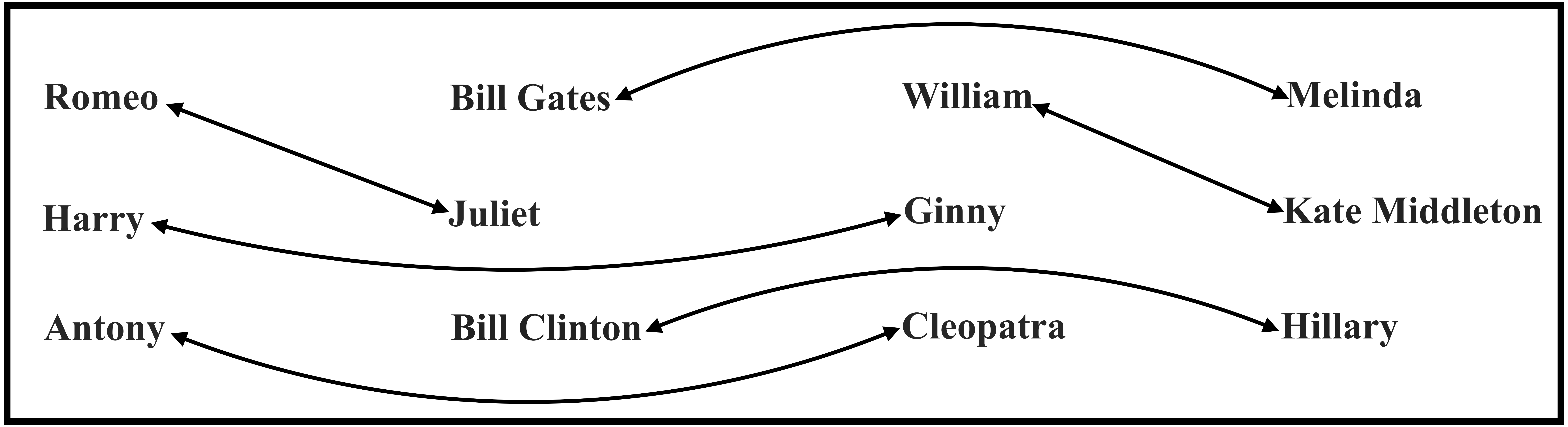
S = { Romeo, Bill Gates, William, Melinda, Harry, Juliet, Ginny, Kate Middleton, Antony,
Bill Clinton,Cleopatra, Hillary}



Example relation *spouse* on S

Relation

S = { Romeo, Bill Gates, William, Melinda, Harry, Juliet, Ginny, Kate Middleton, Antony, Bill Clinton,Cleopatra, Hillary}



Example relation *spouse* on S

spouse = { (Romeo, Juliet), (Bill Gates, Melinda), (William, Kate Middleton) (Harry, Ginny), (Antony, Cleopatra), (Bill Clinton, Hillary) }

Relation

Relation

S = { 1, 2, 3, 4 }

1 **2** **3** **4**

Relation

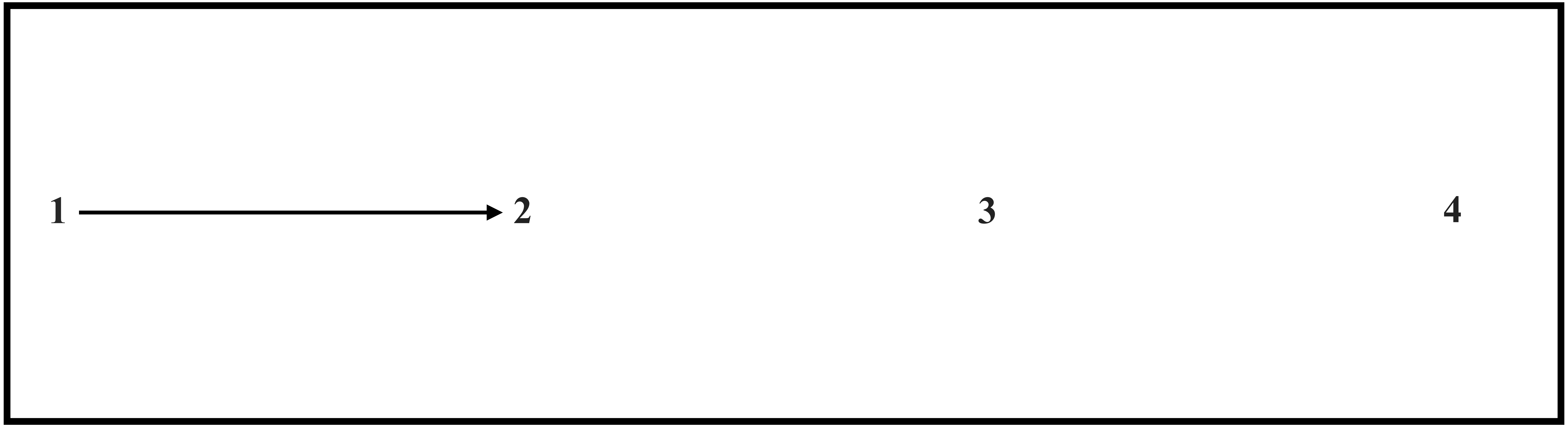
S = { 1, 2, 3, 4 }

1 2 3 4

Example relation *less_than* on S

Relation

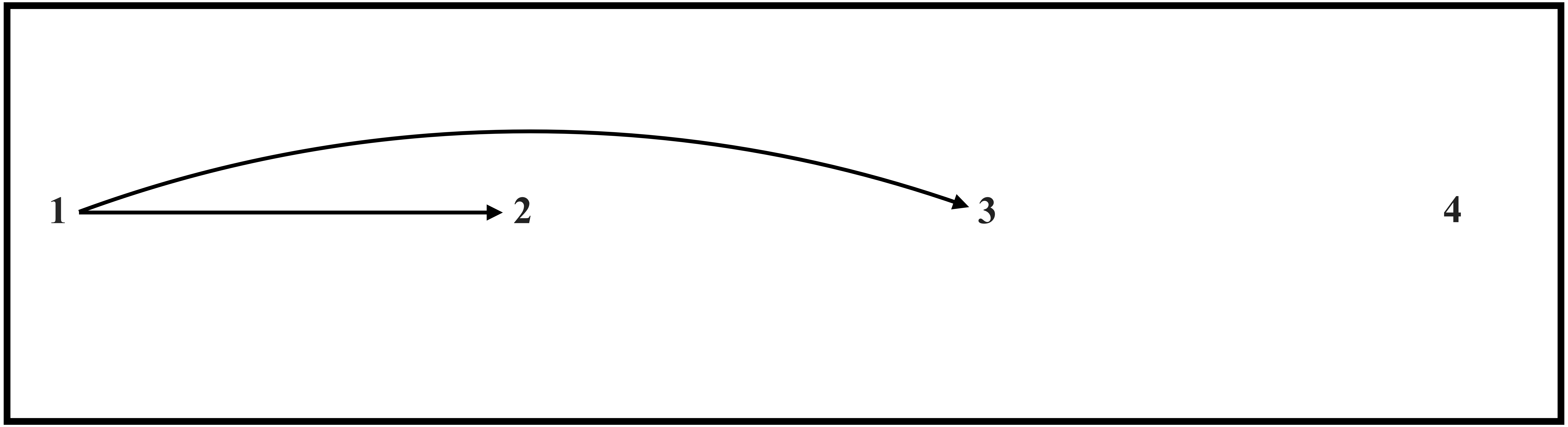
S = { 1, 2, 3, 4 }



Example relation *less_than* on S

Relation

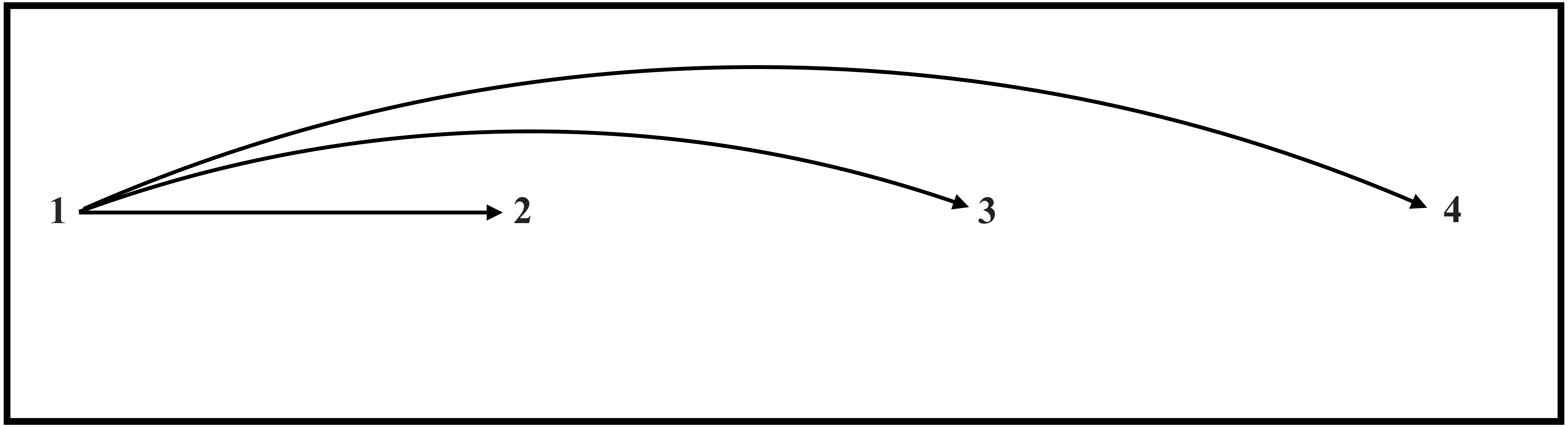
S = { 1, 2, 3, 4 }



Example relation *less_than* on S

Relation

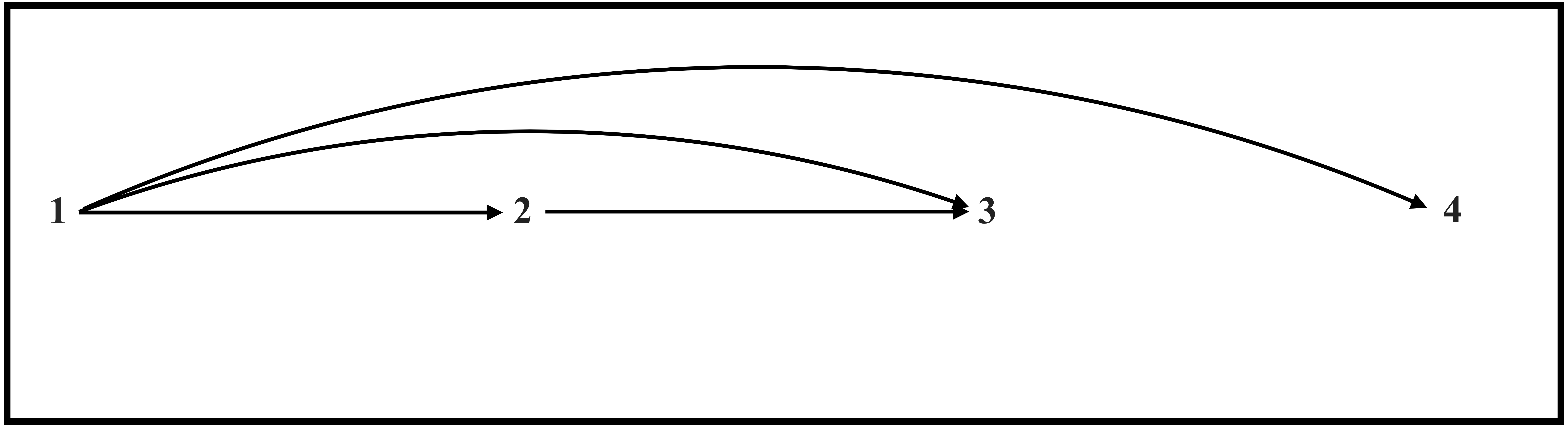
S = { 1, 2, 3, 4 }



Example relation *less_than* on S

Relation

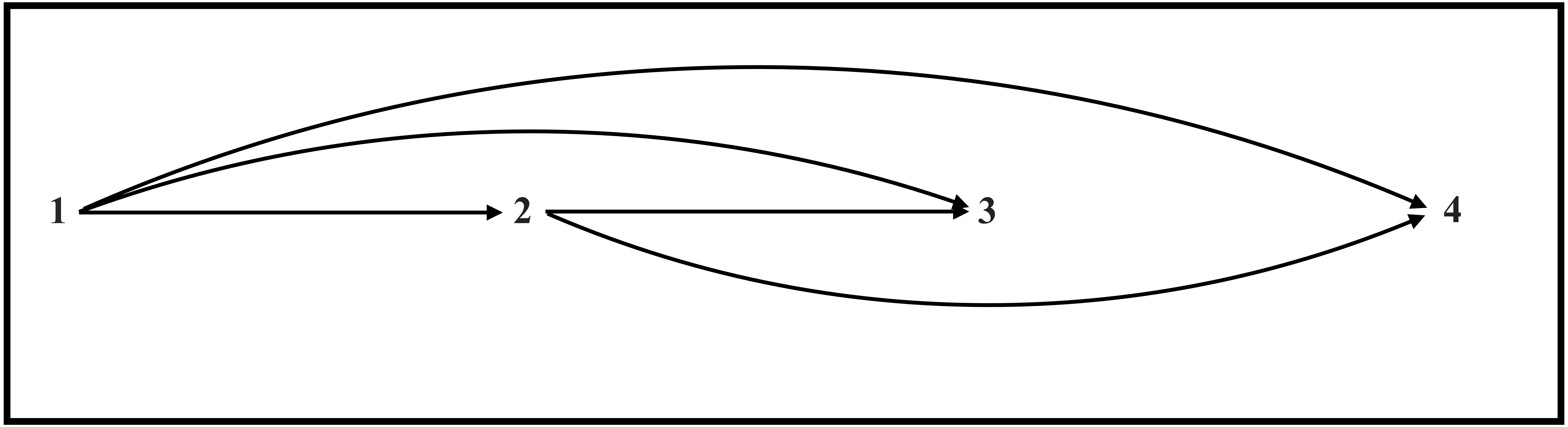
S = { 1, 2, 3, 4 }



Example relation *less_than* on S

Relation

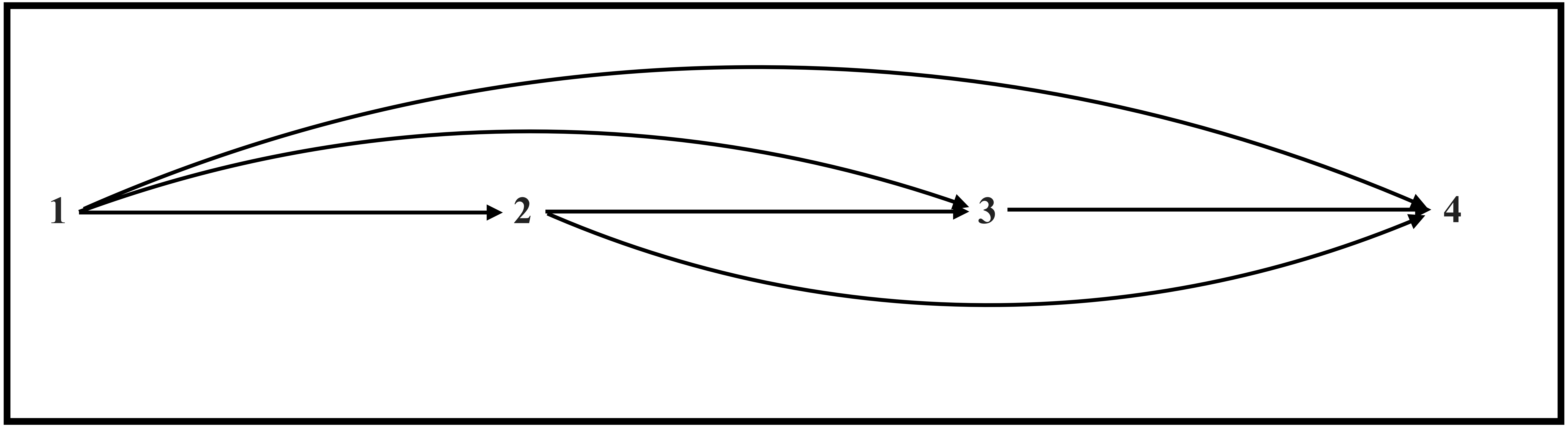
S = { 1, 2, 3, 4 }



Example relation *less_than* on S

Relation

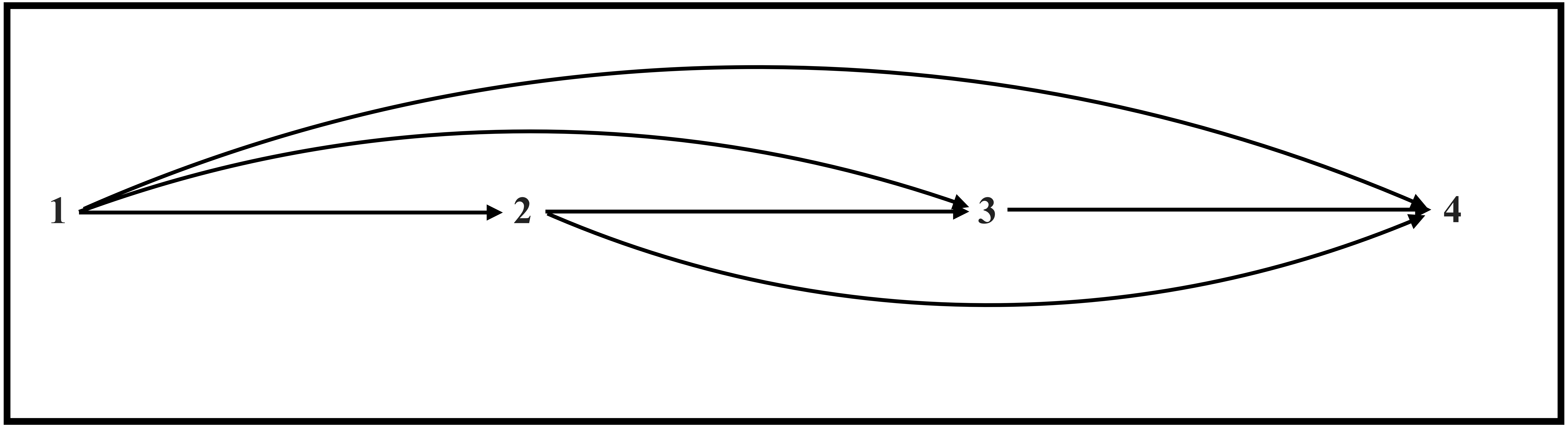
S = { 1, 2, 3, 4 }



Example relation *less_than* on S

Relation

S = { 1, 2, 3, 4 }



Example relation *less_than* on S

less_than = { (1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4) }

Relation

Relation

Cartesian product

Let A and B be two sets. Then, their cartesian product is defines as:

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

Relation

Cartesian product

Let A and B be two sets. Then, their cartesian product is defines as:

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

Cartesian product - Example

$A = \{1,2,3\}, B = \{x, y\}$:

$$A \times B = \{(1,x), (2,x), (3,x), (1,y), (2,y), (3,y)\}$$

Relation

Cartesian product

Let A and B be two sets. Then, their cartesian product is defines as:

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

Cartesian product - Example

$A = \{1,2,3\}, B = \{x, y\}$:

$$A \times B = \{(1,x), (2,x), (3,x), (1,y), (2,y), (3,y)\}$$

Relation

Let A and B be two sets. Then, a binary relation R between A and B is any subset of $A \times B$.

Relation

Cartesian product

Let A and B be two sets. Then, their cartesian product is defines as:

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

Cartesian product - Example

$A = \{1,2,3\}, B = \{x, y\}$:

$$A \times B = \{(1,x), (2,x), (3,x), (1,y), (2,y), (3,y)\}$$

Relation

Let A and B be two sets. Then, a binary relation R between A and B is any subset of $A \times B$.

Relation - Example

$A = \{1,2,3\}, B = \{x, y\}$:

$$R_1 = \{(1,x), (3,y)\}$$

Relation

Cartesian product

Let A and B be two sets. Then, their cartesian product is defines as:

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

Cartesian product - Example

$A = \{1,2,3\}, B = \{x, y\}$:

$$A \times B = \{(1,x), (2,x), (3,x), (1,y), (2,y), (3,y)\}$$

Relation

Let A and B be two sets. Then, a binary relation R between A and B is any subset of $A \times B$.

Relation - Example

$A = \{1,2,3\}, B = \{x, y\}$:

$$R_1 = \{(1,x), (3,y)\} \qquad R_2 = \{(3,x)\}$$

Relation

Cartesian product

Let A and B be two sets. Then, their cartesian product is defines as:

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

Cartesian product - Example

$A = \{1,2,3\}, B = \{x, y\}$:

$$A \times B = \{(1,x), (2,x), (3,x), (1,y), (2,y), (3,y)\}$$

Relation

Let A and B be two sets. Then, a binary relation R between A and B is any subset of $A \times B$.

Relation - Example

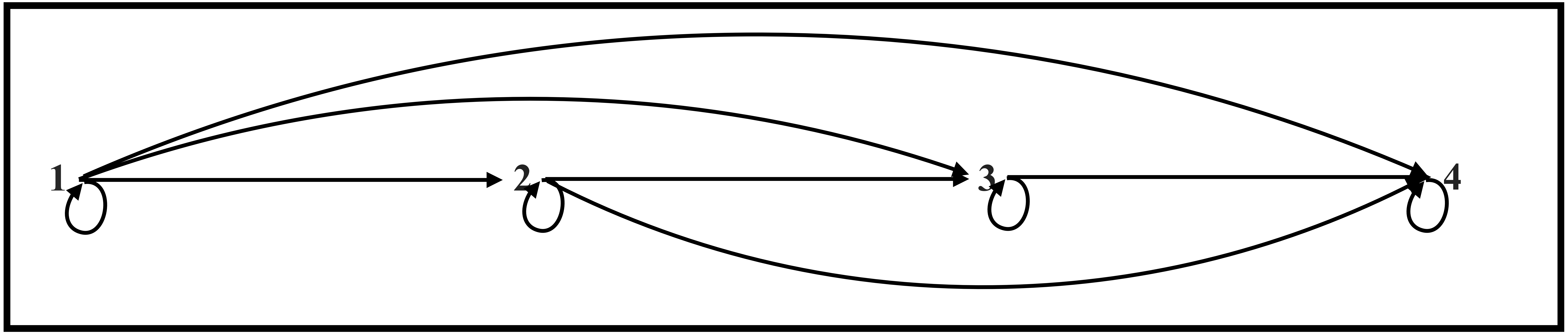
$A = \{1,2,3\}, B = \{x, y\}$:

$R_1 = \{(1,x), (3,y)\}$ $R_2 = \{(3,x)\}$ $R_3 = \{\}$

Reflexive Relation

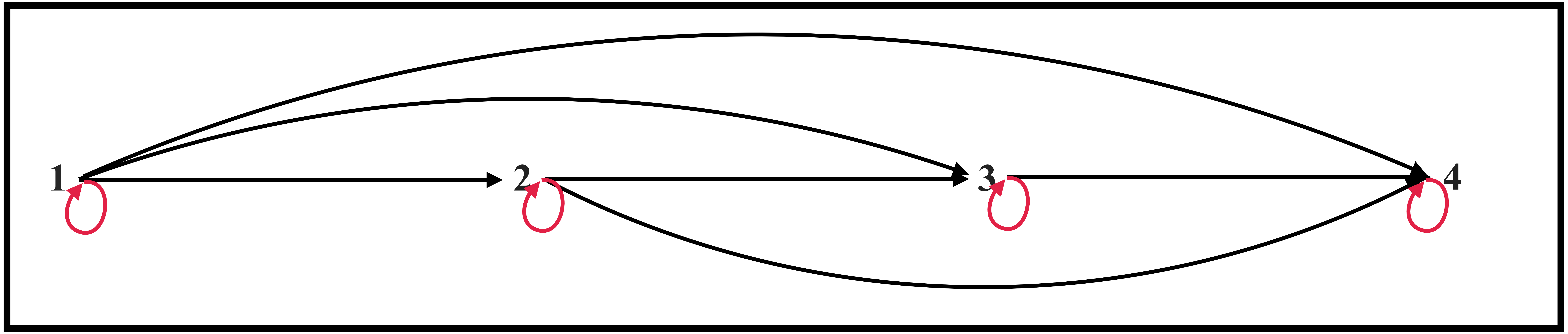
1	2	3	4

Reflexive Relation



≤: {(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)}

Reflexive Relation

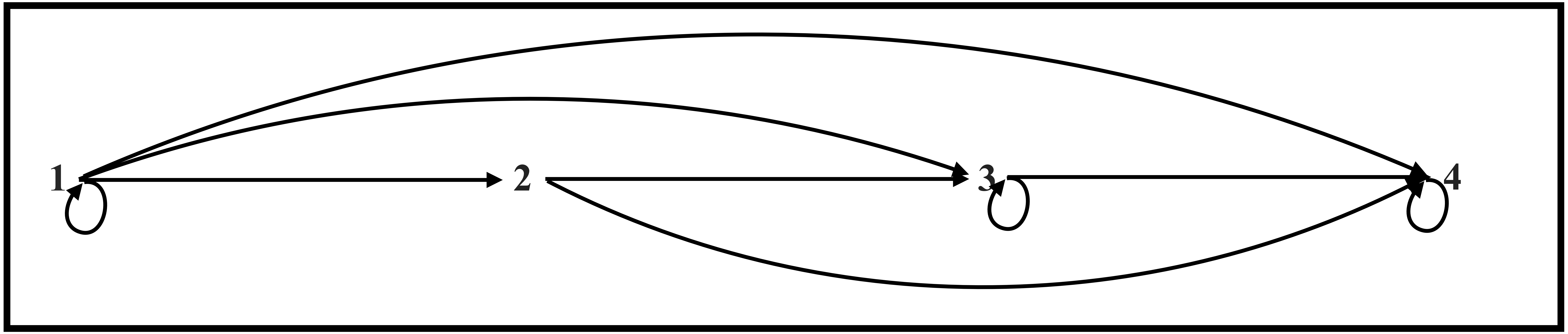


$\leq$: {(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)}

Reflexive Relation

A relation $R \subseteq A \times A$ is said to be reflexive iff $\forall a \in A \mid (a, a) \in R$.

Reflexive Relation



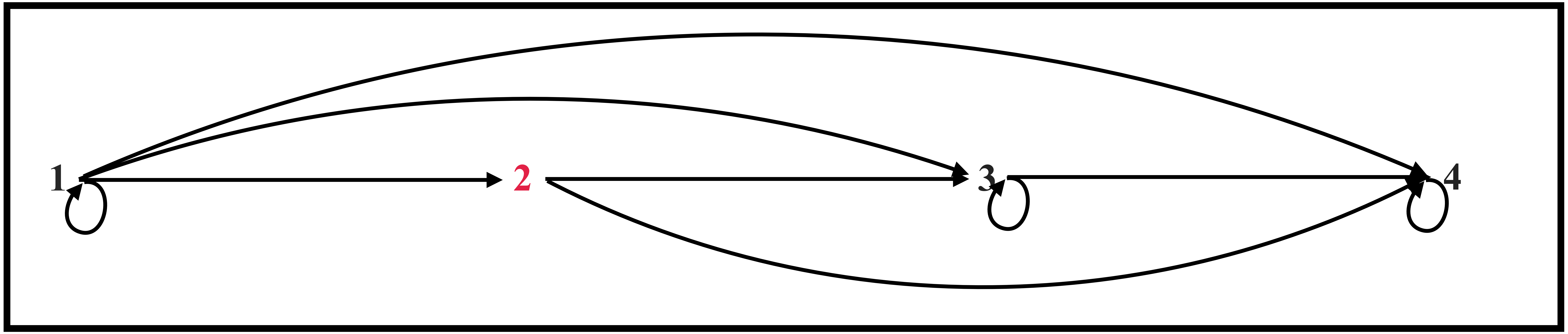
Reflexive Relation

A relation $R \subseteq A \times A$ is said to be reflexive iff $\forall a \in A \mid (a, a) \in R$.

Examples:

$R_1 = \{(1,1), (1,2), (1,3), (1,4), (2,3), (2,4), (3,3), (3,4), (4,4)\}$

Reflexive Relation



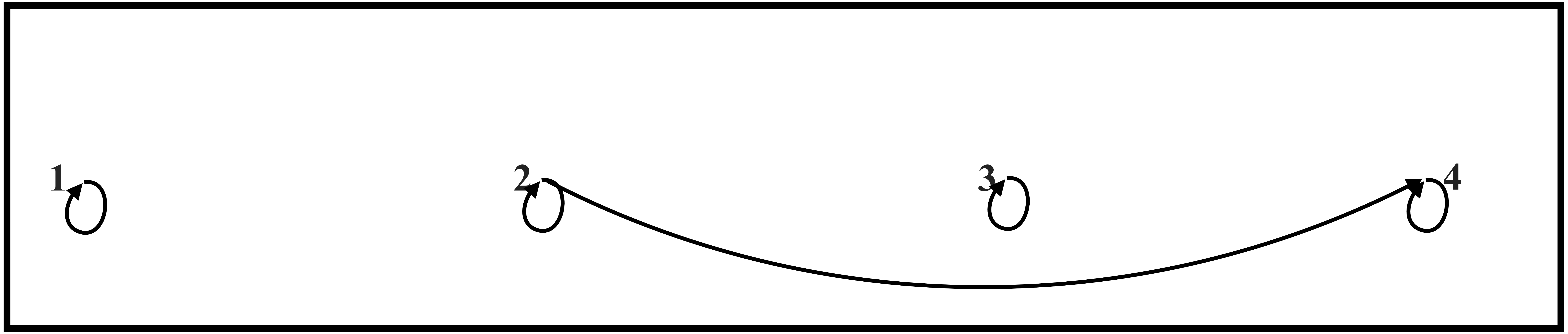
Reflexive Relation

A relation $R \subseteq A \times A$ is said to be reflexive iff $\forall a \in A \mid (a, a) \in R$.

Examples:

$R_1 = \{(1,1), (1,2), (1,3), (1,4), (2,3), (2,4), (3,3), (3,4), (4,4)\}$ ✕

Reflexive Relation



Reflexive Relation

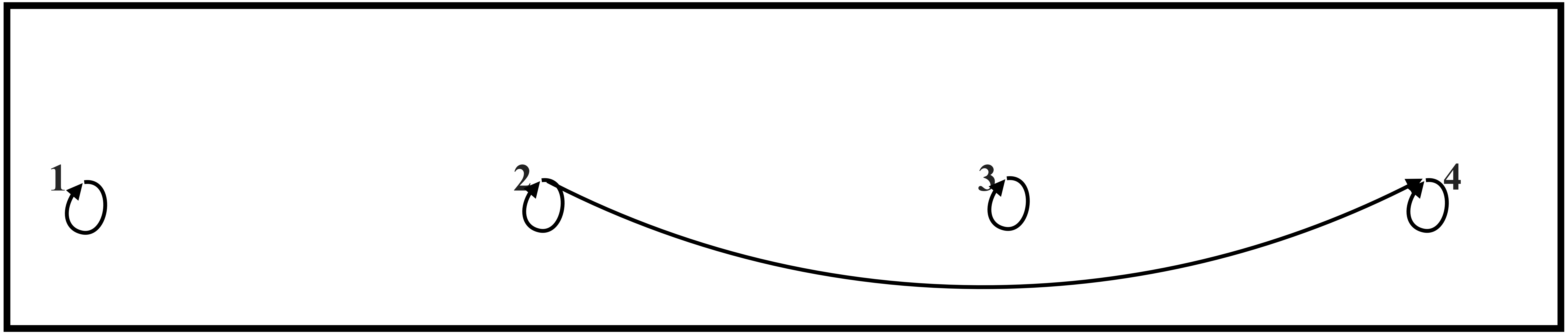
A relation $R \subseteq A \times A$ is said to be reflexive iff $\forall a \in A \mid (a, a) \in R$.

Examples:

$R_1 = \{(1,1), (1,2), (1,3), (1,4), (2,3), (2,4), (3,3), (3,4), (4,4)\}$ ✗

$R_2 = \{(1,1), (2,2), (2,4), (3,3), (3,4), (4,4)\}$

Reflexive Relation



Reflexive Relation

A relation $R \subseteq A \times A$ is said to be reflexive iff $\forall a \in A \mid (a, a) \in R$.

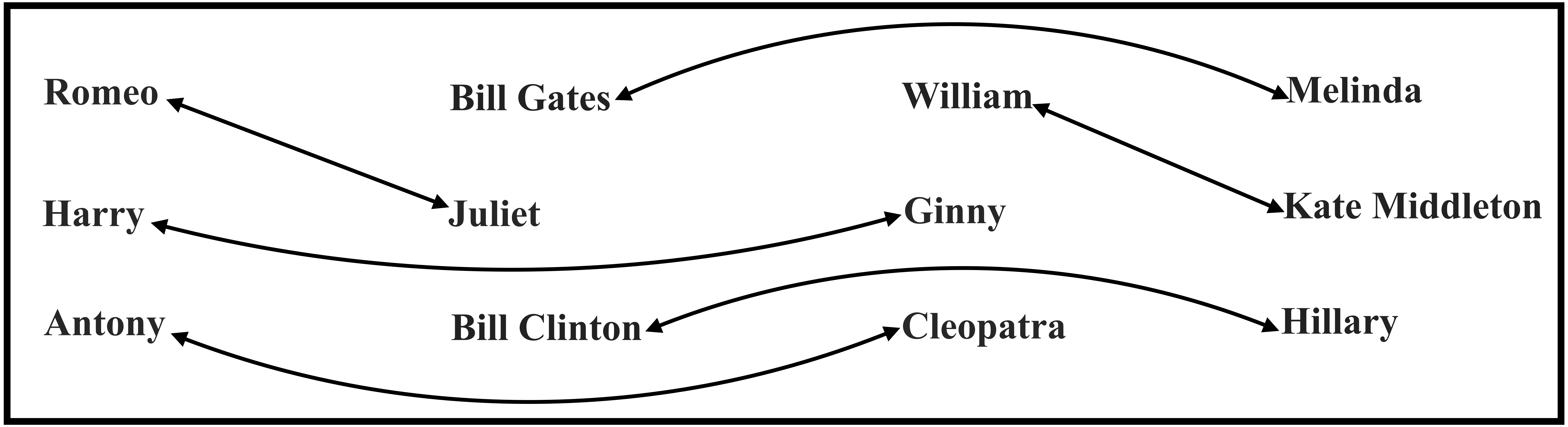
Examples:

$R_1 = \{(1,1), (1,2), (1,3), (1,4), (2,3), (2,4), (3,3), (3,4), (4,4)\}$ ✗

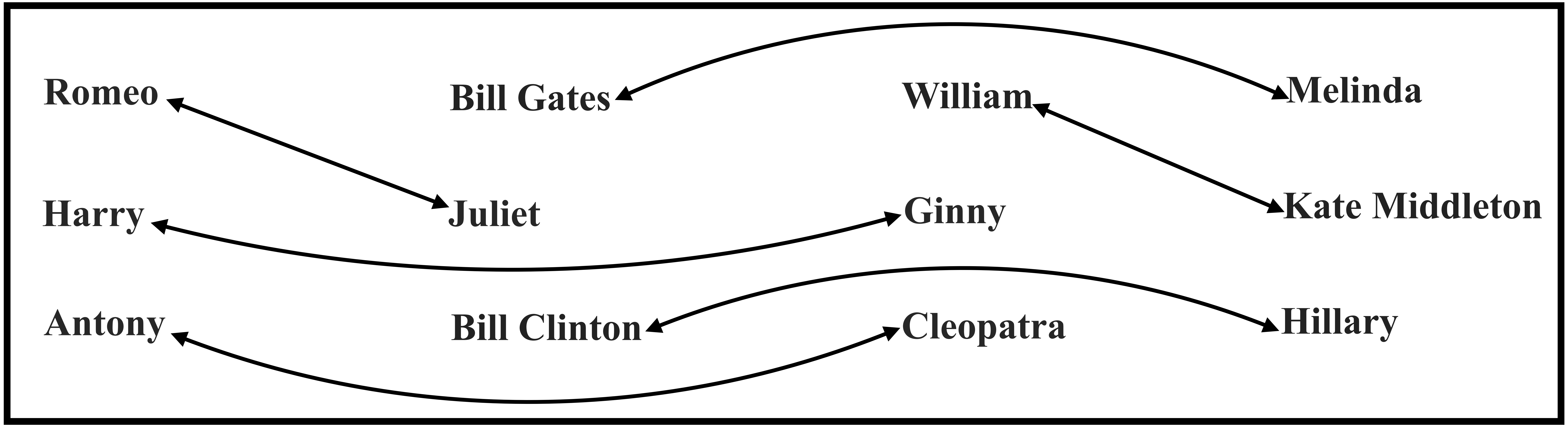
$R_2 = \{(1,1), (2,2), (2,4), (3,3), (3,4), (4,4)\}$ ✓

Symmetric Relation

Symmetric Relation



Symmetric Relation

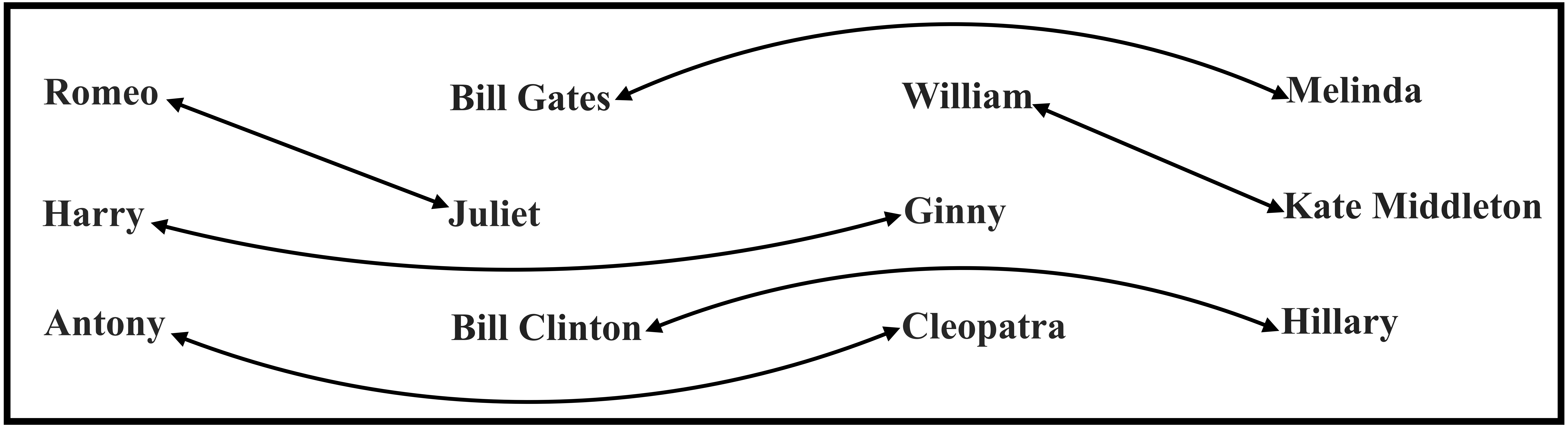


Symmetric Relation

A relation $R \subseteq A \times A$ is said to be symmetric if:

$$\forall (a, b) \in A \times A \mid (a, b) \in R \implies (b, a) \in R.$$

Symmetric Relation



Symmetric Relation

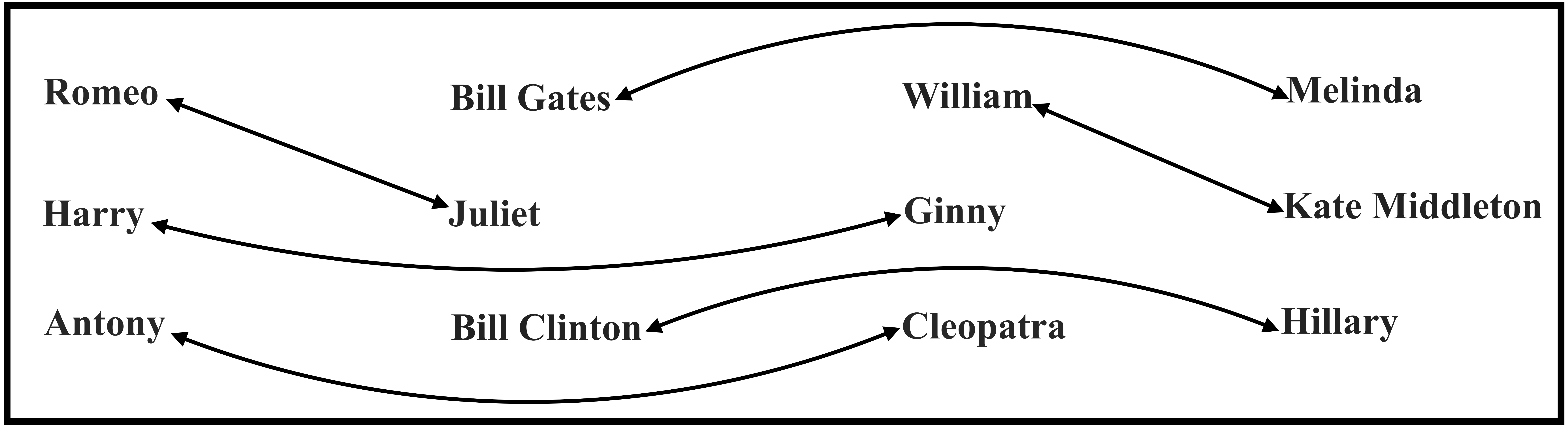
A relation $R \subseteq A \times A$ is said to be symmetric if:

$$\forall (a, b) \in A \times A \mid (a, b) \in R \implies (b, a) \in R.$$

Examples:

$\leq$ **on** $\mathbb{N}$

Symmetric Relation



Symmetric Relation

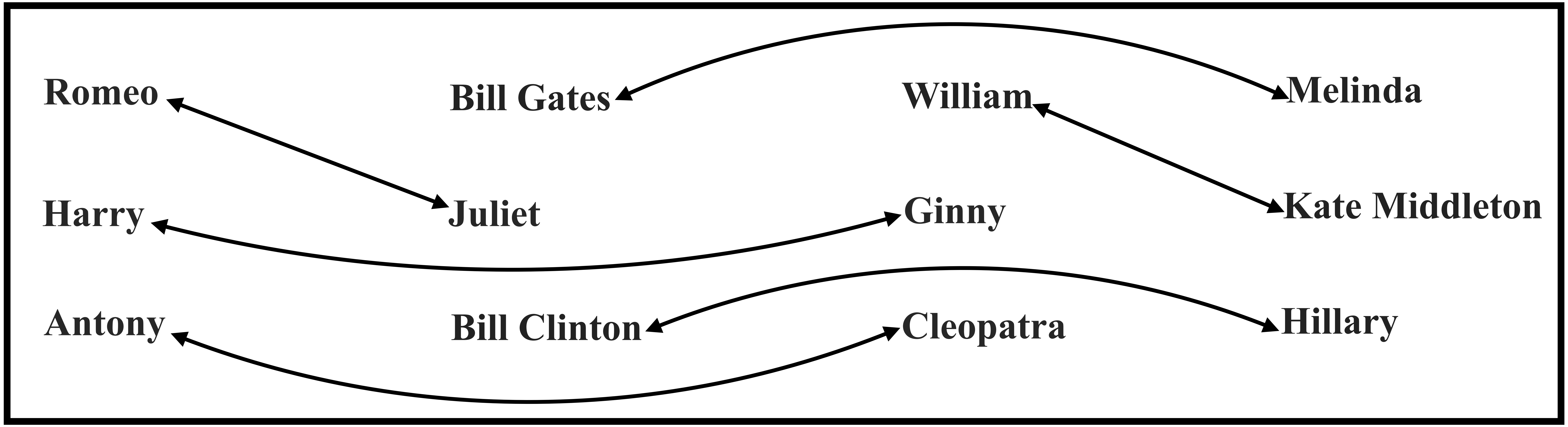
A relation $R \subseteq A \times A$ is said to be symmetric if:

$$\forall (a, b) \in A \times A \mid (a, b) \in R \implies (b, a) \in R.$$

Examples:

$\leq$ **on** $\mathbb{N}$

Symmetric Relation



Symmetric Relation

A relation $R \subseteq A \times A$ is said to be symmetric if:

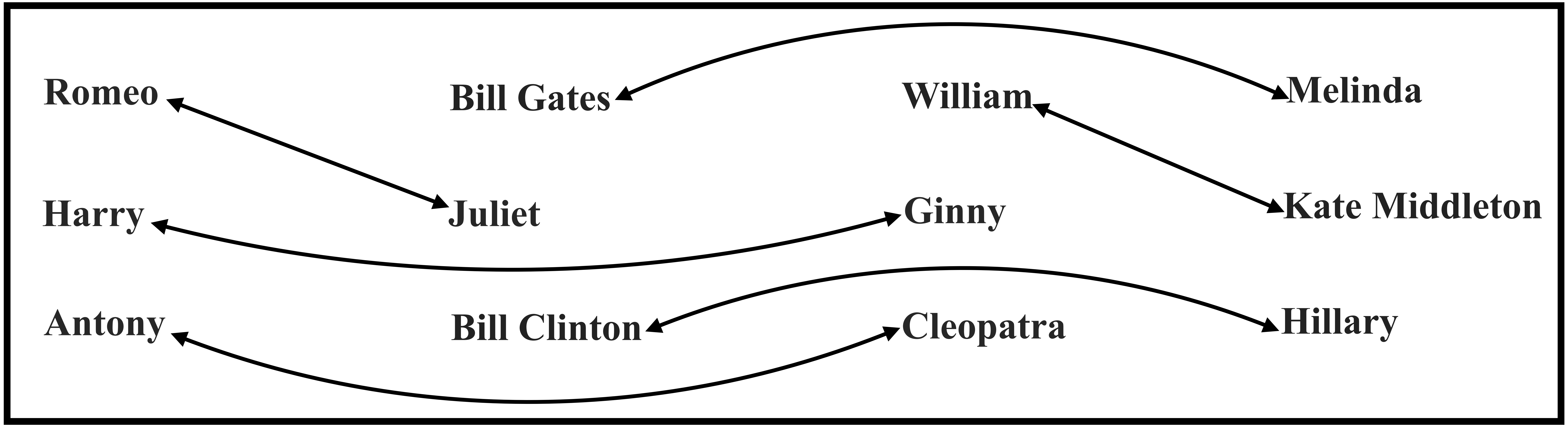
$$\forall (a,b) \in A \times A \mid (a,b) \in R \implies (b,a) \in R.$$

Examples:

$\leq$ **on** $\mathbb{N}$

$=$ **on** $\mathbb{N}$

Symmetric Relation



Symmetric Relation

A relation $R \subseteq A \times A$ is said to be symmetric if:

$$\forall (a,b) \in A \times A \mid (a,b) \in R \implies (b,a) \in R.$$

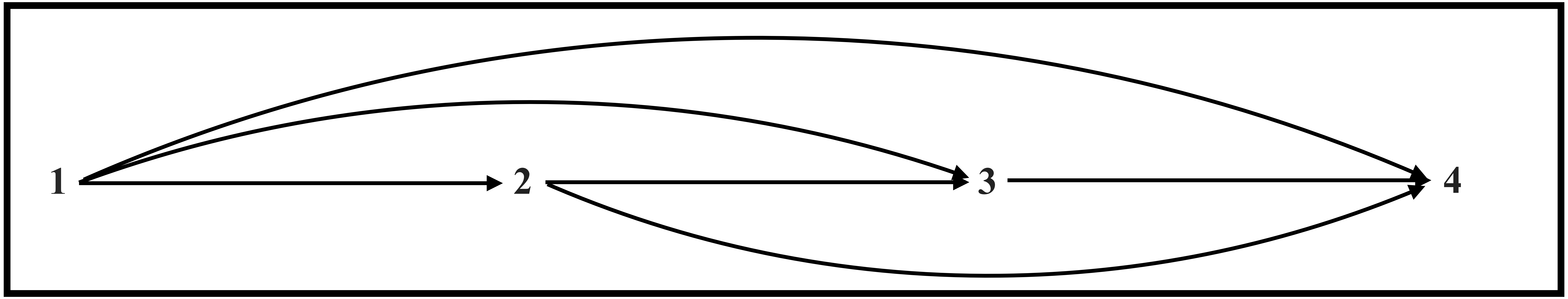
Examples:

$\leq$ **on** $\mathbb{N}$

$=$ **on** $\mathbb{N}$

Transitive Relation

Transitive Relation

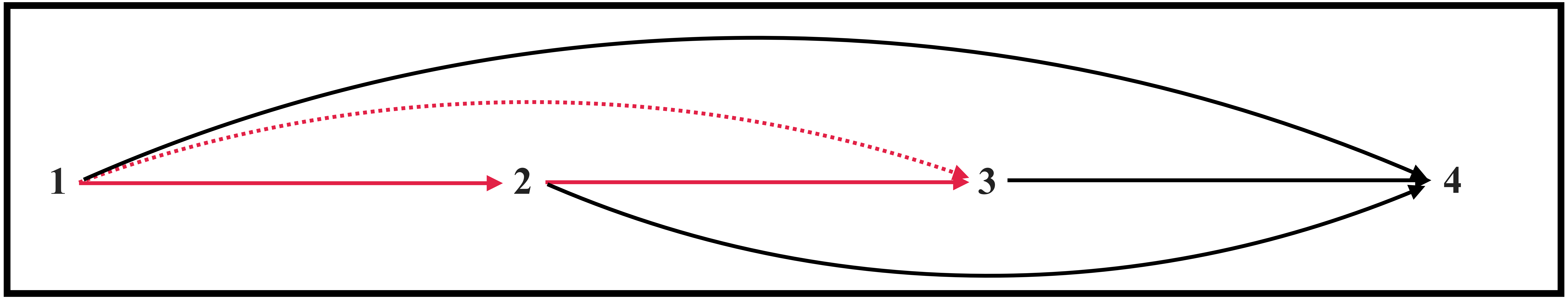


Transitive Relation

A relation $R \subseteq A \times A$ is said to be transitive if:

$$\forall (a, b) \in A \times A, (b, c) \in A \times A \mid (a, b) \in R \text{ and } (b, c) \in R \implies (a, c) \in R.$$

Transitive Relation

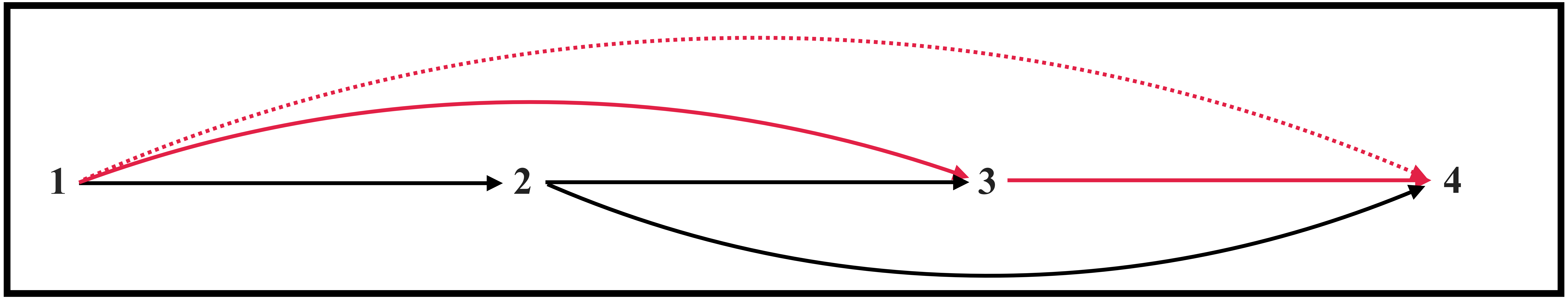


Transitive Relation

A relation $R \subseteq A \times A$ is said to be transitive if:

$$\forall (a,b) \in A \times A, (b,c) \in A \times A \mid (a,b) \in R \text{ and } (b,c) \in R \implies (a,c) \in R.$$

Transitive Relation

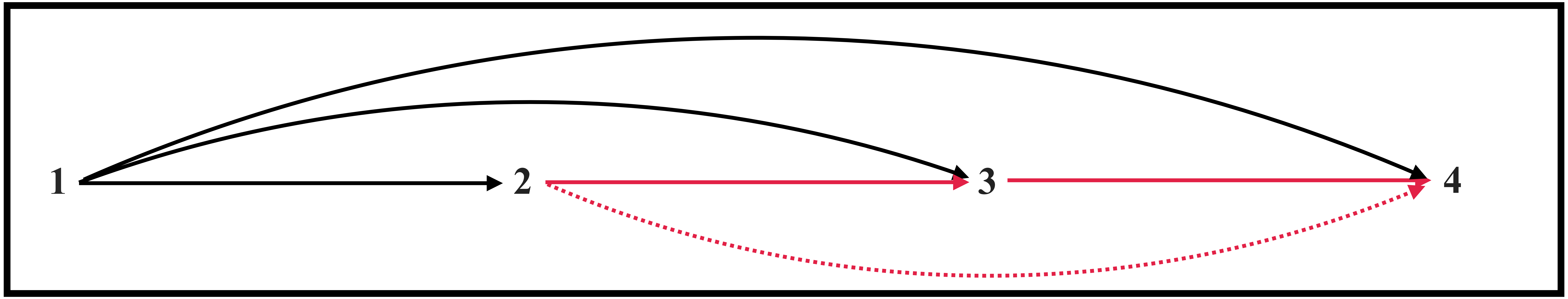


Transitive Relation

A relation $R \subseteq A \times A$ is said to be transitive if:

$$\forall (a, b) \in A \times A, (b, c) \in A \times A \mid (a, b) \in R \text{ and } (b, c) \in R \implies (a, c) \in R.$$

Transitive Relation



Transitive Relation

A relation $R \subseteq A \times A$ is said to be transitive if:

$$\forall (a, b) \in A \times A, (b, c) \in A \times A \mid (a, b) \in R \text{ and } (b, c) \in R \implies (a, c) \in R.$$

Transitive Relation

Transitive Relation

A relation $R \subseteq A \times A$ is said to be transitive if:

$$\forall (a, b) \in A \times A, (b, c) \in A \times A \mid (a, b) \in R \text{ and } (b, c) \in R \implies (a, c) \in R.$$

Examples:

$\leq$ **on** $\mathbb{N}$

Transitive Relation

Transitive Relation

A relation $R \subseteq A \times A$ is said to be transitive if:

$$\forall (a, b) \in A \times A, (b, c) \in A \times A \mid (a, b) \in R \text{ and } (b, c) \in R \implies (a, c) \in R.$$

Examples:

$\leq$ **on** $\mathbb{N}$ ✓

Transitive Relation

Transitive Relation

A relation $R \subseteq A \times A$ is said to be transitive if:

$$\forall (a,b) \in A \times A, (b,c) \in A \times A \mid (a,b) \in R \text{ and } (b,c) \in R \implies (a,c) \in R.$$

Examples:

$\leq$ **on** $\mathbb{N}$ ✓

$\neq$ **on** $\mathbb{N}$

Transitive Relation

Transitive Relation

A relation $R \subseteq A \times A$ is said to be transitive if:

$$\forall (a, b) \in A \times A, (b, c) \in A \times A \mid (a, b) \in R \text{ and } (b, c) \in R \implies (a, c) \in R.$$

Examples:

$\leq$ **on** $\mathbb{N}$ ✓

$\neq$ **on** $\mathbb{N}$ ✗

Transitive Relation

Transitive Relation

A relation $R \subseteq A \times A$ is said to be transitive if:

$$\forall (a,b) \in A \times A, (b,c) \in A \times A \mid (a,b) \in R \text{ and } (b,c) \in R \implies (a,c) \in R.$$

Examples:

$\leq$ **on** $\mathbb{N}$ ✓

$\neq$ **on** $\mathbb{N}$ ✗ $(2,3) \in \neq$ **and** $(3,2) \in \neq$ **, but** $(2,2) \notin \neq$

Transitive Relation

Transitive Relation

A relation $R \subseteq A \times A$ is said to be transitive if:

$$\forall (a, b) \in A \times A, (b, c) \in A \times A \mid (a, b) \in R \text{ and } (b, c) \in R \implies (a, c) \in R.$$

Examples:

$\leq$ **on** $\mathbb{N}$ ✓

$\neq$ **on** $\mathbb{N}$ ✗

$=$ **on** $\mathbb{N}$

Transitive Relation

Transitive Relation

A relation $R \subseteq A \times A$ is said to be transitive if:

$$\forall (a,b) \in A \times A, (b,c) \in A \times A \mid (a,b) \in R \text{ and } (b,c) \in R \implies (a,c) \in R.$$

Examples:

$\leq$ **on** $\mathbb{N}$ ✓

$\neq$ **on** $\mathbb{N}$ ✗

$=$ **on** $\mathbb{N}$ ✓

Equivalence Relation

Equivalence Relation

Transitive Relation

A relation $R \subseteq A \times A$ is said to be an equivalence relation if it satisfies the following 3 properties:

(i) R is reflexive

(ii) R is symmetric

(iii) R is transitive

Equivalence Relation

Transitive Relation

A relation $R \subseteq A \times A$ is said to be an equivalence relation if it satisfies the following 3 properties:

(i) R is reflexive

(ii) R is symmetric

(iii) R is transitive

Examples on $\{1,2,3\}$**:**

$R_1 = \{(1,1), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2), (3,3)\}$

Equivalence Relation

Transitive Relation

A relation $R \subseteq A \times A$ is said to be an equivalence relation if it satisfies the following 3 properties:

(i) R is reflexive

(ii) R is symmetric

(iii) R is transitive

Examples on $\{1,2,3\}$**:**

$R_1 = \{(1,1), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2), (3,3)\}$

Equivalence Relation

Transitive Relation

A relation $R \subseteq A \times A$ is said to be an equivalence relation if it satisfies the following 3 properties:

(i) R is reflexive

(ii) R is symmetric

(iii) R is transitive

Examples on $\{1,2,3\}$:

$R_1 = \{(1,1), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2), (3,3)\}$ ✗ Not reflexive since $(2,2) \notin R_1$

Equivalence Relation

Transitive Relation

A relation $R \subseteq A \times A$ is said to be an equivalence relation if it satisfies the following 3 properties:

(i) R is reflexive

(ii) R is symmetric

(iii) R is transitive

Examples on $\{1,2,3\}$**:**

$R_1 = \{(1,1), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2), (3,3)\}$ ✗

$R_2 = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,2), (3,3)\}$

Equivalence Relation

Transitive Relation

A relation $R \subseteq A \times A$ is said to be an equivalence relation if it satisfies the following 3 properties:

(i) R is reflexive

(ii) R is symmetric

(iii) R is transitive

Examples on $\{1,2,3\}$**:**

$R_1 = \{(1,1), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2), (3,3)\}$ ✗

$R_2 = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,2), (3,3)\}$ ✗

Equivalence Relation

Transitive Relation

A relation $R \subseteq A \times A$ is said to be an equivalence relation if it satisfies the following 3 properties:

(i) R is reflexive

(ii) R is symmetric

(iii) R is transitive

Examples on $\{1,2,3\}$**:**

$R_1 = \{(1,1), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2), (3,3)\}$ ✗

$R_2 = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,2), (3,3)\}$ ✗

Not symmetric since $(1,3) \in R_2$, but $(3,1) \notin R_2$

Equivalence Relation

Transitive Relation

A relation $R \subseteq A \times A$ is said to be an equivalence relation if it satisfies the following 3 properties:

(i) R is reflexive

(ii) R is symmetric

(iii) R is transitive

Examples on $\{1,2,3\}$**:**

$R_1 = \{(1,1),(1,2),(1,3),(2,1),(2,3),(3,1),(3,2),(3,3)\}$ ✗

$R_2 = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,2),(3,3)\}$ ✗

$R_3 = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$

Equivalence Relation

Transitive Relation

A relation $R \subseteq A \times A$ is said to be an equivalence relation if it satisfies the following 3 properties:

(i) R is reflexive

(ii) R is symmetric

(iii) R is transitive

Examples on $\{1,2,3\}$**:**

$R_1 = \{(1,1), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2), (3,3)\}$ ✗

$R_2 = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,2), (3,3)\}$ ✗

$R_3 = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$ ✓

Equivalence Relation

Transitive Relation

A relation $R \subseteq A \times A$ is said to be an equivalence relation if it satisfies the following 3 properties:

(i) R is reflexive

(ii) R is symmetric

(iii) R is transitive

Examples on $\{1,2,3\}$**:**

$R_1 = \{(1,1),(1,2),(1,3),(2,1),(2,3),(3,1),(3,2),(3,3)\}$ ✗

$R_2 = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,2),(3,3)\}$ ✗

$R_3 = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$ ✓

$R_4 = \{(1,1),(2,2),(3,3)\}$

Equivalence Relation

Transitive Relation

A relation $R \subseteq A \times A$ is said to be an equivalence relation if it satisfies the following 3 properties:

(i) R is reflexive

(ii) R is symmetric

(iii) R is transitive

Examples on $\{1,2,3\}$**:**

$R_1 = \{(1,1),(1,2),(1,3),(2,1),(2,3),(3,1),(3,2),(3,3)\}$

$R_2 = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,2),(3,3)\}$

$R_3 = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$

$R_4 = \{(1,1),(2,2),(3,3)\}$

Equivalence Relation

Equivalence Relation

NIMISHA S. NAIR	IMTIYAZ HUSSAIN	ABHIJITH K S	ABHISHEK S
ATHIRA K S	RAGESH O.R.	ABHISHEK MANOHARAN	GOKUL K
PAURNAMI PRADEEP	DEVIKA M S	FAZNA HAREES	ABHILA KRISHNAN K

Equivalence Relation

NIMISHA S. NAIR	IMTIYAZ HUSSAIN	ABHIJITH K S	ABHISHEK S
ATHIRA K S	RAGESH O.R.	ABHISHEK MANOHARAN	GOKUL K
PAURNAMI PRADEEP	DEVIKA M S	FAZNA HAREES	ABHILA KRISHNAN K

Example Relation

Let A be the above set of students in CSE. Consider a relation R on A such that $(s_1, s_2) \in R$ is the students s_1 and s_2 are studying in the same semester. Is R an equivalence relation?

Equivalence Relation

NIMISHA S. NAIR	IMTIYAZ HUSSAIN	ABHIJITH K S	ABHISHEK S
ATHIRA K S	RAGESH O.R.	ABHISHEK MANOHARAN	GOKUL K
PAURNAMI PRADEEP	DEVIKA M S	FAZNA HAREES	ABHILA KRISHNAN K

Example Relation

Let A be the above set of students in CSE. Consider a relation R on A such that $(s_1, s_2) \in R$ is the students s_1 and s_2 are studying in the same semester. Is R an equivalence relation? Yes

Equivalence Relation

NIMISHA S. NAIR	IMTIYAZ HUSSAIN	ABHIJITH K S	ABHISHEK S
ATHIRA K S	RAGESH O.R.	ABHISHEK MANOHARAN	GOKUL K
PAURNAMI PRADEEP	DEVIKA M S	FAZNA HAREES	ABHILA KRISHNAN K

Example Relation

Let A be the above set of students in CSE. Consider a relation R on A such that $(s_1, s_2) \in R$ is the students s_1 and s_2 are studying in the same semester. Is R an equivalence relation? Yes

Reflexive, since $\forall s$, if s is studying in semester$_i$, then s is studying in semester$_i$, hence $(s, s) \in R$

Equivalence Relation

NIMISHA S. NAIR	IMTIYAZ HUSSAIN	ABHIJITH K S	ABHISHEK S
ATHIRA K S	RAGESH O.R.	ABHISHEK MANOHARAN	GOKUL K
PAURNAMI PRADEEP	DEVIKA M S	FAZNA HAREES	ABHILA KRISHNAN K

Example Relation

Let A be the above set of students in CSE. Consider a relation R on A such that $(s_1, s_2) \in R$ is the students s_1 and s_2 are studying in the same semester. Is R an equivalence relation? Yes

Reflexive, since $\forall s$, if s is studying in semester$_i$, then s is studying in semester$_i$, hence $(s, s) \in R$

Symmetric since,

$(s_i, s_j) \in R \implies s_i$ and s_j are in the same semester $\implies s_j$ and s_i are in the same semester $\implies (s_j, s_i) \in R$

Equivalence Relation

NIMISHA S. NAIR	IMTIYAZ HUSSAIN	ABHIJITH K S	ABHISHEK S
ATHIRA K S	RAGESH O.R.	ABHISHEK MANOHARAN	GOKUL K
PAURNAMI PRADEEP	DEVIKA M S	FAZNA HAREES	ABHILA KRISHNAN K

Example Relation

Let A be the above set of students in CSE. Consider a relation R on A such that $(s_1, s_2) \in R$ is the students s_1 and s_2 are studying in the same semester. Is R an equivalence relation? Yes

Reflexive, since $\forall s$, if s is studying in semester$_i$, then s is studying in semester$_i$, hence $(s, s) \in R$

Symmetric since,

$(s_i, s_j) \in R \implies s_i$ and s_j are in the same semester $\implies s_j$ and s_i are in the same semester $\implies (s_j, s_i) \in R$

Transitive since,

$(s_i, s_j) \in R, (s_j, s_k) \in R \implies s_i$ and s_j are in the same semester and s_j and s_k are in the same semester $\implies s_i$ and s_k are in the same semester $\implies (s_i, s_k) \in R$

Equivalence Classes

Example Relation

Let A be the above set of students in CSE. Consider a relation R on A such that $(s_1, s_2) \in R$ is the students s_1 and s_2 are studying in the same semester. Is R an equivalence relation? Yes

Equivalence Classes

Example Relation

Let A be the above set of students in CSE. Consider a relation R on A such that $(s_1, s_2) \in R$ is the students s_1 and s_2 are studying in the same semester. Is R an equivalence relation? Yes

[NIMISHA S. NAIR] = **{NIMISHA S. NAIR, RAJESH O.R., FAZNA HAREES, GOKUL L. K.}** $= S_5$

Equivalence Classes

Example Relation

Let A be the above set of students in CSE. Consider a relation R on A such that $(s_1, s_2) \in R$ is the students s_1 and s_2 are studying in the same semester. Is R an equivalence relation? Yes

[NIMISHA S. NAIR] = **{NIMISHA S. NAIR, RAJESH O.R., FAZNA HAREES, GOKUL L. K.}** $= S_5$

[ATHIRA K. S.] = **{ATHIRA K. S., DEVIKA M. S., ABHISHEK MANOHARAN, ABHISHEK S.}** $= S_7$

Equivalence Classes

Example Relation

Let A be the above set of students in CSE. Consider a relation R on A such that $(s_1, s_2) \in R$ is the students s_1 and s_2 are studying in the same semester. Is R an equivalence relation? Yes

[NIMISHA S. NAIR] = {NIMISHA S. NAIR, RAJESH O.R., FAZNA HAREES, GOKUL L. K.} $= S_5$

[ATHIRA K. S.] = {ATHIRA K. S., DEVIKA M. S., ABHISHEK MANOHARAN, ABHISHEK S.} $= S_7$

[POURNAMI PRADEEP] = {POURNAMI PRADEEP, IMITIYAZ HUSSAIN, ABHISHEK K. S., ABHILA KRISHNAN K.} $= S_3$

Equivalence Classes

Example Relation

Let A be the above set of students in CSE. Consider a relation R on A such that $(s_1, s_2) \in R$ is the students s_1 and s_2 are studying in the same semester. Is R an equivalence relation? Yes

[NIMISHA S. NAIR] = {NIMISHA S. NAIR, RAJESH O.R., FAZNA HAREES, GOKUL L. K.} $= S_5$

[ATHIRA K. S.] = {ATHIRA K. S., DEVIKA M. S., ABHISHEK MANOHARAN, ABHISHEK S.} $= S_7$

[POURNAMI PRADEEP] = {POURNAMI PRADEEP, IMITIYAZ HUSSAIN, ABHISHEK K. S., ABHILA KRISHNAN K.} $= S_3$

- These are called the **equivalences classes** induced by an equivalence relation

Equivalence Classes

An equivalence relation R on a set A ***partition*** A into a number disjoint subsets or equivalence classes $[a_i]$ such that:

$$A = \bigcup_{i} [a_i]$$

where, $a_i \in A$ and $[a_i] = \{x \in A \mid (a_i, x) \in R\}$.

Homework Exercise

- Let $\mathbb{Z}$ be the set of integers and R be a binary relation on $\mathbb{Z}$ defined as follows:

$$\forall (a, b) \in \mathbb{Z} \times \mathbb{Z}, (a, b) \in R \text{ iff } 3 \text{ divides } a - b$$

  Prove that R is an equivalence relation. Also find the equivalence classes induced by R.