

# Safe Autonomy

Lee - 3

① Spec & Verif

② Control theory 101.



# Verification

$S \models \Phi \longrightarrow M \models \Phi$  : Most general automata that captures all behaviors that satisfy  $\Phi$ .

(\*) LTL  
linear temporal logic

(\*) CTL  
Computational Tree Logic

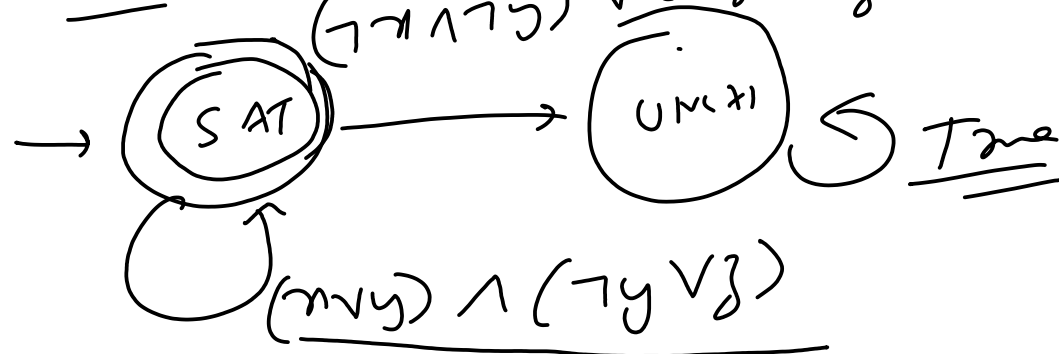
(\*) CTL\*

(\*) Finite Model Theory  
→ Modal logics.

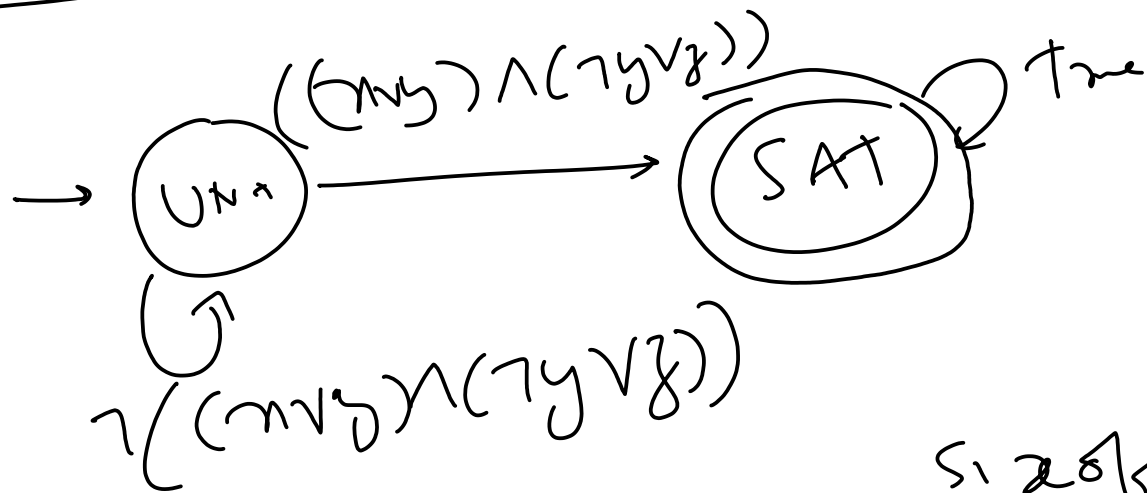
$x, y, z$

$\neg((x \vee y) \wedge (\neg y \vee z))$

$= (\neg x \wedge \neg y) \vee (y \wedge \neg z)$



$$\frac{F((\neg \forall y) \wedge (\neg \exists y \vee z))}{S}$$



Monitor for LTL

→ Kupferman & Vardi

Automata theoretic  
Model Checking 1987

LTL → 1977, CTL - 1983, →

size of Monitor  $\leftarrow O(2^{|\Phi|})$

→ Special Cases

Safety Prop

$G(\Phi)$   $\Phi$  only

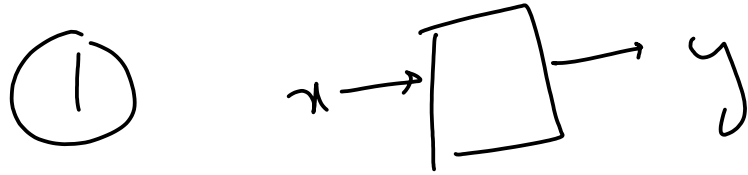
Eventually

$F(\Phi)$

$\Phi$  only has F

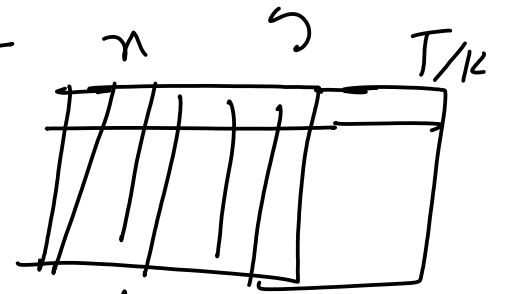
1987 - 2003

# Symbolic Representation



$$\exists x \boxed{S(x, y)} \wedge \neg \boxed{R(I, \phi)}$$

$$\boxed{R(I, \phi)}$$



305.1b

Symbolic rep

1993

BPPs

Binar =  
bear =  
pigram

100-1000 kb



2003

SAT

Symbolically op  
of formulas

$$\frac{10^6}{2^{10^6}}$$

② Program Verif

{Input} Pgm {output}

2017

Polychedral / Decomposed polychedral

1 SMT

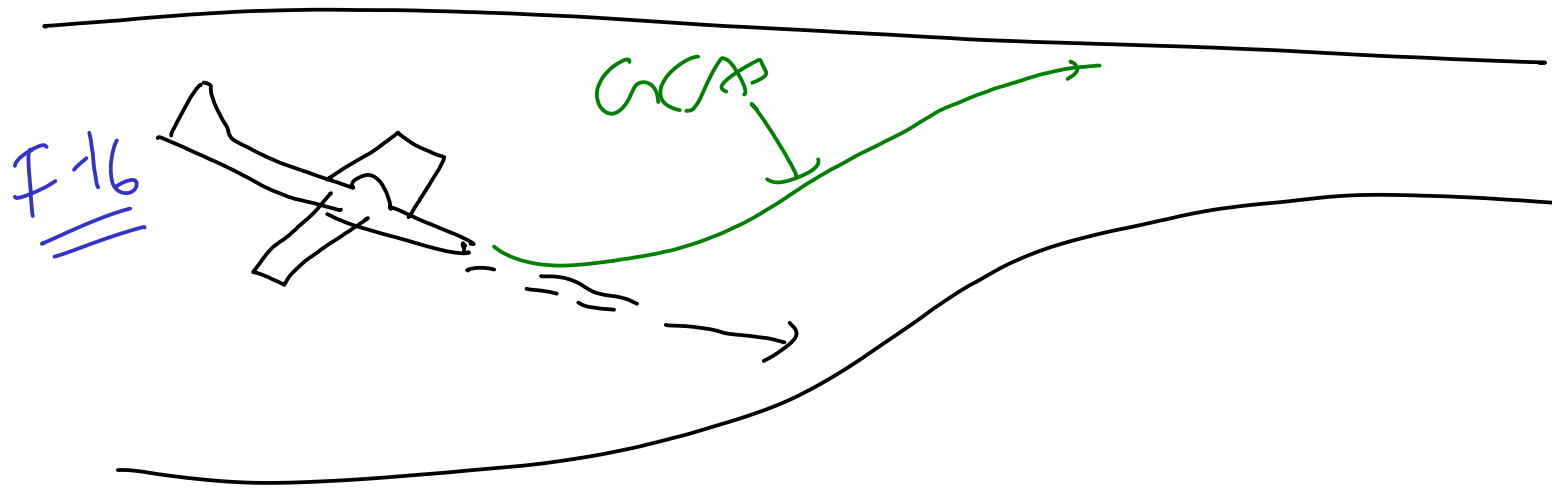


- ① Details of model checking / SAT / -
- ② Theoretical aspects of spec
- ③ Z, Petri nets, Concurrent sys,

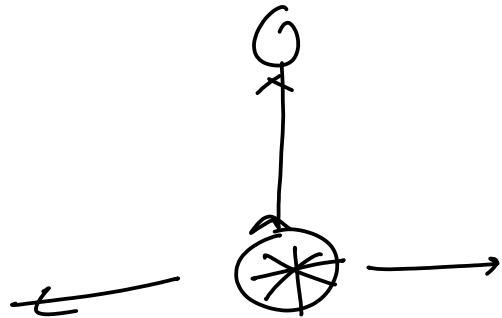
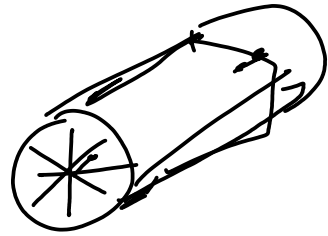
# Foundation of Control

→ Easy to understand, but hard to achieve

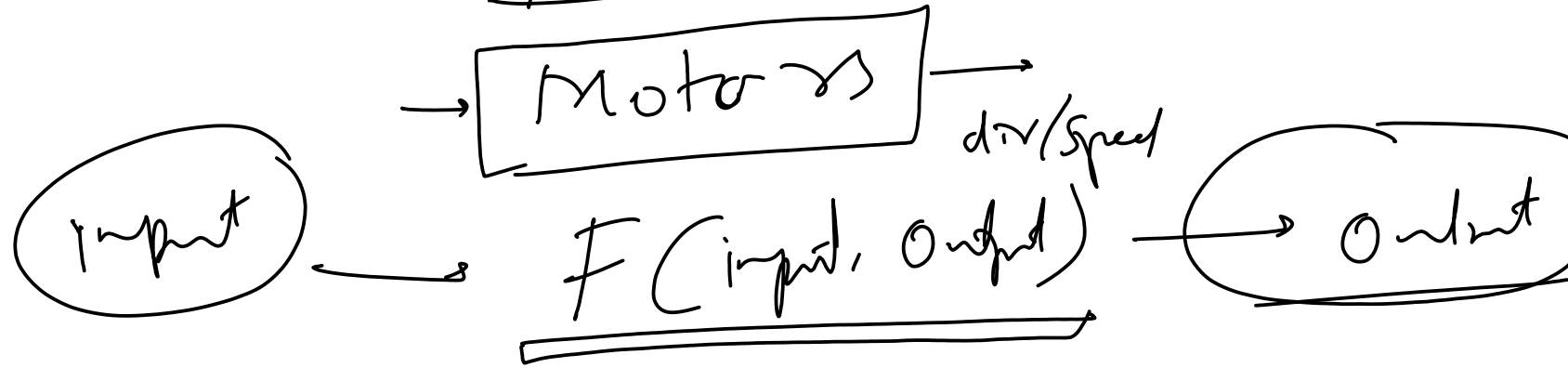
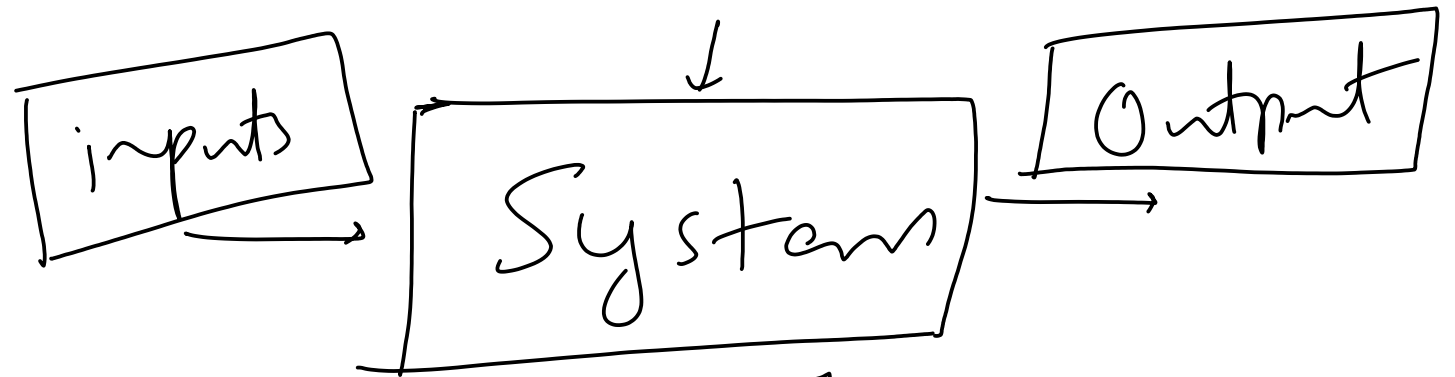
- (\*) What is control
- (\*) Models
- (\*) Feedback control (o)
- (\*) Various dimensions of control



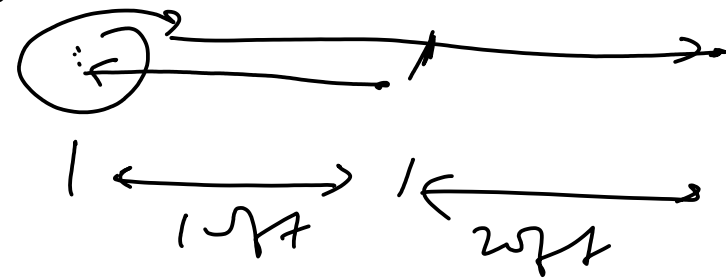
Hover board



Control



What are the "inputs" to  
achieve a certain "output"  
given the system.



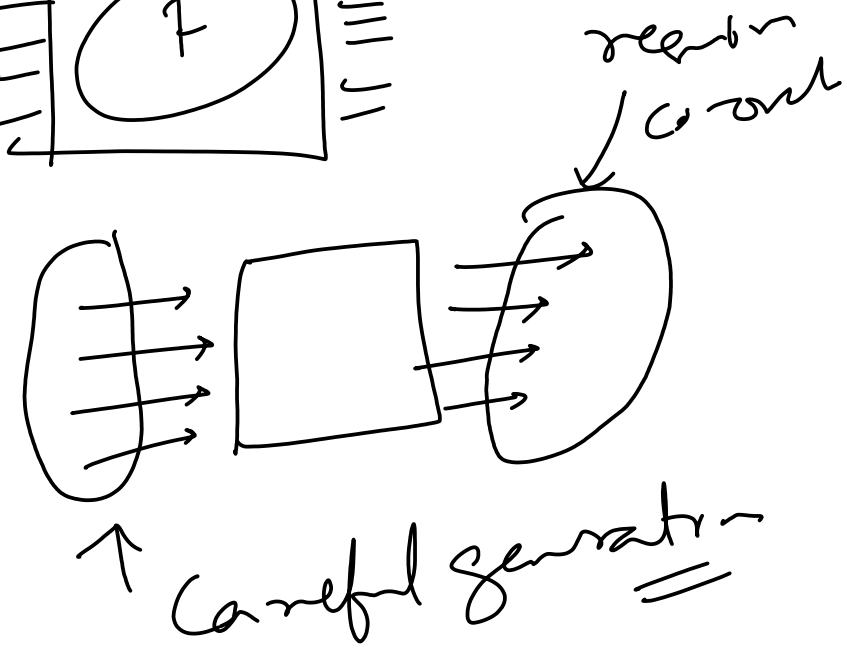
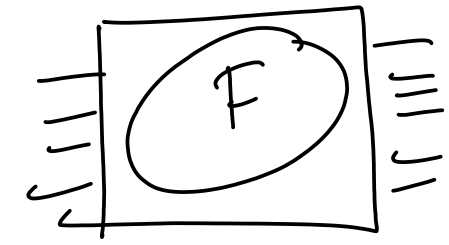
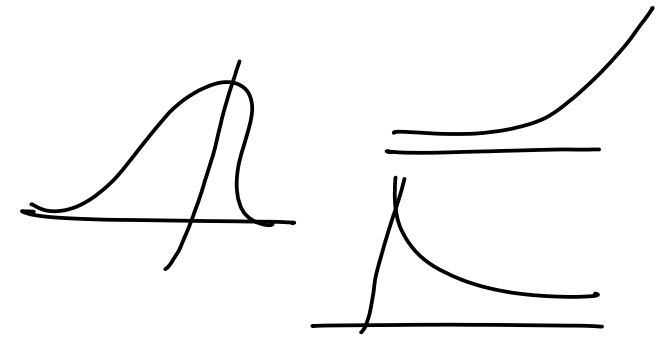
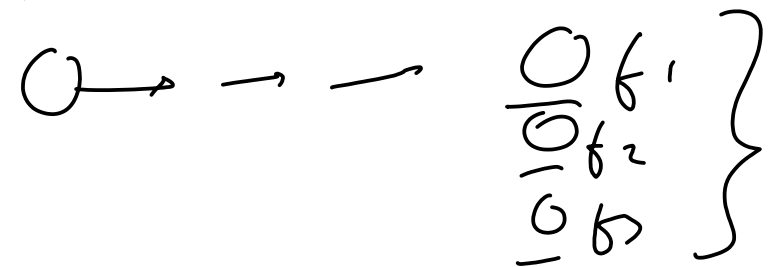
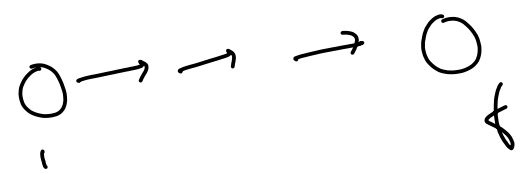


# Classification

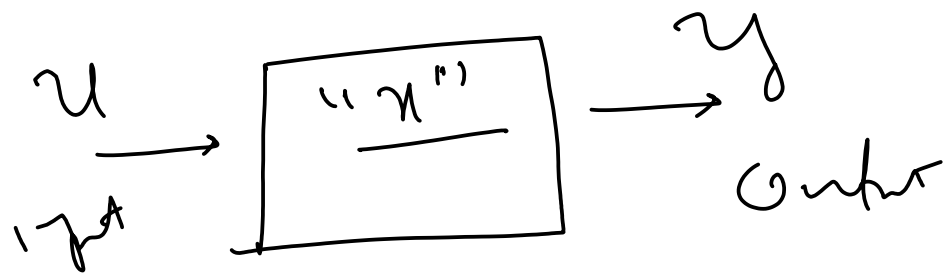
→ <sup>1960s</sup> Discrete vs Continuous <sup>1980s</sup> HCI

→ SISO vs MIMO  
Single input  
Single output      Multiple input  
Multiple output

→ Deterministic vs Stochastic



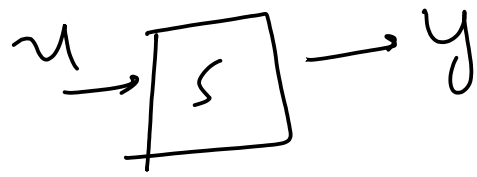
# Common Model for Control



State space model of  
Control  
1960s Kalman

$$\begin{aligned} \dot{x} &= f(x, u) \\ y &= g(x, u) \\ &\quad \downarrow \\ &\quad \underline{g(x)} \end{aligned}$$

# Application

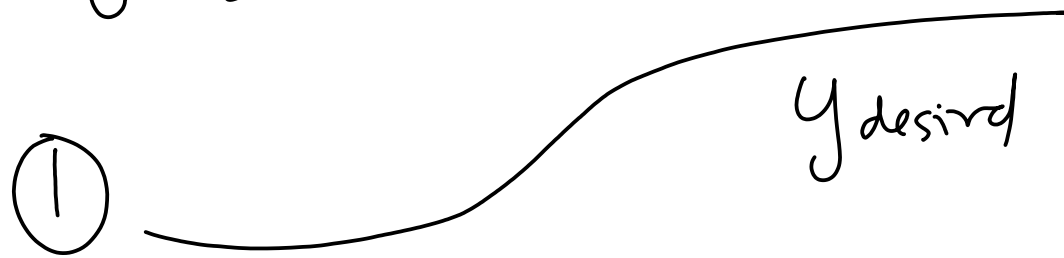


$$\dot{x} = f(x, u)$$

$$y = g(x, u)$$

① Tracking

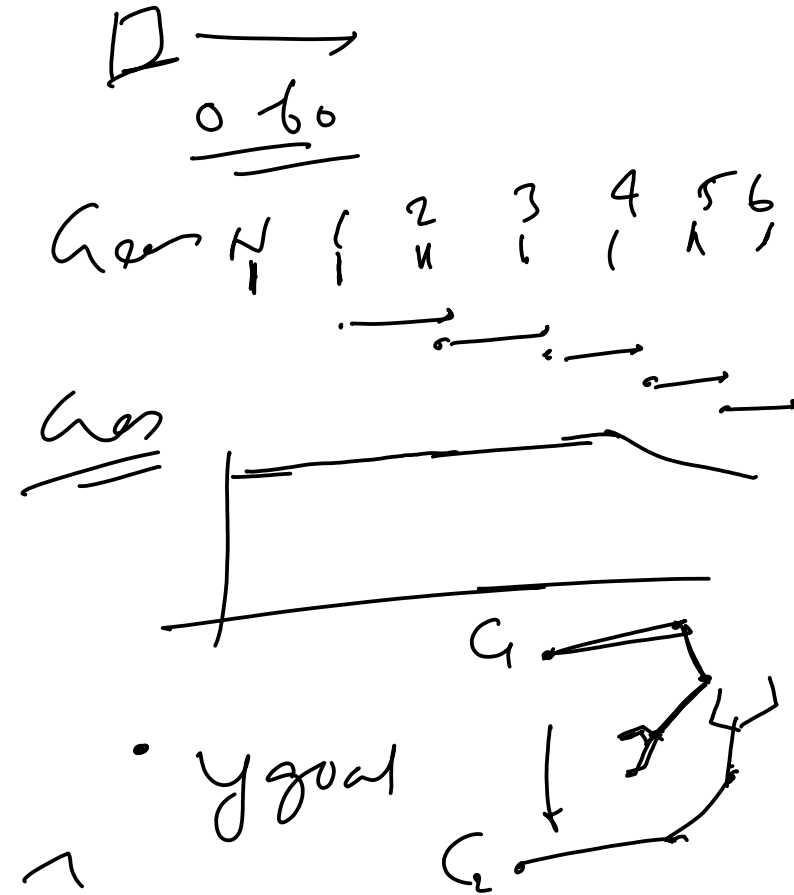
② Regulation.



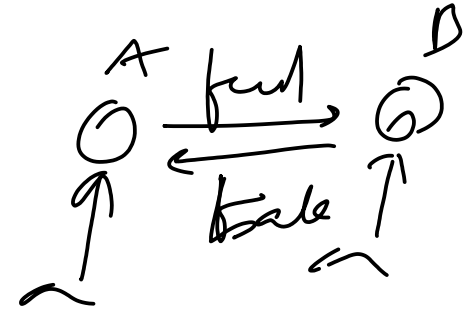
②

Synthesize the inputs ( $\hat{u}$ )  
such  $y = g(\hat{x}, \hat{u}) \sim y_{desired}$

Synthesize  $\hat{u}$   
such that  $\hat{y}(t) = y_{goal}$



# Feedback



- ⊗ Suggestion
- ⊗ Mic / Speaker

→ Doesn't exist by itself

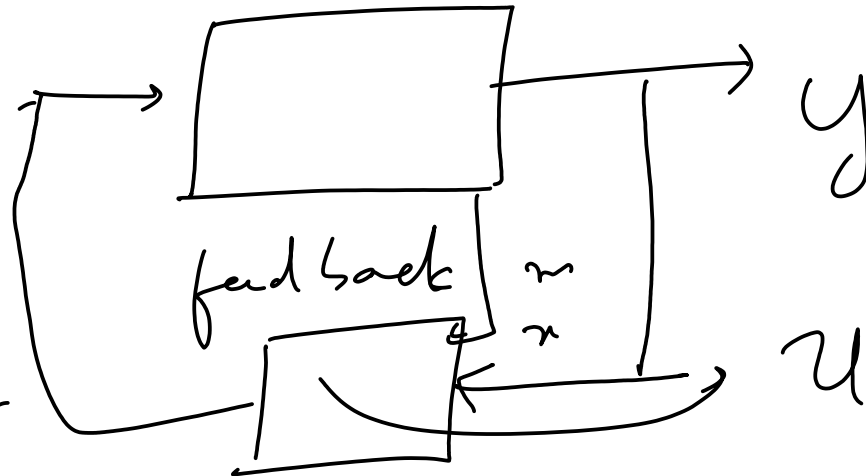
→ Reaction to an independent action that was performed.

action that output feed

closed loop

Autonomous Control System

Car Trans CVT



$$u = h(y, x)$$

$$u = h(y)$$

$$u = h(x)$$

state feedback

