



## **Sequential mixed-integer dynamic optimization for integrated design and control**

M. Alizadeh

M. Ramezani

**Supervisor:**

Dr. Sahlodin

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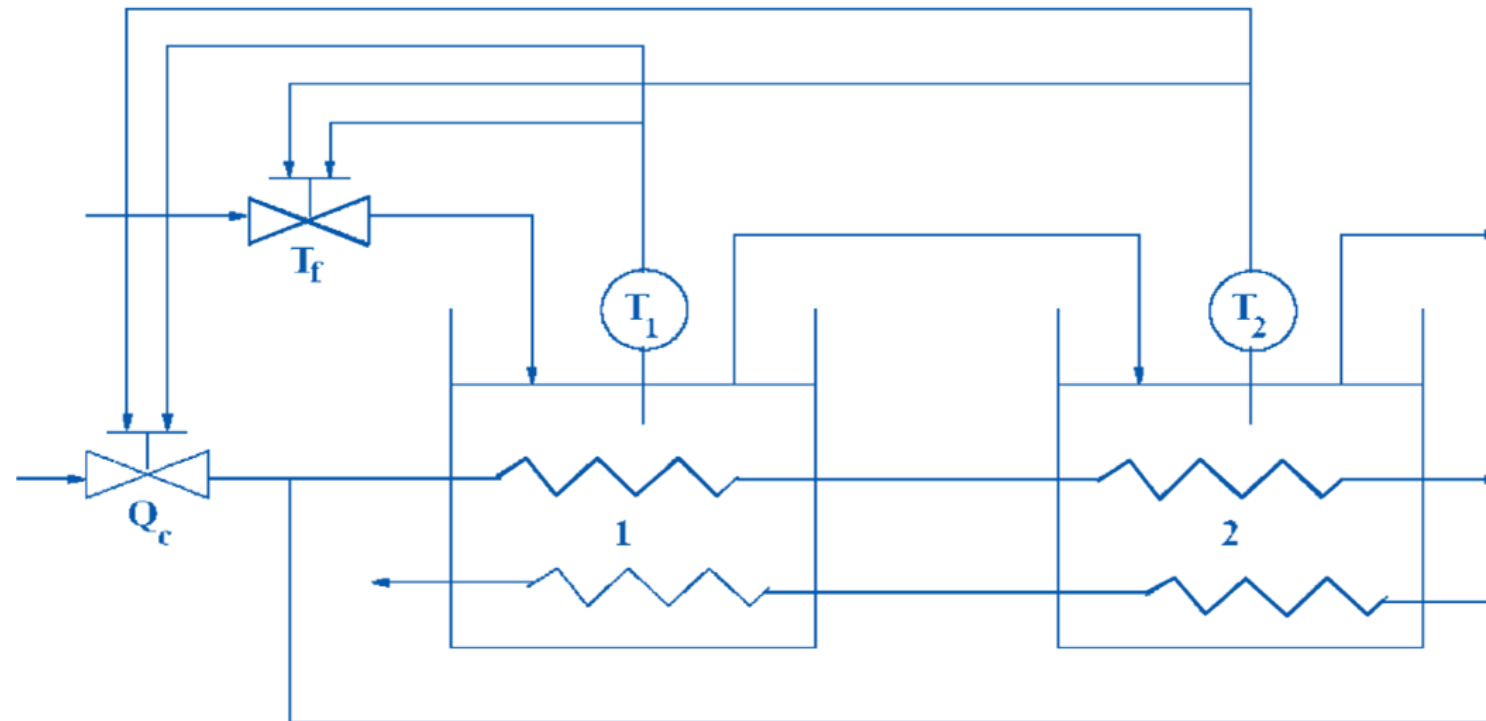


Fig1. Superstructure of processing alternatives<sup>1</sup>



$$\min \frac{1}{t_f} \left( \int_0^{t_f} (T_2^{sp} - T_2)^2 dt + \int_0^{t_f} (T_1^{sp} - T_1)^2 dt \right)$$

-10% Step change for Setpoints

$$C_f = C_f^{\text{nominal}} + \alpha_d(e^{-\lambda t} - 1)$$

$$T_{c1}^{\text{in}} = T_{cf}y_c + (1 - y_c)T_{c2}$$

$$T_{c2}^{\text{in}} = T_{c1}y_c + (1 - y_c)T_{cf}$$

$$Q_c = Q_c^{\text{bias}} - (1 - y_i)(K_{p1}P_1 + K_{i1}I_1) - y_i \times (K_{p2}P_2 + K_{i2}I_2)$$

$$T_f = T_f^{\text{bias}} + y_i \times (K_{p1}P_1 + K_{i1}I_1) + (1 - y_i)(K_{p2}P_2 + K_{i2}I_2)$$

*Mass and Energy Equations*

$$I_1 = \int P_1 dt$$

$$I_2 = \int P_2 dt$$

*Processing constraints*



$$\min \frac{1}{t_f} \left( \int_0^{t_f} (T_2^{sp} - T_2)^2 dt + \int_0^{t_f} (T_1^{sp} - T_1)^2 dt \right)$$

-10% Step change for Setpoints

$$C_f = C_f^{\text{nominal}} + \alpha_d(e^{-\lambda t} - 1)$$

$$T_{c1}^{\text{in}} = T_{cf} y_c + (1 - y_c) T_{c2}$$

$$T_{c2}^{\text{in}} = T_{c1} y_c + (1 - y_c) T_{cf}$$

$$Q_c = Q_c^{\text{bias}} - (1 - y_i)(K_{p1} P_1 + K_{i1} I_1) - y_i \times (K_{p2} P_2 + K_{i2} I_2)$$

$$T_f = T_f^{\text{bias}} + y_i \times (K_{p1} P_1 + K_{i1} I_1) + (1 - y_i)(K_{p2} P_2 + K_{i2} I_2)$$

Mass and Energy Equations



$$I_1 = \int P_1 dt$$

$$I_2 = \int P_2 dt$$

Processing constraints

$$\frac{dc_1}{dt} = \frac{C_f - C_1}{\theta} + r_{A1}$$

$$\frac{dT_1}{dt} = \frac{T_f - T_1}{\theta} + \beta r_{A1} - \alpha(T_1 - T_{c1})$$

$$\frac{dc_2}{dt} = \frac{C_1 - C_2}{\theta} + r_{A2}$$

$$\frac{dT_2}{dt} = \frac{T_1 - T_2}{\theta} + \beta r_{A2} - \alpha(T_2 - T_{c2})$$

$$\frac{dT_1}{dt} = \frac{Q_c(T_{c1}^{\text{in}} - T_{c1})}{V_c} + \alpha(T_1 - T_{c1})$$

$$\frac{dT_2}{dt} = \frac{Q_c(T_{c2}^{\text{in}} - T_{c2})}{V_c} + \alpha(T_2 - T_{c2})$$



$$\min \frac{1}{t_f} \left( \int_0^{t_f} (T_2^{sp} - T_2)^2 dt + \int_0^{t_f} (T_1^{sp} - T_1)^2 dt \right)$$

-10% Step change for Setpoints

$$C_f = C_f^{\text{nominal}} + \alpha_d(e^{-\lambda t} - 1)$$

$$T_{c1}^{\text{in}} = T_{cf}y_c + (1 - y_c)T_{c2}$$

$$T_{c2}^{\text{in}} = T_{c1}y_c + (1 - y_c)T_{cf}$$

$$Q_c = Q_c^{\text{bias}} - (1 - y_i)(K_{p1}P_1 + K_{i1}I_1) - y_i \times (K_{p2}P_2 + K_{i2}I_2)$$

$$T_f = T_f^{\text{bias}} + y_i \times (K_{p1}P_1 + K_{i1}I_1) + (1 - y_i)(K_{p2}P_2 + K_{i2}I_2)$$

*Mass and Energy Equations*

$$I_1 = \int P_1 dt$$

$$I_2 = \int P_2 dt$$

*Processing constraints*



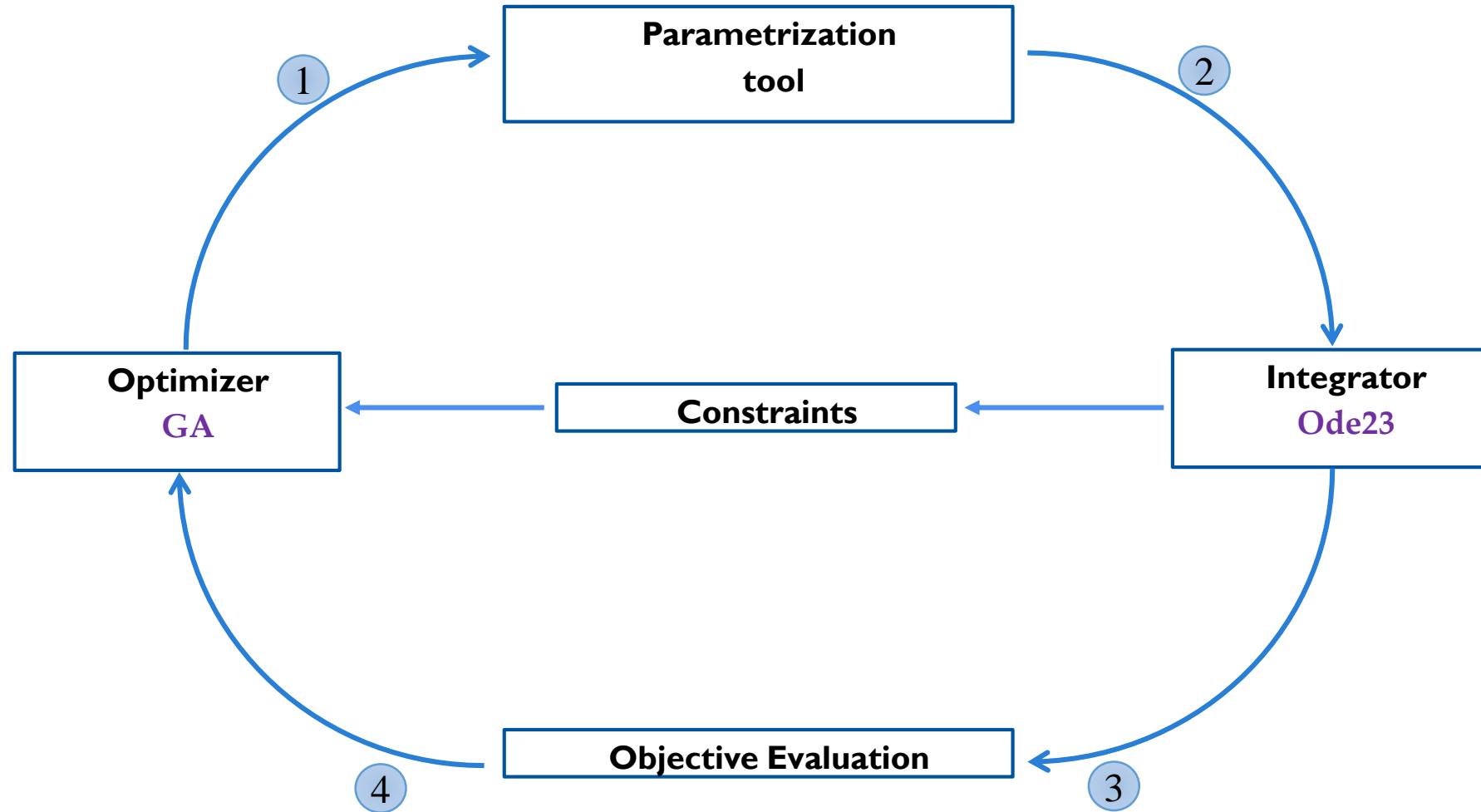
$$\int \max(0, T_F - 60) dt \leq tol$$

$$\int \max(0, -(T_F)) dt \leq tol$$

$$\int \max(0, Q_c - 8) dt \leq tol$$

$$\int \max(0, -Q_c) dt \leq tol$$

# Single shooting method (sequential)





$$y_c = 0$$



Counter-Current flow

$$y_i = 1$$



$(T_2 - Q_c) \quad (T_1, T_f)$

$$V_1 = 1079.996$$

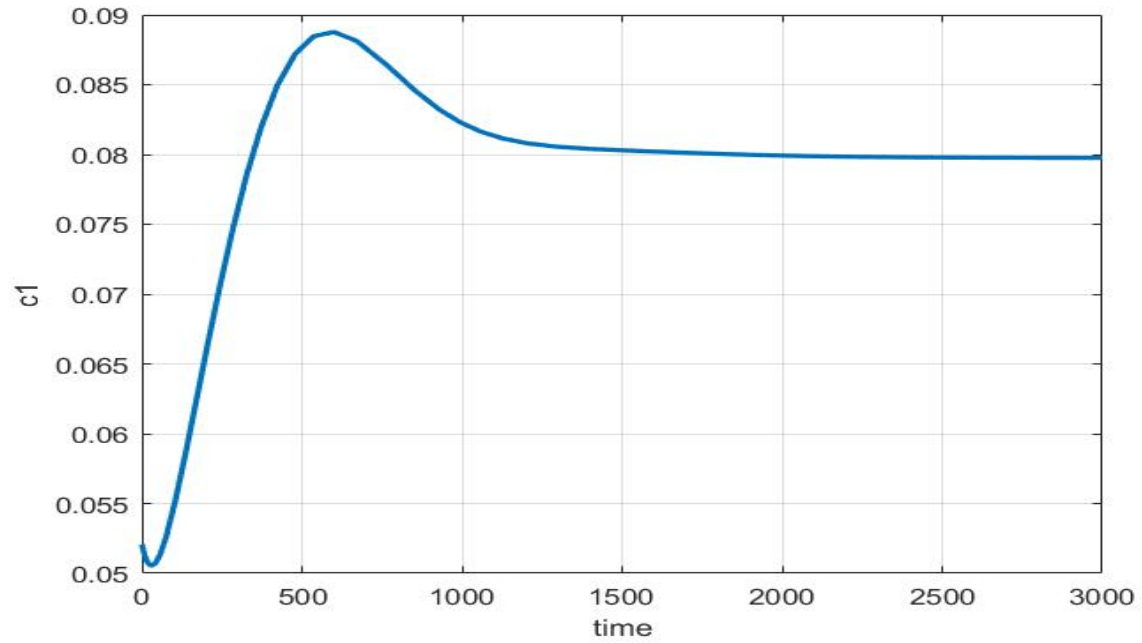
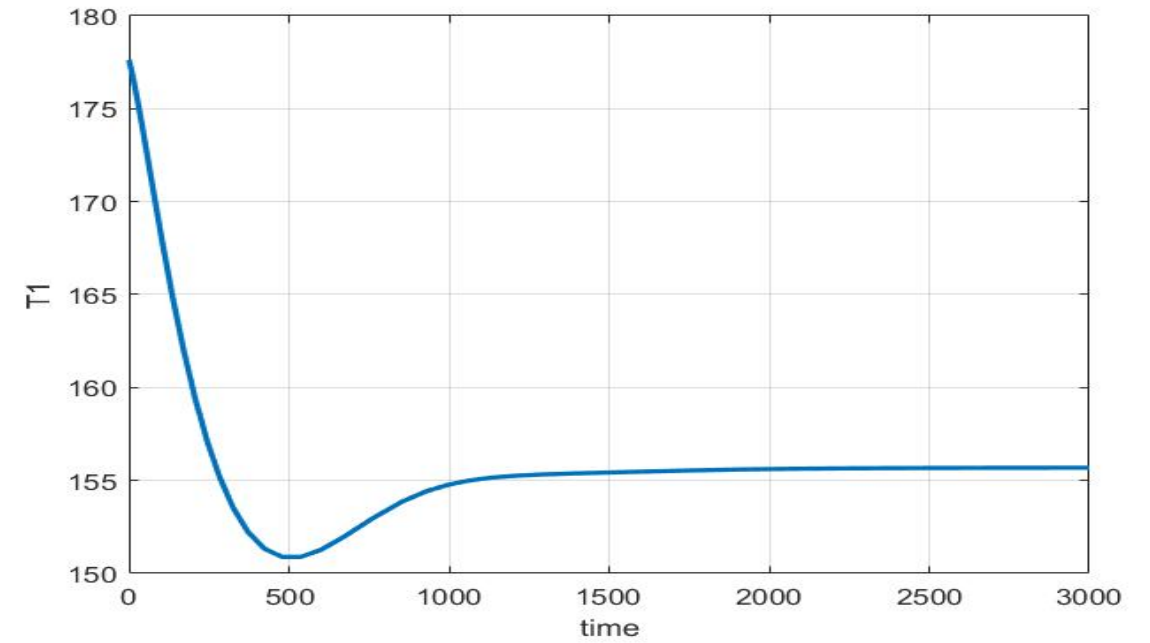
$$V_2 = 720.000$$

$$K_{c1} = 1.32260$$

$$K_{i1} = 0.0036670$$

$$K_{c2} = 0.3321133$$

$$K_{i2} = 0.0004051$$

Fig2.  $C_1$  vs TimeFig2.  $T_1$  vs Time



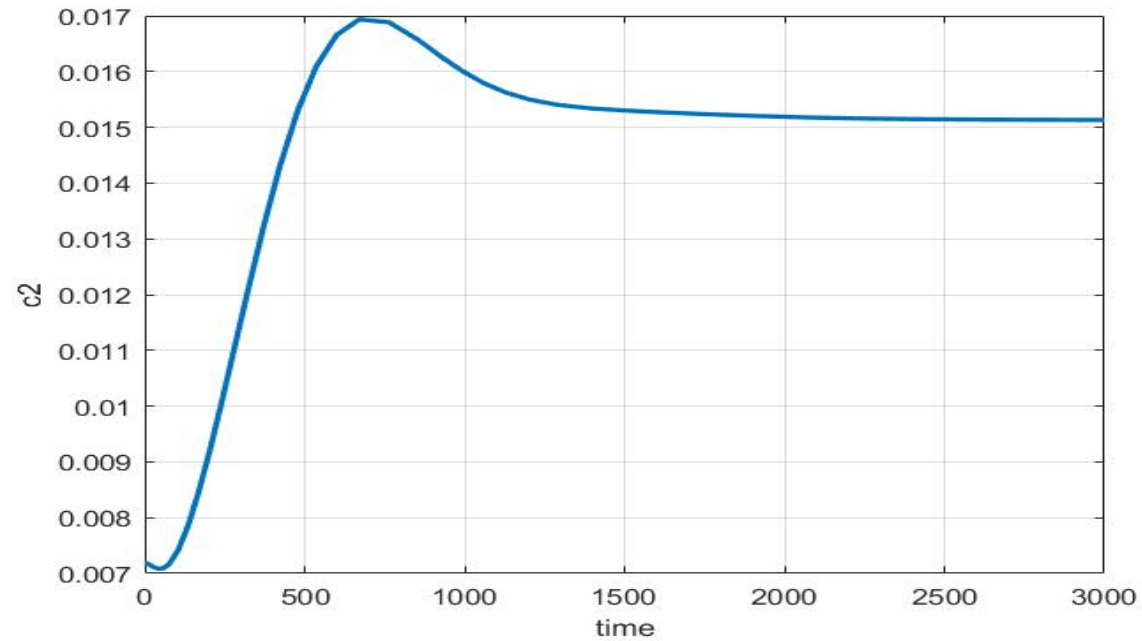


Fig3. C2 vs Time

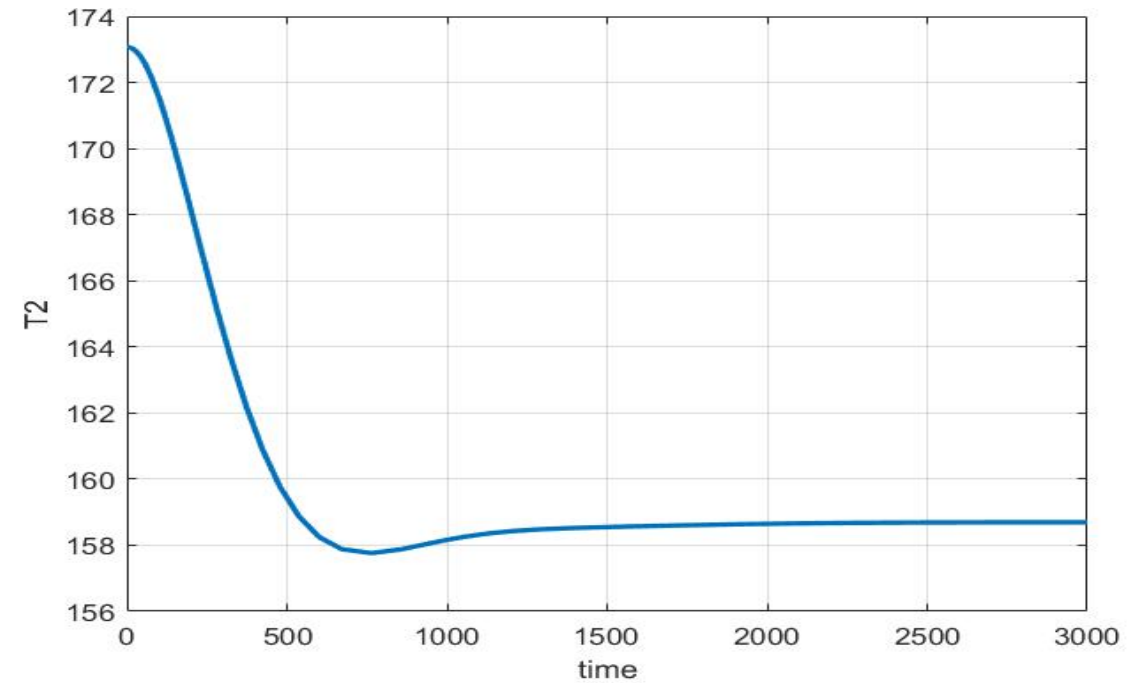


Fig4. T2 vs Time

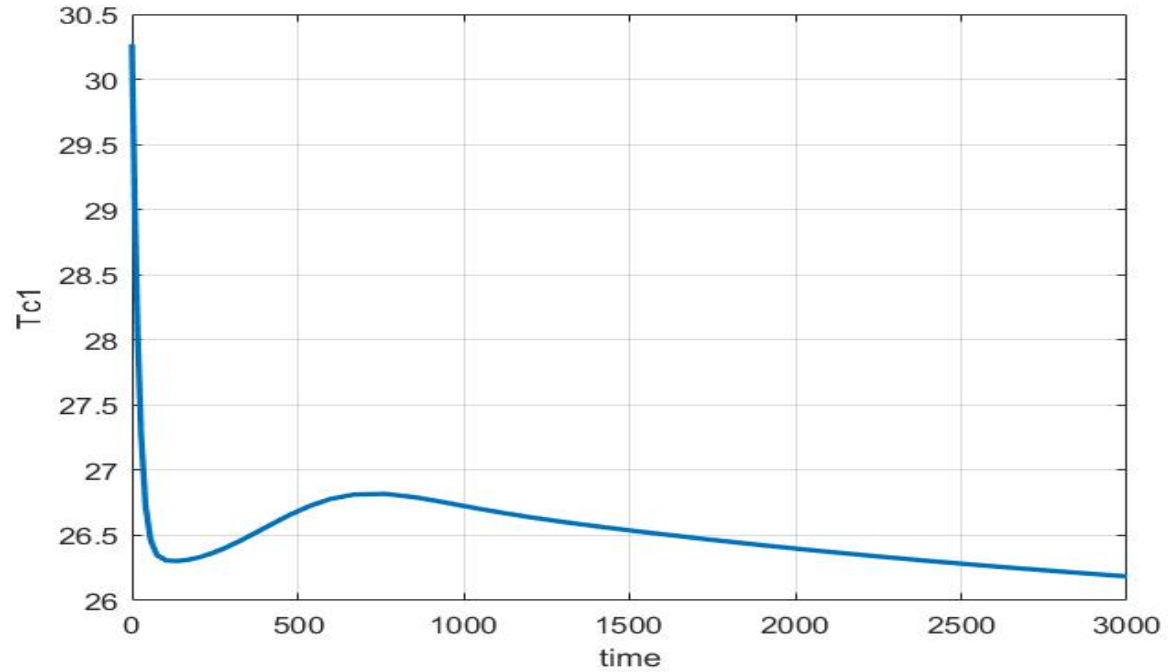


Fig5. Tc1 vs Time

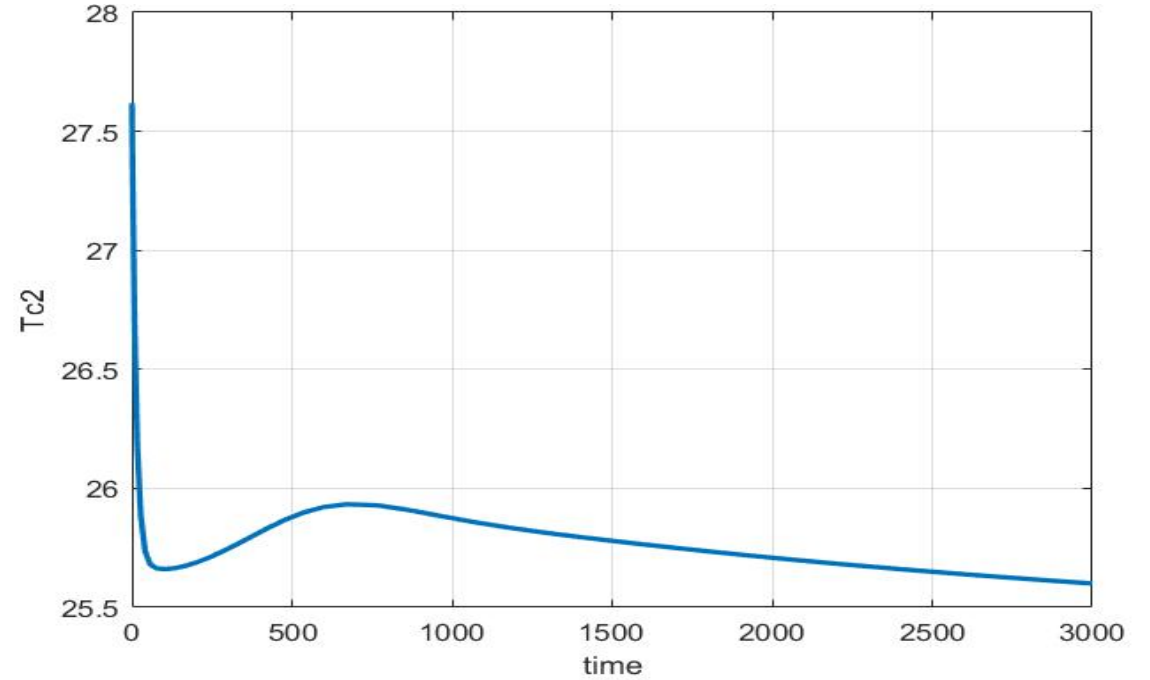
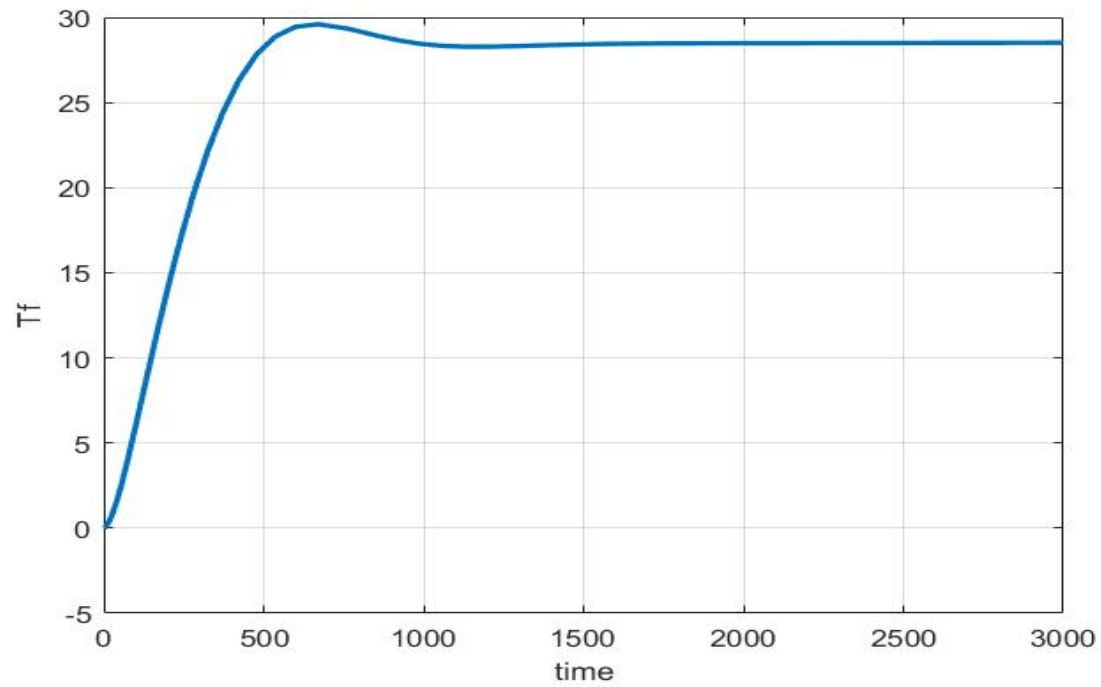
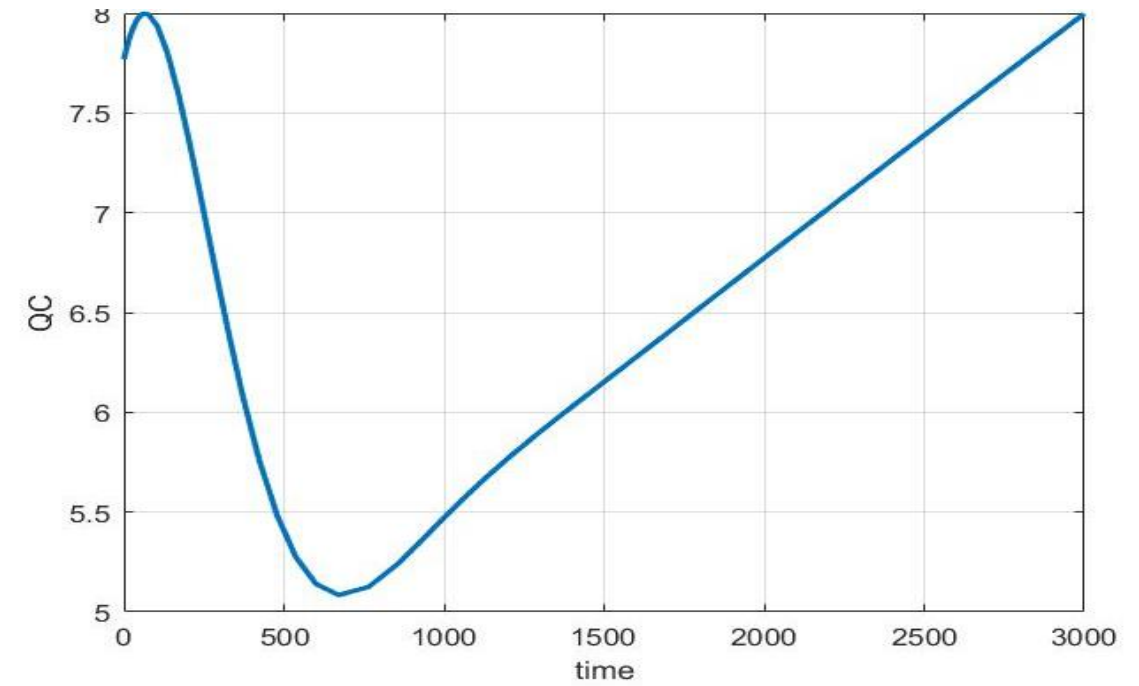


Fig6. Tc2 vs Time

Fig7.  $T_f$  vs TimeFig8.  $Q_c$  vs Time



Setpoint Kick issue!

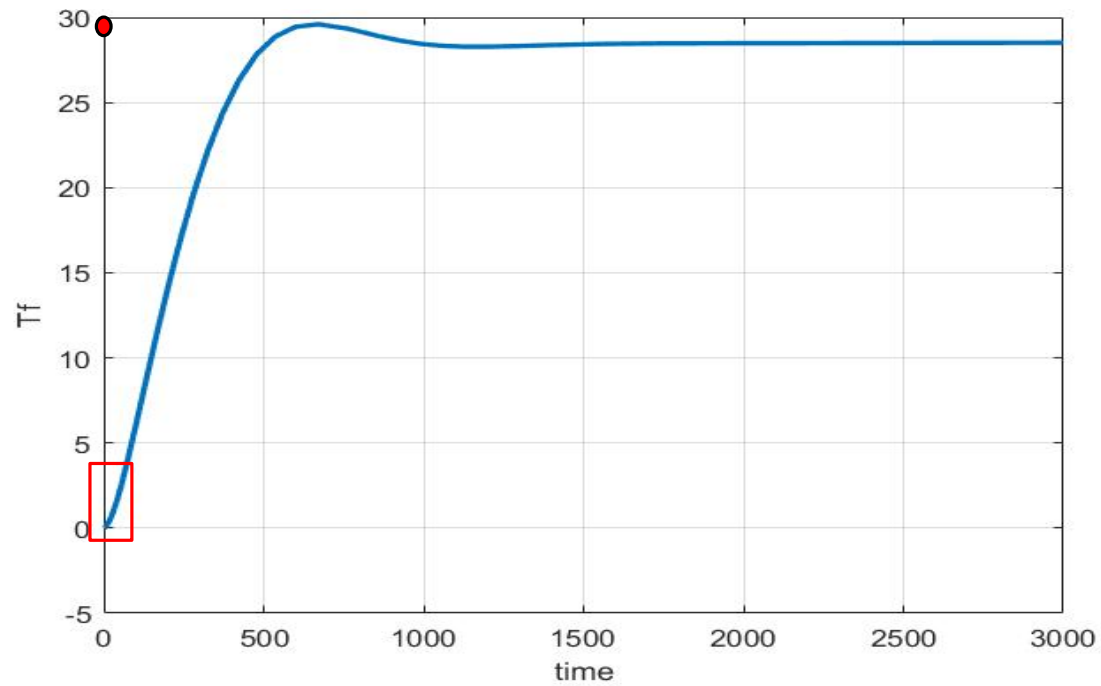


Fig7. Tf vs Time

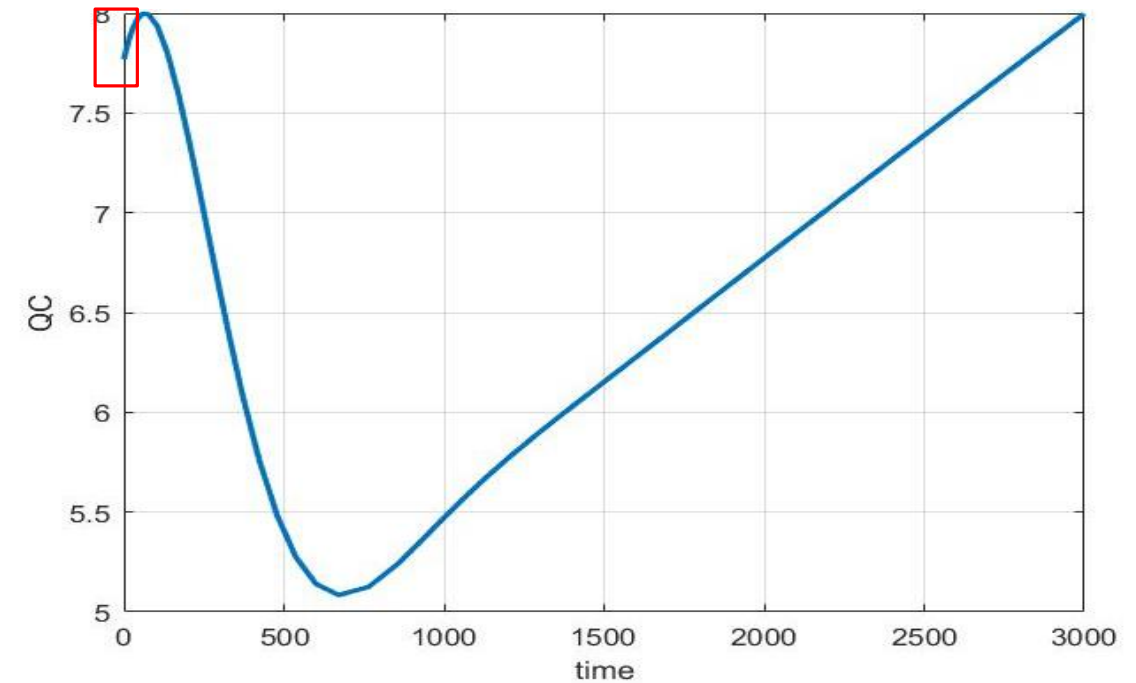


Fig8. Qc vs Time

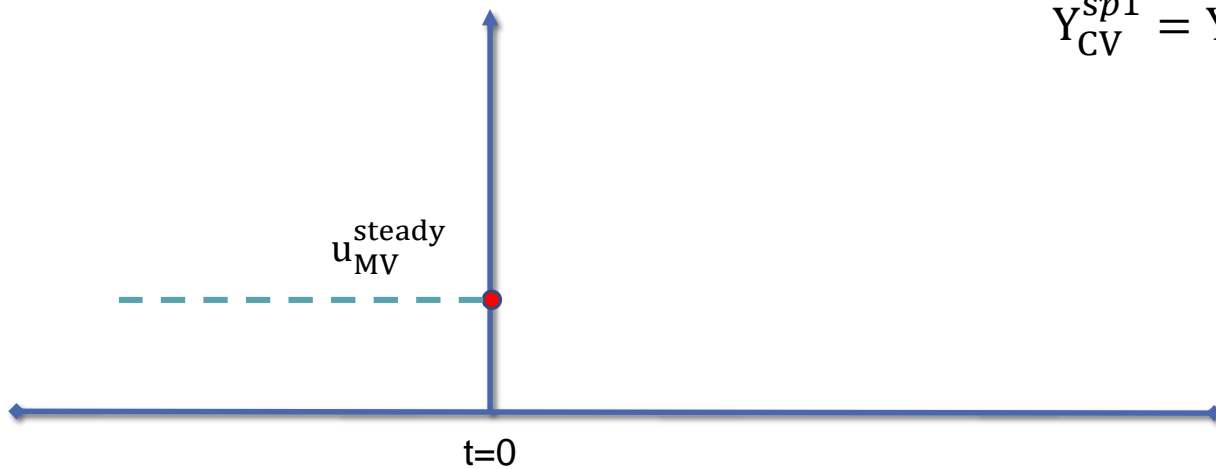
$$u_{MV} = u_{MV}^{\text{steady}} + \left( K_{p1} (Y_{CV}^{sp} - Y_{CV}) + K_{i1} \int (Y_{CV}^{sp} - Y_{CV}) dt \right)$$



$$u_{MV} = u_{MV}^{\text{steady}} + \left( K_{p1} (Y_{CV}^{sp} - Y_{CV}) + K_{i1} \int (Y_{CV}^{sp} - Y_{CV}) dt \right)$$

Before setpoint change:

$$Y_{CV}^{sp1} = Y_{CV}(t = 0) = Y_{CV}^{\text{steady}}$$



## Initial Discontinuity / Setpoint Kick<sup>1</sup> (cont'd)

$$u_{MV} = u_{MV}^{\text{steady}} + \left( K_{p1} (Y_{CV}^{sp} - Y_{CV}) + K_{i1} \int (Y_{CV}^{sp} - Y_{CV}) dt \right)$$

Before setpoint change:

$$Y_{CV}^{sp1} = Y_{CV}(t = 0) = Y_{CV}^{\text{steady}}$$

After setpoint change:

$$Y_{CV}^{sp2} \neq Y_{CV}^{\text{steady}}$$

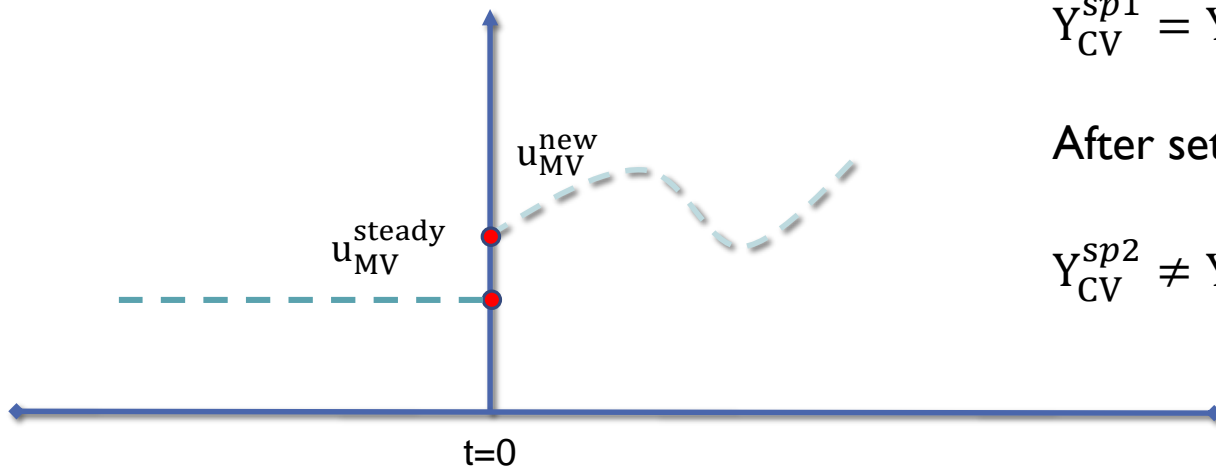


Fig9. schematic of the discontinuity at the initial time

# Initial Discontinuity / Setpoint Kick<sup>1</sup> (cont'd)

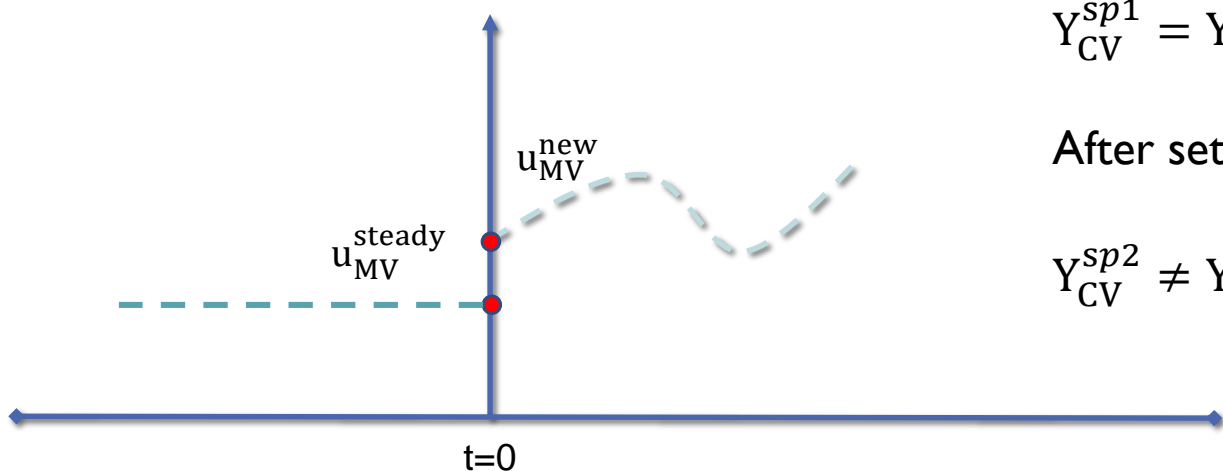
$$u_{MV} = u_{MV}^{\text{steady}} + \left( K_{p1} (Y_{CV}^{sp} - Y_{CV}) + K_{i1} \int (Y_{CV}^{sp} - Y_{CV}) dt \right)$$

Before setpoint change:

$$Y_{CV}^{sp1} = Y_{CV}(t = 0) = Y_{CV}^{\text{steady}}$$

After setpoint change:

$$Y_{CV}^{sp2} \neq Y_{CV}^{\text{steady}}$$



Proposed Solution:

$$(u_{MV}^{\text{steady}} - u_{MV}(t = 0))^2 \leq tol$$

Fig9. schematic of the discontinuity at the initial time



$$y_c = 0$$



Counter-Current flow

$$y_i = 1$$



$(T_2 - Q_c) (T_1, T_f)$

$$V_1 = 938.894$$

$$V_2 = 720.2794$$

$$K_{c1} = 0.0002030$$

$$K_{i1} = 2.9940214e-10$$

$$K_{c2} = 0.00018071$$

$$K_{i2} = 2.8542763e-06$$



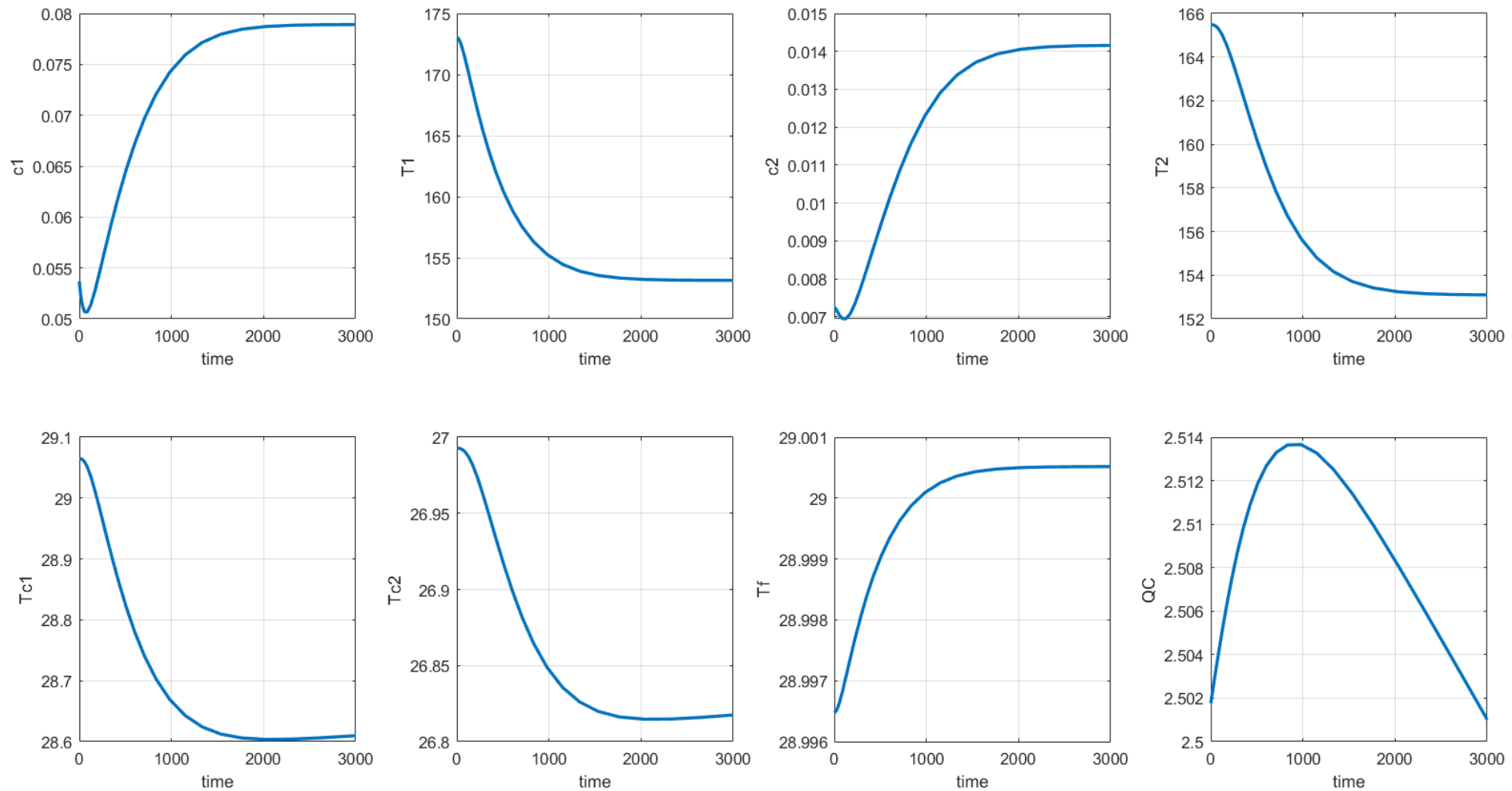


Fig9. schematic of the discontinuity at the initial time