

# Application of Nonlinear Control to a Collision Avoidance System

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## SUMMARY

The design of a collision avoidance (CA) algorithm will be presented. Previous work relied on a simple CA algorithm which used on/off brake control. This work is extended by using nonlinear control techniques to improve brake control and minimize driver discomfort. First, a quarter car model suitable for control design will be described. Then, a multi-surface sliding controller with on/off hysteresis will be developed. The on/off hysteresis will be modified to include a switch with 1<sup>st</sup> order dynamics in an attempt to minimize large initial accelerations. Finally, the simulation results of the CW/CA algorithms will be given.

## INTRODUCTION

The potential for increased vehicle safety motivates the development of collision avoidance systems. Half of the 1.5+ million rear-end crashes that occurred in 1994 could have been prevented by a collision avoidance system [1]. Collision avoidance systems can react to situations that humans can not (due to slow response) or do not (due to driver error). Therefore, they are able to reduce the severity of accidents. A commercially viable CA system must meet several (often conflicting) criteria. The CA system must balance the tradeoff between performance and driver interference. A driver who is attempting an avoidance maneuver, such as steering, may be startled and possibly lose vehicle control if the system automatically applies the brakes [3]. Therefore, a CA system could be designed to be conservative and avoid any collision, but this system would be more likely to apply the brakes during normal driving maneuvers. The system must also account for individual driving styles [2] and a variety of weather conditions.

A previously designed Binary CA Algorithm will now be briefly described [4]. When the vehicle-to-vehicle spacing drops below a pre-defined critical braking distance,  $d_{br}$ , the brakes are fully applied until the vehicle comes to a stop. There is a good chance that a full-on application of the brakes will startle the driver; thus small critical braking distances must be used with the Binary CA Algorithm to ensure that the brakes are only applied at the last possible moment of an impending collision. As a result, most extreme collisions will not be avoided; they will only have their impact velocity reduced.

The objective of this paper is to extend the Binary Algorithm by using a slip-based controller. The brake controller will allow the brakes to be applied smoothly and to be modulated to prevent collisions. The benefit of brake modulation is that the critical

distance can be more conservative (i.e. larger braking critical distance) since smooth application of the brakes will be less likely to startle the driver. A more conservative system leads to better collision avoidance performance.

The system performance will be studied in two fundamentally different scenarios. In the first scenario, the CA vehicle is following another vehicle with a negligible relative velocity when the lead car brakes at its maximum deceleration rate ( $8 \text{ m/s}^2$ ). This will be referred to as the critical scenario. In this situation, the system should initially react slowly to give the driver time to override the controller if desired. If the driver does not override the controller, then the system should react quickly to avoid the collision.

In the second scenario, the CA vehicle is again following another vehicle with no relative velocity when the lead car slows down gently from  $27.8 \text{ m/s}$  to  $24 \text{ m/s}$  over a  $3.5$  second period. This scenario will be referred to as a drift scenario. In this situation, the goal is to prevent the vehicle-to-vehicle spacing from dropping to an unsafe level. The brakes should be gently applied so that the driver is aware that the CA controller has taken over, but is not surprised by this action. In the 1960's, Goldman and von Gierke reported that automotive decelerations up to  $2.5 \text{ m/s}^2$  were comfortable to human passengers [8]. Thus, by "gently applied", we mean that the vehicle deceleration should not exceed this limit.

## CONTROL MODEL

The problem of brake control for vehicle following has been studied greatly in the last decade. Applications range from Intelligent Cruise Control to Automated Highways [5,6]. However, most of the previous work on vehicle following assumes that the tire slip can be neglected so that the relation between tire slip and longitudinal force does not enter the control problem. The CA scenario is fundamentally different in that the brakes are used to generate large longitudinal tractive forces. Therefore, the no-slip assumption fails to hold. Thus, several assumptions need to be given to arrive at a quarter car model suitable for CA brake control design.

First, the prototype vehicle will be fitted with a brake actuator based on a design developed by the Partners for Advanced Transit and Highways (PATH), described in [5],[6]. The design relies on an intermediate cylinder placed between the vacuum booster and master cylinder. An electro-hydraulic servovalve is used to connect the intermediate cylinder to a high pressure node (a pump and accumulator) and force movement of the master cylinder. In this setup, the driver is still able to increase the level of braking through the brake pedal. Furthermore, the actuator response is fast enough that the master cylinder pressure,  $P_{mc}$ , can be considered the control input. A major drawback of this design is that it does not allow control of brake pressure at individual wheels.

Since we can only control the aggregate brake pressure,  $P_{mc}$ , we need to assume similar behavior all on four wheels. Specifically, we will assume that all wheels are operating below the peak slip of the force versus slip curve. Furthermore, the force-slip curve is fairly linear up to its peak, so we will assume that tractive force is proportional to tire slip.

If the brake controller attempts to track a tire slip which causes any tire to lock up, then the ABS will release pressure and cause the tire to stay at the peak slip.

Gerdes [5] and Maciuca [6] extensively modeled the brake system with this actuator setup. Using  $P_{mc}$  as a control input, they developed a nonlinear sliding controller to track a desired wheel pressure,  $P_{w,des}$ , which was measured with a pressure sensor on one of the front wheels. This paper will use this controller and assume that it can track desired wheel pressures fast enough to assume that wheel pressure can be used as a control input.

Using these assumptions, the vehicle dynamics reduce to the following two state quarter-car model:

$$\begin{aligned}\dot{v} &= \frac{1}{m} [-c \cdot v^2 - F_r - k_\lambda \cdot \lambda] \\ \dot{\lambda} &= \frac{(1-\lambda)}{m \cdot v} [-c \cdot v^2 - F_r - k_\lambda \cdot \lambda] - \frac{r^2 \cdot k_\lambda}{J \cdot v} \lambda + \frac{r \cdot k_b}{J \cdot v} P_w\end{aligned}\quad (1)$$

where  $m$  = vehicle mass,  $c$  = aerodynamic drag coefficient,  $F_r$  = rolling resistance,  $k_\lambda$  = lumped tractive force gain,  $\lambda = (v - r \cdot \omega) / v$  = tire slip,  $v$  = velocity,  $r$  = tire radius,  $J$  = tire rotational inertia,  $k_b$  = lumped coefficient relating wheel pressure and brake torque.

Finally, it will be assumed that a reasonable slip estimate can be accurately calculated using only vehicle velocity and wheel angular velocity information. The wheel angular velocity can be obtained from a wheel speed sensor. The absolute velocity can be obtained using the wheel speed sensor and accelerometer measurements. Just before the braking maneuver, the undriven wheel on an actual vehicle has virtually no slip. Therefore, the wheel speed measurement on this wheel can be used to calculate the true vehicle velocity. Whenever a collision avoidance maneuver is being performed, the accelerometer measurements can be integrated (using the last undriven wheel speed measurement as an initial condition) to obtain true vehicle velocity. Collision avoidance maneuvers will typically have short time spans so that integrator error buildup should not be a problem. Once the maneuver ends, the vehicle, if not stopped, will return to a steady cruising speed and true velocity can again be calculated from undriven wheel speed measurements.

## CONTROL ALGORITHM DEVELOPMENT

A two-surface sliding controller will be used to control the velocity of a quarter-car vehicle. The upper level (the velocity controller) uses the tire slip as a synthetic input for velocity control. This means that the tire slip is treated as a control input which can be used to track a desired velocity. In reality, the upper level commands a desired tire slip which will result in the necessary velocity tracking.

It is the job of the lower level (slip controller) to track this desired tire slip. It is assumed that the wheel angular dynamics are much faster than the vehicle velocity dynamics.

Therefore, the slip controller should force  $\lambda \rightarrow \lambda_d$  very quickly from the view of the velocity controller. In fact, it is hoped that this convergence will happen so quickly that the velocity controller can be designed using the tire slip as the control input, hence the term synthetic input.

### BRAKING CRITICAL DISTANCE

The range at which the brakes are applied by the CA system,  $d_{br}$ , can be defined in many ways. Typically,  $d_{br}$  is a function of vehicle velocity and relative velocity between vehicles. For this analysis,  $d_{br}$  will be derived from the kinematics of two vehicles braking at their maximum decelerations until they come to a stop. The definition will be:

$$d_{br} = \frac{1}{2} \left( \frac{v^2}{\alpha} - \frac{v_p^2}{\alpha} \right) + v \cdot \tau + d_o \quad (2)$$

where  $\alpha$  = deceleration rate of both vehicles =  $6 \text{ m/s}^2$ ,  $\tau$  = system (0.2 sec) + driver delays (1.0 sec) = 1.2 sec, and  $d_o$  = safety offset = 5m. This critical distance definition is very conservative, meaning it results in large following distances. In actual implementation, the driver will be allowed to scale  $d_{br}$  (within bounds) to obtain a critical distance suitable to their driving style.

### VELOCITY CONTROLLER

The velocity controller described in this section follows the usual sliding controller development given in [7]. First, define  $S_1$  to be:

$$S_1 \equiv (v - v_d) + \Lambda \cdot (x - x_d) = (v - (v_p - v_{rel,des})) + \Lambda \cdot (x - (x_p - d_{br})) \quad (3)$$

The first sliding surface will be given by  $S_1=0$ . Define  $r = x_p - x$  and  $v_{rel} = v_p - v$  and rewrite  $S_1$  as:

$$S_1 = (v_{rel,des} - v_{rel}) + \Lambda \cdot (d_{br} - r) \quad (4)$$

Furthermore, we can determine  $v_{rel,des}$  by inverting the critical braking distance relation,  $d_{br} = f(v, v_{rel,des})$ . Thus,  $v_{rel,des} = f^{-1}(r, v)$ .

Differentiating  $S_1$ , using Equation 1 and assuming  $\dot{v}_p = 0$  yields:

$$\dot{S}_1 = \frac{1}{m} [-c \cdot v^2 - F_r - k_\lambda \cdot \lambda] + \dot{v}_{rel,des} + \Lambda \cdot (\dot{d}_{br} - \dot{r}) = -b_1 \cdot \lambda + h_1(v, d_{br}, \dot{r}) \quad (5)$$

where  $b_1$  and  $h_1$  have the implied definitions. If we knew  $b_1$  and  $h_1$  exactly, we could cancel out these nonlinear dynamics and replace them with more desirable dynamics. Since both of these terms have uncertainty, we can only use our best estimate to cancel these terms out:

$$\bar{\lambda}_d = -\frac{1}{\hat{b}_1} [\hat{h}_1 - K_1 \cdot S_1] \quad (6)$$

The hat is used to denote the best estimate of the uncertain quantities. The overbar on the desired slip implies that this desired slip will be filtered before its use by the slip controller. The necessary filtering is described in the next section.

For the moment, assume that the filtering dynamics are fast and the slip controller causes fast convergence so that  $\lambda \rightarrow \lambda_d \rightarrow \bar{\lambda}_d$ . Then, the first term of Equation 7 cancels out the unwanted nonlinear dynamics with the best model estimate while the second term attempts to overcome any errors between the modeled and true values. For example, if  $\lambda = \bar{\lambda}_d$ , Equation 6 becomes:

$$\dot{S}_1 = \frac{b_1}{\hat{b}_1} \left[ -\hat{h}_1 - K_1 \cdot S_1 \right] + h_1 = -\beta_1 \cdot K_1 \cdot S_1 + (1 - \beta_1) \cdot \hat{h}_1 + \Delta h_1 \quad (7)$$

where  $\beta_1 = b_1/\hat{b}_1 > 0$  and  $\Delta h_1 = h_1 - \hat{h}_1$ . The assumption that  $\beta_1 > 0$  means that we must know the sign of  $b_1$ .

Finally, if  $\lambda = \bar{\lambda}_d$ , the gain needed to guarantee convergence to a boundary layer of the surface,  $|S_1| \leq \Phi_1$ , is:

$$\frac{(1 - \beta_{1,\min}) \cdot |\hat{h}_1| + \max(\Delta h_1)}{\beta_{1,\min} \cdot \Phi_1} < K_1 \quad (8)$$

### DESIRED SLIP FILTER

In the development of the velocity controller,  $d_{br}$  and  $v_{rel,des}$  were differentiated. Similarly, the desired slip will need to be differentiated during the slip controller design. Notice that the derivative of  $\bar{\lambda}_d$  (Equation 6) results in many terms. To avoid this explosion of terms, the following filter is used to obtain  $\lambda_d$ :

$$\tau_f \cdot \dot{\lambda}_d + \lambda_d = \bar{\lambda}_d \quad \text{with } \lambda_d(0) = \bar{\lambda}_d(0) \quad (9)$$

The use of this filter allows  $\dot{\lambda}_d$  to be easily computed. It should be noted that  $d_{br}$  and  $v_{rel,des}$  are also filtered for the same reason before their use by the velocity controller.

### SLIP CONTROLLER

Following the velocity controller development, define:

$$S_2 = \lambda - \lambda_d \quad (10)$$

Differentiating  $S_2$  and using Equation 1 gives:

$$\dot{S}_2 = \frac{(1 - \lambda)}{m \cdot v} \left[ -c \cdot v^2 - F_r - k_\lambda \cdot \lambda \right] - \frac{r^2 \cdot k_\lambda}{J \cdot v} \lambda + \frac{r \cdot k_b}{J \cdot v} P_w - \dot{\lambda}_d = b_2 \cdot P_w + h_2(v, \lambda, \dot{\lambda}_d) \quad (11)$$

To cancel the unwanted nonlinear dynamics, define the desired wheel brake pressure to be:

$$\bar{P}_{w,des} = \frac{1}{\hat{b}_2} \left[ -\hat{h}_2 - K_2 \cdot S_2 \right] \quad (12)$$

Again,  $\bar{P}_{w,des}$  will be filtered before being used by the brake controller designed by Maciuca and Gerdes. Similar to above, the gain needed to converge to  $|S_2| \leq \Phi_2$  if  $P_w = \bar{P}_{w,des}$  is:

$$\frac{(1 - \beta_{2,\min}) \cdot |\hat{h}_2| + \max(\Delta h_2)}{\beta_{2,\min} \cdot \Phi_2} < K_2 \quad (13)$$

### ON/OFF HYSTERESIS

The CA controller will switch on whenever  $r \leq d_{br}$  and it will try to force  $r \rightarrow d_{br}$ . Switching logic with hysteresis is necessary to prevent the controller from turning on and off when  $r \rightarrow d_{br}$ . The first iteration switching logic turned on the controller when  $r \leq d_{br}$  and turned off the controller when  $r > d_{br} + 5$ . Unfortunately, this logic caused the brakes to be applied harshly when the controller turned on resulting in large initial accelerations. We wanted to eliminate these large initial accelerations without hindering controller performance over the remainder of the collision avoidance maneuver. Instead of turning on the controller abruptly based on the above logic, the desired wheel pressure was scaled by the following first order switch whenever the controller turned on:

$$\begin{aligned} \text{if } (t_{CA} < 1) \quad & d(\text{SWITCH})/dt = 0.2 \cdot (-\text{SWITCH} + 1) \\ \text{else} \quad & d(\text{SWITCH})/dt = 1.0 \cdot (-\text{SWITCH} + 1) \end{aligned} \quad (14)$$

For the first second after the controller turns on, the SWITCH rises slowly. This scaling factor prevents the desired wheel pressure from rising too rapidly and startling the driver. After the first second of the CA maneuver, the SWITCH rises rapidly and until its scaling effect is negligible. This allows high gains to be used in the controller without the consequence of large initial accelerations.

### RESULTS

The critical and drift scenarios described above were simulated on a half car vehicle model. The vehicle parameters used by the controller ( $m$ ,  $J$ ,  $r$ ,  $c$ ,  $k_b$ , and  $F_r$ ) were given 10% error. The tire slope,  $k_\lambda$ , is difficult to determine and was given 25% error. Furthermore, the front tire slip (%) was used for feedback. To simulate the use of a slip estimate, noise and a large bias were added to the front tire slip.

Figure 1 shows the controller performance in the critical scenario without the first order hysteresis switch. When the range drops below the critical braking distance (upper right plot), the controller switches on (upper left). This causes the slip controller to command a large step increase in wheel pressure, which is then tracked by the brake controller (lower right). As a result, the passenger experiences a large initial acceleration (upper left). After this large initial acceleration, the controller performs well, forcing the relative velocity to zero and the range to the critical braking distance. Also notice that the brake controller faithfully tracks the filtered desired wheel pressure produced by the slip controller, so that the assumption for the multi-surface controller holds. Finally, notice that the estimated slip used by the slip controller is noisy and biased as described above. The slip controller causes the estimated slip to converge to the filtered desired slip, but the true slip does not track the desired value. Yet, the velocity controller is able to overcome this error and still perform well.

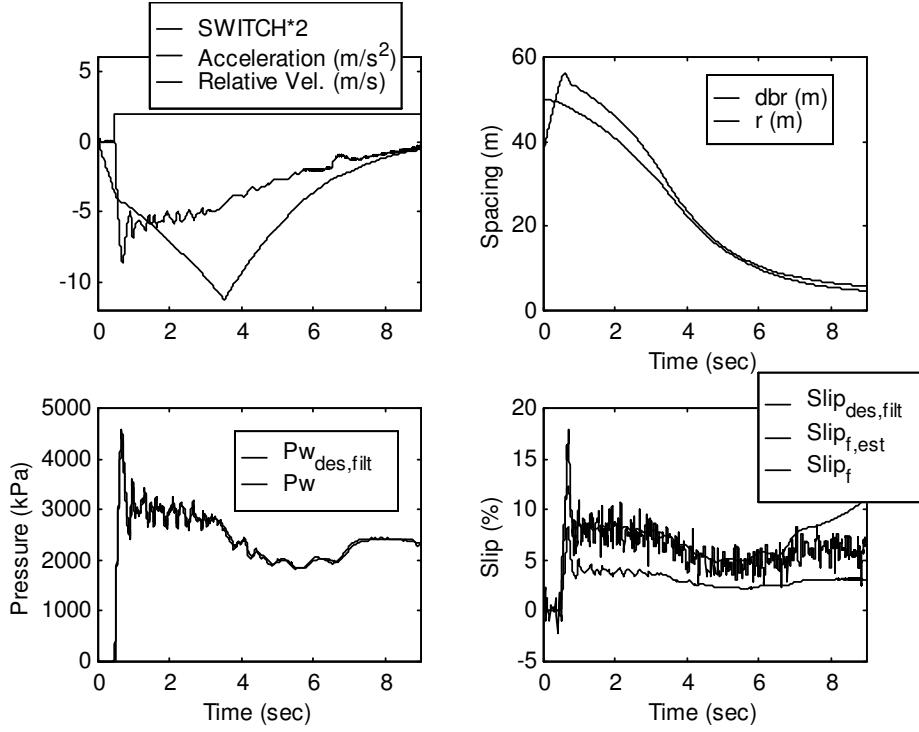


Figure 1: Critical Scenario On/Off Switch

Figure 2 shows the drift scenario without the first order switch. As soon as the range drops below the critical braking distance, the controller harshly applies the brakes. This large initial acceleration would not be acceptable to most drivers considering the mildness of the situation. This first order switch was then implemented to reduce this initial harshness. The results (Figure 3) show that the switch turns on very slowly for the first second after the controller turns on. As a result, the initial accelerations are mild. After the first second, the switch rises rapidly to a gain, allowing the controller performance to improve as the error decreases.

One concern about using the switch is that it may hinder the vehicle performance in critical situations. The results of the critical scenario with the first order switch (Figure 4) show the controller is still able to prevent the collision. Furthermore, the controller does not slam on the brakes, making the results more desirable from a human factors point of view.

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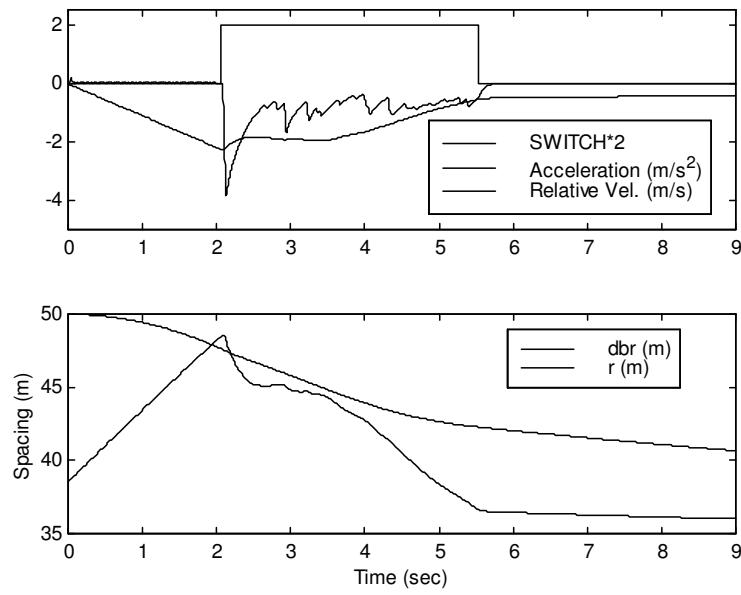


Figure 2: Drift Scenario with On/Off Switch

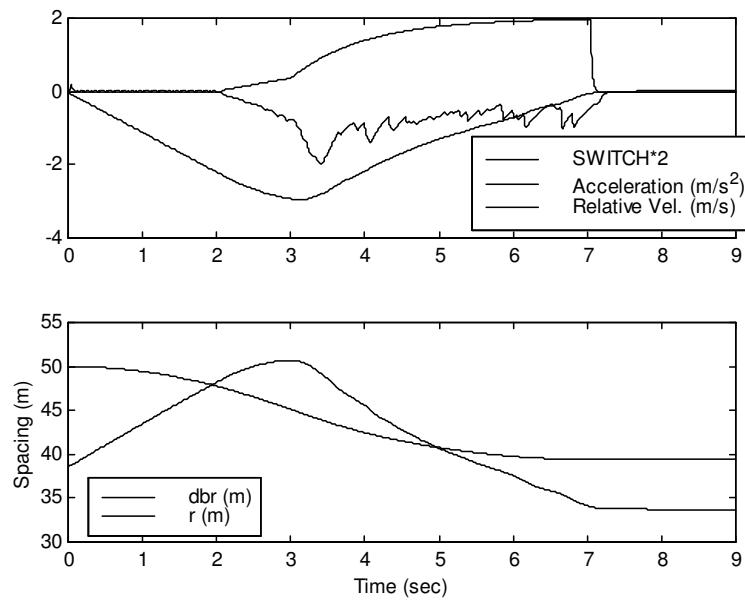


Figure 3: Drift Scenario with 1<sup>st</sup> Order Switch

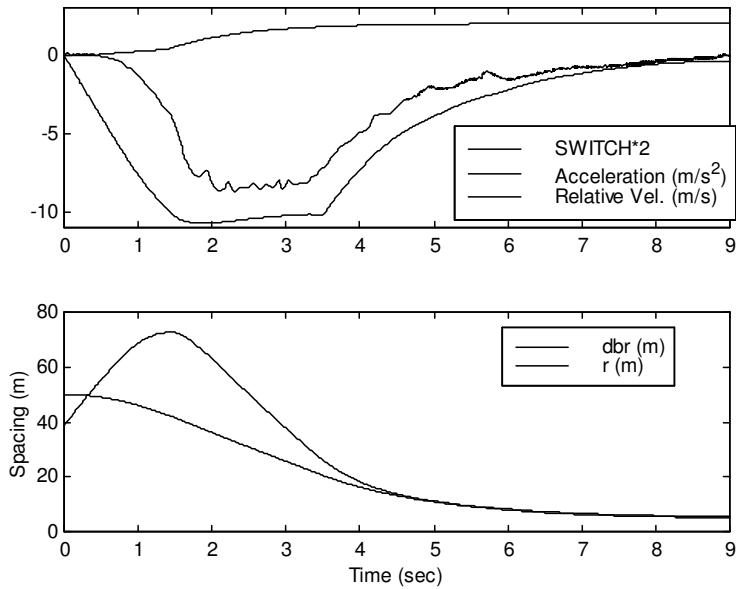


Figure 4: Critical Scenario with 1<sup>st</sup> Order Switch

## REFERENCES

1. NHTSA. Report to Congress on the National Highway Traffic Safety Administration ITS Program. Technical Report, U.S. Department of Transportation, January 1997.
2. A.D. Horowitz and T.A. Dingus. "Warning Signal Design: A Key Human Factors Issue in an In-Vehicle Front-to-Rear Collision Warning System" *Proceedings of the Human Factors Society*, vol. 2, p. 1011-1013, Atlanta, Georgia, October 1992.
3. Y. Fujita, K. Akuzawa, and M. Sato. "Radar Brake System" *1995 Annual Meeting of ITS America*, vol. 1, p. 95-101, Washington, D. C., March 1995.
4. P. Seiler, B. Song and J. K. Hedrick. "Development of a Collision Avoidance System" *SAE Conference*, p. 97-103, Detroit, Michigan, February 1998.
5. J.C. Gerdes. "Decoupled Design of Robust Controllers for Nonlinear Systems: As Motivated by and Applied to Coordinated Throttle and Brake Control for Automated Highways" Ph.D. Thesis, University of California at Berkeley, Berkeley, CA, 1996.
6. D.B. Maciucă. "Nonlinear Robust Control with Application to Brake Control for Automated Highway Systems" Ph.D. Thesis, University of California at Berkeley, Berkeley, CA, 1997.
7. J.J.E. Slotine and W. Li. *Applied Nonlinear Control*, Prentice Hall, 1991.
8. D.E. Goldman and H.E. von Gierke. Chapter 44 in *Shock and Vibration Handbook*, Cyril M. Harris (ed.), McGraw-Hill Book Company, 3<sup>rd</sup> edition, 1988.