

- 1) Take one pill from the first bottle, 2 pills from the second, 3 pills from the third bottle and so on..

Put all these pills together and weight them.

Subtract this weight from $(1+2+3+\dots+20) = (20 \cdot 21)/2 = 210$

Divide the difference by 0.1 . The answer will indicate the bottle number containing the overweighed pills.

- 2) Game 1: Probability of winning the game: p

Game 2: the number of ways you win the game:

$(WWL), (WLW), (LWW), (WWW)$ where W is the event when you win

L is the event when you lose

Probability of $(WWL) = p \cdot p \cdot (1-p)$

Probability of $(WWW) = p^3$

Hence probability of winning the game $= p^3 + p^2(1-p) \cdot 3$

If the probability of the winning the first game is higher than the probability of winning the second game:

$$p > p^3 + p^2(1-p) \cdot 3$$

$p^2 + 3p(1-p) < 1$ since p cannot be less than zero

$$3p - 2p^2 < 1$$

$$(2p-1)(p-1) > 0$$

Since $p-1$ cannot be positive hence $2p-1 < 0$

Therefore $p < 0.5$

Therefore when $p < 0.5$ then it is better to play game 1 else play game 2

- 3) When the two squares are taken from the diagonally opposite corners, two opposite edges will have just seven squares. Thus one of the squares will pair up with the a square in the next layer and this continues. Thus 31 dominoes cannot be formed from the board.
Another way of describing is that , the chessboard initially had 32 white and 32 black squares. Now, after the removal of diagonally opposite squares, the number of squares of one colour has reduced to 30 and the number of squares of another color remains 32. Hence under such condition it is not possible for 31 dominos to be formed

- 4) The probability that all the n ants located on n vertices are going at a single direction: $(1/2)^N$
The probability that all the n ants located on n vertices are going in the anti-clockwise direction: $(1/2)^N$

Therefore probability of no collision: $2 \cdot (1/2)^N$

$$= (1/2)^{(N-1)}$$

Probability of collision: $1 - (1/2)^{(N-1)}$

- 5) **In the first pass:**

Fill the 5 quart jar to the brim. Pour water to the 3 quart jar from 5 quart jar and hence $(5-3 = 2)$ quart remains in the five quart jar

In the second pass:

Fill the 3 quart jar with the quantity of water left in the 5 quart(2quart of liquid). Fill the 5 quart of jar. Now it contains 5 quart of water.

Since the 3 quart of jar is already filled with 2 quart of fluid. Pour fluid from 5 quart to 3 quart jar so as to fill the 3 quart jar completely. Thus 1quart flows out from 5 quart jar and four quart remains behind.

6) N=1

If the number of blue eyed people is one , everyone other than the blue eyed people can see one person having blue eyes and hence the person having the blue eyes will reason this out and will leave on the first night

N=2

If the number of people having blue eyes are two then each of the said person can see the other person having blue eyes. But after the first night they will realize that the blue eyed person must be seeing another person as having blue eyes and they will reason that this will continue for sometime.

This is the precise process that will happen for all the number of blue eyes. Hence if the number of blue eyed people are I, then all will realize on the (i-1) day and will then leave the ship.

7) B: event when a boy is born = 0.5

G: event when a girl is born = 0.5

Hence the event of having children in the family is – G+BG+BBG+BBBG+BBBBG+..

Event	Probability of the event	Probability of the boy
G	0.5	$0 \cdot 0.5$
BG	0.25	$1 \cdot \frac{1}{4}$
BBG	$\frac{1}{8}$	$2 \cdot \frac{1}{8}$
BBBG	$\frac{1}{16}$	$3 \cdot (\frac{1}{16})$
BBBBG	$\frac{1}{32}$	$4 \cdot \frac{1}{32}$
BBBBBG	$\frac{1}{64}$	$5 \cdot (\frac{1}{64})$
BBBBBBG	$\frac{1}{128}$	$6 \cdot (\frac{1}{128})$

The summing up all the events for a boy: $(\frac{32}{128} + \frac{32}{128} + \frac{24}{128} + \frac{16}{128} + \frac{10}{128} + \frac{6}{128}) = (\frac{120}{128})$ which is equivalent to 1 indicating that the gender ratio remains the same .

8) Since the load factor for both the eggs should be balanced, therefore, the interval gap for egg1 should be progressively decreased.

$$N+(N-1)+(N-2)+\dots+1 = N(N+1)/2 = 100$$

Therefore solving for N=14

Therefore the first drop should be at a height of 14 ft

- 9) From the problem it is very evident that those doors will remain open which has odd number of factors. Except for the square numbers, every other number has even number of factors. Hence only square numbers will have the door toggled to open.

(1,4,9,25,36,49,64,81) = 9 doors will remain open

- 10) The bottles are divided into batches of 100 bottles.

The first day:

The first strip will be soaked by bottles having zero in their MSB

Second strip will be soaked by bottles having 1 in their MSB and so on ..

The second day:

The first strip will be soaked by bottles having zero in their ten's position

Second strip will be soaked by bottles having 1 in their ten's position and so on..

The third day:

The first strip will be soaked by bottles having zero in their LSB

Second strip will be soaked by bottles having 1 in their LSB and so on..

So if the poisoned bottle has different digits in its id then this scheme will suffice

However if the poisoned bottle has an id that has repeated digits then this scheme will not help as there wont be any change in one of the strip showing repetition. To counteract that, the strips for the LSB should be shifted and used on the fourth day

So the first strip should be soaked by all bottles whose id ends in a 9

The second strip should be soaked by all bottles whose id ends in a 0

The third strip should be soaked by all bottles whose id ends in a 1

The fourth strip should be soaked by all bottles whose id ends in a 2 and so on..

If any new strip shows a change then the LSB was the previous numbered strip and this can be easily comprehended

Example: ID – 353 is poisoned

Day 1 : Strip 4 will have a change

Day 2: Change in strip 6

Day 3: No change and hence the LSB can be either 3/5

Day 4: Strip 5 changes. Since there is a shift for using the strips hence it is clear that the LSB is 3