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Representations in mathematics Textbooks, Learning and Teaching: Multiple perspectives

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Introduction

- In East Asian regions such as Hong Kong, it is known that textbooks play a significant influence in the teaching in the classrooms.

- Textbook has an important interim role between the intended curriculum and the implemented curriculum.

Introduction

- What counted as good textbooks?
- How can textbooks be used in the teaching inside the classrooms?
  - Enactive, iconic and symbolic representations (Bruner)
  - Functions of multiple external representations (Ainsworth, 1999)
  - Representation of problem types (Zhu, 2003)
  - A lesson in Hong Kong via the framework of variation
  - Conclusion & Discussion

A problem in Secondary 2 in70's: To change a denary number to a binary number

Express $11_{(10)}$ as binary numbers. **$11_{(10)}$** **$= 1x\ 2^3 + 0x2^2 + 1x2^1 + 1$** **$=1011_{(2)}$**	**$2\underline{)\ 11}$** **$2\underline{)\ 5}$ ……1** **$2\underline{)\ 2}$ ……1** **1……0**

The topic had been removed and is now back.

Questions for my student teachers:

- What does each "1" in the repeated division mean?
- Why does the algorithm work?

11	
5….1	2) 11 5 ……1
2 groups of 4 1 group of 2 1	2) 11 2) 5 ……1 2 ……1
1 group of 8 0 group of 4 1 group of 2 1	2) 11 2) 5 ……1 2) 2 ……1 1……0

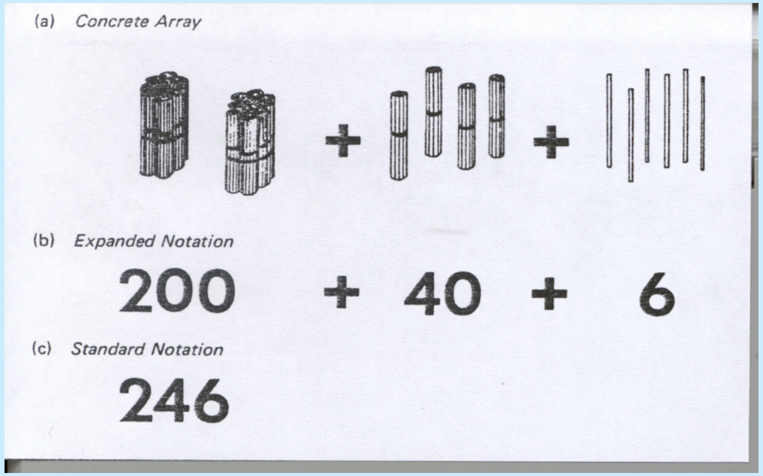
Doing division with real objects. What does dividing by two mean?

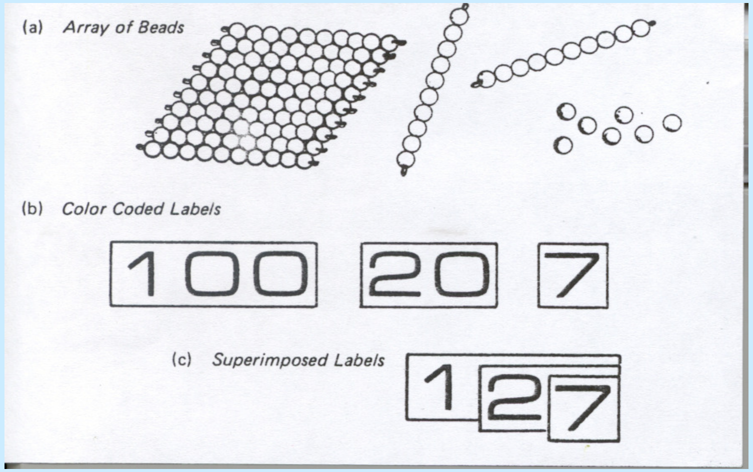
- Some keep dividing the group of coins into two groups and end up in nowhere.
- Hint: Besides dividing into two groups, division by two can also be a way to answer the question: How many groups of two are there?

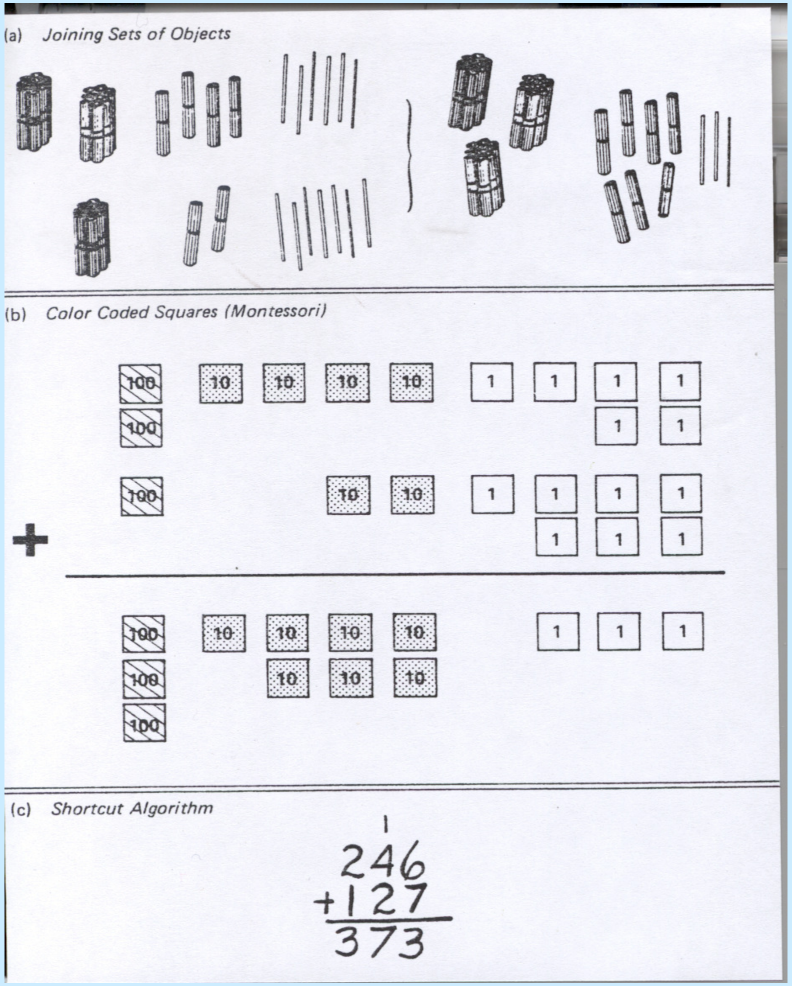
Bruner

- Enactive representation
- Iconic representation
- Symbolic representation
- Spiral Approach

A sequence used often in the case of early learning of numbers: Enactive -> Iconic -> Symbolic







Different sequence of experiencing

$11_{(10)}$

$= 1x\ 2^3 + 0x2^2 + 1x2^1 + 1$

$=1011_{(2)}$

2) 11

2) 5 ……1

2) 2 ……1

1……0

• •••••• • •• • •• •• •• •• •• •

1
1

5….1

•• •• •• ••
••
•

2 groups of 4
1 group of 2
1

•• ••
•• ••
••
•

1 group of 8
0 group of 4
1 group of 2
1

What may they learn? Different representations, relationship between different representations, better understanding

Multiple External Representations MERs (Ainsworth, 1999, 2006)

- Complementary function
- Constrain interpretation
- Construct deeper understanding

Complementary function: the equation of $y=x^2+5x+3$ [y=f(x)] & graphs

Each of the representations provides different information or support different mental processing.

1. Y=f(x): putting some values of x into the “formula” to obtain the corresponding y value.
   - a formula with an instrumental purpose for finding the value of y locally for a single value, rather than an apprehension of a relationship between two variables as a whole.
   - The experience is computational, i.e., linked with numbers calculation.
2. Graphs: Those properties associated with the whole picture of the functions such as increasing / decreasing trends, maximum or minimum

Constrain understanding

- A representation would constrain the interpretation of another. This can be achieved either by the use of a familiar representation to constrain the interpretation of an unfamiliar, or by the use of the inherent properties of representation to constrain the interpretation of a second one.

E.g. To transform the information of a question to table, or from table to equations; or from verbal description to a diagram.

Construct deeper understanding:

- By ‘deep understanding’, it refers to an understanding of generalized / abstract ideas, the relationship between representations, or the extension of meaning from one representation to another.
- E.g., Subtraction from Dienes blocks to written numerals; multiple readings of 3(x+5)+1 (sequence of procedures to abstract expression of a function); algebraic method to graphical method for solving equations.

Representation of Problem types

1. Zhu's study
2. The Hong Kong Study – Junior secondary algebra

Problem Solving in China, Singapore, and US Mathematics Textbooks: A Comparative Study

Zhu, Y. (2003)
yanzhu@hku.hk

Research Question

How problem solving is represented in three series of mathematics textbooks at the lower secondary level in China, Singapore, and the United States?

- **What and how different types of problems are represented in the three countries' mathematics textbooks?**
- **What and how different problem solving strategies, including heuristics, are represented in the three countries' mathematics textbooks?**

COUNTRIES

United States
China
Singapore
80°
40°
0°
40°
80°
160°
120°
80°
40°
0°
40°
80°
120°
160°

Background and Rationale

- **Many available cross-national comparisons revealed that, overall, the performance of East Asian students in mathematics was better than that of US students.**
- **However, when assessment tasks included higher-order thinking problems, US students either performed equally well as or better than East Asian students.**

•A wide array of problems could better assess students' ability in problem solving!

•A better understanding of textbooks and their roles enable teachers to make wiser decisions!

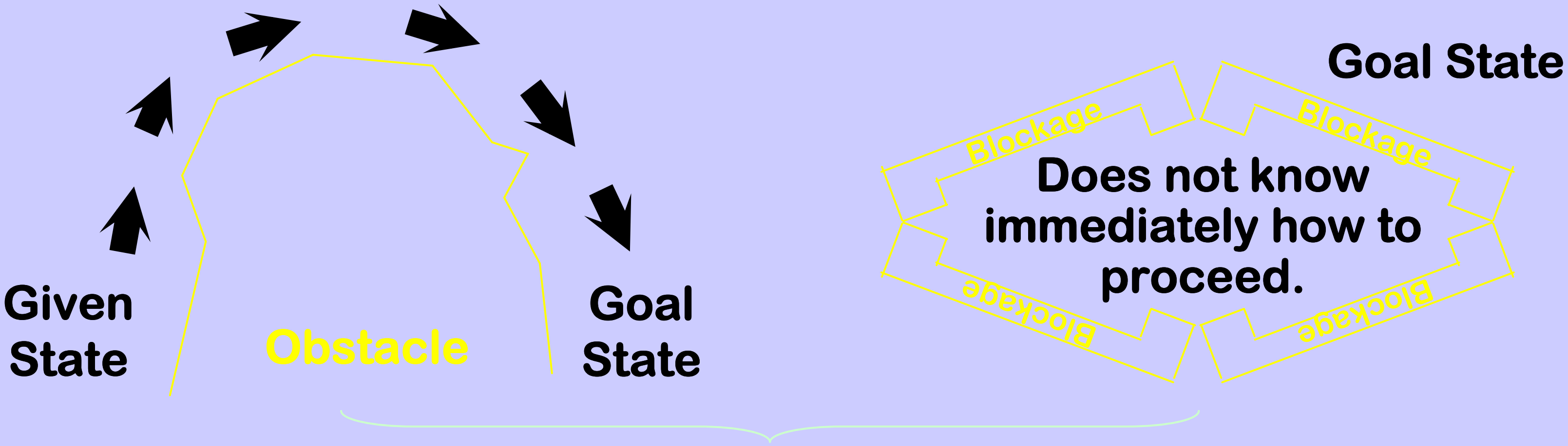
Conceptual Framework

Definition of problems

Classification of Problems

Problem Solving Procedure

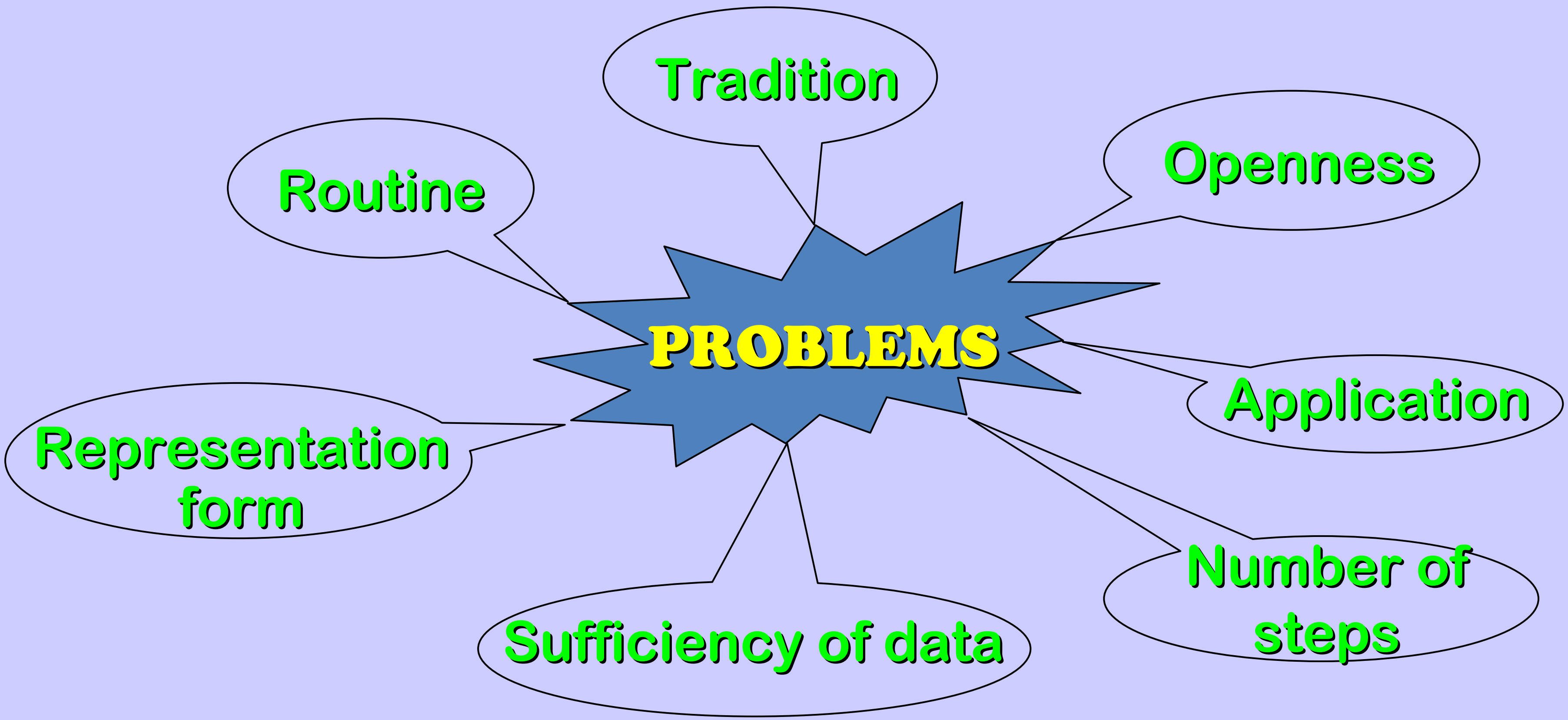
Definition of Problems



Problem

A situation that requires a decision and/or answer, no matter the solution is readily available or not to the problem solver.

Classification of Problems



General Description

- The US textbooks contain many more problems than the Asian books, in terms of the total number.

  Chinese: 6850   Singapore: 9914   US: 13286

- In the two Asian series, many chapters set problems on opening pages, whereas the US series does so less frequently.

- The ratio of exercise problems to text problems in the US series is higher than that in the other two series.

Representation of Problems in Various Types

- Routine problems and traditional problems comprise the majority of problems in all the three textbook series.

- The non-traditional problems in the US textbooks are more varied than those in the Asian books.

- More than 90% of problems in the three examined series are close-ended problems.

- The majority of problems in all the textbooks are not contextualized in the real world situations.

Conclusions (I)

- Among the large number of problems in each textbook series, most are routine, traditional, and close-ended.

- Compared to the Asian textbooks, the US books provide a larger variety of problems (e.g., richer contexts and various non-traditional problems).

- Problems in the Asian textbooks are overall more challenging than those in the US textbooks, in terms of the number of steps involved in problem solutions.

Conclusions (II)

- The US textbooks model Pólya’s four stages through the representation of problem solutions more frequently than the Asian books.

- The Singapore textbooks introduce the most number of specific problem solving heuristics (China: 11, Singapore: 16, US: 14).

- However, all the textbooks do not illustrate the use of heuristics in solving problems frequently.

The Hong Kong Study: Junior Secondary Algebra

Mok, Zhu, Yau

Textbook analysis

- Grade : Junior Secondary 1 – 3
- Topics selected: Algebra
- Textbook selected: Top three seller

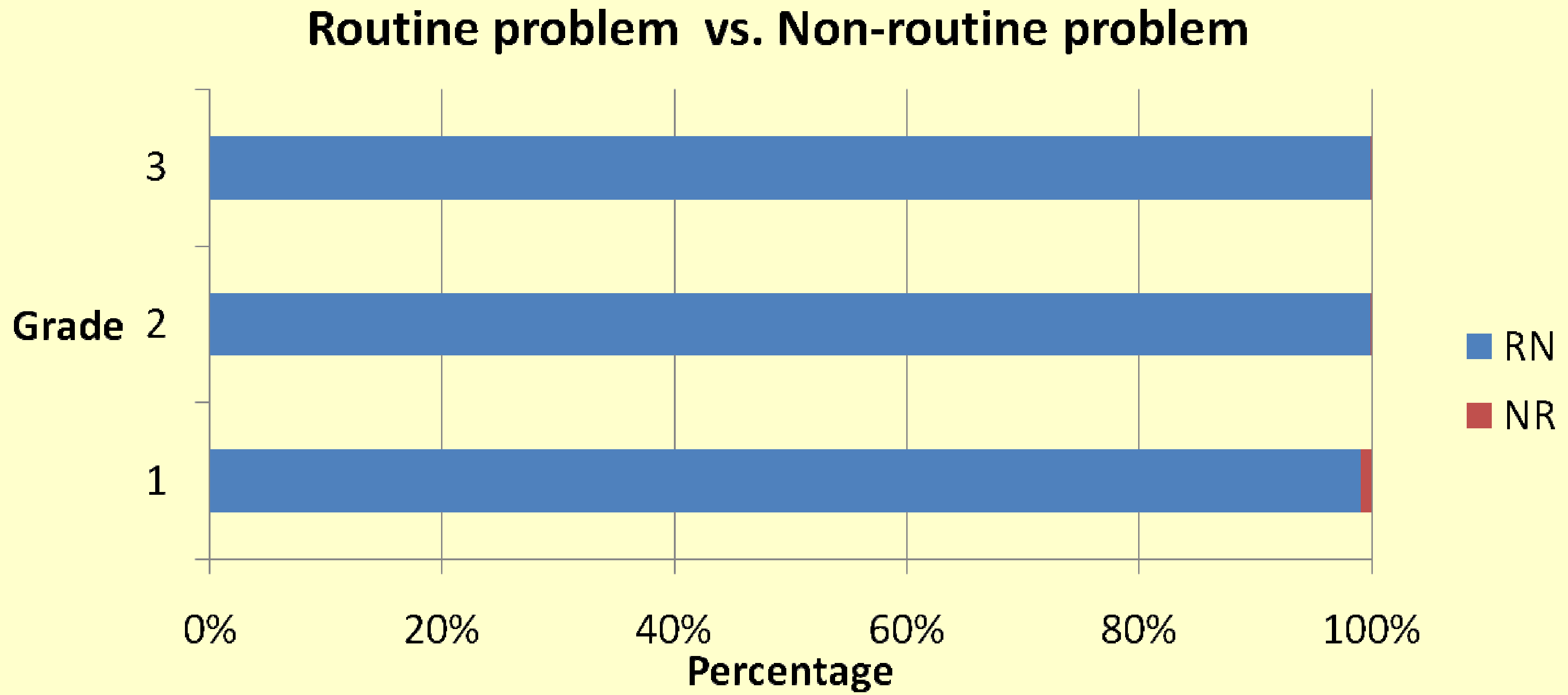
Topics

S1	S2	S3
Basic Mathematics	Approximation and Errors	More about Factorization of Polynomials
Directed Numbers	Manipulations and Factorization of Polynomials	Law of Indices
Using Algebra to Solve Problems (I)	Identities	Percentages (II)
Percentages (I)	Formulae	Linear Inequalities in One Unknown
Using Algebra to Solve Problems (II)	Linear Equations in Two Unknown	
Estimation in Numbers and Measurement	Rate and Ratio	
	Pythagoras' Theorem (algebra only)	

Coding

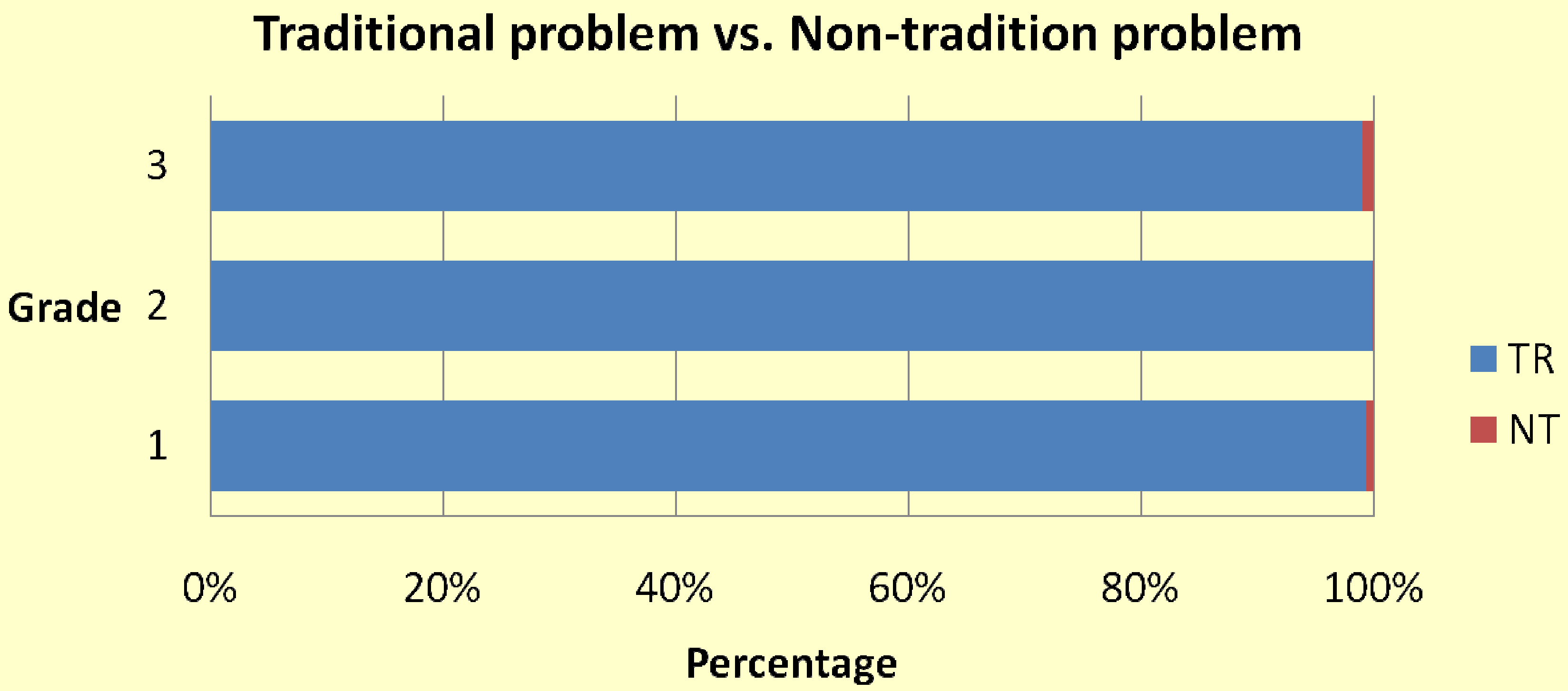
- 7 Codes
  - Routine (RN) vs. Non-routine (NR)
  - Traditional Problems (TR) vs. Non-traditional Problems (NT)
  - Open-ended Problems (OE) vs. Closed-ended Problems (CE)
  - Applications Problems (FAP or AAP) vs. Non-application Problems (NA)
  - Single-step Problems (SS) vs. Multiple-step Problems (MS)
  - Sufficient Data Problems (SD), Extraneous Data Problems (ED) and Insufficient Data Problems (ID)
  - Problems in a Purely Mathematics Form (PM), Problems in a Verbal Form (VB), Problems in a Visual Form (VF), and Problems in a Combined Form (CF)

Finding



	S1	S2	S3
Routine problem (RN)	1618 (99.08 %)	2350 (99.75%)	1270 (99.84%)
Non-routine problem (NR)	15 (0.92%)	6 (0.25%)	2 (0.16%)

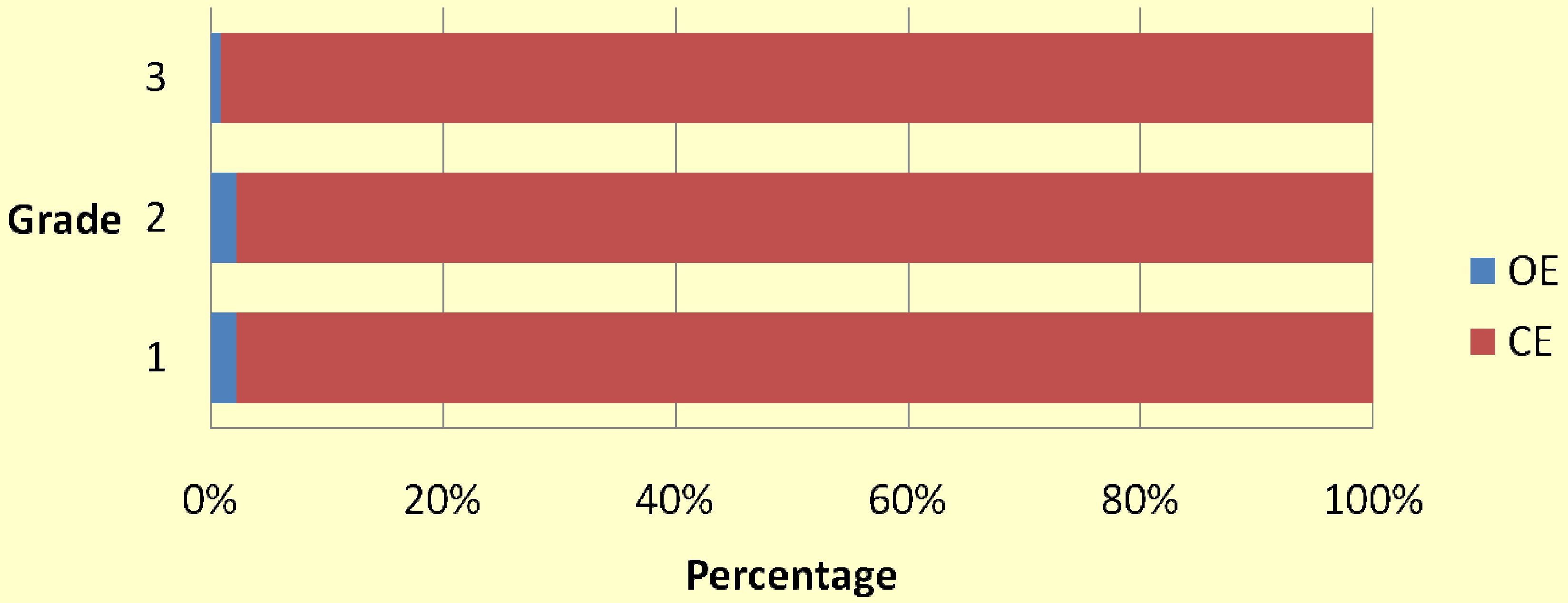
Finding



	S1	S2	S3
Traditional problem (TR)	1619 (99.26 %)	2352 (99.83%)	1259 (98.98%)
Non-traditional problem (NR)	12 (0.74%)	4 (0.17%)	13 (1.02%)

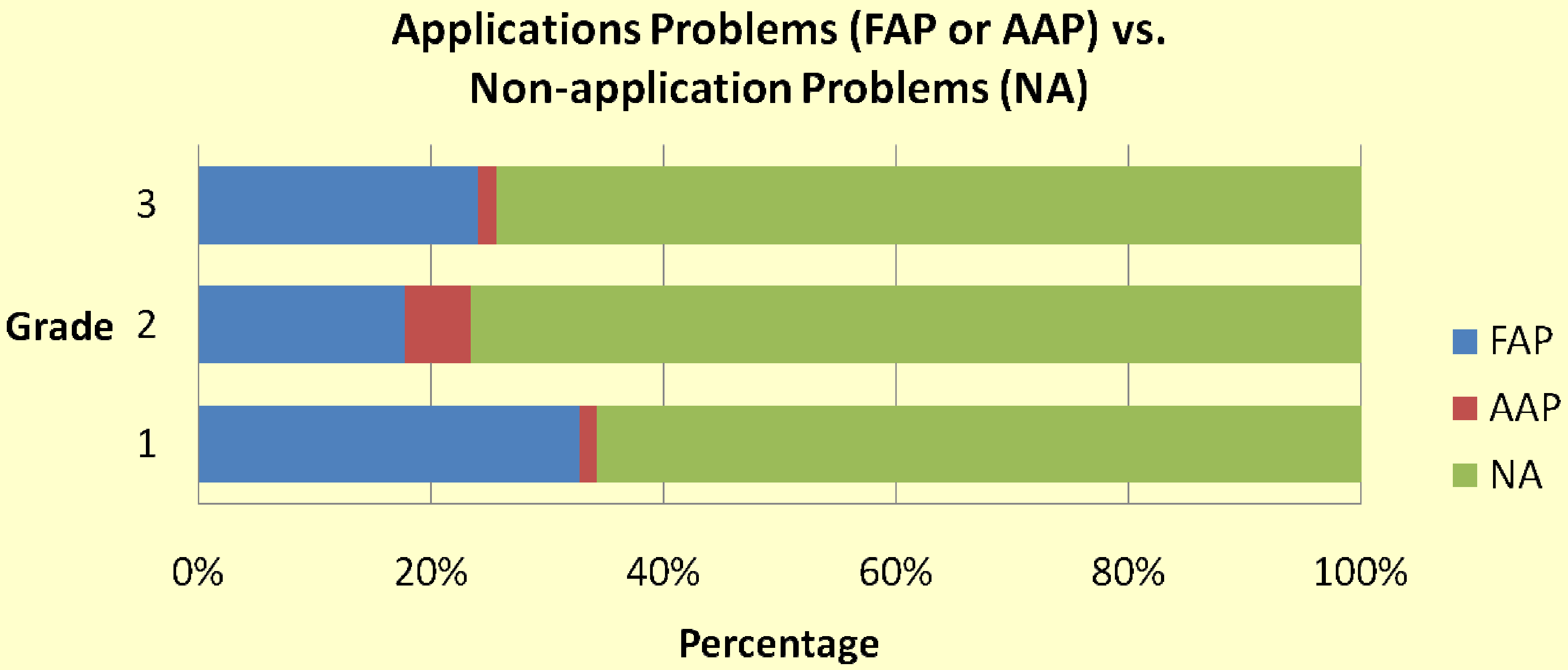
Finding

Open-ended problem vs. Closed-ended problem



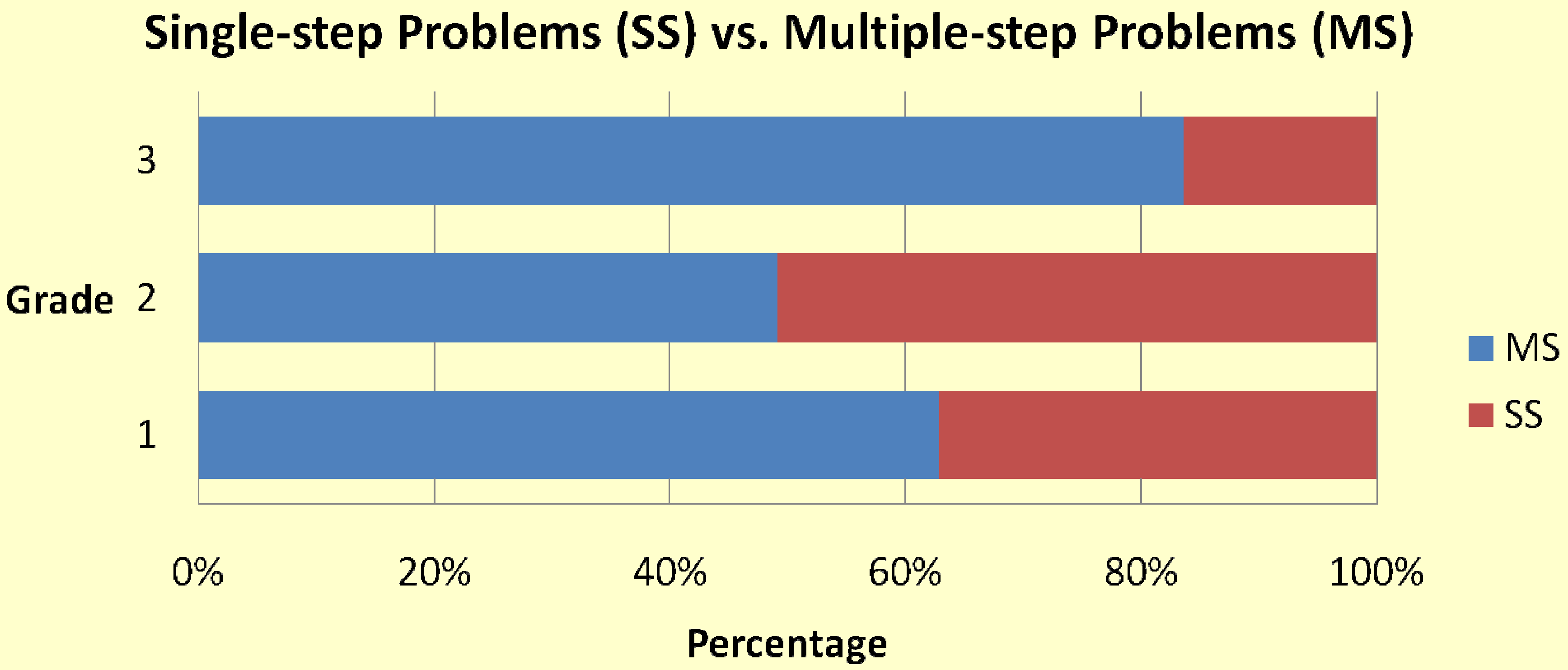
	S1	S2	S3
Open-ended problem (OE)	1594 (97.73%)	2303 (97.75%)	1260 (99.06%)
Closed-ended problem (CE)	37 (2.27%)	53 (2.25%)	12 (0.94%)

Finding



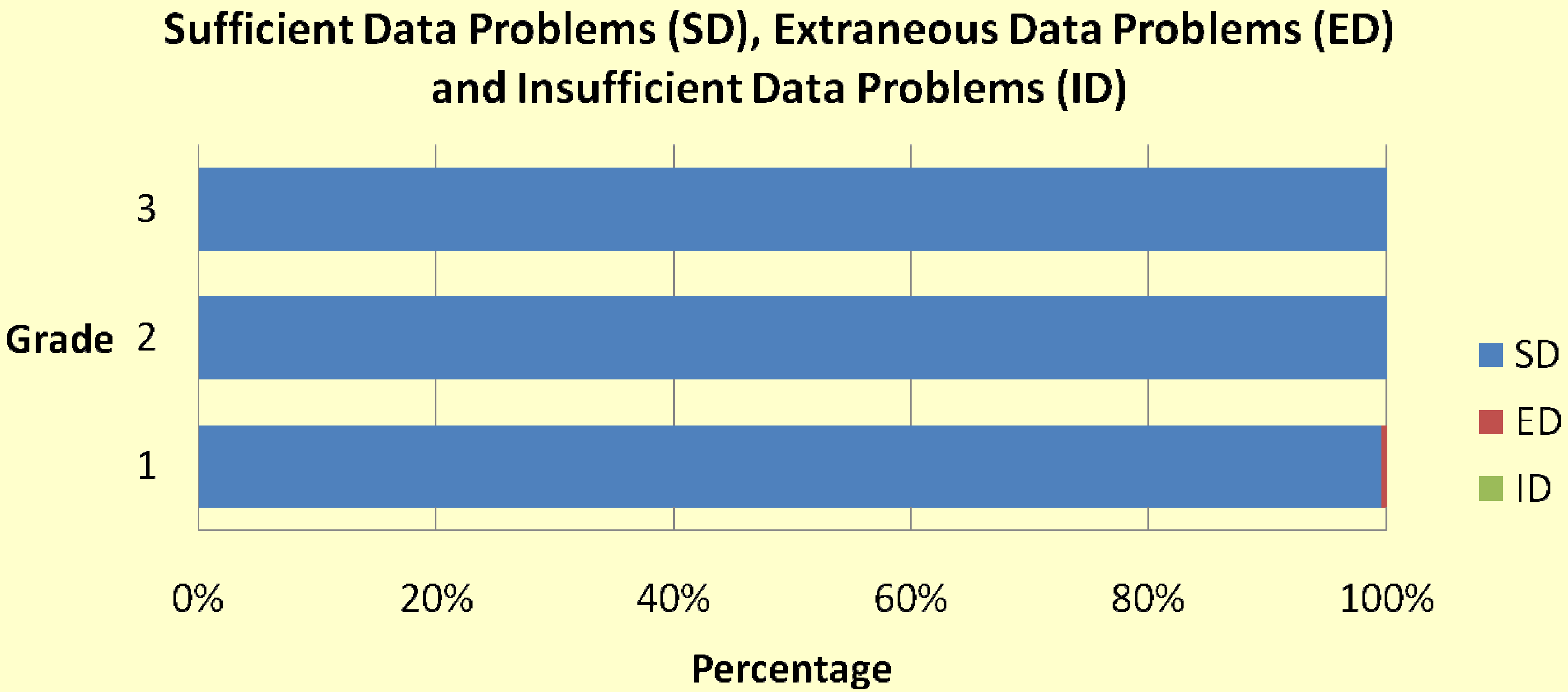
	S1	S2	S3
Fictitious application problems (FAP)	536 (32.86%)	416 (17.66%)	306 (24.06%)
Authentic application problems (AAP)	23 (1.41%)	134 (5.69%)	21 (1.65%)
non-application problems (NA)	1072 (65.73%)	1806 (76.66%)	945 (74.29%)

Finding



	S1	S2	S3
Single-step problem (SS)	608 (37.28%)	1199 (50.08)	206 (16.19)
Multiple-step problem (MS)	1023 (62.72%)	1157 (49.11)	1066 (83.81)

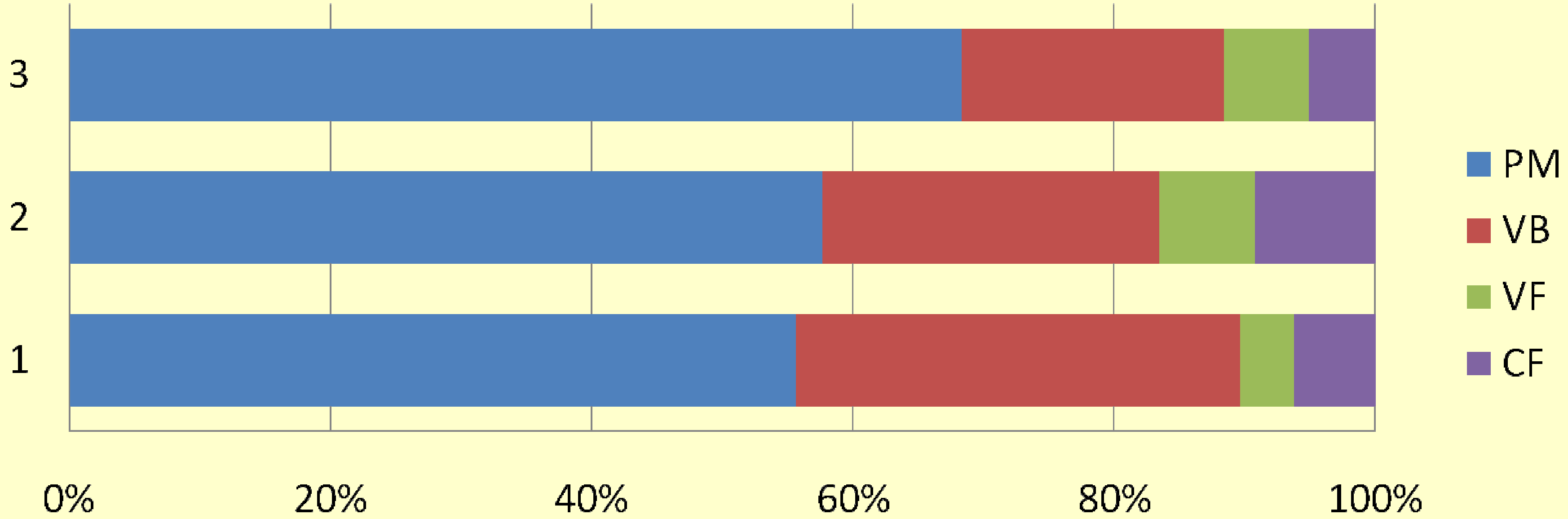
Finding



	S1	S2	S3
Sufficient data problem (SD)	1623 (99.51%)	2356 (100%)	1272 (100%)
Extraneous data problem (ED)	8 (0.49%)	0 (0%)	0 (0%)
Insufficient data problem (ID)	0 (0%)	0 (0%)	0 (0%)

Finding

Problems in a Purely Mathematic Form (PM), Problems in a Verbal Form (VB), Problems in a Visual Form (VF), and Problems in a Combined Form (CF)



	S1	S2	S3
Purely mathematic form (PM)	906 (55.55%)	1575 (66.85%)	868 (68.24%)
Verbal form (VB)	555 (34.03%)	385 (16.34%)	256 (20.13%)
Visual form (VF)	68 (4.17%)	169 (7.17%)	83 (6.53%)
Combined form (CF)	102 (6.25%)	227 (9.63%)	65 (5.11%)

Finding

- Most questions are routine
- Most of them are traditional; only two types of non-traditional questions are found (journal and puzzle)
- The open-ended questions decreases from Grade 2 to Grade 3
- Non-applicable problems shares 65 – 76% of questions;
- Multiple steps questions becomes dominant in Grade 3
- The majority of data is sufficient data problems; Extra data problems only find in Grade 1; No insufficient data problems find among three Grades
- 55 – 68 % of questions are in purely mathematical form; 20 – 33% are in verbal form; Visual Form and Combined Form are more or less the same.