Mok-Santiago Part II

An Algebra Lesson on Factorization of Polynomials

Mok

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Objectives of this presentation

What may a lesson with quality instruction look like in the Hong Kong education system?

- A researcher's perspective (Variation Theory): How the teacher's teaching may possibly create learning opportunities for the learners to see the mathematical object in a certain way.
- The researcher's analysis was placed in contrast with the teacher's and students' commentary of the same lesson.

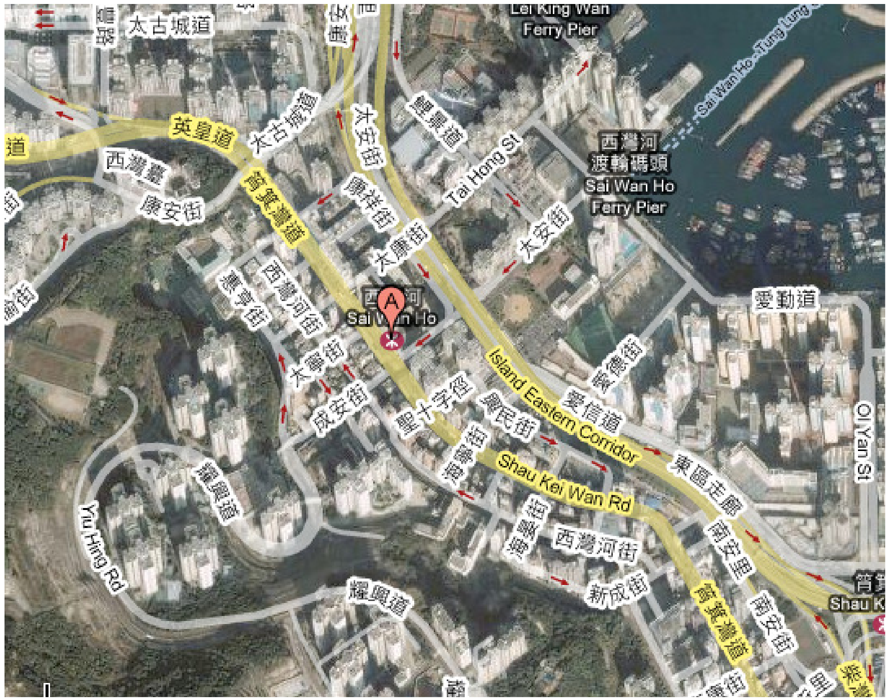
The HK Education System: 6-3-2-2

- Hong Kong education system consists of 6 years of primary (ALLOCATION)
- 3 years of junior secondary (ALLOCATION)
- 2 years of senior secondary (EXAM)
- 2 years of Advanced-level (EXAM)
- University

The Learners' Perspective Study: The source of data

- Three competent teachers of each region were selected and at least ten consecutive grade-8 lessons were video-recorded.
- Two students were interviewed after each lesson.
- Each teacher participated in three video-stimulated interviews.
- Copies were also made of student written materials, textbook pages, and worksheets used in class.

The neighborhood of the school



School HK1

- The school uses Chinese as the medium of teaching, which was the choice of the majority of the schools in Hong Kong.
- Among the schools in the city, the school belonged to average standard.
- According to the teacher, the students could be counted as a combination of average and slightly above average in mathematics based on the internal screening of the school.
- The average International Benchmark Test IBT score was 40.26 out of 50.

The teacher

- Mr. X is a teacher with more than twenty years of experience in both primary and secondary mathematics teaching. He is active in teaching, curriculum development and research activities. He is recommended as a very good teacher by local mathematics educators, his school principal, colleagues and students.

The students liked their lessons.

- Out of the 34 student interviews, 31 students expressed that they liked their mathematics lesson, two did not express any opinion and only one said that he did not like the lesson.

The theory of variation

- Marton, et al. (2004): Learning is a kind of experiencing in which the learners develop a way of seeing or experiencing.
- Learning in lessons is centred on objects of learning and it is very important for the learners to discern critical features of the object of learning. A key feature enhancing learning and awareness is variation.

Looking for variations in the analysis

- Contrast – a comparison between what the object is and what it is not, e.g., “three” and not three, such as “two” or “four”.
- Separation – separating a certain aspect of the object from the other aspects. To experience this, one aspect must vary while other aspects remain invariant.
- Fusion – an experience of taking several critical aspects into account.

The lesson: Factorization of Polynomials

- Introduction in the form of teacher-led whole class discussion (4 minutes).
- Main part (16 minutes) of teaching in the form of teacher-led whole class discussion, which included a very brief small group discussion (less than one minute).
- Supporting activities included individual seatwork, teacher's between-desk-instruction students working on the board and the teacher's comments on the students' board work (7 minutes).

Snapshots of the lesson events

Introduction of the name of the topic “Factorization of Polynomials”

T: “What does it remind you of?”

Associated meaning of the name and old knowledge

Patrick: Addition and subtraction.

T: It reminds you of adding and subtracting numbers. Thank you. Why do you think about addition and subtraction?

Invitation of elaboration

Patrick: Simplify complicated things.

T: Who’s more imaginative? What comes to your mind when you’re reading this question, factorizing polynomials? What do you know and what do you want to know?

Invitation of participation & probing

Mark: Learning factors.

Starting from a very simple example "m(a+b)=ma+mb"

T: This is a very simple expansion of polynomial. From here to here [the teacher writing two arrows of opposite directions on the blackboard] multiplication starts from left to right. But from right to left…you know it's correct once you see it. This has a name. What's the name? [The teacher wrote the word "factorization" below the identity on the blackboard.]

T: The name is factorization. Expanding two or more polynomials. If you do it in the reversed way and make the answer…make the answer into the question, this is called factorization.

A contrast between multiplication and factorization: iconic (arrows), reading (L-R and R-L)

Correction/Feedback for a student's example: "2(m+s)=2m+2s"

T: It's absolutely correct. Can you explain which direction you should go in factorization? You've given me the example, but it's just an example of expansion. I won't count it as an example of factorization.

Probing for the student's explanation

Mark: Two is their common factor.

T: Is it called factorization if I go from left to right or from right to left?

Probing for the contrast

Mark: From right to left.

T: Thank you. I want to emphasize this. In order to…explain this more clearly…we use to read from left to right. This is our habit. When we look at the graphs, whether it ascends or descends, we read from the left to the right. In order to show you what is factorization, we write in this way. [The teacher wrote 2m+2s=2(m+s) replacing the student's on the blackboard.]

T: From left to right. We simply call this factorization. You may think that this is very easy. You only have to reverse it and change the answer into a question. If the question asks you to multiply…eight, that's it. Is it that simple? Theoretically, it is, but there're many techniques in the questions.

Starting from the simple but more to follow

Variation in a sequence of 7 examples

Q.1 $na+nb$

Q.2 $2a+2b$

Q.3 $2na+2nb$

Q.4 $-2na-2nb$

Q.5 $-2na+2nb$

Q.6 $2na+2n^2b$

Q.7 $2na+2n^2b^2$

Variation is built in the design of the tasks: the contrast between the questions

Fusion: Observing different aspects of the process of factorization together

T: Pay attention now! Questions one and two are easy! Questions one and two are similar to Question three. It involves "two", and it involves "n". Is it embarrassing? Which one should we deal with first?

- [After getting the three answers, “n(2a+2b)”, “2n(a+b)” and “2(na+nb)”, from the students,]

T: n brackets two a plus two b. Okay! First of all, I would like to know…these three answers…are they the same as the original formula?

S: The same!

T: Then are all three answers correct?

S: No!

T: No? You said they are the same and now they are not all correct? How come? Discuss with your classmates and then tell me your conclusions!

Q.1 na+nb, Q.2 2a+2b, Q.3 2na+2nb

Expecting more…than intuition

T:[The discussion lasted for less than one minute.] Time's up. Some of you have no conclusion! Well…in fact…these three answers…leave out those things, they are actually the same as the original. Which one is the factorization then? The first one, second, or the third?

S:The first one!

T:The first one! Why is it the first one?

S:It seems to be! [All laughing]

Q.1 $na+nb$
Q.2 $2a+2b$
Q.3 $2na+2nb$

Contrast with $12=4\times3$

"$n(2a+2b)$",
"$2n(a+b)$",
"$2(na+nb)$"

T: It seems to be. Okay, Thank you. Let's think back when we were in primary school. How did we do the factorization? …Can we write in that way? [The teacher wrote $12=4\times3$ on the blackboard]

T: Can we? That's factorization, right? What then? How to rewrite it then? Two times two times three. Okay! We can factorize further, and further. The question is that whether this question can be further factorized. [The teacher pointing at $2(na+nb)$.]

E: Yes!

T: And this one, can it be further factorized?

E: Yes!

So, which one should we choose?

“n(2a+2b)”,
“2n(a+b)”,
“2(na+nb)”

LEO: The middle one!

T: The middle one. Give me a reason.

LEO: Because it is the simplest one!

T: The biggest?

LEO: The simplest!

T: The simplest? You should say it is…what factor? It shouldn’t be that complicated. (…)

LEO: True factor.

T: It should be …it should be what factor? Didn't you learn this in the previous lesson?

T: What factor? How to spell it in English?

LEO: LCM.

LEO: HCF.

T: To say in simple words…in fact…it is a HCF. But this is not a HCF, just a CF, that is the common factor. This is the common factor, but not the highest common factor! If you want to have a complete factorization, you must have the HCF.

The teacher's perspective

- The topic as basic and important for paving the way of learning in future.

  I have to build up the meaning of factorization of polynomial in their mind and pave the way of future. This is a very important beginning. That is if the beginning is not good, they do not know what is the use of factorization of.

  Well, actually, mm, for a new topic, this is the basic work. Well… if we can do it well, this would be… helpful to the smooth flow of …the teaching of the lesson.

Summarizing his pedagogical style

- Developing new knowledge from the old knowledge
- Developing habits in mathematics
- Developing a method of comparison
- An awareness of the difficult points
- Catering for the need of the students
- Being reflective on his lesson

The Student: Naomi

- He saw the sequence of the six questions from easy to difficult as important.

  Naomi: From now on…from question 1 to 6…these examples help us to understand how to solve these problems step by step.

  Int.: How do you understand?

  Naomi: He goes through the examples from the easier ones to the difficult ones very slowly. It's easier to understand. He won't give you difficult examples at the beginning. He explains well too. The students…I think his explanation is very clear and thorough.

The Student: Leo

Leo had some kinds of general rules for seeing some forms of the lesson activity important.

- The revision and the beginning was important.
- Difficult questions were important.
- The checking of answers was important.

Class Atmosphere: T-Ss

T: We want to teach the students to understand all the knowledge and make them happy and relaxed.

Naomi: It’s not related to mathematics. We’re just laughing, so that we are not that bored, just to relax a bit.

Leo: We’re doing a lot of other things, but the teacher doesn’t see it. …Such as…chatting.

A directive approach

- The teacher talk was a major input component of teaching.
- The teacher was directive in most part of the discourse showing a clear guidance for students to observe certain features.
- The guidance was planned by a careful design of the mathematical problems and the key oral questions.

- This directive approach: keeps the focus on the mathematical content; following the teacher’s plan; good time control; and relatively efficient.

The technique of using variation

- The technique of variation was used in the design of the mathematical problems.
- Further dimensions of variation were created in the class interaction.
- These variations were obviously well picked up by the teacher in the actual lesson as he pointed to the contrast between questions and answers in the class discourse.
- They were also mentioned in his reflection of the lesson and they were referred to as advanced planning.

Similarity and diversity in the students' perspectives

- Naomi may have focused on the variation of the levels of difficulty in a simplistic way, whereas Leo may have focused on the same episodes based on his principles: revision, difficult point, checking (reflective).
- In both cases, the students were very attentive and ready to receive clear explanation from the teacher.

Further thoughts

- Mathematical enculturation is an interpersonal process and therefore it is an interactive process between people. (Bishop, 1991)
- The shaping process is a result of the interactive process between the teacher and the students. Within this process, concepts, meanings, processes and values are what are being shaped and eventually belong to the students.
- Is this what we want?

A culture

To promote **critical reflection** and **a positive attitude** that implies modesty to prepare for ratification and correction, open-mindedness to receive alternative views and possibility, and a caring heart to share their learning process with the others.

A challenge for many teachers in their daily teaching!!!!

Thank You