Mathematics for Teachers:

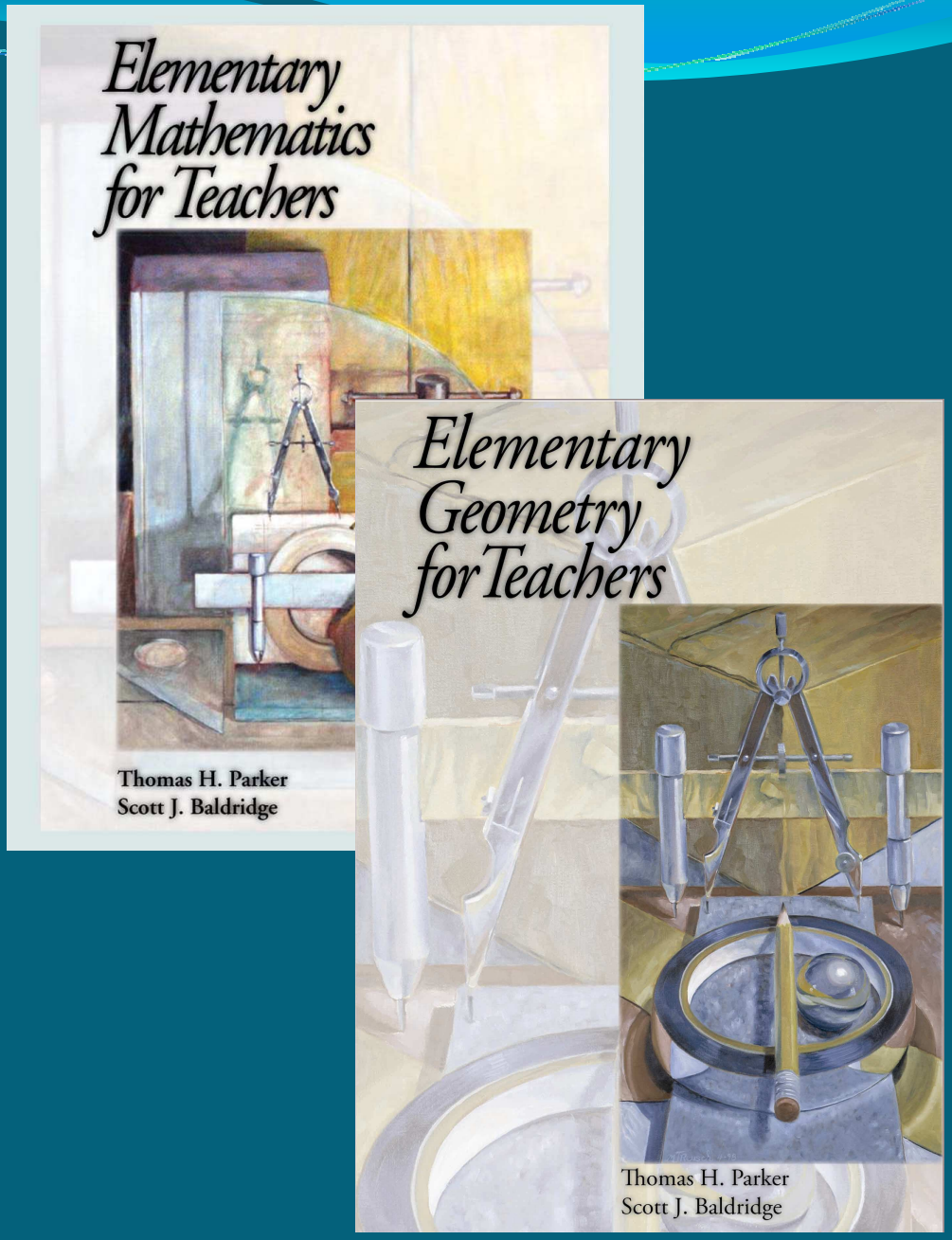
Content Knowledge and Pedagogical Content Knowledge

Scott Baldridge
Louisiana State University

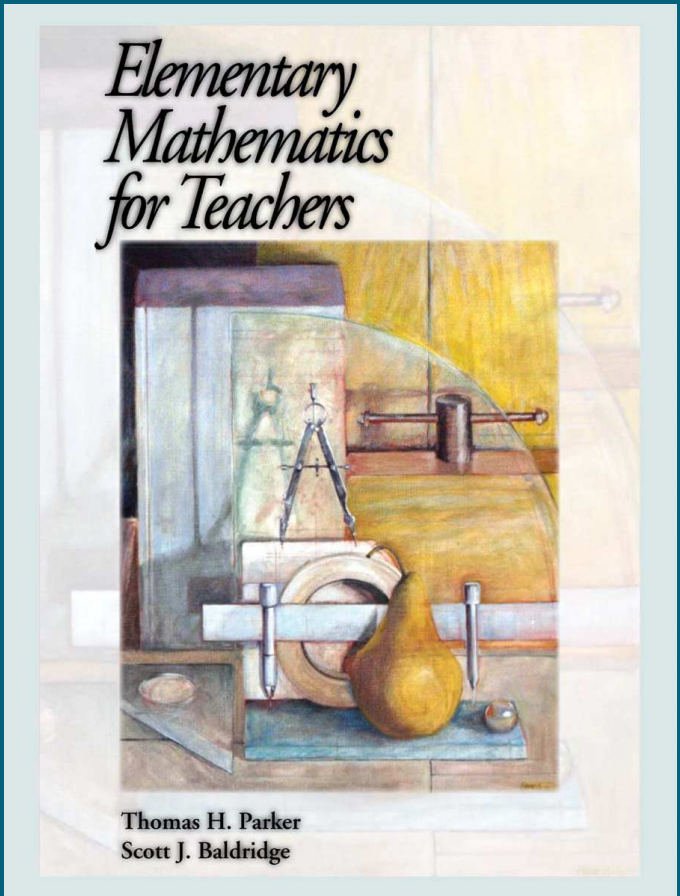
sbaldrid@math.lsu.edu
http://math.lsu.edu/~sbaldrid

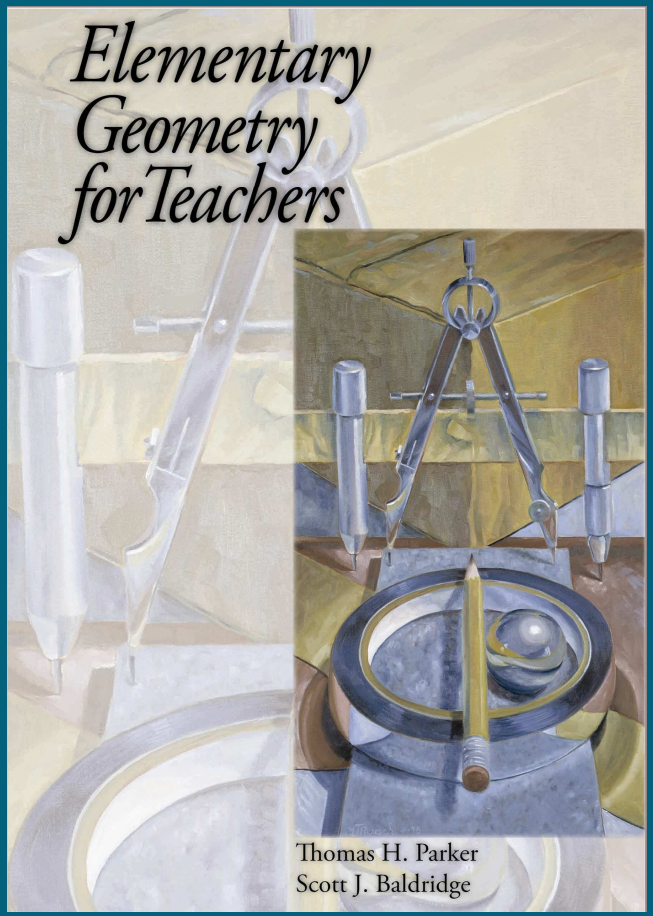
Overview

- The "Tease"
- The Theory
- The Evidence
- Our Program



Do our textbooks prepare preservice teachers to teach math?

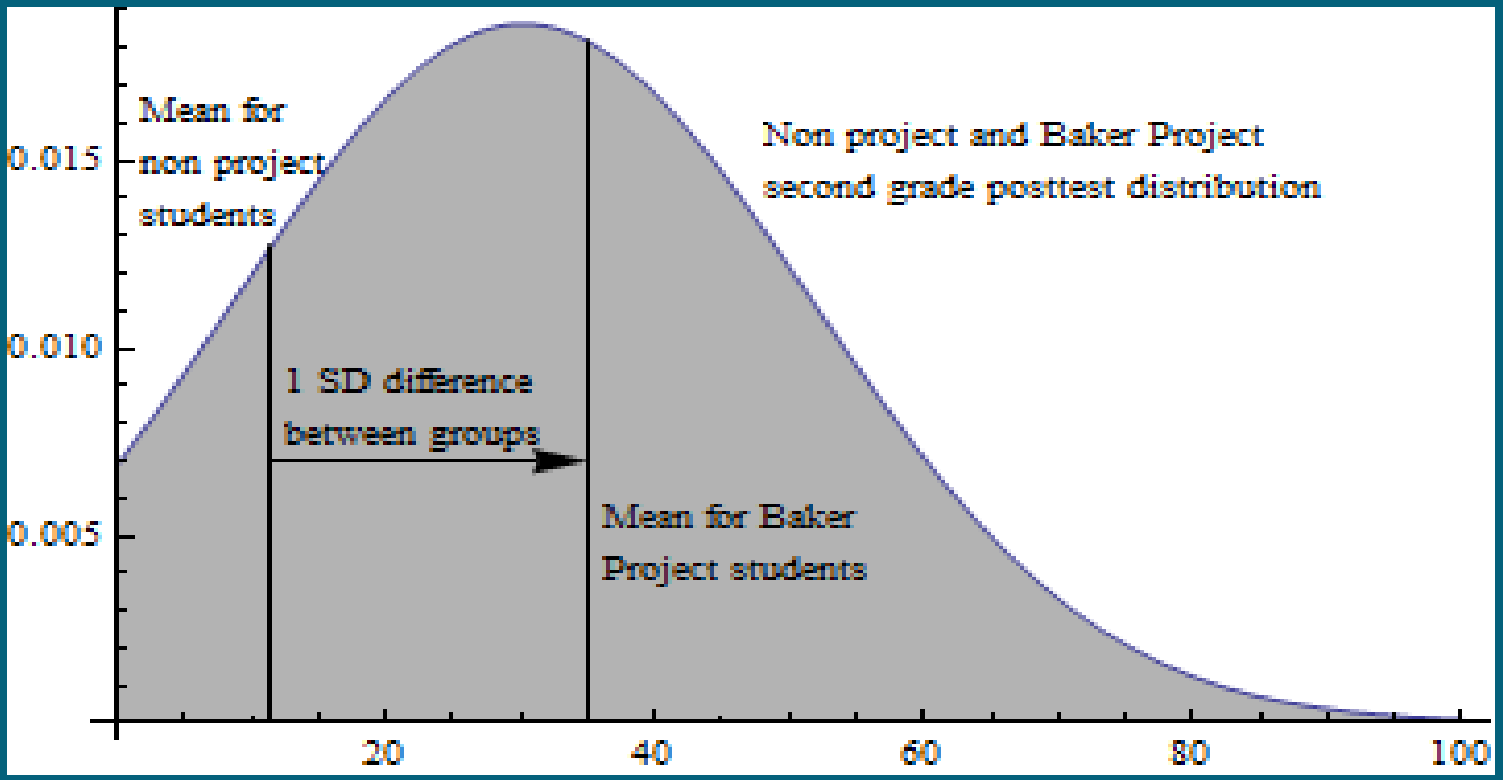




Evidence of our textbooks working

- The Baker Project
- Timeline
  - 2 week professional development during two summers using our textbooks
  - Twice a month in-service training using our textbooks during the school year
  - Teachers used the Primary Mathematics textbooks from Singapore during the school year
  - Teachers and students were evaluated regularly, including both summers

First, the students: 2nd Grade Comparison



- Non project and Baker Project combined normal distribution.

Next, the teachers:

- Teachers were evaluated both summers on content and pedagogical content knowledge.
- Teachers who were in the project during the academic year were compared to teachers starting in the second summer.
- Teachers who learned from EMT/EGT and taught from the Primary Mathematics Textbooks during the year scored two times better than the new teachers to the project on the content/pedagogical content test.

The Theory

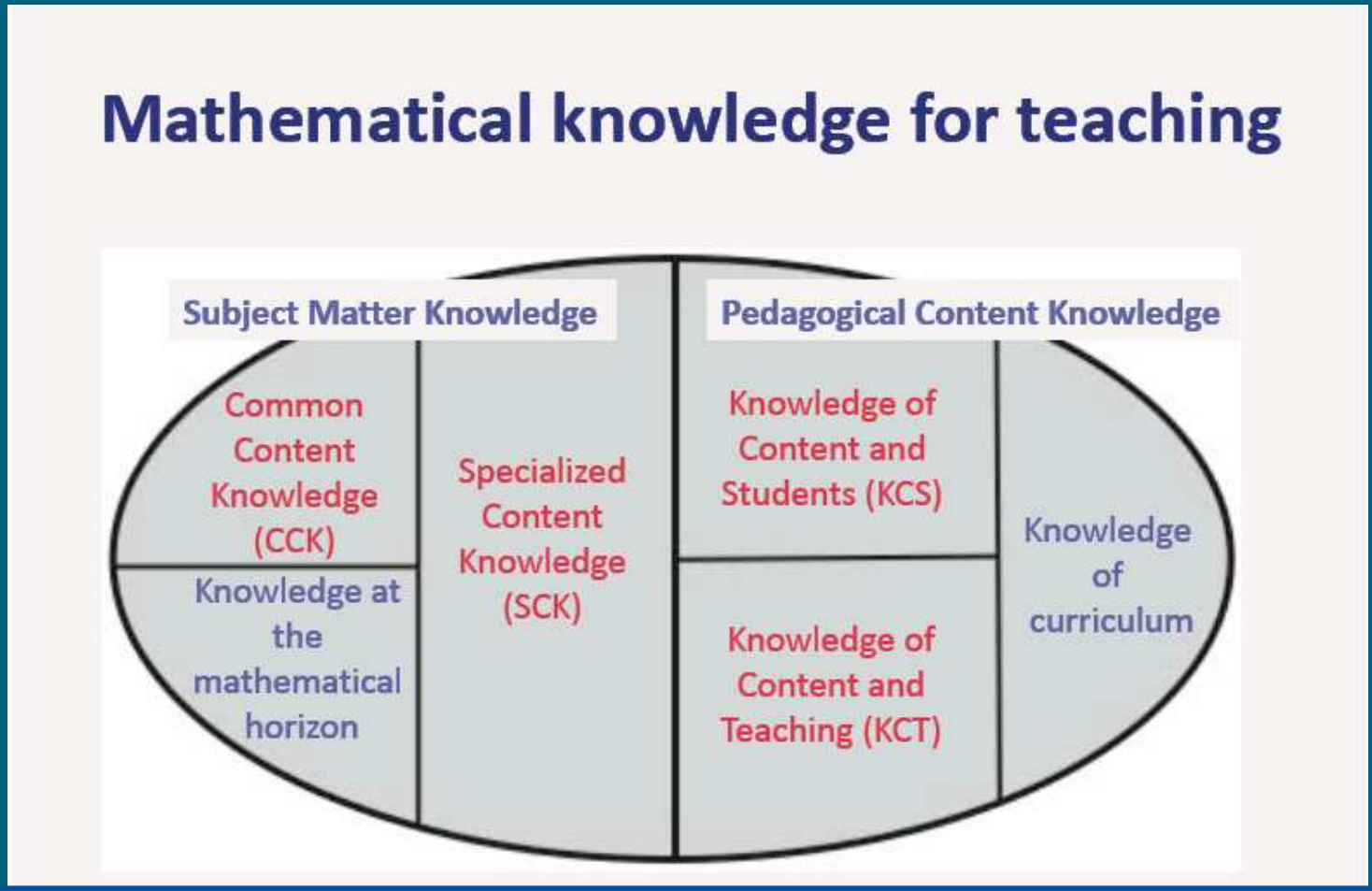
- Shulman (1986) distinguished 3 dimensions of teacher knowledge:
  - Content Knowledge (CK)
  - Pedagogical Content Knowledge (PCK)
  - Generic Pedagogical Knowledge
- Both CK and PCK make unique contributions to explaining differences in the quality of instruction and student progress in *mathematics*.

The Theory

- Ball (1990), Ma (1999), and many others have shown qualitatively:
  - Insufficient Math CK limits a teacher's ability to explain and represent mathematics to students
  - That this inability cannot be offset by pedagogical skill
- Other qualitative findings suggest:
  - $CK(T_1) = CK(T_2)$ but $PCK(T_1) \neq PCK(T_2)$

The Theory (MKT)

- A useful model:



Deborah Ball, Hyman Bass, et al., October, 2008

The Theory: Examples

- CK: Common Knowledge

Calculate:

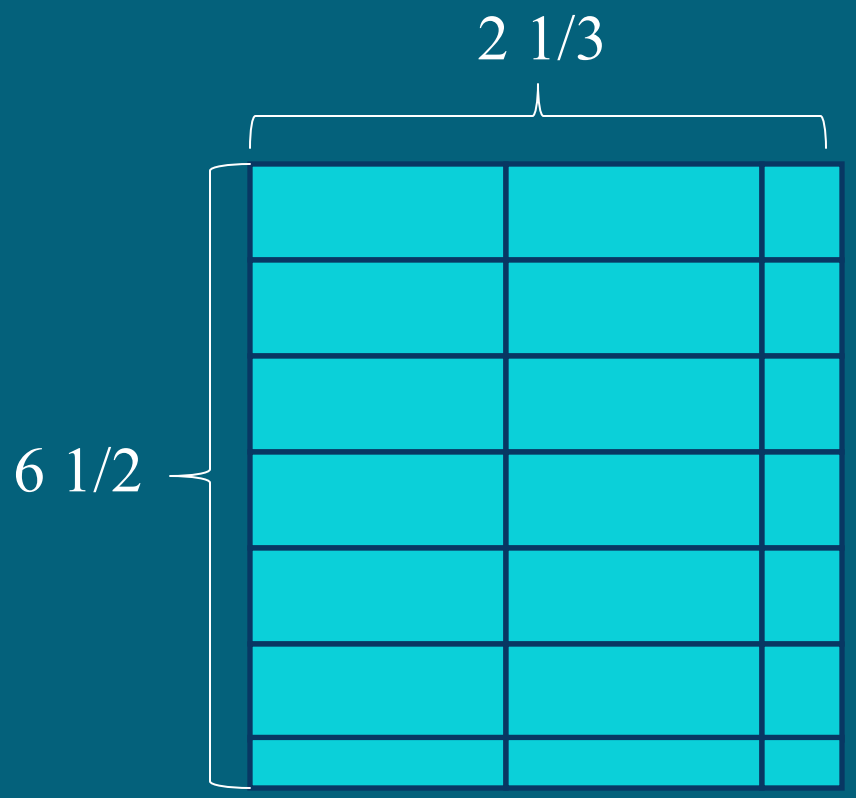
$$2\frac{1}{3} \times 6\frac{1}{2}$$

The Theory: Examples

- CK: Specialized Content Knowledge

Find the following product by drawing an area model:

$$2\frac{1}{3}\times 6\frac{1}{2}$$



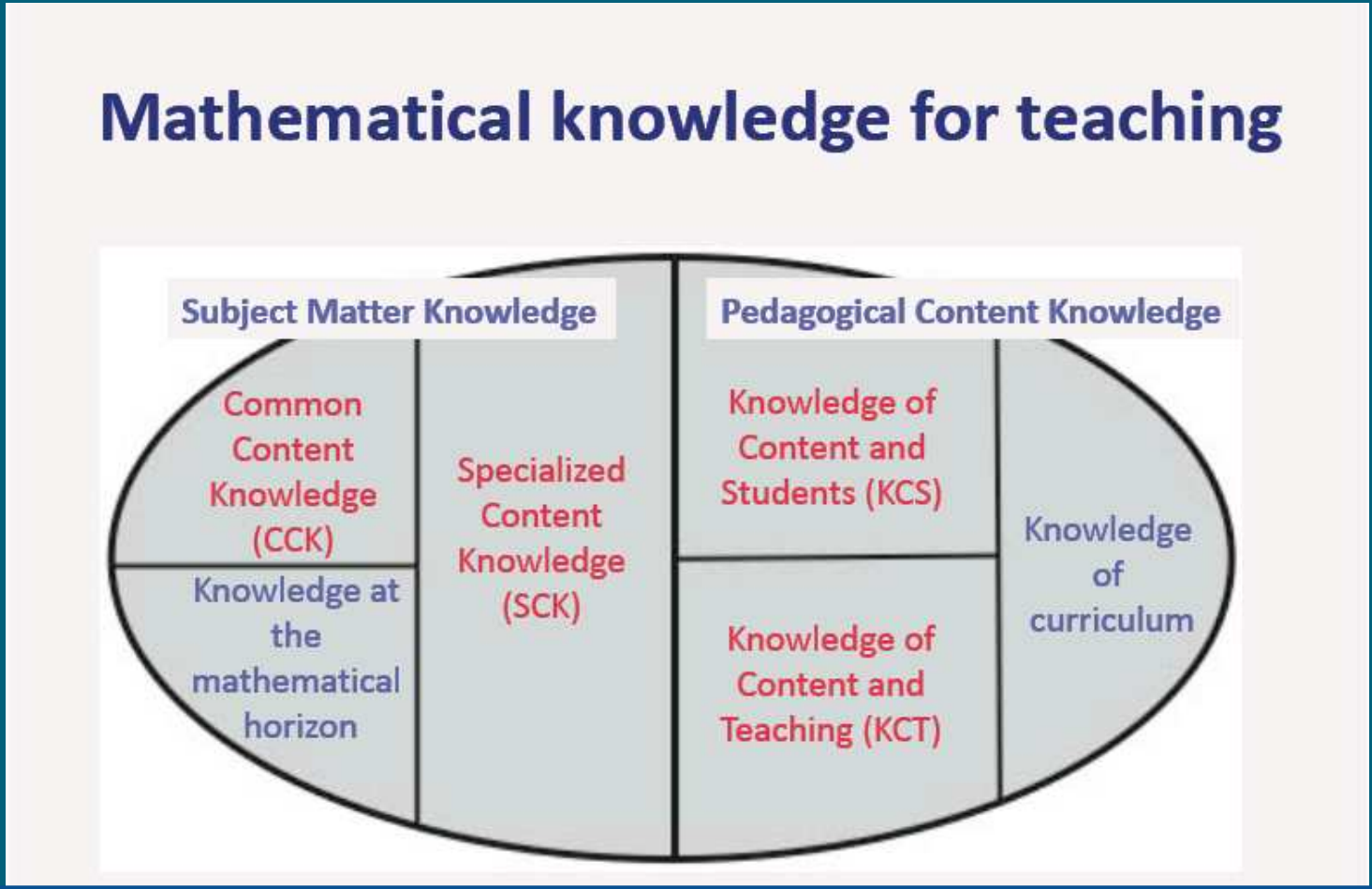
The Theory: Examples

- CK: Specialized Content Knowledge

Make up a short non-area word problem for $2\ 1/3 \times 6\ 1/2$.

- If it takes 2 1/3 days to asphalt 1 mile of road, how many days will it take to asphalt 6 ½ miles?

The Theory



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The Theory: Examples

- PCK: Knowledge of Content and Students

  One common student error is to write

$$2\frac{1}{4}\times 1\frac{1}{3}=2\frac{1}{12}.$$

  Give an area model and brief explanation which simultaneously shows both the error this student is making and what the correct solution is.

The Theory: Examples

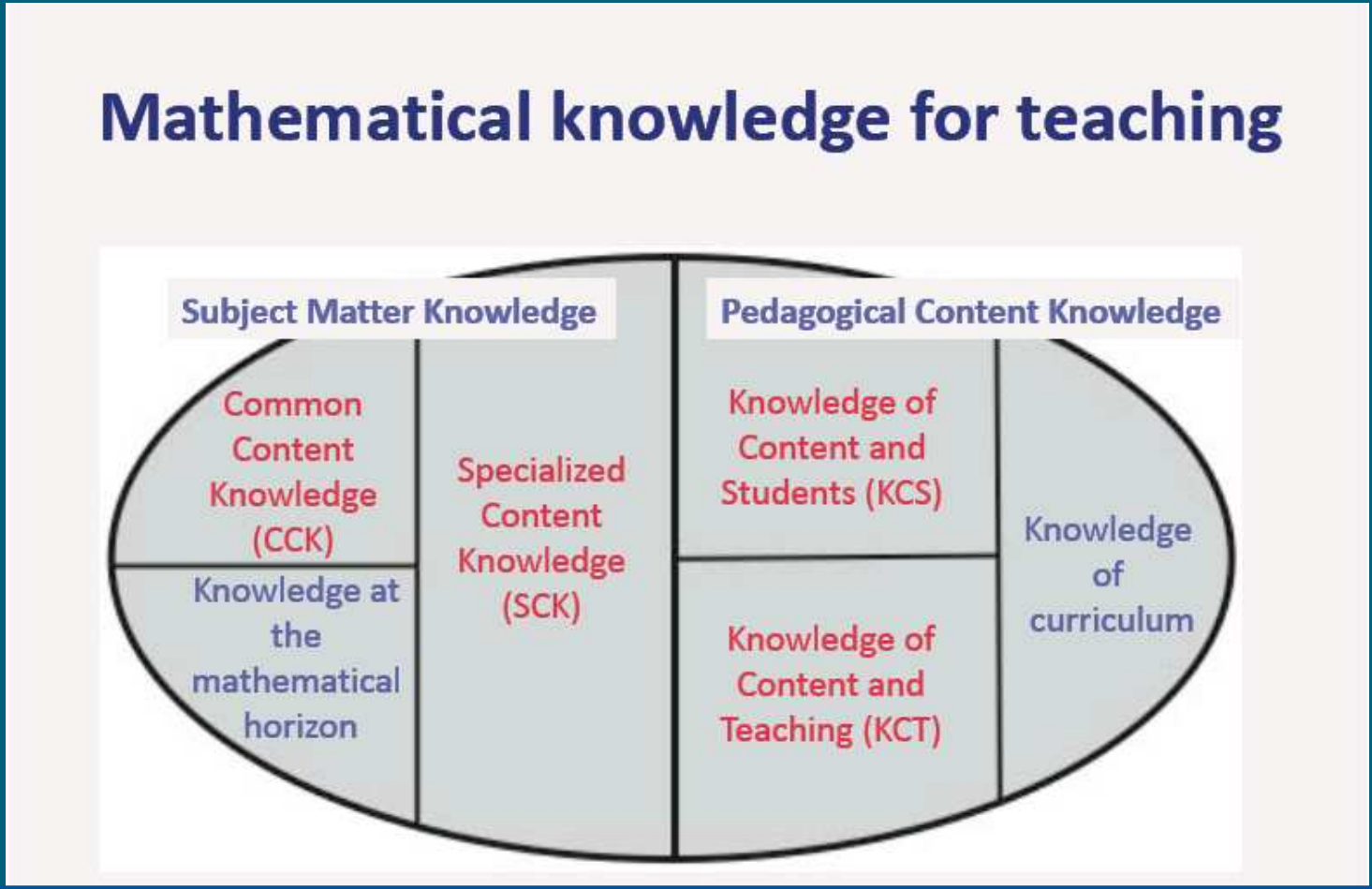
- PCK: Knowledge of Content and Teaching

  A Teaching Sequence for introducing fraction multiplication is:

  1. Whole number × Fraction
  2. Fraction × Whole number
  3. Fraction × Fraction

  Why? In particular, in which step do students make a conceptual shift in their understanding of multiplication? What is the conceptual shift?

The Theory



Deborah Ball, Hyman Bass, et al., October, 2008

The Theory: Examples

- PCK: Knowledge of Curriculum

  How does a 5th grade Teacher's Solution for the following problem fit into Singapore's K-8 curriculum for teaching problem solving?

  Todd spent 2/5's of his money on a shirt and 1/5 of the remainder on a book. If he has $36 left, how much did the shirt cost?

A mini-lesson: Teacher's Solutions

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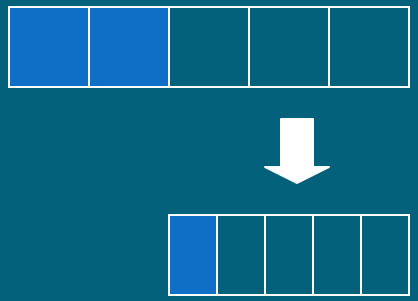

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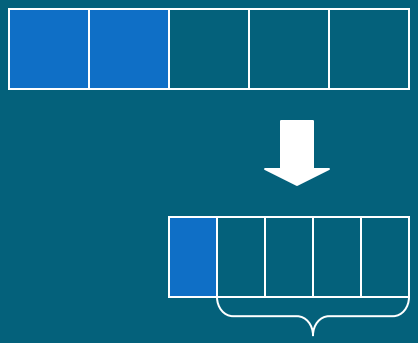

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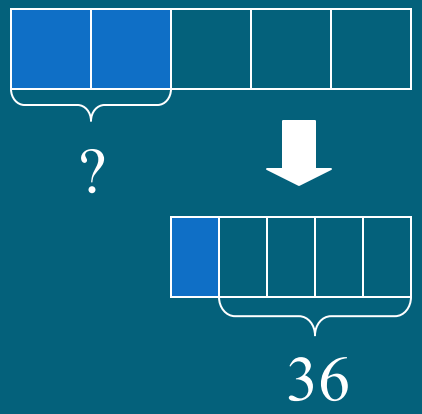

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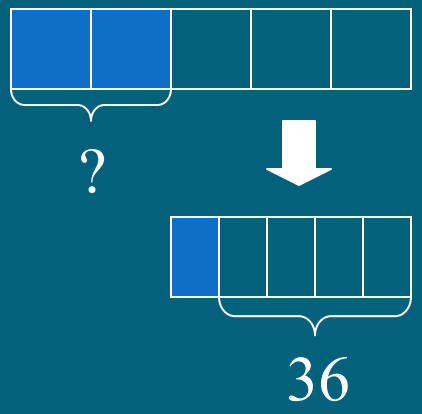

4 units = 36
1 unit = 9
5 units = 45

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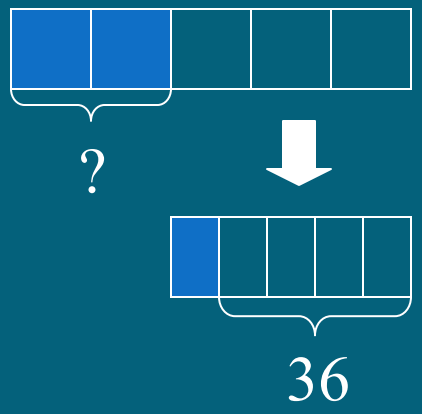

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3 units = 45
1 unit = 15
2 units = 30

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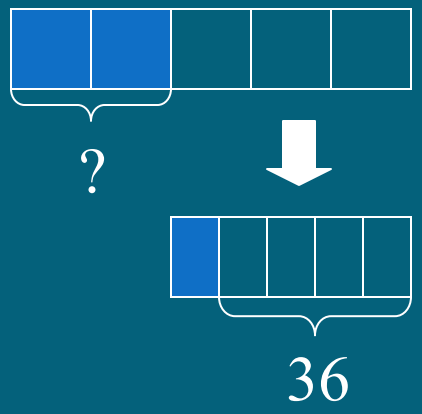

4 units	=	36
1 unit	=	9
5 units	=	45

3 units	=	45
1 unit	=	15
2 units	=	30

The shirt cost $30.

The Theory: Examples

- PCK: Knowledge of Curriculum

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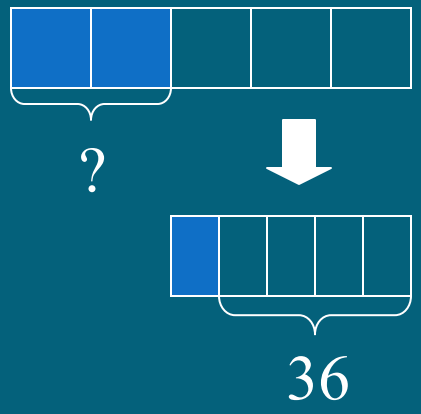

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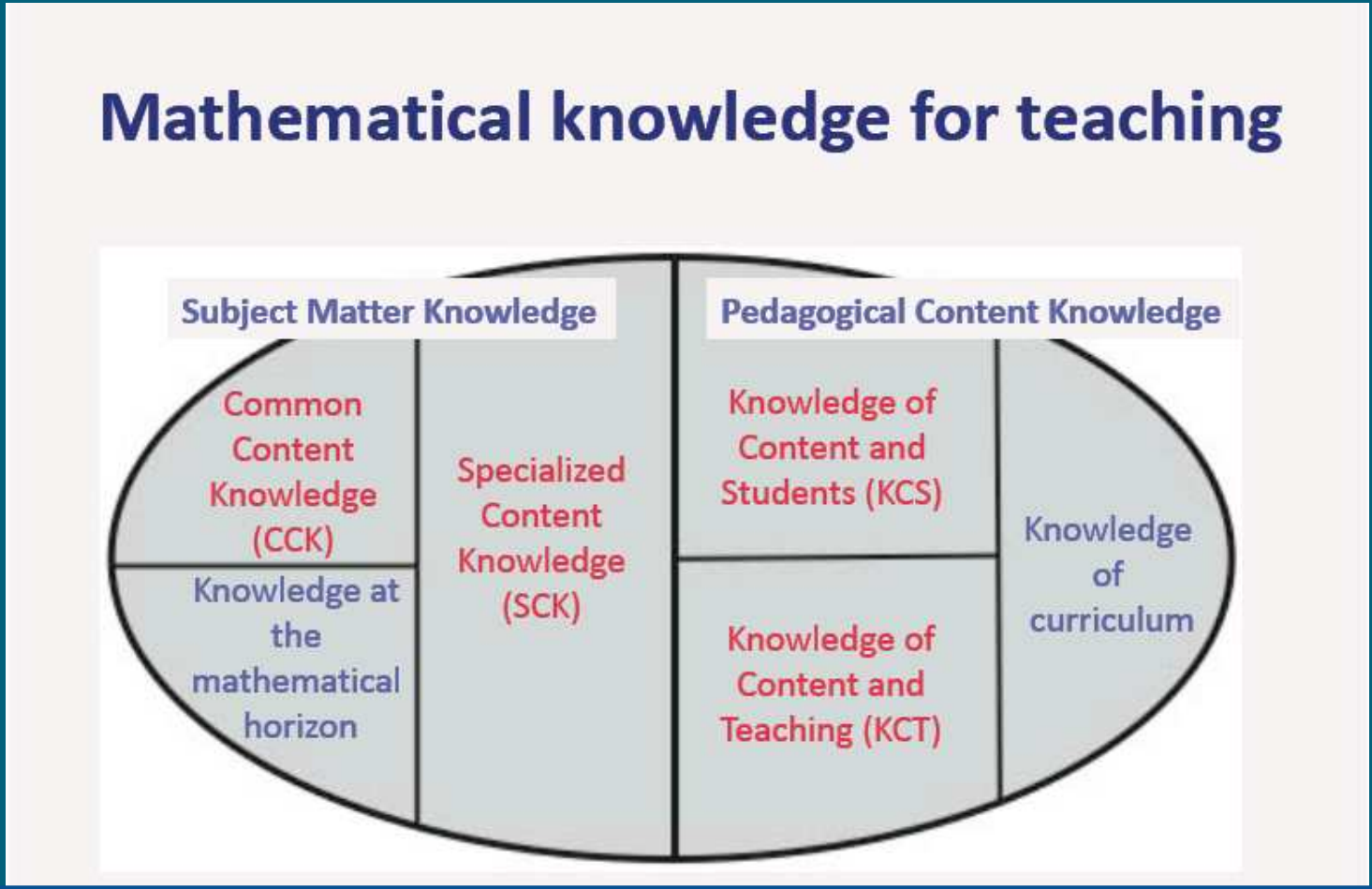
The shirt cost $30.

The Theory



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The Theory: Examples

- CK: Knowledge at the Mathematical Horizon

  Advanced knowledge that equips teachers with perspective for their work; it is not knowledge of the kind they need to explain to pupils.

The Theory: Examples

- CK: Knowledge at the Mathematical Horizon

  Why do the fraction rules depend on the arithmetic properties and not on the models and interpretations used to introduce the rules?

The Evidence

- "Teacher's Mathematical Knowledge, Cognitive Activation in the Classroom, and Student Progress"
  - Authors: J. Baumert, M. Kunter, W. Blum, M. Brunner, T. Voss, A. Jordan, U. Klusmann, S. Krauss, M. Neubrand, Y.M. Tsai.
  - Published: American Education Research Journal, 2010, Volume 47.
  - Online at: aer.sagepub.com/cgi/content/abstract/47/1/133

The Evidence

- Sample
  - 4,353 Students in Germany spanning a 1-year period from the end of grade 9 to the end of grade 10.
  - 181 grade 10 Teachers in 194 classes

The Evidence

- Measures
  - Teachers took both a CK test and a PCK test. The two factors show substantial inter-correlation, but still clearly distinguishable.
  - Teacher instruction was assessed via
    - Curricular level of tasks
    - Cognitive level of tasks
    - Individual learning support
    - Classroom management
  - Students' mathematics achievement was assessed at the end of grade 9 using PISA literacy tests and again at the end of grade 10 based upon the state curriculum. Social-economic background assessed through a questionnaire.

The Evidence: Results

- Do CK and PCK have an impact on student achievement?

Class mathematics achievement, end of grade 10 on	Content Knowledge Model, controlled for track	Pedagogical Content Knowledge Model, controlled for track
CK as the variable	0.30	
PCK as the variable		0.42
R^2 (% of the variance)	44%	54%

The Evidence: Results

- PCK is inconceivable without a substantial level of CK
- Previous findings have also shown substantial correlations between CK and PCK to increase as a function of teacher expertise
- Are CK and PCK interchangeable?

The Evidence: Results

- Mediation Model:

	Content Knowledge Model, controlled for track	**Pedagogical Content Knowledge Model, controlled for track**
Cognitive level on CK (PCK)	0.01	**0.24**
Curricular level on CK (PCK)	**0.32**	**0.33**
Individual learning support on CK (PCK)	-0.06	**0.26**
Effective classroom management on CK (PCK)	-0.01	0.14

The Evidence: Results

- Controlled for tracks?
  - Two different training programs in Germany
    - Academic track
    - Nonacademic track
  - Academic track training requires teachers take 1/3 more subject matter courses than the non-academic track
  - Both tracks take the same number of methods courses from the same professors

The Evidence: Results

	Academic	Nonacademic	Difference
CK	0.737	-0.524	1.26 SD
PCK	0.427	-.007	0.43 SD

The Evidence

Conclusions for training teachers

- CK and PCK of a teacher has a direct impact on the instructional quality of their teaching and the mathematical achievement of their students
- Despite the high correlation, PCK has a higher predictive power than CK on student progress, probably due to higher cognitive level of tasks and better individual learning support.

The Evidence

Conclusions for training teachers:

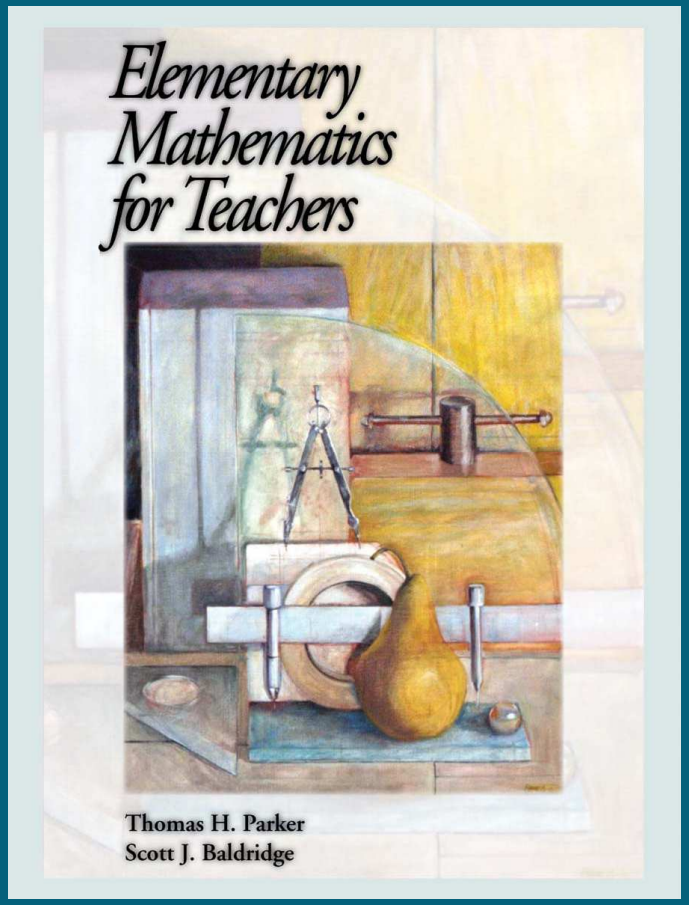
- CK and PCK are largely dependent on the type of training program
- Limited CK training have detrimental effects on PCK and consequently negative effects on instructional quality and student progress
- Differences in CK that emerge during preservice training persist across the entire teaching career

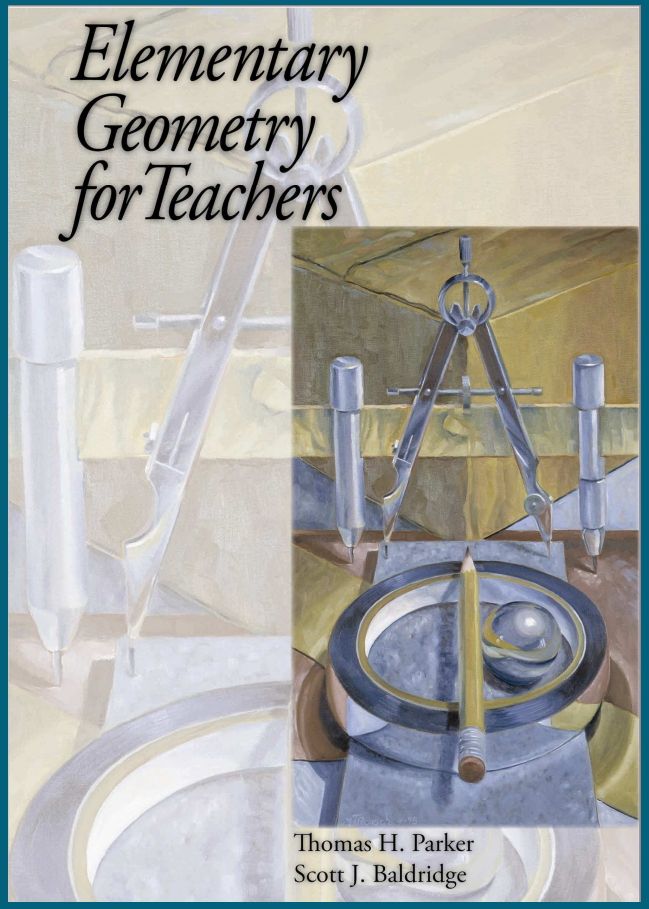
What does this mean for Elementary Teacher Prep Programs?

We need to get a *lot* better at teaching teachers:

- Elementary teachers need *strong content courses* at the level they will be teaching
- Math and Education courses must achieve a proper balance between CK and PCK
- To achieve that balance, mathematicians and math educators need to work together to build a structurally coherent program

Our Program


Elementary
Mathematics
for Teachers
Thomas H. Parker
Scott J. Baldridge


Elementary
Geometry
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The Overall Program

- Freshman Year:
  - Elementary Mathematics for Teachers (MKT)
  - Elementary Geometry for Teachers (MKT)
- Sophomore and Junior Years:
  - Rigorous PCK pedagogy courses that are tied specifically to the K-6 mathematics curriculum, including
    - Student teaching experiences with mentor teachers
    - MKT labs with mathematicians and math educators
- Senior Year:
  - Future book: Elementary Algebra for Teachers (Mature MKT)

EMT and EGT content in a nutshell

- EMT: The first course tells the story of numbers—whole numbers, fractions, rational numbers, real numbers.

- EGT: The study of measurement and how that leads naturally to logical deduction.

  Underlying themes: forming and using units, measurement, and problem solving

Something to keep in mind…

- All examples of CK and PCK given earlier are from our textbooks.

The Main Feature:

- Our courses/textbooks are ***unique*** in that they approach teaching mathematics through the study of an actual curriculum—The Primary Mathematics Series from Singapore.

Why?

Our mathematics courses for teachers seek to prepare prospective teachers by duplicating, ***as much as possible, the actual classroom environment***.

Of course, we must do this without actually involving kids (that comes later). But we can go surprisingly far in this direction by using actual K-8 textbooks and studying them from a teachers' perspective.

The Question: WHICH TEXTBOOKS?

Not any curriculum will work for teaching teachers. One needs a series that

- Develops K-8 math as a coherent story,
- Is exceptional in its fidelity to mathematics,
- Has a proven track record of success,
- Prepares students for teaching from any elementary school materials.
- Is inexpensive and available (in English).

Why the Primary Math Series?

- The elementary textbooks continually remind students that one day they will be teaching mathematics.
- The textbooks lay out the content of elementary mathematics, clearly organized by grade, showing:
  - How material can be presented
  - How models and representations can be used
  - Pace
  - Order of topics & within topic
  - How to keep focus on the MATH

Why the Primary Math Series?

Preservice teachers learn directly from the elementary textbooks. They

- Solidify their knowledge of K-8 math (and learn some material for the first time!)
- Clearly see that the material is relevant
- Gain ***confidence***
- Instill habits of mind.

After working with the Primary Math books, students come away with raised expectations about the mathematical capabilities of elementary students.

Features

- The material focuses directly and exclusively on mathematics that is relevant to elementary teachers (CK)
- Develops mathematics by following the Primary Mathematics Curriculum (PCK)
- Presents mathematics in terms of the students' future teaching rather than their learning (PCK)

Features

- Students work hundreds of challenging problems. They master the material they will eventually teach. (CK/PCK)
- Students learn to present clear explanations ("Teacher's Solutions") in homework and in front of the class. (PCK)
- Students learn to love solving word problems!

Features

- Students also learn to create their own math problems.
  - Build up to multi-step word problems and geometry problems (CK)
- Presents the mathematics behind
  - Teaching Sequences (PCK)
  - Correct modeling when explaining mathematics (CK)
  - Student errors (PCK)
- Students see first-hand how a coherent curriculum "folds" old ideas into new ones. (PCK)

Elementary Algebra for Teachers

- Polya's elephant in the room
- More than/less than
- Coordinate geometry and algebra

Profound Understanding Warning

- Avoid textbooks that say they give their students "Profound Understanding of Fundamental Mathematics"
  - PUFM usually takes at least 7 years past graduation to develop.
  - We specifically designed our textbooks to instill certain habits that put preservice teachers on the path to PUFM.

National Council on Teacher Quality

"Uncommon Denominator"

Implementation

- IREMT.MATH.MSU.EDU