Lecture 2

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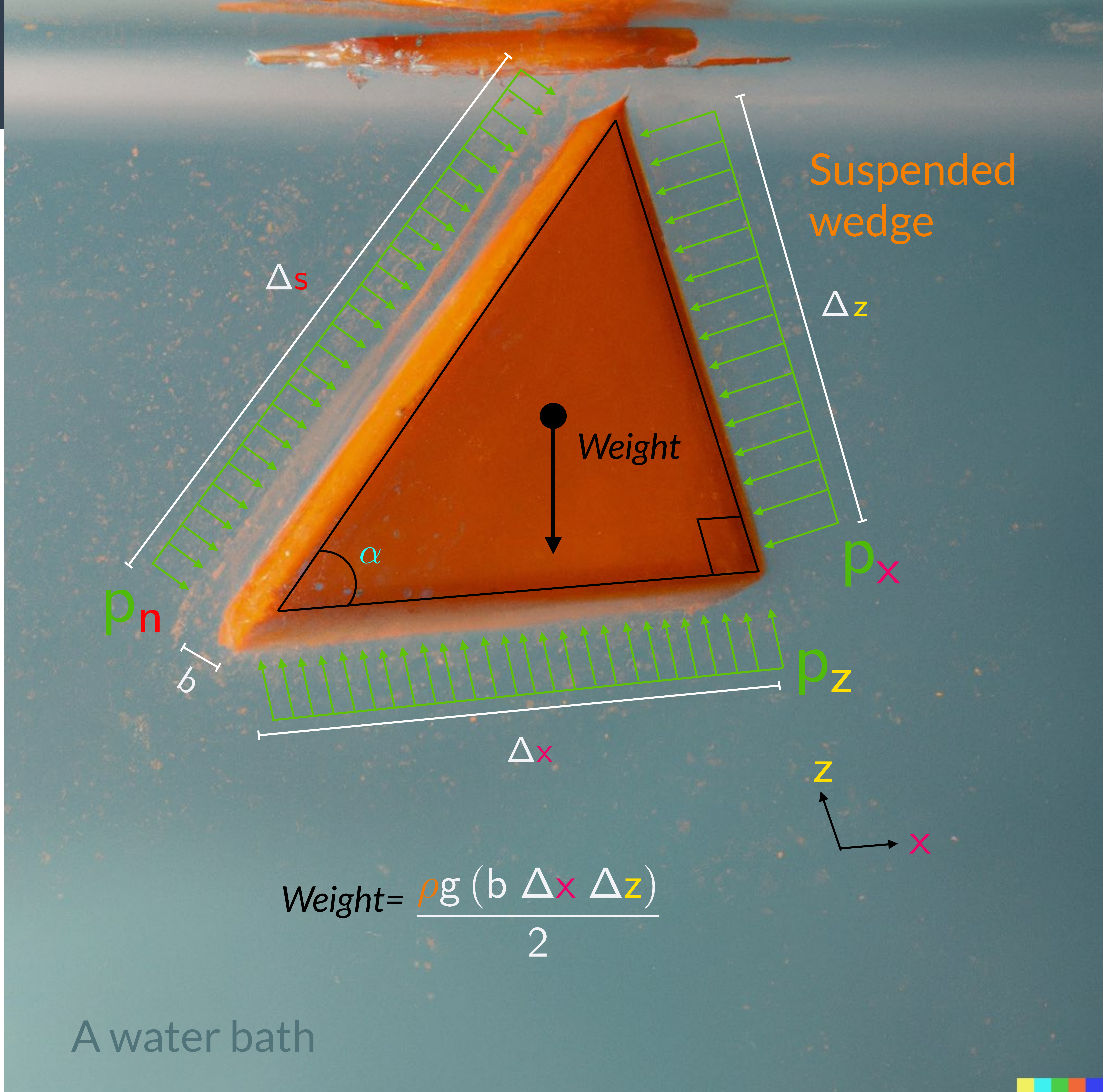
Fluid Statics

- When a fluid is at rest, there can be no velocity, and this implies there are no shear stresses present.
    - Later, we shall study the connection between shear and velocity gradients.

- To demonstrate the above, consider the balance of forces in a suspended fluid element.

Fluid Statics

I. *Since the fluid is at rest, the small element doesn't alter its position and thus*

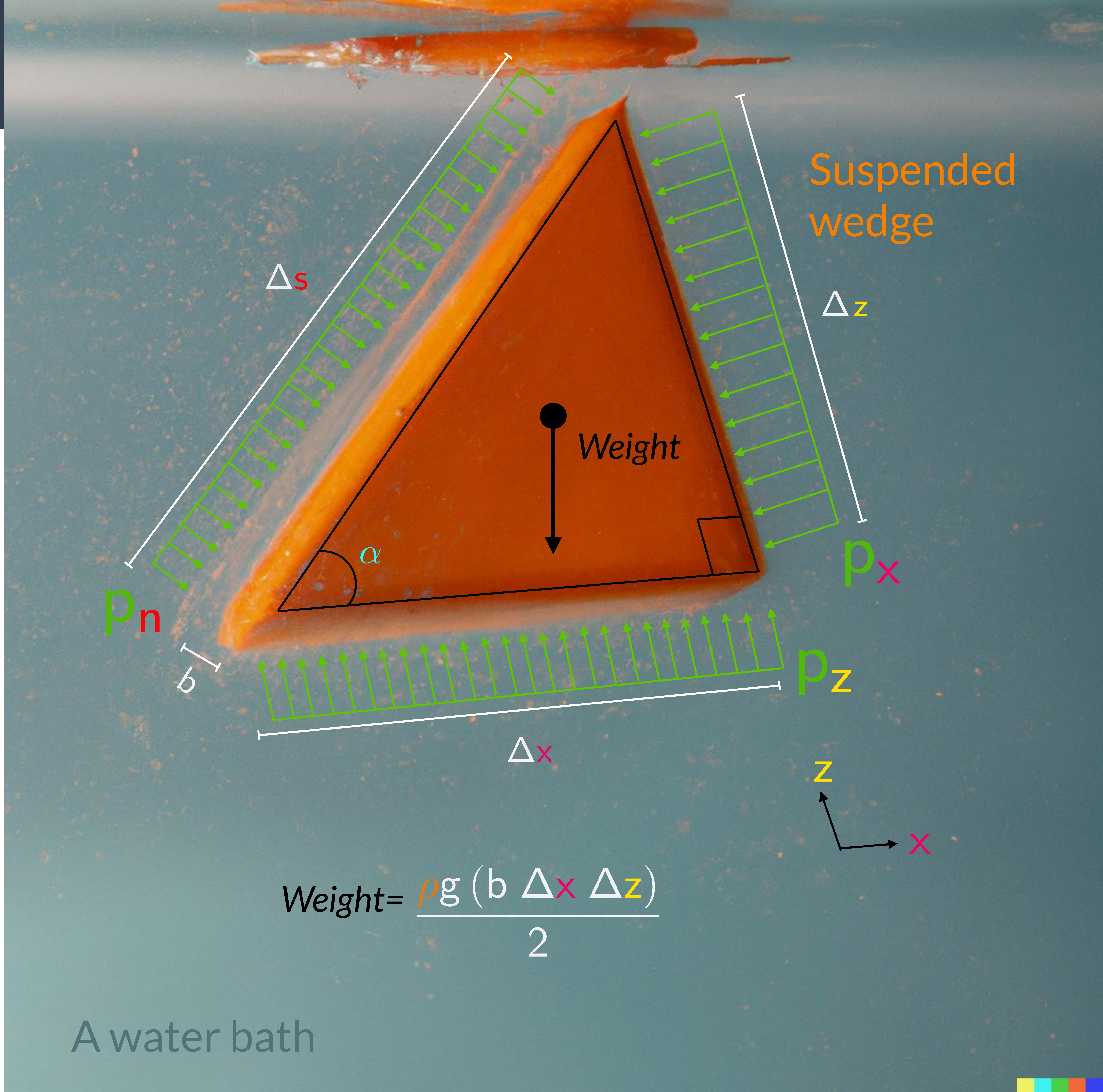


Fluid Statics

II. *Consider the forces along the two directions...*

$$\sum F_x =$$

$$\sum F_z =$$



Fluid Statics

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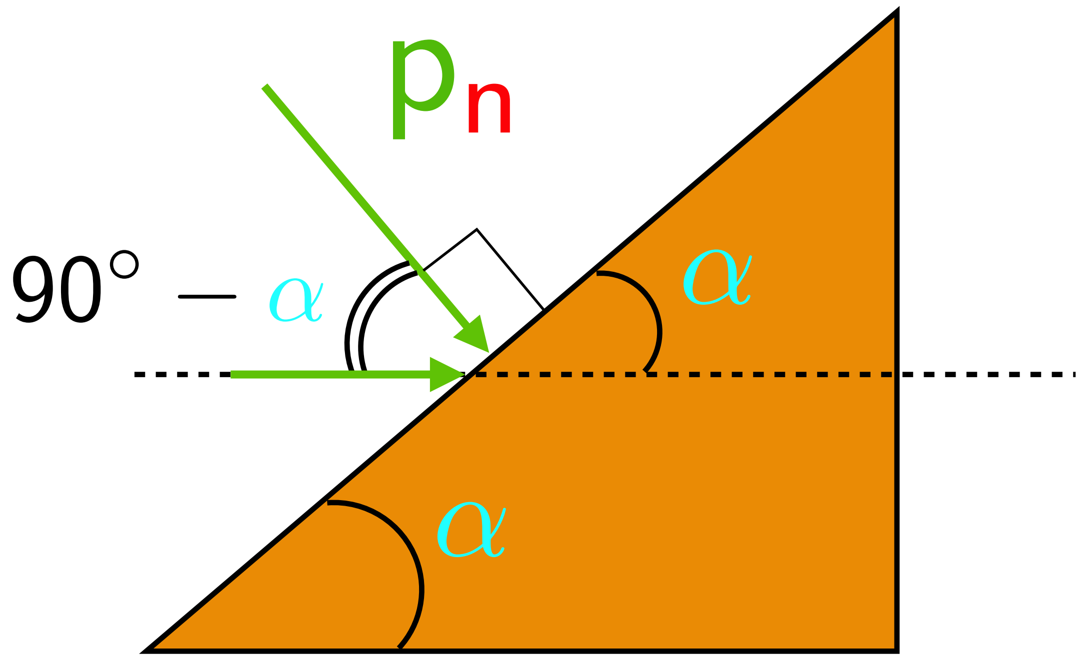
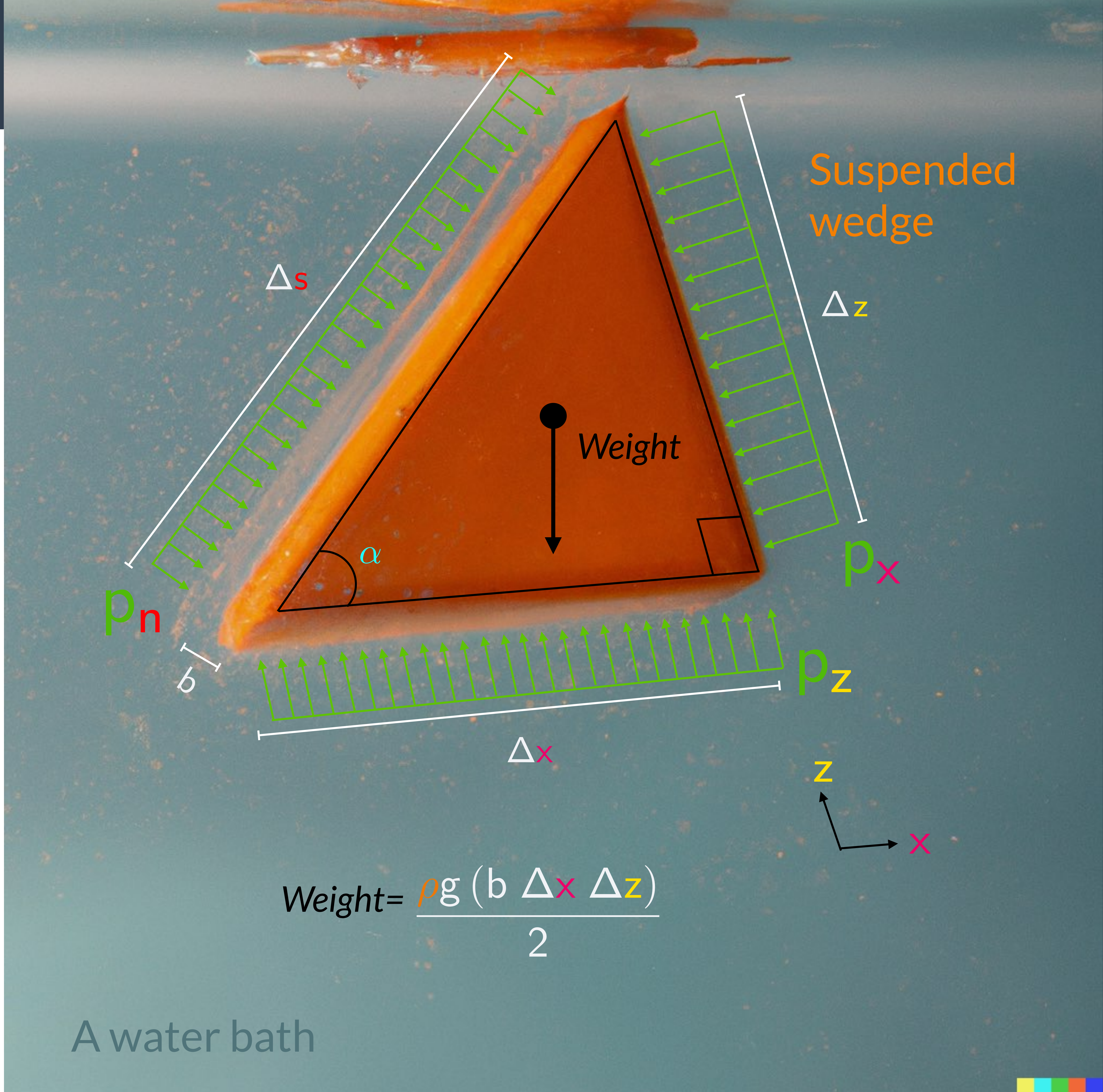
Aside

$$\cos(90^\circ - \alpha) = \frac{p_{n,x}}{p_n}$$

$$\Rightarrow \sin(\alpha) = \frac{p_{n,x}}{p_n}$$

$$\Rightarrow p_n \sin(\alpha) = p_{n,x}$$





Fluid Statics

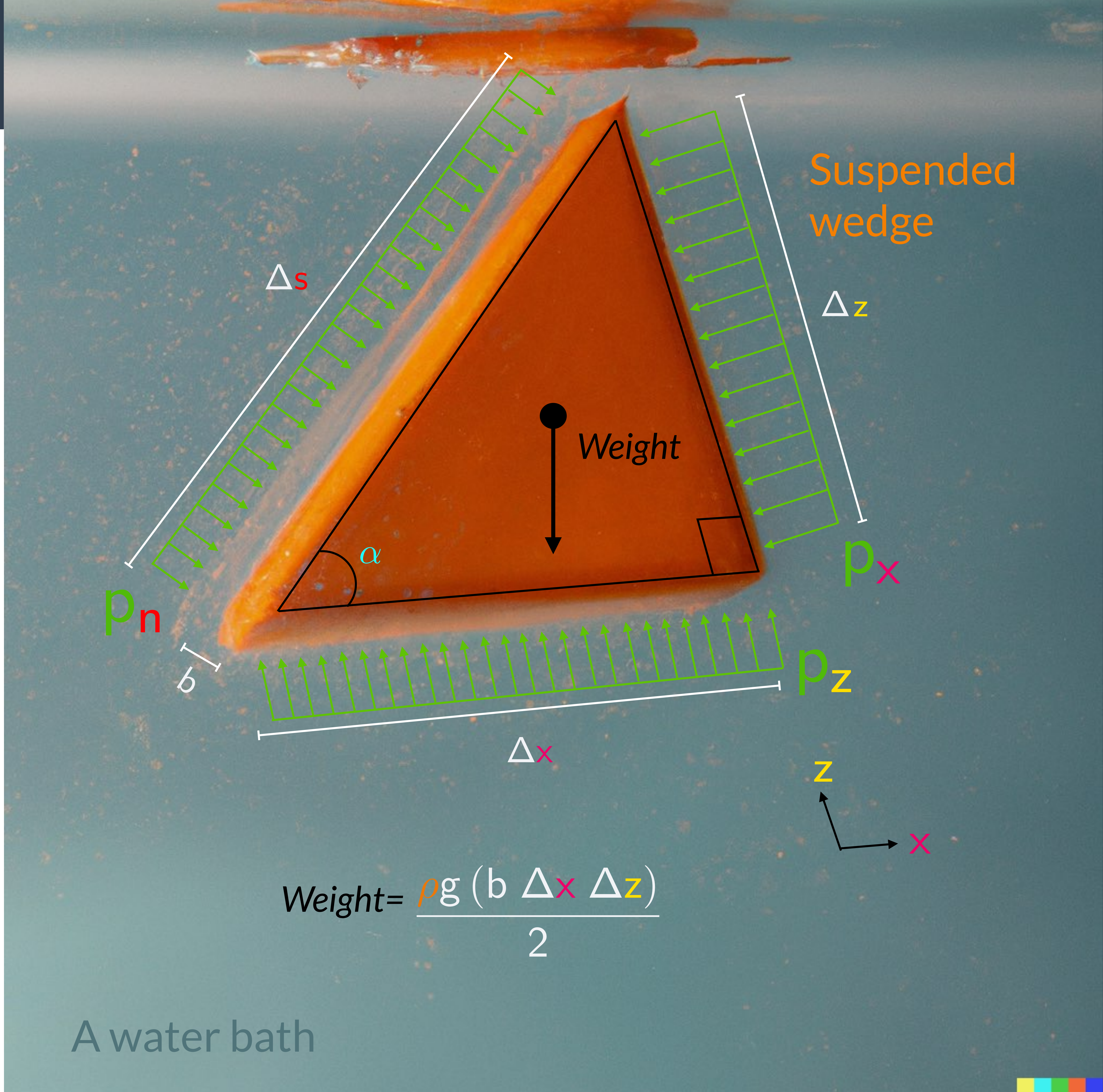
III. *From the wedge geometry, we have*

$$\Delta s \sin(\alpha) = \Delta z$$

Plugging this into the first equation...

$$\sum F_x = -(p_x b\ \Delta z) + (p_n \sin(\alpha)\ b\ \Delta s) = 0$$

$$\Rightarrow p_n = p_x$$



Fluid Statics

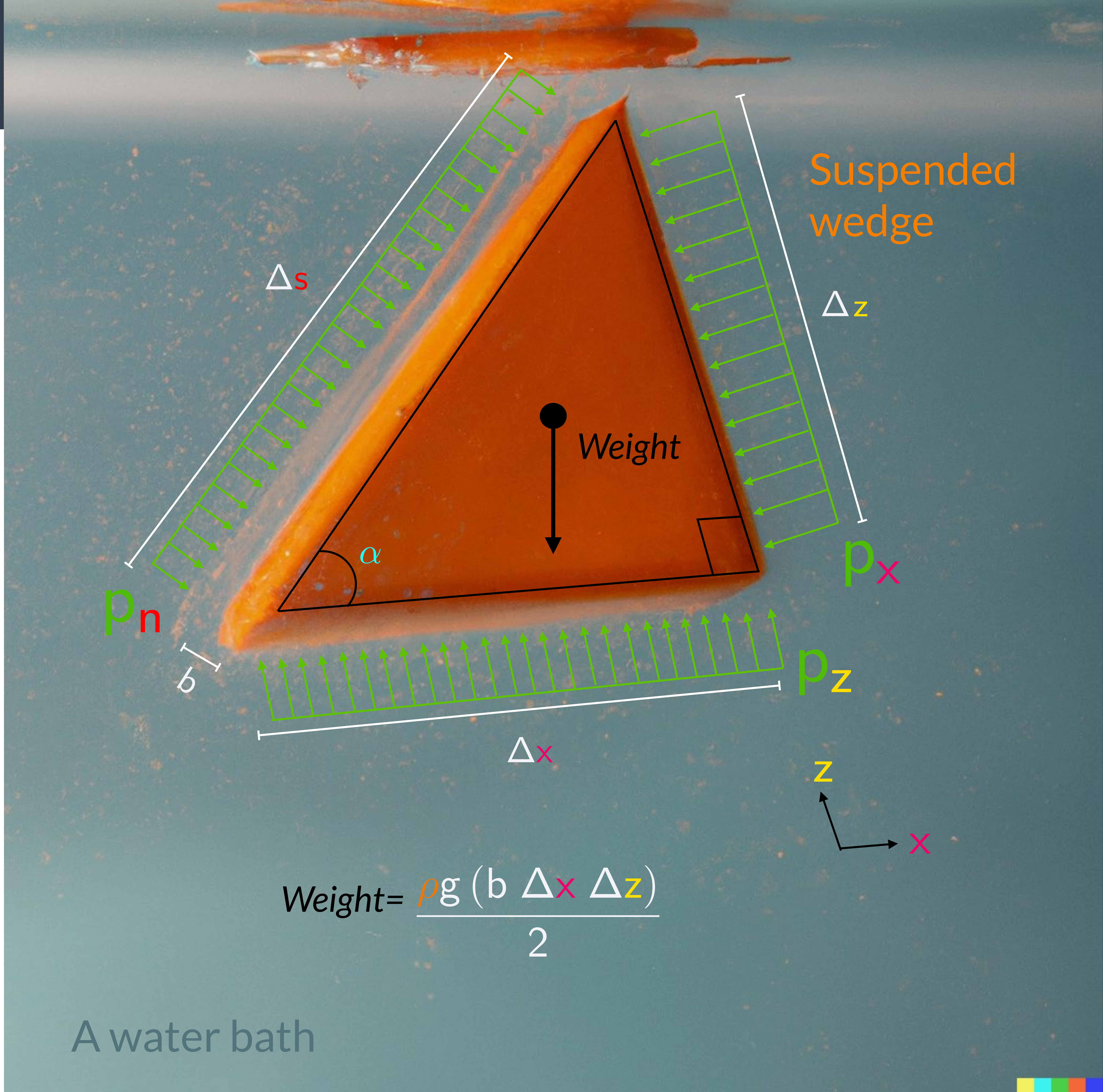
IV. *From the wedge geometry, we have*

$$\Delta s \cos(\alpha) = \Delta x$$

Plugging this into the second equation...

$$\sum F_z = p_z b \Delta x - p_n \cos(\alpha)\, b \Delta s - \text{weight} = 0$$

$$\Rightarrow p_z = p_n + \frac{1}{2}\rho g \Delta z$$



Fluid Statics

- So we have:

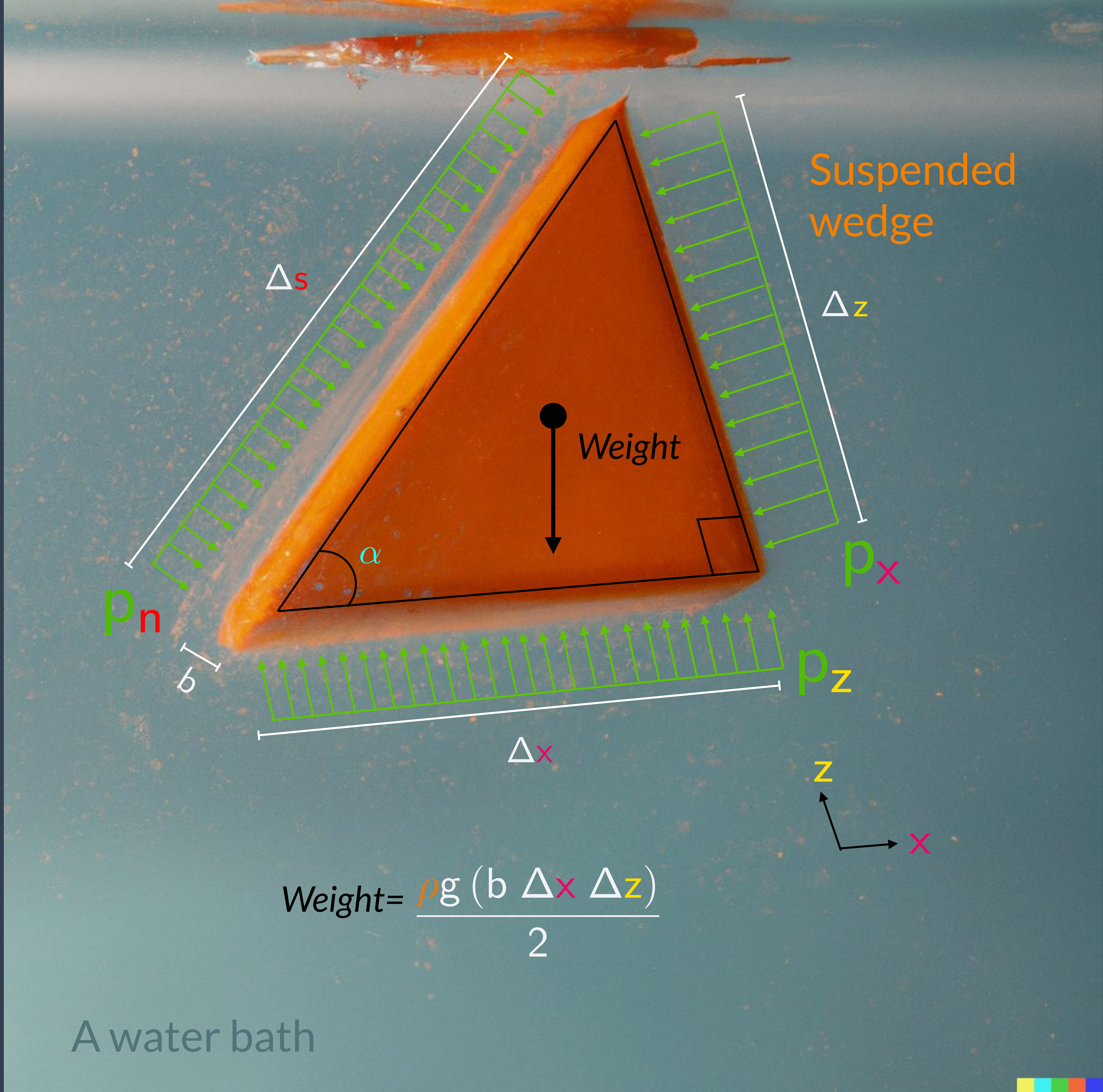
$$p_x = p_n$$

$$p_n = p_z + \frac{1}{2}\rho g \Delta z$$

- And in the limit $\Delta z \to 0$, i.e., if the element is shrunk to a very small one, then

$$p_x = p_n = p_z$$

- This is a very important result!



Fluid Statics

- Demonstrates that the pressure on any surface at a given point is the same.

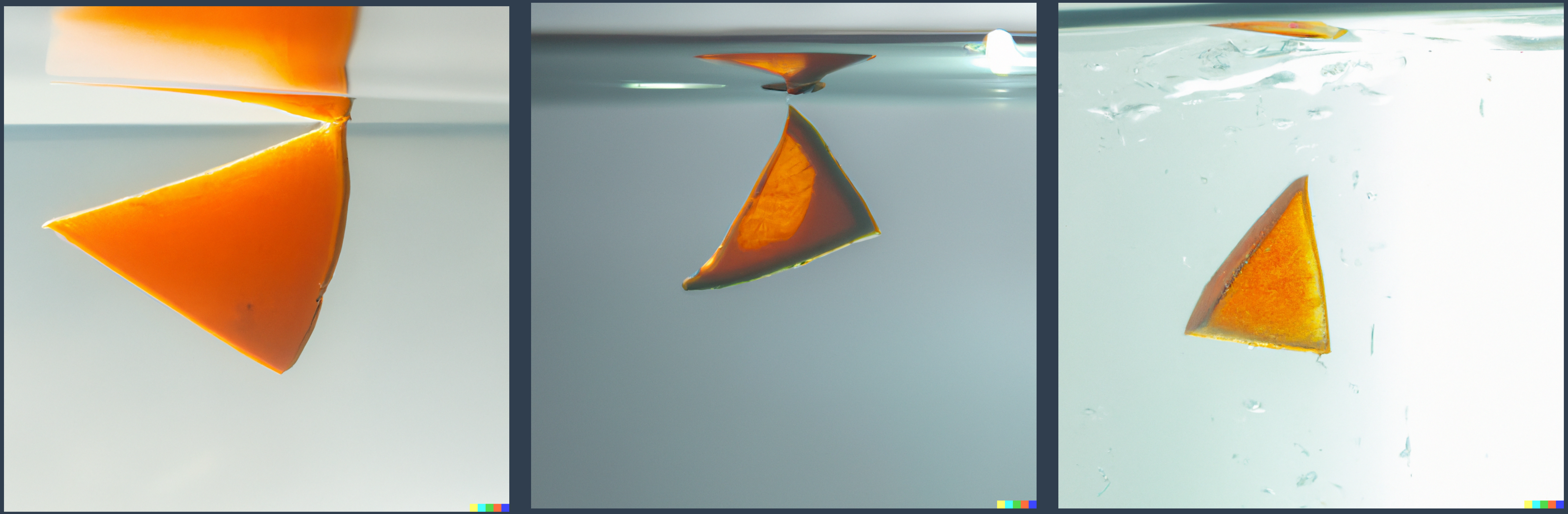
Pressure acts uniformly in all directions.

Pressure is set by the normal component of force with respect to the surface.

Pressure vs. depth

- It is a common experience that pressure increases with depth in a fluid.
- Consider the pressures acting on a submarine as it goes deeper into the ocean.

Varying depth

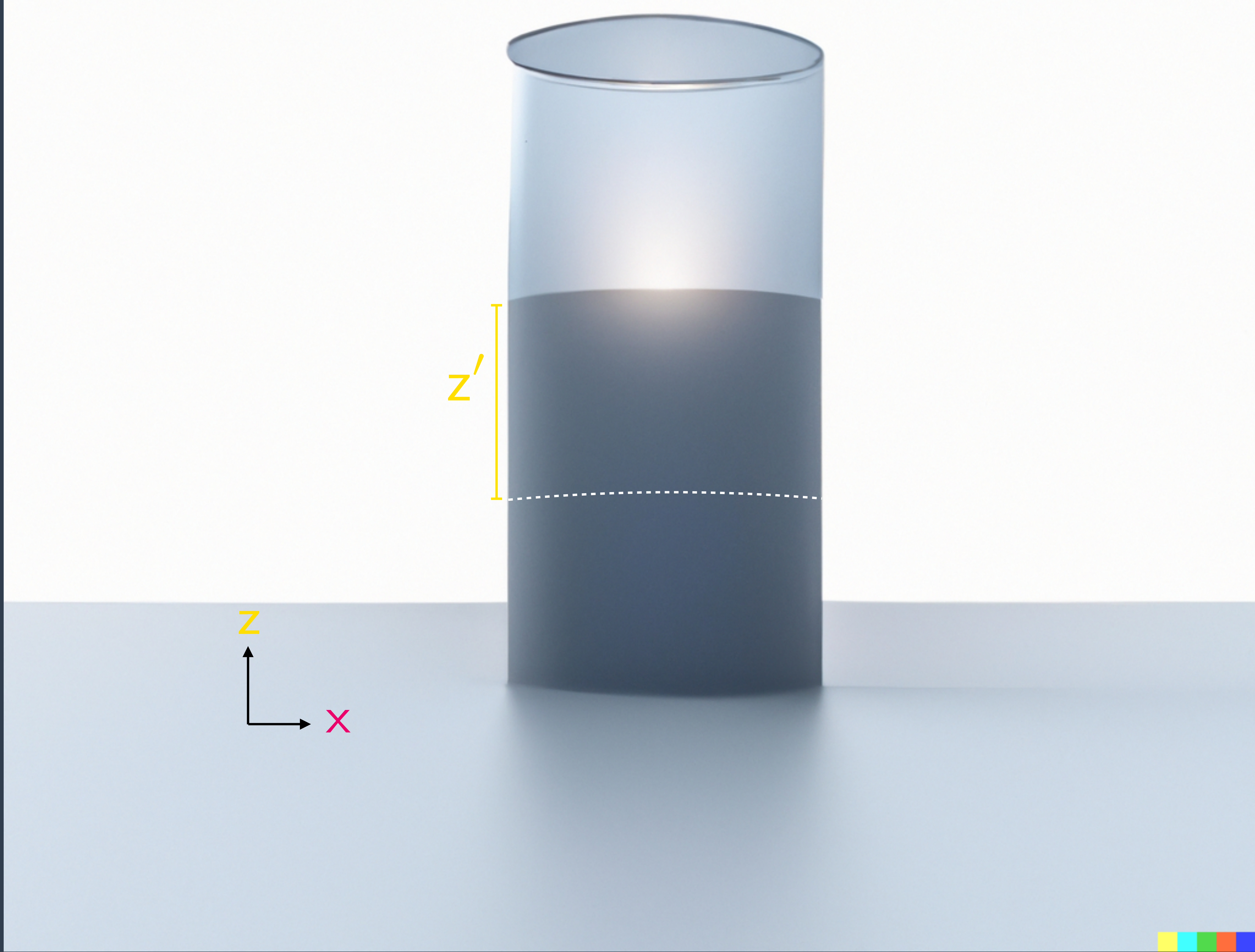


Pressure vs. depth

- Consider a cylindrical volume of fluid.
- The pressure at the free surface (top) is atmospheric.
- We wish to determine the pressure at a given depth.

Pressure vs. depth

- Consider a cylindrical volume of fluid.
- The pressure at the free surface (top) is atmospheric.
- We wish to determine the pressure at a given depth.
- We will consider the vertical equilibrium of a section within this cylinder as a free body.

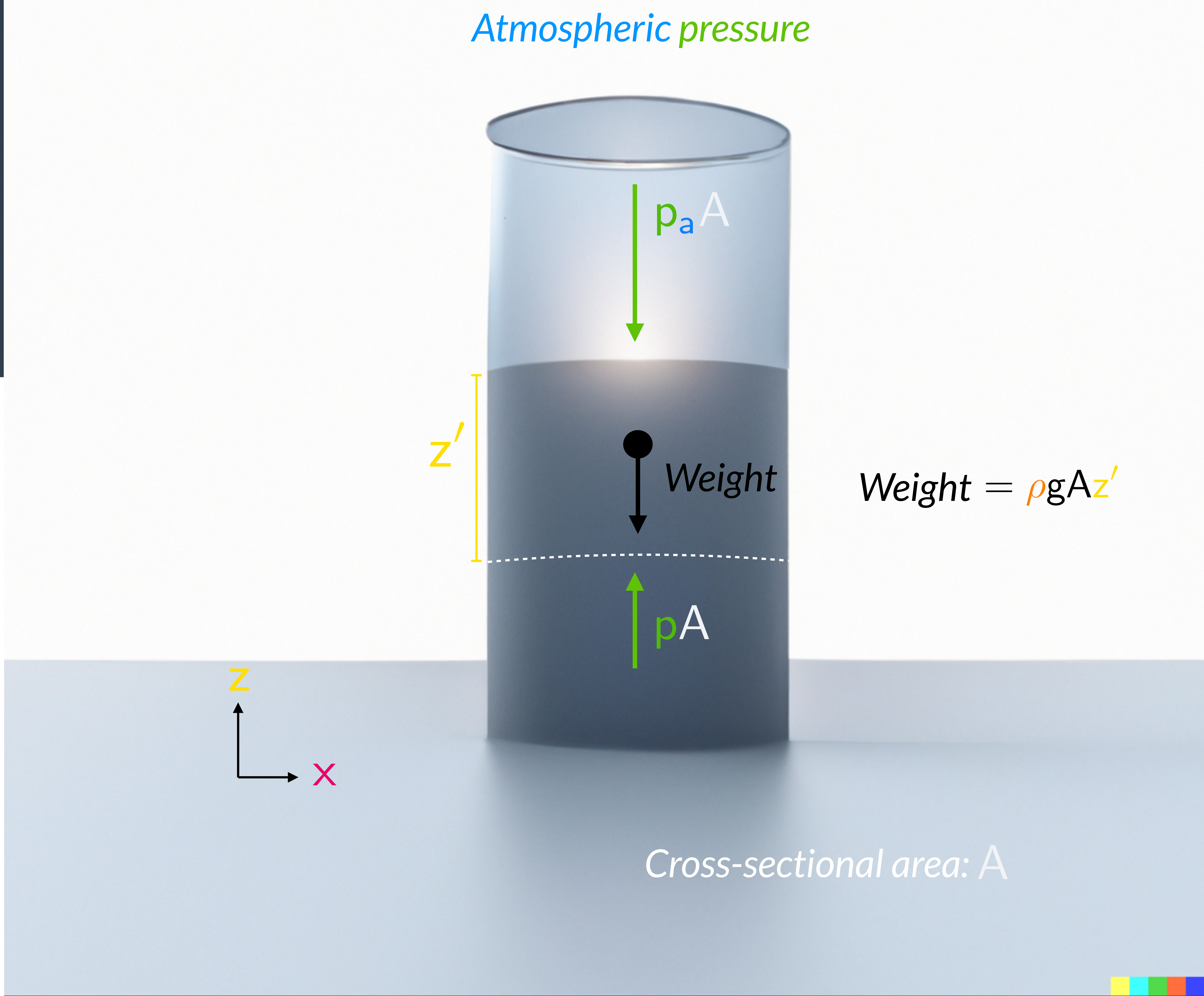


Pressure vs. depth

- What we know, at least intuitively, is that the pressure force acting on the bottom surface, pA, must balance the pressure force at the top surface, $p_a A$, in addition to the weight of the column.

V. *Summing the forces in the z direction:*

$$\sum F_z = 0$$



Pressure vs. depth

- Pressure increases with depth in order to support the weight of the liquid above it.

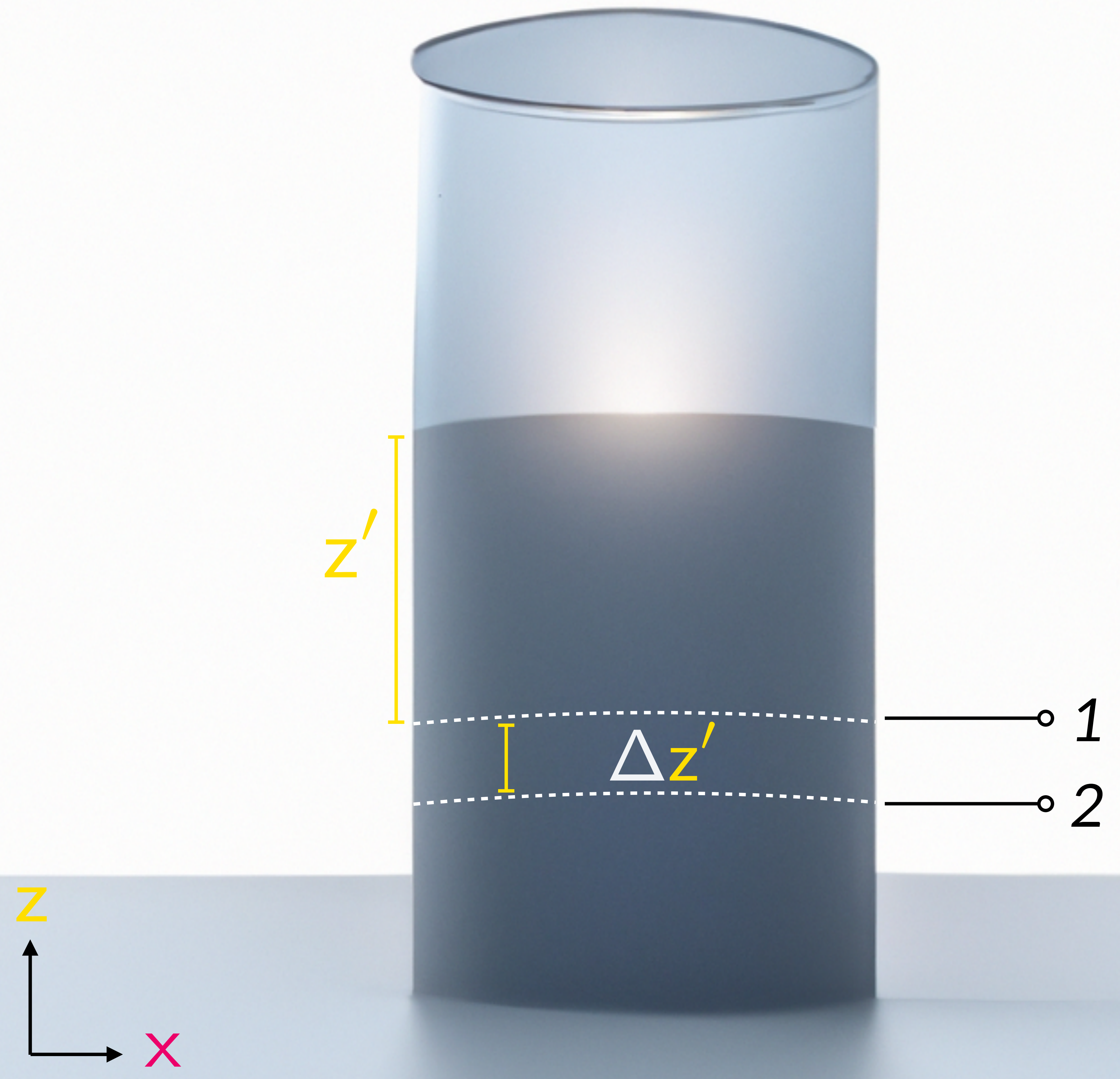
- Consider applying this notion at two different depths.

$$p_2 - p_1 = \rho g \Delta z'$$

$$\Rightarrow \Delta p = \rho g \Delta z'$$

- In the limit $\Delta z' \to dz$, we find the incremental pressure variation, dp, to be

$$dp = -\rho g \, dz$$



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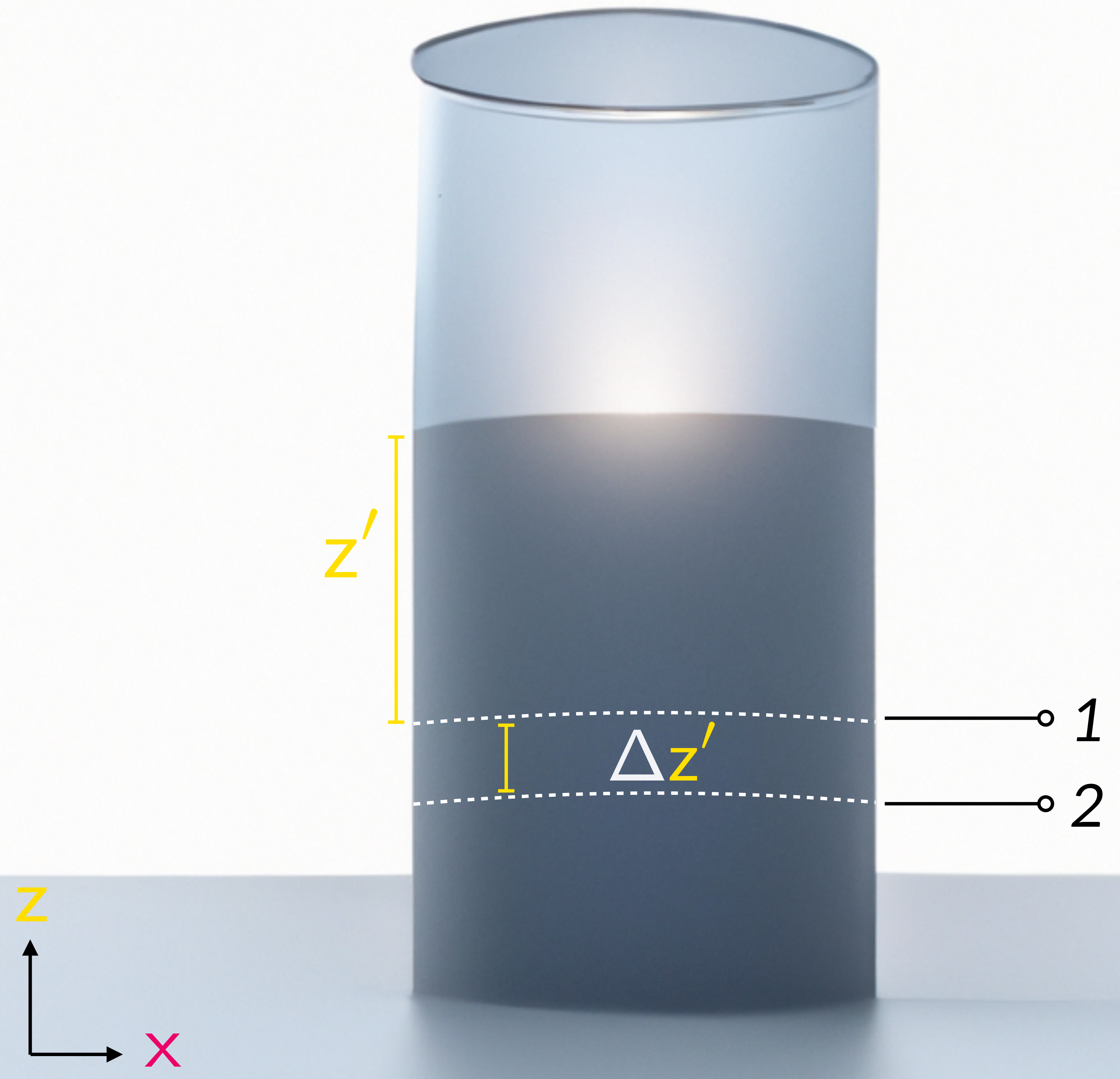
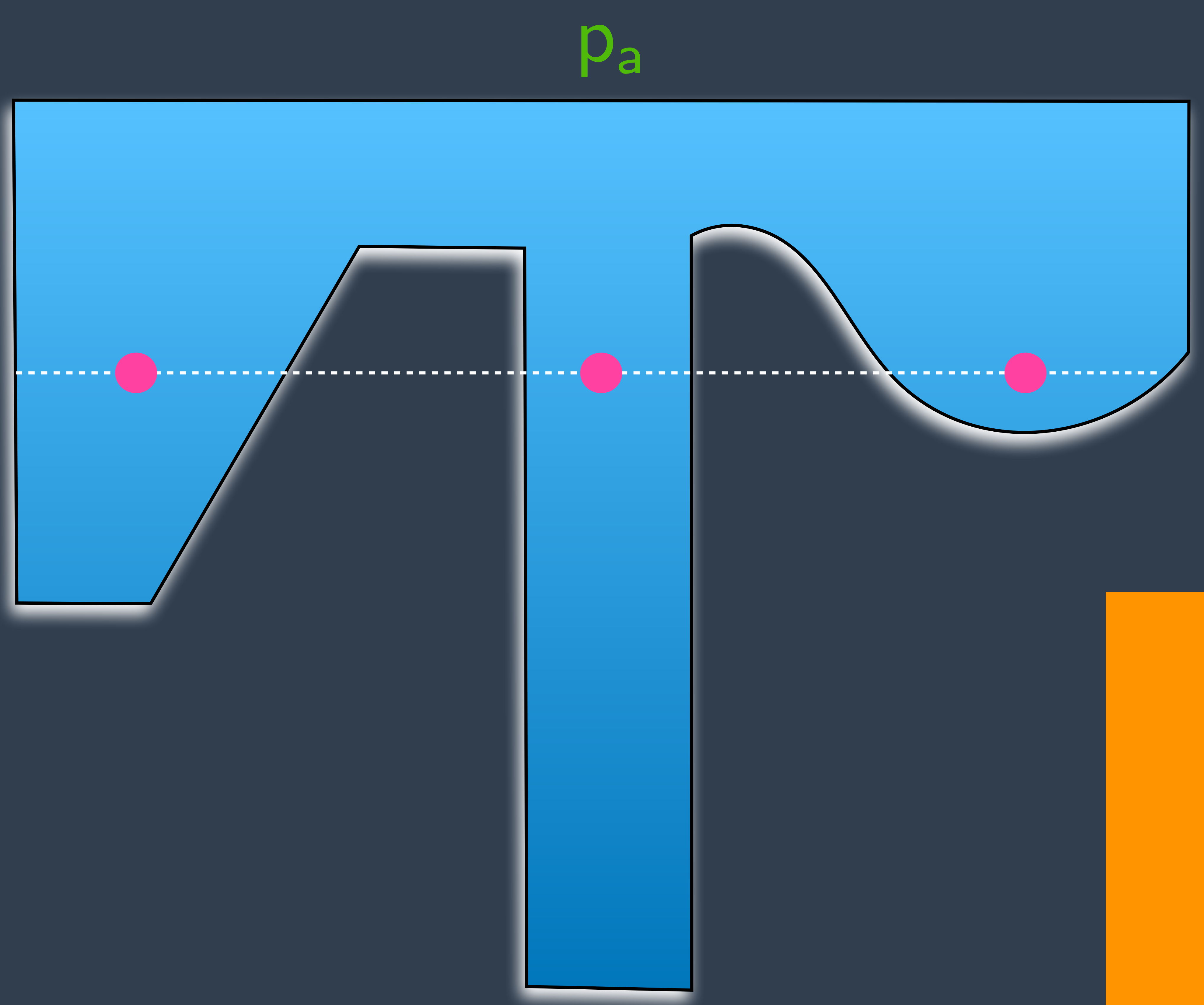


The negative sign arises because we count dz positive in the upward direction.

Pressure vs. depth

- Assuming the fluid is "connected", i.e., there are no bubbles, then pressure at all points, ●, is the same.

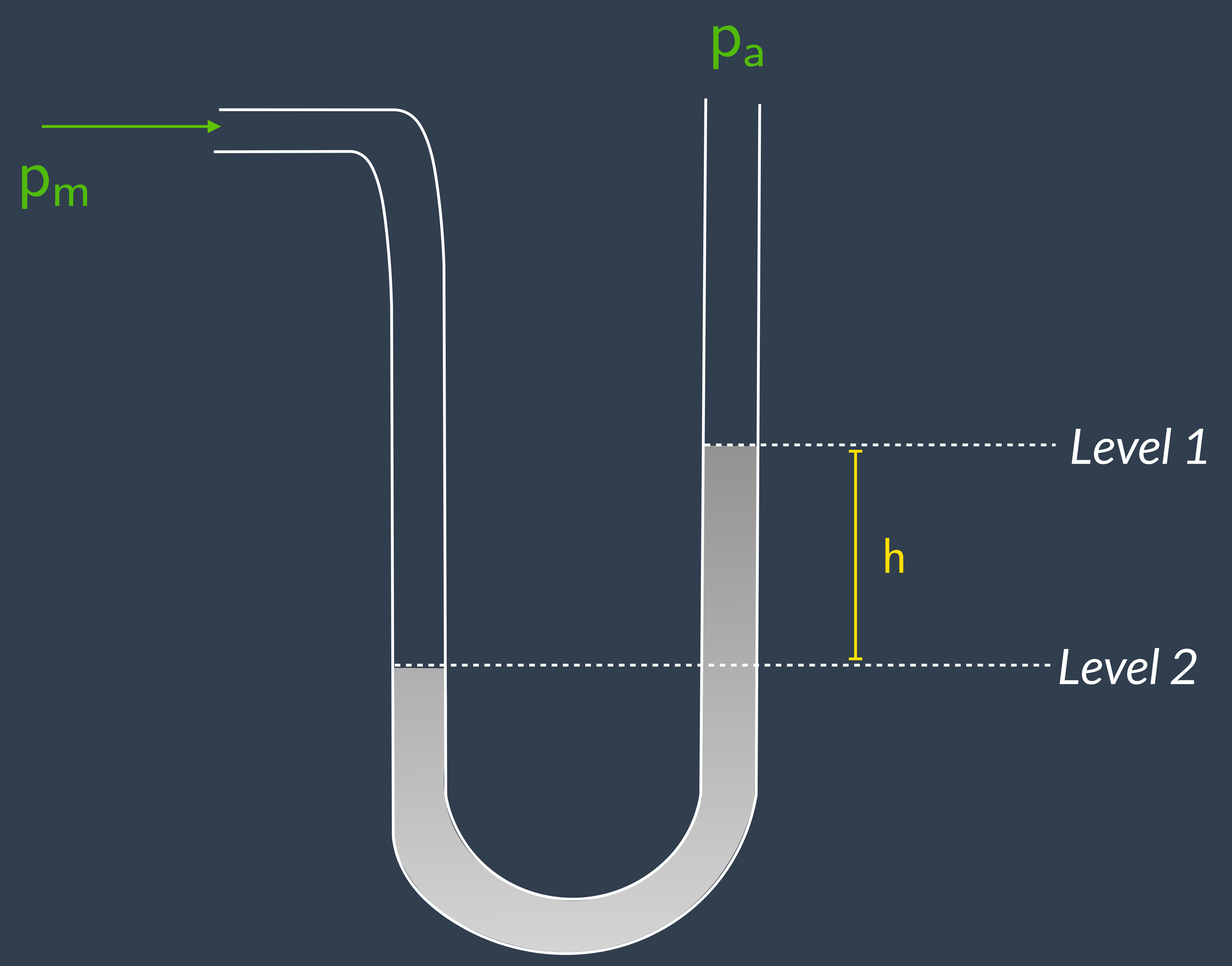


$$p = \rho g h + p_a$$

Pressure in a fluid increases linearly with depth.

Manometers

- The idea that pressure varies as a function of depth is used in a U-tube manometer to measure the pressure.



Manometers

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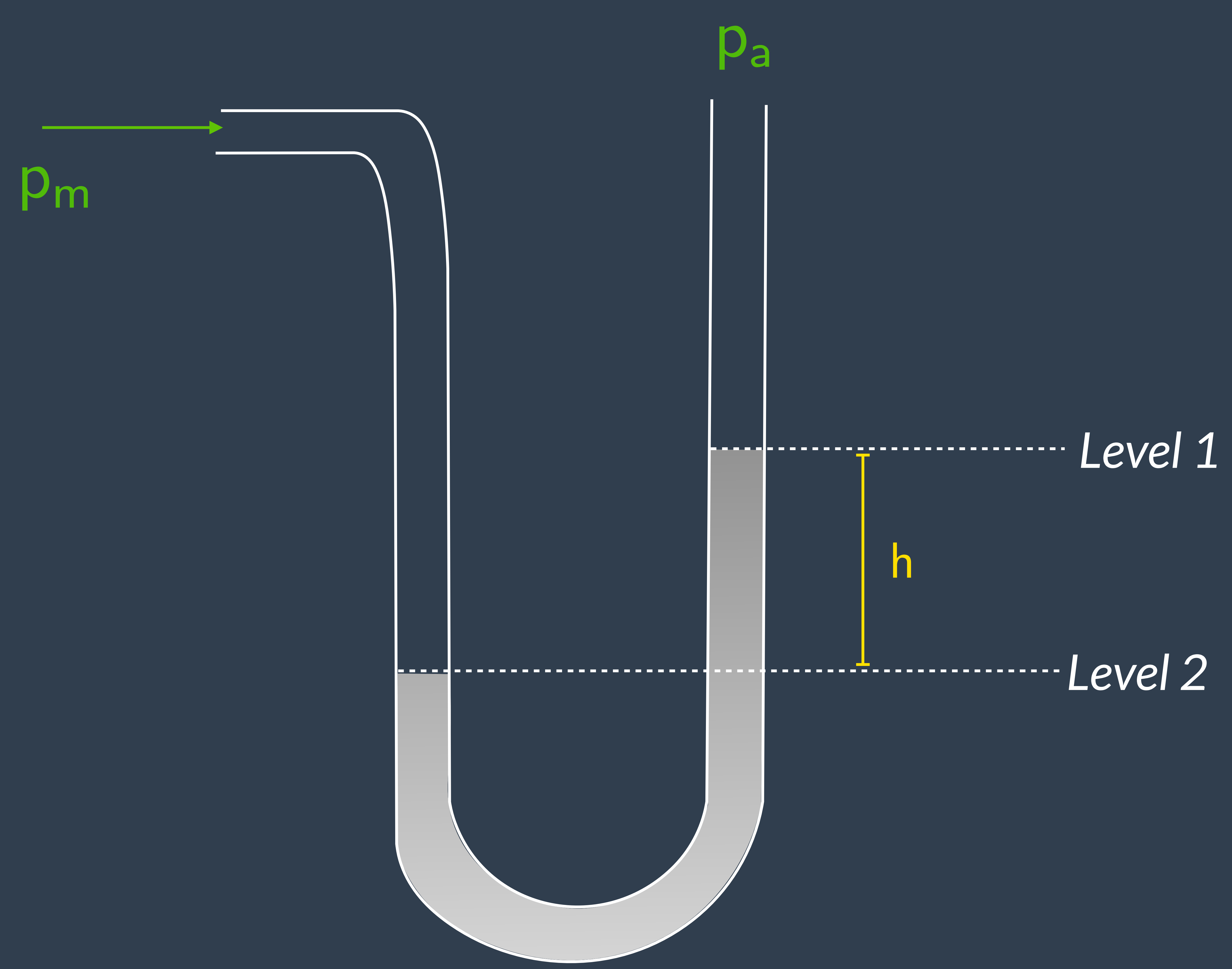
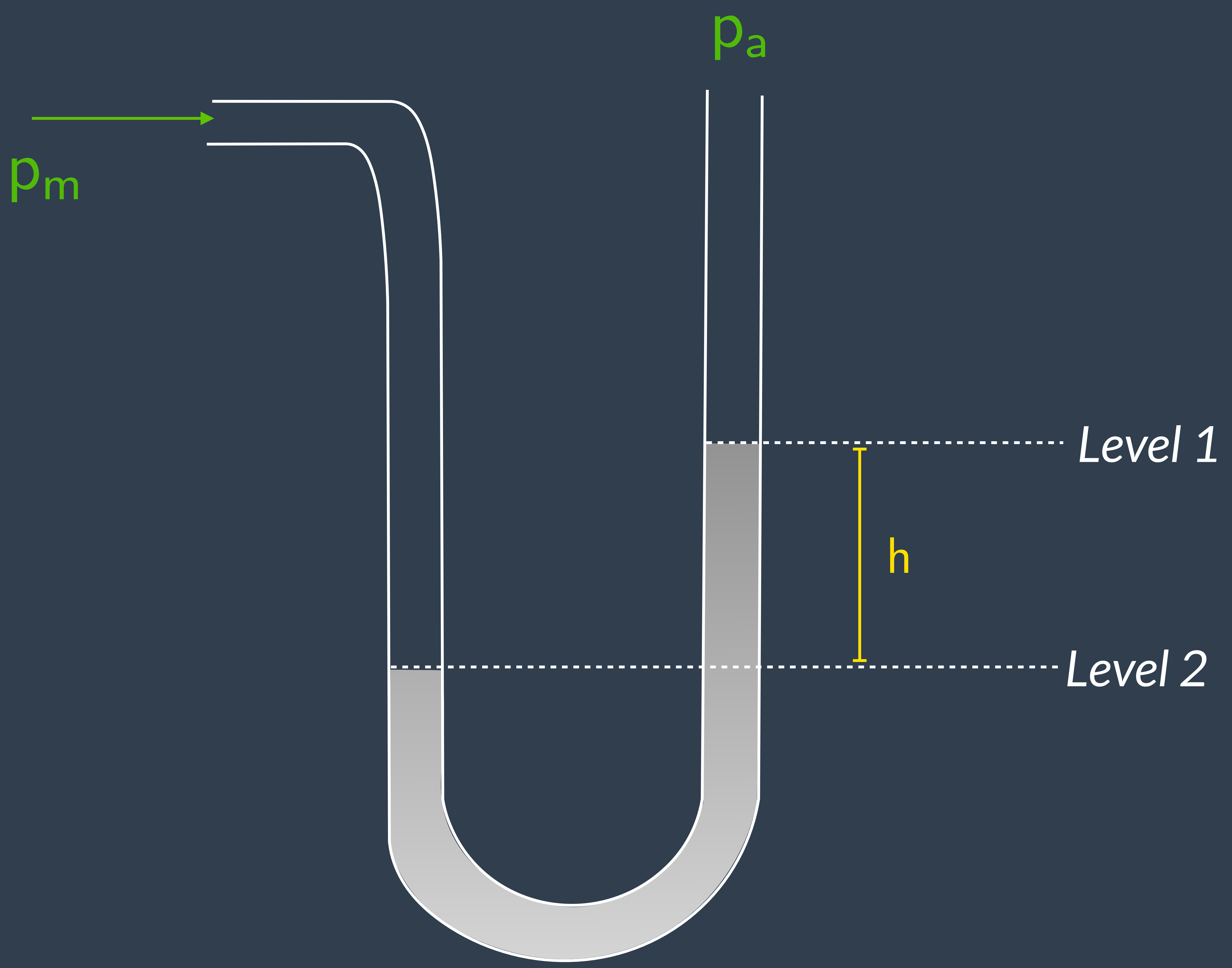


- The left-end sees an unknown pressure, p_m, while the right-end sees atmospheric pressure, p_a.

- The tube itself will house a liquid with a density greater than air, and the difference in the height, h, between the two parallel tubes is used to work out the unknown pressure.

Manometers



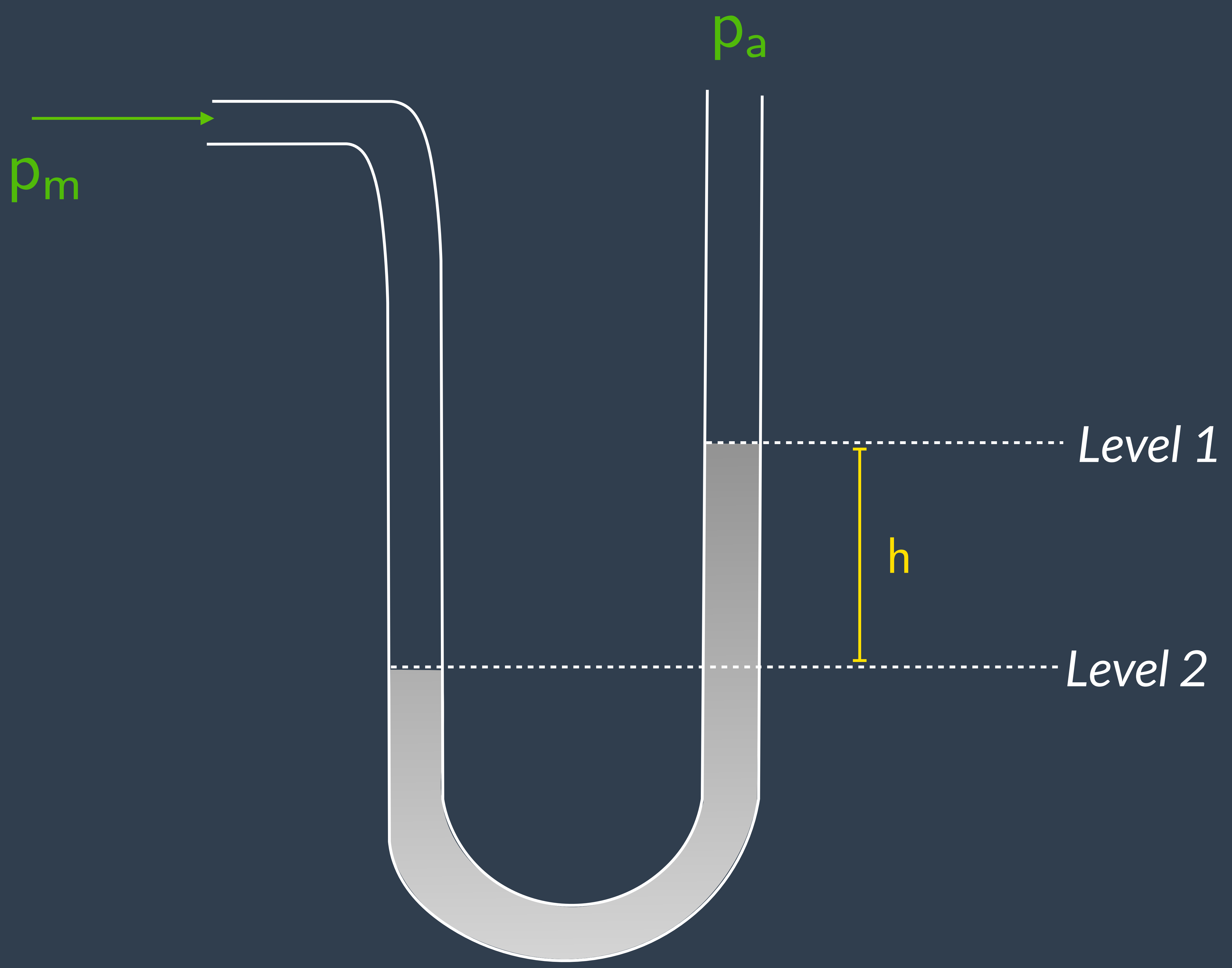
VI.

On the left-hand side, at level 2, the pressure is

On the right-hand side, at level 2, the pressure is

This yields...

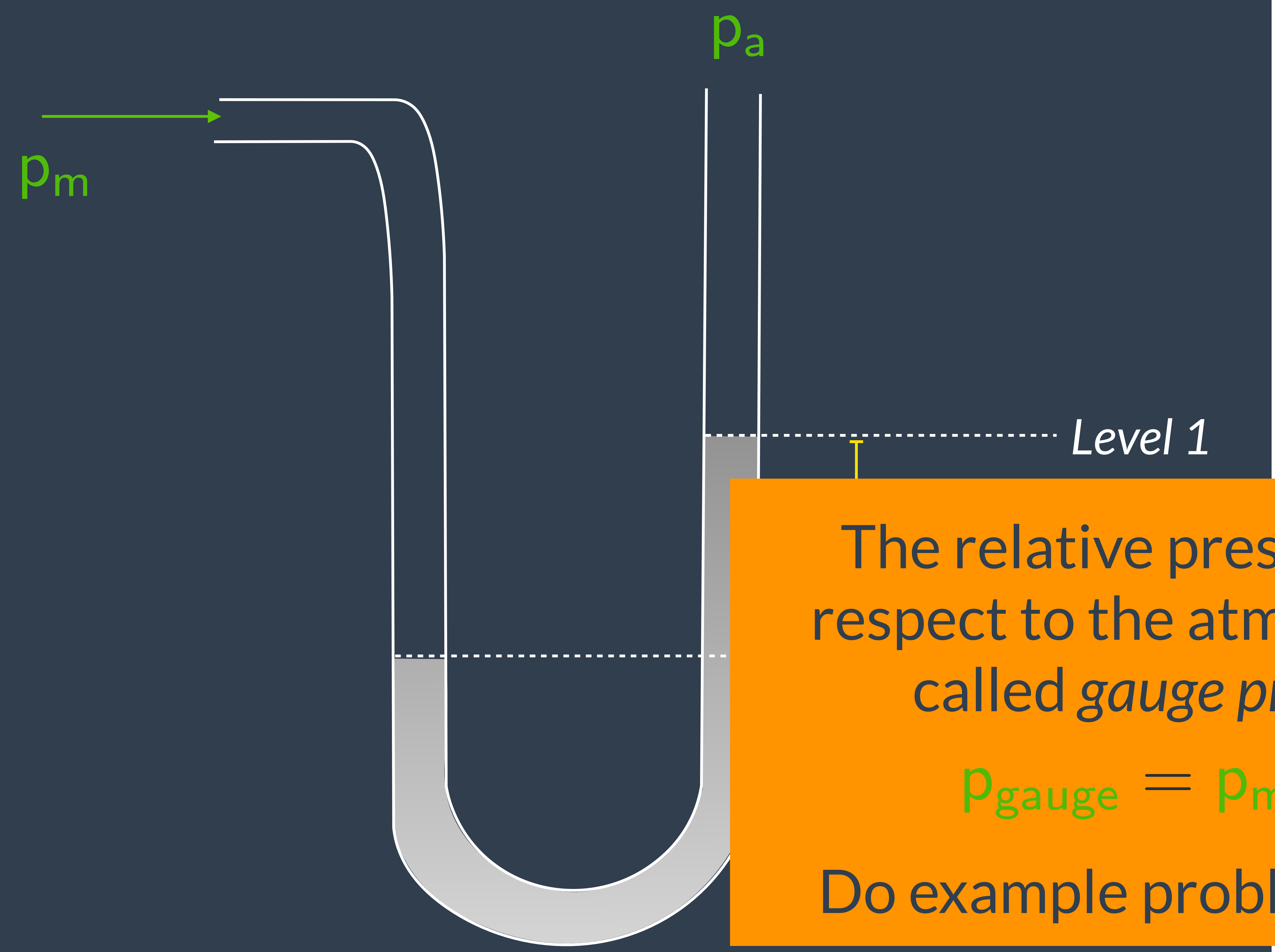
Manometers



VII.

Typically, the liquid used is much denser, thus, we can ignore the density of air. This yields

Manometers



The relative pressure with respect to the atmosphere is called *gauge pressure.*

$$p_{gauge} = p_m - p_a$$

Do example problem #1 & #2

VII.

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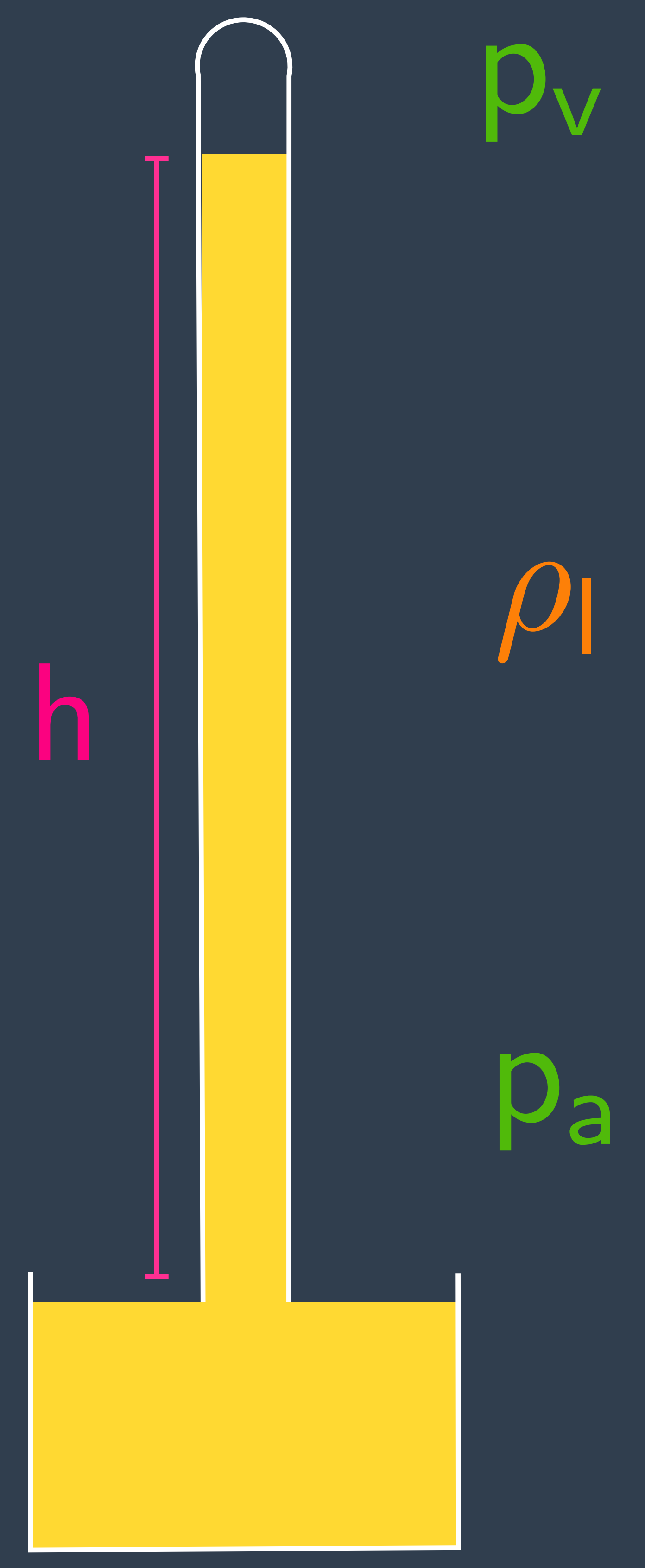
Barometers

- Barometers are simpler U-tube manometers, used to measure the atmospheric pressure.
- The pressure at the top of the tube, p_v, denotes a region of vacuum. This permits the level of the mercury (although today other fluids are used), to vary.
- As the atmospheric pressure, p_a, increases, the height, h, of the fluid rises.



Do example problem #3

Archimedes' principle

- Any object that is immersed in a fluid receives an upward thrust (force), F_u, in the direction opposite to gravity.

- This arises as the fluid pressure acting at the bottom of the object is larger than at the top.

  Recall: Pressure increases with depth!

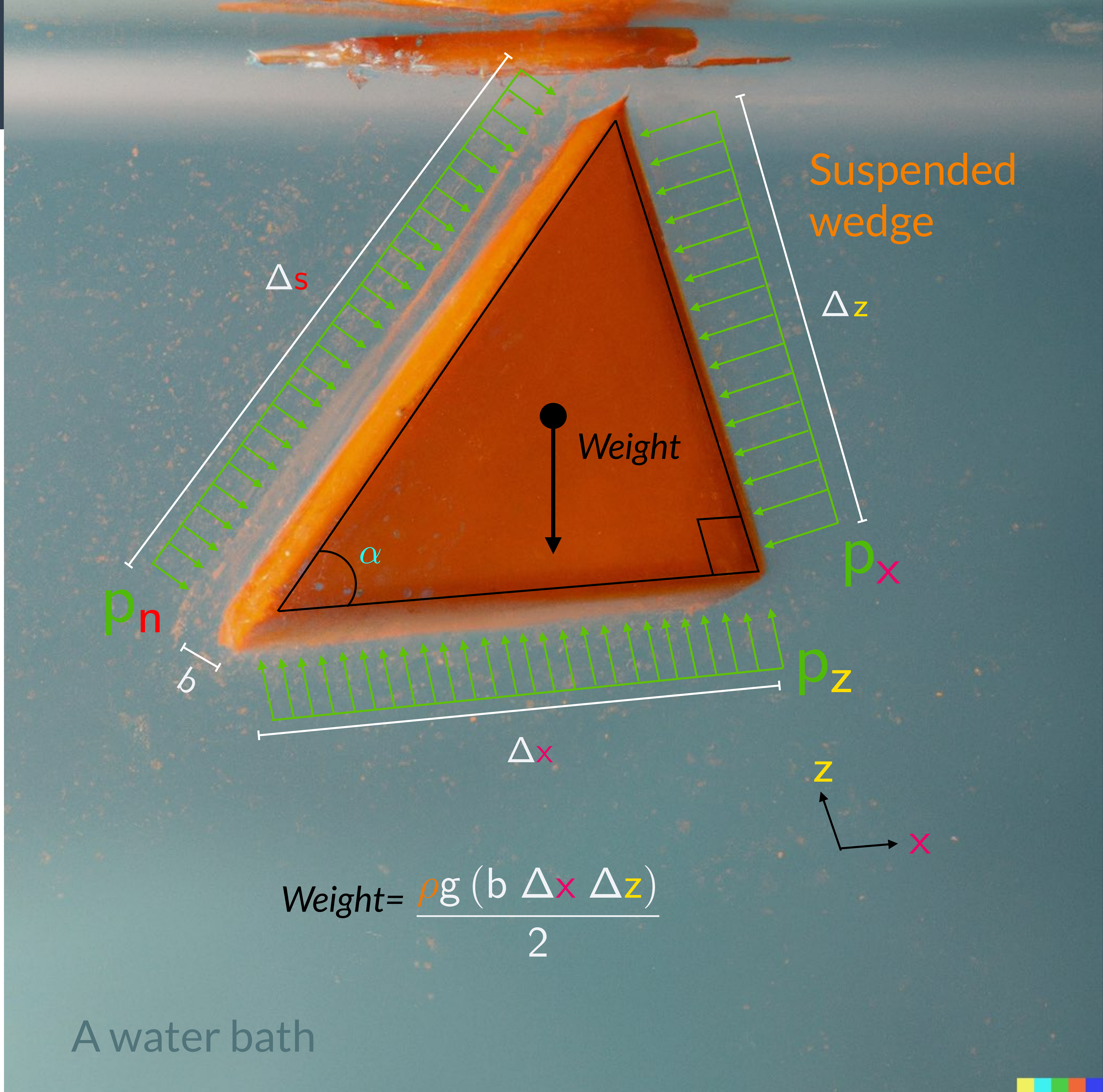
- Archimedes principle states that if a body is partially or wholly submerged, then it receives an upward thrust equivalent to the displaced fluid.

Archimedes' principle

VIII.

From earlier, we had

$$\sum F_z = p_z b \Delta x - p_n \cos(\alpha)\, b \Delta s - \text{weight} = 0$$



Archimedes' principle

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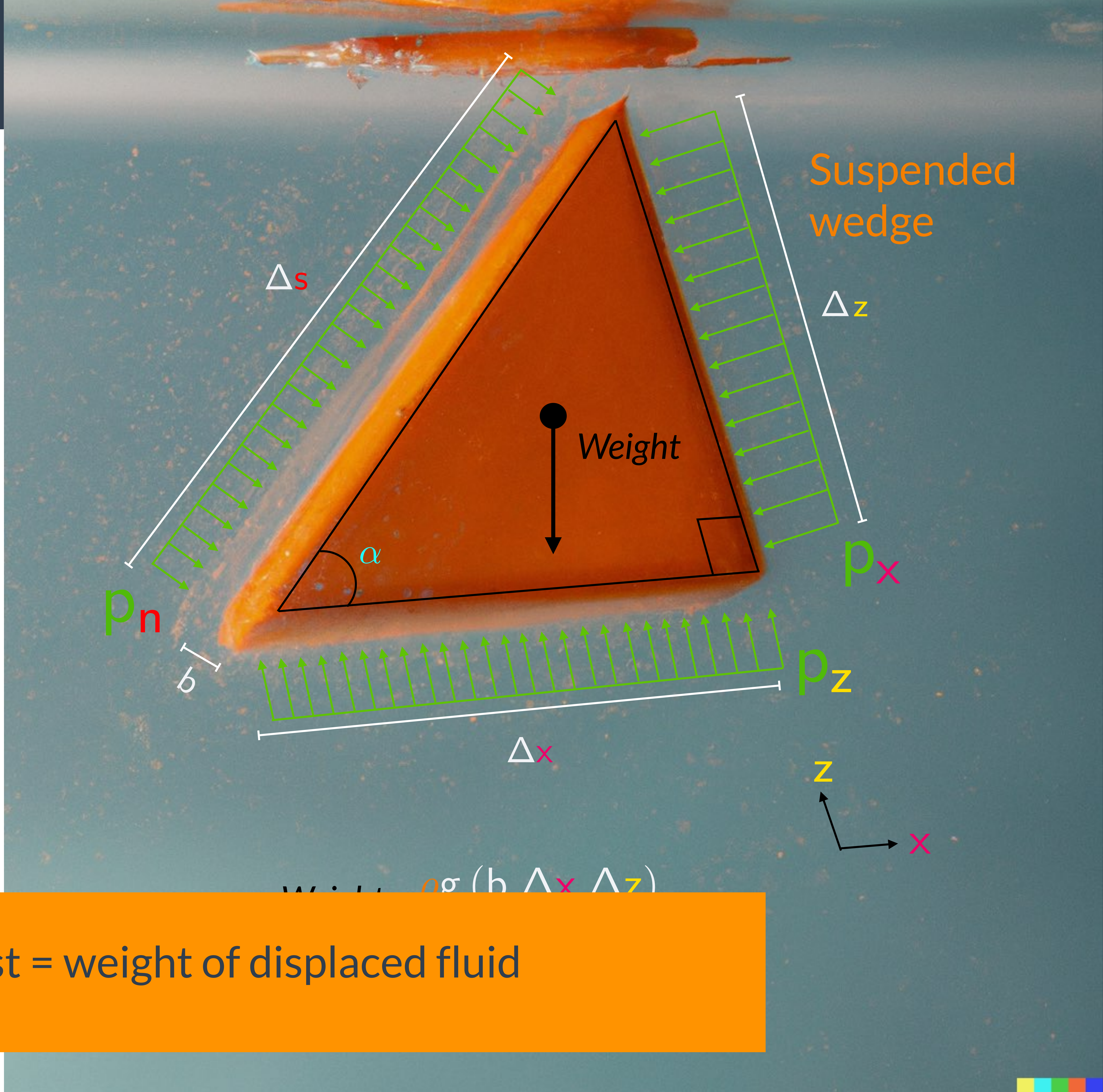
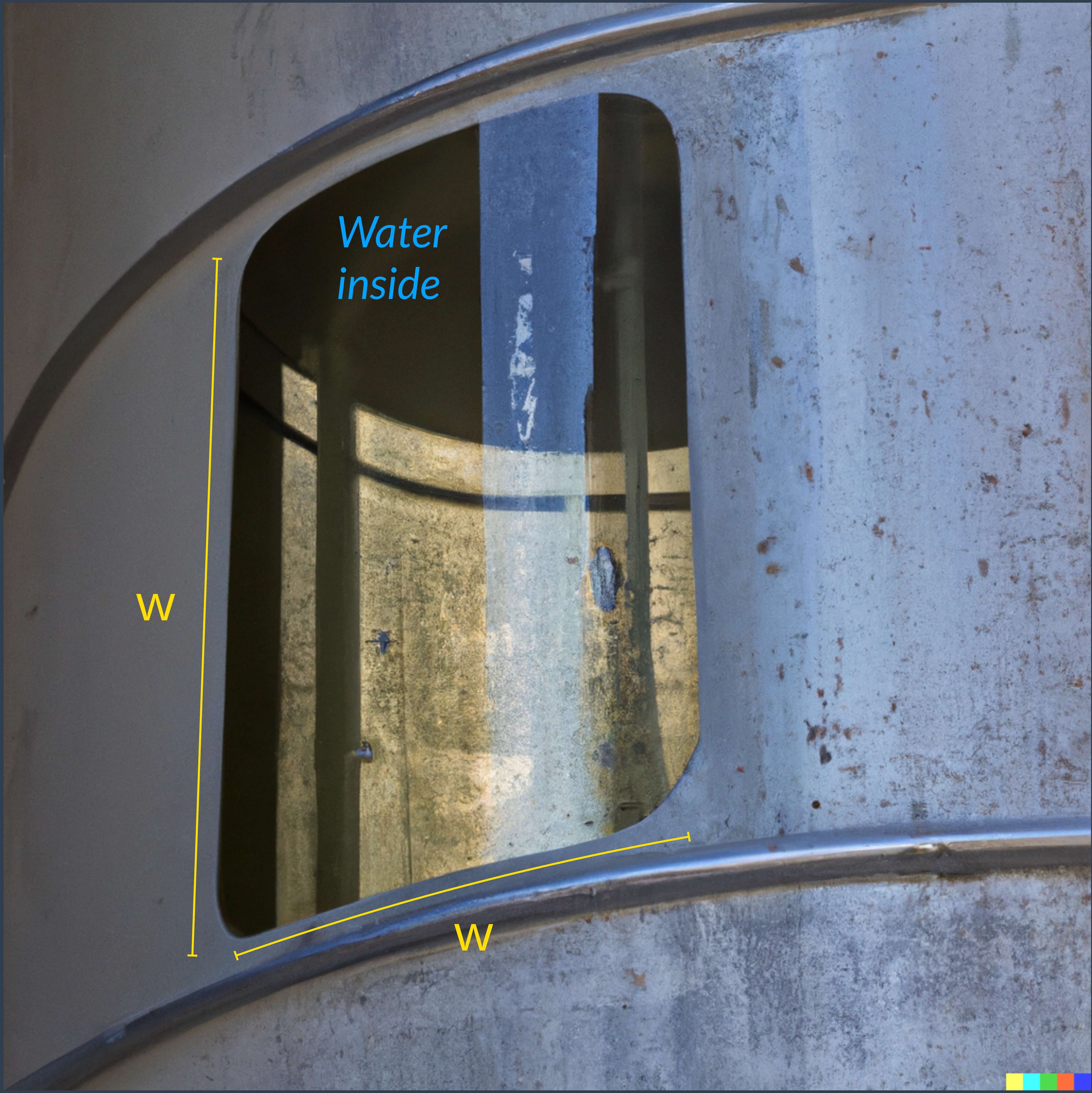


Upward thrust = weight of displaced fluid

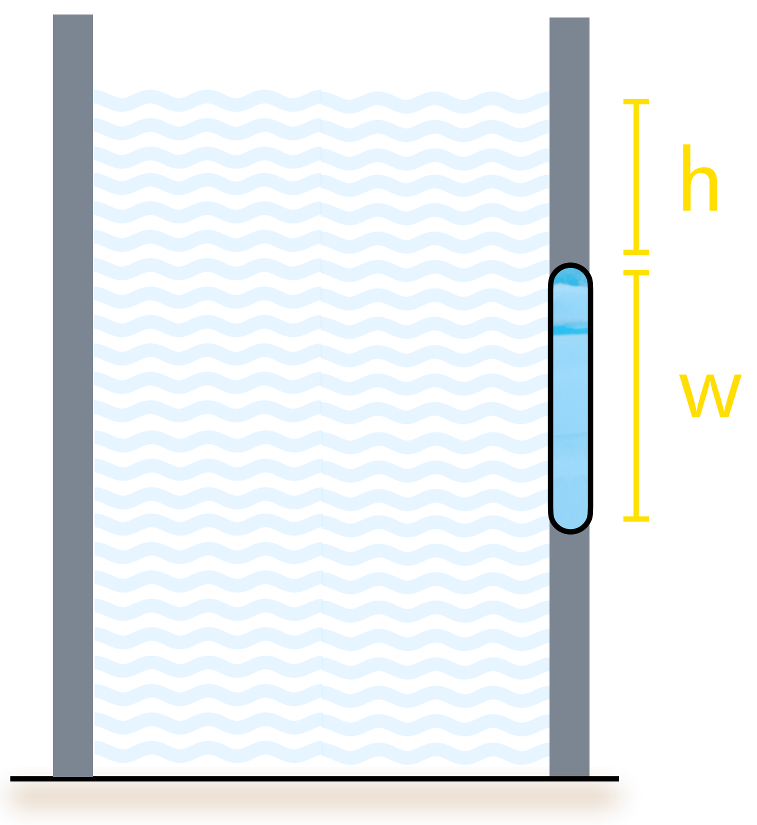
Forces on submerged bodies

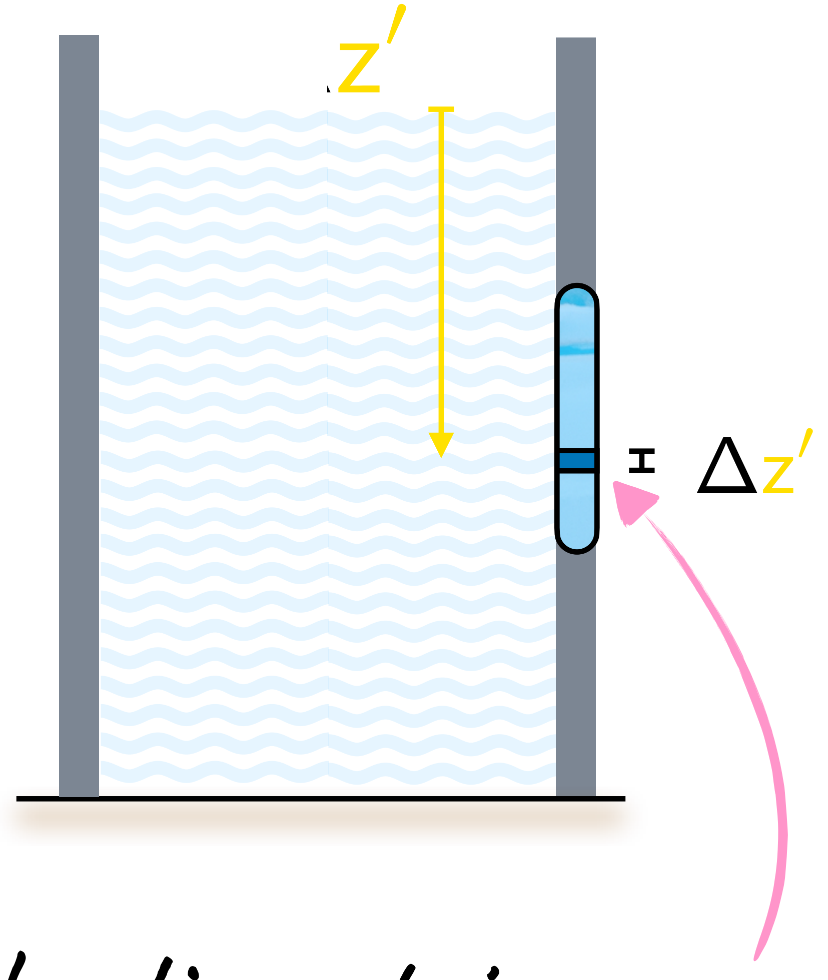
- A square (well nearly) window is embedded in the face of a large container that holds water.
- We want to quantify the net hydrostatic force on this window.
- The challenge is that the pressure varies over the window, so we cannot simply find the force by multiplying the window area by some pressure.



Forces on submerged bodies

$\mathbb{IX}.$





Steps required:

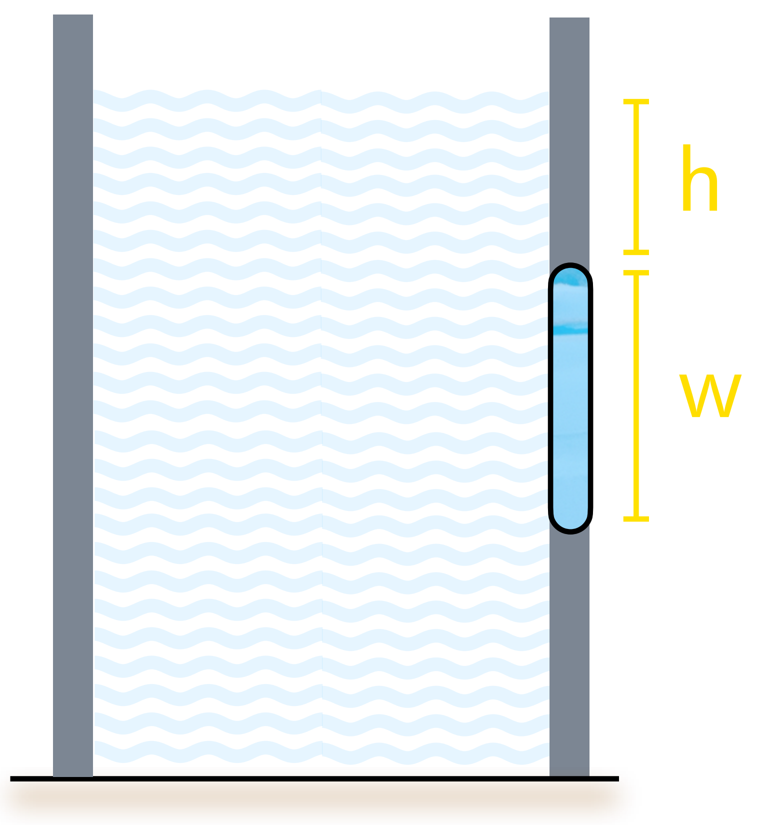
1. *Consider the tiny strip,* ▬ .

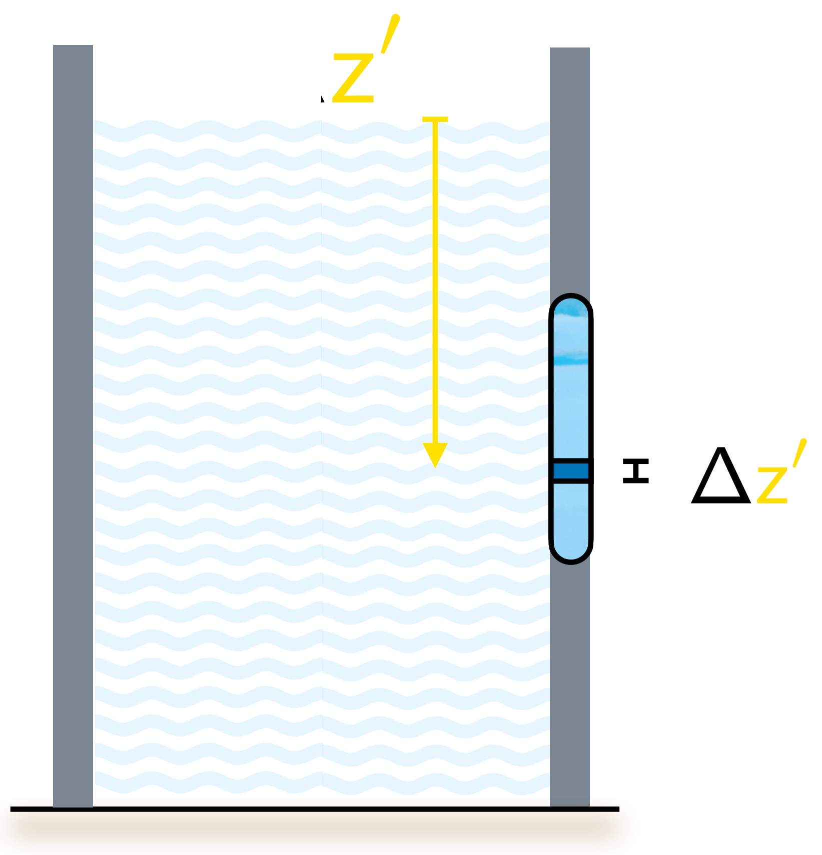
Force on the tiny strip, ▬ *, is:*

$$\Delta F = p\ \Delta A = (\rho g z')\ (w\ \Delta z')$$

Forces on submerged bodies

$\mathbb{X}$.





Total force on window:

$$F = \sum (\rho g z') \, w \, \Delta z'$$ *(summation over the window)*

Forces on submerged bodies

- Consider an architecturally savvy swimming pool where the edges are formed of inclined glass planes.

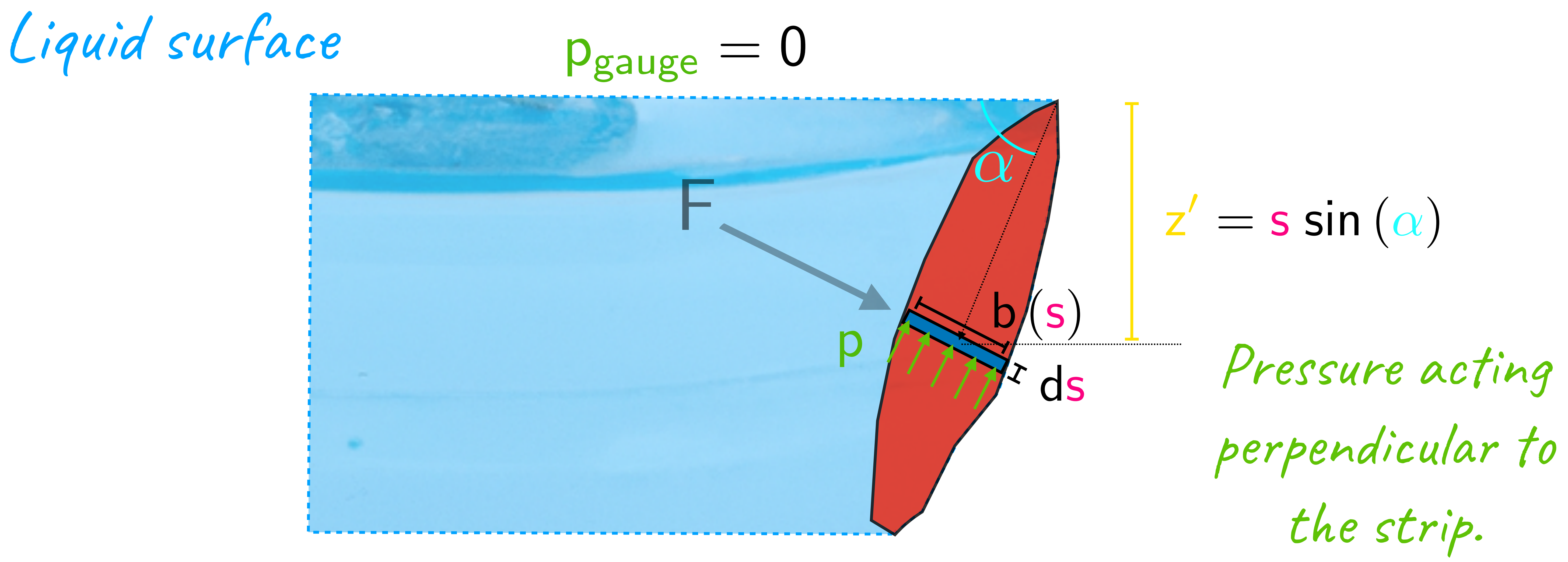
- Naturally, the glass needs to withstand the hydrostatic force, which will vary depending on the depth of the pool.

Forces on submerged bodies

- Consider an architecturally savvy swimming pool where the edges are formed of inclined glass planes.
- Naturally, the glass needs to withstand the hydrostatic force, which will vary depending on the depth of the pool.
- You want to avoid a scenario where the glass collapses under the weight of water.

Forces on submerged bodies

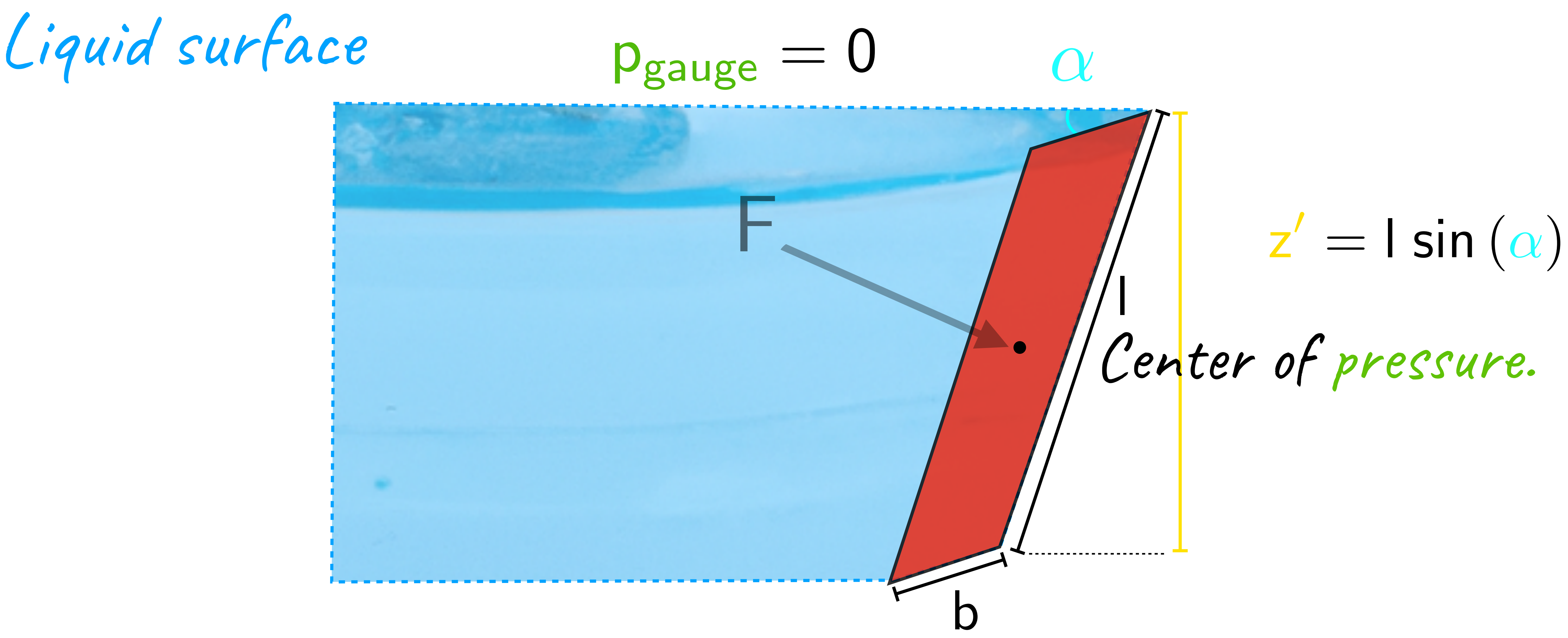
XI. *Assume the glass plate is not rectangular at all, but has some irregular shape.*



$$F = \int_{\text{plate}} dF = \int_{\text{plate}} p(s)\, b(s)\, ds =$$

Forces on submerged bodies

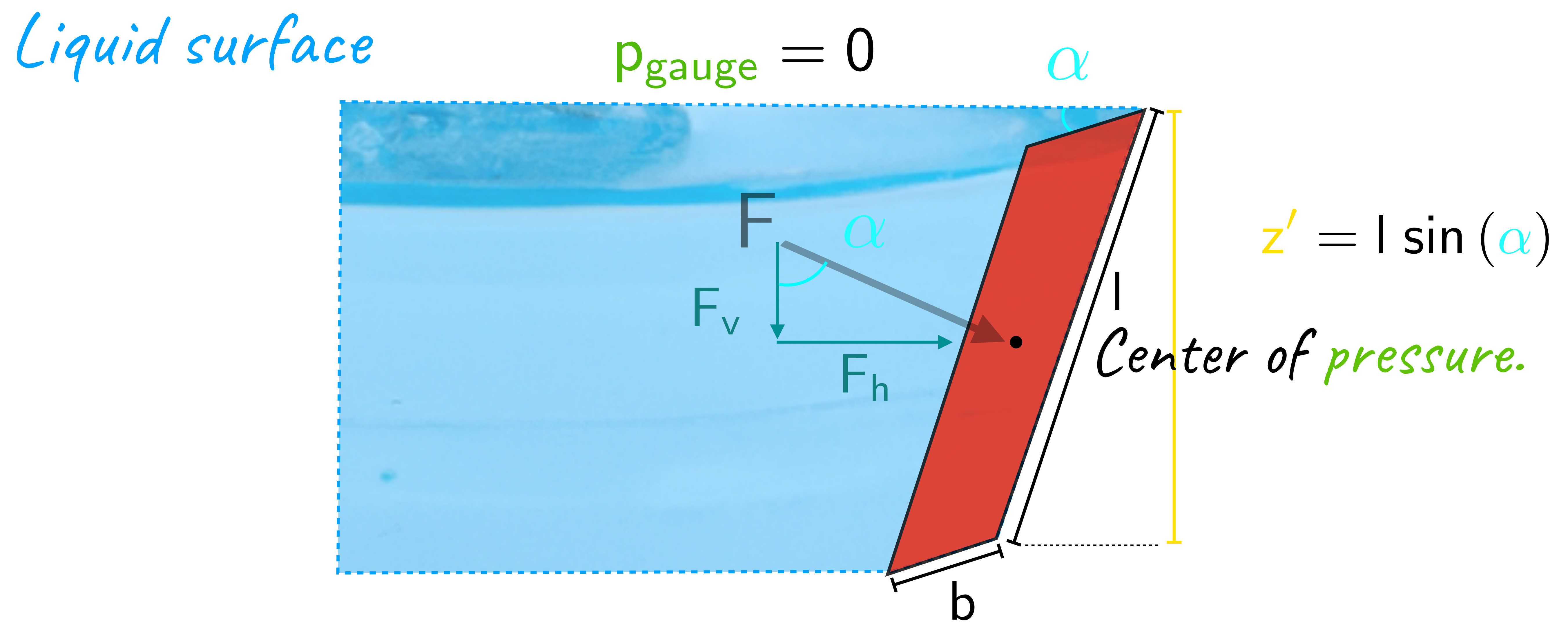
XII. *For rectangular plates with constant width, this simplifies to*



$$F = \rho g \sin(\alpha)\, b \int_{plate} s \, ds =$$

Forces on submerged bodies

XIII. *The horizontal and vertical components of this force can be calculated rather easily...*



$$F = \rho g \sin(\alpha)\, b \int_{plate} s\, ds = \rho g \sin(\alpha)\, b \frac{l^2}{2}$$

$F_h =$ & $F_v =$

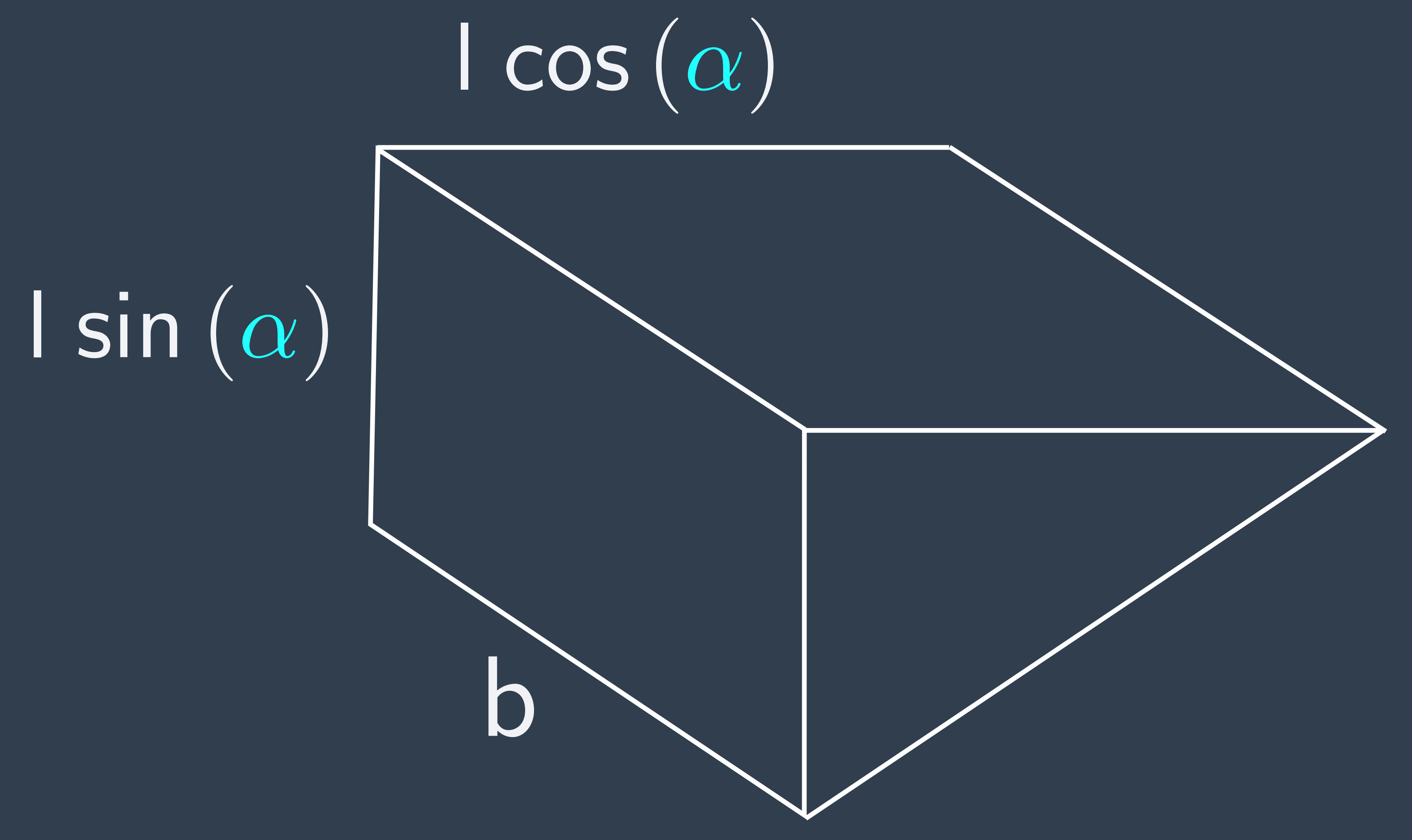
Forces on submerged bodies

- The horizontal component is equivalent to the force experienced by a vertical surface of the same projected area, i.e., $b \times l \sin(\alpha)$.

- To see this consider that

$$F_h = \rho g b \int_0^{l \sin(\alpha)} s \, ds$$

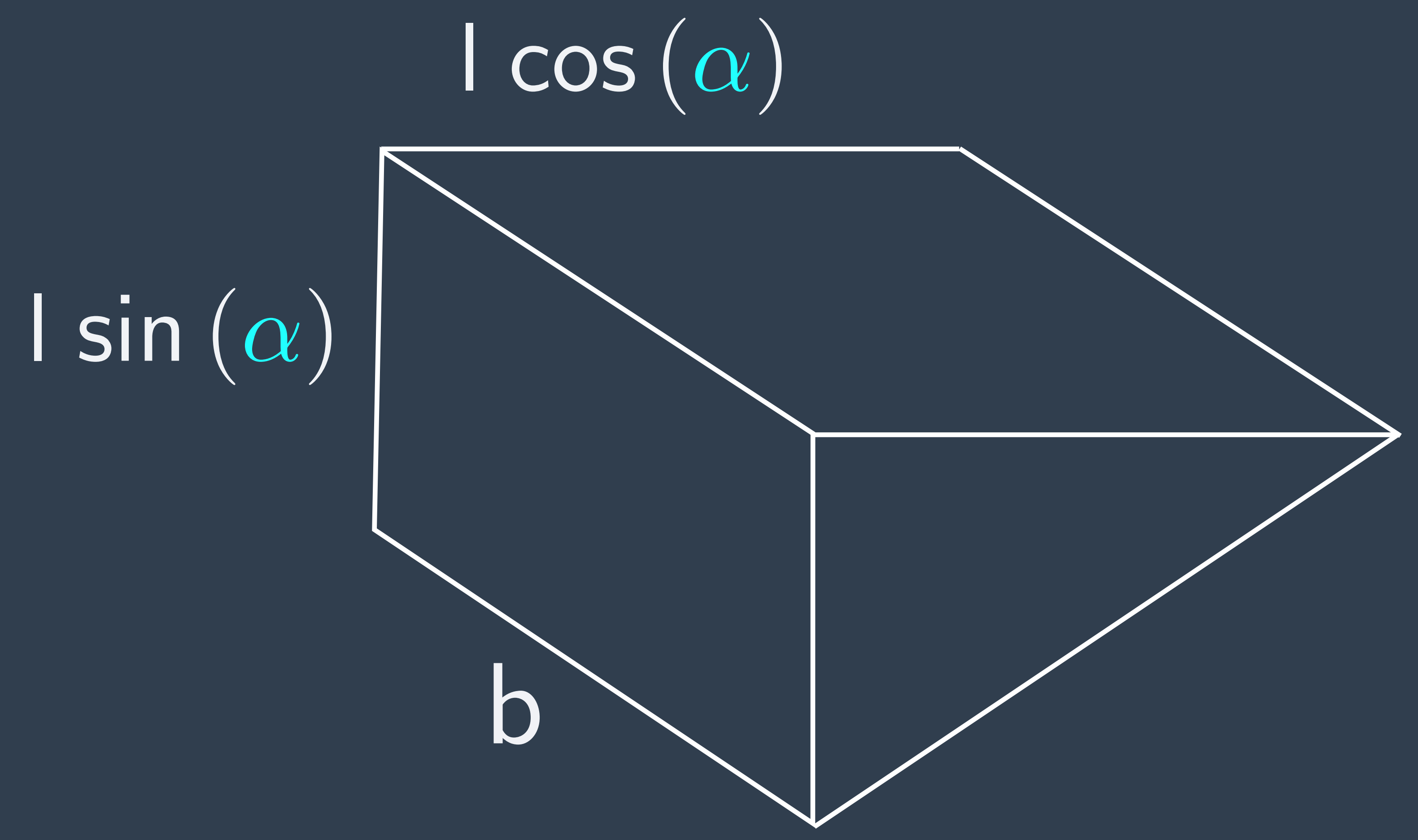
$$= \rho g b \frac{(l \sin(\alpha))^2}{2}$$



Forces on submerged bodies

- By inspection, the vertical force component, F_v, is equivalent to the weight of the volume of fluid above the plate, i.e.,

$$\text{Weight} = \text{Volume} \times \rho g = \frac{1}{2} l\cos(\alpha)\ \ l\sin(\alpha)\ \ b\ \rho g$$



- This is generally the case for *any* submerged surface.

- During recitation next week we will cover a more complex version of these type of problems.