Lecture 3

Pranay Seshadri
Georgia Institute of Technology

prse@gatech.edu

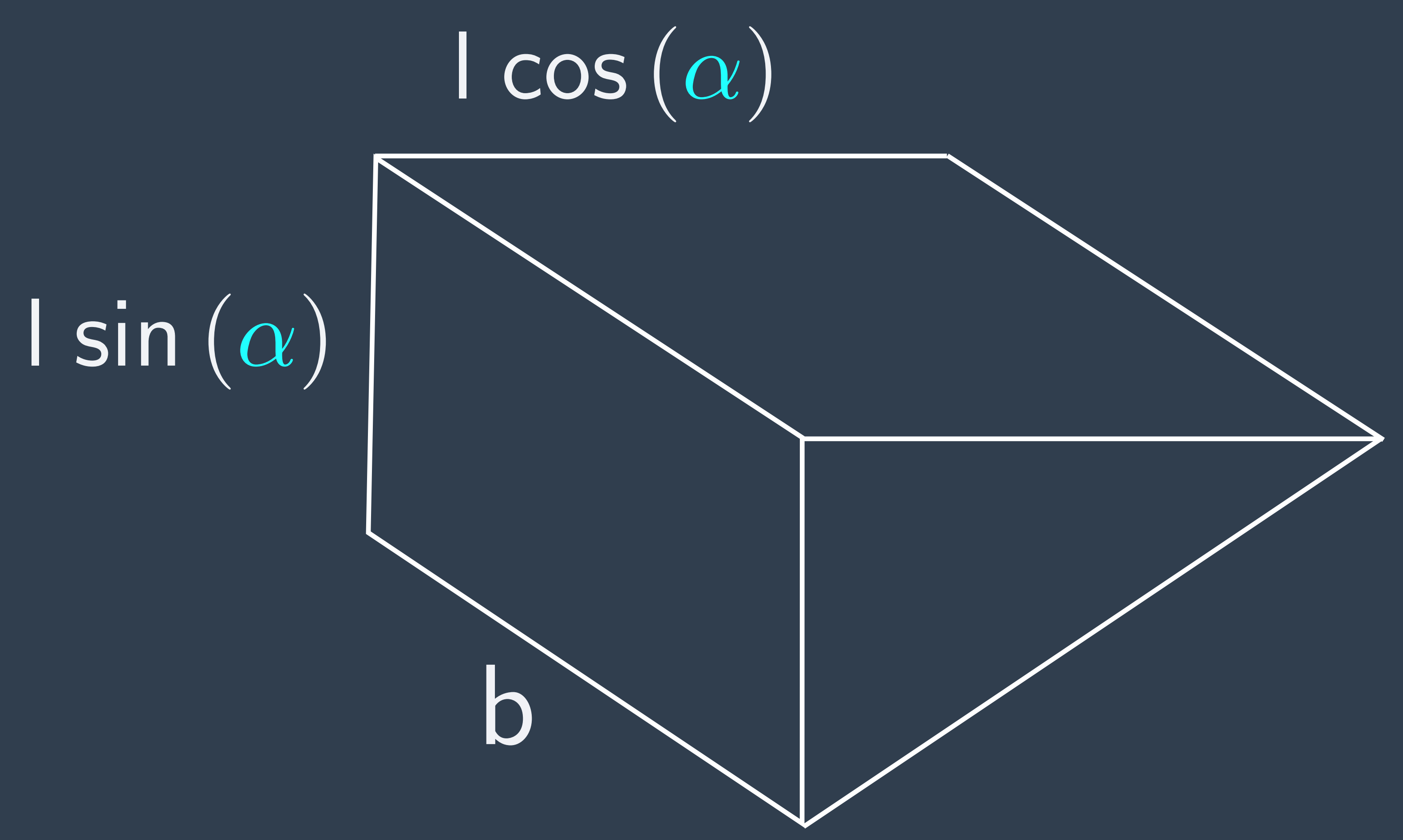
Forces on submerged bodies

- The horizontal component is equivalent to the force experienced by a vertical surface of the same projected area, i.e., $b \times l \sin(\alpha)$.

- To see this consider that

$$F_h = \rho g b \int_0^{l \sin(\alpha)} s \, ds$$

$$= \rho g b \frac{(l \sin(\alpha))^2}{2}$$



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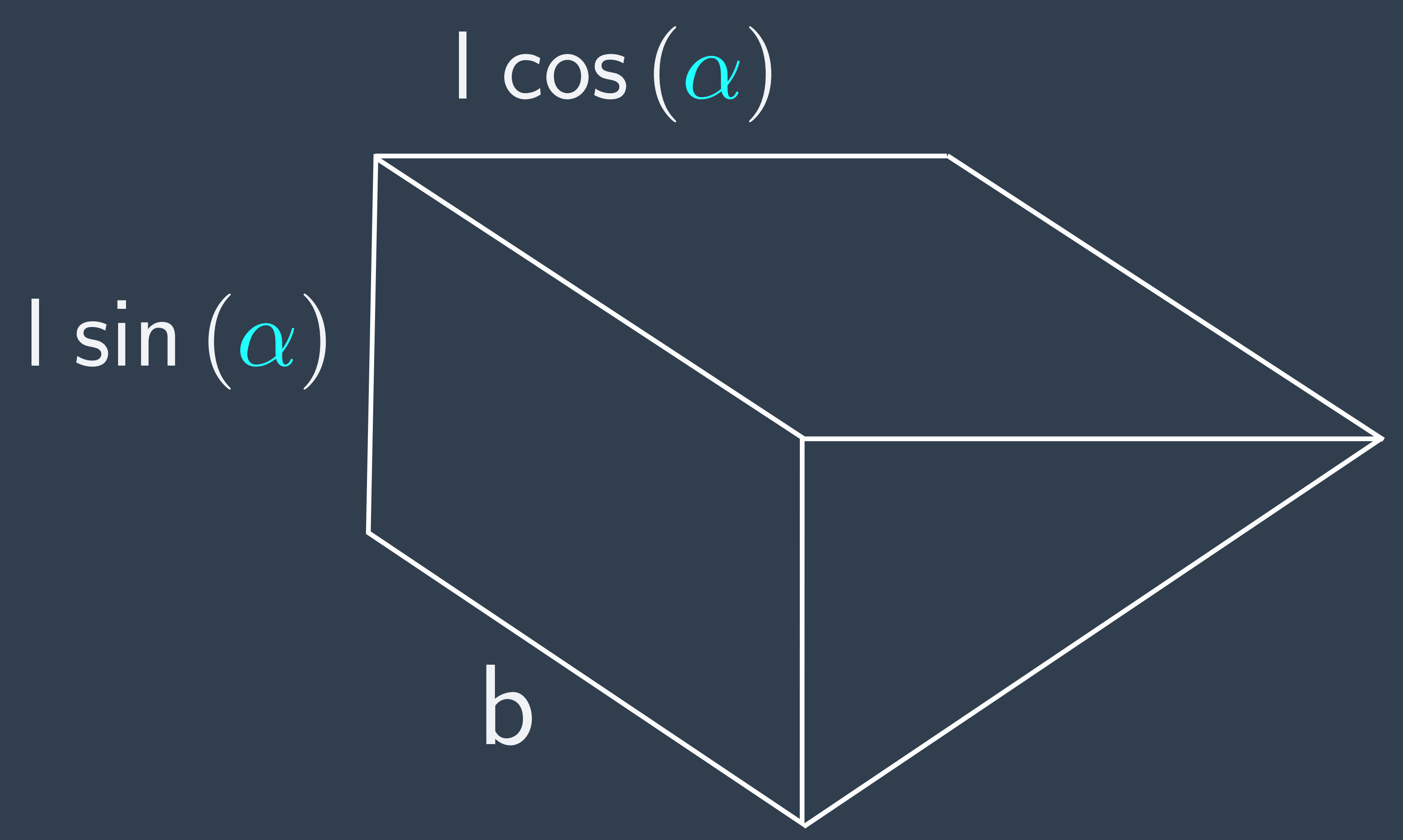


- By inspection, the vertical force component, F_v, is equivalent to the weight of the volume of fluid above the plate, i.e.,

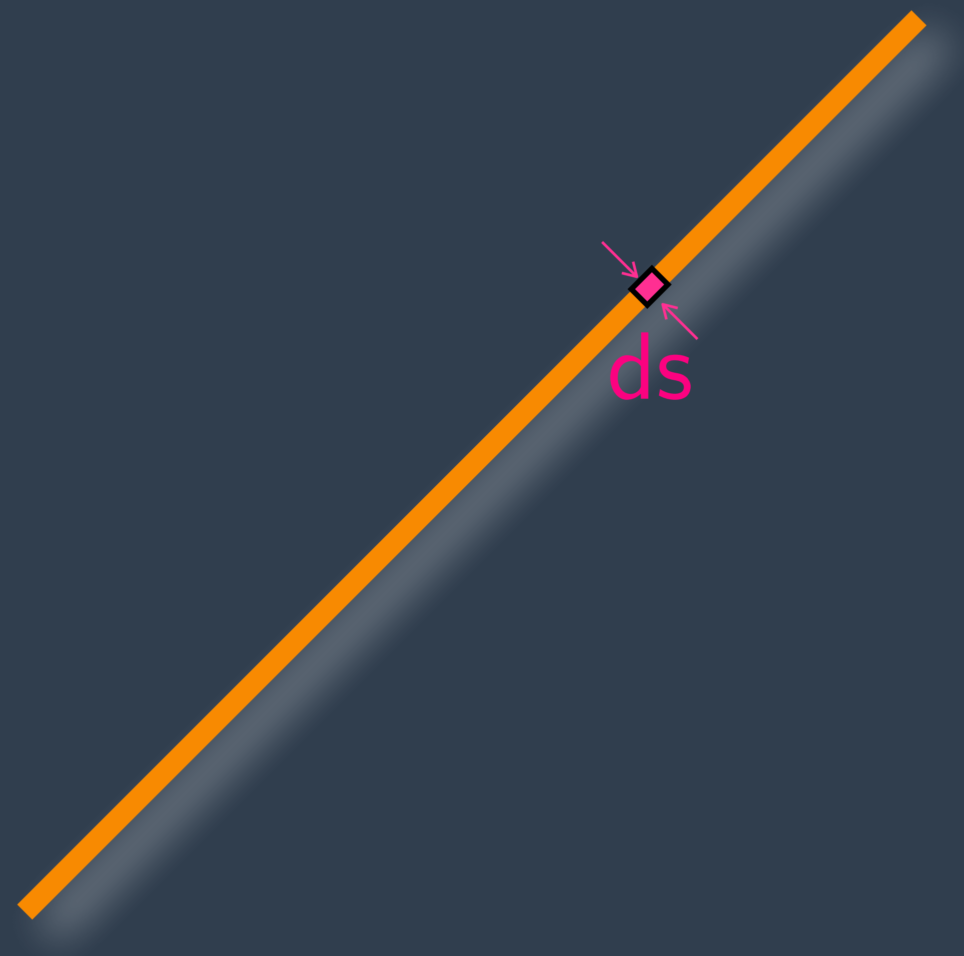
$$\text{Weight} = \text{Volume} \times \rho g$$

$$= \frac{1}{2} l\cos(\alpha)\ l\sin(\alpha)\ b\ \rho g$$

Forces on submerged bodies

- What remains is to locate the point of action for this force. The moment of the net force, F, about a suitable reference axis must equal the sum of the moments of all the elemental forces for each strip.

- Let s_p be the distance from the trace to the effective point of application of F. This point , •, is known as the center of pressure.



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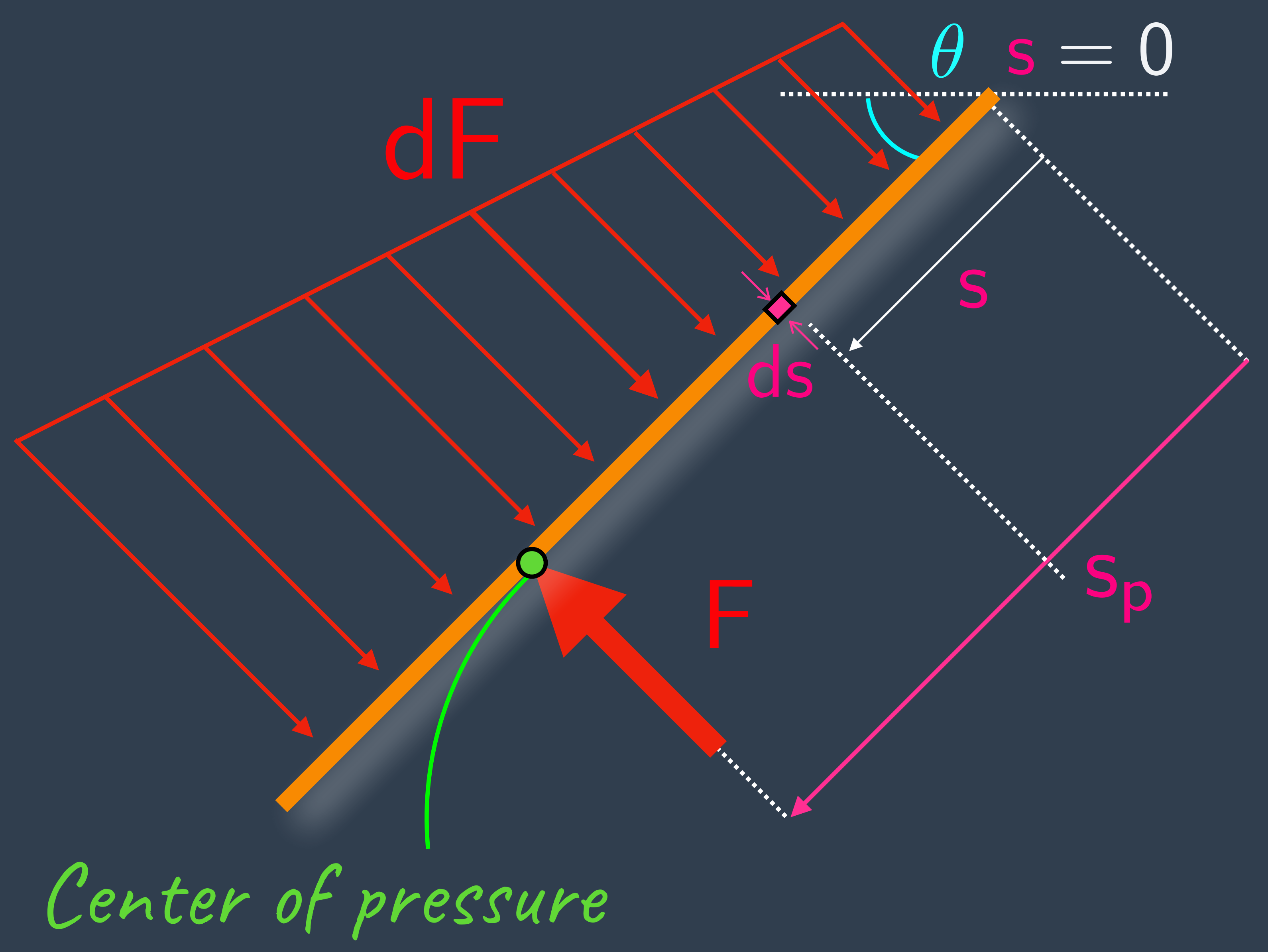

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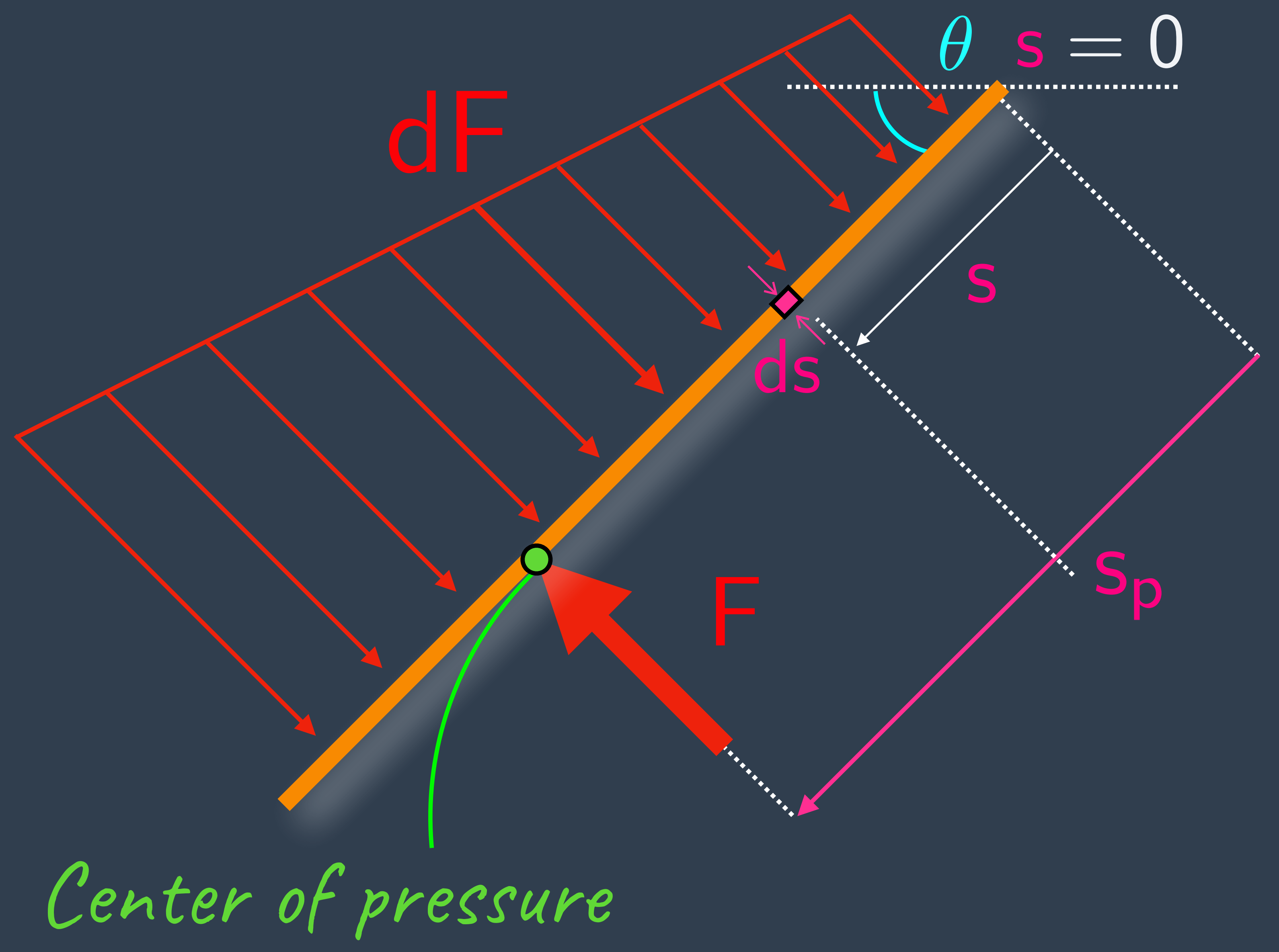

$$Fs_p = \int_{\text{plate}} s\, dF = \int_{\text{plate}} dp(s)\, b(s)\, ds$$

$$\Rightarrow Fs_p = \rho g \sin(\theta) \int_{\text{plate}} b(s)\, s^2 ds$$

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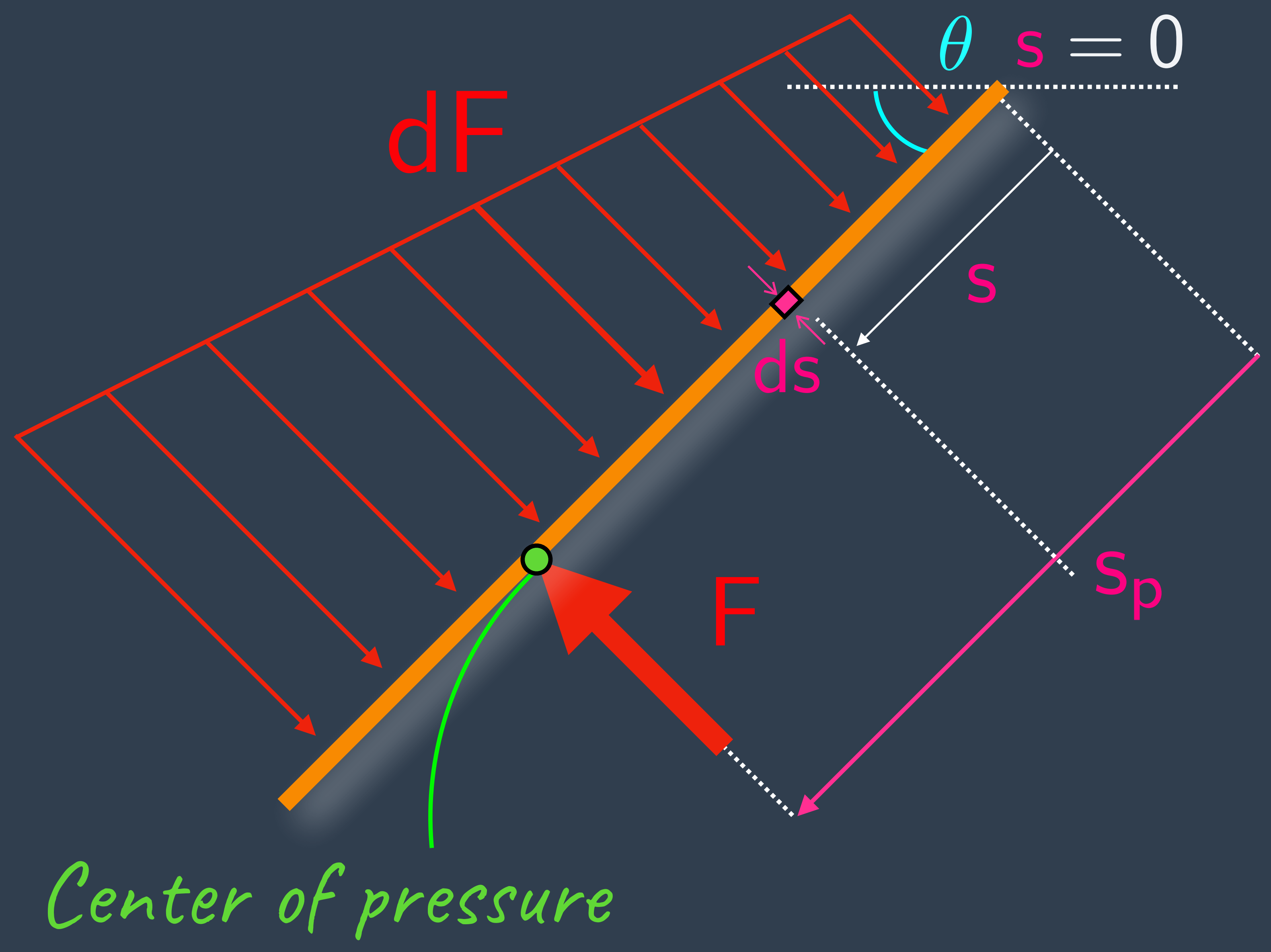

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- For rectangular plates:

$$F s_p = \rho g \sin(\theta)\, b \int_{\text{plate}} s^2 ds$$

Forces on submerged bodies

- Note that the actual force is indeed distributed, but it is useful to identify the effective point of application (for calculating moments).

- Also note that the pressure acting over the plate varies linearly with depth.

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The total pressure force on a plate with a general shape in a fluid is worked out by integrating the forces along the plate's surface.

The center of pressure is obtained by integrating the forces and moment arms acting on the surface & diving by the total force.

Fluid dynamics terminology

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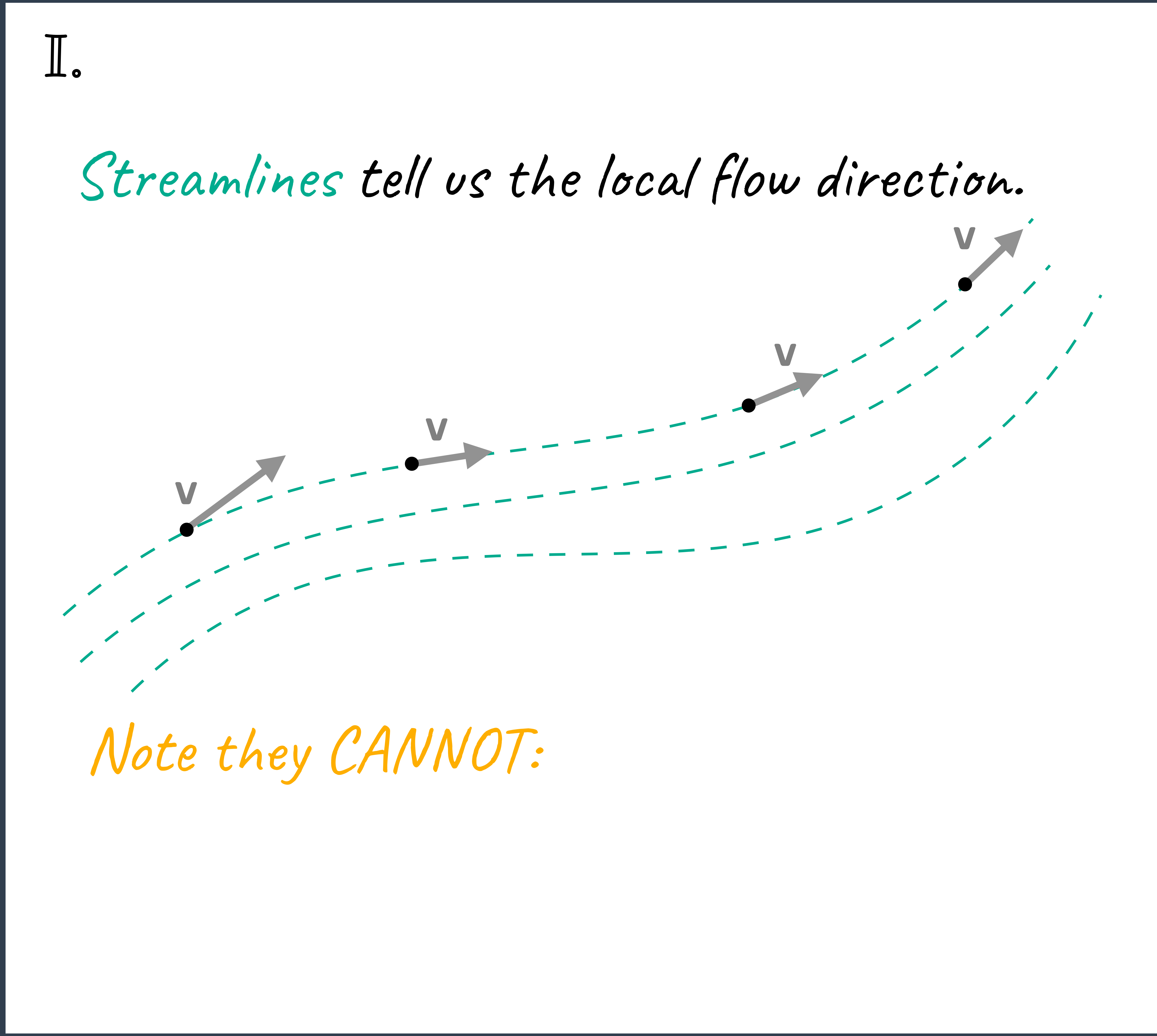

- Streamlines are curves that are tangential to the velocity vectors at each point in a flow.

Frame from video titled, "Aerodynamics | Pressure around airfoil"
Image source: Youtube
Image by: NACA (original)

https://tinyurl.com/2x99mrtf

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II.

At a stagnation point the velocity is 0.

- Any object immersed in a flow has one or more stagnation points, which delineate points on the surface where the flow velocity is zero.

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- Typically, streamlines will follow the surface of an object because the velocity vectors next to the surface are tangential to it.

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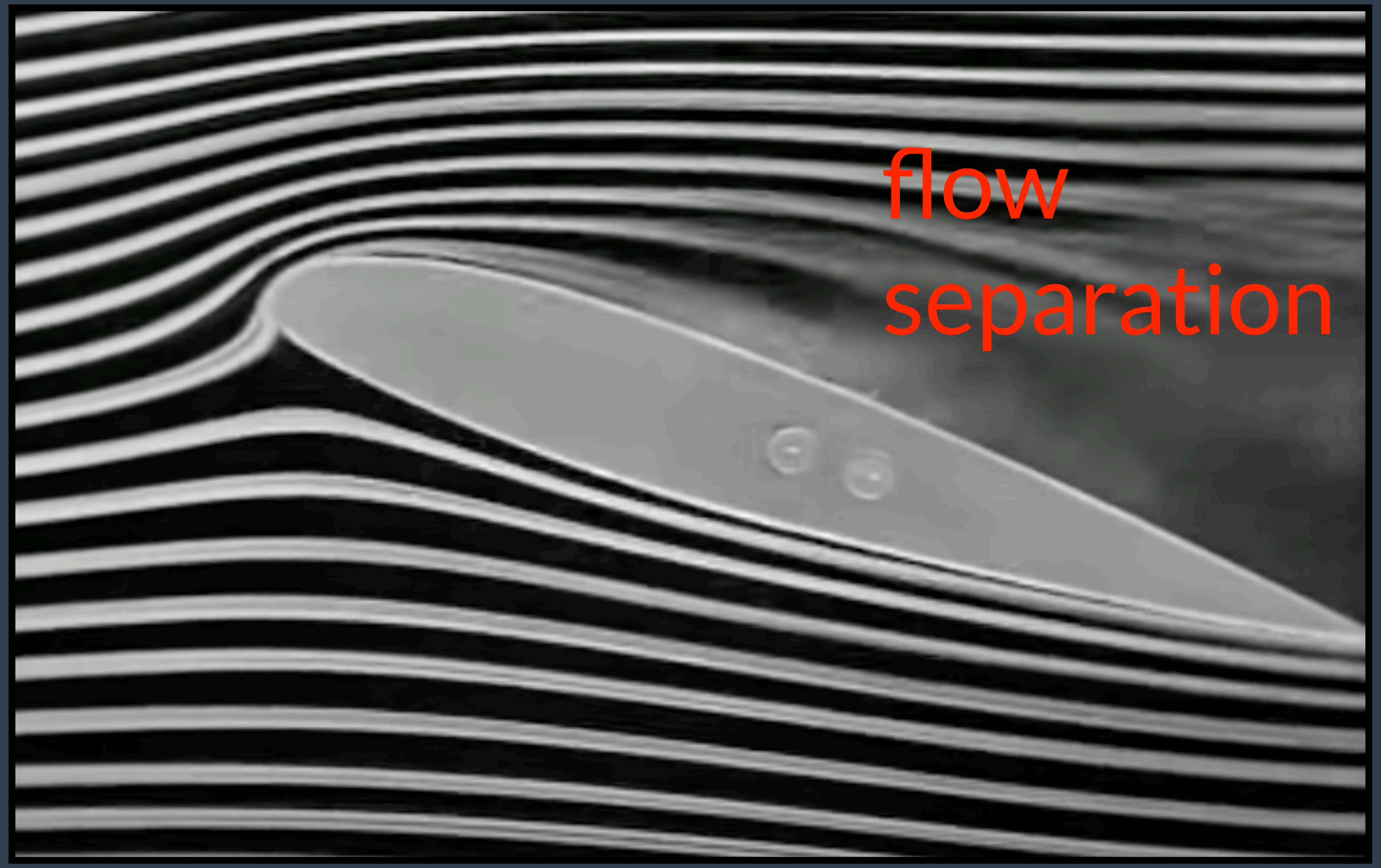
- Typically, streamlines will follow the surface of an object because the velocity vectors next to the surface are tangential to it.

- However, it very common to observe flow apparently departing from the surface, because it cannot follow the [typically adverse] curvature, resulting in flow separation.



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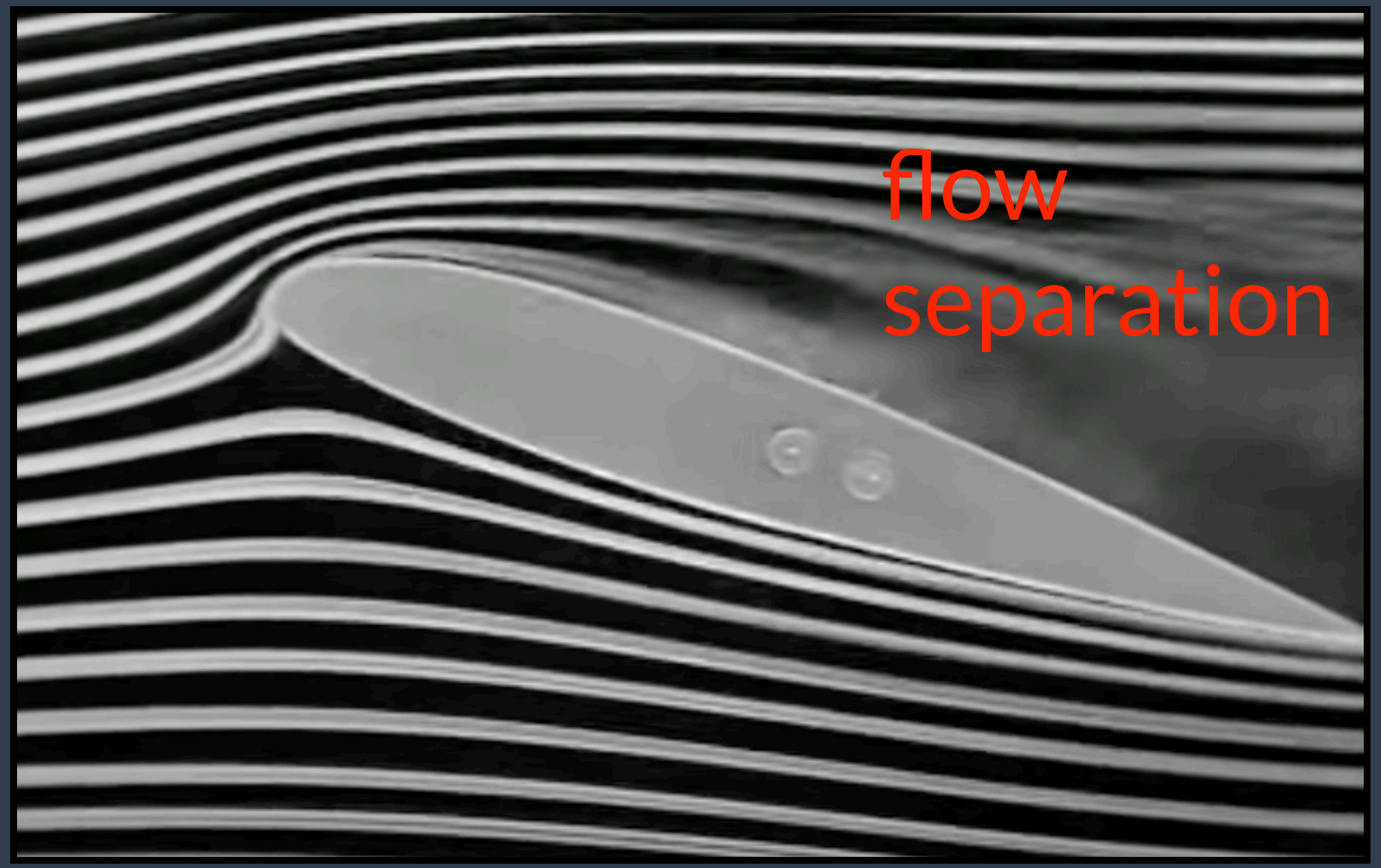
Play video



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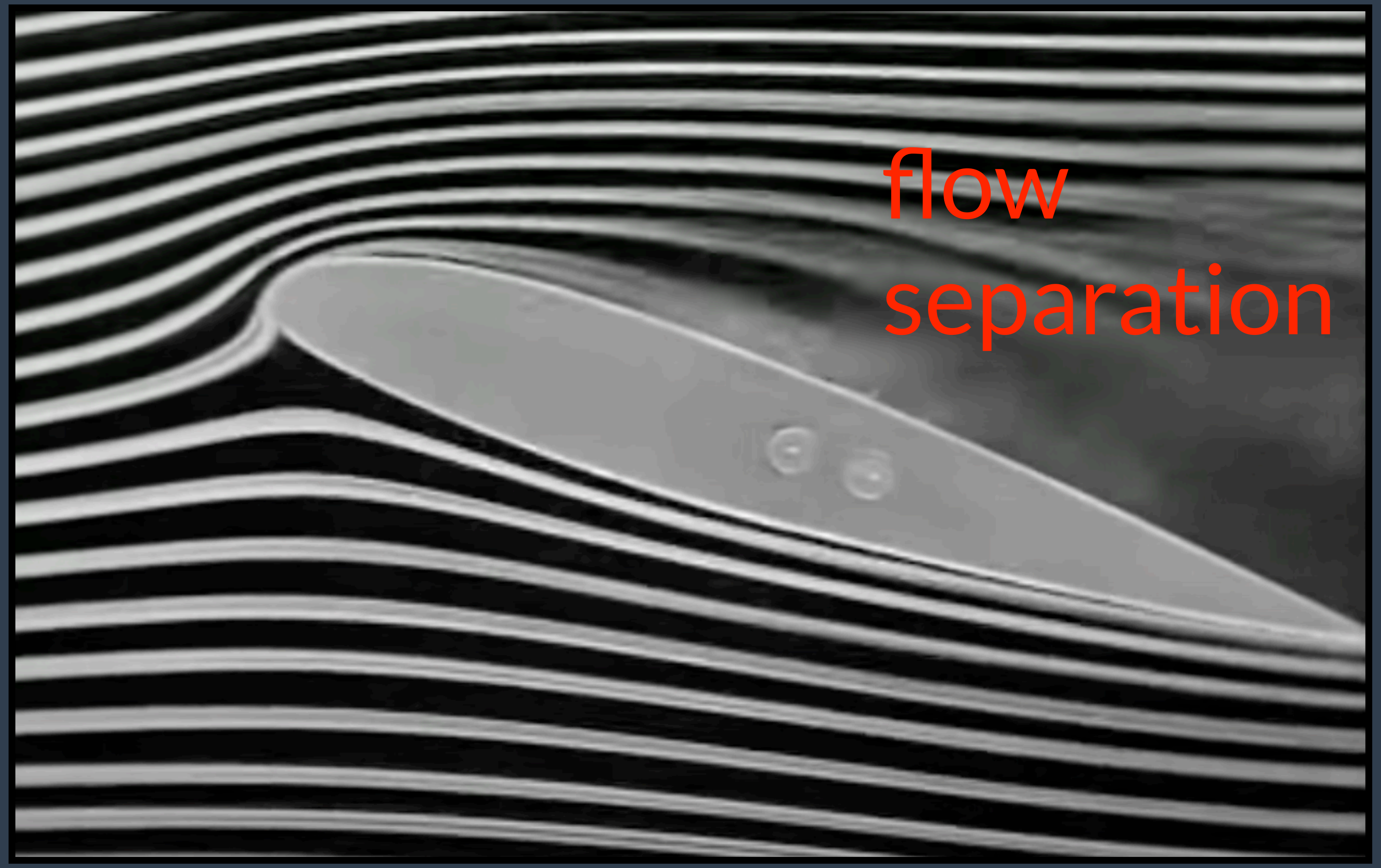
- From the video, you will have noticed that for certain angles (termed angles of attack) the flow was very steady and had a predictable trajectory. However, for higher angles, the flow seemed chaotic.

- Many practical flows change with time, i.e., they are unsteady. In this case streamlines are not always obvious (because they change very rapidly) and it is not useful to draw them.



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III.

Flow tends to be steady if airfoil is:

Flow tends to be unsteady if airfoil is:

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- Many problems that we will encounter will assume that the flow is incompressible, i.e., the density is constant.

- While this may seem quite reasonable when we consider fluids such as water, in air this is far from obvious. For instance, consider a bicycle bump or a party balloon.

- To understand why we can ignore density variations in applications, remember that for constant temperature, density $\propto$ pressure.

Fluid dynamics terminology

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Cargolux 747-4 powering out of Calgary International
Image source: Unsplash
Image by: Justin Hu

https://tinyurl.com/mvv4n8bj

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- For many civil transport aircrafts, e.g., Boeing 747, the wing loading (weight / wing area) is roughly 500 N/m^2. The weight is equivalent to lift and this is achieved by a pressure difference between the two surfaces.

- But this pressure is only 5% of atmospheric pressure. The density variation is also approximately of this magnitude and therefore it is reasonable to neglect it.

- At supersonic speeds, the pressure differences are far greater, and the assumption of constant density (incompressibility) cannot be ignored.

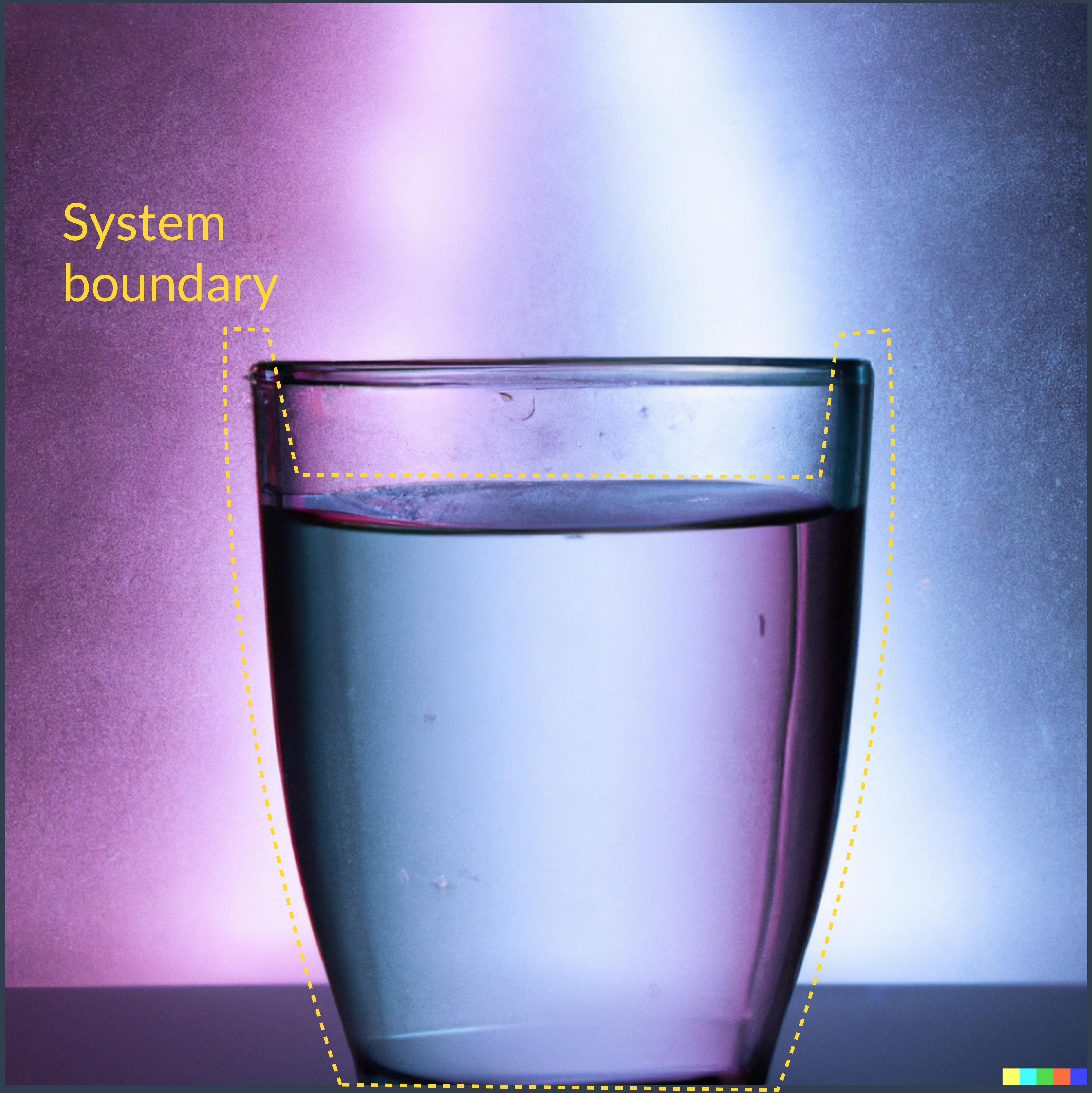
Cargolux 747-4 powering out of Calgary International
Image source: Unsplash
Image by: Justin Hu

https://tinyurl.com/mvv4n8bj

Systems and conservation principles

- In fluids and thermodynamics, we often use the concept of a system — i.e., an imaginary boundary.

- Mass *cannot* cross a system boundary. Energy in the form of work or heat *can* cross a system boundary.



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- If the ambient pressure decreases, then the volume of water may shrink and the system boundary will have to be altered to accommodate this change.

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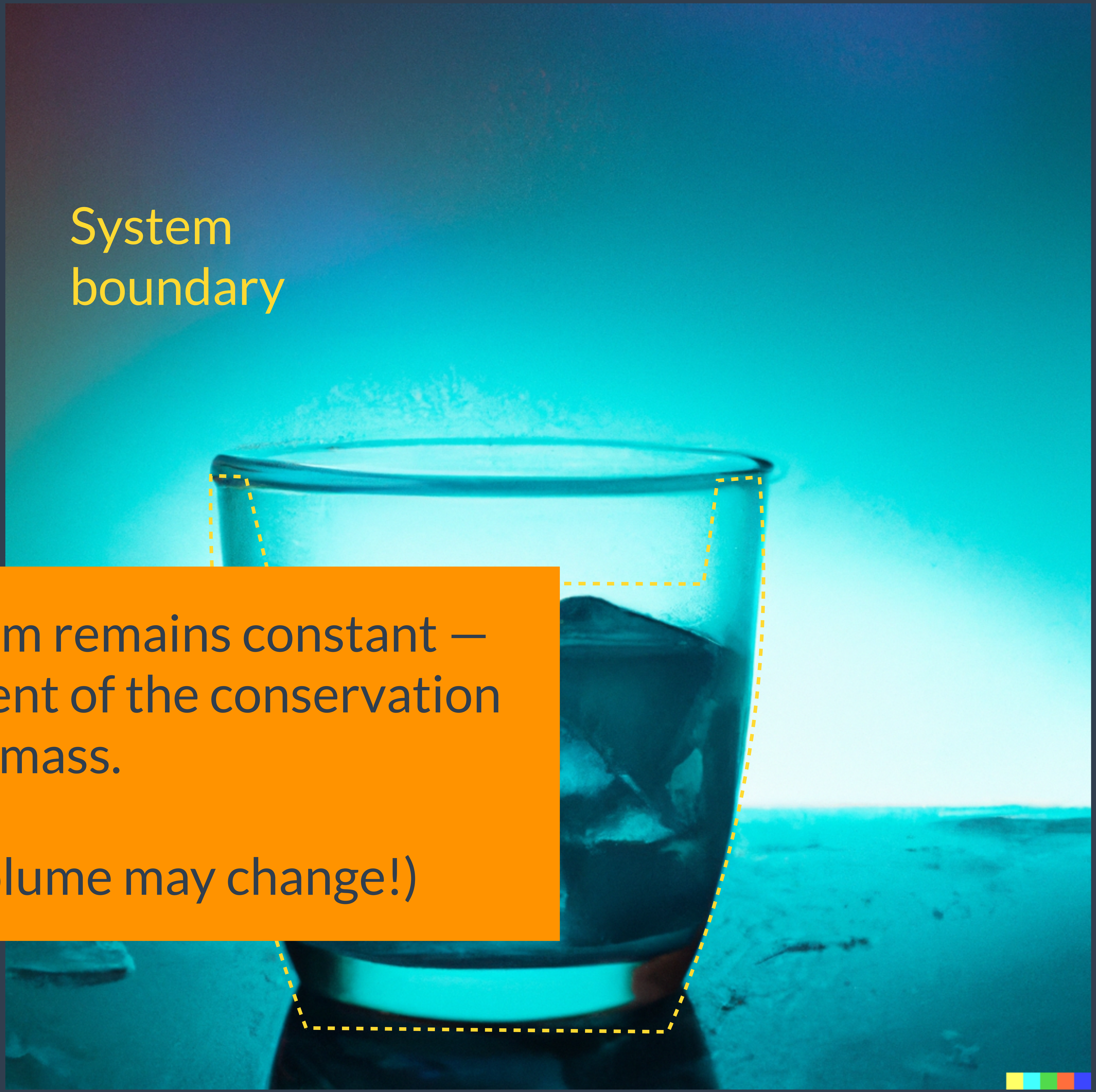

The mass of a system remains constant — and this is a statement of the conservation of mass.

(However, its volume may change!)

Conservation of mass

- The idea that mass is conserved forms one of the fundamental ideas in science.
- A similar idea is that the momentum of a system remains constant, unless a force is applied to it. This is another way of describing Newton’s second law.

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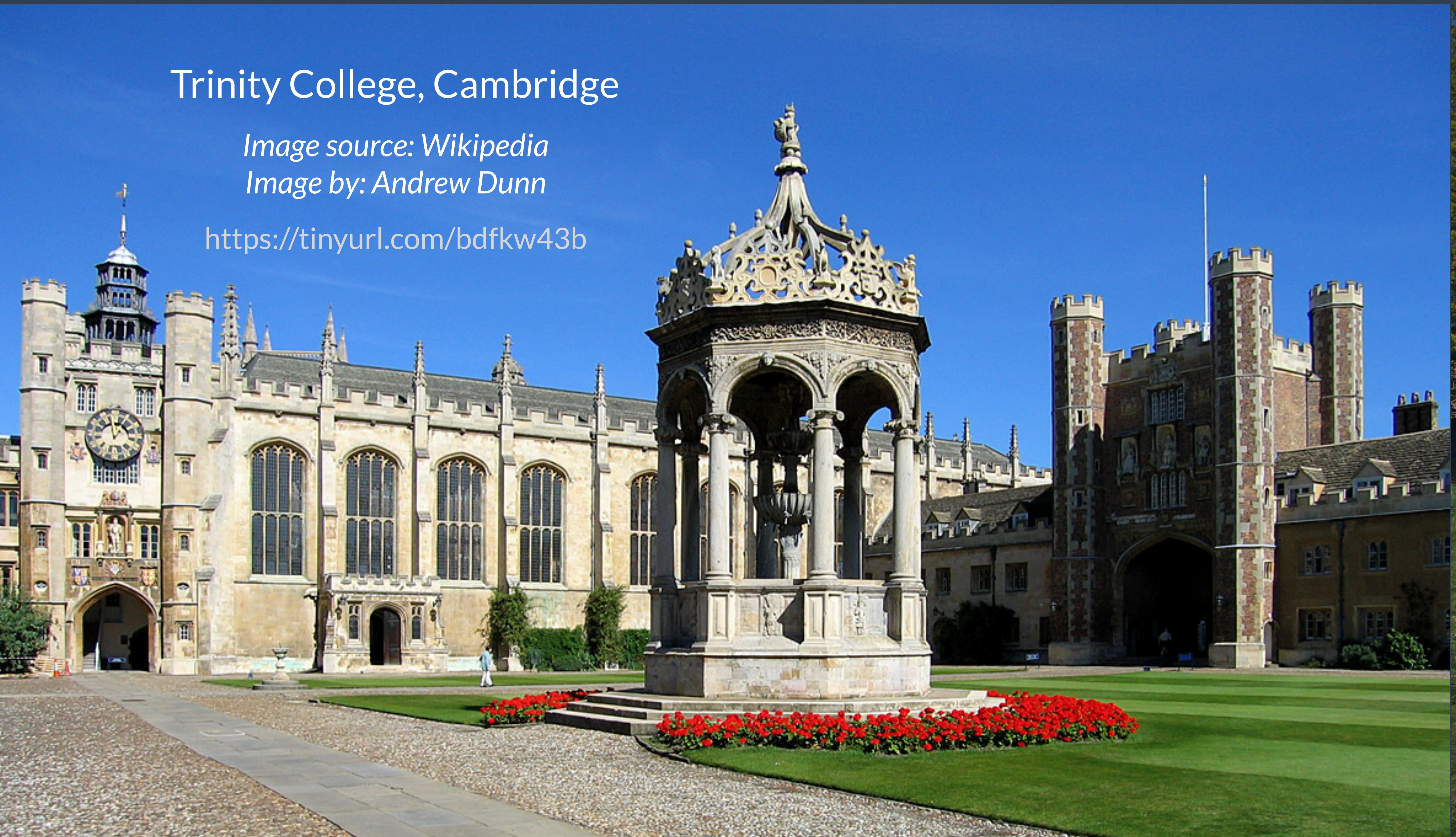
- A similar idea is that the momentum of a system remains constant, unless a force is applied to it. This is another way of describing Newton's second law.

- Newton's second law requires an entire lecture to itself (Lecture 4). It is a journey that we will take together next time, and that journey cuts across Cambridgeshire & Lincolnshire.



Trinity College, Cambridge

Image source: Wikipedia
Image by: Andrew Dunn

https://tinyurl.com/bdfkw43b

Woolsthorpe Manor, Lincolnshire

Image source: Wikipedia
Image by: Bs0u10e01

https://tinyurl.com/2p93tahb

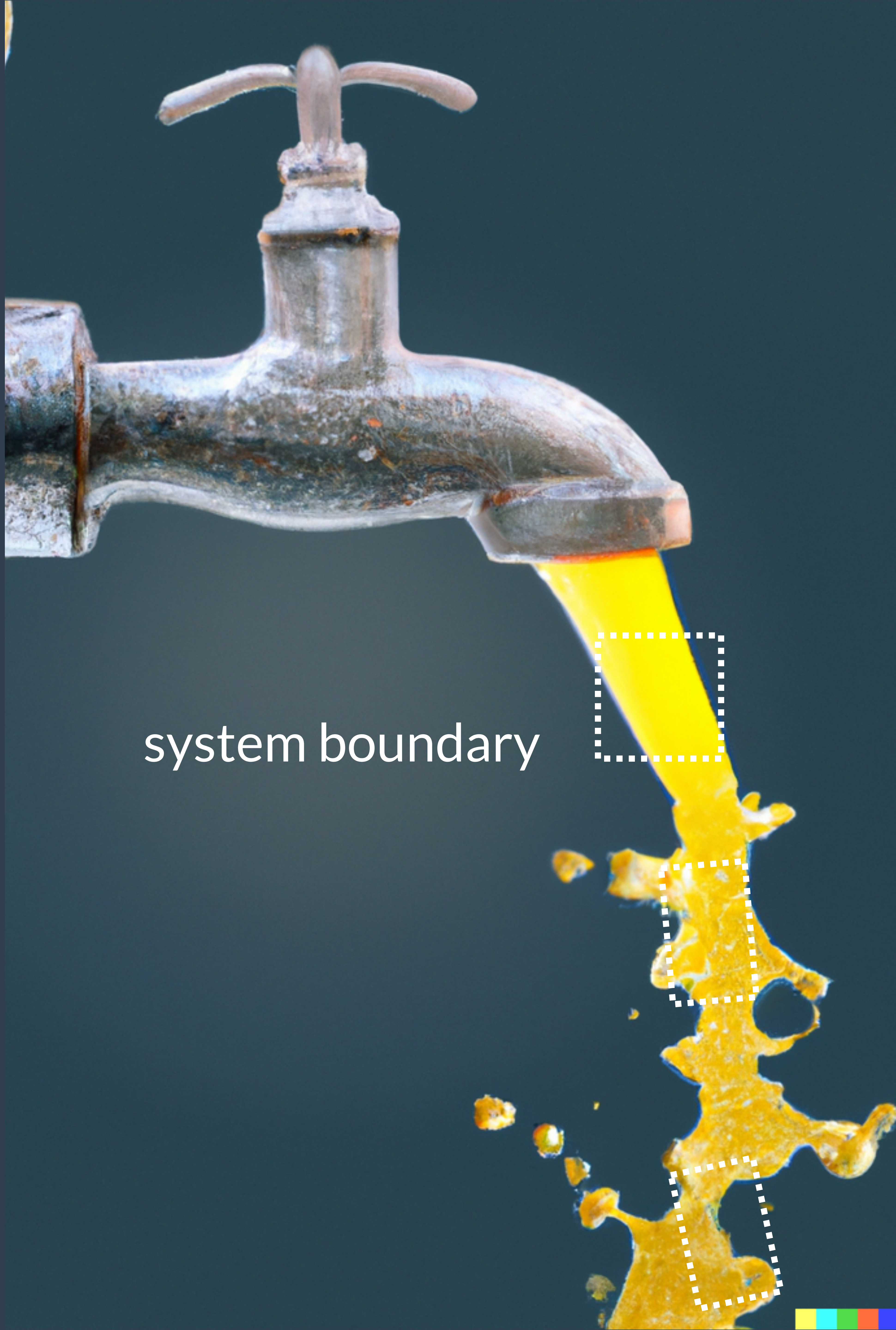
Conservation of mass

- In fluids, one is typically interested in problems where there is continuous flow. Recall, when defining a system we had said that:

    - “Mass *cannot* cross a system boundary. Energy in the form of work or heat *can* cross a system boundary”.

- This implies that the system must move with the fluid.

- However, *following* the fluid is tedious and therefore it is useful to introduce the notion of a control volume.



Conservation of mass in control volumes

Conservation of mass in control volumes

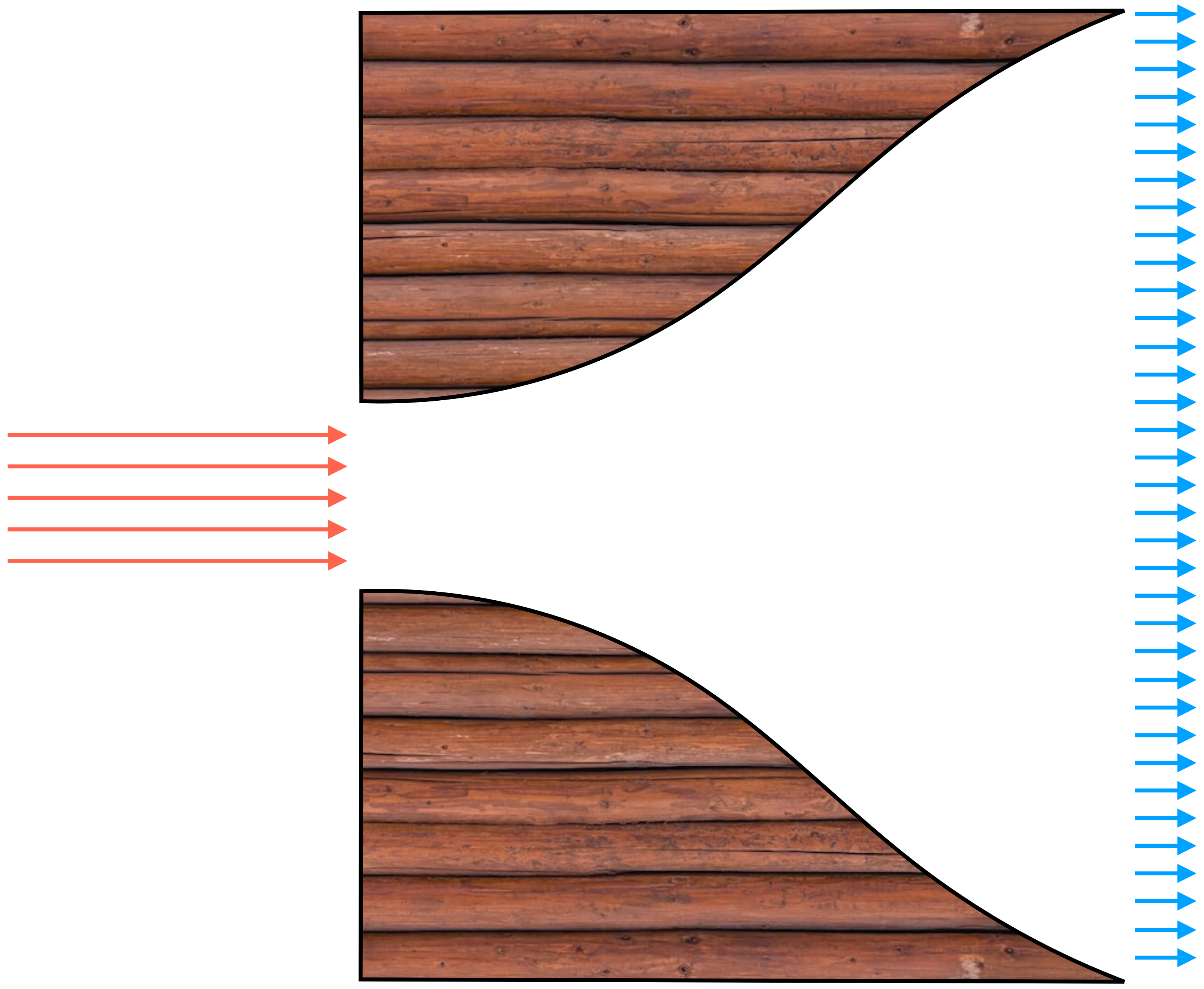
- A control volume is similar to a system (imaginary boundary), but mass is permitted to cross its boundaries.

- Unlike a system, a control volume does not alter its shape, however it can move at a constant speed.

- Mass flowing into a control volume must be equal to the mass flowing out of it.

Conservation of mass in control volumes

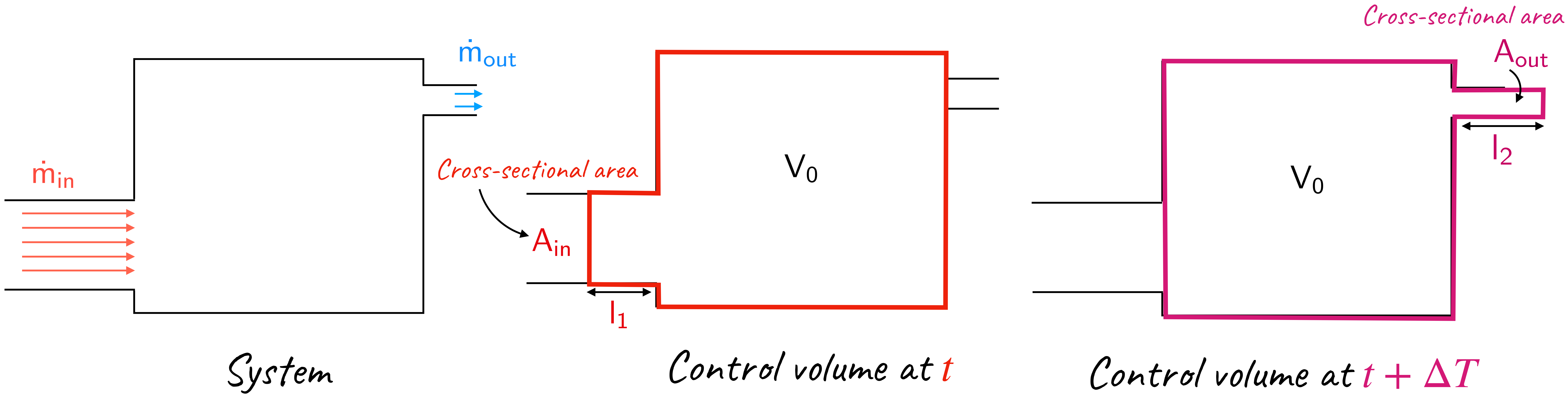
IV. *The conservation of mass in a control volume may be written as*



Conservation of mass in control volumes

$\mathbb{V}$.

Conservation of mass in action



Conservation of mass in control volumes

VI.

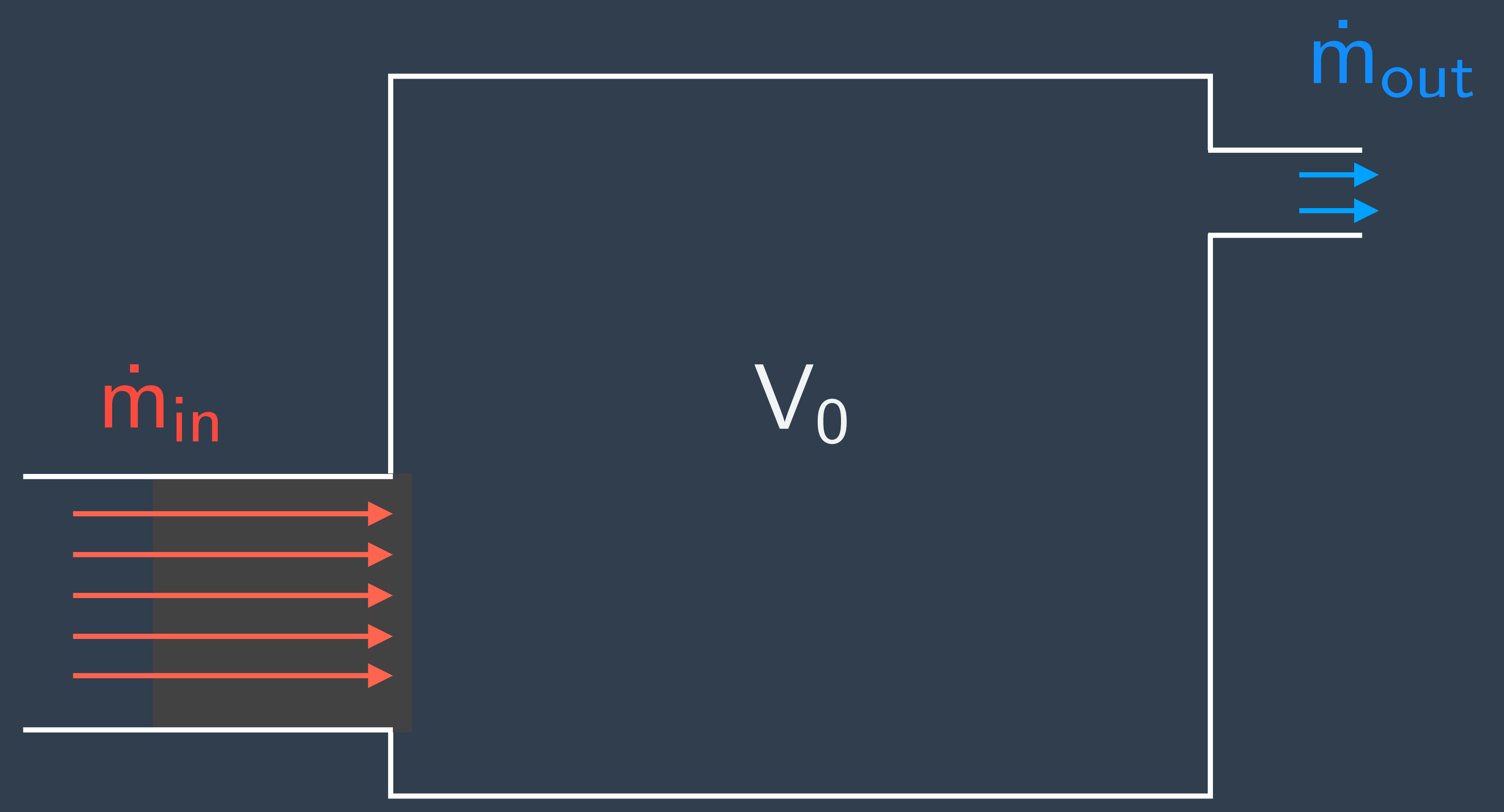
Setting the masses to be equal to each other...

1 = 2

Conservation of mass in control volumes

- The principle of mass conservation applied to the reservoir has lead to the conclusion that the mass flow rate into and out of the reservoir are equal.

- In fluids it is more convenient to consider *mass fluxes.*



Conservation of mass in control volumes

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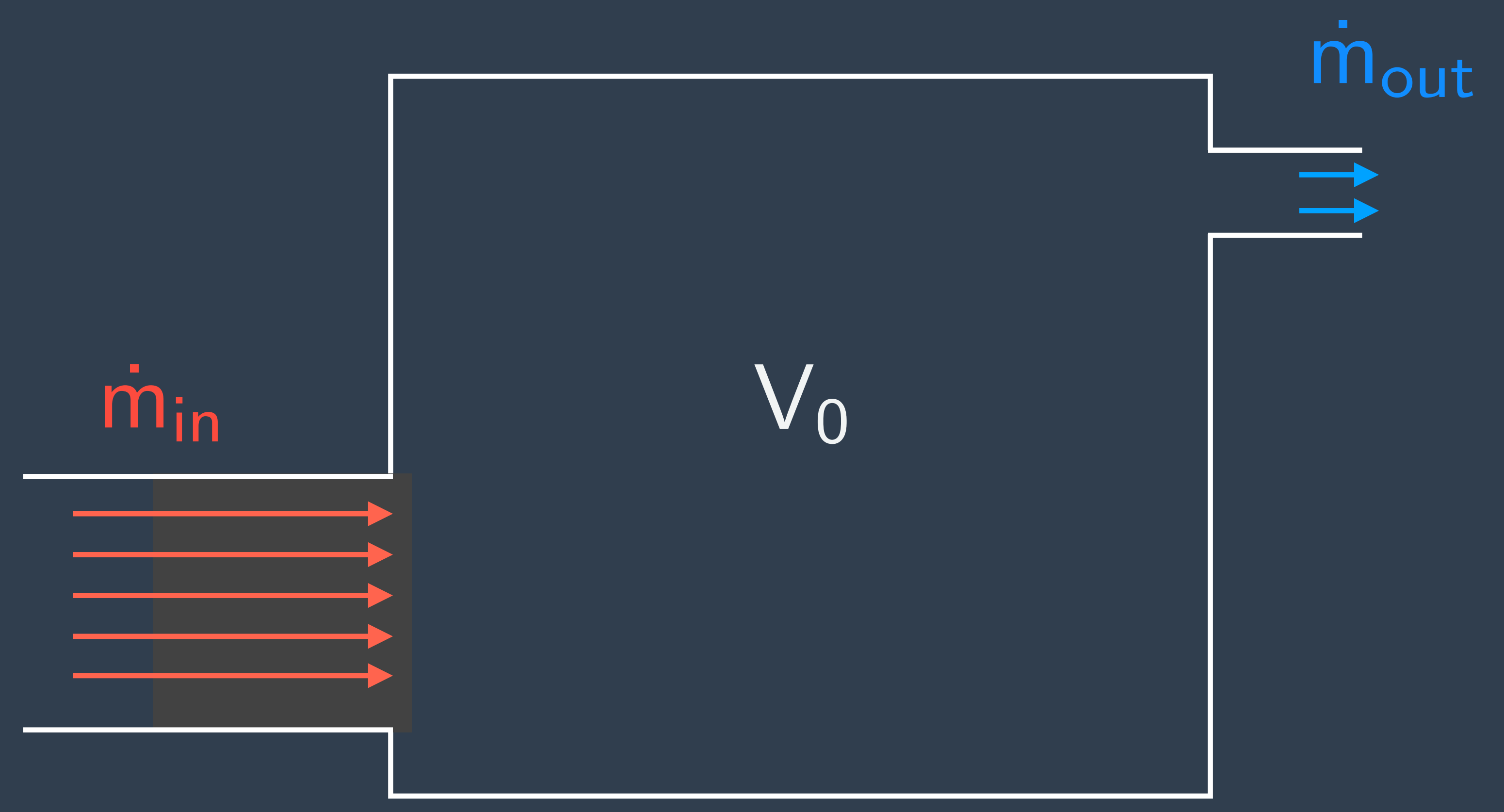
- Recall, we considered a control volume that remains fixed in space with flow moving in and out of the control volume at various places.

- As a general rule, we write

$$\sum_i \dot{m}_i = 0 \quad \text{where} \quad \begin{cases} \dot{m} > 0 & \text{for flow out of a control volume} \\ \dot{m} < 0 & \text{for flow into a control volume} \end{cases}$$



Conservation of mass in control volumes

- Note that the conservation of mass equation is often called the continuity equation, or even the steady flow mass equation. The *unsteady* version is for next time.

- We now consider an example where we have a body (or half a body) in the wind tunnel.

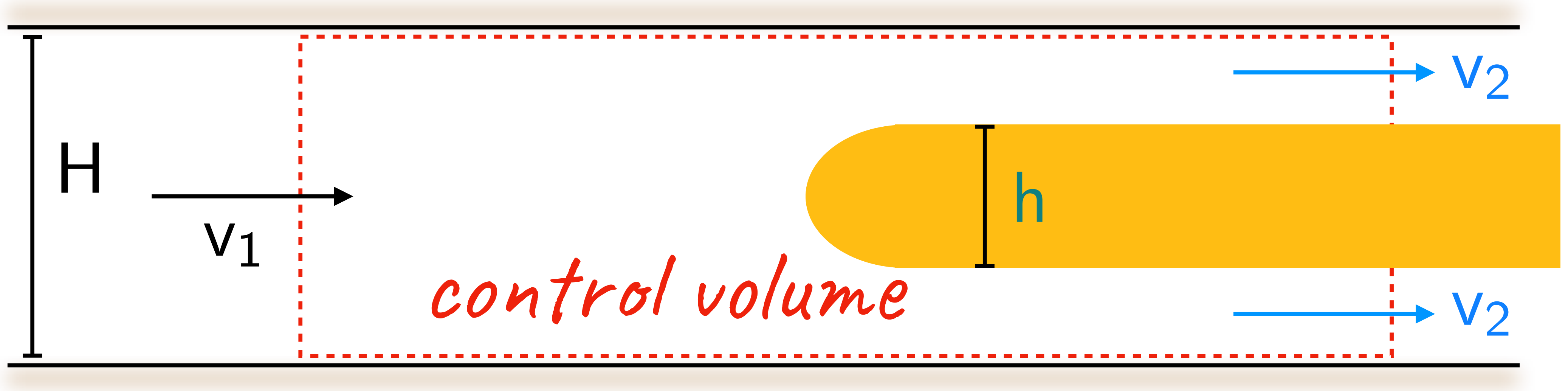
John Harper Wind Tunnel (Georgia Institute of Technology)
Image source: Georgia Institute of Technology
Image by: Rob Felt

https://tinyurl.com/fkawbt9j

Conservation of mass in control volumes

VII.

Half-body in a wind tunnel (width w)



Mass flux in:

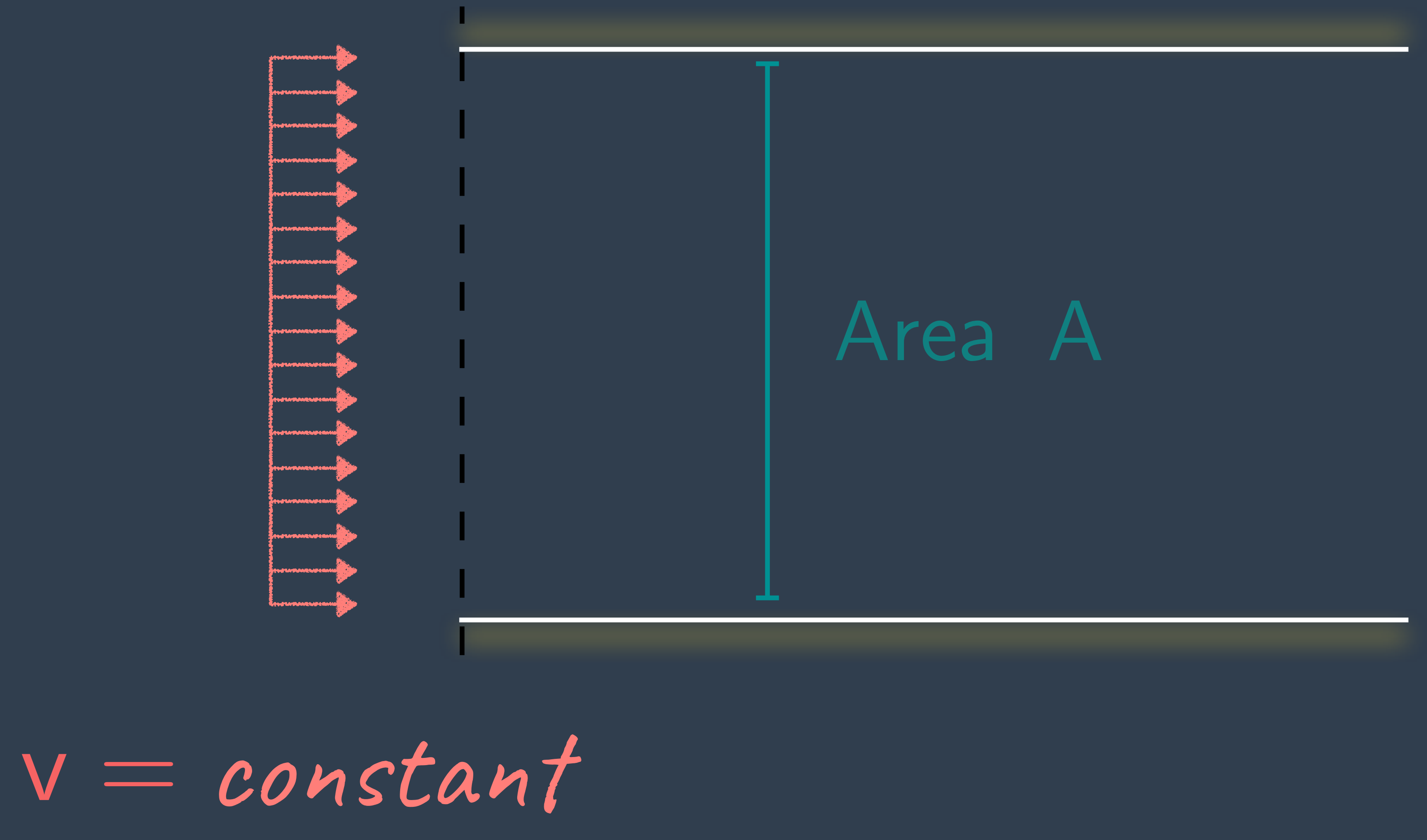
Mass flux out:

Conservation of mass:

Conservation of mass in control volumes

- We have already considered the simplest case for mass flux through an area where the velocity is perpendicular to the control surface.

$$\dot{m} = \rho A v$$



- For a control volume with many in-flows & out-flows, with uniform velocities, this implies

$$\sum_{i}^{\text{no. of control surfaces}} \rho_i A_i v_i = 0$$

- Note that if we limit our focus to fluids with the same density, this yields…

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VIII.

For incompressible flows where

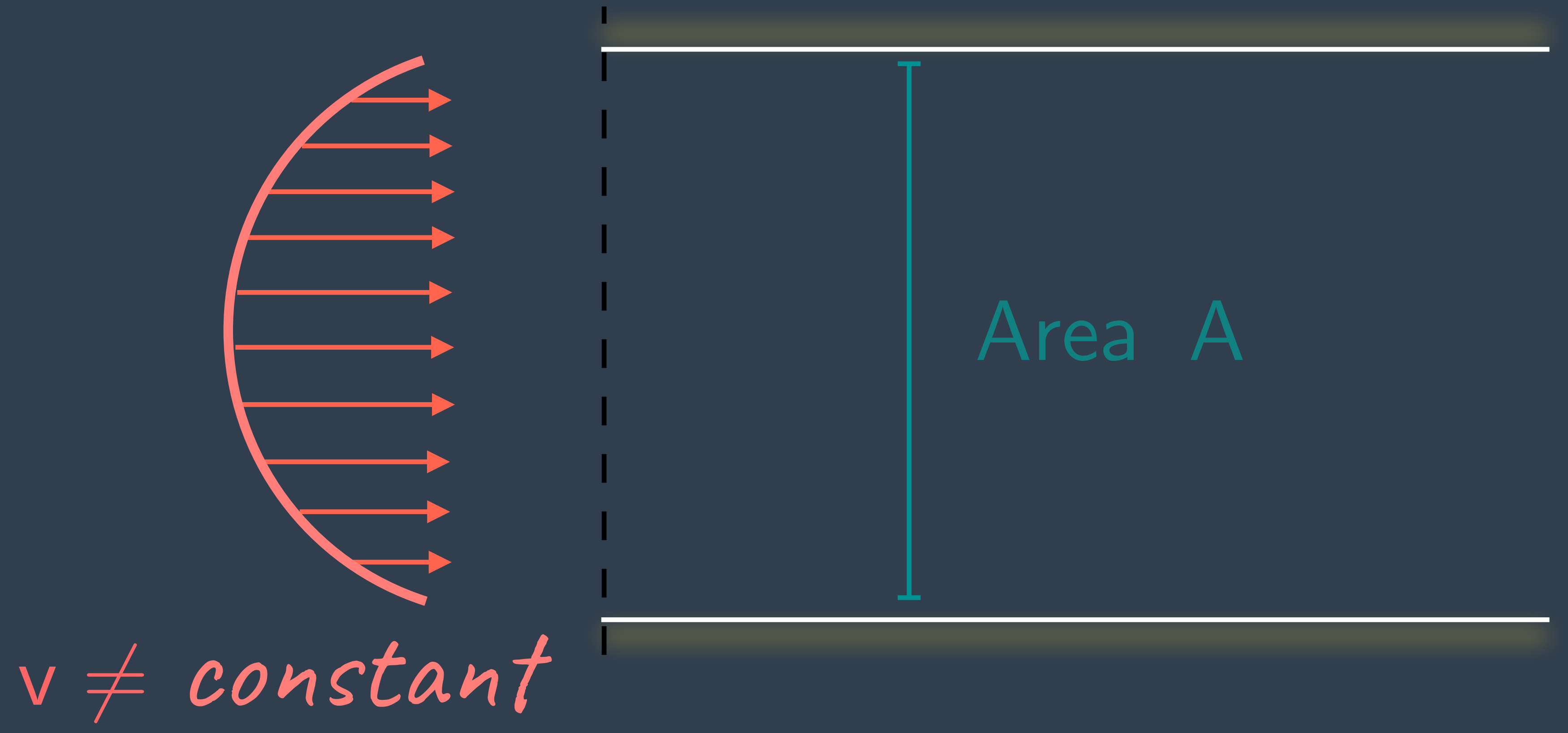
$$\rho = \textit{constant}$$

We can state that the VOLUMETRIC FLOW RATE is constant

Conservation of mass in control volumes

- For situations where the velocity across an area is not uniform, the mass flux through the area can be calculated as:

$$\dot{m} = \int_A \rho v \, dA$$

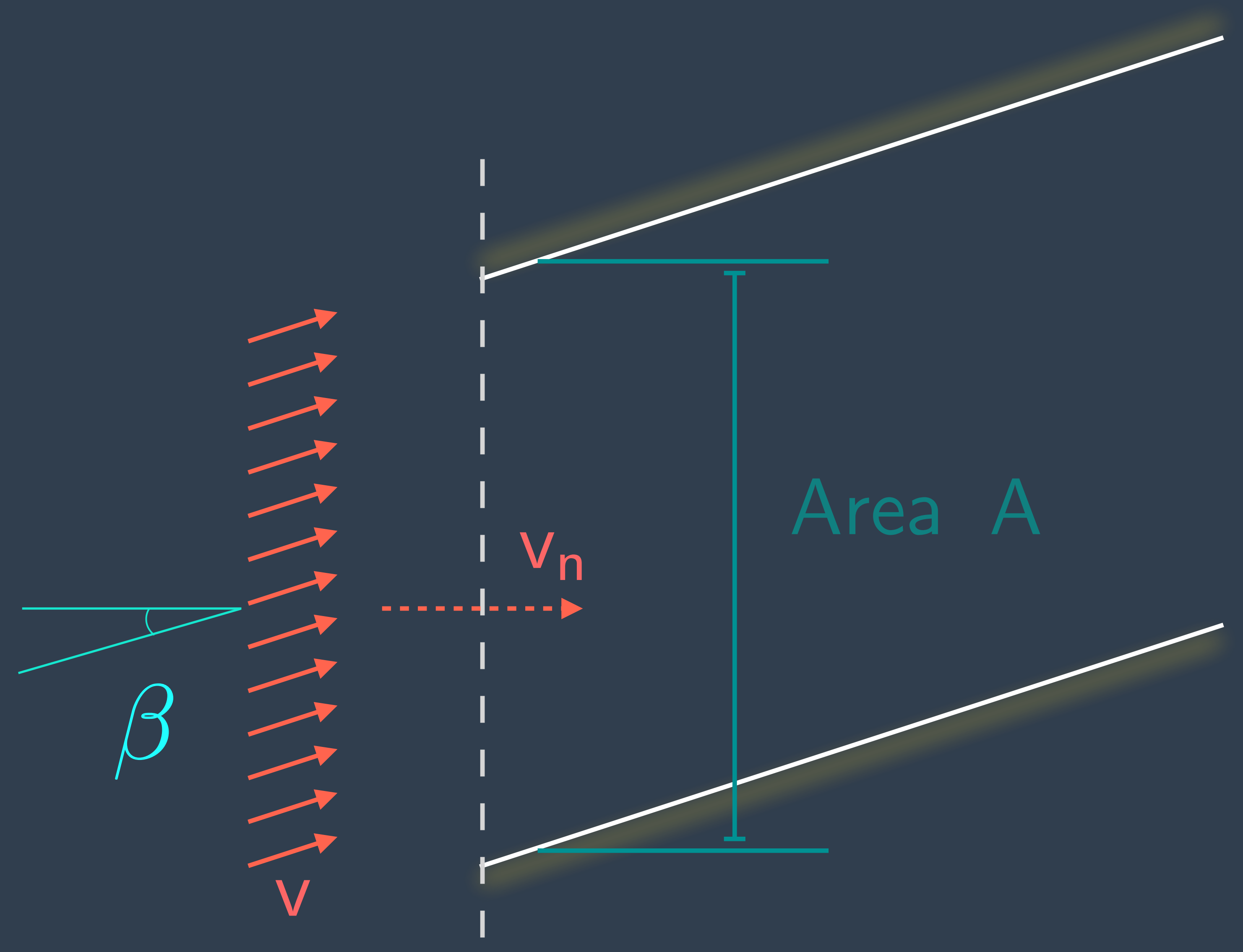


Conservation of mass in control volumes

- When the flow direction into or out of the control volume is not perpendicular to the control surface, we need only consider the normal component of the velocity:

$$\dot{m} = \rho A v_n$$

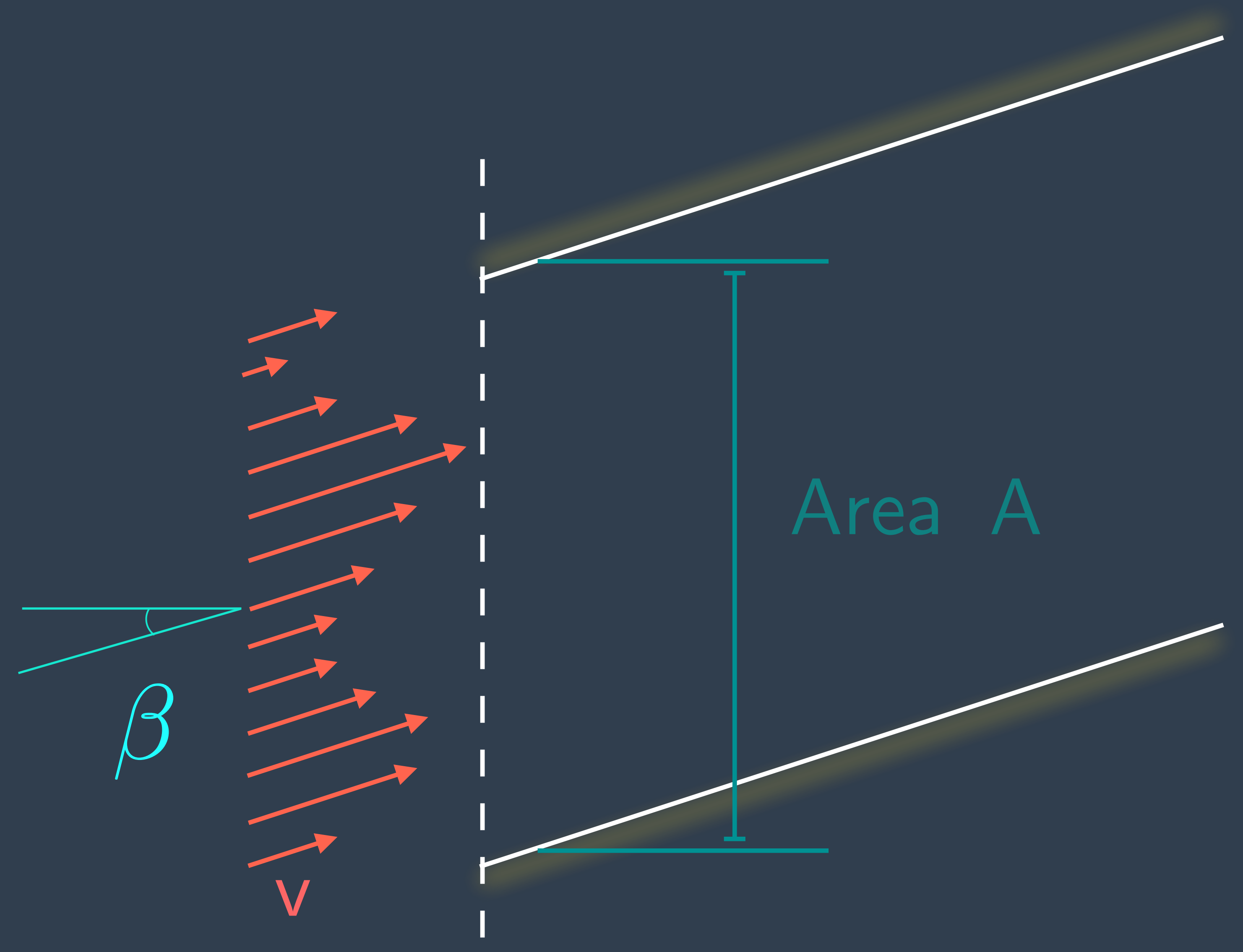
$$\dot{m} = \rho A v \cos\left(\beta\right)$$



Conservation of mass in control volumes

- In cases where both the magnitude and direction of the flow across the control surface vary, we need to integrate

$$\dot{m} = \int_A \rho v_n dA$$



Conservation of mass in control volumes

- Sometimes it is more elegant to define a surface vector which points out of the control volume in a direction normal to the control surface. Here the mass flux is:

$$\oiint_{\mathrm{CS}} \rho \mathbf{v} \cdot \mathrm{d}\mathbf{A} = 0$$