1.

(a)

(i) $f = inv(a) \equiv f \leftrightarrow a' \equiv (f \to a')(f' \to a) \equiv (a' + f')(a + f)$

(ii) $f = and(a, b') \equiv f \leftrightarrow (a \wedge b') \equiv (f \to a)(f \to b')(f' \to (a' \vee b))$
$\equiv (a + f')(b' + f')(a' + b + f)$

(iii) $f = nand(a, b) \equiv f \leftrightarrow (a \wedge b)' \equiv f \leftrightarrow (a' \vee b') \equiv (f \to (a' \vee b'))(f' \to a)(f' \to b)$
$\equiv (a' + b' + f')(a + f)(b + f)$

(iv) $f = or(a, b) \equiv f \leftrightarrow (a \vee b) \equiv (f \to (a \vee b))(f' \to a')(f' \to b')$
$\equiv (a + b + f')(a' + f)(b' + f)$

(v) $f = nor(a, b) \equiv f \leftrightarrow (a + b)' \equiv (f \to a')(f \to b')(f' \to (a + b))$
$\equiv (a' + f')(b' + f')(a + b + f)$

(b) CNF formula: (The following numbers denote the literal corresponding to the gate)

$23(21+22+23')(21'+23)(22'+23)(20'+19'+22)(20+22')(19+22')$
$(20'+21')(18'+21')(18+20+21)(17+16+20')(17'+20)(16'+20)$
$(14+15+19')(15'+19)(14'+19)(11+13+18')(11'+18)(13'+18)$
$(12'+17')(11'+17')(11+12+17)(10+16')(1'+16')(10'+1+16)(10'+15')$
$(2+15')(9+15')(10+2'+9'+15)(2'+14')(10+14')(1+14')(2+10'+1'+14)$
$(8+10+13)(10'+13')(8'+13')(7'+6'+12)(7+12')(6+12')(7+6+11)$
$(7'+11')(6'+11')(4+3+10)(4'+10')(3'+10')(4'+5'+9')(4+9)(5+9)$
$(2'+1'+8)(2+8')(1+8')(4'+3'+7)(4+7')(3+7')(1+2+6)(1'+6')(2'+6')$
$(3+1+5)(3'+5')(1'+5')$

number of clauses: 60

number of literals: 140

(c) From (b), and PG encoding

$23(2|+22+23')(21'+23)(22'+23)(20'+19'+22)(20+22')(19+22')$
$(20'+21')(18'+21')(18+20+21)(17+16+20')(17'+20)(16'+20)$
$(14+15+19')(15'+19)(14'+19)(11+13+18')(11'+18)(13'+18)$
$(12'+17')(11'+17')(11+12+17)(10+16')(1'+16')(10'+1+16)(10'+15')$
$(2+15')(9+15')(10+2'+14'+15)(2'+14')(10+14')(1+14')(2+10'+1'+14)$
$(8+10+13)(10'+13')(8'+13')(7'+6'+12)(7+12')(6+12')(7+6+11)$
$(7'+11')(6'+11')(4+3+10)(4'+10')(3'+10')(4'+5'+9')(4'+9)(5+9)$
$(2'+1'+8)(2+8')(1+8')(4'+3'+7)(4+7')(3+7')(1+2+6)(1'+6')(2'+6')$
$(3+1+5)(3'+5')(1'+5')$

Number of clauses: 44, percentage of reduction: $\frac{60-44}{60} \approx 26.67\%$

Number of literals: 100, percentage of reduction: $\frac{140-100}{140} \approx 28.57\%$

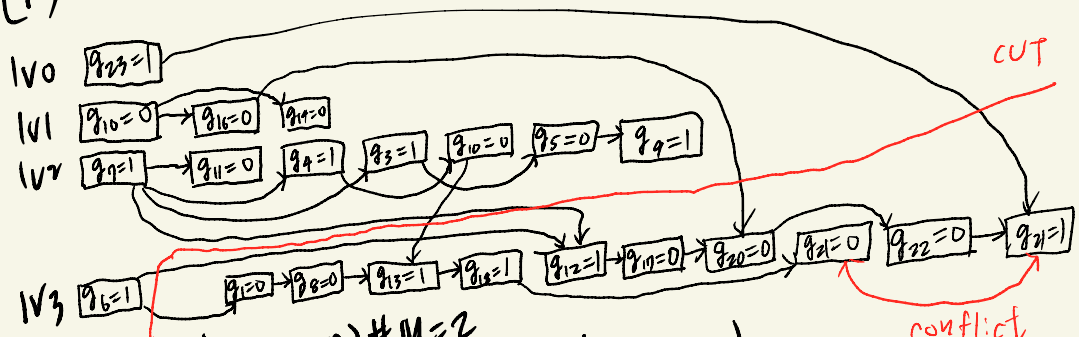
(d)

$g_1:(2,5)^7_3$	$g_5:(1,1)^2_0$	$g_9:(1,1)^2_0$	$g_{13}:(1,1)^2_0$	$g_{17}:(3,2)^5_1$	$g_{21}:(2,1)^3_1$
$g_2:(2,3)^5_1$	$g_6:(4,3)^7_1$	$g_{10}:(4,4)^8_0$	$g_{14}:(3,1)^4_2$	$g_{18}:(1,2)^3_1$	$g_{22}:(2,1)^3_1$
$g_3:(2,3)^5_1$	$g_7:(4,3)^7_1$	$g_{11}:(4,2)^6_2$	$g_{15}:(3,1)^4_2$	$g_{19}:(1,1)^2_0$	$g_{23}:(1,1)^2_0$
$g_4:(3,2)^5_1$	$g_8:(2,1)^3_1$	$g_{12}:(3,2)^5_1$	$g_{16}:(3,2)^5_1$	$g_{20}:(2,3)^5_1$	

(e) First, I mark the score diff in the previous problem.

The top 7 decision gates are: $g_{10}, g_7, g_6, g_1, g_{11}, g_{20}$ and g_{17}, from the top to the bottom.

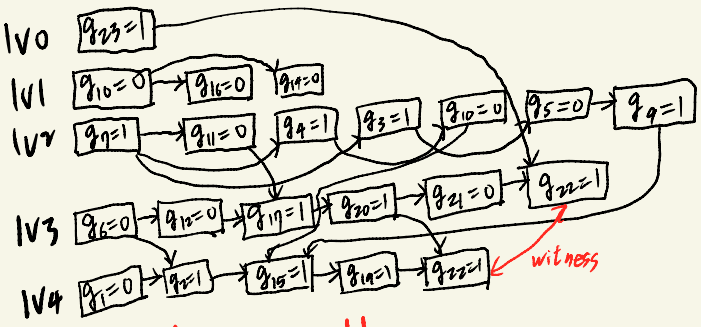
(f)



Mark $(g_{21}=1)$, $(g_{21}=0)$ #M=2
unmark $(g_{21}=1)$, mark $(g_{22}=0)$, add $(g_{23}=1)$ #M=2
unmark $(g_{22}=0)$, mark $(g_{20}=0)$ #M=2
unmark $(g_{21}=0)$, mark $(g_{18}=1)$ #M=2
unmark $(g_{20}=0)$, mark $(g_{17}=0)$, add $(g_{16}=0)$ #M=2
unmark $(g_{17}=0)$, mark $(g_{12}=1)$ #M=2
unmark $(g_{12}=1)$, mark $(g_6=1)$, add $(g_7=1)$ #M=2
unmark $(g_{18}=1)$, mark $(g_{13}=1)$ #M=2
unmark $(g_{13}=1)$, mark $(g_8=0)$, add $(g_{10}=0)$ #M=2
unmark $(g_8=0)$, mark $(g_1=0)$ #M=2
unmark $(g_1=0)$, mark $(g_6=1)$ #M=1, terminate

Find UIP: $(g_6=1)$, conflict source: $\{(g_6=1), (g_{10}=0), (g_7=1), (g_{16}=0), (g_{23}=1)\}$
Constructed learned gate: $(g_6' + g_{10} + g_7' + g_{16} + g_{23}')$

(f) Backtrack to lv2, the derived implication is $g_6=0$



the witness is found!