IDC410
A course on Image Processing and Machine Learning
(Lecture 02)

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Reference for Image Processing: MACHINE VISION. by Ramesh Jain, Rangachar Kasturi, Brian G. Schunck Published by McGraw-Hill, Inc., ISBN 0-07-032018-7, 1995

https://cse.usf.edu/~r1k/MachineVisionBook/MachineVision.files/

What is Image?

What is an Image?

- Image is a large matrix (N x M) of pixels
- Each pixel is represented by it's color
- Color is a mixture of 3 fundamental colors namely Red, Green and Blue (RGB) as per intensity of each color
- Intensity of each of the color is represented by a single byte, thus having a value between 0-255
- For a Grayscale image: Pixel intensity is denoted by a single byte varying between 0 (black) to 255 (white).

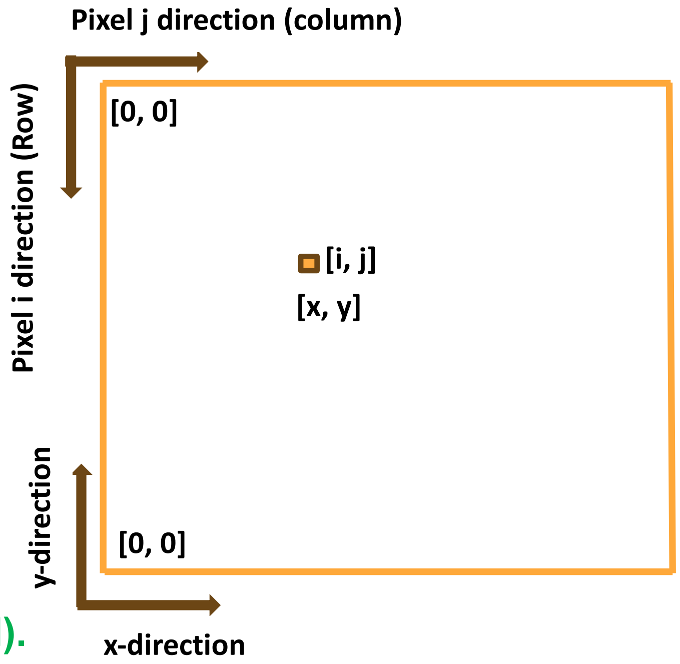
The image read with the OpenCV function imread(), provides the colours in order of BGR (Blue, Green and Red).

```
img = cv2.imread("C:\\Users\\Darshita\\Desktop\\OpenCV\\flower.jpg")
```

What is an Image?

Image Size: Image denoted by n X m pixels, where n is number of pixels in vertical direction (or number of rows) and m is number of pixels in horizontal direction (or number of columns) correspond to a **size of 3nm bytes** (before compression) .

Pixel Location: A location of pixel in an image M is denoted by M(i, j) where, i is a row number (vertical direction) and j is a column number (horizontal direction)



The image read with the OpenCV function imread(), provides the colours in order of BGR (Blue, Green and Red).

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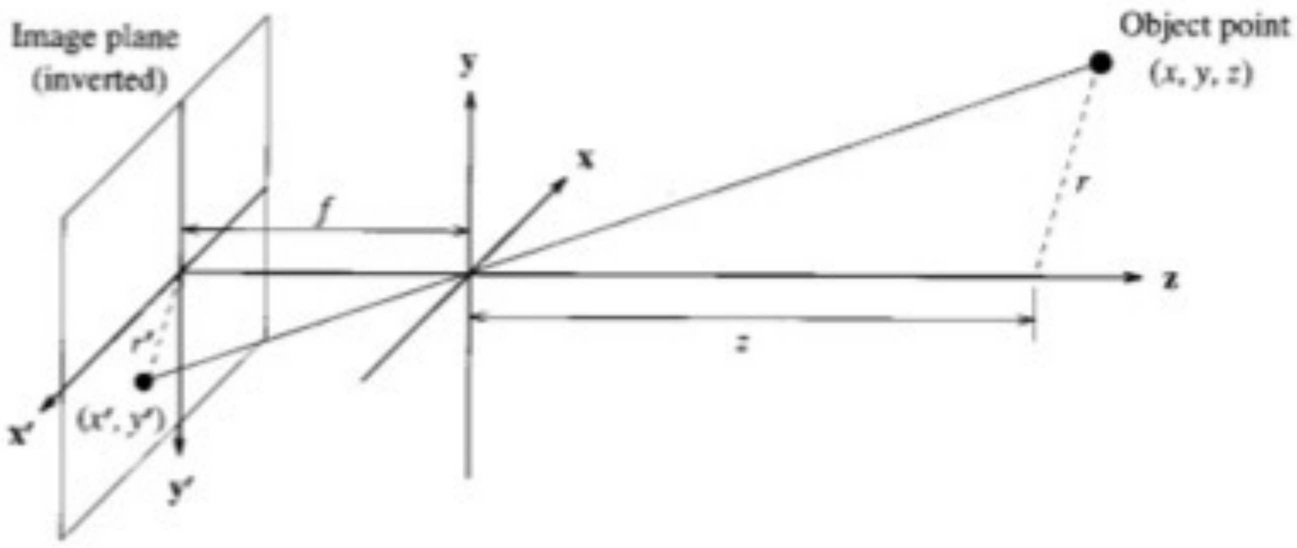
```

Physical Object Geometry and Image Geometry



$$\frac{r'}{r} = \frac{z}{f}, \quad x' = \frac{f}{z}x \quad \text{and} \quad y' = \frac{f}{z}y$$

$$x' = j - \frac{m-1}{2} \quad \text{and} \quad y' = -\left(i - \frac{n-1}{2}\right)$$

$r = \sqrt{x^2 + y^2}$ and $r' = \sqrt{x'^2 + y'^2}$

Image size: N X M

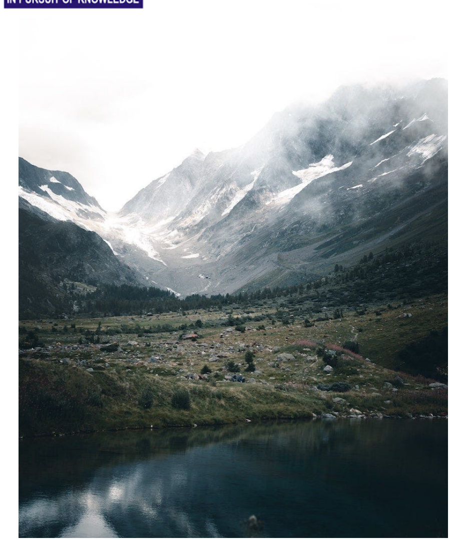
Application	**Ideal Image Size (in pixels)**	**Resolution (in pixels/inch)**
For mobile phones and portable devices with small screens:	320 x 240 pixels	72 ppi
For emails, online sharing sites and viewing on standard computer monitors:	1024 x 768 pixels	72 ppi
Viewing at full-screen size on LCD monitors with 5:4 aspect ratio	1280 x 1024 pixels	72 ppi
Viewing on standard definition TV sets with 4:3 aspect ratios:	720 x 576 pixels	72 ppi
Viewing on widescreen standard definition TV sets:	1280 x 720 pixels	72 ppi
Viewing on high definition TV sets:	1920 x 1080 pixels	72 ppi

Brightness and Contrast

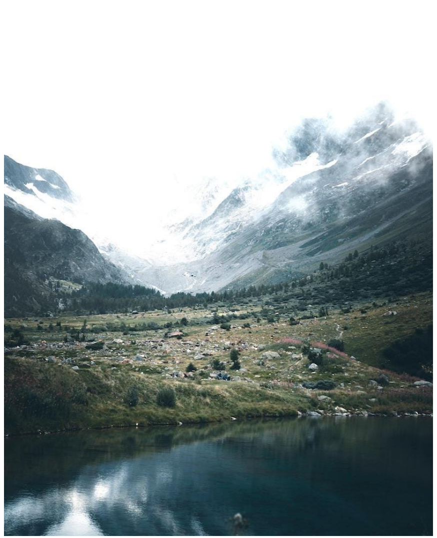
Image Brightness and Contrast link

- Brightness of image describes the overall lightness or darkness of an image. Increasing the brightness makes dark colours lighter and light colour whiter!
- Contrast of image describes the overall differences the difference in brightness between objects or regions of an image. A high-contrast image has very bright highlights and very dark shadows. For example, a black dog against a white background has good contrast, while a white rabbit running across a snowy field has poor contrast.

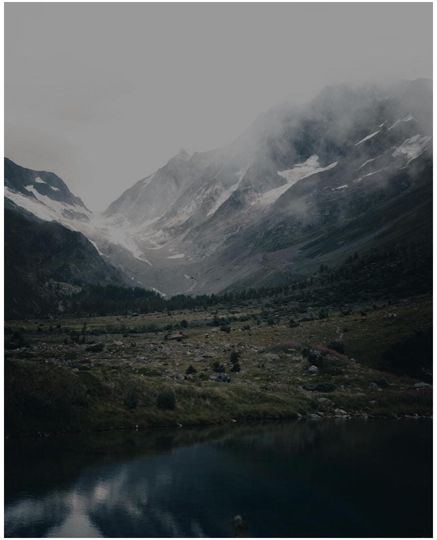
Image Brightness



Original



Brightness: +40%



Brightness: -40%

I'[i, j] = sF* I[i, j] **if(I'[i, j])>255) then I'[i, j]=255**

Scale Factor sF <1 implies make image darker

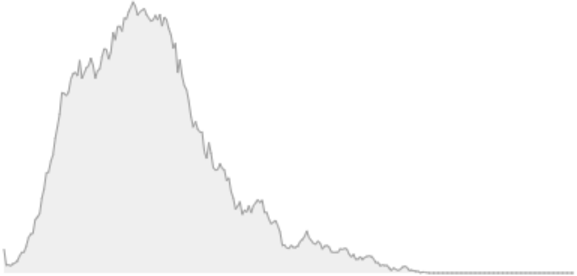
Image Contrast

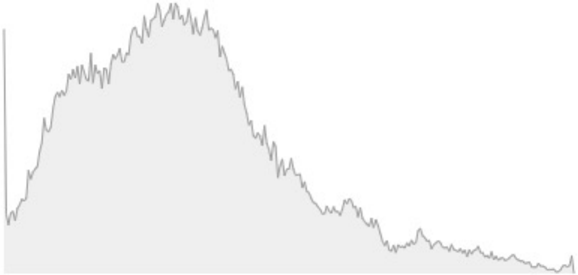
Min = Minimum(M[i,j])

Max = Maximum(M[i,j])

Diff = Max – Min

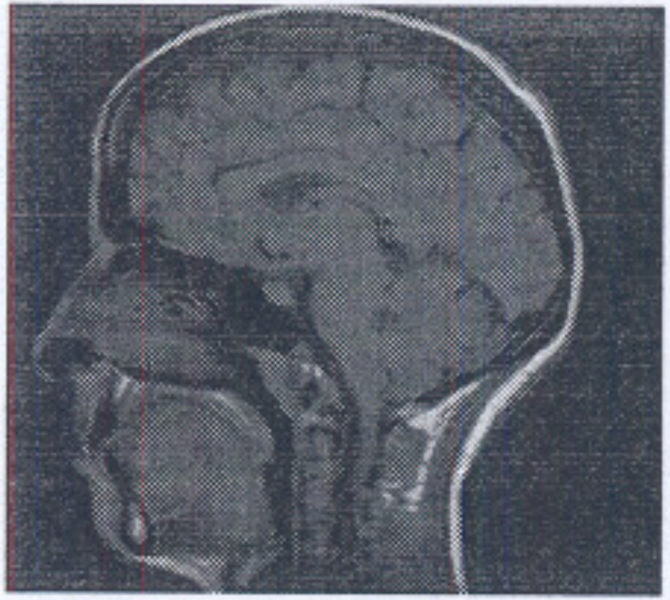
$$M'[i,j] = \frac{255 \ * (M[i,j] - Min)}{Diff}$$

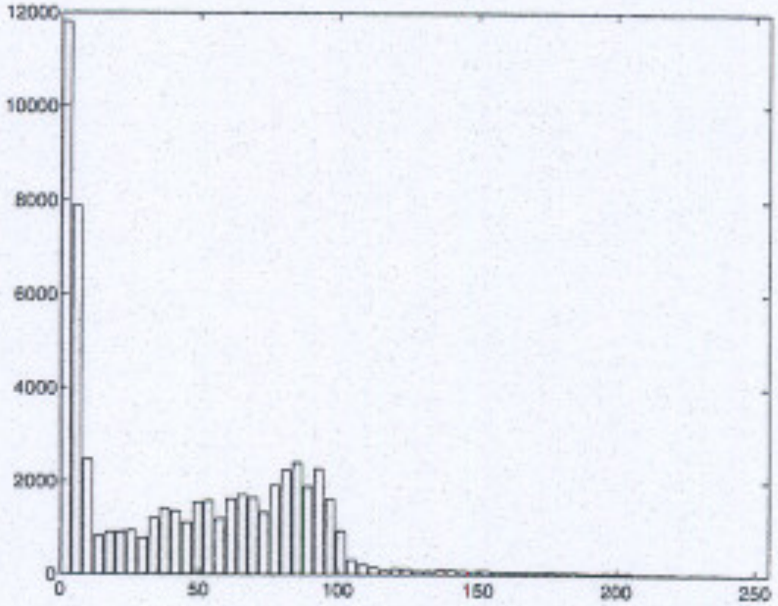




Other methods:
Histogram Equalization…

Contrast Algorithms





- The original image has very poor contrast since the gray values are in a very small range.
- Histogram scaling improves the contrast but leaves gaps in the final histogram.

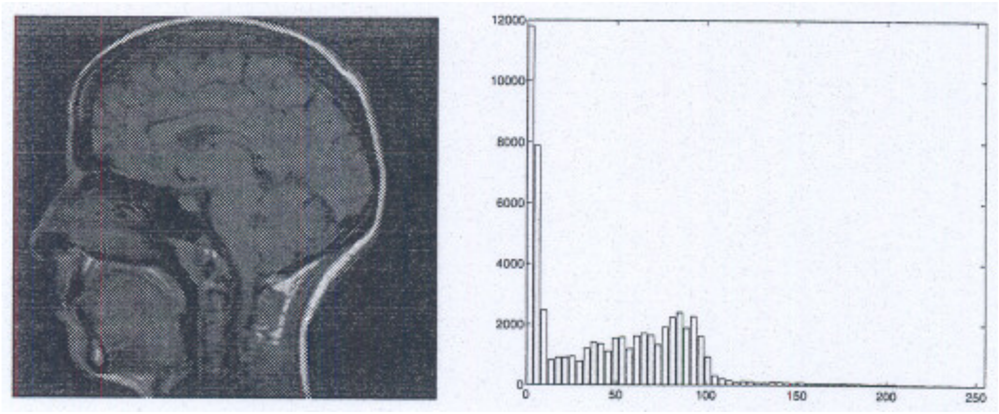
Contrast Algorithm: Image Scaling

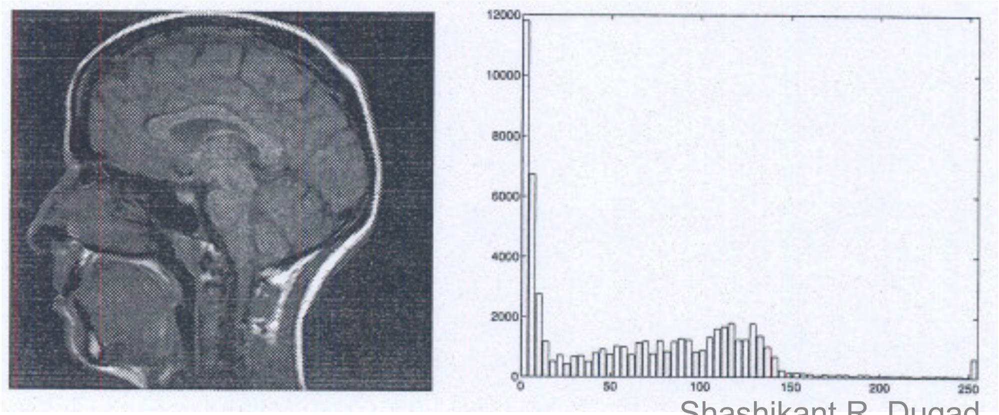
- A simple example of histogram modification is image scaling:
- Original image has the pixel intensity in the range of *[a,b]*
- In scaled-up image the pixel intensities are expanded to fill the range $[z_1, z_k]$.
- The formula for mapping a pixel intensity value I in the original image range into a pixel intensity value I′ in the new image intensity range is:

$$I' = \frac{z_k - z_1}{b - a}(I - \mathrm{a}) + z_1$$

- Note: a, b, z_1 *and* z_k are constants
- The problem with this scheme is that when the histogram is stretched according to this formula, the resulting histogram has gaps between bins

Image Contrast: Before and After Scaling





The original image has very poor contrast since the gray values are in a very small range.

Histogram scaling improves the contrast but leaves gaps in the final histogram.

***Top:* Original image and histogram.**

***Bottom:* Histogram Scaled Image and resulting histogram after image scaling.**

Image Contrast: Histogram Equalization

- Consider a gray scale image of size N x N, thus, having N^2 pixels. Position of a pixel is denoted by (y, x)
- If the intensity, I(y, x), of a pixel is represented by m bits then.
  - the maximum intensity is $I_{max} = (2^m - 1)$
- Image contrast is optimized by doing smart intensity transformation function!
  - $I'(y,x) = T(I(x,y))$ T is a function of intensity transformation
- For a given original image, obtain intensity histogram, n = h(I), where, a) n is number of pixels having intensity I and b) range of intensity is $0 <= I < (2^m - 1)$ ➔Total # of bins in histogram are $(2^m - 1)$
  - Note: $\sum n = \sum h(I) = N^2$
- Probability of a pixel having intensity I is: $p(I) = \frac{n}{N^2}$ where, $\sum p(I) = 1$

Image Contrast: Histogram Equalization

- Let us apply following constraints on te intensity transformation function
  - T(I) must be a strictly increasing function. This makes it an injective (one-to-one) function.
  - $0 \leq T(I) \leq L-1$. This makes T(I) surjective.
- The above two conditions make T(I) a bijective function, thus inversible
  - Therefore, there exist a function that provides, $I = T^{-1}(I')$
- Cumulative Distribution Function (CDF) for input image can be defined as:

$$F_r(I) = P(r \leq I) = \sum_{i=0}^{I} p_r(i) = \frac{1}{N^2}\sum_{i=0}^{I} n_i$$

- CDF for final image can be written as,

$$F_s(x) = P(s \leq x) = P(T(r) \leq x) = P\left(r \leq T^{-1}(x)\right) = F_r(T^{-1}(x))$$

Image Contrast: Histogram Equalization

We put the first condition of $T(r)$ precisely to make the above step hold true. The second condition is needed as s is the intensity value for the output image and so must be between o and (L-1).

So, a pdf of s can be obtained by differentiating $F_S(\ x\ )$ with respect to x. We get the following relation:

$$p_s(s) = p_r(r)\frac{\mathrm{d}r}{\mathrm{d}s}$$

Now, if we define the transformation function as follows:

$$s = T(r) = (L-1)\int_0^r p_r(x)dx$$

Then using this function gives us a uniform pdf for s.

$$\frac{\mathrm{d}s}{\mathrm{d}r} = (L-1)\frac{\mathrm{d}}{\mathrm{d}r}\int_0^r p_r(x)dx = (L-1)p_r(r)$$

Image Contrast: Histogram Equalization

The above step used Leibnitz's integral rule. Using the above derivative, we get:

$$p_s(s) = p_r(r)\frac{\mathrm{d}r}{\mathrm{d}s} = p_r(r)\frac{1}{(L-1)p_r(r)} = \frac{1}{L-1}$$

So the pdf of s is uniform. This is what we want.

Now, we extend the above continuous case to the discrete case. The natural replacement of the integral sign is the summation. Hence, we are left with the following histogram equalization transformation function.

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = \frac{(L-1)}{N^2}\sum_{j=0}^{k} n_j$$

Since s must have integer values, any non-integer value obtained from the above function is rounded off to the nearest integer.

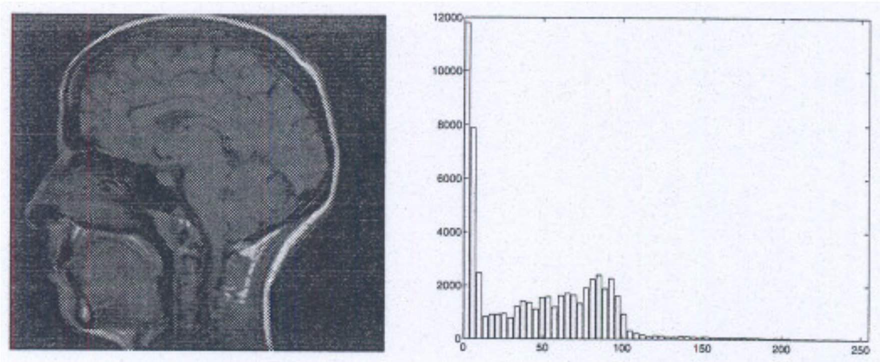
Histogram Equalization: Actual Calculation

Image Size: 8 x 9	Pixel intensity: 3 bits	L=8

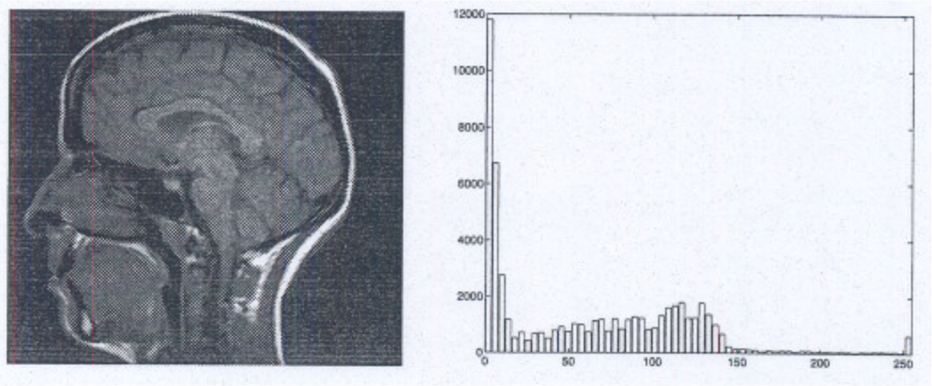
Pixel Intensity in Original Image (I)	Frequency (f)	I*f	Probbility Distribution Function (PDF)	Cumulative PDF (CPDF)	(L-1) *CPDF	Pixel Intensity (I') in Final Image	I' * f
0	2	0	0.02778	0.02778	0.19444	0	0
1	4	4	0.05556	0.08333	0.58333	1	4
2	6	12	0.08333	0.16667	1.16667	1	6
3	8	24	0.11111	0.27778	1.94444	2	16
4	10	40	0.13889	0.41667	2.91667	3	30
5	12	60	0.16667	0.58333	4.08333	4	48
6	14	84	0.19444	0.77778	5.44444	5	70
7	16	112	0.22222	1.00000	7.00000	7	112
Total =	**72**	**336**	**1**			**Total =**	**286**
Average Intensity =	**4.6667**				**Average Intensity =**		**4**

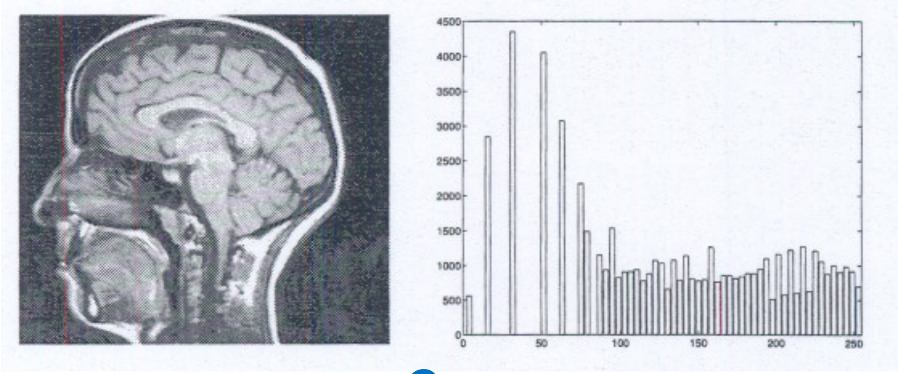
Image Contrast Results

A



B





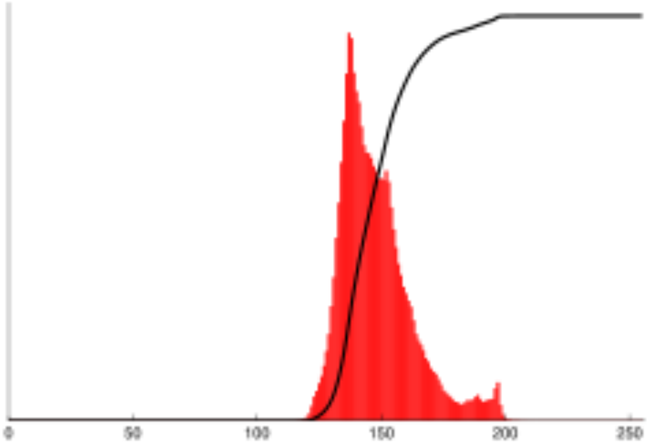
C

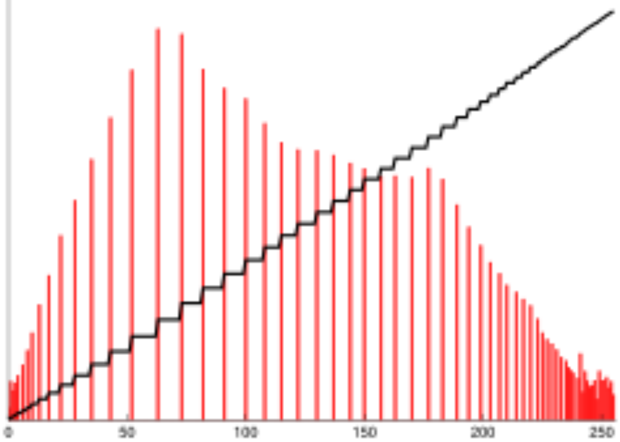
A: Original

B: Histogram Scaling

C: Histogram Equalization

Image before and after Histogram Equalization





Source: https://en.wikipedia.org/wiki/Histogram_equalization