MECHANICS ME1L001

1. FORCE SYSTEMS

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- Moment of force about an axis
- Couple moment
- Reduction of force system to a force and a couple

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- Equations of Equilibrium
- Problems in two and three dimensions
- Plane Frames
- Trusses

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- Square threaded screws
- Belt friction
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INTRODUCTION TO BASIC LAWS

- Force represents the action of one body on another. A force is characterized by its point of application, magnitude, and direction. Therefore, force is a vector quantity.
- Now let us quickly go through some basics laws involving forces first. These basic laws are none other than the three laws of motion given by Newton. The first law of motion, the second law of motion, the third law of motion and the law of Gravitation.

NEWTON'S FIRST LAW OF MOTION

- Newton's first law states that, if a body is at rest or moving at a constant speed in a straight line, it will remain at rest or keep moving in a straight line at constant speed unless it is acted upon by a force. This postulate is also known as the law of Inertia.
- For example, if you kick a football in the air, it keeps on rising until unless the force of gravitation pulls it down. Had the gravitation been not present, the ball would continue its motion of rising up forever. Here, gravitational force is the external force.
- Similarly, if the football is at rest, you need to kick it in order to move it. So, if you do not kick it (external force), it will remain at rest.

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NEWTON'S SECOND LAW OF MOTION

- Newton's second law of motion states that the rate of change of momentum of a body over time is directly proportional to the force applied, and occurs in the same direction as the applied force.
- It basically tells us that acceleration of a body is in the same sense in which the force is applied.
- This law leads us to the equation : **F = ma** where F is force applied, m is the mass of the body and a is the acceleration produced due to the applied force.

NEWTON'S THIRD LAW OF MOTION

- Newton's third law of motion states that every action has an equal and opposite reactions. All forces between two objects exist in equal magnitude and opposite direction.
- For example, if the road exerts a forward frictional force on an accelerating car's tires, then it is also a frictional force that Newton's third law predicts for the tires pushing backward on road.

NEWTON'S LAW OF UNIVERSAL GRAVITATION

- Newton's law of universal gravitation is usually stated as that every particle attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
- $F = G\frac{m1m2}{r^2}$

PRINCIPLE OF TRANSMISSIBILITY OF FORCES

- The principle of transmissibility states that the point of application of a force can be moved anywhere along its line of action without changing the external reaction forces on a rigid body.
- Conditions of equilibrium or motion are not affected by transmitting a force along its line of action.

MOMENT OF FORCE ABOUT A POINT

- A force vector is defined by its magnitude and direction. Its effect on the rigid body also depends on its point of application. Based on it, either translational or rotational or both effects are produced.
- The moment vector M (calculated about a point) is responsible for the rotational effect. This vector is perpendicular to the plane containing that point and the force.
- Moment = (distance from point) x (force) (vector cross product)
- M = Fd (where d is the perpendicular distance from point to the line of action of force)

Question 1:

What is the moment that Force A exerts about point A? What is the moment that Force B exerts about Point A?

Calculations:

F_A

$$M_A = F \times d = (100\,N)(2\,m) \circlearrowleft$$ clockwise is ← negative

$$M_A = -200\,Nm$$

F_B

$$d = 2\sin(30^\circ)$$

$$M_A = F \times d = (200\,N)(2\sin(30)\,m) \circlearrowleft$$ counter-clockwise is positive

$$M_A = 200\,Nm$$

Solution

From F_A $M_A = -200\,Nm$ From F_B $M_A = 200\,Nm$

MOMENT OF FORCE ABOUT A GIVEN AXIS

- Scalar moment about an axis is the projection of the vector moment M (calculated about a point on the given axis) onto the axis.
- The result is independent of the point on the axis.

COUPLE MOMENT

- Two forces F and –F having same magnitude, parallel lines of action and opposite sense are said to form a couple.
- The two couples lie in parallel planes and have the same sense or tendency to cause rotation in the same direction.
- The moment vector of the couple is independent of the choice of the origin of the co-ordinate axes, i.e., it is a free vector that can be applied at any point with the same effect.

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REDUCTION OF FORCE SYSTEM

- Force vector F can not be simply moved to O' from O without modifying its action on the body.
- Moving the force couple system from O to O' requires the addition of moment about O and O'.

Fig. 3.36 Reducing a system of forces to a force-couple system. (*a*) Initial system of forces; (*b*) all the forces moved to act at point *O*, with couple vectors added; (*c*) all the forces reduced to a resultant force vector and all the couple vectors reduced to a resultant couple vector.

FORCE SYSTEMS

CONTENTS

- Introduction to Basic Laws
- Principle of Transmissibility of forces
- Varignon’s Theorem
- Moment of force about a point
- Moment of force about an axis
- Couple Moment
- Reduction of force system to a force and couple

EQUILIBRIUM

CONTENTS

- Free Body Diagram
- Equations of Equilibrium
- Problems in two and three dimensions
- Plane frames
- Trusses

FREE BODY DIAGRAM (FBD)

- Free Body Diagram is a diagrammatic representation of the isolated system treated as single body.
- Only after such a diagram has been carefully drawn should the equations be written.
- Because of its critical importance, we emphasize here that the free body diagram is the most important single step in the solution of problems in Mechanics.

. A book is at rest on a tabletop. A free-body diagram for this situation looks like this:

2. A gymnast holding onto a bar, is suspended motionless in mid-air. The bar is supported by two ropes that attach to the ceiling. Diagram the forces acting on the combination of gymnast and bar. A free-body diagram for this situation looks like this:

3. An egg is free-falling from a nest in a tree. Neglect air resistance. A free-body diagram for this situation looks like this:

4. A flying squirrel is gliding (no *wing flaps*) from a tree to the ground at constant velocity. Consider air resistance. A free-body diagram for this situation looks like this:

7. A college student rests a backpack upon his shoulder. The pack is suspended motionless by one strap from one shoulder. A free-body diagram for this situation looks like this:

EQUATIONS OF EQUILIBRIUM

- Equilibrium means that sum of all external forces is zero. The system or body may or may not exhibit motion during the equilibrium position.
- By equilibrium, we mean both – translational as well as rotational equilibrium.
- So, at equilibrium the sum of forces in each directions as well as sum of the moments of these forces about a point should also be zero.

Equations of Equilibrium

Vector Equations of Equilibrium

$$\sum F = 0$$
$$\sum M_o = 0$$

Scalar Equations of Equilibrium

$$\sum F_x = 0 \qquad \sum M_x = 0$$
$$\sum F_y = 0 \qquad \sum M_y = 0$$
$$\sum F_z = 0 \qquad \sum M_z = 0$$

PROBLEMS IN TWO AND THREE DIMENSIONS

The uniform 7-m steel shaft has a mass of 200 kg and is supported by a ball-and-socket joint at A in the horizontal floor. The ball end B rests against the smooth vertical walls as shown. Compute the forces exerted by the walls and the floor on the ends of the shaft.

Vector solution. We will use A as a moment center to eliminate reference to the forces at A. The position vectors needed to compute the moments about A are

$$\mathbf{r}_{AG} = -1\mathbf{i} - 3\mathbf{j} + 1.5\mathbf{k}\ \text{m} \qquad \text{and} \qquad \mathbf{r}_{AB} = -2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}\ \text{m}$$

where the mass center G is located halfway between A and B.

The vector moment equation gives

$$[\Sigma \mathbf{M}_A = \mathbf{0}] \qquad \mathbf{r}_{AB} \times (\mathbf{B}_x + \mathbf{B}_y) + \mathbf{r}_{AG} \times \mathbf{W} = \mathbf{0}$$

$$(-2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}) \times (B_x\mathbf{i} + B_y\mathbf{j}) + (-\mathbf{i} - 3\mathbf{j} + 1.5\mathbf{k}) \times (-1962\mathbf{k}) = \mathbf{0}$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & -6 & 3 \\ B_x & B_y & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -3 & 1.5 \\ 0 & 0 & -1962 \end{vmatrix} = \mathbf{0}$$

$$(-3B_y + 5890)\mathbf{i} + (3B_x - 1962)\mathbf{j} + (-2B_y + 6B_x)\mathbf{k} = \mathbf{0}$$

Equating the coefficients of $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ to zero and solving give

$$B_x = 654\ \text{N} \qquad \text{and} \qquad B_y = 1962\ \text{N} \qquad \textit{Ans.}$$

The forces at A are easily determined by

$$[\Sigma \mathbf{F} = \mathbf{0}] \qquad (654 - A_x)\mathbf{i} + (1962 - A_y)\mathbf{j} + (-1962 + A_z)\mathbf{k} = \mathbf{0}$$

and $\quad A_x = 654\ \text{N} \qquad A_y = 1962\ \text{N} \qquad A_z = 1962\ \text{N}$

Finally $\quad A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$$= \sqrt{(654)^2 + (1962)^2 + (1962)^2} = 2850\ \text{N} \qquad \textit{Ans.}$$

Scalar solution. Evaluating the scalar moment equations about axes through A parallel, respectively, to the x- and y-axes, gives

$$[\Sigma M_{A_x} = 0] \qquad 1962(3) - 3B_y = 0 \qquad B_y = 1962 \text{ N}$$

$$[\Sigma M_{A_y} = 0] \qquad -1962(1) + 3B_x = 0 \qquad B_x = 654 \text{ N}$$

The force equations give, simply,

$$[\Sigma F_x = 0] \qquad -A_x + 654 = 0 \qquad A_x = 654 \text{ N}$$

$$[\Sigma F_y = 0] \qquad -A_y + 1962 = 0 \qquad A_y = 1962 \text{ N}$$

$$[\Sigma F_z = 0] \qquad A_z - 1962 = 0 \qquad A_z = 1962 \text{ N}$$

PLANE FRAMES

- Frames are structures with at least one multi force member. Frames are designed to support loads and are usually stationary.
- A free body diagram of the complete frame is used to determine the external forces acting on the frame.
- Internal forces are determined by dismembering the frame and creating free body diagrams for each component.

P
C
Q
A
B
Ax
Ay
Bx
By
Cx
Cy
-Cy
-Cx

TRUSSES

- A truss consists of straight members connected at joints. No member is continuous through a joint.
- Most structurer are made of several trusses joined together to form a space framework. Each truss carries those loads which act in its plane and may be treated as two-dimensional structures.
- Members of a truss are slender and not capable of supporting large lateral loads. Loads must be applied at the joint.
- The weights of the members of the truss are assumed to be applied to the joints, half of the weight of each member being applied to each of the two joints the member connects.

SIMPLE TRUSS

- A simple truss is constructed by successively adding two members and one connection to the basic triangular truss.
- In a simple truss, $m = 2n - 3$ where m is total number of members and n is number of joints.
- No. of bars + No. of reactions = 2(No. of joints).

5 kN
5 m
3 m
3 m

SPACE TRUSSES

- An elementary space truss consists of 6 members connected at 4 joints to form a tetrahedron.
- In a space truss, m = 3n - 6

FIG. 4.28 Simple Space Truss

METHOD OF JOINTS

- Dismember the truss and create a FBD for each member and pin.
- The two forces exerted on each member are equal, have the same line of action and opposite sense.
- Forces exerted by a member on the pins or joints at its ends are directed along the member and are equal and opposite.
- Conditions of equilibrium on the pins provide 2n equations for 2n unknowns.
- Conditions for equilibrium for the entire truss provide 3 additional equations which are not independent of the pin equations.

PROBLEM 6.2

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Entire truss

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

$$\circlearrowleft + \ \Sigma M_B = 0: \quad C(5.25\text{ m}) - (5\text{ kN})(4\text{ m}) = 0$$

$$\mathbf{C} = \frac{80}{21}\text{ kN} \uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + \frac{80}{21}\text{ kN} - 5\text{ kN} = 0$$

$$\mathbf{B}_y = \frac{25}{21}\text{ kN} \uparrow$$

Free body: Joint B:

$$\frac{F_{AB}}{5} = \frac{F_{BC}}{4} = \frac{25/21}{3}\text{ kN} = \frac{25}{63}$$

$$F_{AB} = \frac{125}{68}\text{ kN} = 1.9841\text{ kN} \qquad F_{AB} = 1.984\text{ kN} \quad C \blacktriangleleft$$

$$F_{BC} = \frac{100}{63}\text{ kN} = 1.5873\text{ kN} \qquad F_{BC} = 1.587\text{ kN} \quad T \blacktriangleleft$$

Free body: Joint C:

$$\frac{F_{AC}}{3.25} = \frac{F_{BC}}{1.25} = \frac{80/21}{3}\text{ kN} = \frac{80}{63}\text{ kN}$$

$$F_{AC} = \frac{100}{63}\text{ kN} = 1.5873\text{ kN} \qquad F_{AC} = 1.587\text{ kN} \quad C \blacktriangleleft$$

$$F_{BC} = \frac{100}{63}\text{ kN} \quad T \quad \text{(Checks)}$$

METHOD OF SECTIONS

- When the force in only one member or the forces in a very few members are desired, the method of sections work very well.
- To determine the force in member BD, pass a section through the truss as shown and create a free body diagram for the left side.
- With only three members cut by the section, the equations for static equilibrium may be applied to determine the unknown member forces, including the force in member BD.

P1
P2
P3
A
B
D
G
C
E
n
n
P1
P2
A
B
FBD
C
E
FCE

FRICTION

CONTENTS

- Laws of Coulomb Friction
- Problems involving large and small contact surfaces
- Square threaded screws
- Belt friction
- Rolling resistance

LAWS OF COULOMB FRICTION

- In reality, no frictionless surface exists. For two surfaces in contact, tangential forces, called friction forces, will develop if one attempts to move one relative to the other.
- However, the friction forces are limited in magnitude and will not prevent motion if sufficiently large forces are applied.
- There are two types of friction : dry or coulomb friction and fluid friction. We would limit ourselves to the discussion of dry friction or coulomb friction only.

- Block of weight W placed on horizontal surface. Forces acting on block are its own weight and reaction of surface N.
  Small horizontal force P applied to block. For block to remain stationary, in equilibrium, a horizontal component F of the surface reaction is required. F is a static friction force.
  As P increases, the static friction force increases as well until it reaches a maximum value Fmax. Fmax = μsN.
  Further increase in P causes the block to begin to move as F drops to a smaller kinetic friction force Fk = μkN.

CO-EFFECIENTS OF FRICTION

- Four situations can occur when a rigid body is in contact with a horizontal surface:

ANGLES OF FRICTION

- It is sometimes convenient to replace normal force N and friction force F by their resultant R :

- No friction
- No motion
- Motion impending
- Motion

$$\tan\phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N}$$

$$\tan\phi_s = \mu_s$$

$$\tan\phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N}$$

$$\tan\phi_k = \mu_k$$

ANGLES OF REPOSE

- Consider block of weight W resting on board with variable inclination angle θ

The moveable bracket shown may be placed at any height on the 3-in. diameter pipe. If the coefficient of friction between the pipe and bracket is 0.25, determine the minimum distance x at which the load can be supported. Neglect the weight of the bracket.

SOLUTION:

- When W is placed at minimum x, the bracket is about to slip and friction forces in upper and lower collars are at maximum value.

$$F_A = \mu_s N_A = 0.25 N_A$$

$$F_B = \mu_s N_B = 0.25 N_B$$

- Apply conditions for static equilibrium to find minimum x.

$$\sum F_x = 0: \quad N_B - N_A = 0 \qquad N_B = N_A$$

$$\sum F_y = 0: \quad F_A + F_B - W = 0$$

$$0.25 N_A + 0.25 N_B - W = 0$$

$$0.5 N_A = W \qquad N_A = N_B = 2W$$

$$\sum M_B = 0: \quad N_A(6\text{ in.}) - F_A(3\text{ in.}) - W(x - 1.5\text{ in.}) = 0$$

$$6N_A - 3(0.25N_A) - W(x - 1.5) = 0$$

$$6(2W) - 0.75(2W) - W(x - 1.5) = 0$$

$$\boxed{x = 12\text{ in.}}$$

Given: Dresser weight = 90 lb, man's weight = 150 lb. μ_s = 0.25.

Find: The smallest magnitude of **F** needed to move the dresser if $\theta = 30°$.

Also determine the smallest coefficient of static friction between his shoes and the floor so that he does not slip.

Plan: a) Draw FBDs of the dresser and man.

b) Determine the unknowns.

d) Apply E-of-E to solve for the unknowns.

FBD of the man

FBD of the dresser

Man : $+\rightarrow \sum F_X = \mu_m N_m - 30.363\,(\cos 30) = 0$

$+\uparrow \sum F_Y = N_m - 150 + 30.363\,(\sin 30) = 0$

These two equations give:

$N_m = 134.8$ lb

$\underline{\mu_m = 0.195}$

SQUARE-THREADED SCREWS

- Square-threaded screws frequently used in jacks, presses, etc. Analysis similar to block on inclined plane. Recall friction force does not depend on area of contact.
- Thread of base has been "unwrapped" and shown as straight line. Slope is $2\pi r$ horizontally and lead L vertically.
- Moment of force Q is equal to moment of force P. $Q = Pa/r$

- Impending motion upwards. Solve for Q.
- $\phi_s > \theta$, Self-locking, solve for Q to lower load.
- $\phi_s > \theta$, Non-locking, solve for Q to hold load.

A clamp is used to hold two pieces of wood together as shown. The clamp has a double square thread of mean diameter equal to 10 mm with a pitch of 2 mm. The coefficient of friction between threads is $\mu_s = 0.30$.

If a maximum torque of 40 N*m is applied in tightening the clamp, determine (a) the force exerted on the pieces of wood, and (b) the torque required to loosen the clamp.

SOLUTION

- Calculate lead angle and pitch angle. For the double threaded screw, the lead L is equal to twice the pitch.

$$\tan\theta = \frac{L}{2\pi r} = \frac{2(2\text{ mm})}{10\pi\text{ mm}} = 0.1273 \qquad \theta = 7.3°$$

$$\tan\phi_S = \mu_S = 0.30 \qquad \phi_S = 16.7°$$

- Using block and plane analogy with impending motion up the plane, calculate clamping force with force triangle.

$$Qr = 40\text{ N}\cdot\text{m} \qquad Q = \frac{40\text{ N}\cdot\text{m}}{5\text{ mm}} = 8\text{ kN}$$

$$\tan(\theta + \phi_S) = \frac{Q}{W} \qquad W = \frac{8\text{ kN}}{\tan 24°}$$

$$\boxed{W = 17.97\text{ kN}}$$

- Relate T_1 and T_2 when belt is about to slide to right.
- Draw free-body diagram for element of belt

$$\sum F_x = 0: \quad (T+\Delta T)\cos\frac{\Delta\theta}{2} - T\cos\frac{\Delta\theta}{2} - \mu_s \Delta N = 0$$

$$\sum F_y = 0: \quad \Delta N - (T+\Delta T)\sin\frac{\Delta\theta}{2} - T\sin\frac{\Delta\theta}{2} = 0$$

- Combine to eliminate ΔN, divide through by $\Delta\theta$,

$$\frac{\Delta T}{\Delta\theta}\cos\frac{\Delta\theta}{2} - \mu_s\left(T + \frac{\Delta T}{2}\right)\frac{\sin(\Delta\theta/2)}{\Delta\theta/2}$$

- In the limit as $\Delta\theta$ goes to zero,

$$\frac{dT}{d\theta} - \mu_s T = 0$$

- Separate variables and integrate from $\theta = 0$ to $\theta = \beta$

$$\ln\frac{T_2}{T_1} = \mu_s \beta \quad \text{or} \quad \frac{T_2}{T_1} = e^{\mu_s \beta}$$

CONCEPT OF STRESS AND STRAIN

CONTENTS

- Normal Stress
- Shear Stress
- State of stress at a point
- Ultimate stress
- Allowable stress
- Factor of safety
- Normal Strain
- Sheer Strain
- Hooke’s Law
- Poisson’s ratio
- Generalized Hooke’s Law
- Analysis of axially loaded members

- The main objective of the study of mechanics of materials is to provide the future engineer with the means of analyzing and designing various machines and load bearing structures.
- Design of new structure requires selection of appropriate materials and component dimensions to meet performance requirements.
- For reasons based on weight, cost, availability, etc., the choice is made to construct the structure from some material.

NORMAL STRESS

- The resultant of the internal forces for an axially loaded member is normal to a section cut perpendicular to the member axis.
- The force intensity on that section is defined as normal stress.

$$P = \sigma_{ave} A = \int dF = \int_A \sigma \, dA$$

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \qquad \sigma_{ave} = \frac{P}{A}$$

CENTRIC AND ECCENTRIC LOADING

- A uniform distribution of stress passing through centroid of structure is centric loading.
- In eccentrically loaded structure, the stress is non-uniformly distributed and the line of action does not pass through centroid.

(a)
P
A
C
P'
(b)
P
d
C
P'
(c)
P
d
M
P'

SHEAR STRESS

- Corresponding internal forces act in the plane of section C are called shear stress.
- Shear stress distribution varies from zero at the member surfaces to maximum values that may be much larger than the average value.

Difference between Normal Stress & Shear Stress

$$\sigma = \frac{F}{A} = \frac{10 \times 10^3 N}{(100 \times 10^{-3} m)(10 \times 10^{-3} m)}$$

$$= 10{,}000{,}000 \text{ Pa}$$

$$= \underline{10 \text{ MPa.}}$$

$$\tau = \frac{V}{A} = \frac{15 \times 10^3 N}{(100 mm)(10 mm)}$$

$$= \underline{15 \text{ MPa}}$$

NOTE: $\frac{N}{mm^2} = \frac{N}{10^{-6} m^2} = 10^6 \text{Pa} = \text{MPa}$

→ IF YOU LEAVE UNITS IN <u>N</u> AND <u>mm</u>, YOU WILL GET <u>MPa.</u>

NORMAL STRAIN

- When the force acts perpendicular to the surface of an object, it exerts a normal stress. Due to this, deformation of the body takes place. The ratio of this deformation to the original length is known as normal strain.

HOOKE'S LAW

- Hooke's law states that under proportionality limit stress and stress are directly proportional to each other.
- **Stress = E*Strain** where E is the constant of proportionality also known as Young's Modulus.
- This behaviour is described as Hooke's Law.

STRESS-STRAIN DIAGRAM FOR DUCTILE MATERIALS

- Stress Strain have linear relationship till a point. In this limit, Hooke's law is valid. In this region, even at a very high stress, there is a little deformation. If we remove this load, the material will reform to its original position. In this region, slope = Young's Modulus.
- In yield region, there is very significant change in stress but large deformation can be observed. It is because it reforms and re-orients itself.
- Beyond this point, it re-orients itself to offer more resistance to increasing stress. It hardens itself. At the peak of this region, it is able to handle the maximum stress although now it gets permanently deformed. This is plastic region. Beyond this it will never reform fully.
- Then in necking region if we increase load, curve goes down and this causes a failure and the material breaks.

Stress, σ
Strain hardening
Necking
Ultimate strength
Fracture
Yield strength
Rise
Run
Young's modulus = Slope = Rise / Run
Strain, ε

STRESS-STRAIN DIAGRAM FOR BRITTLE MATERIALS

- So, ductile materials can handle normal stress but not brittle materials.
- For making structures or component, we choose a material within yield region.
- Allowable stress <= Stress at yield point
- Allowable stress = $\frac{Ultimate\ Stress}{Factor\ of\ safety}$
- Factor safety is always greater than 1.

A wire 2 m long and 2 mm in diameter, when stretched by weight of 8 kg has its length increased by 0.24 mm. Find the stress, strain and Young's modulus of the material of the wire. g = 9.8 m/s²

$$\text{Stress} = F / A = mg / \pi r^2$$

$$\therefore \text{Stress} = (8 \times 9.8) / (3.142 \times (1 \times 10^{-3})^2)$$

$$\therefore \text{Stress} = (8 \times 9.8) / (3.142 \times 1 \times 10^{-6})$$

$$\therefore \text{Stress} = 2.5 \times 10^{7}\ N/m^2$$

$$\text{Strain} = l / L = 0.24 \times 10^{-3} / 2$$

$$\therefore \text{Strain} = 0.12 \times 10^{-3} = 1.2 \times 10^{-4}$$

Now, Young's modulus of elasticity= Y = Stress / Strain

$$\therefore Y = (2.5 \times 10^{7}) / (1.2 \times 10^{-4})$$

$$\therefore Y = 2.08 \times 10^{11}\ N/m^2$$

Ans.: Stress = $2.5 \times 10^{7}\ N/m^2$, Strain $= 1.2 \times 10^{-4}$, Yong's modulus of elasticity= $2.08 \times 10^{11}\ N/m^2$

A brass wire of length 2 m has its one end, fixed to a rigid support and from the other end a 4 kg wt is suspended. If the radius of the wire is 0.35 mm, find the extension produced in the wire. $g = 9.8\ m/s^2$, $Y = 11 \times 10^{10}\ N/m^2$

Given: Initial length of wire = L = 2 m, Radius of wire = 0.35 mm = 0.35×10^{-3} m = 3.5×10^{-4} m, Load attached = F = 4 kg wt = 4×9.8 N, $g = 9.81\ m/s^2$, $Y = 11 \times 10^{10}\ N/m^2$.

To Find: Extension =?

Solution:

$$Y = FL / A\,l$$

$$\therefore\ l = F\,L / \pi\, r^2\, Y$$

$$\therefore\ l = (4 \times 9.8 \times 2) / (3.142 \times (3.5 \times 10^{-4})^2 \times 11 \times 10^{10})$$

$$\therefore\ l = (4 \times 9.8 \times 2) / (3.142 \times 12.25 \times 10^{-8} \times 11 \times 10^{10})$$

$$\therefore\ l = 1.85 \times 10^{-3}\ m = 0.185 \times 10^{-2}\ m = 0.185\ cm$$

Ans.: Extension of wire is 0.185 m

THERMAL STRESS

- There will be a change in the length of the material if there is a change in temperature.
- A temperature change results in a change in length or thermal strain. There is no stress associated with the thermal strain unless the elongation is restrained by the supports.
- Treat the additional support as redundant and apply the superposition principle. The thermal deformation and the deformation from the redundant support must be compatible.

THERMAL STRESSES

- Temperature change causes thermal strain

(a) at $T = T_{ref}$ (b) at $T = T_{ref} + \Delta T$

- Constraints cause thermal stresses
- Thermo-elastic stress-strain relationship

$$\sigma = E(\varepsilon - \alpha\Delta T)$$

Thermal expansion coefficient

$$\varepsilon = \frac{\sigma}{E} + \alpha\Delta T$$

POISSON'S RATIO

- The elongation in the x-direction is accompanied by a contraction in other two directions although there is no stress in that direction.
- This effect is known as poisson's effect.
- Poisson's ratio = $\frac{Lateral\ Strain}{Axial\ Strain} = -\frac{\varepsilon y}{\varepsilon x} = -\frac{\varepsilon z}{\varepsilon x}$

POISSON'S RATIO
Lateral Strain
Longitudinal Strain
Longitudinal Strain
Lateral Strain
Poisson's Ratio = Lateral Strain / Longitudinal Strain
shutterstock
IMAGE ID: 1718230171
www.shutterstock.com

The rigid bar ABC in Fig. P-268 is pinned at B and attached to the two vertical rods. Initially, the bar is horizontal and the vertical rods are stress-free. Determine the stress in the aluminum rod if the temperature of the steel rod is decreased by 40°C. Neglect the weight of bar ABC.

Figure P-268

Contraction of steel rod, assuming complete freedom:

$$\delta_{T(st)} = \alpha\, L\, \Delta T$$
$$\delta_{T(st)} = (11.7 \times 10^{-6})(900)(40)$$
$$\delta_{T(st)} = 0.4212 \text{ mm}$$

The steel rod cannot freely contract because of the resistance of aluminum rod. The movement of A (referred to as δ_A), therefore, is less than 0.4212 mm. In terms of aluminum, this movement is (by ratio and proportion):

$$\frac{\delta_A}{0.6} = \frac{\delta_{al}}{1.2}$$
$$\delta_A = 0.5\delta_{al}$$

$$\delta_{T(st)} - \delta_{st} = 0.5\delta_{al}$$
$$0.4212 - \left(\frac{PL}{AE}\right)_{st} = 0.5\left(\frac{PL}{AE}\right)_{al}$$
$$0.4212 - \frac{P_{st}(900)}{300(200\,000)} = 0.5\left[\frac{P_{al}(1200)}{1\,200(70\,000)}\right]$$
$$28080 - P_{st} = 0.4762P_{al} \qquad \rightarrow \text{Equation (1)}$$

$$\Sigma M_B = 0$$
$$0.6P_{st} = 1.2P_{al}$$
$$P_{st} = 2P_{al} \qquad \rightarrow \text{Equation (2)}$$

Equations (1) and (2)

$$28\,080 - 2P_{al} = 0.4762P_{al}$$
$$P_{al} = 11\,340 \text{ N}$$

$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{11\,340}{1200}$$
$$\sigma_{al} = 9.45 \text{ MPa} \qquad \textit{answer}$$

A steel rod of length L and uniform cross sectional area A is secured between two walls, as shown in the figure. Use L=1.5m, E=200 GPa, $\alpha = 11.7 \times 10^{-6}$ /°C and $\Delta T = 80$ °C. Calculate the stress for a temperature increase of ΔT for:

a) The walls are fixed.
b) The walls move apart a distance 0.5mm.

a) $\delta_{th} - \delta_R = 0$

$$\alpha(\Delta T)L - \frac{RL}{AE} = 0$$

$\therefore$ $R = A\,E\,\alpha\,(\Delta T)$

$$\sigma = \frac{R}{A} = E\alpha(\Delta T)$$

$= 200 \times 10^9 \times 11.7 \times 10^{-6} \times 80 = 187.2$ MPa (Answer)

b) $\delta_{th} - \delta_R = \delta_w$

$$\alpha(\Delta T)L - \frac{RL}{AE} = \delta_w$$

$R = AE\,(\alpha\,\Delta T - \frac{\delta_w}{L})$

The compressive stress is then,

$$\sigma = \frac{R}{A} = E\left(\alpha\Delta T - \frac{\delta_w}{L}\right)$$

$= 200 \times 10^9 \left(11.7 \times 10^{-6} \times 80 - \frac{0.5\times10^{-3}}{1.5}\right) = 120.52$ MPa (Answer)

GENERALIZED HOOKE'S LAW

- For an element subjected to multi-axial loading, the normal strain components resulting from the stress components may be determined from the principle of superposition. This requires :
- Strain is linearly related to stress
- Deformations are small

$$\varepsilon_1 = \frac{1}{E}\left(\sigma_1 - \nu(\sigma_2 + \sigma_3)\right),$$
$$\varepsilon_2 = \frac{1}{E}\left(\sigma_2 - \nu(\sigma_1 + \sigma_3)\right),$$
$$\varepsilon_3 = \frac{1}{E}\left(\sigma_3 - \nu(\sigma_1 + \sigma_2)\right),$$

TORSION

CONTENT

- Torsion of cylindrical bars
- Torsional Stress
- Modulus of rigidity and deformation

TORSION OF CYLINDRICAL BARS

- Let us take up an example of a circular shaft in the generator and motor for understanding torsion in cylindrical bars.
- Turbine exerts a torque T on the shaft. Shaft transmits the torque to the generator. Generator creates an equal and opposite torque T'.
- Net of the internal shearing stress is an internal torque equal and opposite to the applied torque.
- Torque applied to shaft produces shearing stresses on the face perpendicular to the axis. From observation, the angle of twist of the shaft is proportional to the applied torque and the shaft length.

- When subjected to torsion, every cross section of a circular shaft remains plane and undistorted. Cross section for hollow and solid circular shaft remains plain and undistorted because a circular shaft is axisymmetric.
- Cross sections of non-circular shafts are distorted when subjected to tension.
- That is why we observe that almost every rotating part has a circular cross-section.

(a) Torsion of a circular shaft

(b) Isolated element abcd

(c) Strain of a circular shaft due to torsion

TORSIONAL STRESS

- Instead of causing any deformations in its length or area, torsional stress causes angle to twist.

The shaft in Fig. (a) consists of a 3-in. -diameter aluminum segment that is rigidly joined to a 2-in. -diameter steel segment. The ends of the shaft are attached to rigid supports, Calculate the maximum shear stress developed in each segment when the torque $T = 10$ kip in. is applied. Use $G = 4\times10^6$ psi for aluminum and $G = 12\times10^6$ psi for steel.

(a)

(b) FBD

Compatibility the two segments must have the same angle of twist; that is, $\theta_{st} = \theta_{al}$ From Eq. (3.4b), this condition between.

$$\left(\frac{TL}{GJ}\right)_{st} = \left(\frac{TL}{GJ}\right)_{al} \qquad \frac{T_{st}(3\times 12)}{(12\times 10^6)\frac{\pi}{32}(2)^4} = \frac{T_{al}(6\times 12)}{(4\times 10^6)\frac{\pi}{32}(3)^4}$$

from which

$$T_{st} = 1.1852\ T_{al} \qquad \text{(b)}$$

Solving Eqs. (a) and (b), we obtain

$$T_{al} = 4576\ lb \cdot \text{in.} \qquad T_{st} = 5424\ lb \cdot \text{in.}$$

the maximum shear stresses are

$$(\tau_{\max})_{al} = \left(\frac{16T}{\pi d^3}\right)_{al} = \frac{16(4576)}{\pi(3)^3} = 863\,psi \qquad \textit{Answer}$$

$$(\tau_{\max})_{st} = \left(\frac{16T}{\pi d^3}\right)_{st} = \frac{16(5424)}{\pi(2)^3} = 3450\,psi \qquad \textit{Answer}$$

Sample problem 3.3

The four rigid gears, loaded as shown in Fig. (a), are attached to a 2-in.-diameter steel shaft. Compute the angle θ of rotation of gear A relative to gear D. Use $G = 12\times10^6$ psi for the shaft.

Assume that the internal torques T_{AB}, T_{BC}, and T_{CD} are positive according to the sign convention introduced earlier (positive torque vectors point away from the cross section). Applying the equilibrium condition $\Sigma M_x = 0$ to each FBD, we obtain

$$500 - 900 + 1000 - T_{CD} = 0$$

$$500 - 900 - T_{BC} = 0$$

$$500 - T_{AB} = 0$$

$$T_{AB} = 500\ lb\cdot\ \text{ft},$$

$$T_{BC} = -400\ lb\cdot\ \text{ft}$$

$$T_{CD} = 600\ lb\cdot\ \text{ft}$$

(b) FBDs

The minus sign indicates that the sense of T_{BC} is opposite to that shown on the FBD.A is gear D were fixed.

This rotation is obtained by summing the angles of twist of the three segments:

$$\theta_{A/D} = \theta_{A/B} + \theta_{B/C} + \theta_{C/D}$$

Using Eq.(3.4b), we obtain (converting the lengths to inches and torques to pound-inches)

$$\theta_{A/D} = \frac{T_{AB}L_{AB} + T_{BC}L_{BC} + T_{CD}L_{CD}}{GJ}$$

$$= \frac{(500\times12)(5\times12) - (400\times12)(3\times12) + (600\times12)(4\times12)}{\left[\pi(2)^4/32\right](12\times10)^6}$$

$$= 0.02827 \text{ rad} = 1.620^\circ$$

Answer

MODULUS OF RIGIDITY AND DEFORMATION

- Modulus of rigidity is the ratio of shear stress to the displacement per unit sample length (shear strain). More specifically, modulus of rigidity is the co-efficient of elasticity for a shearing force.

$$\text{Shear Stress} = \frac{\text{Shearing Force}}{\text{Area Under Shear}} = \frac{F}{A}$$

$$\text{Shear Strain} = \frac{\text{Relative displacement of any layer}}{\text{Its distance from a fixed layer}}$$

$$\text{Shear strain} = \tan\theta = \theta = \frac{x}{h}$$

Now modulus of rigidity

$$\eta = \frac{\text{Shear stress}}{\text{Shear strain}}$$

$$\eta = \frac{F/A}{\theta} = \frac{F}{A\theta} = \frac{Fh}{Ax}$$

This is an expression for modulus of rigidity.

FLEXURAL LOADING

CONTENT

- Shear and moment in Beams
- Load, shear and moment relationship
- Shear and Moment Diagrams
- Flexure formula
- Shear stress in beams
- Deflection equation of elastic curve
- Deflection of beams

SHEAR AND MOMENT IN BEAMS

- Beam is a structural member designed to support loads applied at various points along its length.
- Beam can be subjected to concentrated loads or distributed loads or combination of both.

- Wish to determine bending moment and shearing force at any point in a beam subjected to concentrated and distributed loads.
- Determine reactions at supports by treating whole beam as free-body.
- Cut beam at C and draw free-body diagrams for AC and CB. By definition, positive sense for internal force-couple systems are shown.
- From equilibrium considerations, determine M and V or M' and V'.

- Variation of shear and bending moment may be plotted.
- Determine reactions at supports.
- Cut beam at C and consider member AC, V = +P/2 and M = +Px/2.
- Cut beam at E and consider member EB, V = -P/2 and M = +P(L-x)/2
- For a beam subjected to concentrated loads, shear is constant between loading points and moments vary linearly.

Draw the shear and bending moment diagrams for the beam and loading shown.

SOLUTION:

- Taking entire beam as a free-body, calculate reactions at *B* and *D*.
- Find equivalent internal force-couple systems at sections on either side of load application points.

$$\sum F_y = 0: \quad -20\,\text{kN} - V_1 = 0 \qquad \boxed{V_1 = -20\,\text{kN}}$$

$$\sum M_2 = 0: \quad (20\,\text{kN})(0\,\text{m}) + M_1 = 0 \qquad \boxed{M_1 = 0}$$

Similarly,

$$\boxed{\begin{array}{ll} V_3 = 26\,\text{kN} & M_3 = -50\,\text{kN}\cdot\text{m} \\ V_4 = 26\,\text{kN} & M_4 = -50\,\text{kN}\cdot\text{m} \\ V_5 = 26\,\text{kN} & M_5 = -50\,\text{kN}\cdot\text{m} \\ V_6 = 26\,\text{kN} & M_6 = -50\,\text{kN}\cdot\text{m} \end{array}}$$

- Plot results.

  Note that shear is of constant value between concentrated loads and bending moment varies linearly.

REALTION AMONG LOAD, SHEAR AND MOMENT

- Reactions at supports, $R_A = R_B = \dfrac{wL}{2}$
- Shear curve,

$$V - V_A = -\int_0^x w\,dx = -wx$$

$$V = V_A - wx = \frac{wL}{2} - wx = w\left(\frac{L}{2} - x\right)$$

- Moment curve,

$$M - M_A = \int_0^x V dx$$

$$M = \int_0^x w\left(\frac{L}{2} - x\right)dx = \frac{w}{2}\left(Lx - x^2\right)$$

$$M_{\max} = \frac{wL^2}{8} \quad \left(M \text{ at } \frac{dM}{dx} = V = 0\right)$$

Draw the shear and bending-moment diagrams for the beam and loading shown.

SOLUTION:

- Taking entire beam as a free-body, determine reactions at supports.

$$\sum M_A = 0:$$

$$D(24\text{ ft}) - (20\text{ kips})(6\text{ ft}) - (12\text{ kips})(14\text{ ft}) - (12\text{ kips})(28\text{ ft}) = 0$$

$$\boxed{D = 26\text{ kips}}$$

$$\sum F_y = 0:$$

$$A_y - 20\text{ kips} - 12\text{ kips} + 26\text{ kips} - 12\text{ kips} = 0$$

$$\boxed{A_y = 18\text{ kips}}$$

- Between concentrated load application points, $dV/dx = -w = 0$ and shear is constant.
- With uniform loading between D and E, the shear variation is linear.

- Between concentrated load application points, $dM/dx = V = \text{constant}$. The change in moment between load application points is equal to area under the shear curve between points.

$$M_B - M_A = +108 \qquad M_B = +108\ \text{kip}\cdot\text{ft}$$
$$M_C - M_B = -16 \qquad M_C = +92\ \text{kip}\cdot\text{ft}$$
$$M_D - M_C = -140 \qquad M_D = -48\ \text{kip}\cdot\text{ft}$$
$$M_E - M_D = +48 \qquad M_E = 0$$

- With a linear shear variation between D and E, the bending moment diagram is a parabola.

SHEAR STRESS IN BEAMS

- For a linearly elastic material,

$$\sigma_x = E\varepsilon_x = -\frac{y}{c}E\varepsilon_m$$

$$= -\frac{y}{c}\sigma_m \quad \text{(stress varies linearly)}$$

- For static equilibrium,

$$F_x = 0 = \int \sigma_x \, dA = \int -\frac{y}{c}\sigma_m \, dA$$

$$0 = -\frac{\sigma_m}{c}\int y \, dA$$

First moment with respect to neutral plane is zero. Therefore, the neutral surface must pass through the section centroid.

- For static equilibrium,

$$M = \int(-y\sigma_x \, dA) = \int(-y)\left(-\frac{y}{c}\sigma_m\right)dA$$

$$M = \frac{\sigma_m}{c}\int y^2 \, dA = \frac{\sigma_m I}{c}$$

$$\sigma_m = \frac{Mc}{I} = \frac{M}{S}$$

Substituting $\sigma_x = -\frac{y}{c}\sigma_m$

$$\sigma_x = -\frac{My}{I}$$

(a) S-beam (b) W-beam

- The maximum normal stress due to bending,

$$\sigma_m = \frac{Mc}{I} = \frac{M}{S}$$

$I =$ section moment of inertia

$S = \dfrac{I}{c} =$ section modulus

A beam section with a larger section modulus will have a lower maximum stress

- Consider a rectangular beam cross section,

$$S = \frac{I}{c} = \frac{\frac{1}{12}bh^3}{h/2} = \frac{1}{6}bh^3 = \frac{1}{6}Ah$$

Between two beams with the same cross sectional area, the beam with the greater depth will be more effective in resisting bending.

- Structural steel beams are designed to have a large section modulus.

DEFLECTION OF BEAMS AND ELASTIC CURVE

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI}$$

(*a*) Simply supported beam

(*b*) Overhanging beam

(*c*) Cantilever beam

Concept Application 9.1

The cantilever beam AB is of uniform cross section and carries a load **P** at its free end A (Fig. 9.9a). Determine the equation of the elastic curve and the deflection and slope at A.

Using the free-body diagram of the portion AC of the beam (Fig. 9.9b), where C is located at a distance x from end A,

Fig. 9.9 (*a*) Cantilever beam with end load. (*b*) Free-body diagram of section *AC*. (*c*) Deformed shape and boundary conditions.

dx^2

Integrating in x,

$$EI\frac{dy}{dx} = -\tfrac{1}{2}Px^2 + C_1 \qquad \textbf{(2)}$$

Now observe the fixed end B where $x = L$ and $\theta = dy/dx = 0$ (Fig. 9.9*c*). Substituting these values into Eq. (2) and solving for C_1 gives

$$C_1 = \tfrac{1}{2}PL^2$$

which we carry back into Eq. (2):

$$EI\frac{dy}{dx} = -\tfrac{1}{2}Px^2 + \tfrac{1}{2}PL^2 \qquad \textbf{(3)}$$

Integrating both members of Eq. (3),

$$EI\,y = -\tfrac{1}{6}Px^3 + \tfrac{1}{2}PL^2x + C_2 \qquad \textbf{(4)}$$

But at B, $x = L$, $y = 0$. Substituting into Eq. (4),

$$0 = -\tfrac{1}{6}PL^3 + \tfrac{1}{2}PL^3 + C_2$$

$$C_2 = -\tfrac{1}{3}PL^3$$

Carrying the value of C_2 back into Eq. (4), the equation of the elastic curve is

$$EI\,y = -\tfrac{1}{6}Px^3 + \tfrac{1}{2}PL^2x - \tfrac{1}{3}PL^3$$

or

$$y = \frac{P}{6EI}(-x^3 + 3L^2x - 2L^3) \qquad \textbf{(5)}$$

The deflection and slope at A are obtained by letting $x = 0$ in Eqs. (3) and (5).

$$y_A = -\frac{PL^3}{3EI} \qquad \text{and} \qquad \theta_A = \left(\frac{dy}{dx}\right)_A = \frac{PL^2}{2EI}$$

TRANSFORMATION OF STRESS AND STRAIN

y'
y
$\tau_{x'y'}\Delta A$
$\sigma_{x'}\Delta A$
x'
θ
x
$\sigma_x(\Delta A\cos\theta)$
$\tau_{xy}(\Delta A\cos\theta)$
$\tau_{xy}(\Delta A\sin\theta)$
$\sigma_y(\Delta A\sin\theta)$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$(\sigma_{x'} - \sigma_{ave})^2 + \tau_{x'y'}^2 = R^2$$

where

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Maximum shearing stress occurs for $\sigma_{x'} = \sigma_{ave}$

$$\tau_{\max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

Note : defines two angles separated by 90° and offset from θ_p by 45°

$$\sigma' = \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

Fig. 7.13

For the state of plane stress shown, determine (a) the principal panes, (b) the principal stresses, (c) the maximum shearing stress and the corresponding normal stress.

SOLUTION:

- Find the element orientation for the principal stresses from

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

- Determine the principal stresses from

$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

- Calculate the maximum shearing stress with

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma' = \frac{\sigma_x + \sigma_y}{2}$$

A single horizontal force P of 150 lb magnitude is applied to end D of lever ABD. Determine (a) the normal and shearing stresses on an element at point H having sides parallel to the x and y axes, (b) the principal planes and principal stresses at the point H.

SOLUTION:

- Determine an equivalent force-couple system at the center of the transverse section passing through H.

$$P = 150\,\text{lb}$$

$$T = (150\,\text{lb})(18\,\text{in}) = 2.7\,\text{kip}\cdot\text{in}$$

$$M_x = (150\,\text{lb})(10\,\text{in}) = 1.5\,\text{kip}\cdot\text{in}$$

- Evaluate the normal and shearing stresses at H.

$$\sigma_y = +\frac{Mc}{I} = +\frac{(1.5\,\text{kip}\cdot\text{in})(0.6\,\text{in})}{\frac{1}{4}\pi(0.6\,\text{in})^4}$$

$$\tau_{xy} = +\frac{Tc}{J} = +\frac{(2.7\,\text{kip}\cdot\text{in})(0.6\,\text{in})}{\frac{1}{2}\pi(0.6\,\text{in})^4}$$

$$\boxed{\sigma_x = 0 \quad \sigma_y = +8.84\,\text{ksi} \quad \tau_y = +7.96\,\text{ksi}}$$

- Determine the principal planes and calculate the principal stresses.

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(7.96)}{0 - 8.84} = -1.8$$

$$2\theta_p = -61.0°, 119°$$

$$\boxed{\theta_p = -30.5°,\ 59.5°}$$

$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0 + 8.84}{2} \pm \sqrt{\left(\frac{0 - 8.84}{2}\right)^2 + (7.96)^2}$$

$$\boxed{\begin{aligned}\sigma_{\max} &= +13.52\,\text{ksi}\\ \sigma_{\min} &= -4.68\,\text{ksi}\end{aligned}}$$