EE1101
SIGNALS AND SYSTEMS

Course Summary

OBJ : Develop a language to describe Signals and Systems and Tools to analyse them.

OVERVIEW

1. The Basics : PART 1 - SIGNALS
2. The Basics : PART 2 - SYSTEMS
3. Linear Time Invariant Systems
4. The Fourier Series
5. The Fourier transforms

BIBLIOGRAPHY

1. PROF. Gaurav Raina, “EE1101 Lectures and Notes”
2. PROF. Shanti Bhattacharya , “EE1101 Lectures”
3. PROF. Umesh , “EE1101 Lectures”
4. PROF. Venkatesh Ramaiyan, “EE1101 Lectures”
5. PROF. V.G.K. Murti, “NPTEL Lectures”
6. A.V. Oppenheim, “Signals and Systems”

WHAT ARE SIGNALS AND SYSTEMS?

- SYSTEM :

- SIGNAL : A Mathematical function that has some independent variable. (Not necessarily time)

CLASSIFICATION OF SIGNALS

CT

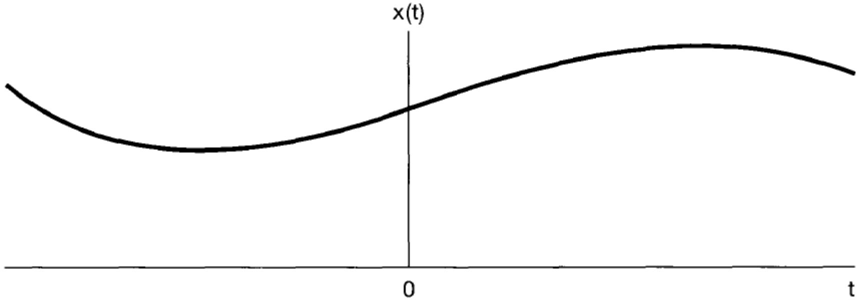
DT

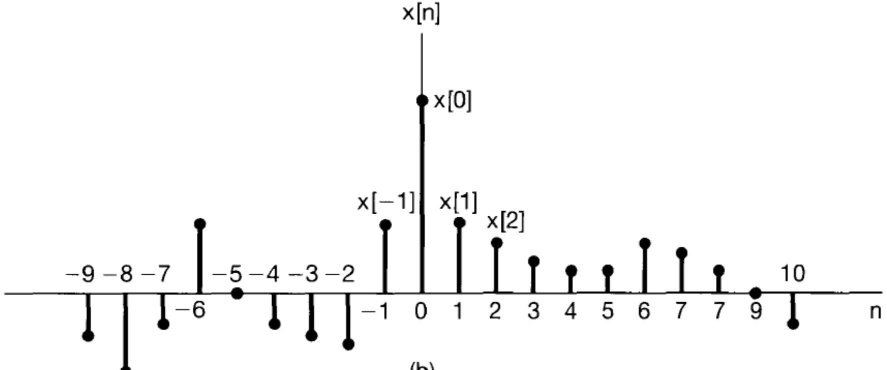
CT:

- Independent variable is continuous.
- Signal is defined for a continuum of values.

DT:

- Independent variable takes only discrete set of values.
- Signal is defined only at discrete times.

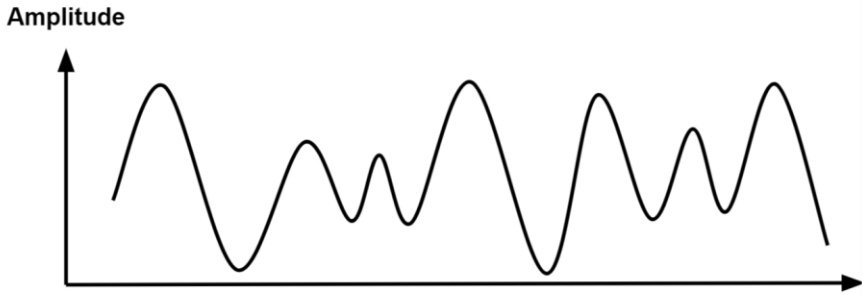




ANALOG
DIGITAL

- Amplitude takes a continuum of values

- Amplitude can take only finite values.
  (Binary → 2 Levels)



EVEN

ODD

- Identical to its time-reversed counterpart.
- CT : $X(t) = X(-t)$
- DT : $X[n] = X[-n]$

- Negative of its time-reversed counterpart.
- CT : $X(t) = X(-t)$
- DT : $X[n] = X[-n]$

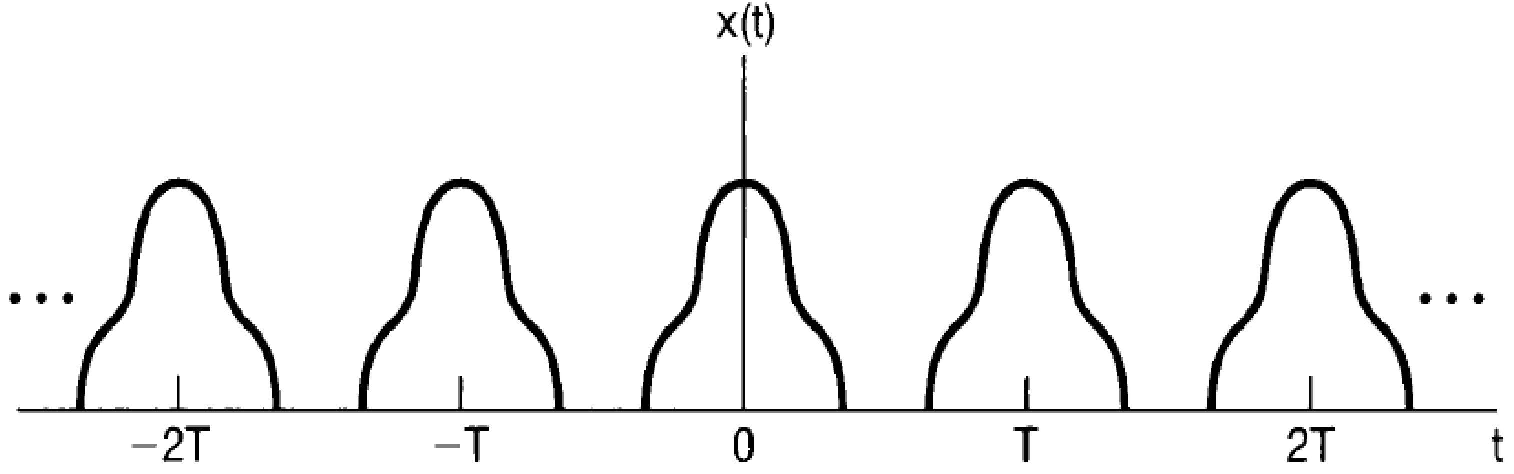
ANY SIGNAL CAN BE WRITTEN AS A SUM OF EVEN AND ODD SIGNAL.

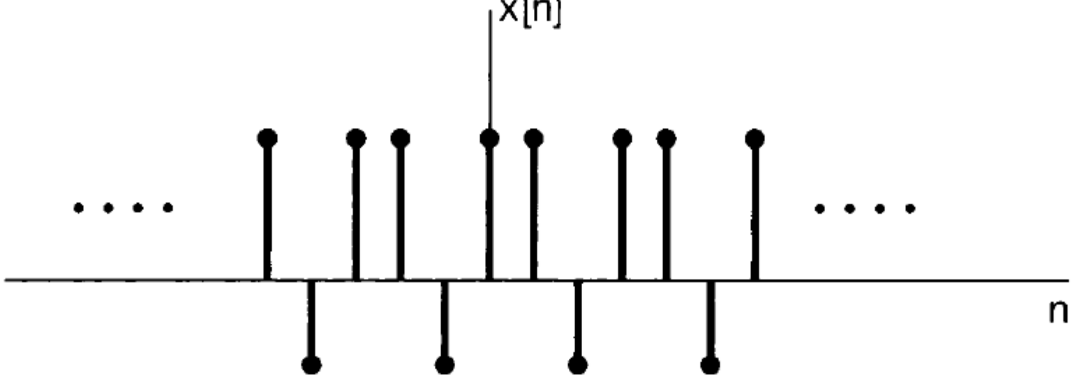
PERIODIC SIGNAL

- A Signal that has a property of remaining unchanged when time-shifted by T(in case of CT) and N (in case of DT).

$$x(t) = x(t+T) \qquad\qquad x[n] = x[n+N]$$

- FUNDAMENTAL PERIOD → Smallest Value of T / Smallest positive of N


x(t)
−2T
−T
0
T
2T
t


x[n]
n

CAUSALITY IN SIGNALS

- CAUSAL : $x(t) = 0$ For all $t < 0$
- NON CAUSAL : $x(t) \neq 0$ For all $t < 0$
- ANTI CAUSAL : $x(t) = 0$ For all $t \geq 0$

ENERGY AND POWER OF SIGNALS

ENERGY OF A SIGNAL

$$E_\infty \triangleq \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \, dt = \int_{-\infty}^{+\infty} |x(t)|^2 \, dt,$$

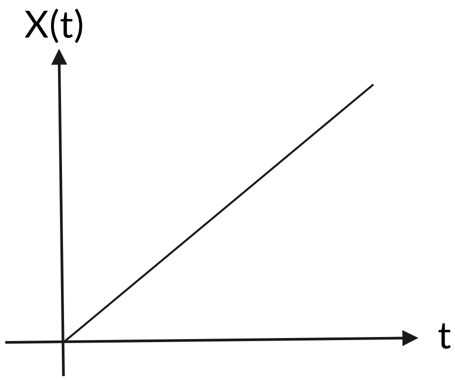
$$E_\infty \triangleq \lim_{N \to \infty} \sum_{n=-N}^{+N} |x[n]|^2 = \sum_{n=-\infty}^{+\infty} |x[n]|^2$$

POWER OF A SIGNAL

$$P_\infty \triangleq \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$P_\infty \triangleq \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x[n]|^2$$

- ENERGY SIGNAL : $0 \leq E < \infty$ [POWER is zero]
- POWER SIGNAL : $0 \leq P < \infty$ [ENERGY tends to infinity]
- NEITHER POWER NOR ENERGY SIGNALS : Neither of E and P are finite.



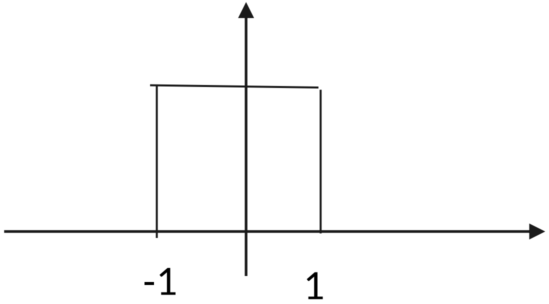
THE BASICS : PART 2 - SYSTEMS

“We’re analog beings living in a digital world, facing a quantum future.”

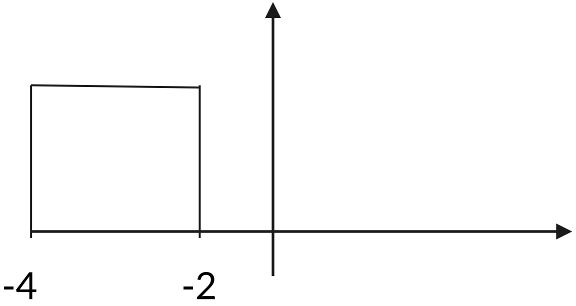
SIGNAL TRANSFORMATION

CT

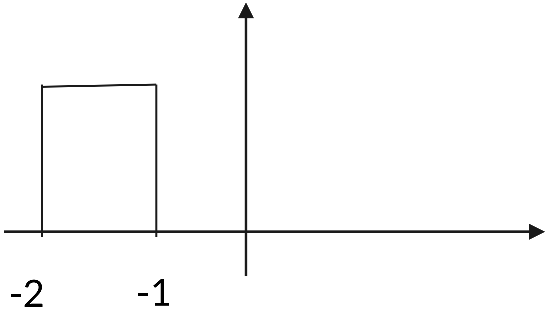
- TIME SHIFTING : $X(t) \longrightarrow X(t \pm T)$
- TIME SCALING : $X(t) \longrightarrow X(at)$
- TIME REVERSAL : $a = -1$
- AMPLITUDE SCALING : $X(t) \longrightarrow -X(t)$



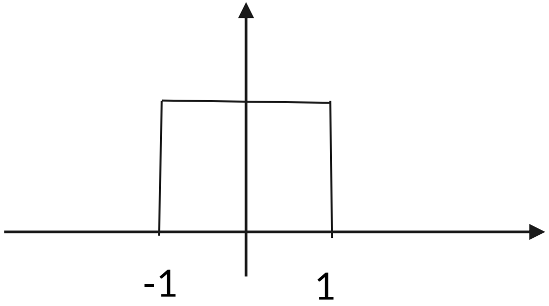
X(t) → X(2*t + 3)



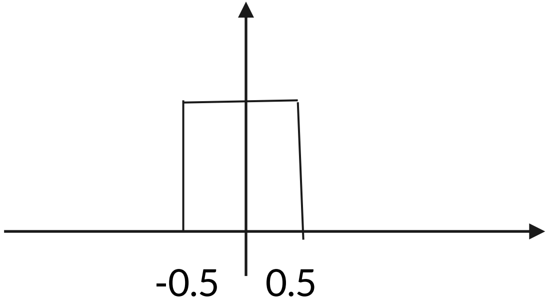
Shift By -3



Scale by 0.5



X(t) → X(2*(t + 1.5))



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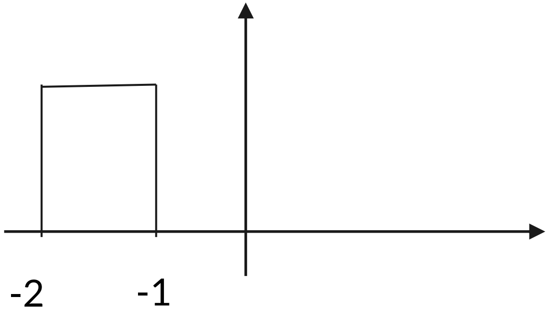


Shift By -1.5

TIME SCALING IN DT

$X[n] \rightarrow X[a*n]$

- a > 1 : Throw out data from original sequence
- a < 1 : Fill the gaps in resulting signal using interpolation

ELEMENTARY FUNCTIONS

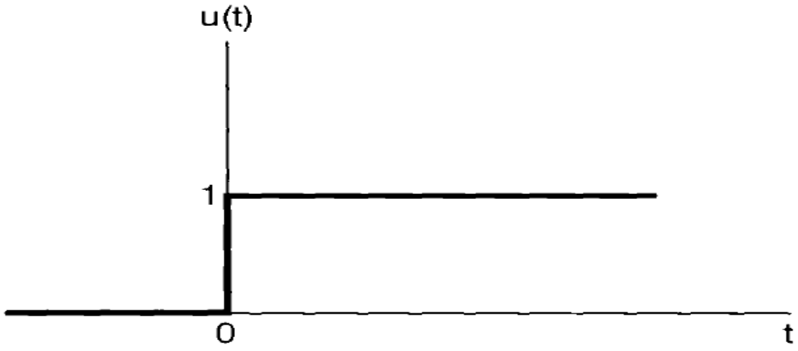
UNIT STEP FUNCTION

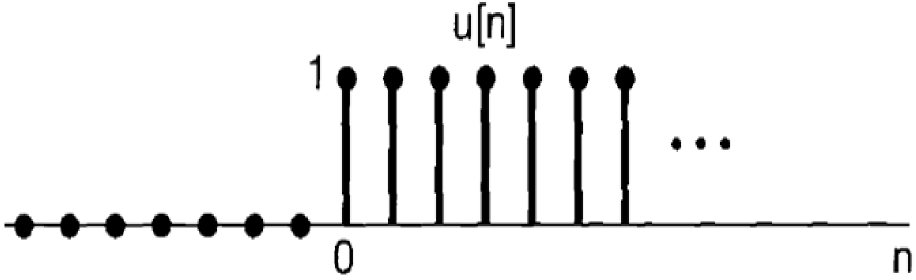
CT

DT

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases},$$

$$u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}.$$



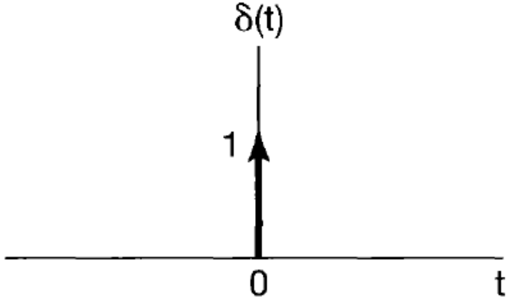


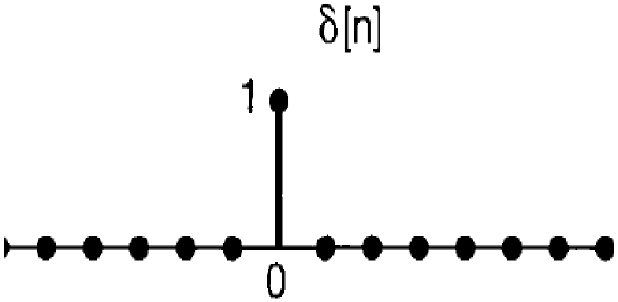
UNIT IMPULSE FUNCTION

CT

DT

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$





THEIR RELATION

- UNIT IMPULSE is the first difference of UNIT STEP.

$$\delta[n] = u[n] - u[n-1].$$

- UNIT STEP is the running sum of UNIT IMPULSE.

$$u[n] = \sum_{m=-\infty}^{n} \delta[m].$$

- UNIT IMPULSE is the first derivative of UNIT STEP.

$$\delta(t) = \frac{du(t)}{dt}.$$

- UNIT STEP is the running sum of UNIT IMPULSE.

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau.$$

CLASSIFICATION OF SYSTEMS

WITH MEMORY

- The concept of memory in a system corresponds to the presence of a mechanism in the system that retains or stores information about input values.
- In many physical systems, memory is directly associated with the storage of energy.

$$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)\, d\tau,$$

$$y[n] = x[n-1].$$

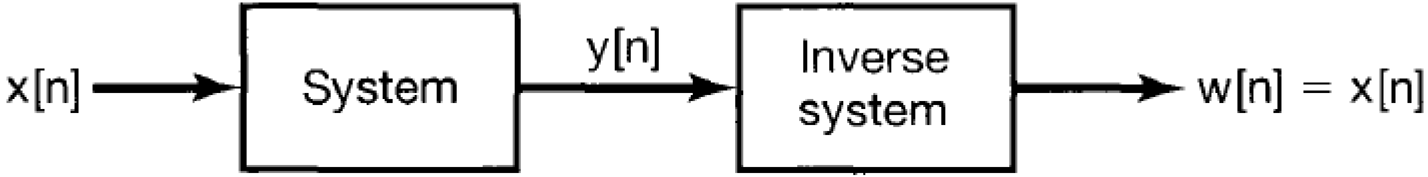
MEMORYLESS

- Output for each value of the independent variable at a given time is dependent only on the input at that same time.
- Shouldn't have a memory of future too.

$$y[n] = (2x[n] - x^2[n])^2$$

$$y(t) = Rx(t),$$

INVERTIBILITY



A system is said to be invertible if distinct inputs lead to distinct outputs.

CAUSALITY

- The output at any time depends only on values of the input at the present time and in the past.
- NON - Anticipative
- All memory less systems are causal.

CAUSAL SYSTEMS

NON-CAUSAL SYSTEMS

$$w[n] = y[n] - y[n-1],$$

$$y[n] = x[-n].$$

$$y(t) = x(t)\cos(t+1).$$

STABILITY

- A stable system is one in which small inputs lead to responses that do not diverge.
- BIBO Stability:

If $|X(t)| < B$ and $|Y(t)| < B'$, where B and B' are finite, Then the system is stable.

TIME INVARIANCE

- A system is time invariant if the behavior and characteristics of the system are fixed over time.
- In a Time-invariant system, A time shift in the input signal results in an identical time shift in the output signal.

$$y[n] = nx[n].$$

TIME VARIANT

$$y(t) = x(2t).$$

LINEARITY

- Possess the property of SUPERPOSITION : If an input consists of the weighted sum of several signals, then the output is the weighted sum of the responses of the system to each of those signals.
- That is has ADDITIVITY and HOMOGENEITY.

$$\text{continuous time: } ax_1(t) + bx_2(t) \rightarrow ay_1(t) + by_2(t),$$

$$\text{discrete time: } ax_1[n] + bx_2[n] \rightarrow ay_1[n] + by_2[n].$$

Where ‘a’ is complex scalar.