1 i) 1 Mark including correctly coded scheme

Using a first order forward difference on the Robin BC gives

$$u_0 - \frac{u_1 - u_0}{h} = \alpha$$

this gives the first row of the Matrix system

Now the Finite difference formula is

$$\frac{u_{i-1} - 2u_i + u_{i+1}}{h^2} + p\,\frac{u_{i+1} - u_{i-1}}{2h} + q u_i = r$$

Giving rows 2 to N+1 of the Matrix system

The N+2 row is

$$u_{N+1} = \beta$$

ii) 1.5 marks including correctly coded scheme

If we insert a ghost node at x_{-1} then the Robin BC becomes

$$u_0 - \frac{u_1 - u_{-1}}{2h} = \alpha$$

$$\Rightarrow \quad u_{-1} = 2h(\alpha - u_0) + u_1$$

The ODE applied at x_0 is

$$\frac{u_{-1} - 2u_0 + u_1}{h^2} + p\frac{u_1 - u_{-1}}{2h} + qu_0 = r$$

So row 1 is

$$\frac{-2(1+h)u_0 + 2u_1}{h^2} + pu_0 + qu_0 = r + \alpha\left(p - \frac{2}{h}\right)$$

Rows 2 to N $(i \in [1, N-1])$

$$\frac{u_{i-1} - 2u_i + u_{i+1}}{h^2} + p\frac{u_{i+1} - u_{i-1}}{2h} + qu_i = r$$

Row $N+1$ $(i=N)$ is

$$\frac{u_{i-1}-2u_i}{h^2} - \frac{p\,u_{i-1}}{2h} + q u_i = r - \beta\left(\frac{1}{h^2}-\frac{p}{2h}\right)$$

Q2 a) See code

1.5 marks for correct code

b) 0.5 marks

c) 0.5 marks

Must do a change of variables

$\bar{x} = \frac{2x-a-b}{b-a}$ and also map ODE

to these new variables. Then solve on $\bar{x} \in (-1,1)$

using our collocation scheme.