

School of Mathematics and Statistics
MAST90026 Computational Differential Equations
2026

Homework 4

Due: 11:59PM Thursday, 2nd April.

This homework is worth 5% of the total assessment in this subject. Submit your hand working and published MATLAB code* as a combined PDF file through Canvas.

*Use the publish command (or use the GUI) to run your script and save it as a PDF.

Late submissions will not be marked and a grade of 0 will be awarded. If there are extenuating circumstances apply for an extension or special consideration (more information available in the assessment adjustments page in the student support module).

1. For the case $D(x) = 1, q(x) = 0$ and $\{\phi_j\}$ are the hat functions on a non-uniform mesh, show that the stiffness matrix \mathbf{K} takes the form

$$\begin{aligned} K_{ii} &= \frac{1}{h_i} + \frac{1}{h_{i-1}}, \\ K_{i,i-1} = K_{i-1,i} &= -\frac{1}{h_{i-1}}, \\ K_{ij} &= 0, \quad |i - j| > 1, \end{aligned}$$

i.e. \mathbf{K} is tridiagonal and symmetric.

2. Write a MATLAB function `FEj=elem_load(f, xj, xjp1)` that computes, using Gaussian quadrature of order 3, the element load vector for linear elements

$$\mathbf{F}_{E_j} = \begin{bmatrix} \int_{x_j}^{x_{j+1}} f(x) \phi_j \, dx \\ \int_{x_j}^{x_{j+1}} f(x) \phi_{j+1} \, dx \end{bmatrix}.$$

You should submit your code. Hint: the Gaussian quadrature on $[0, 1]$ of order 3 is:

$$(x_0, \omega_0) = \left(-\frac{1}{2}\sqrt{\frac{3}{5}} + \frac{1}{2}, \frac{5}{18} \right), \quad (x_1, \omega_1) = \left(\frac{1}{2}, \frac{8}{18} \right), \quad (x_2, \omega_2) = \left(\frac{1}{2}\sqrt{\frac{3}{5}} + \frac{1}{2}, \frac{5}{18} \right).$$

3. Solve the Problem

$$u'' - \sin(u) = -1; \quad u(0) = 0; \quad u(1) = 1,$$

using a finite difference method and newton method. Submit your code and a plot of the solution.