School of Mathematics and Statistics
MAST90026 Computational Differential Equations
2024

Student Talks:

The presentations are worth 20% of the total assessment in this subject.

Your task is to find out something about one of the following topics and tell us about it!

You will be working in pairs, which you can choose yourself; anyone who doesn't choose a partner will be assigned one at random. You can also select up to 5 preferred topics (there is no guarantee you will be assigned one of these but I will try my best to optimise everyone's happiness). Please fill in the below link by 5pm Monday 15th April and I will release the allocations and presentation timetable on Wednesday 17th April during the tutorial.

pollev.com/mast90026

For your assigned topic, you are expected to show a numerical demonstrations. You may use MATLAB code that you find on the MathWorks file exchange as long as you appropriately site this during your presentation. You must however do a reasonable quantity of coding yourself. For example, if you are assigned one of the easier topics (e.g. 2) it would make sense to write code from scratch whereas if you are assigned one of the harder topics (e.g., 16 or 17) then it makes sense to use existing code from the MathWorks file exchange and write scripts/functions to call that code. If you are unsure on the difficulty level of your assigned topic then please ask me.

Your presentation must be between 10–13 minutes long and you should each aim to speak for about half the time. I will cut you off at 13 minutes if you are not finished and you will loose a mark if it is not at least 10 minutes long; there will be a timer at the front of the room that you can see. There are an additional 2 minutes for questions.

You must submit your presentation, code and a brief statement (2-4 sentences) on what you contributed to both the coding and the presentation slides. You should state which slides you created/contribute to and which parts of the code you wrote etc. These should all be uploaded to Canvas. Note only one group member needs to upload the code/presentation slides but all group members should upload a statement on your individual contribution.

Grading (20 marks total): slides (5 marks), presentation (5 marks), numerical demonstration (5 marks), difficulty/depth of the content covered (4 marks), attendance at all talks (1 mark).

1. **Shooting methods for BVPs**

   Mention: simple shooting, superposition

   No need for: multiple shooting

   Some references: AMR §4.1, 4.2, 4.6, A & P §7.1

2. **Gaussian quadrature**

   Investigate: Gauss-Legendre quadrature, Gauss-Lobatto quadrature and Gauss-Radau

   Some references: A & H §5.3, Heath §8.3, S & M Ch 10

3. **Gaussian quadrature on unbounded domains**

   Investigate: Gauss-Hermite, Gauss-Laguerre

   Some references: G, M & R pp. 185–208

4. **Cubic splines**

   Piecewise polynomial interpolation, splines, natural/clamped/not-a-knot splines

   No need for: B-splines, Bezier splines, shape-preserving splines

   Some references: S & M Ch 11, Heath §7.4, Moler §3.2-3.6, KC §6.3-6.4

5. **Root-finding**

   Bisection, Newton's method, secant method, IQI, hybrid method (`fzero`)

   1D only

   Some references: S & M Ch1, A & H §3.2-3.3, Moler §4.2-4.8

6. **Direct methods for solving sparse linear systems**

   Mention: ordering — sparse Cholesky, minimum degree,

   No need for: sparse LU, nested dissection

   Some references: Iserles Ch. 11, Gockenbach Ch. 10

7. **Stationary iterative methods for solving linear systems**

   Mention: Jacobi method; Gauss-Seidel; SOR

   Some references:

   KC §4.6; BF §7.3; Gockenbach Ch 12; Iserles §12.1–12.3

8. **Iterative methods for nonsymmetric linear systems**

   Mention: CGNR, CGNE, GMRES (basics)

   No need for: details on implementation, preconditioned GMRES

   References: Kelley §2.6, 3.1–2, 3.4; Trefethen & Bau Lecture 35.

9. **Multigrid iteration**

   Mention: smoothers, V-cycle, W-cycle, full MG

   No need for: nonlinear multigrid (FAS)

   Some references: Iserles Ch. 13; Gockenbach Ch. 13; Briggs

10. **Simple methods for finding eigenvalues/eigenvectors**

    Mention: Jacobi method; power method; deflation; inverse iteration

    No need for: QR method; SVD

    Some references:

    KC §5.1; Heath §4.1-4.3; BF §9.2; Atkinson §9.2,9.6; NR1 §11.1,11.7

11. **Adaptive Method of Lines/moving mesh**

    1D only, error equidistribution

    Some references: Vande Wouwer, Saucez & Schiesser Ch 1

12. **Stiff solvers**

    Mention: A-stability, L-stability, RKC methods.

    No need for: Implicit RK methods

    Some references: Iserles Ch 4; A & P: §3.3–3.7; Leveque Ch. 8

13. **Spectral methods for elliptic PDEs**

    Discuss at least Chebyshev methods

    Some references: Iserles Ch 10

14. **Discontinuous Galerkin methods**

    1D case only

    Some references: Hesthaven & Warburton §2.1-2.2, 3.1-3.5

15. **Finite volume methods (Generalized difference methods)**

    1D case only

    Some reference: Li, Chen & Wu §2.1-2.2.

16. **Mixed finite elements for Stokes flows**

    Mention: Mass and momentum conservation, choice of basis functions

    No need for: Navier Stokes, existence of solutions

    Some References: Elman §5, 6

17. **FEM for linear elasticity**

    Mention: governing equations, weak formulation, mesh locking

    No need for: convergence results, plate theory

    Some references: Johnson §5.1; Gockenbach §2.5, 9.2, 9.3

18. **Methods for stochastic ODEs**

    Mention : Brownian paths, Euler-Maruyama method, strong vs weak convergence.

    No need for: Milstein method

    Some references: Higham

19. **Fast Poisson solvers**

    Mention: FFT based fast solver for 5-point stencil

    No need for: FFT based fast solver for 9-point stencil

    Some reference: Iserles §15.1, Demmel's Lecture 24

References (note these are not compulsary, just a place to start looking if you're stuck):

AMR: Ascher, Mattheij & Russell, *Numerical Solution of BVPs for ODEs*, SIAM, 1987.

A & P: Ascher & Petzold, *Computer Methods for ODE and DAEs*, SIAM, 1998.

A & H: Atkinson & Han, *Elementary Numerical analysis,* 3rd ed., Wiley, 2004.

Briggs, *A multigrid tutorial*, SIAM, 1987.

BF: Burden & Faires, *Numerical Analysis*, 5th ed., 1993.

Demmel, https://people.eecs.berkeley.edu/~demmel/cs267/lecture24/lecture24.html

Elman, Silvester & Wathen, *Finite Elements and Fast Iterative Solvers : with Applications in Incompressible Fluid Dynamics* OUP, 2005.

G, M & R: Gautschi, Mastroianni and Rassias, *Approximation and Computation*, Springer, 2011

Gockenbach, *Understanding and implementing the finite element method*, SIAM, 2006.

Heath, *SCIENTIFIC COMPUTING: AN INTRODUCTORY SURVEY*, SIAM, Revised Second Edition, 2018.

Hesthaven & Warburton, *Nodal Discontinuous Galerkin Methods*, Springer, 2008.

Higham , *An Algorithmic Introduction to Numerical Simulation of SDEs*, SIAM Review, 2001.

Li, Chen, & Wu,*Generalized Difference Methods for Differential Equations*, CRC Press, 2000.

Iserles, *A first course in the numerical analysis of differential equations*, CUP, 2nd ed, 2008.

Johnson, *Numerical solution of partial differential equations by the finite element method*, CUP, 1992.

Kelley, *Iterative methods for Linear and Nonlinear Equations*, SIAM, 1995.

KC: Kincaid & Cheney, *Numerical Analysis*, 3rd ed., Brooks-Cole, 2002.

Leveque, *Finite Difference Methods for Ordinary and Partial Differential Equations*, SIAM 2007.

Lor, C.Y. *Moving Mesh Methods for Time-Dependent PDEs*, Ph. D. Thesis, 2010.

Moler, *Numerical Computing with MATLAB*, SIAM, 2004.

NR1: Press et. al. *Numerical Recipes in C*, 1st ed., 1988.

S & M: Süli & Myers, *An introduction to Numerical Analysis*, CUP, 2003.

Trefethen & Bau, *Numerical Linear Algebra*, SIAM 1997.

Vande Wouwer, Saucez & Schiesser , *The Adaptive method of Lines*, Chapman & Hall Ch 1.