The following problem will be used to test if your codes are working

$$u'' + 6\tan(x)\,u' - 9u = 21\sin x$$

$$\Rightarrow \quad -\left(\sec^6(x)\,u'\right)' + 9\sec^6 x\,u = -21\sec^5(x)\tan(x)$$

$$\begin{cases} u'(-3/2) = \frac{3}{8}\left(\cos\left(\frac{3}{4}\right) - 5\cos\left(\frac{9}{4}\right)\right) \\ u(1/2) = -\frac{1}{4}\left(1 + 5\cos(1)\right)\sin(1/2) \end{cases}$$

or
$$\begin{cases} u'(-1) = \frac{3}{8}\left(\cos(1) - 5\cos(3)\right) \\ u(1) = -\frac{1}{4}\left(1 + 5\cos(2)\right)\sin(1) \end{cases}$$

Gives $\quad u = \frac{1}{8}\left(3\sin x - 5\sin 3x\right)$

Q1 3 marks (1 for a 2 for b)

a)

$$-(D(x)u')' + q(x)u = f(x)$$

$$\Rightarrow \quad u'' + \frac{D'(x)}{D(x)}u' - \frac{q(x)}{D(x)}u = -\frac{f(x)}{D(x)}$$

$$\Rightarrow \quad \tilde{p}(x) = \frac{D'(x)}{D(x)} \quad \Rightarrow \int_{x_0}^{x} \tilde{p}(x) = \log(D) + C$$

where $x(a) = x_0$

we choose $c = \exp(1)$ so that

$D(x) = \exp\left(\int_{x_0}^{x} \tilde{p}(\mathcal{X})\,d\mathcal{X}\right)$ and so we see

$D(x) > 0$

(Note any choice of constant is correct as long as it is carried through to q & f)

$$\tilde{q} = \frac{-q}{D}$$

$$\Rightarrow \quad q = -\tilde{q}(x)\exp\left(\int_{x_0}^{x} \tilde{p}(\mathcal{X})\,d\mathcal{X}\right)$$

similarly $f = -\tilde{f}(x)\exp\left(\int_{x_0}^{x} \tilde{p}(\mathcal{X})\,d\mathcal{X}\right)$

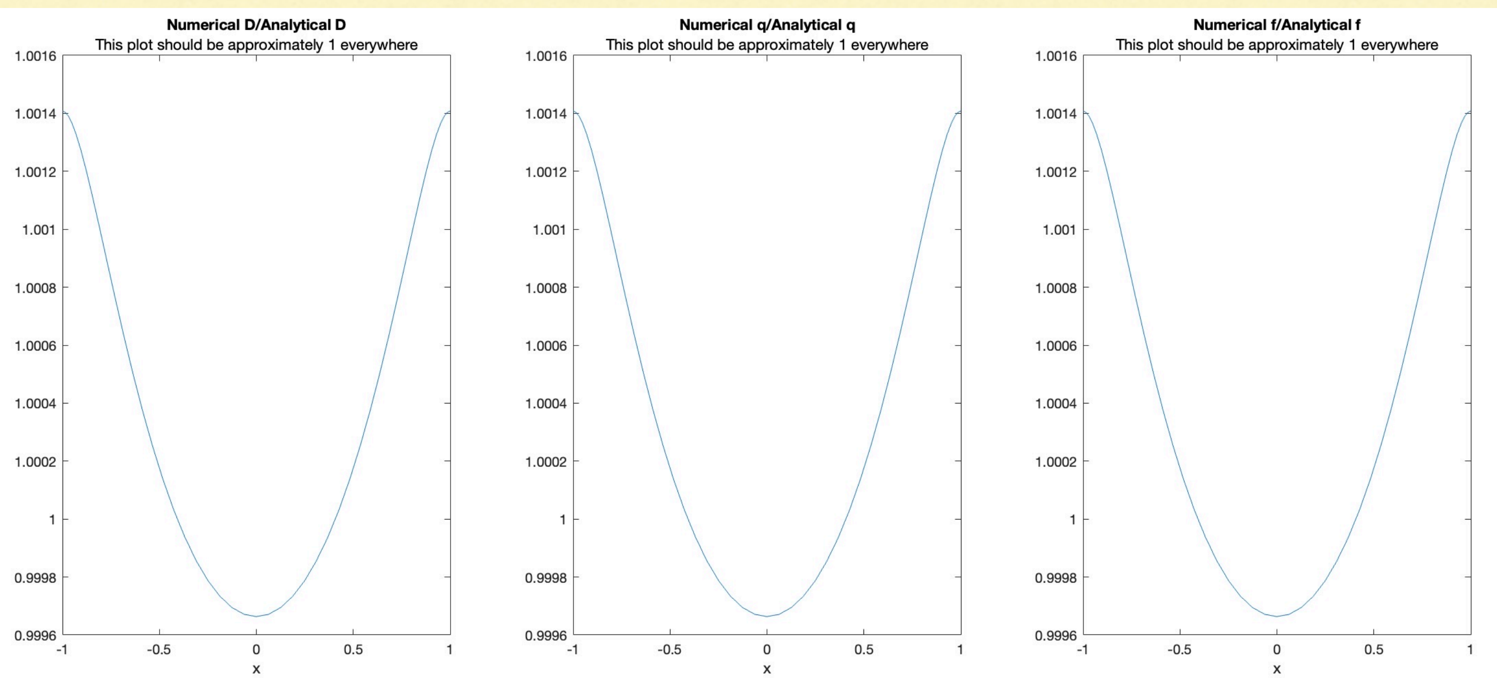
b)

Correct plots when running

Assignment_1_Q1b_Assess.m

Note if plots are incorrect I will look through your code to see where I can give you marks. This holds for all questions.

Should look something like

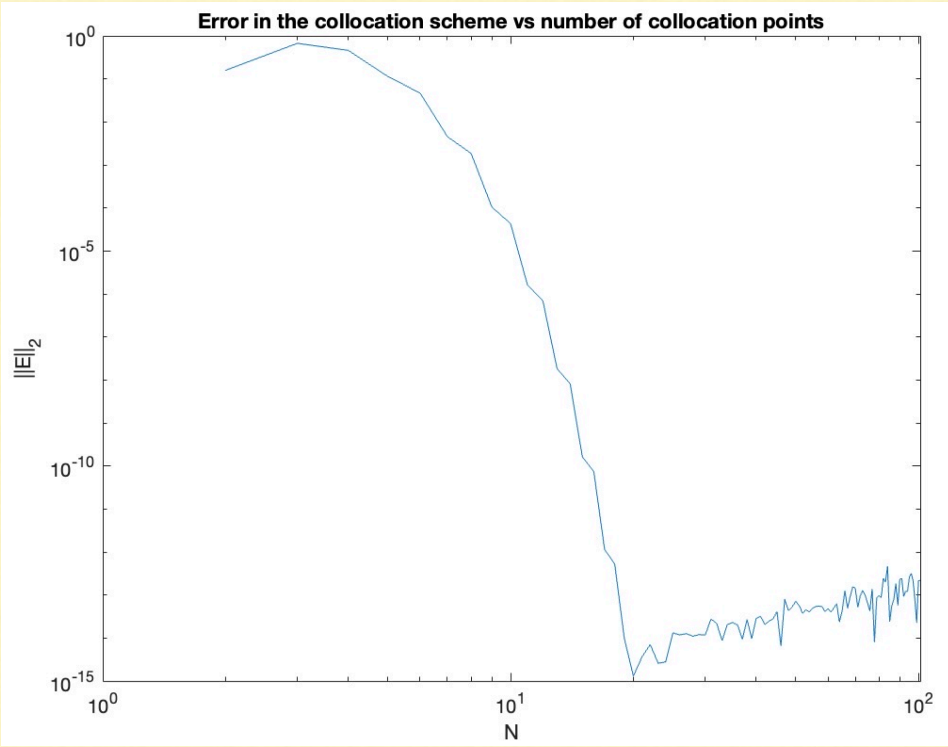


Q2 3 marks

Correct plot when running

Assignment_1_Q2_Assess.m

should look something like



Q3 3 marks

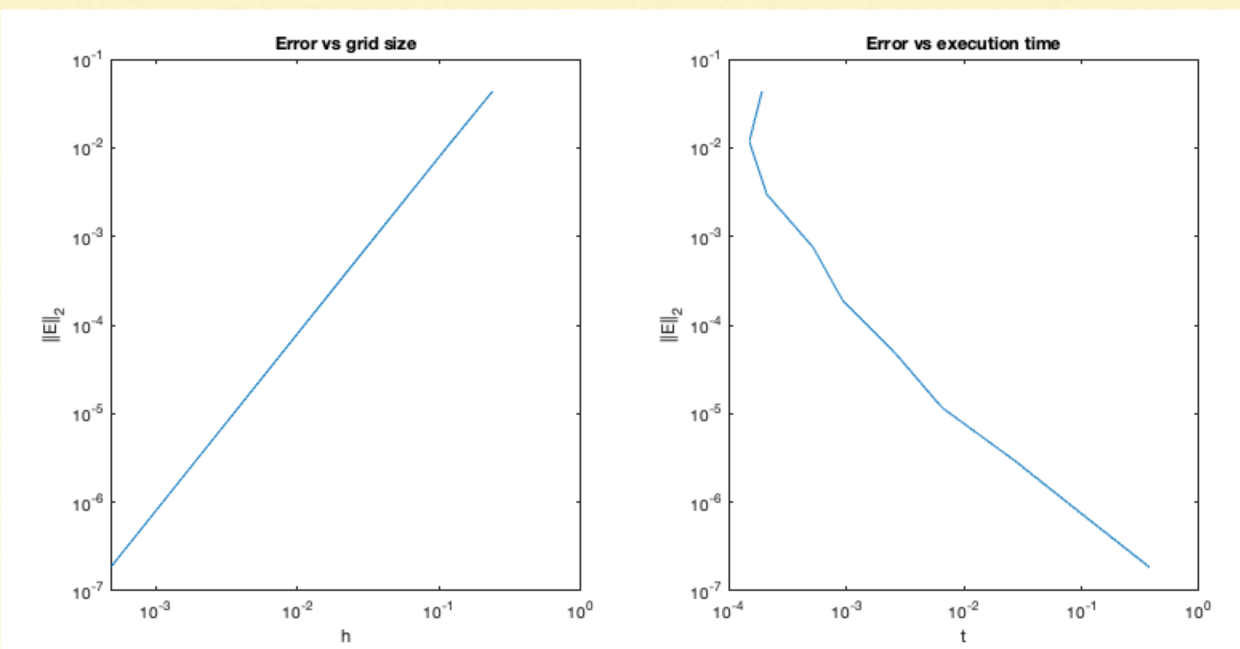
Because our coefficient functions are only defined at the x-values in node, we should use the trapezoidal rule to calculate all integrals, i.e.

$$\int_{x_i}^{x_{i+1}} g(x)\,dx = \frac{1}{2h_i}\left(g(x_i) + g(x_{i+1})\right)$$

where $h_i = x_{i+1} - x_i$.

Correct plot when running Assignment_1_Q3_Asses.m

Plot should look something like



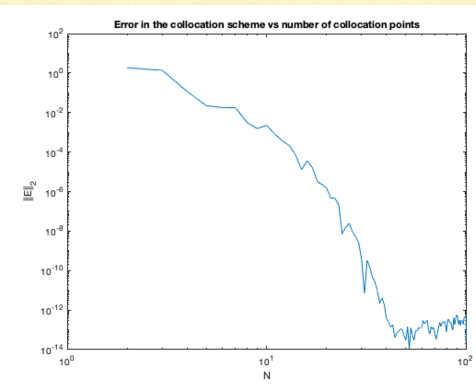
Error vs execution time for your interest only. It is not graded.

Q4 4 marks

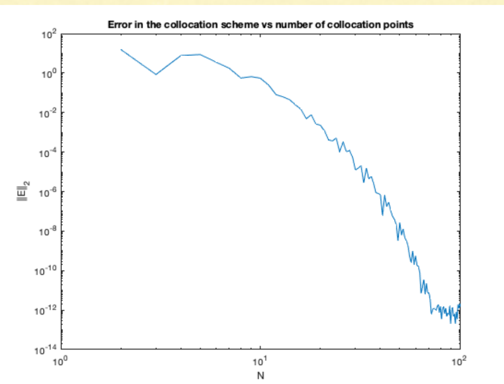
There is a lot of freedom in how you approach this question and marks are awarded based on how interesting and comprehensive your investigation is. Below is what I found.

First, using the collocation method

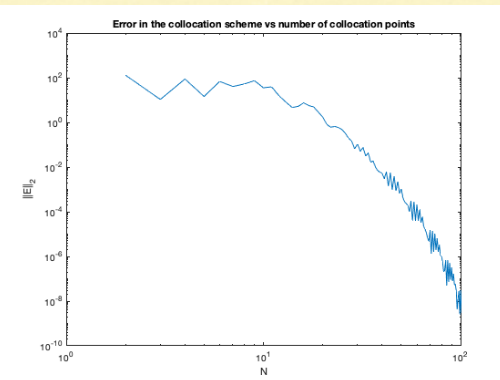
$p = 1$



$p = 0.1$



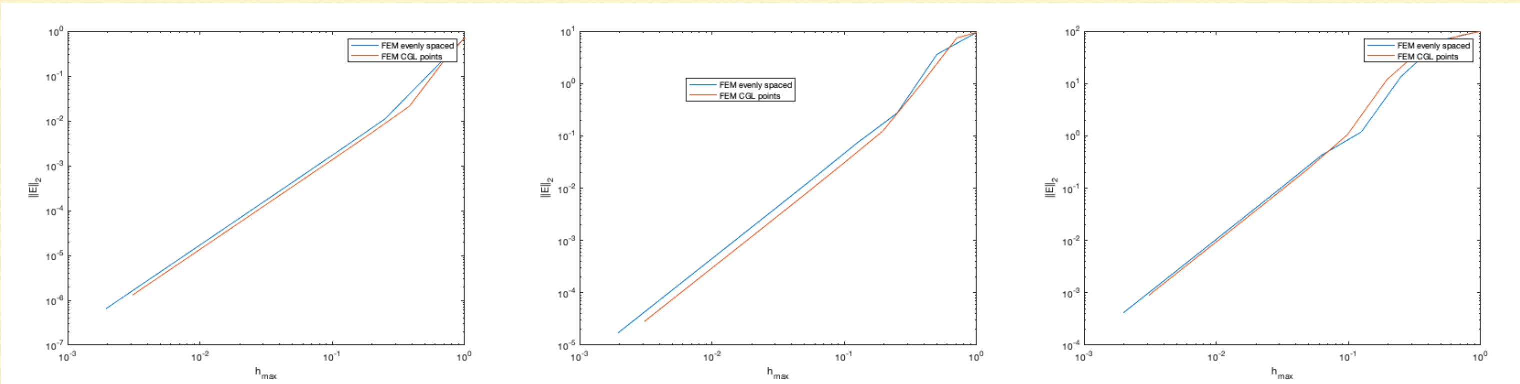
$p = 0.01$



We see that as p becomes small we need a larger number of collocation points to calculate the solution accurately. This makes sense as the solution becomes sharper and so we need higher order polynomials to approximate the exact solution

Now using FEM

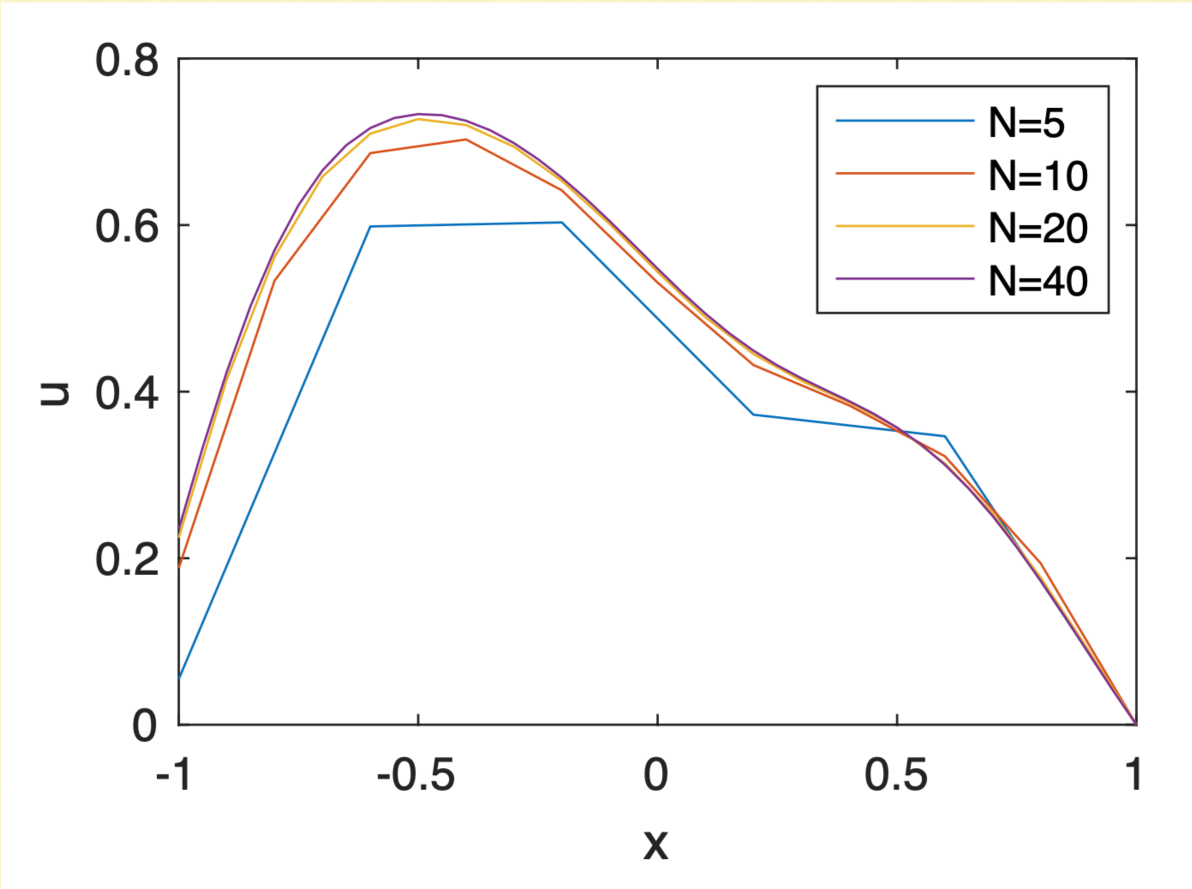
$P=1$ $\qquad\qquad$ $P=0.1$ $\qquad\qquad$ $P=0.01$



Overall there is very little difference between the two kinds of discretisation when using FEM. At the smallest value of p when the discretisation is small there is a slight advantage to using evenly spaced points. This is likely because CGL concentrates points near the edge of the domain but this is were the true solution varies the slowest and so is not where we need additional points.

Q5 2 marks

Here's what I get. Looks like it's converging



Q6 5 marks (2 for a, 2 for code in b, 1 for discussion of b)

a) let $x = L\tilde{x}$

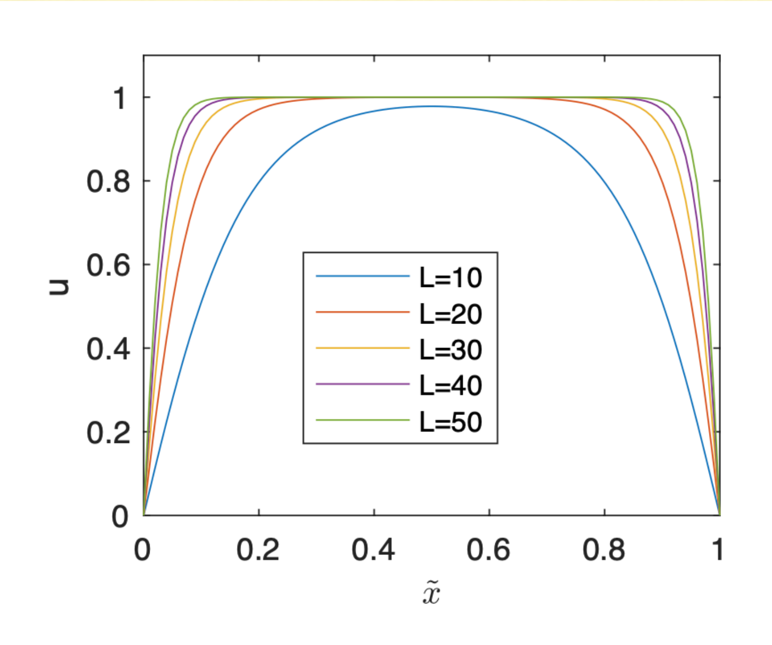
$$\Rightarrow \quad \frac{du}{dx} = \frac{d\tilde{x}}{dx}\frac{du}{d\tilde{x}} = \frac{1}{L}\frac{du}{d\tilde{x}}$$

Similarly $\dfrac{d^2u}{dx^2} = \dfrac{1}{L^2}\dfrac{d^2u}{d\tilde{x}^2}$

So we have

$$-\frac{d^2u}{d\tilde{x}^2} + L^2 u(u-1) = 0 \qquad u(0) = u(1) = 0$$

b)



As L increases, a boundary layer develops around $\tilde{x} = 0$ & 1. See MAST90064 for how to solve this in the limit $L \to \infty$