School of Mathematics and Statistics
MAST90026 Computational Differential Equations
2024

Homework 4
Due: 11:00AM Wednesday, 27th March.

This homework is worth 5% of the total assessment in this subject. You should submit copies of MATLAB programs (include all files necessary for the programs to run) and sufficient relevant output online through LMS (You may find the Matlab command publish useful!). Any hand written working should be scanned and converted to a PDF.

All files should be compressed into a single zip file *with your student ID number in the file name.*

1. For the case $D(x) = 1, q(x) = 0$ and $\{\phi_j\}$ are the hat functions on a non uniform mesh, show that the stiffness matrix $\mathbf{K}$ takes the form

$$\begin{aligned} K_{ii} &= \frac{1}{h_i} + \frac{1}{h_{i-1}}, \\ K_{i,i-1} = K_{i-1,i} &= -\frac{1}{h_{i-1}}, \\ K_{ij} &= 0, \quad |i-j| > 1, \end{aligned}$$

   i.e. $\mathbf{K}$ is tridiagonal and symmetric.

2. Write a MATLAB function `FEj=elem_load(f,xj,xjp1)` that computes, using Gaussian quadrature of order 3, the element load vector for linear elements

$$\mathbf{F}_{E_j} = \begin{bmatrix} \int_{x_j}^{x_{j+1}} f(x)\phi_j \, dx \\ \int_{x_j}^{x_{j+1}} f(x)\phi_{j+1} \, dx \end{bmatrix}.$$

   You should submit your code. (Hint: the Gaussian quadrature on $[0,1]$ of order 3 is: $(x_0, \omega_0) = (-\frac{1}{2}\sqrt{\frac{3}{5}} + \frac{1}{2}, \frac{5}{18}), (x_1, \omega_1) = (\frac{1}{2}, \frac{8}{18}), (x_2, \omega_2) = (\frac{1}{2}\sqrt{\frac{3}{5}} + \frac{1}{2}, \frac{5}{18}))$

3. Solve the Problem

$$u'' - sin(u) = -1; \quad u(0) = 0; \; u(1) = 1,$$

   using a finite difference method and newton method. Submit your code and a plot of the solution.