School of Mathematics and Statistics
MAST90026 Computational Differential Equations
2026

Homework 1
Due: 11:59PM Friday, 13th March.

This homework is worth 5% of the total assessment in this subject. Submit your hand working and published MATLAB code* as a combined PDF file through Canvas.

*Use the `publish` command (or use the GUI) to run your script and save it as a PDF.

Late submission will not be marked and a grade of 0 will be awarded. If there are extenuating circumstances apply for an extension or special consideration (more information available in the assessment adjustments page in the student support module).

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1. Consider the matrix $A \in \mathbb{R}^{N\times N}$ which is obtained by using central finite difference method to discretise $u'' = f(x), u(0) = u(1) = 0$, i.e.

$$A = \frac{1}{h^2}\begin{bmatrix} -2 & 1 & & & & \\ 1 & -2 & 1 & & & \\ & & \ddots & \ddots & \ddots & \\ & & & 1 & -2 & 1 \\ & & & & 1 & -2 \end{bmatrix} \tag{1}$$

   (a) Denote the kth eigenvalue of A by λ^k and the corresponding eigenvalue vector is denoted by $\mathbf{u}^k$. Find λ^k by assuming $\mathbf{u}^k_j = \sin(\frac{k\pi j}{N+1})$, $j = 1, \ldots, N$;

   (b) Use the result in (a) to prove that $\|A^{-1}\|_2 = \frac{1}{\pi^2} + O(h^2)$.

2. Consider the finite difference scheme for the 1D steady state convection-diffusion equation

$$\epsilon u'' - u' = -1, \quad 0 < x < 1$$
$$u(0) = 1, \quad u(1) = 3$$

   (a) Verify the exact solution is

$$u(x) = 1 + x + \left(\frac{e^{x/\epsilon} - 1}{e^{1/\epsilon} - 1}\right)$$

   (b) Compare the following two finite difference methods for $\epsilon = 0.3, 0.1, 0.05$, and 0.0005.

   (i) Central finite difference scheme:

$$\epsilon\frac{U_{i-1} - 2U_i + U_{i+1}}{h^2} - \frac{U_{i+1} - U_{i-1}}{2h} = -1$$

(ii) Central-upwind finite difference scheme:

$$\epsilon \frac{U_{i-1} - 2U_i + U_{i+1}}{h^2} - \frac{U_i - U_{i-1}}{h} = -1$$

Do a grid refinement analysis for each case to determine the order of accuracy. Include plots of $\|\boldsymbol{E}\|_2$ vs h.

(c) From your observations, in your opinion which method is better?