Q1.

a)

$$-(D(x)u')' + q(x)u = f(x)$$

$$-D'u' - Du'' + qu = f$$

$$u'' + \frac{D'}{D}u' - \frac{q}{D}u = -\frac{f}{D}$$

$$u'' + \tilde{p}u' + \tilde{q}u = \tilde{f}$$

So $\frac{D'}{D} = \tilde{p}$, $\frac{-q}{D} = \tilde{q}$, $-\frac{f}{D} = \tilde{f}$

$$\frac{D'}{D} = \tilde{p} \Rightarrow \left(De^{-\int p(x)dx}\right)' = 0$$

So $D(x) = Ce^{\int_a^x p(t)dt}$

$$q(x) = Ce^{\int_a^x p(t)dt}\tilde{q}(x)$$

$$f(x) = Ce^{\int_a^x p(t)dt}\tilde{f}(x)$$

(b) In this question, see the Q1. in the zip file. As 1.

The matlab file to selfAdjointForm.m is the code.

Q2.

In this question, see the Q2 in the zip file As1. the BvpSpec matlab file is the code needed. The file cheb is used to generate corresponding matrix D and node x.

Tips: First run cheb file
then run BvpSpec. file.

Q3.

In this question, see the Q3 in zip file As1. the matlab file BvpFe is needed.

Q4.

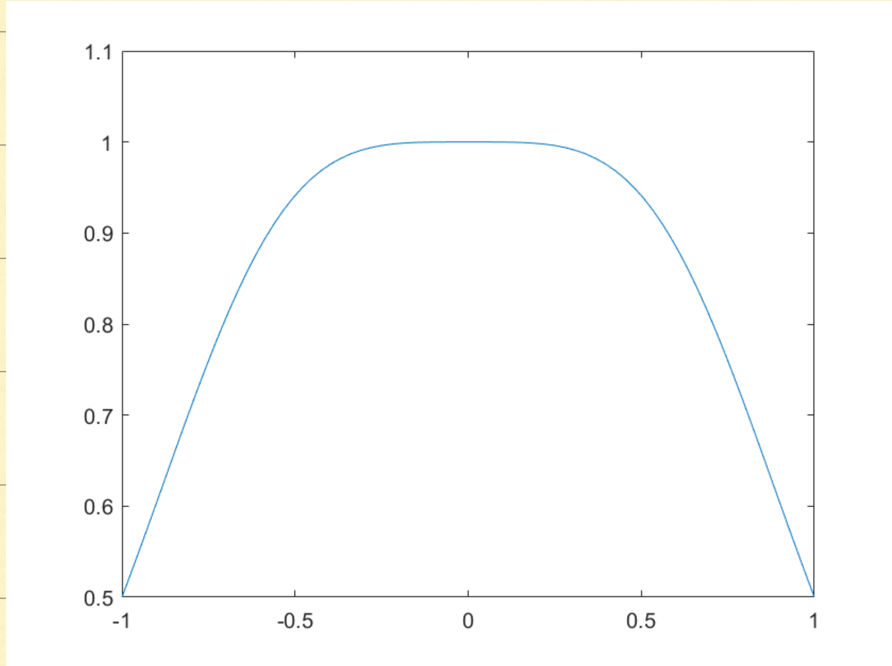
In this problem. see Q4 in 22p As1.

① file BvpSpec and BvpFe are same with Q2, Q3.

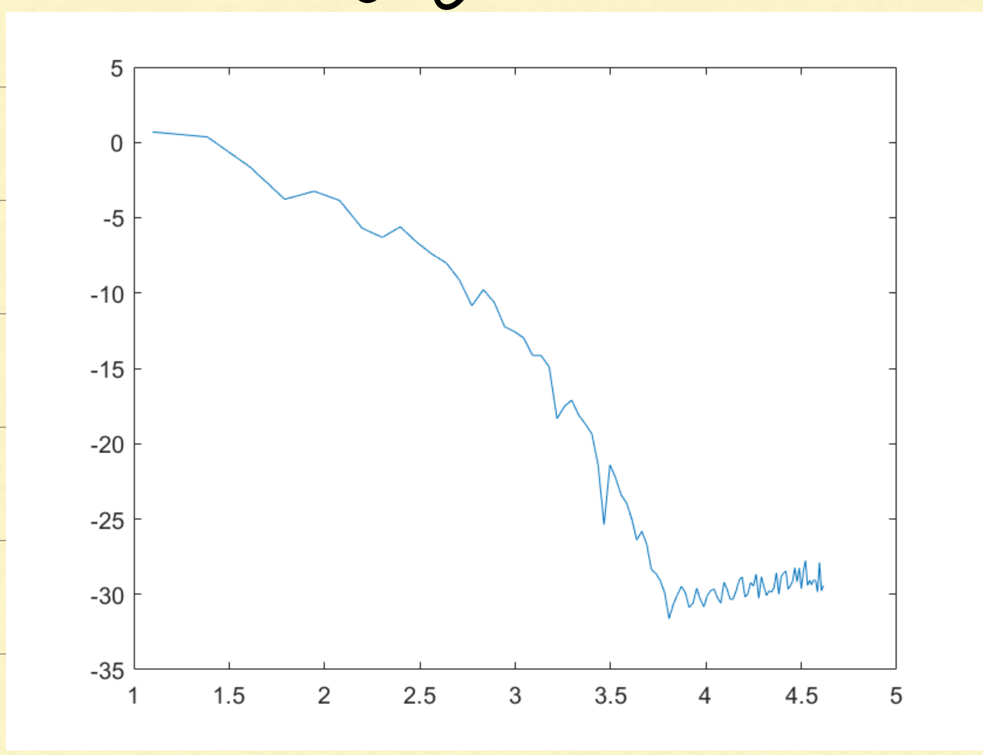
② test94 and test94spec files are used to solve this problem.

1. For Spectral collection using Chebyshev-Gauss-Lobatto points

① when $p=1$, in test94spec file set $p=1$, $N=101$. we can get x and solution U, shown in figure.



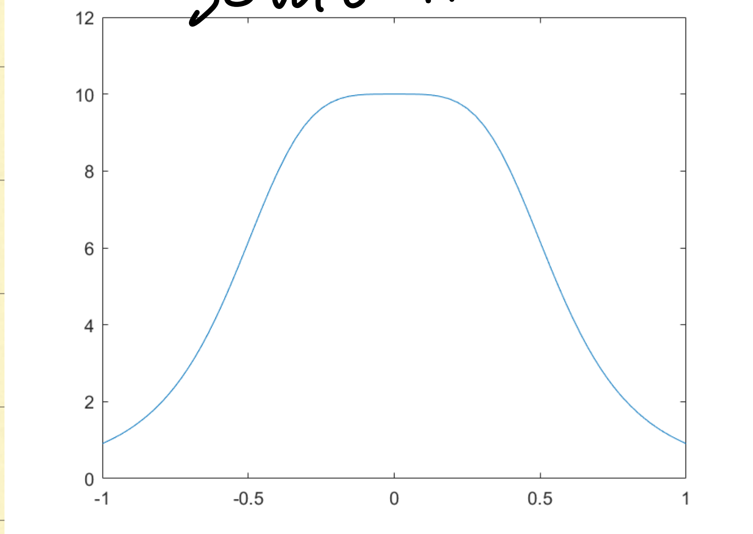
Run ERR file set N=101 and p=1, get the error and N. see figure.

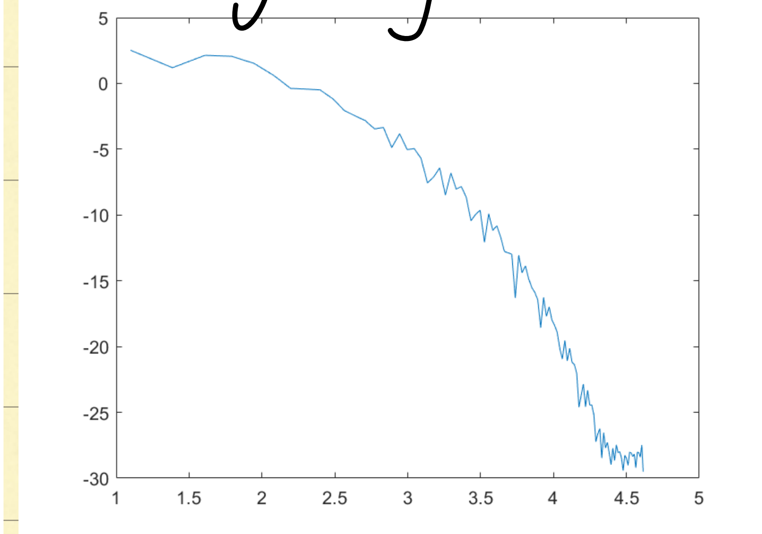


② Similarly, for $P = 0.1$, choose $N=101$, the solution and loglog plot are.

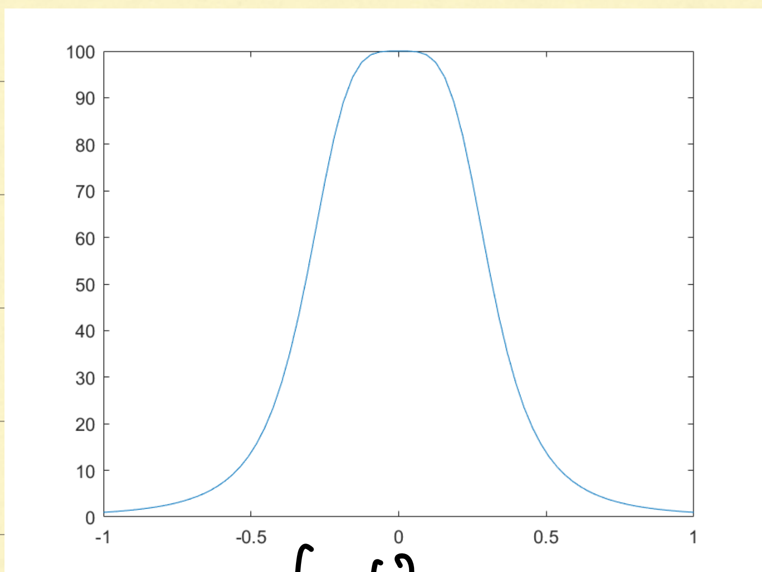
Solution

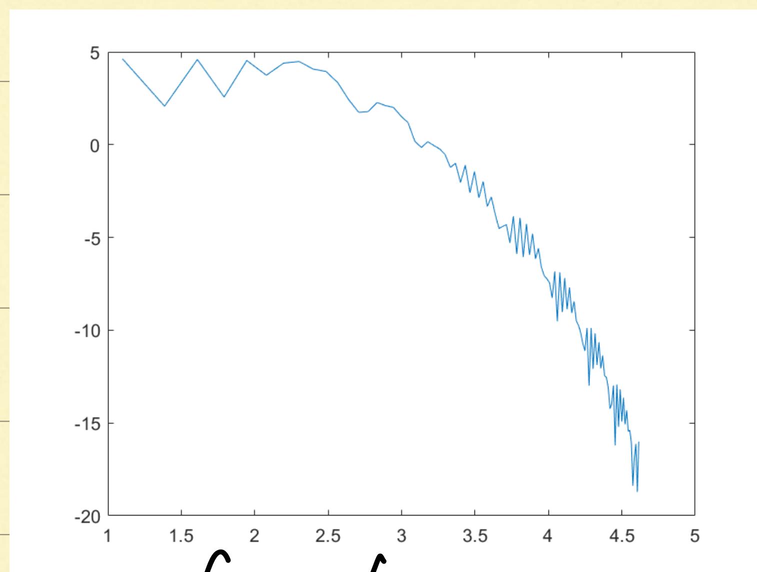
Log-log.





③ For $P = 0.01$, $N=101$, we have





Solution

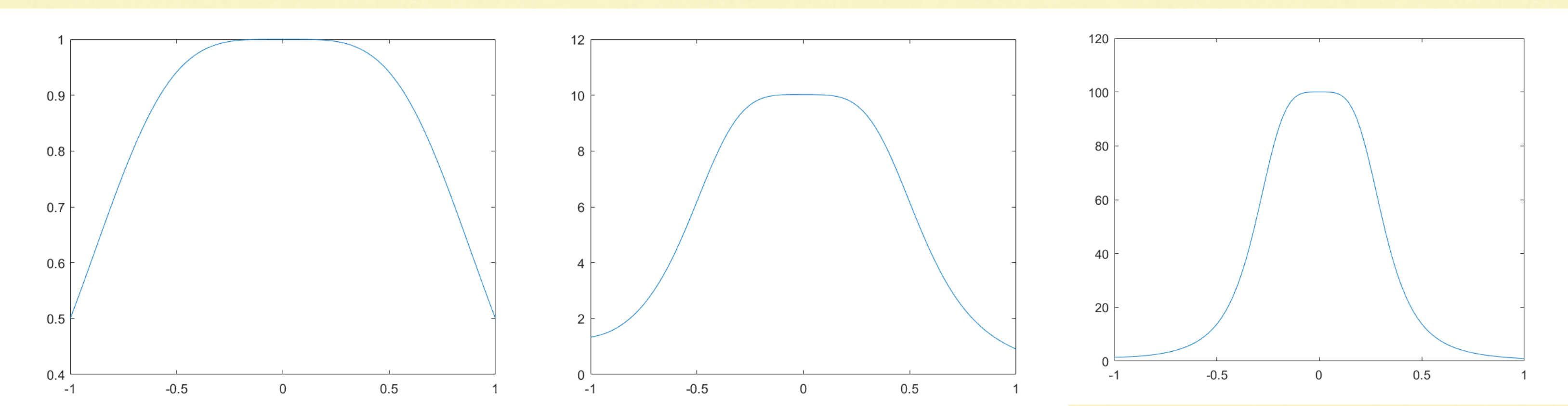
Log Log

2. For finite element method with equal space.

① In test94 file, set $N=100$, $P=0.01, 0.1, 1$, will get solution of u.

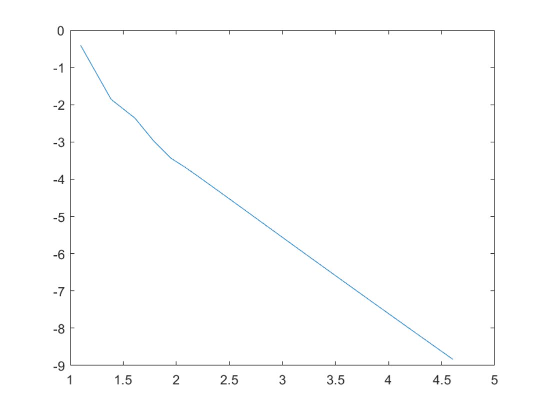
Run Loglogg file you will have error and N Log-log plot. See below:

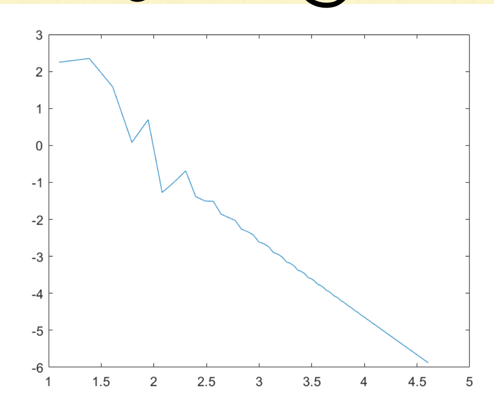
For the solution

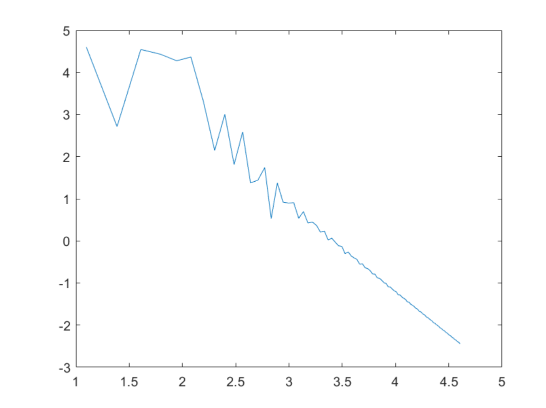


$P=1$ $P=0.1$ $P=0.01$

For the Log-Log plot.







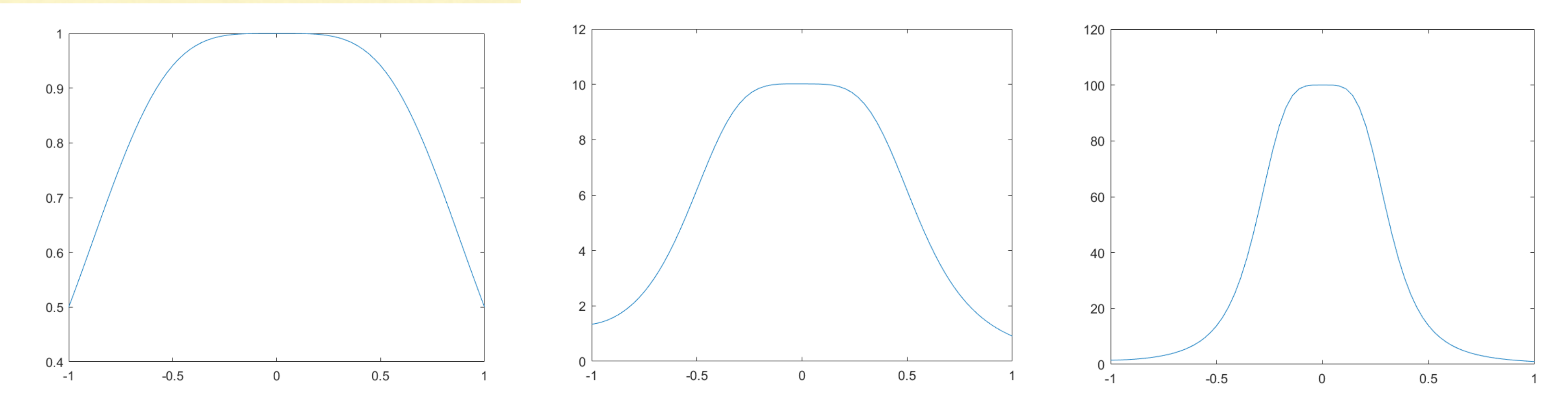
$P=1$ $P=0.1$ $P=0.01$.

3. For the finite element use Chebyshev-Gauss-Lobatto points.

In . test94 file replace $x = \text{linespace}[-1, 1, N]$ with cheb(N),

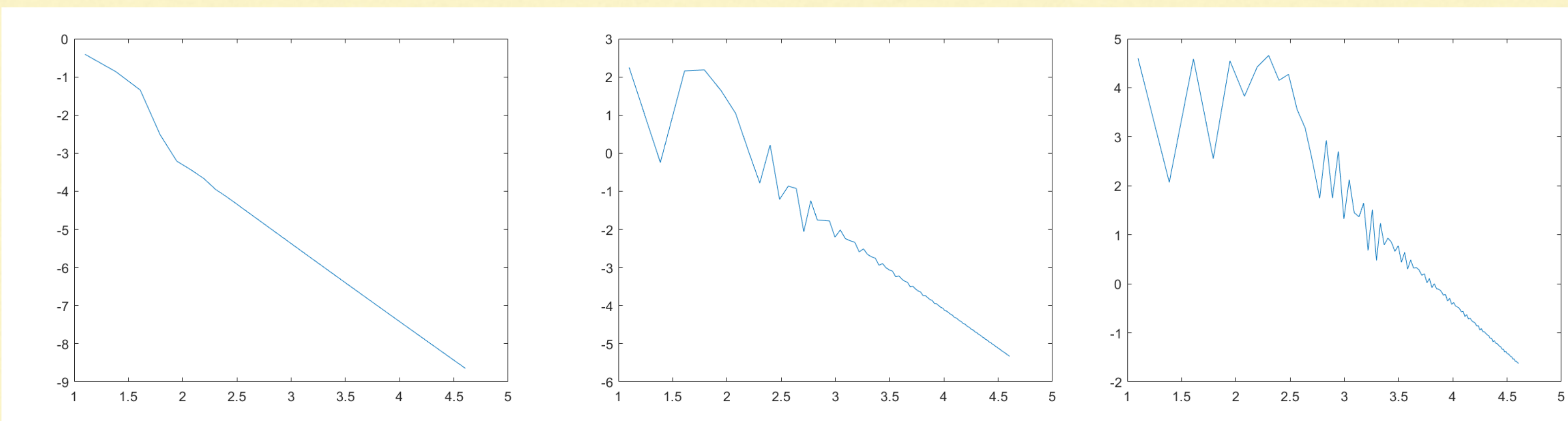
Also, flip(x) is needed! but for linespace it does not need!

After that, run testq4 with $N=100$, $\varepsilon=1, 0.1, 0.01$, solution is.



$P=1$ $P=0.1$ $P=0.01$

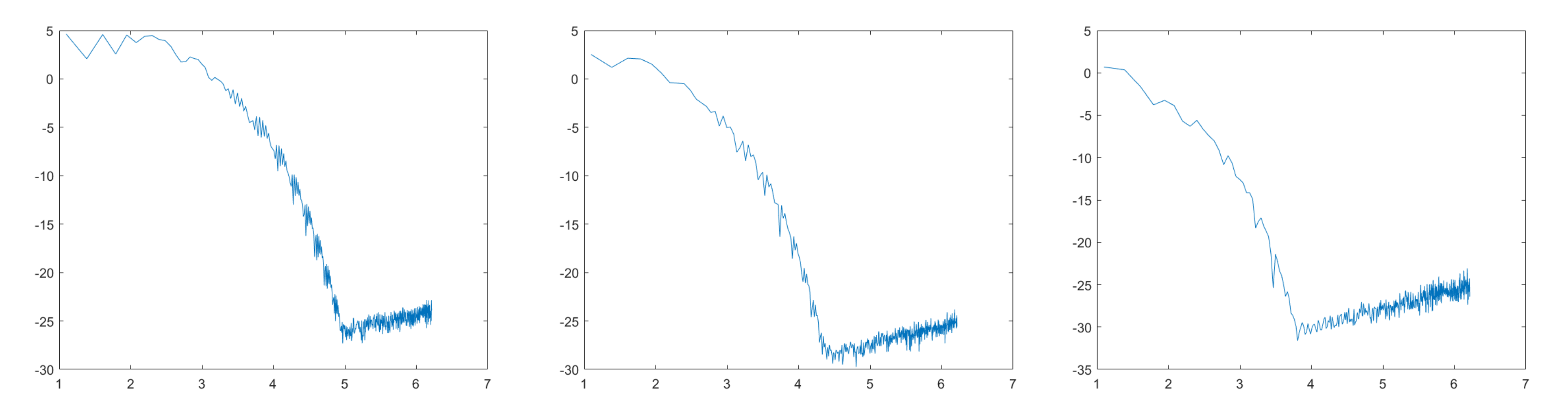
Next, Run, loglogg2 for $N=100$, $P=1, 0.1, 0.01$, we have.



$P=1$ $P=0.1$ $P=0.01$

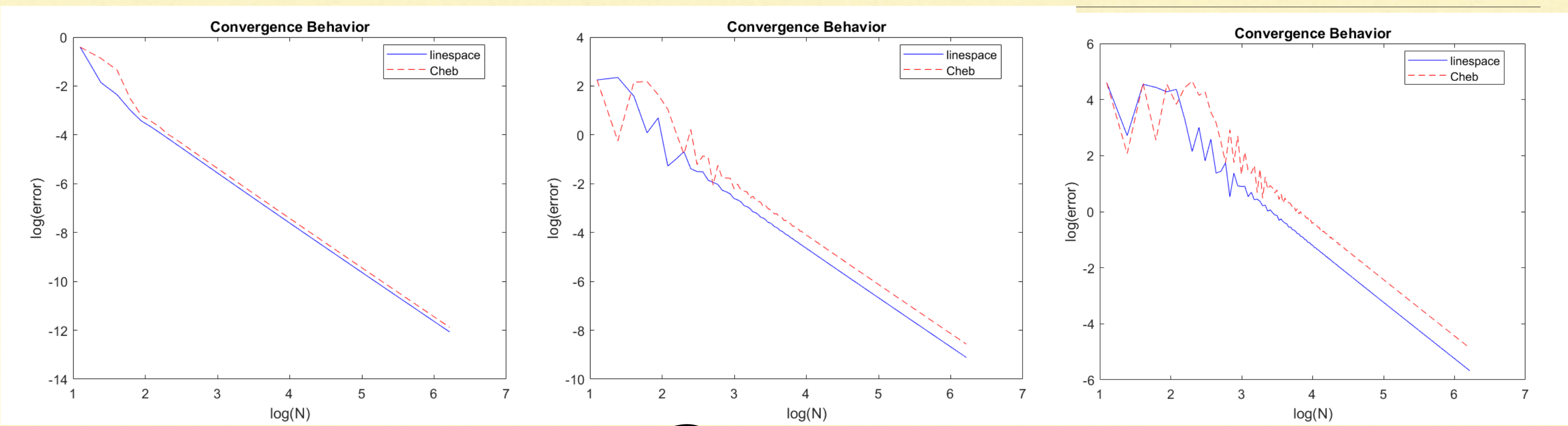
For the error computation,
The output of ERR, loglogg
and loglogg2 will give you
the error from $N=3$ to 101
for spectral method
and error from $N=3$ to 100
for finite element method.

4. Finally, for the spectral
method and finit element method,
for chebyshev-Gauss-Lobatto points,
Let $N=500$ and $P=1, 0.1, 0.01$
from the log-log plot.



P=1 P=0.1 P= 0.01

Spectral method



P=1 P=0.1 P=0.01

Finite element method.

We can find.

① For spectral method, the accurancy first increase as N increase, after

Some N, the accuracy decrease.

For finite element method, both two type of points, the accurancy increase as N increase. Also Spectral accurancy is higher than finite element method.

② We find in finite element method, the linespace with equal has higher accurancy than chebyshef points for fixed N.

Q5.

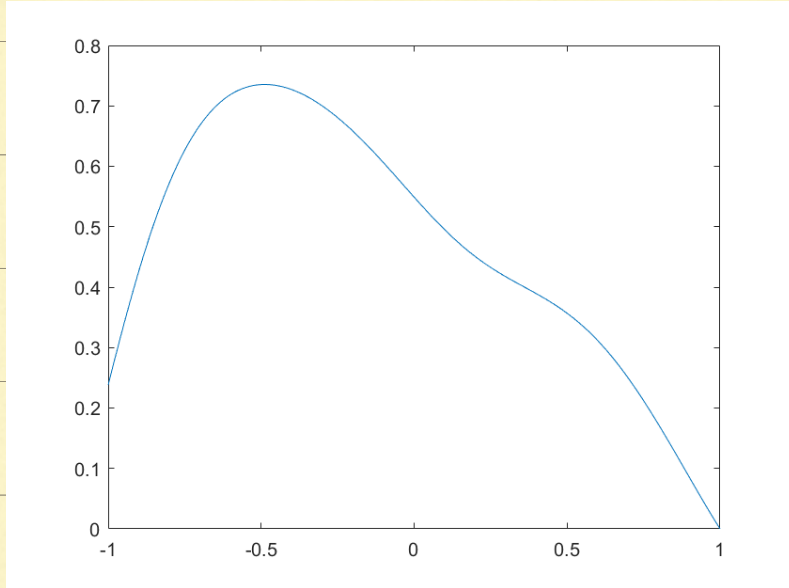
In this question, q5 test file is used to obtain solution, Assignment_1_Q5_BVP_Functions is used to obtain pt, qt, ft at given node x, to selfAdjoint is used to compute p, q, f at given x based on pt, qt, ft.

Finally, BvpFE is used to solve the BVP.

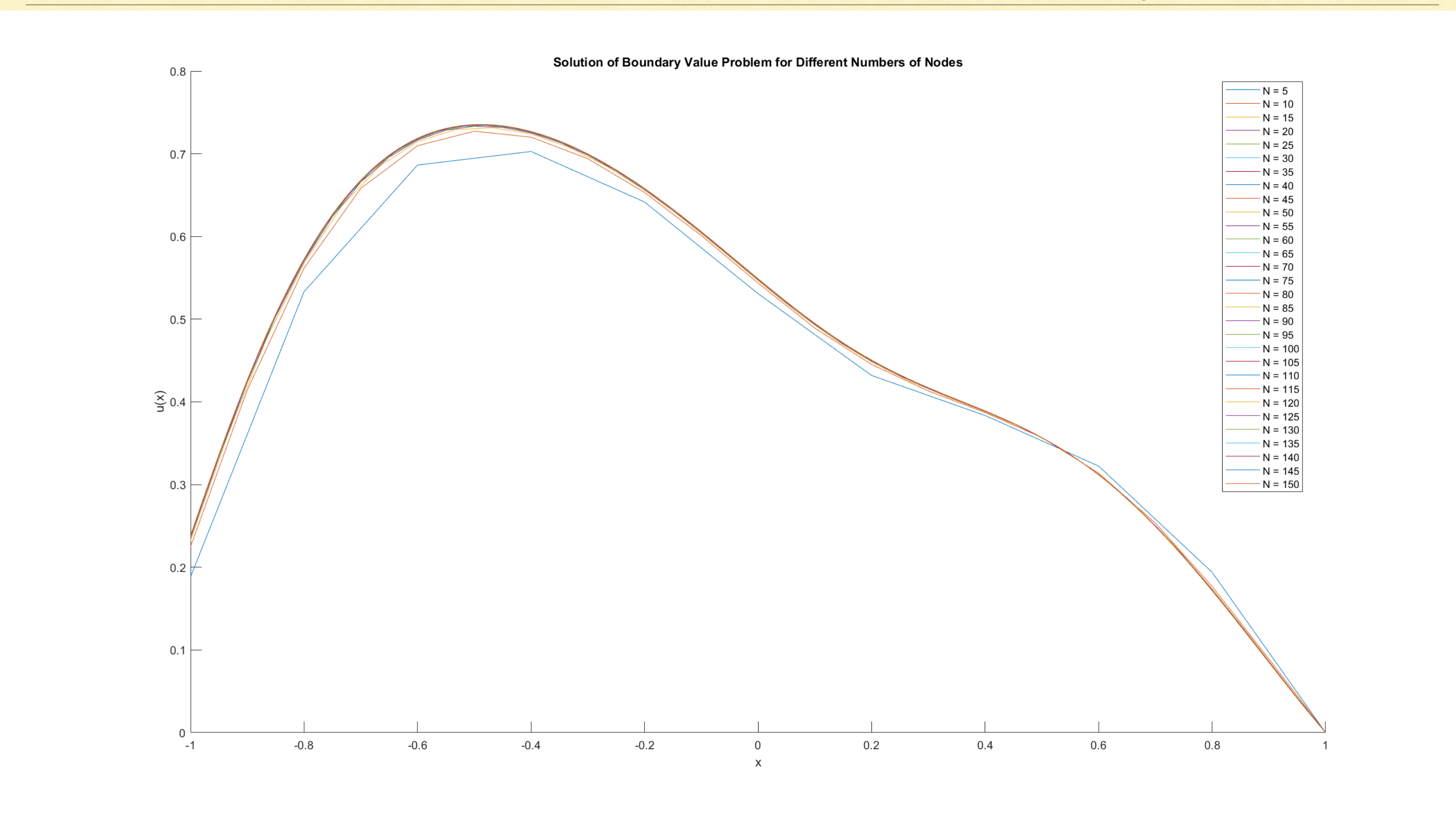
If choose $x = -1:0.01:1$

Run q5-test, we get the

Solution of u in figure



Also, to verify the convergence, we use graph to express this, for different N, the solution is following



From the figure, we can find the solution converges as N increase.

Q6.

(a) Let $t = \frac{x}{L}$, then we have

$t \in [0, 1]$

$$\frac{du}{dx} = \frac{du}{dt} \cdot \frac{dt}{dx} = \frac{1}{L} \frac{du}{dt}$$

$$\frac{d^2u}{dx^2} = \frac{1}{L} \frac{dt}{dx} \frac{d}{dt}\left(\frac{du}{dt}\right) = \frac{1}{L^2} \frac{d^2u}{dt^2}$$

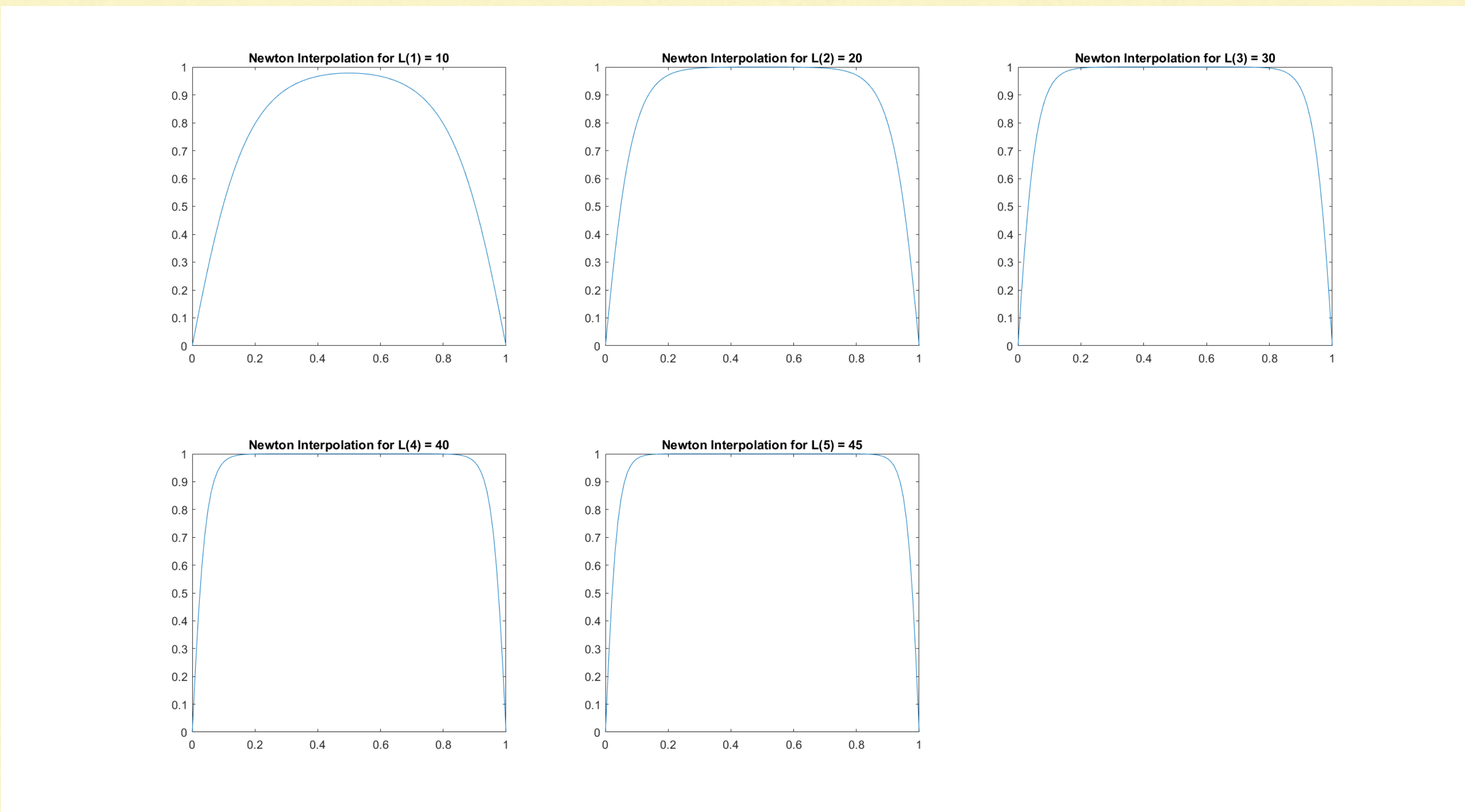
So the BVP now is

$$\frac{1}{L^2} u''(t) + u(u-1) = 0$$

$$u(0) = u(1) = 0.$$

(b) For solving this non-linear BVP, the file Newton in Q6, is used to solve this.

For the initial guess, we set $u_0(t) \equiv 1$. Next, take $N=100$, for different value of $L = 10, 20, 30, 40, 45$ we plot the solution.



From the figure we can find when L increase, the solution

$u(x)$ tends to $u(x) \equiv 1, x \neq 0, 1$

but for the boundary point

$u(0) = u(1)$ still 0.