School of Mathematics and Statistics
MAST90026 Computational Differential Equations
2026

Homework 3
Due: 11:59PM Friday, 27th March.

This homework is worth 5% of the total assessment in this subject. Submit your hand working and published MATLAB code* as a combined PDF file through Canvas.

*Use the `publish` command (or use the GUI) to run your script and save it as a PDF.

Late submissions will not be marked and a grade of 0 will be awarded. If there are extenuating circumstances apply for an extension or special consideration (more information available in the assessment adjustments page in the student support module).

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1. Consider the following BVP:

$$-u''(x) + u(x) = f(x), \quad 0 < x < 1, \quad u(0) = u(1) = 0.$$

Derive the linear system of the equations for the finite element approximation

$$u_h = \sum_{j=1}^{3} \alpha_j \phi_j(x)$$

with the following information:

- $f(x) = 1$;
- the nodal points and the elements are indexed as

$$x_0 = 0, \quad x_1 = \frac{1}{4}, \quad x_2 = \frac{1}{2}, \quad x_3 = \frac{3}{4}, \quad x_4 = 1$$
$$E_1 = [x_0, x_1], \quad E_2 = [x_1, x_2], \quad E_3 = [x_2, x_3], \quad E_4 = [x_3, x_4]$$

- the basis functions are the hat functions

$$\phi_i(x_j) = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{otherwise} \end{cases}$$

- assemble the stiffness/mass matrix and the load vector element by element.

What do you find the linear system to be?

2. Derive the Galerkin equations (for a given basis $\{\phi_j\}$) for the case of mixed BCs

$$u(a) - u'(a) = \alpha, u(b) = \beta.$$

3. Write code to use finite elements with linear basis functions on uniform mesh to solve the constant coefficient Dirichlet BVP:

$$-u'' + qu = r; \;\; u(a) = \alpha, u(b) = \beta,$$

where $q, r, a, b, \alpha, \beta$ are constants.

Test your code on problem from Homework 2:

$$-u'' + u = 0; \;\; u(0) = 1, u(1) = \exp(1),$$

and plot the maximum grid error $\max |e_j|$ versus N as a log-log plot. What is the rate of convergence?

Submit the MATLAB code and the resulting error plot.