School of Mathematics and Statistics
MAST90026 Computational Differential Equations
2024

Homework 2
Due: 11:00AM Wednesday, 13th March.

This homework is worth 5% of the total assessment in this subject. You should submit copies of MATLAB programs (include all files necessary for the programs to run) and sufficient relevant output online through LMS (You may find the Matlab command publish useful!). Any hand written working should be scanned and converted to a PDF.

All files should be compressed into a single zip file *with your student ID number in the file name.*

1. Write a code to solve the mixed BC problem

$$u'' + pu' + qu = r;\ u(a) - u'(a) = \alpha,\ u(b) = \beta,$$

where $p, q, r, a, b, \alpha, \beta$ are constants.

Use

(i) a first order FD formula
(ii) a 2nd order method

to handle the Robin BC at $x = a$.

Test your code on problem

$$u'' - u = 0;\quad u(0) - u'(0) = 0, u(1) = \exp(1),$$

whose exact solution is $u(x) = \exp(x)$ and plot the maximum grid error $\max|e_j|$ versus N as a log-log plot, for each method.

2. Write code to use collocation at Legendre Gaussian Lobatto points to solve the constant coefficient Dirichlet BVP:

$$u'' + pu' + qu = r;\quad u(-1) = \alpha, u(1) = \beta,$$

where p, q, r, α, β are constants. You may use the file '`cheb.m`'.

   (a) Test your code on problem A:

$$u'' - u = 0;\quad u(-1) = 1, u(1) = 3,$$

   whose exact solution is $\hat{u}(x) = 2\cosh(x)/\cosh(1) + \sinh(x)/\sinh(1)$. Inspect the solution visually by plotting both the numerical and exact solution on the same axes.

   (b) Approximate the error using $\|E\|_2 = \sqrt{\int_{-1}^{1}(u - \hat{u})^2 dx}$ and an appropriate quadrature rule. Plot the error vs N.

   (b) What change would you have to make to handle the problem with $u(a) = \alpha, u(b) = \beta$.