School of Mathematics and Statistics
MAST90026 Computational Differential Equations
2024

Assignment 1: Boundary value problems
Due: 11AM Wednesday, 17th April.

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1. Self-adjoint form of BVPs.

(a) $-(D(x)u')' + q(x)u = f(x)$

$-D'u' - Du'' + q(x)u = f(x)$

$u'' + \frac{D'}{D}u' - \frac{q}{D}u = -\frac{f(x)}{D}$

$u'' + \tilde{p}(x)u' + \tilde{q}(x)u = \tilde{f}(x)$

$\frac{D'}{D} = \tilde{p}(x), \quad \frac{-q}{D} = \tilde{q}(x), \quad \frac{-f}{D} = \tilde{f}(x)$

$D' = \tilde{p}(x)D \qquad D' - \tilde{p}(x)D = 0$

$D'e^{-\int \tilde{p}(t)dt} - \tilde{p}(t)De^{-\int \tilde{p}(t)dt} = 0$

$(e^{-\int \tilde{p}(t)dt}D)' = e^{-\int \tilde{p}(t)dt}(-\tilde{p}(t))D + D'e^{-\int \tilde{p}(t)dt}$

$De^{-\int \tilde{p}(t)dt} = C \qquad \text{let } C = 1$

$D(x) = C \cdot e^{\int \tilde{p}(x)dx}$

$q(x) = -D\tilde{q}(x) = -C \cdot e^{\int \tilde{p}(x)dx}\,\tilde{q}(x)$

$f(x) = -\tilde{f}(x)D = -C \cdot e^{\int \tilde{p}(x)dx}\,\tilde{f}(x)$

(b) The file toselfAdjointForm.m with 'x', 'pt', 'qt', and 'ft' as inputs to compute the coefficients in the self-adjoint form of the ODE.

The file testq1.m to test the toselfAdjointForm function.

2. Coding the spectral collocation method

The file cheb.m computes the Chebyshev - Gauss - Lobatto points.

The file BvpSpecnew.m solve the BVP using spectral collocation at chebyshev points.

The file testq4spec.m use to test the defined function.

3. Coding the finite element method

The file BvpFEnew.m intended to solve BVP problem using the Finite Element Method with linear basis functions.

Te file testq4.m test the function BvpFEnew.

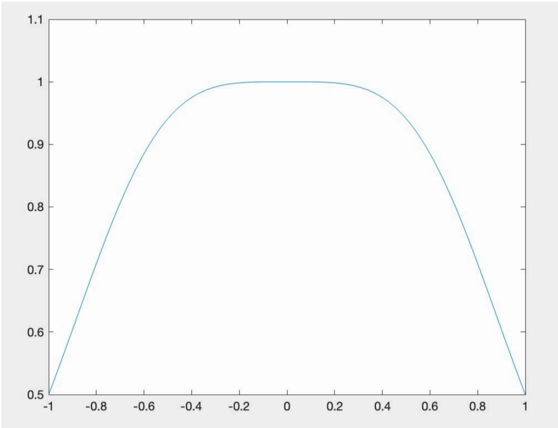
4. Linear BVPs

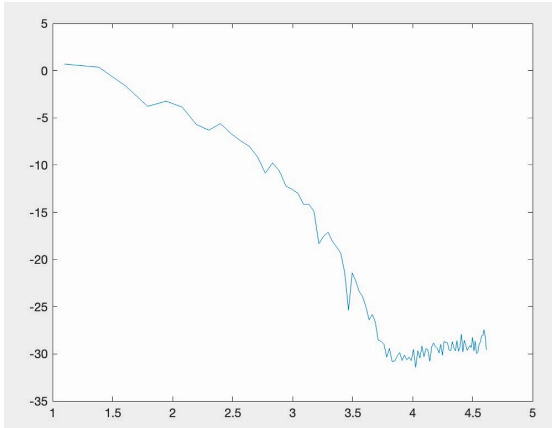
The file BvpSpec.m and BvpFE.m need to be used.

The file testq4.m and Testq4spec.m are used.

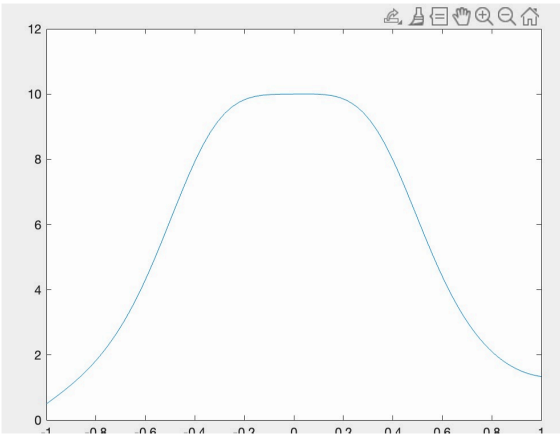
For spectral collection using chebyshev-Gauss Lobatto points

when $P=1$, $N=101$, run testq4spec.m   run ERR.m we get:

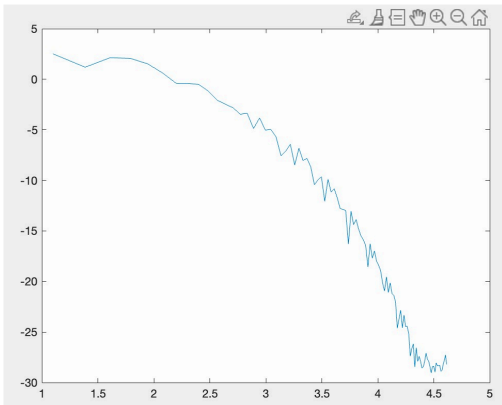




Now chang $p=0.1$ $N=101$ we get:

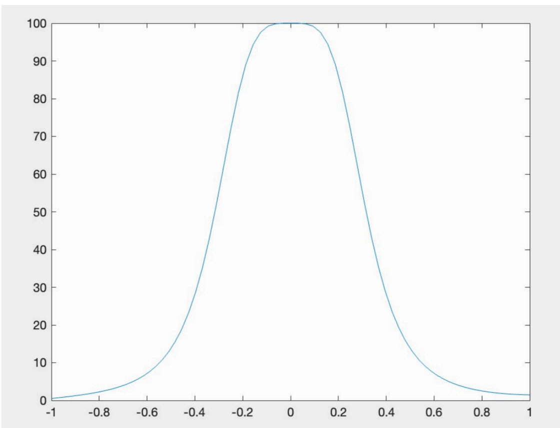


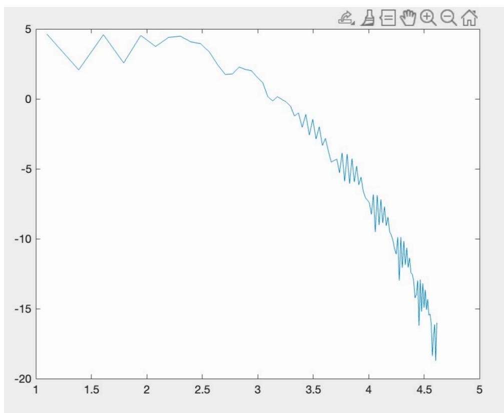
solution (testq4spee.m)



loglogg.

Now chang $p=0.01$ $N=101$ we get:



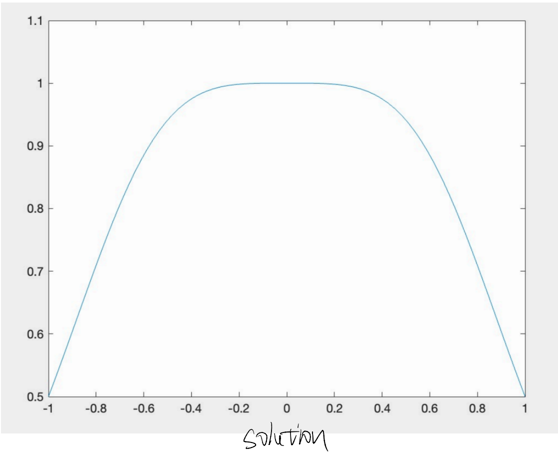


solution (testq4spee.m)

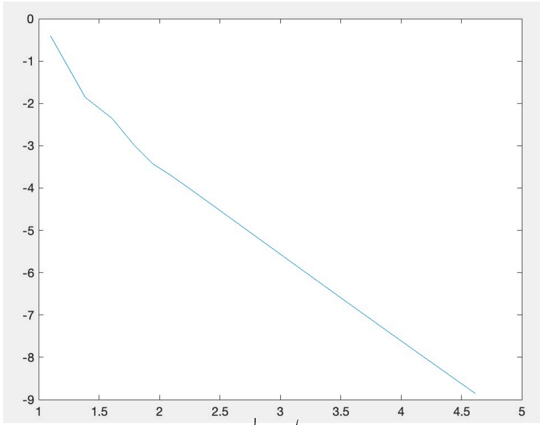
② For the finite element method with equal space.

set N=100 P=0.01, 0.1, 1 and run loglogg.m we get:

when P=1, N=101.

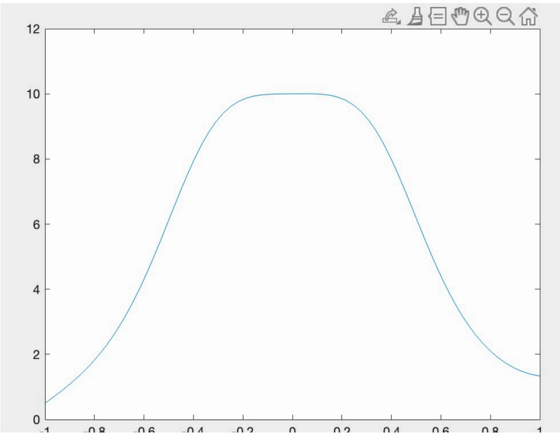


solution

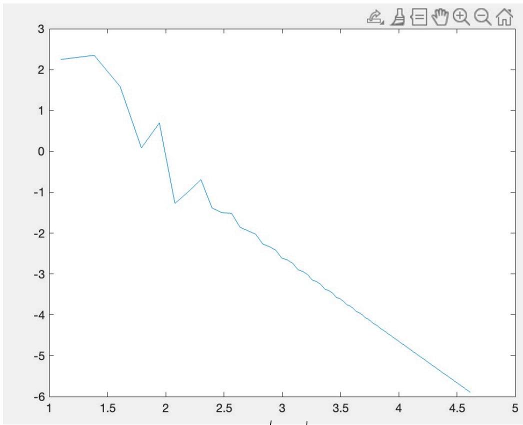


log-log

Now chang p = 0.1 N=101 we get:

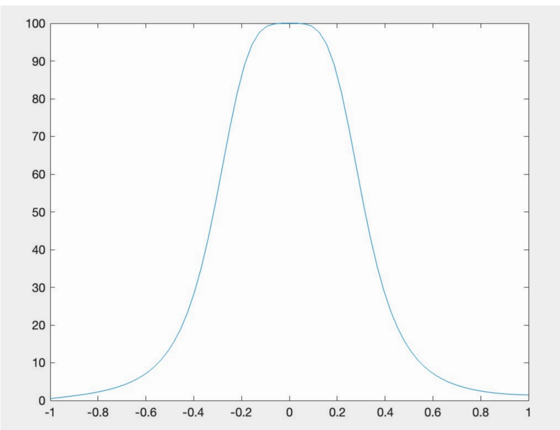


solution

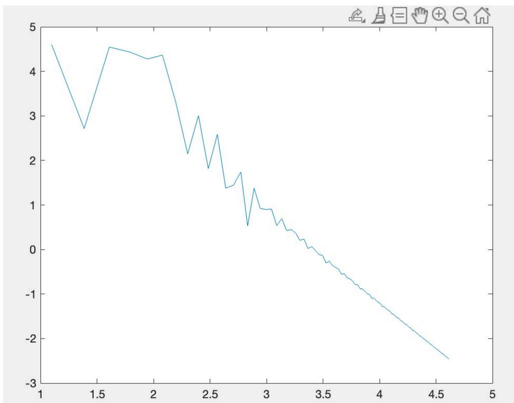


log-log

Now chang p = 0.01 N=101 we get:



solution

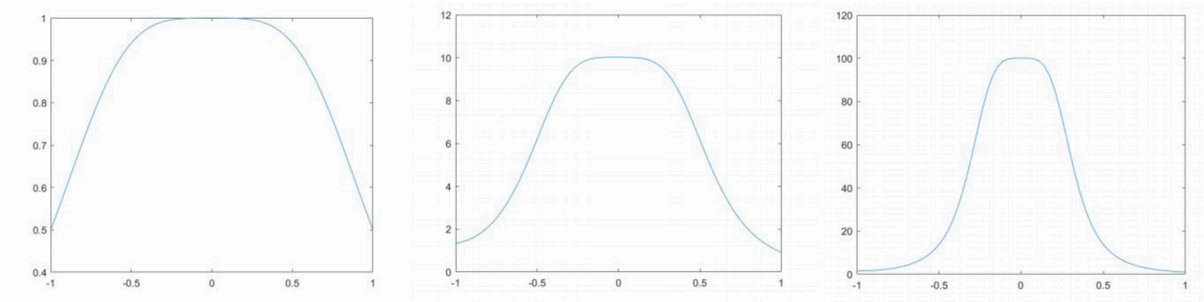


log-log

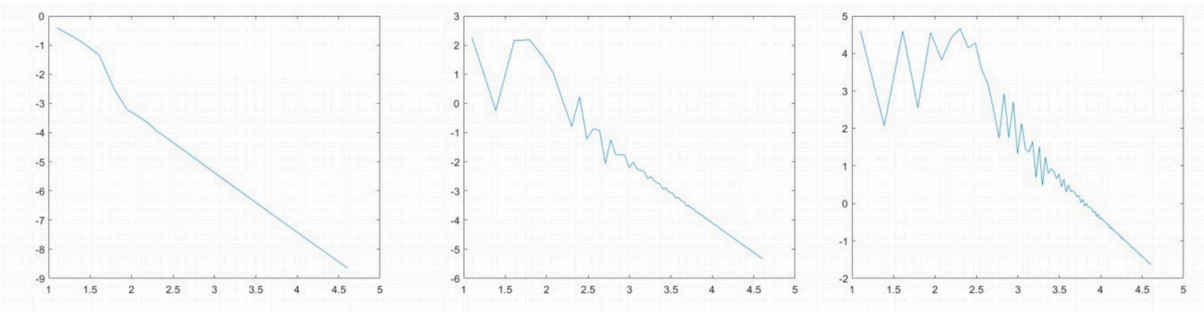
③ For the finite element use Chebyshev - Gauss - Lobatto points.

use testq4.m file with $x =$ linespace $[-1, 1, N]$ with cheb

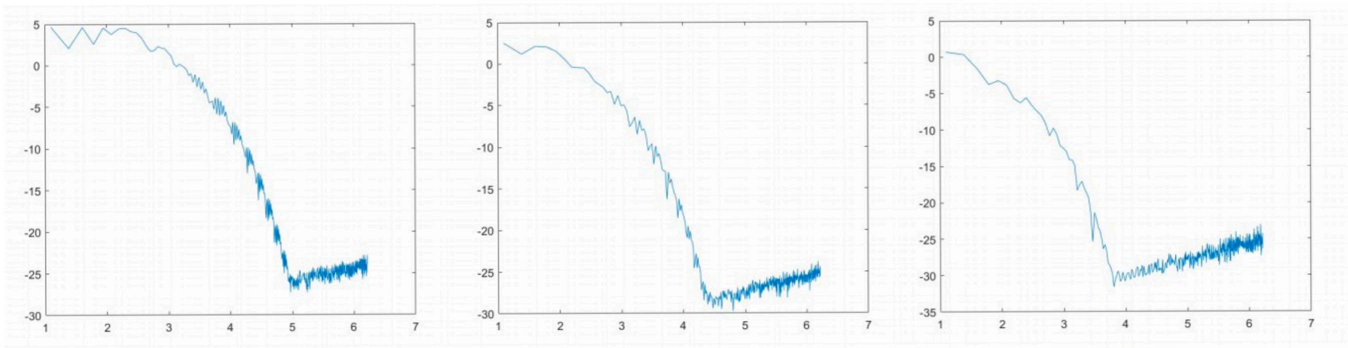
run testq4 and loglogq2.m with $N=100$ $q=1, 0.1, 0.01$, we get:



$p=1$ $p=0.1$ $p=0.01$



④ For the spectral method and finite element method, for Chebyshev - Gauss - Lobatto points.
Let $N=500$ $p=1, 0.1, 0.01$. we get the log-log plots:



$p=1$ $p=0.1$ $p=0.01$

we can find:

for spectral method, as N increase the accurancy increase, and after some N the accurany decrease.
for finite element method, as N increase the accurancy increase.
spectral accurancy is higher than finite element's.

5. Linear BVPs

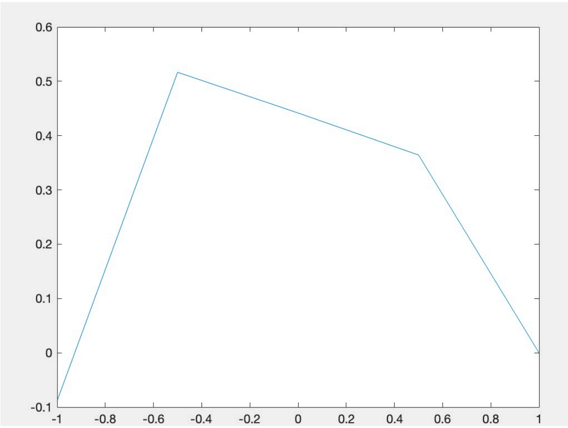
The file q5test.m use to obtain solution.

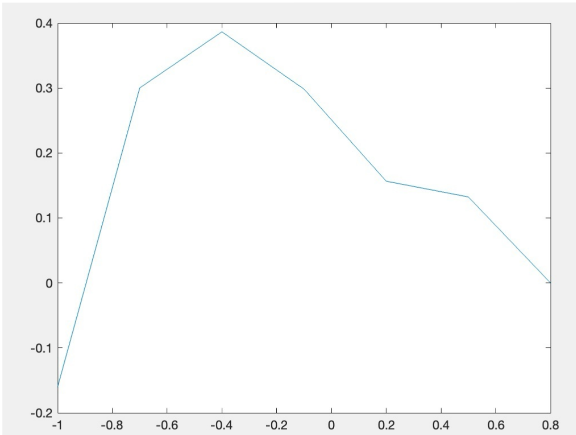
The file Assignment_1_Q5_BVP_Functions.m is used to obtain Pt, qt, ft at a given node x.

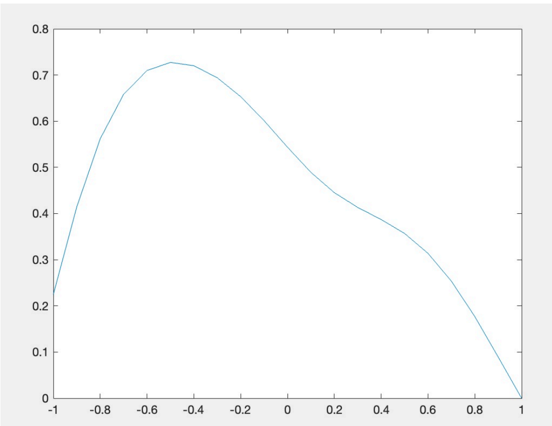
The file toselfAdjointForm.m is used to compute D, q, f at given x based on Pt, qt, ft.

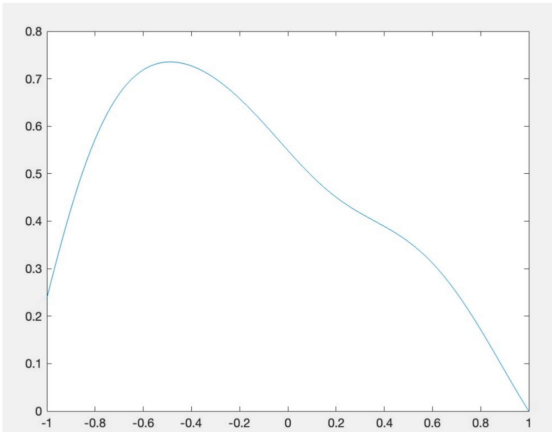
The file Bvp_FEnew.m is used to solve the BVP.

I try different N from N=5 to N=201 The solution of u:









we can see that, as N increase, the solution coverges.

6. Nonlinear BVP

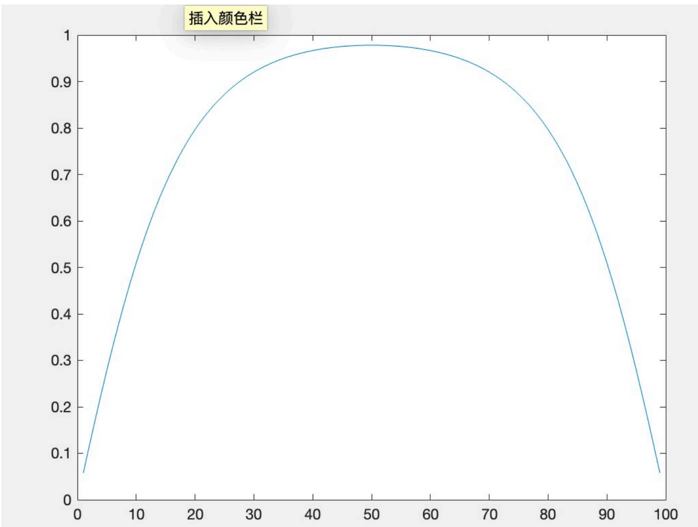
(a) Let $t=\frac{x}{L}$, then we have $t\in[0,1]$

$$\frac{du}{dx}=\frac{du}{dt}\cdot\frac{dt}{dx}=\frac{1}{L}\frac{du}{dt}$$

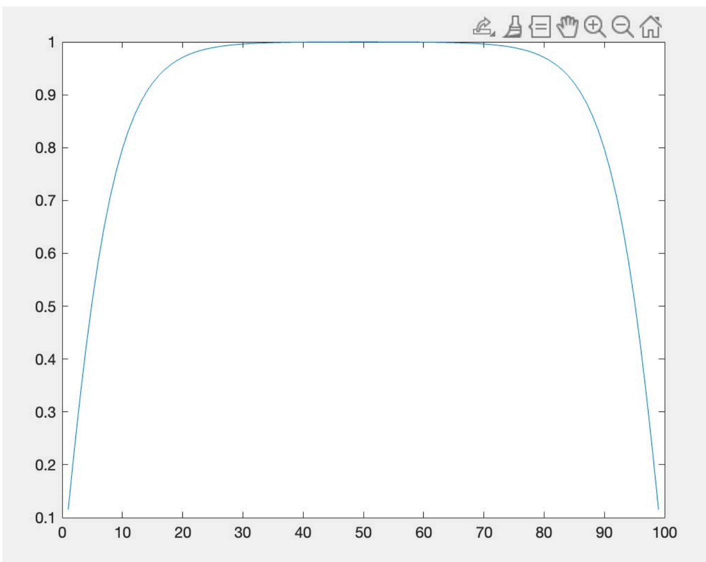
$$\frac{d^2u}{dx^2}=\frac{1}{L}\frac{dt}{dx}\frac{d}{dt}\left(\frac{du}{dt}\right)=\frac{1}{L^2}\frac{d^2u}{dt^2}$$

so the BVP is $-u''(t)+u(u-1)=0 \quad u(0)=u(1)=0$

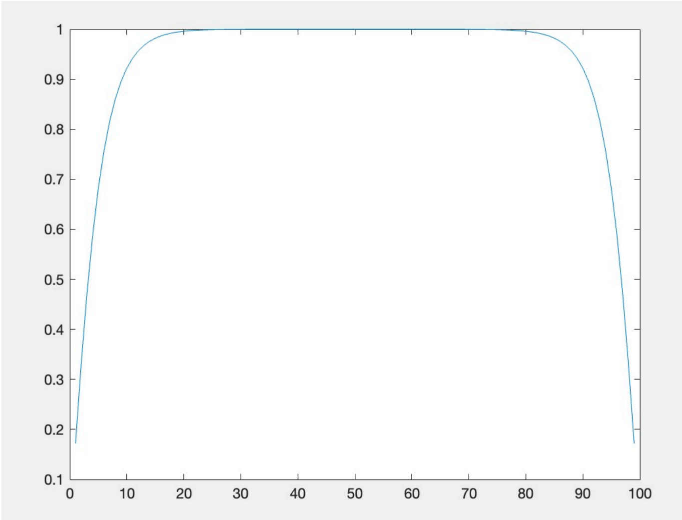
(b) The file Newton.m implement a numerical solution to a nonlinear BVP.
I take $N=100$. and different value of L. $L=10$. $L=20$, $L=30$, $L=40$, $L=50$.
And plot the solution:



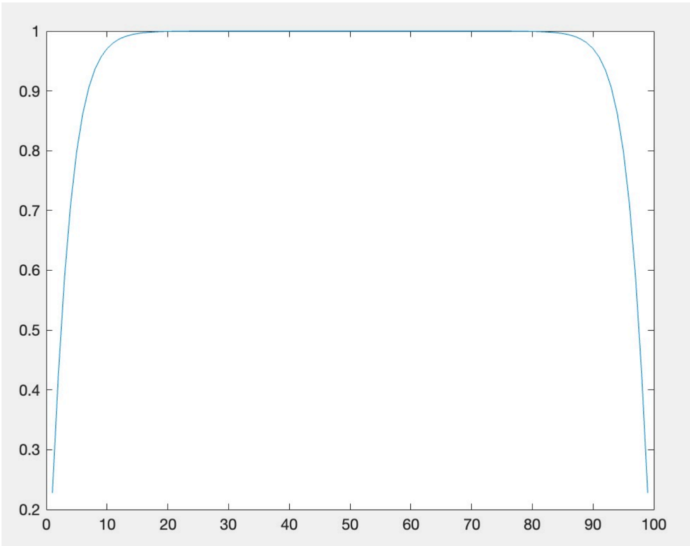
$L=10$



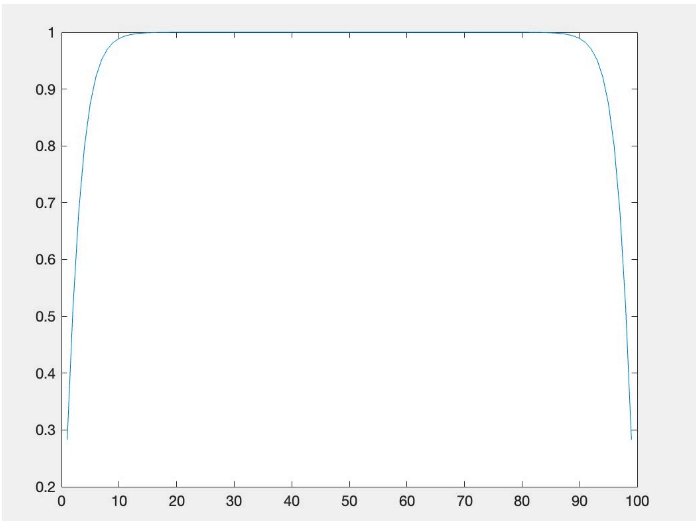
$L=20$



$L = 30$



$L = 40$



$L = 50$

As L increase from 10 to 50, we observe that:

The solution tends to $u(x)=1$, $x \neq 0,1$ and falls back to $u(x)=0$ at the other boundary.