School of Mathematics and Statistics
MAST90026 Computational Differential Equations
2024

Homework 3
Due: 11:00AM Wednesday, 20th March.

This homework is worth 5% of the total assessment in this subject. You should submit copies of MATLAB programs (include all files necessary for the programs to run) and sufficient relevant output online through LMS (You may find the Matlab command publish useful!). Any hand written working should be scanned and converted to a PDF.

All files should be compressed into a single zip file *with your student ID number in the file name.*

1. Consider the following BVP:

$$-u''(x) + u(x) = f(x), \quad 0 < x < 1, \quad u(0) = u(1) = 0.$$

Derive the linear system of the equations for the finite element approximation

$$u_h = \sum_{j=1}^{3} \alpha_j \phi_j(x)$$

with the following information:

- $f(x) = 1$;
- the nodal points and the elements are indexed as

$$x_0 = 0, \quad x_1 = \frac{1}{4}, \quad x_2 = \frac{1}{2}, \quad x_3 = \frac{3}{4}, \quad x_4 = 1$$
$$E_1 = [x_0, x_1], \quad E_2 = [x_1, x_2], \quad E_3 = [x_2, x_3], \quad E_4 = [x_3, x_4]$$

- the basis functions are the hat functions

$$\phi_i(x_j) = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{otherwise} \end{cases}$$

- assemble the stiffness/mass matrix and the load vector element by element.

What do you find the linear system to be?

2. Derive the Galerkin equations (for a given basis $\{\phi_j\}$) for the case of mixed BCs

$$u(a) - u'(a) = \alpha, u(b) = \beta.$$

3. Write code to use finite elements with linear basis functions on uniform mesh to solve the constant coefficient Dirichlet BVP:

$$-u'' + qu = r; \quad u(a) = \alpha, u(b) = \beta,$$

where $q, r, a, b, \alpha, \beta$ are constants.

Test your code on problem from Homework 2:

$$-u'' + u = 0; \quad u(0) = 1, u(1) = \exp(1),$$

and plot the maximum grid error $\max |e_j|$ versus N as a log-log plot. What is the rate of convergence?

Submit the MATLAB code and the resulting error plot.