Using $t \to bl^+\nu_l$ to Search for New Physics

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New physics (NP) is considered, contributing to the decay of $t \to bl^+\nu_l$. A calculation of the decay width from a semileptonic decay of the top quark is performed. From this, NP is considered by considering a separate particle similar to the W boson (W') taking it to be much heavier than the top quark. We calculate the partial rate asymmetry by considering top decays and anti-top decays. We show that the presence of NP can be seen through the measurement of the partial rate asymmetry.

I. INTRODUCTION

The top quark, the last quark to be discovered, was found in April 1995 at Fermilab [1]. This quark was predicted before its discovery from Glashow's symmetry due to the six quark and six lepton symmetry [2]. With only five quarks found up to the discovery, early searches were made but were unsuccessful. The difficulty of finding the top quark comes from the extraordinarily heavy mass alongside it being short-lived. The top quark was discovered through the collaboration of the CDF and D0 experiments. To find the top quark, proton and anti-proton collisions were evaluated at high energies revolving around 1800 GeV [1], finally completing the six-quark picture in the Standard Model.

The Standard Model (SM) is the theory of particles, fields, and the forces that govern their interactions [3]. The SM has explained experimental results and phenomena and has stood testing for over 50 years. The SM is one of the most well-tested theories, yet many believe the SM is not the end of the story. Physics beyond the SM is anticipated due to many issues that it does not address. An example to explain this comes from the quark and lepton masses. The SM, unlike other theories, does not provide us with a way to calculate quark and lepton masses. In the SM, quarks and leptons are assigned mass values experimentally. In total, there are over 20 parameters in the SM, leading many to think that this is not a final theory[2]. Other issues that the SM does not address are gravity, dark matter, and neutrino oscillations[4].

Since the SM's failure to address concepts, many postulate that there is physics beyond the SM. Gravity is a concept not pictured in the SM, but many believe that at the quantum level, there must be a carrier of the gravitational field. From this, it is proposed that a hypothetical carrier exists called the 'graviton' [4]. Going further, on April 7, 2022, the W boson was recorded to be approximately 70 MeV higher than what the SM predicts it to be [5]. A measurement of the mass this far off could infer that things are missing from the SM. In search of new physics, we consider the W', taking it to be much heavier than the W boson and top quark. Though W' is taken to be extremely heavy, the W' has a current limit. With standard coupling, the mass limit of W' is 6000 Gev [6]. Considering this new particle, we examine $t \to bl^+\nu_l$, a decay that involves the W, and introduce W'. We then calculate the partial rate asymmetry and measure top decays and anti-top decays. The expression we obtain for the partial rate asymmetry infers that there is physics beyond the SM since it does not equal zero. The remainder of this paper is organized as follows. In Sec. II we determine the Standard Model (SM) contributions to $t \to bl^+\nu_l$, and also consider NP contributions to this decay, taking the W boson much heavier than the top quark. After, we calculate the partial-rate asymmetry as we consider NP. In Sec. III we conclude with a discussion of our results. Appendices A and B contain technical details that are relevant to calculating SM and NP contributions to $t \to bl^+\nu_l$. Appendix A contains Feynman diagram rules that are used to calculate the amplitude for multiple decays. Appendix B assists in determining SM and NP contributions to $t \to bl^+\nu_l$ by calculating Γ_{total} for the W decay.

II. STANDARD MODEL AND NEW-PHYSICS CONTRIBUTIONS

In this section, we determine the SM contributions to $t \to bl^+\nu_l$, and observe NP contributions to this decay. Our NP analyzes another decay process of the top quark, taking the W boson much heavier than the top quark. After, we calculate the partial-rate asymmetry as we consider our NP.

1. Calculation of $\Gamma(t \to bW^+)$

To calculate the width for $t \to bW^+$, we begin by creating a Feynman diagram. Appendix A includes Feynman rules while Fig. (3) shows the Feynman diagram for $t \to bW^+$. The resulting amplitude is given by

$$\mathcal{M} = \frac{g_W \epsilon_{\mu*} V_{tb}^*}{\sqrt{2}} \left[\bar{u}_\nu \gamma^\mu P_L u_l\right], \tag{1}$$

where V is the Cabibbo–Kobayashi–Maskawa (CKM) matrix. To calculate the spin-averaged amplitude, we complex conjugate the amplitude in Eq. (1) and obtain

$$\mathcal{M}^* = \frac{g_W \epsilon_\nu V_{tb}}{\sqrt{2}} \left[\bar{u}_l \gamma^\nu P_L u_\nu\right]. \tag{2}$$

It is important to note that a new letter is used for the index of the gamma matrix since it got complex conjugated. Now, we proceed by multiplying Eq. (1) and Eq. (2) together to derive the spin-average amplitude. Upon multiplication, we add a summation for all the spin states with a factor of one-half (spin states of the top quark) and a summation for polarization and obtain,

$$\langle|\mathcal{M}|^2\rangle = \frac{g_W^2 \epsilon_{\mu*}\epsilon_\nu |V_{tb}|^2}{4} \times \sum_{\substack{spins\\polariz.}} \left[\bar{u}_\nu \gamma^\mu P_L u_l\right]\left[\bar{u}_l \gamma^\nu P_L u_\nu\right]. \tag{3}$$

Using Casimir's trick after summing over all spins, we must multiply the matrices and take the trace. It can be noted that the mass of the bottom quark can be neglected. With these simplifications, Eq. (3) is as follows,

$$\langle|\mathcal{M}|^2\rangle = \frac{g_W^2 \epsilon_{\mu*}\epsilon_\nu |V_{tb}|^2}{4} \times \sum_{polariz.} \mathrm{Tr}(\gamma^\mu P_L(\not{p}_l + m_l c)\gamma^\nu P_L \not{p}_\nu). \tag{4}$$

Now, $\epsilon_{\mu*}\epsilon_\nu$ with the polarization summation is given by Eq. (59) found in Appendix B. Using this identity yields

$$\langle|\mathcal{M}|^2\rangle = \frac{g_W^2 |V_{tb}|^2}{4}\left[-g_{\mu\nu} + \frac{p_{W\mu}p_{W\nu}}{m_W^2 c^2}\right] \times \mathrm{Tr}(\gamma^\mu P_L(\not{p}_l + m_l c)\gamma^\nu P_L \not{p}_\nu). \tag{5}$$

To get to the final expression of $\langle|\mathcal{M}|^2\rangle$ in terms of momentum four vectors, we must compute the trace. After this, we must multiply by the polarization term derived as the completeness relation. The final amplitude is given by,

$$\langle|\mathcal{M}|^2\rangle = \frac{g_W^2 |V_{tb}|^2}{2} \times \left[2p_l \cdot p_\nu + \frac{1}{m_W^2}\left(2p_l \cdot p_b p_\nu \cdot p_\nu - 2p_l \cdot p_\nu p_b^2\right)\right]. \tag{6}$$

Now, we may work out the kinematics of our decay system. According to the conservation of momentum, the following relation can be obtained, $p_t = p_b + p_W$. This allows us to derive the following relations,

$$p_l^2 = m_b^2 c^2 + 2p_\nu \cdot p_b, \tag{7}$$

$$p_l \cdot p_b = p_\nu \cdot p_b = \frac{(m_l c)^2 - (m_b c)^2}{2}, \tag{8}$$

and

$$p_l \cdot p_\nu = \frac{(m_l c)^2 + (m_b c)^2}{2}. \tag{9}$$

It is important to take notice that from now on, we will set $\hbar$ and c equivalent to 1. With the momentum relations being inserted into the amplitude equation, we have the following expression,

$$\langle|\mathcal{M}|^2\rangle = \frac{g_W^2 |V_{tb}|^2}{4m_W^2}(m_t^4 + m_W^2 m_t^2 - 2m_W^4). \tag{10}$$

Next, we can use the two-body decay formula for the width,

$$\Gamma = \frac{S|p|}{8\pi\hbar m_1^2 c}|\mathcal{M}|^2, \tag{11}$$

where 'S' is the statistical factor (which corrects for double-counting when there are identical particles in the final state) equivalent to 1. The magnitude of either outgoing momentum, $|p|$, can also be found utilizing the conservation of energy (in this case we choose $|p_\nu|$). Doing so obtains,

$$|p_\nu| = -\frac{m_W^2 - m_t^2}{2m_t}. \tag{12}$$

We choose to have the expression in terms of G_F instead of g_W, where $g_W^2 = \frac{8G_F m_W^2 c^4}{\sqrt{2}}$. It is traditional to express weak interaction formulas in terms of the Fermi coupling constant. Finally, inserting all the relations and expressions that we have derived earlier into Eq. (11) and simplifying, we obtain

$$\Gamma(t \to b + W^+) = \frac{|V_{tb}|^2 G_F}{8\sqrt{2}\pi} m_t^3 (1 - 3\zeta^4 + 2\zeta^6). \tag{13}$$

where $\zeta = \frac{m_W}{m_t}$.

2. Approximation of $\Gamma(t \to bl^+\nu_l)$

We can use the branching ratio from Appendix B to calculate $\Gamma(t \to bl^+\nu_l)$. To a good approximation,

$$\Gamma(t \to bl^+\nu_l) = \Gamma(t \to b + W^+) BR(W^+ \to l^+ + \nu_l). \tag{14}$$

Since we have calculated $\Gamma(t \to b + W^+)$ and have previously determined $BR(W^+ \to l^+ + \nu_l)$, found in Appendix B, we can use this as a shortcut to our final answer. Either way will yield the same answer. An approach we could take is evaluating the problem as a whole instead of solving separate pieces. It is up to preference which problem we could solve, but we will also show the method of calculating $\Gamma(t \to bl^+\nu_l)$. We will do this since the calculation of $\Gamma(t \to bl^+\nu_l)$ will provide us shortcuts when considering NP.

3. Calculation of $\Gamma(t \to bl^+\nu_l)$

To calculate $\Gamma(t \to bl^+\nu_l)$, we begin by creating a Feynman diagram to derive the amplitude. Appendix A provides Feynman rules while Fig, (7) shows the Feynman diagram for $t \to bl^+\nu_l$. We begin by setting up the expression for the amplitude where,

$$-i\mathcal{M} = \bar{u}_b \left(-\frac{ig_W V_{tb}^*}{\sqrt{2}}\gamma^\alpha P_L\right) u_t \bar{u}_\nu \left(-\frac{ig_W}{\sqrt{2}}\gamma^\beta P_L\right) v_l \times \left(\frac{-i(g_{\alpha\beta} - \frac{q_\alpha q_\beta}{m_W^2})}{q^2 - m_W^2 + i\Gamma_W m_W}\right) \tag{15}$$

in which $-\frac{q_\alpha q_\beta}{m_W^2}$ is approximately zero. Since q_β is contracted with a gamma matrix, we may use the Dirac equation to express it in terms of masses, which we are taking to be negligible. To carry out $|\mathcal{M}|^2$, we set

$$\mathcal{M} = \frac{-g_W^2 V_{tb}^*}{2(q^2 - m_W^2 + i\Gamma_W m_W)} \times \left[\bar{u}_b \gamma^\alpha P_L u_t \bar{u}_\nu \gamma_\alpha P_L v_l\right], \tag{16}$$

and

$$\mathcal{M}^* = \frac{-g_W^2 V_{tb}}{2(q^2 - m_W^2 - i\Gamma_W m_W)} \times \left[\bar{u}_t \gamma^\beta P_L u_b \bar{v}_l \gamma_\beta P_L u_\nu\right]. \tag{17}$$

We obtain $|\mathcal{M}|^2$ by multiplying Eq. (16) and Eq. (17) and averaging over the spin states and obtain,

$$\langle|\mathcal{M}|^2\rangle = \frac{g_W^4 |V_{tb}|^2}{8((q^2 - m_W^2)^2 + \Gamma_W^2 m_W^2)} \times \sum_{spins} \mathrm{Tr}(u_b \bar{u}_b \gamma^\alpha P_L u_t \bar{u}_t \gamma^\beta P_L) \times \mathrm{Tr}(u_\nu \bar{u}_\nu \gamma_\alpha P_L v_l \bar{v}_l \gamma_\beta P_L). \tag{18}$$

Next, we find $\langle|\mathcal{M}|^2\rangle$ by computing the trace and simplifying it, which yields

$$\langle|\mathcal{M}|^2\rangle = \frac{g_W^4 |V_{tb}|^2}{8((q^2 - m_W^2)^2 + \Gamma_W^2 m_W^2)} \times 16(p_l \cdot p_t)(p_\nu \cdot p_b). \tag{19}$$

Looking ahead, we must take an integral that is given by the Particle Data Group (PDG). Parts in Eq. (19) can be expressed in terms of q^2 and ρ^2. These are defined as follows:

$$\rho^2 = (p_\nu + p_b)^2 = (p_t - p_l)^2, \tag{20}$$

$$p_\nu \cdot p_b = \frac{1}{2}\rho^2, \tag{21}$$

and

$$p_t \cdot p_l = \frac{m_t^2 - \rho^2}{2}. \tag{22}$$

Using the relations of four vectors of momentum above, the final expression can be achieved in terms of q^2, ρ^2, and G_F. Doing so yields,

$$\langle|\mathcal{M}|^2\rangle = \frac{16G_F^4 |V_{tb}|^2 m_W^4}{((q^2 - m_W^2)^2 + \Gamma_W^2 m_W^2)} \rho^2(m_t^2 - \rho^2). \tag{23}$$

We now integrate Eq. (23) over ρ^2 and q^2. Substituting $\langle|\mathcal{M}|^2\rangle$ into the three-body decay formula, which is given by PDG, we obtain,

$$d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32m_t^3} \frac{16G_F^4 |V_{tb}|^2 m_W^4}{((q^2 - m_W^2)^2 + \Gamma_W^2 m_W^2)} \times \rho^2(m_t^2 - \rho^2) d\rho^2 dq^2. \tag{24}$$

To do the integral, we must figure out the bounds of integration. q^2 goes from 0 to m_t^2 and ρ^2 can be found by looking at the region of integration in Fig. ?. Integral over ρ^2 is a simple one. However, for integrating over q^2 we realize that $\frac{1}{((q^2 - m_W^2)^2 + \Gamma_W^2 m_W^2)}$ is approximately a delta function with a normalization constant. Once the normalization constant, A is found, conveniently, we can truncate the integration.

$$\int_{-\infty}^{\infty} \frac{dq^2}{((q^2 - m_W^2)^2 + \Gamma_W^2 m_W^2)} = A \int_{-\infty}^{\infty} \delta(q^2 - m_W^2) dq^2. \tag{25}$$

$$A = \frac{\pi}{\Gamma_W m_W}. \tag{26}$$

Inserting the normalization constant, A, and a delta function into Eq. (24) yields,

$$d\Gamma = \int_0^{m_t^2} \frac{1}{(2\pi)^3} \frac{1}{32m_t^3} 16G_F^4 |V_{tb}|^2 m_W^4 \times \left(\frac{m_t^6}{6} - \frac{m_t^2 q^4}{2} + \frac{q^6}{3}\right)\left(\frac{\pi}{\Gamma_W m_W}\right) \delta(q^2 - m_W^2) dq^2, \tag{27}$$

which can be simplified. The final is found using the delta function that truncates the integration and yields,

$$\Gamma = \left(\frac{\pi}{\Gamma_W m_W}\right)\left(\frac{1}{(2\pi)^3}\right)\left(\frac{1}{32m_t^3}\right) \times 16G_F^4 |V_{tb}|^2 m_W^4 \left(\frac{m_t^6}{6} - \frac{m_t^2 q^4}{2} + \frac{q^6}{3}\right). \tag{28}$$

The width for $t \to b + l^+ + \nu_l$ is approximately equal to the width for $t \to b + W$ times the branching ratio for $W \to l^+\nu_l$. Respectively, we obtain 0.164862 for $\Gamma(t \to b+l^+ +\nu_l)$ and 0.164859 for $\Gamma(t \to b+W)BR(W \to l^+\nu_l)$. Eventually, the full width for the top quark decay can be calculated by multiplying $\Gamma(t \to b+l^+ +\nu_l)$ by the inverse of the branching ratio.

$$\Gamma_t = 9 * 0.164862 = 1.48376 GeV. \tag{29}$$

In comparison to the reference value obtained from the PDG, the above value differs by 4.5% from the accepted value, which is 1.42GeV.

Additionally, we can integrate $\frac{d\Gamma}{dq^2}$ in Fig. (1) numerically for the SM case. The calculated area under the curve of $\frac{d\Gamma}{dq^2}$ is $0.159492 GeV$, and it differs by 3.26% from the the value that was calculated using a delta function. It can be noted that the narrow-width approximation is more accurate than the value in Eq. (29), which is $0.164862 GeV$ when multiplied by the branching ratio.

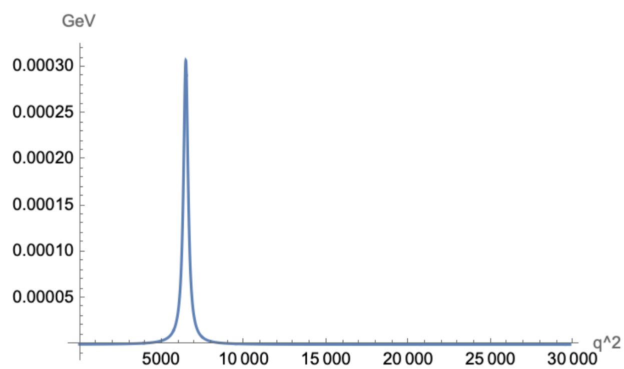

FIG. 1. $\frac{d\Gamma}{dq^2}$

4. Solving for New Physics With the W and W'

In this section, we calculate the total $\langle|\mathcal{M}|^2\rangle$ with the NP contributions of both W and W'. So far, we calculated the full width for $\Gamma(t \to bl^+\nu_l)$ with W. Setting up the amplitude and its complex conjugate for W', we can add $\mathcal{M}_W$ and $\mathcal{M}'_W$ together. Eventually, the $< \langle|\mathcal{M}|^2\rangle$ can be calculated for the top quark. To start the calculation we can create a Feynman diagram. Appendix A includes Feynman rules while Fig. (**??**) shows the Feynman diagram. Now, the mediating particle is W'. Following the Feynman rules, we obtain

$$\begin{aligned}-i\mathcal{M}_{W'} &= \bar{u}_b\left(-\frac{ig_W V_{tb}^*\beta e^{i\alpha}}{\sqrt{2}}\gamma^\mu P_L\right)u_t\bar{u}_\nu \\ &\times\left(-\frac{ig_W V_{tb}^*\beta e^{-i\alpha}}{\sqrt{2}}\gamma^\nu P_L\right)v_l\left(\frac{-i(g_{\alpha\beta}-\frac{q_\alpha q_\beta}{m_{W'}^2})}{q^2-m_{W'}^2+i\Gamma_{W'}m_{W'}}\right),\end{aligned} \tag{30}$$

where β is a real constant and $e^{i\alpha}$ and $e^{-i\alpha}$ are phase factors. Since $m_{W'}$ is much larger compared to other terms, $m_{W'}^2$ becomes a dominating term, and $-\frac{q_\alpha q_\beta}{m_{W'}^2}$ is approximately zero. From this, we obtain,

$$\mathcal{M}_{W'} = \frac{g_W^2\beta^2 V_{tb}^* e^{i\alpha}}{2m_{W'}^2}\left[\bar{u}_b\gamma^\mu P_L u_t\bar{u}_\nu\gamma_\mu P_L v_l\right], \tag{31}$$

and

$$\mathcal{M}_{W'}^* = \frac{g_W^2\beta^2 V_{tb} e^{-i\alpha}}{2m_{W'}^2}\left[\bar{u}_t\gamma^\nu P_L u_b\bar{v}_l\gamma_\nu P_L u_\nu\right]. \tag{32}$$

Now, we have $\mathcal{M} = \mathcal{M}_W + \mathcal{M}_{W'}$, which it follows that

$$\begin{aligned}|\mathcal{M}|^2 &= |\mathcal{M}_W|^2 \\ &+\mathcal{M}_W\mathcal{M}_{W'}^* + \mathcal{M}_W^*\mathcal{M}_{W'} + |\mathcal{M}_{W'}|^2.\end{aligned} \tag{33}$$

We define the trace as Ω, which is common factor for all four terms, and we can set the equation for $|\mathcal{M}|^2$ as follows,

$$\begin{aligned}|\mathcal{M}|_t^2 = (\frac{g_W^4|V_{tb}|^2\Omega}{8})\Bigg[&\frac{1}{((q^2-m_W^2)^2+\Gamma_W^2 m_W^2)} \\ &-\frac{\beta^2 e^{-i\alpha}}{m_{W'}^2(q^2-m_W^2+i\Gamma_W m_W)} \\ &-\frac{\beta^2 e^{i\alpha}}{m_{W'}^2(q^2-m_W^2-i\Gamma_W m_W)} \\ &+\frac{\beta^4}{m_{W'}^4}\Bigg],\end{aligned} \tag{34}$$

where

$$\Omega = \mathrm{Tr}(\gamma^\nu P_L u_b\bar{u}_b\gamma^\mu P_L u_t\bar{u}_t\gamma_\mu P_L v_l\bar{v}_l\gamma_\nu P_L u_\nu\bar{u}_\nu).$$

A simplification can be made in Eq. (34) by integrating Ω and employing Euler's identity to obtain trigonometric functions. The final $|\mathcal{M}|^2$ is as follows,

$$\begin{aligned}\langle|\mathcal{M}|^2\rangle = 2g_W^4|V_{tb}|^2(p_l\cdot p_t)(p_b\cdot p_\nu)\Bigg[&\frac{1}{(q^2-m_W^2)^2+\Gamma_W^2 m_W^2} \\ +\frac{2\beta^2(\Gamma_W m_W\sin\alpha+\cos\alpha(m_W^2-q^2))}{m_{W'}^2((q^2-m_W^2)^2+\Gamma_W^2 m_W^2)} &+ \frac{\beta^4}{m_{W'}^4}\Bigg].\end{aligned} \tag{35}$$

Similarly to the approach outlined in Eq. (23), $\langle|\mathcal{M}|^2\rangle$ can be substituted into the three-body decay rate formula. We take a similar step we took earlier in Eq. (26), except that being aware that $\frac{\beta^4}{m_W^4}$ doesn't have any delta function in it we must split this integral into two parts, one with a delta function and the other without it. Note that here we evaluate the same integral over ρ^2 in Eq. (24). By doing this, we obtain

$$\Gamma = \int_0^{m_t^2} \left(\frac{1}{(2\pi)^3}\right)\left(\frac{1}{32m_t^3}\right)\left(\frac{g_W^4|V_{tb}|^2}{2}\right)$$
$$\left[\frac{m_t^6}{6} - \frac{m_t^2q^4}{2} + \frac{q^6}{3}\right]\left[\frac{1}{(q^2-m_W^2)^2+\Gamma_W^2m_W^2}\right. \quad (36)$$
$$\left.+\frac{2\beta^2(\Gamma_W m_W\sin\alpha + \cos\alpha(m_W^2-q^2))}{m_{W'}^2((q^2-m_W^2)^2+\Gamma_W^2m_W^2)} + \frac{\beta^4}{m_{W'}^4}\right] dq^2$$

This is the point at which we split our expression into two parts, SM and NP. Let us consider that the normalization constant, A, is the same for the delta function. With the delta function, we can evaluate the integral over q^2. The width for the SM is as follows,

$$\Gamma_{SM+cross} = \left(\frac{1}{(2\pi)^3}\right)\left(\frac{1}{32m_t^3}\right)\left(\frac{g_W^4|V_{tb}|^2}{2}\right)$$
$$\times\left[\frac{m_t^6}{6} - \frac{m_t^2q^4}{2} + \frac{q^6}{3}\right]\left[\left(\frac{\pi}{\Gamma_W m_W}\right)\left(1+\frac{2\beta^2}{m_{W'}^2}(\Gamma_W m_W\sin\alpha)\right)\right], \quad (37)$$

which can be further simplified as,

$$\Gamma_{SM+cross} = \frac{G_F|V_{tb}|^2m_t^3}{72\sqrt{2}\pi}(1-3\zeta^4+2\zeta^6)$$
$$\times\left(1+\frac{3\beta^2G_F\sin\alpha m_W^4}{\sqrt{2}\pi m_{W'}^2}\right). \quad (38)$$

For the NP, we must integrate $\frac{\beta^2}{m_{W'}^4}$ over q^2 by hand

$$d\Gamma_{NP} = \int_0^{m_t^2}\left(\frac{1}{(2\pi)^3}\right)\left(\frac{1}{32m_t^3}\right)\left(\frac{g_W^4|V_{tb}|^2}{2}\right)$$
$$\times\left[\frac{m_t^6}{6} - \frac{m_t^2q^4}{2} + \frac{q^6}{3}\right]\frac{\beta^4}{m_{W'}^4}dq^2 \quad (39)$$

The final expression for the NP contributions can be calculated and simplified in terms of G_F as follows,

$$\Gamma_{NP} = \frac{G_F^2m_W^4|V_{tb}|^2m_t^5\beta^4}{192\pi^3m_{W'}^4} \quad (40)$$

In the last section of this paper, we will delve into the discussion of partial rate asymmetry between the top quark and the anti-top quark. A comparable calculation needs to be undertaken for the anti-top quark with both W and W'. Following the Feynman rules in Appendix A, we obtain,

$$-i\mathcal{M}_W = \bar{v}_t\left(-\frac{ig_WV_{tb}}{\sqrt{2}}\gamma^\mu P_L\right)v_b$$
$$\times\bar{u}_l\left(-\frac{ig_W}{\sqrt{2}}\gamma^\nu P_L\right)v_\nu\left(\frac{-i(g_{\mu\nu}-\frac{q_\mu q_\nu}{m_W^2})}{q^2-m_W^2+i\Gamma_Wm_W}\right), \quad (41)$$

$-\frac{q_\mu q_\nu}{m_W^2}$ becomes negligible because q_ν contracts with γ^ν, and the amplitudes can be expressed as follows,

$$\mathcal{M}_W = \frac{-g_W^2V_{tb}}{2(q^2-m_W^2+i\Gamma_Wm_W)}\left[\bar{v}_t\gamma^\mu P_Lv_b\bar{u}_l\gamma_\mu P_Lv_\nu\right], \quad (42)$$

and

$$\mathcal{M}_W^* = \frac{-g_W^2V_{tb}^*}{2(q^2-m_W^2-i\Gamma_Wm_W)}\left[\bar{v}_b\gamma^\nu P_Lv_t\bar{v}_\nu\gamma_\nu P_Lu_l\right]. \quad (43)$$

The amplitudes for W' can also be obtained as follows,

$$-i\mathcal{M}_{W'} = \bar{v}_t(-\frac{ig_WV_{tb}\beta e^{-i\alpha}}{\sqrt{2}}\gamma^\mu P_L)v_b$$
$$\times\bar{u}_l(-\frac{ig_W\beta}{\sqrt{2}}\gamma^\nu P_L)v_\nu\left(\frac{-i(g_{\mu\nu}-\frac{q_\alpha q_\beta}{m_{W'}^2})}{q^2-m_{W'}^2+i\Gamma_{W'}m_{W'}}\right). \quad (44)$$

Since $m_{W'}^2$ is much larger compared to other terms, $m_{W'}^2$ becomes a dominating term, and $-\frac{q_\mu q_\nu}{m_{W'}^2}$ and $i\Gamma_{W'}m_{W'}$ become negligible. We now can express the amplitudes as,

$$\mathcal{M}_{W'} = \frac{g_W^2\beta^2V_{tb}e^{-i\alpha}}{2m_W^2}\left[\bar{v}_t\gamma^\mu P_Lv_b\bar{u}_l\gamma_\mu P_Lv_\nu\right], \quad (45)$$

and

$$\mathcal{M}_{W'}^* = \frac{g_W^2\beta^2V_{tb}^*e^{i\alpha}}{2m_W^2}\left[\bar{v}_b\gamma^\nu P_Lv_t\bar{v}_\nu\gamma_\nu P_Lu_l\right]. \quad (46)$$

Now we can calculate the $|\mathcal{M}|_{\bar{t}}^2$ as in Eq. (34) where,

$$|\mathcal{M}|_{\bar{t}}^2 = (\frac{g_W^4|V_{tb}|^2\Omega}{8})\left[\frac{1}{((q^2-m_W^2)^2+\Gamma_W^2m_W^2)}\right.$$
$$-\frac{\beta^2e^{i\alpha}}{m_{W'}^2(q^2-m_W^2+i\Gamma_Wm_W)}$$
$$-\frac{\beta^2e^{-i\alpha}}{m_{W'}^2(q^2-m_W^2-i\Gamma_Wm_W)} \quad (47)$$
$$\left.\frac{\beta^4}{m_{W'}^4}\right],$$

where

$$\Omega = \mathrm{Tr}(\gamma^\nu P_Lu_b\bar{u}_b\gamma^\mu P_Lu_t\bar{u}_t\gamma_\mu P_Lv_l\bar{v}_l\gamma_\nu P_Lu_\nu\bar{u}_\nu).$$

It can be noted that what is different from Eq. (35) is the negation of α. Advancing towards the final expression and negating the α's in the phase factors, we obtain,

$$\langle|\mathcal{M}|^2\rangle = 2g_W^4|V_{tb}|^2(p_l\cdot p_t)(p_b\cdot p_\nu)\left[\frac{1}{(q^2-m_W^2)^2+\Gamma_W^2 m_W^2}\right.$$
$$+\frac{2\beta^2(\Gamma_W m_W\sin(-\alpha)+\cos(-\alpha)(m_W^2-q^2))}{m_{W'}^2((q^2-m_W^2)^2+\Gamma_W^2 m_W^2)} \quad (48)$$
$$\left.+\frac{\beta^4}{m_{W'}^4}\right].$$

Given that the cosine function is even, it follows that $\cos(-\alpha)=\cos(\alpha)$. In contrast, the sine function is odd, resulting in $\sin(-\alpha)=-\sin(\alpha)$. The final expression of the SM for the anti-top quark is given by,

$$\bar{\Gamma}_{SM} = \frac{G_F|V_{tb}|^2 m_t^3}{72\sqrt{2}\pi}(1-3\zeta^4+2\zeta^6) \times\left[1-\frac{3\beta^3 G_F\sin(\alpha)m_W^4}{\sqrt{2}\pi m_{W'}^2}\right]. \quad (49)$$

5. Calculation of Partial Rate Asymmetry (PRA)

In this section, we calculate the partial rate asymmetry, which is the difference between the top quark and the anti-top quark. It could cause a violation of charge conjugation parity symmetry if the value turns out to be non-zero.

$$\mathcal{A} = \frac{\Gamma(t\to bl^+\nu_l)-\Gamma(\bar{t}\to\bar{b}l^-\bar{\nu}_l)}{\Gamma(t\to bl^+\nu_l)-\Gamma(\bar{t}\to\bar{b}l^-\bar{\nu}_l)} \quad (50)$$

A simplification can be done by expressing Eq. (50) in terms of Γ_{SM}, Γ_{cross}, and Γ_{NP} as follows,

$$\mathcal{A} = \frac{\Gamma_{cross}}{\Gamma_{SM}+\Gamma_{NP}} \quad (51)$$

Inserting the Γ's into Eq. (51) and simplifying yields the final expression below,

$$\mathcal{A} = \frac{12\pi\beta^2 G_F m_W^4 m_{W'}^2(1-3\zeta^4+2\zeta^6)sin(\alpha)}{3\beta^4 G_F m_t^2 m_W^4+4\sqrt{2}\pi^2 m_{W'}^4(1-3\zeta^4+2\zeta^6)} \quad (52)$$

For further research, we calculated the partial rate asymmetry (PRA) for the case in which a W' is included (b = 0.5, 1, 2; mW' = 400 GeV, 600 GeV, 1 TeV; a = pi/2)

$m_{W'}/\beta$	400GeV	600GeV	1TeV
0.5	0.000514	0.000228	0.0000822
1.0	0.00205	0.000913	0.000329
2.0	0.00821	0.00365	0.00132

We also plot the PRA as a function of the phase α for several different W' masses as shown in Fig. (2) below,

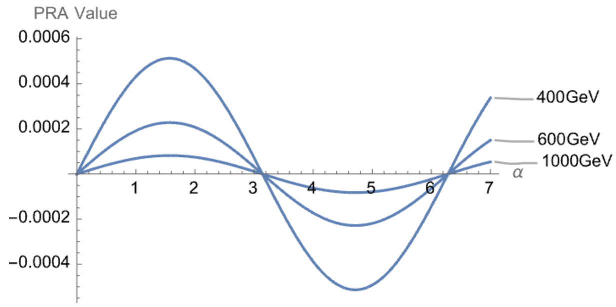

FIG. 2. PRA as a function of α

III. DISCUSSIONS AND CONCLUSION

In this paper, we considered new physics, examining the decay of $t\to bl^+\nu_l$. Using the calculation of the decay width for the top quark, we consider a new, mediating particle, taking it to be much heavier. We then calculated the partial rate asymmetry by considering top decays and anti-top decays. We showed that the presence of new physics can be seen through the measurement of the PRA. Future work would include considering other mediating particles of $t\to bl^+\nu_l$. For example, a charged Higgs can be introduced, and we can follow Sec. (II) and Sec. (II). When considering a PRA, if the value is not zero, then we can consider NP. If the value of the PRA is zero, then we may assume that this mediating particle cannot be considered in this decay. More decays can be considered in the search for NP.

ACKNOWLEDGMENTS

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Appendix A: Feynman Diagram Rules

This Appendix contains relevant Feynman rules for weak interactions and new physics.

1. A.1 External Lines

This section assigns spinors to incoming and outgoing spin-$\frac{1}{2}$ and spin-1 particles.

$$\text{Spin }\frac{1}{2}:\begin{cases}\text{Incoming Particle: } u\\ \text{Incoming Antiparticle: } \bar{v}\\ \text{Outgoing Particle: } \bar{u}\\ \text{Incoming Antiparticle: } v\end{cases}$$

$$\text{Spin } 1: \begin{cases} \text{Incoming: } \epsilon_\mu \\ \text{Incoming: } \epsilon^*_\mu \end{cases}$$

2. A.2 Propagators

This section assigns propagator values for W and W'.

$$\text{Spin } 1: \begin{cases} W\text{: } \frac{\left[-i\left(g^{\mu\nu} - \frac{g^\mu g^\nu}{m_w^2 c^2}\right)\right]}{q^2 - m_w^2 c^2 + i\hbar\Gamma_w m_w} \\ \\ W'\text{: } \frac{\left[-i\left(g^{\mu\nu} - \frac{g^\mu g^\nu}{m_w^2 c^2}\right)\right]}{-m_w'^2 c^2} \end{cases}$$

3. A.3 Vertex Factors

This section assigns vertex factors for vertices in their respective Feynman Diagram.

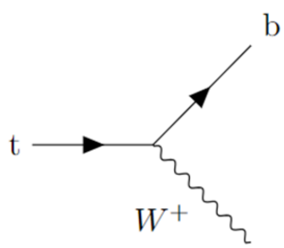

FIG. 3. Feynman Diagram of $\Gamma(t \to bW^+)$

Figure (3) yields the vertex factor:

$$\left[\frac{-ig_W}{\sqrt{2}} V^*_{tb} \gamma_\mu P_L\right].$$

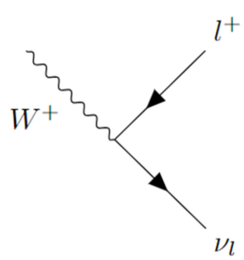

FIG. 4. Feynman Diagram of $\Gamma(W^+ \to l^+\nu_l)$

Figure (4) yields the vertex factor:

$$\left[\frac{-ig_W}{\sqrt{2}} \gamma_\mu P_L\right].$$

Figure (5) yields the vertex factor:

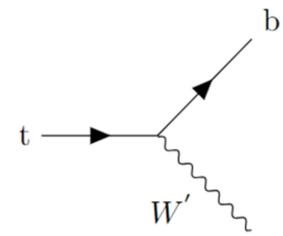

FIG. 5. Feynman Diagram of $\Gamma(t \to bW)$

$$\left[\frac{-ig_W \beta}{\sqrt{2}} V^*_{tb} e^{i\alpha} \gamma_\mu P_L\right].$$

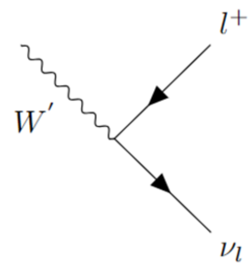

FIG. 6. Feynman Diagram of $\Gamma(W \to l^+\nu_l)$

Figure (6) yields the vertex factor:

$$\left[\frac{-ig_W}{\sqrt{2}} \beta \gamma_\mu P_L\right].$$

4. A.4 Feynman Diagrams

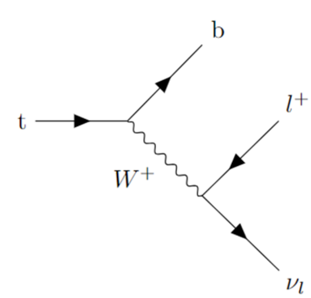

FIG. 7. Feynman Diagram of $\Gamma(t \to bl^+\nu_l)$ for W^+

The Feynman Diagram assisting the amplitude calculation in Sec. (II) is shown in Fig. (7).

The Feynman Diagram assisting the New Physics calculation in Sec. (II) is shown in Fig. (8).

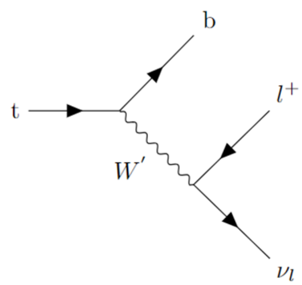

FIG. 8. Feynman Diagram of $\Gamma(t \to bl^+\nu_l)$ for W

Appendix B: Calculation of Γ_{total} for the W Decay

This Appendix contains the calculation of Γ_{total} for the W decay, a relevant calculation and building block for semi-leptonic top decay. An essential place to start this calculation is solving for $\Gamma(W^- \to e^- + \nu_e)$, providing shortcuts for the following components of the Γ_{total} calculation. $\Gamma(W^- \to e^- + \nu_e)$ can be modeled using a Feynman diagram as shown:
Following our Feynman diagram rules from Appendix A, we find

$$-i\mathcal{M} = \bar{u}_e \left[\frac{-ig_W}{\sqrt{2}}\gamma_\alpha P_L\right] v_\nu \epsilon^\alpha \tag{53}$$

where

$$P_L = \frac{1}{2}(1-\gamma^5), \tag{54}$$

using α as the Greek index for the vertex. To find $|\mathcal{M}|^2$, we must first simplify Eq. (53) and take the complex conjugate, which yields

$$\mathcal{M}^* = \frac{g_W}{\sqrt{2}}\epsilon^{\beta*}\bar{v}_\nu\gamma_\beta P_L u_e, \tag{55}$$

in which we have changed the name of the summation index from α to β. By multiplying Eq. (53) and Eq. (55) and averaging over the spins, we find

$$\begin{aligned}\langle|\mathcal{M}|^2\rangle = \frac{1}{3}\sum_{spins}[\frac{g_W}{\sqrt{2}}\epsilon^\alpha\bar{u}_e\gamma_\alpha P_L v_\nu]\\ \times[\frac{g_W}{\sqrt{2}}\epsilon^{\beta*}\bar{v}_\nu\gamma_\beta P_L u_e].\end{aligned} \tag{56}$$

Equation (56) can be simplified by treating ϵ^α and $\epsilon^{\beta*}$ as scalar quantities, setting $m_{v_e} \approx 0$, and taking the trace of the matrices to obtain

$$\langle|\mathcal{M}|^2\rangle = \frac{g_W^2}{6}\epsilon^\alpha\epsilon^{\beta*}\mathrm{Tr}(\not{p}_e\gamma_\alpha P_L(\not{p}_\nu + m_\nu c)\gamma_\beta P_L). \tag{57}$$

We can simplify the inside of the trace knowing that ${P_L}^2 = P_L$. Our 'slashed' momenta can also be expanded, taking the scalar quantity outside the trace while leaving the contracted matrix inside, where we obtain

$$\begin{aligned}\langle|\mathcal{M}|^2\rangle = \frac{g_W^2}{6}\epsilon^\alpha\epsilon^{\beta*}\\ \times(\frac{1}{2}p_{e\mu}p_{\nu\nu}\mathrm{Tr}(\gamma^\mu\gamma_\alpha\gamma^\nu\gamma_\beta(1-\gamma^5)).\end{aligned} \tag{58}$$

It can be noted that μ and ν are dummy indices used to represent the contraction between the scalar and matrix quantities. Now, $\epsilon^\alpha\epsilon^{\beta*}$ can be expressed using the identity:

$$\sum_{s=1,2,3}\epsilon^\alpha\epsilon^{\beta*} = \left[-g^{\alpha\beta} + \frac{p_W^\alpha p_W^\beta}{m_W^2c^2}\right]. \tag{59}$$

Inserting the above expression into Eq. (58) yields

$$\begin{aligned}\langle|\mathcal{M}|^2\rangle = \frac{g_W^2}{6}\left[-g^{\alpha\beta} + \frac{p_W^\alpha p_W^\beta}{m_W^2c^2}\right]\\ \times\frac{1}{2}p_{e\mu}p_{\nu\nu}\mathrm{Tr}\left[\gamma^\mu\gamma_\alpha\gamma^\nu\gamma_\beta(1-\gamma^5)\right].\end{aligned} \tag{60}$$

To carry the expression further, we must manipulate the corresponding α and β indices. Since they are contracted together, the α and β indices may be lowered and raised. It is important to note that this can only be done since they are contracted together. Doing this provides the following,

$$\begin{aligned}\langle|\mathcal{M}|^2\rangle = \frac{g_W^2}{12}\left[-g_{\alpha\beta} + \frac{p_W\alpha p_W\beta}{m_W^2c^2}\right]\\ \times p_{e\mu}p_{\nu\nu}\mathrm{Tr}\left[\gamma^\mu\gamma^\alpha\gamma^\nu\gamma^\beta(1-\gamma^5)\right].\end{aligned} \tag{61}$$

The following identities will prove helpful in simplifying Eq. (58):

$$\begin{aligned}\mathrm{Tr}(\gamma^\mu\gamma^\alpha\gamma^\nu\gamma^\beta(1-\gamma^5)) =\\ 4[g^{\mu\alpha}g^{\nu\beta} - g^{\nu\mu}g^{\alpha\beta} + g^{\mu\beta}g^{\alpha\nu} - i\epsilon^{\mu\alpha\nu\beta}],\end{aligned} \tag{62}$$

$$p_{e\mu}g^{\mu\alpha} = p_e^\alpha, \tag{63}$$

and

$$g_{\alpha\beta}\epsilon^{\alpha\beta\mu\nu} = 0. \tag{64}$$

Equation (62) provides a way to expand the trace in Eq. (61). From here, we can utilize Eq. (63) as a simplification, producing dot products between the momenta through the contracted indices. Equation (64) provides another simplification for a term that will appear when Eq. (61) is expanded given the identity in Eq. (62). Since there is an anti-symmetric term multiplying a symmetric term, the following product of these terms is equivalent to zero. Now, we can rewrite Eq. (61) as follows,

$$\begin{aligned}\langle|\mathcal{M}|^2\rangle = \frac{g_W^2}{3} 2(p_\nu \cdot p_e) + \frac{g_W^2}{3}\frac{1}{m_W^2 c^2} \\ \times\left[2(p_W \cdot p_\nu)(p_W \cdot p_e) - (p_\nu \cdot p_e)(p_W \cdot p_W)\right].\end{aligned} \tag{65}$$

A simplification can be made in Eq. (65) by expressing the momenta using their respective four vector quantities. The dot products of the momenta are as follows,

$$p_\nu \cdot p_e = \frac{1}{2} m_W^2 c^2, \tag{66}$$

$$p_W \cdot p_\nu = \frac{1}{2} m_W^2 c^2, \tag{67}$$

$$p_W \cdot p_e = \frac{1}{2} m_W^2 c^2, \tag{68}$$

and

$$(p_W \cdot p_W) = m_W^2 c^2. \tag{69}$$

Inserting these expressions into Eq. (65) and simplifying produces

$$\langle|\mathcal{M}|^2\rangle = \frac{g_W^2}{3}(m_W^2 c^2). \tag{70}$$

Now that we have found the amplitude squared, we may insert Eq. (70) into the decay width equation for the two-particle decay given as,

$$\Gamma = \frac{S|p|}{8\pi\hbar m_1^2 c}\langle|\mathcal{M}|^2\rangle. \tag{71}$$

Using the final expression given by Eq. (70) and inserting it into Eq. (71) we obtain,

$$\Gamma = \frac{S|p|}{8\pi\hbar m_1^2 c}\frac{g_W^2}{3}(m_W^2 c^2). \tag{72}$$

From here, we may manipulate Eq. (72) by solving for g_W^2, putting it in terms of G_F. Doing so produces

$$\Gamma_e = \frac{G_F}{(\hbar c)^3}\frac{m_W^3 c^6}{6\sqrt{2}\pi\hbar}. \tag{73}$$

It is important to note the e subscript found on Γ in Eq. (73). This subscript notes the decay of $\Gamma(W^- \to e^- + \bar{\nu}_e)$, one of the various decays of the W boson. To find Γ_{total}, we must sum all of the decays that are found by the W boson. This concept can be pictured below,

$$\Gamma_{tot} = \Gamma_e + \Gamma_2 + ... + \Gamma_9, \tag{74}$$

where the 2 and the 9 subscripts refer to other decays of the W boson. There are nine decays of the W boson, three that decay to a lepton with their corresponding neutrinos, and six that decay hadronically. The reason there are not nine hadronic decays is due to the top quark being heavier than the W boson. This calculation may seem extensive with eight more decay widths to calculate, but many shortcuts can be made. Since we assume the masses of the neutrinos to be approximately zero and know that the final expression in Eq. (70) yields no other mass dependence, our semileptonic decays follow the same pattern, obtaining the same expression for Eq. (70). That means every semileptonic decay has the same expression for Eq. (73), i.e.,

$$\begin{aligned}\Gamma(W^- \to e^- + \bar{\nu}_e) = \Gamma(W^- \to \mu^- + \bar{\nu}_\mu) \\ = \Gamma(W^- \to \tau^- + \bar{\nu}_\tau)\end{aligned}. \tag{75}$$

Very similar to the semileptonic decays, the hadronic decays experience no mass dependence since we approximate our masses to be equivalent to zero. There are only two differences now, a color factor and the vertex factor. The color factor is simply introducing the number 3 for all of the different colors at the end of the decay. The vertex factor introduces the Cabibbo–Kobayashi–Maskawa (CKM) matrix. The CKM matrix is a scalar quantity that will not influence the matrix multiplication; therefore, we may move it anywhere within the expression. When multiplying $\mathcal{M}$ and $\mathcal{M}^*$ together, everything follows in addition to a $|V_{ij}|^2$ term being multiplied alongside. Knowing this, we can immediately arrive at Eq. (70), multiplying everything by a 3 and a $|V_{ij}|^2$, obtaining,

$$\langle|\mathcal{M}|^2\rangle = \frac{(3)g_W^2}{3}|V_{ij}|^2(m_W^2 c^2) \tag{76}$$

where every hadronic decay produces this same expression. Solving this symbolically, we can express all of them together as follows,

$$\Gamma(W^- \to \text{hadrons}) = 3\sum_{i,j}|V_{ij}|^2\Gamma_e, \tag{77}$$

where

$$\sum_{i,j}|V_{ij}|^2 = 2. \tag{78}$$

Following Eq. (74) while utilizing concepts from Eq. (75) and Eq. (76), we can solve for Γ_{total} in term of Γ_e. This is expressed as,

$$\Gamma_{total} = 3\Gamma_e + 6\Gamma_e = 9\Gamma_e, \tag{79}$$

where Γ_{total} is equivalent to the decay width of the W boson. Using Eq. (73) we can express the decay width (Γ) as,

$$\Gamma_{total} = \frac{(9)G_F}{(\hbar c)^3}\frac{m_W^3 c^6}{6\sqrt{2}\pi\hbar}. \tag{80}$$

Γ_{total} can be expressed numerically in units of GeV by multiplying Eq. (80) through by $\hbar$ obtaining

$$\hbar\Gamma_{total} = 2.04419GeV. \tag{81}$$

Now, with our given value, we may compare this to the experimental value through a percent difference calculation. Our value differs by 1.96% from the accepted value, which is [ACCEPTED VALUE].

Now that Γ_{total} is obtained, we can evaluate the branching ratios. The equation is given to calculate the branching ratio,

$$BR = \frac{\Gamma_{branch}}{\Gamma_{total}}. \tag{82}$$

With Eq. (82), we may find the branching ratio for the leptons. Using Eq. (75), we can note that solving for the branching ratio for one semileptonic decay will obtain the branching ratio for all semileptonic decays. This is shown as,

$$BR_e = BR_\mu = BR_\tau = \frac{\Gamma_e}{9\Gamma_e} = \frac{1}{9} \tag{83}$$

Following this we can solve for the branching ratio of quarks and antiquarks. We can show the method for solving for the branching ratio of each quark by showing $BR_{\bar{u}d}$. Knowing

$$\Gamma_{\bar{u}d} = 3|V_{ud}|^2\Gamma_e, \tag{84}$$

and using Eq. (82), we obtain

$$BR_{\bar{u}d} = \frac{3|V_{ud}|^2\Gamma_e}{9\Gamma_e}. \tag{85}$$

From here, we can simplify Eq. (85) where $|V_{ud}|^2$ is a value in the CKM. Simplifying yields

$$BR_{\bar{u}d} = 0.316. \tag{86}$$

Following this same method for the quarks and antiquarks, we obtain

$$\begin{aligned} \bar{u}d &= 0.316 \\ \bar{u}s &= 0.017 \\ \bar{u}b &= 5.333 \times 10^{-6} \\ \bar{c}d &= 0.017 \\ \bar{c}s &= 0.316 \\ \bar{c}b &= 5.936 \times 10^{-4} \end{aligned} \tag{87}$$

Now we can compare our values given by the Particle Data Group. The experimental values are given by the Particle Data Group as follows:

$$\begin{aligned} e\nu_e &= 10.71 \pm 0.16 \\ \mu\nu_\mu &= 10.63 \pm 0.15 \\ \tau\nu_\tau &= 11.38 \pm 0.21 \\ hadrons &= 67.41 \pm 0.27 \\ cX &= 33.3 \pm 2.6 \\ \bar{c}s &= 31 \pm \frac{11}{13} \end{aligned} \tag{88}$$

Our theoretical values with our error are as follows:

$$\begin{aligned} e\nu_e &= 11.1 \text{ (3.64\% error)} \\ \mu\nu_\mu &= 11.1 \text{ (4.42\% error)} \\ \tau\nu_\tau &= 11.1 \text{ (2.46\% error)} \\ hadrons &= 66.7 \text{ (1.05\% error)} \\ cX &= 33.4 \text{ (0.30\% error)} \\ \bar{c}s &= 31.6 \text{ (1.94\% error)} \end{aligned} \tag{89}$$

The calculation of $\Gamma(W^- \to e^- + \bar{\nu}_e)$ is a building block of $\Gamma(t \to bl\nu_l)$. This calculation will assist used identities and shortcuts in calculating $\Gamma(t \to bl\nu_l)$.

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