$$\begin{CD} V @>{\mathcal{A}^*}>> V \\ @V{\simeq}VV @VV{\simeq}V \\ V^* @>{\mathcal{A}^*}>> V^* \end{CD}$$