Study of Multi-Objective Optimization using Evolutionary Algorithms

Shivam Pandey | 160010003
School of Mathematics and Computer Science
Indian Institute of Technology Goa

Supervisors : *Prof. Somenath Biswas* (Indian Institute of Technology Goa)
Prof. Chilukuri K. Mohan (Syracuse University, New York)

1 Problem Statement & Motivation

Optimization is a procedure of finding and comparing feasible solutions until no better solution can be found. A multi-objective optimization problem (MOOP) entails more than one objective to be optimized, often conflicting objectives, and has a pareto-optimal set of solutions. ***Pareto-optimal set*** is defined as the *non-dominated*[1] set of the entire feasible search space. Multi-Objective Evolutionary Algorithms (MOEAs) seek to approximate the *pareto front*[2] of the MOOP, by evolving a population of solutions, inspired from the biological evolution using selection, reproduction, recombination and mutation. Classical optimization methods can at best find one solution in one simulation run, thereby making those methods inconvenient to solve MOOPs. Evolutionary approaches to MOOPs, on the other hand, are capable of searching for *multiple optimal solutions concurrently in a single run*, due to their population-approach. This ability of an evolutionary algorithm (EA) to find multiple optimal solutions in one simulation run makes EAs unique in solving MOOPs.

My B.Tech project work focuses on studying and exploring existing MOEAs like *Non-dominated Sorting Genetic Algorithm-II (NSGA-II)* [2] , *Pareto-Archived Evolution Strategy (PAES)* [4], etc., and formulate possible improvement ideas along with combining the best ideas from existing MOEAs to propose a *better* algorithm, in terms of *quality of solutions*[3], *running time of the algorithm*, or preferably both.

2 Literature Review

[1] In the initial phase of my work, the formulation of a MOOP was studied. Most multi-objective optimization algortihms use the concept of *dominance* and *pareto-optimality* in their search. A solution x is said to *dominate* the other solution y, if both the following conditions are true :

- The solution x is no worse than y in all objectives.
- The solution x is strictly better than y in at least one objective.

After studying the terminologies under the literature of EAs, the general structure of a genetic algorithm (GA) was studied and how they are implemented to solve an optimization problem, may it be a single-objective problem or a multi-objective problem.

[2] An existing *elitist* MOEA, particularly known as NSGA-II, with computational complexity $O(MN^2)$ (here, M = number of objectives, N = population size), was studied and implemented on a two-objective optimization benchmark problem: Schaffer's study (SCH) [3] (here, we took a *minimisation* problem). The two objective functions considered for the minimisation problem were x^2 and $(x-2)^2$, where the decision variable space : $x \in (-\infty, +\infty)$. It is evident that the optimal set of solutions for this problem would be $x \in [0, 2]$, hence we would expect the algorithm to converge to the pareto front (non-dominated solutions) in the *objective space* after sufficient number of generations are evolved.

[1] *Among a set of solutions P, the* ***non-dominated*** *set of solutions P', are those that are not dominated by any member of the set P.*

[2] *The curve formed in the objective space by joining these pareto-optimal solutions is known as a* ***pareto front****.*

[3] *The MOEA converges as* ***close*** *as possible to the pareto front and maintains a* ***good diversity*** *amongst the solutions.*

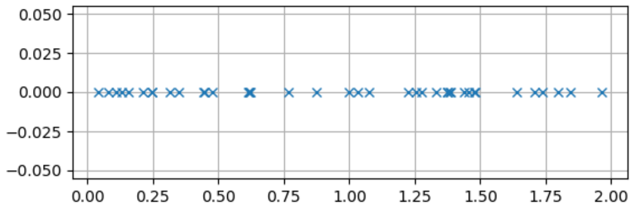

Figure 1: *Visualization of Decision Variable Space (or, Data Space)*

In Figure 1, each of the *crosses* denote one individual in the final population the algorithm outputs, which is evidently close to the optimal solution set, i.e. $x \in [0, 2]$.

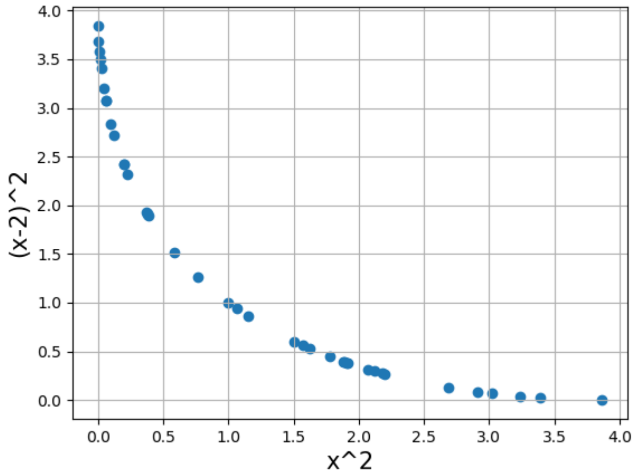

Figure 2: *Visualization of Objective Space*

In Figure 2, a reasonably *good approximation* to the pareto front was obtained in the objective space, along with reasonable diversity which is expected from NSGA-II. Hence, the run of NSGA-II was successful on this particular MOOP. With this successful execution of NSGA-II, a concrete understanding was developed about the working of MOEAs.

3 Work Plan (Fall 2019)

- To study how the concept of non-dominated sorting was evolved and why it is better than the plain genetic algorithm (GA).
- It should be noted that NSGA-II does not take into account the *diversity* in data space, but only in objective space. This motivates further research to consider the diversity (individually and simultaneously) in both, *data space* and *objective space*, and compare the results with traditional NSGA-II.
- To take into account the idea of "archives" [4], that it is useful to keep track of *good solutions* that have been visited in the past, even if they are no longer active in offspring generation. Then, the reproducing population can perform much more active *exploration*, instead of being filled up with old solutions.

References

[1] K. Deb, *Multiobjective Optimization Using Evolutionary Algorithms*. Chichester, U.K.: Wiley, 2001.

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[4] J. Knowles and D. Corne, “The Pareto archived evolution strategy: A new baseline algorithm for multiobjective optimization,” in *Proceedings of the 1999 Congress on Evolutionary Computation*. Piscataway, NJ: IEEE Press, 1999, pp. 98–105.