Worksheet 1 - Signal Transformations

Introduction

For this set of hands-on exercises, you'll work by hand and with MATLAB to analytically and visually see the results of amplitude and time transformations on signals.

Consider the following signal:

$$x(t) = \begin{cases} 0 & t < -1.5 \\ 2 & -1.5 \leq t \leq 0 \\ 2e^{-t/2} & 0 \leq t \leq 3 \\ 0 & t > 3 \end{cases}$$

1. (Analytic) On a piece of paper, sketch a plot of the signal.
2. (Matlab) Recall that there are three approaches to this which are equally valid, but some have advantages over others further down the road. Whichever approach is used, you'll end up having to have values of t as well as values of $x(t)$ to be able to fully plot this signal. The approaches are as follows:

   (a) explictly defining values of t as a Matlab vector, and then explicitly defining values of $x(t)$ as another vector, and then plotting t vs. $x(t)$;

   (b) explictly defining values of t as a Matlab vector, and then defining a `function` for $x(t)$ which takes t as an input and produces $x(t)$ as an output, and then plotting t vs. $x(t)$; or

   (c) explictly defining values of t as a Matlab vector, and then defining an *anonymous function* for $x(t)$, and then plotting t vs. $x(t)$.

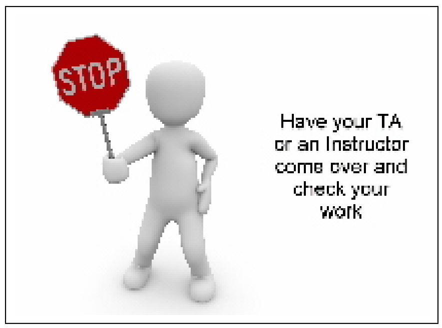
   For this worksheet, I recommend you explicitly define a `function` in MATLAB. Do this to graph and label the function, and make sure that your output matches the one you sketched earlier.

Amplitude Transformations

1. (Matlab exploration) In a Matlab window, run the interactive GUI for amplitude transformations by running `sop_demo1` (just type it in and hit enter; if you path is set correctly it should just run). Take a few minutes to play with the sliders to explore the graphical effect of changing the gain B and offset A.

2. (analytic) Using the same $x(t)$ as above, sketch a plot of $y(t) = 3x(t) - 5$

3. (Matlab) Confirm your sketch of the amplitude transformations by using Matlab (preferably with functions)



Time Transformations

(Matlab exploration) In a Matlab window, run the interactive GUI for amplitude transformations by running `sop_demo2`. Note that this GUI allows you to explore several different signal operations: time scaling, time shifting, adding two signals, and multiplying two signals.

Take a few minutes to play with the sliders to explore the graphical effect of changing time scaling and time shifting.

Time Scaling

1. Consider $y(t) = x(3t)$. Do you expect this to be a compressed or expanded version of the original signal? Draw a quick sketch of what you think the signal should look like and then confirm it with a Matlab function call again.

2. Consider $y(t) = x(t/2)$. Do you expect this to be a compressed or expanded version of the original signal? Draw a quick sketch of what you think the signal should look like and then confirm it with a Matlab function call again.

Time Shifting

1. Consider $y(t) = x(t-3)$. Do you expect this to be a version of $x(t)$ which is shifted in the positive$-t$ direction or the negative$-t$ direction? Draw a quick sketch of what you think the signal should look like and then confirm it with a Matlab function call again.

2. Consider $y(t) = x(t+4)$. Do you expect this to be a version of $x(t)$ which is shifted in the positive$-t$ direction or the negative$-t$ direction? Draw a quick sketch of what you think the signal should look like and then confirm it with a Matlab function call again.

Combined Time Transformations

One of the more difficult and non-intuitive types of transformations that exists is when both time-scaling and time-shifting are present. If you would like to get a better handle on WHY you must consider time-shifting before time-scaling, check out the file "Precedence Rule for Time-Shifting and Time-Scaling" (on D2L). In what follows, you will first walk through a methodology for doing it step-by-step, and then confirm your results with Matlab.

1. (analytic) Consider the signal $y(t) = x(3t-3)$

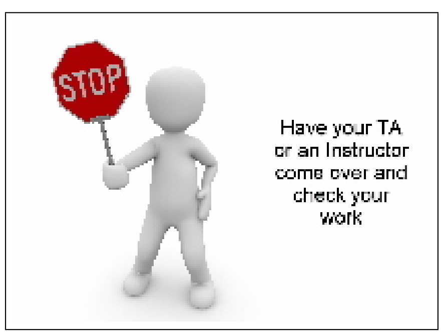
   (a) First, resketch the function x but with the variable t replaced with τ.

   (b) The new variable τ is now set to equal the argument of the input function, in this case $\tau = 3t - 3$. Now rearrange this to solve for t in terms of τ.

   (c) Underneath your sketch of $x(\tau)$, draw another horizontal axis and label it as t. For several (important) values of τ (above it on the graph of $x(\tau)$), indicate the correct value for t right below it.

   (d) Now re-sketch $y(t)$ by using your graph above with the transformed t-axis. Try and make it the same scale as the original $x(t)$ sketch to compare.

2. (Matlab) Now use your defined function for $x(t)$ that you already created in Matlab to see if your final sketch is correct.



Combined Time and Amplitude Transformations

1. (Analytic) Now let's put it all together, but using the techniques discussed to plot $y(t) = 3x\left(1 - \frac{t}{2}\right) - 1$ (Note that there is a time reversal here which may get a little tricky). Take it step by step:

   (a) Do the time transformations first, by determining the transformed t-axis, drawing it underneath the original signal, and sketching the new intermediate signal $y_1(t) = x\left(1 - \frac{t}{2}\right)$

   (b) Finally, do your amplitude scaling and shifting on the intermediate signal and sketch $y(t) = 3y_1(t) - 1$

2. (Matlab) Check your work with Matlab, you should be to go directly to plotting $y(t)$ if you have already defined a function for $x(t)$.

You're all done!!