COL 352 Introduction to Automata and Theory of Computation **Holi 2022**

Problem Set 3

Out: March 9 *Due:* **March 17**

Please attempt all problems. Read the instructions for problem sets posted on the announcement channel in MS Teams (and also here) carefully. Turn in your solutions via Gradescope by 11 pm on the due date.

1. We say that a context-free grammar G is self-referential if for some non-terminal symbol X we have $X \to^* \alpha X \beta$, where $\alpha, \beta \neq \varepsilon$. Show that a CFG that is not self-referential is regular.
2. Prove that the class of context-free languages is closed under intersection with regular languages. That is, prove that if L_1 is a context-free language and L_2 is a regular language, then $L_1 \cap L_2$ is a context-free language. Do this by starting with a DF
3. (**2 points**) Given two languages L, L', denote by
$$L||L' := \{x_1 y_1 x_2 y_2 \dots x_n y_n \mid x_1 x_2 \dots x_n \in L, y_1 y_2 \dots y_n \in L'\}$$
.
Show that if L is a CFL and L' is regular, then $L||L'$ is a CFL by constructing a PDA for $L||L'$. Is $L||L'$ a CFL if both L and L' are CFLs? Justify your answer.
4. For $A \subseteq \Sigma^*$, define
$$\mathit{cycle}(A) = \{yx \mid xy \in A\}$$
For example if $A = \{aaabc\}$, then
$$\mathit{cycle}(A) = \{aaabc, aabca, abcaa, bcaaa, caaab\}$$
. Show that if A is a CFL then so is $\mathit{cycle}(A)$
5. Let
$$A = \{wtw^R \mid w, t, \in \{0,1\}^* \text{ and } |w| = |t|\}$$
. Show that A is not a CFL.
6. Prove the following stronger version of pumping lemma for CFLs: If A is a CFL, then there is a number k where if s is any string in A of length at least k then s may be divided into five pieces $s = uvxyz$, satisfying the conditions:
   - for each $i \geq 0$, $uv^i xy^i z \in A$
   - $v \neq \varepsilon$, and $y \neq \varepsilon$, and
   - $|vxy| \leq k$.
7. Give an example of a language that is not a CFL but nevertheless acts like a CFL in the pumping lemma for CFL (Recall we saw such an example in class while studying pumping lemma for regular languages).