COL 352 Introduction to Automata and Theory of Computation **Holi 2022**

Problem Set 4

Out: March 22 *Due:* **March 31**

Please attempt all problems. Read the instructions for problem sets posted on the announcement channel in MS Teams (and also here) carefully. Turn in your solutions via Gradescope by 11 pm on the due date.

1. Show that every infinite Turing-recognizable language has an infinite decidable subset.
2. Show that single-tape TMs that cannot write on the portion of the tape containing the input string recognize only regular languages.
3. Let C be a language. Prove that C is Turing-recognizable iff a decidable language D exists such that
$$C = \{x \mid \exists y(\langle x, y\rangle \in D)\}$$
4. Say that a variable A in CFL G is *usable* if it appears in some derivation of some string $w \in G$. Given a CFG G and a variable A, consider the problem of testing whether A is usable. Formulate this problem as a language and show that it is decidable.
5. Consider the problem of determining whether a Turing machine M on an input w ever attempts to move its head left when its head is on the left-most tape cell. Formulate this problem as a language and show that it is undecidable.
6. Consider the problem of determining whether a Turing machine M on an input w ever attempts to move its head left at any point during its computation on w. Formulate this problem as a language and show that it is decidable.
7. Let
$$AMB_{CFG} = \{\langle G\rangle \mid G \text{ is an ambiguous CFG }\}$$
Show that AMB_{CFG} is undecidable via a reduction from PCP.
8. In the Silly Post Correspondence Problem (SPCP), the top string in each pair has the same length as the bottom string. Show that the SPCP is decidable.