COL 352 Introduction to Automata and Theory of Computation **Holi 2022**

Problem Set 1

Out: January 21 *Due:* **January 28**

Please attempt all problems. Read the instructions for problem sets posted on the announcement channel in MS Teams (and also here) carefully. Turn in your solutions via Gradescope by 11 pm on the due date.

To prove that regular languages are closed under some binary operation `op`, a straightforward way is to show how to construct, for any two regular languages L_1 and L_2, an NFA N recognizing $L_1 \texttt{op} L_2$ from DFAs D_1, D_2 recognizing L_1, L_2 respectively. Remember to prove that N accepts a string if and only if the string belongs to $\texttt{op}(L_1, L_2)$. You have to decide between whether to define N mathematically or to describe the construction informally in a human language (eg. for closure under concatenation, Connect every accepting state of D_1 to the initial state of D_2 by an "ϵ-transition"). There is a tradeoff here. If the mathematical definition is short, clean, and intuitive, write that and avoid giving a vague informal description. If the mathematical definition is unnecessarily complicated and it hides the main idea, avoid it and describe your construction informally but clearly.

1. Given an alphabet $\Gamma = \{\ell_1, \ldots, \ell_k\}$, construct an NFA that accepts strings that don't have all the characters from Γ. Can you give an NFA with k states?

2. An all-NFA M is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ that accepts $x \in \Sigma^*$ if every possible state that M could be in after reading input x is a state from F. Note, in contrast, that an ordinary NFA accepts a string if some state among these possible states is an accept state. Prove that all-NFAs recognize the class of regular languages.

3. Show that regular languages are closed under the **repeat** operation, where **repeat** operation on a language L is given by
$$\textbf{repeat}(L) = \{\ell_1\ell_1\ell_2\ell_2 \ldots \ell_k\ell_k \mid \ell_1\ell_2 \ldots \ell_k \in L\}$$

4. Design an algorithm that takes as input the descriptions of two DFAs, D_1 and D_2, and determines whether they recognize the same language.

5. For any string $w = w_1w_2 \ldots w_n$ the reverse of w written w^R is the string $w_n \ldots w_2w_1$. For any language A, let $A^R = \{w^R \mid w \in A\}$. Show that if A is regular, then so is A^R. In other words, regular languages are closed under the reverse operation.

6. Let Σ and Γ be two finite alphabets. A function $f : \Sigma^* \to \Gamma^*$ is called a homomorphism if for all $x, y \in \Sigma^*$, $f(x \cdot y) = f(x) \cdot f(y)$. Observe that if f is a string homomorphism, then $f(\epsilon) = \epsilon$, and the values of $f(a)$ for all $a \in \Sigma$ completely determines f. Prove that the class of regular languages is closed under homomorphisms.That is, prove that if $L \subseteq \Sigma^*$ is a regular language, then $f(L) = \{f(x) \in \Sigma^* \mid x \in L\}$ is regular. Try to informally describe how you will start with a DFA for L and get an NFA for $f(L)$.