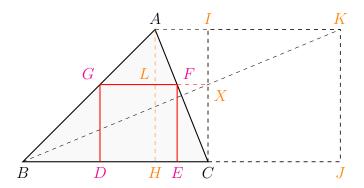
Problem

Given an acute triangle $\triangle ABC$, construct with straightedge and compass square DEFG such that D and E are on \overline{BC} , G is on \overline{AB} and F is on \overline{AC} .

Straightedge and compass can construct the middle point of a line segment, and perpendicular line through a point on a line segment.



Construction steps are:

- 1. draw height AH to BC
- 2. extend BC to J such that CJ = AH
- 3. draw square CJKI
- 4. set intersection of CK and BJ to X
- 5. draw line pass X and parallel to BC, intersect with AB at G, with AC at F
- 6. draw perpendicular lines down from G and F to get D and E

To prove DEFG is a square, since $\triangle BCX \sim \triangle BJK$,

$$\frac{CX}{JK} = \frac{BC}{BJ}$$
 or $\frac{EF}{AH} = \frac{BC}{BC + AH}$ or $EF = \frac{BC * AH}{BC + AH}$ (1)

since $\triangle ABC \sim \triangle AGF$,

$$\frac{GF}{BC} = \frac{AL}{AH} = \frac{AH - EF}{AH} \tag{2}$$

Then we just need to verify that GF = EF. From (2),

$$GF = BC * \frac{AH - EF}{AH} = \frac{BC}{AH} * (AH - EF)$$

$$= \frac{BC}{AH} * (AH - \frac{BC * AH}{BC + AH}), \quad from(1)$$

$$= \frac{BC}{AH} * \frac{AH^2}{BC + AH} = \frac{BC * AH}{BC + AH} = EF$$
(3)

The construction logic is derived from the GF expression.

This

is

a long

solution

 ${\it spilled}$

to

page

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