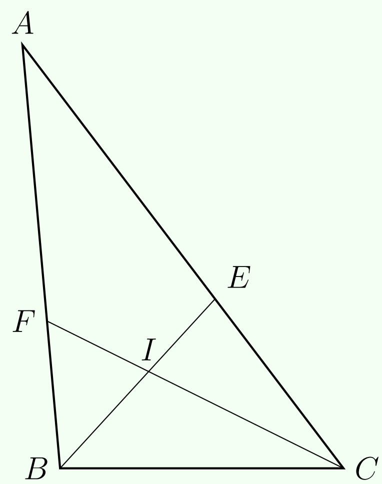
Problem

BE and CF are angle bisectors that meet at I as shown below. $CE = 4$, $AE = 6$, and $AB = 8$.

(a) Prove that $\angle EIC = 90^\circ - \dfrac{\angle A}{2}$

(b) Find BC

(c) Find BF



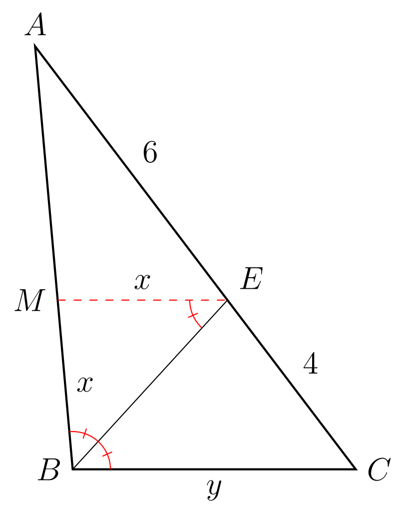
(a) Because $\angle EIC$ is an external angle of the triangle $\triangle EIC$,

$$\begin{aligned}\angle EIC &= \angle IBC + \angle ICB \\ &= \frac{\angle B}{2} + \frac{\angle C}{2}\end{aligned}$$

Hence,

$$\begin{aligned}\frac{\angle A}{2} + \angle EIC &= \frac{\angle A}{2} + \frac{\angle B}{2} + \frac{\angle C}{2} \\ &= \frac{\angle A + \angle B + \angle C}{2} \\ &= \frac{180^\circ}{2} \\ &= 90^\circ\end{aligned}$$

(b) Draw a line passing E and parallel to BC, intersect with AB at M.



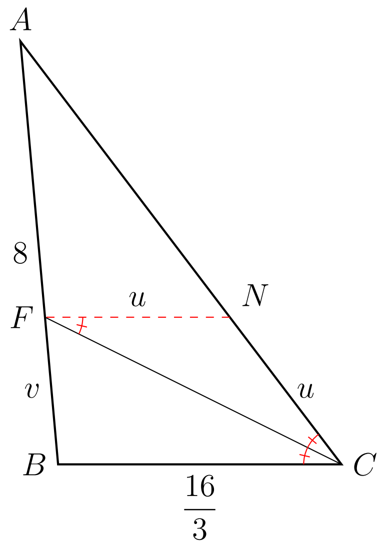
Because $\angle MBE = \angle EBC = \angle MEB$, $ME = MB$. Because $\triangle AME \sim \triangle ABC$,

$$\frac{AM}{AB} = \frac{AE}{AC}$$
$$\frac{8-x}{8} = \frac{6}{10}$$
$$10x = 32$$
$$x = 3.2$$

Then

$$\frac{ME}{BC} = \frac{AE}{AC}$$
$$\frac{x}{y} = \frac{6}{10}$$
$$6y = 32$$
$$y = \frac{16}{3}$$

(c) Draw a line through F and parallel to BC, intersect with AC at N.



Because $\angle NCF = \angle FCB = \angle NFC$, $FN = NC$. Because $\triangle AFN \sim \triangle ABC$,

$$\frac{AN}{AC} = \frac{FN}{BC}$$

$$\frac{10-u}{10} = \frac{u}{\frac{16}{3}} = \frac{3u}{16}$$

$$46u = 160$$

$$u = \frac{80}{23}$$

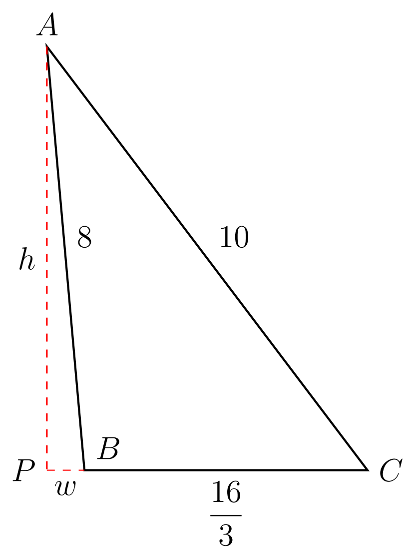
Then

$$\frac{AF}{AB} = \frac{FN}{BC}$$

$$\frac{8-v}{8} = \frac{u}{\frac{16}{3}} = \frac{3u}{16} = \frac{15}{23}$$

$$v = \frac{64}{23}$$

(d) To draw the triangle, we need to figure out the coordinate of A



With the 2 right triangles $\triangle APB$ and $\triangle APC$, the followings are true

$$h^2 + w^2 = 8^2 = 64$$

$$h^2 + (\frac{16}{3} + w)^2 = 10^2 = 100$$

Subtracting these 2 equations,

$$(\frac{16}{3} + w)^2 - w^2 = 10^2 = 36$$

$$w = -\frac{17}{24} = 0.62963$$

$$h = \sqrt{64 - w^2} = 7.96858$$