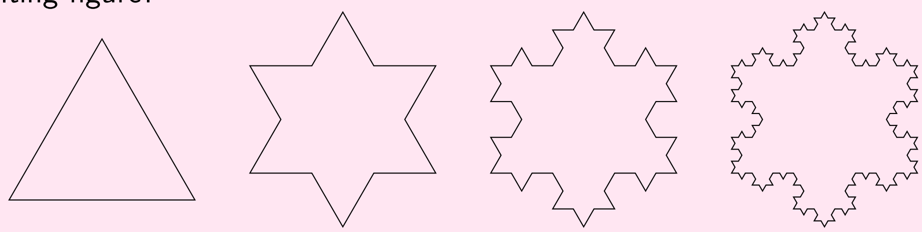
Problem

We begin with an equilateral triangle with side length 1. We divide each side into 3 segments of equal length, and add an equilateral triangle to each side using the middle third as a base. We then repeat this, to get a third figure. If we continue this process forever, what is the area of the resulting figure?



The shape is called Koch Snowflake, check section 4.9 in the book "The Beauty of Fractals Six Different Views", which is in google books. Consider the length changes under such transformations. Everytime, a side is turned into 4 smaller sides and smaller side X 3 = the larger side. So the fractal dimension is

$$\frac{log4}{log3} = 1.26185951 \; or \; 4 = 3^{1.26185951}$$

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Initially, or at level 0, it's just the triangle itself. The area is $A = \frac{\sqrt{3}}{4}$ and the number of sides is 3, or $3X4^0$.

At leve 1, the area added is $3 \; sides \times \frac{A}{9} = \frac{1}{3}A = \frac{1}{3}(\frac{4}{9})^0 A$. The number of sides now is $3 \times 4^1 = 12$.

At leve 2, the area added is $3 \times 4^1 \; sides \times (\frac{1}{9})^2 A = \frac{1}{3}(\frac{4}{9})^1 A$. The number of sides now is 3×4^2.

We compute the total area added from previous side numbers and new triangle area at each level. To add all levels,

$$\sum_{n=1}^{\infty} \frac{1}{3}\left(\frac{4}{9}\right)^{n-1} A = \frac{1}{3}\sum_{n=1}^{\infty}\left(\frac{4}{9}\right)^{n-1} A = \frac{1}{3} \times \frac{1}{1-\frac{4}{9}} A = \frac{3}{5}A$$

So the total area is

$$A + \frac{3}{5}A = \frac{8}{5}A = \frac{8}{5}\frac{\sqrt{3}}{4} = \frac{2\sqrt{3}}{5}$$

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