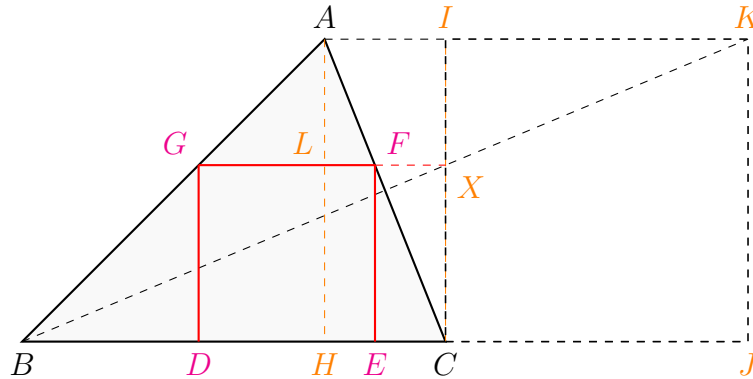


## Problem

Given an acute triangle  $\triangle ABC$ , construct with straightedge and compass square  $DEFG$  such that  $D$  and  $E$  are on  $\overline{BC}$ ,  $G$  is on  $\overline{AB}$  and  $F$  is on  $\overline{AC}$ .

Straightedge and compass can construct the middle point of a line segment, and perpendicular line through a point on a line segment.



Construction steps are:

1. draw height  $AH$  to  $BC$
2. extend  $BC$  to  $J$  such that  $CJ = AH$
3. draw square  $CJKI$
4. set intersection of  $CK$  and  $BJ$  to  $X$
5. draw line pass  $X$  and parallel to  $BC$ , intersect with  $AB$  at  $G$ , with  $AC$  at  $F$
6. draw perpendicular lines down from  $G$  and  $F$  to get  $D$  and  $E$

To prove  $DEFG$  is a square, since  $\triangle BCX \sim \triangle BJK$ ,

$$\frac{CX}{JK} = \frac{BC}{BJ} \quad \text{or} \quad \frac{EF}{AH} = \frac{BC}{BC + AH} \quad \text{or} \quad EF = \frac{BC * AH}{BC + AH} \quad (1)$$

Since  $\triangle ABC \sim \triangle AGF$ ,

$$\frac{GF}{BC} = \frac{AL}{AH} = \frac{AH - EF}{AH} \quad (2)$$

Then we just need to verify that  $GF = EF$ . From (2),

$$\begin{aligned} GF &= BC * \frac{AH - EF}{AH} = \frac{BC}{AH} * (AH - EF) \\ &= \frac{BC}{AH} * \left( AH - \frac{BC * AH}{BC + AH} \right), \quad \text{from (1)} \\ &= \frac{BC}{AH} * \frac{AH^2}{BC + AH} = \frac{BC * AH}{BC + AH} = EF \end{aligned} \quad (3)$$

The construction logic is derived from the  $GF$  expression (1), assuming that  $GF = FE$ . The expression gives hint to construct the square  $CJKI$ .