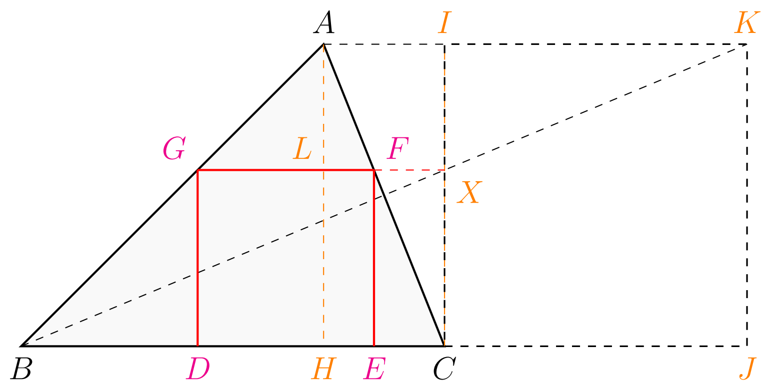
Problem

Given an acute triangle $\triangle ABC$, construct with straightedge and compass square $DEFG$ such that D and E are on $\overline{BC}$, G is on $\overline{AB}$ and F is on $\overline{AC}$.

Straightedge and compass can construct the middle point of a line segment, and perpendicular line through a point on a line segment.



Construction steps are:

1. draw height AH to BC
2. extend BC to J such that $CJ = AH$
3. draw square $CJKI$
4. set intersection of CK and BJ to X
5. draw line pass X and parallel to BC, intersect with AB at G, with AC at F
6. draw perpendicular lines down from G and F to get D and E

To prove $DEFG$ is a square, since $\triangle BCX \sim \triangle BJK$,

$$\frac{CX}{JK} = \frac{BC}{BJ} \quad \text{or} \quad \frac{EF}{AH} = \frac{BC}{BC + AH} \quad \text{or} \quad EF = \frac{BC \cdot AH}{BC + AH} \tag{1}$$

Since $\triangle ABC \sim \triangle AGF$,

$$\frac{GF}{BC} = \frac{AL}{AH} = \frac{AH - EF}{AH} \tag{2}$$

Then we just need to verify that $GF = EF$. From (2),

$$
\begin{aligned}
GF &= BC \cdot \frac{AH - EF}{AH} = \frac{BC}{AH} \cdot (AH - EF) \\
&= \frac{BC}{AH} \cdot (AH - \frac{BC * AH}{BC + AH}), \quad from(1) \\
&= \frac{BC}{AH} \cdot \frac{AH^2}{BC + AH} \\
&= \frac{BC \cdot AH}{BC + AH} = EF
\end{aligned}
\tag{3}
$$

The construction logic is derived from the GF expression (1), assuming that $GF = FE$. The expression gives hint to construct the square $CJKI$.