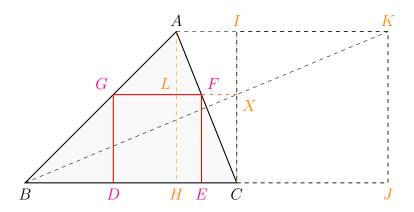
## Problem

Given an acute triangle  $\triangle ABC$ , construct with straightedge and compass square DEFG such that D and E are on  $\overline{BC}$ , G is on  $\overline{AB}$  and F is on  $\overline{AC}$ .

Straightedge and compass can construct the middle point of a line segment, and perpendicular line through a point on a line segment.



Construction steps are:

- 1. draw height AH to BC
- 2. extend BC to J such that CJ = AH
- 3. draw square CJKI
- 4. set intersection of  ${\cal C}{\cal K}$  and  ${\cal B}{\cal J}$  to  ${\cal X}$
- 5. draw line pass X and parallel to BC, intersect with AB at G, with AC at F
- 6. draw perpendicular lines down from G and F to get D and E

To prove DEFG is a square, since  $\triangle BCX \sim \triangle BJK$ ,

$$\frac{CX}{JK} = \frac{BC}{BJ} \quad \text{or} \quad \frac{EF}{AH} = \frac{BC}{BC + AH} \quad \text{or} \quad EF = \frac{BC \cdot AH}{BC + AH} \tag{1}$$

Since  $\triangle ABC \sim \triangle AGF$ ,

$$\frac{GF}{BC} = \frac{AL}{AH} = \frac{AH - EF}{AH} \tag{2}$$

Then we just need to verify that GF = EF. From (2),

$$GF = BC \cdot \frac{AH - EF}{AH} = \frac{BC}{AH} \cdot (AH - EF)$$

$$= \frac{BC}{AH} \cdot (AH - \frac{BC * AH}{BC + AH}), \quad from(1)$$

$$= \frac{BC}{AH} \cdot \frac{AH^2}{BC + AH}$$

$$= \frac{BC \cdot AH}{BC + AH} = EF$$
(3)

The construction logic is derived from the GF expression (1), assuming that GF = FE. The expression gives hint to construct the square CJKI.