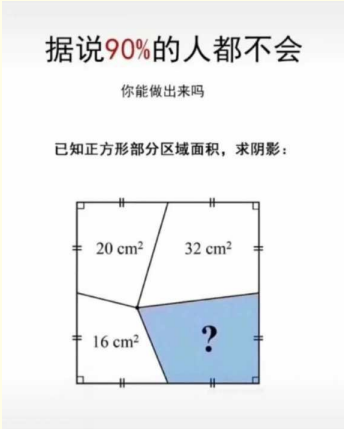
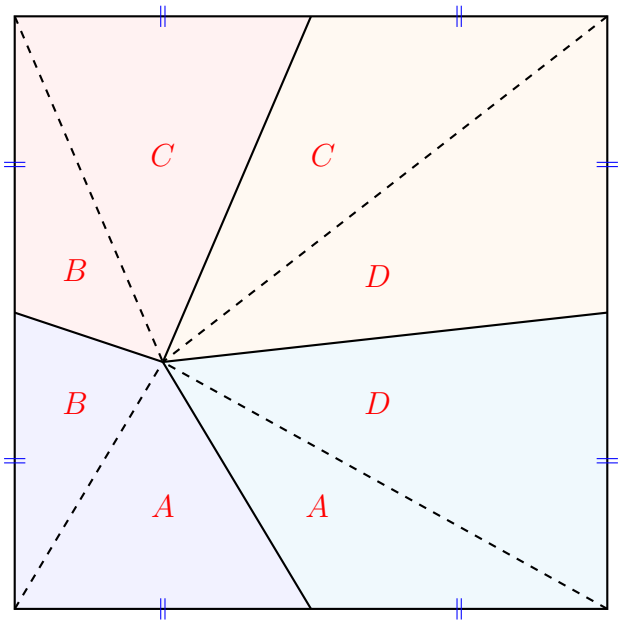
Problem

A point inside a square are connected to all side midpoints. Given the areas of 3 regions, find the area of the shaded.



The key to solve this problem is to divide non-triangle regions to triangle regions in order to see details of the given conditions. Once we do that, we can see the triangle areas are in pairs.



Notice that sum of opposite regions have same triangle areas, $A+B+C+D$

$$\begin{aligned} A+D(rightlower) &= A+B(lower\ left)+C+D(upper\ right)-(B+C)(upper\ left) \\ &= 16+32-20 \\ &= 28 \end{aligned}$$

Now to go one step further, we want to know the length of the square and the coordinates of the internal point(P). Assume square side has length $2a$ and P's coordinate is (x, y). From the given areas of the 3 regions, we derive the following equations

$$\frac{1}{2}xa + \frac{1}{2}ya = 16$$
$$\frac{1}{2}xa + \frac{1}{2}(2a - y)a = 20$$
$$\frac{1}{2}(2a - y)a + \frac{1}{2}(2a - x)a = 32$$

The solution is

$$a = 2\sqrt{6},\ x = \sqrt{6},\ and\ y = \frac{5}{3}\sqrt{6}$$

So x is half of a, a quarter of the square side length. y is near side midpoint. These are used to draw the picture.

Here has the same solution and more similar problems.