

Problem

Given two random variables X and Y over uniform distribution $U(0,1)$. Find the PDF of $Z = \min(X, Y)$.

This [post](#) (for $U(0,1)$), this [post](#) (for $U(a,b)$), this [post](#) (with extended references) and this [post](#) give the solution for n-variable case:

$$F_Z(z) = P(\min_i X_i \leq z) = 1 - (1 - P(X_i \leq z))^n \quad (1)$$

for any i .

Since X_i has uniform distribution, $P(X_i \leq z) = z$ and

$$F_Z(z) = 1 - (1 - z)^n \text{ and } f_Z(z) = n(1 - z)^{n-1} \quad (2)$$

The expected value is

$$\mathbb{E}(Z) = \int_0^1 t f_Z(t) dt = n \int_0^1 t(1 - t)^{n-1} dt = \frac{1}{n+1} \quad (3)$$

Similarly, see [this post](#), if $W = \max_i X_i$, then

$$F_W(w) = w^n \text{ and } f_W(w) = nw^{n-1} \quad (4)$$

The expected value is

$$\mathbb{E}(W) = \int_0^1 t f_W(t) dt = n \int_0^1 t t^{n-1} dt = n \int_0^1 t^n dt = \frac{n}{n+1} \quad (5)$$

This [post](#) discusses the expectation of $\max - \min$.

This [post](#) initiates a thought on spectrum of n .

This [post](#) discusses sums of k smallest (out of n variables).