Case Study 4

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***Abstract*—Numerical methods**

I. Introduction

DIFFERENTIAL

II. Euler's Method

A. First Order Derivation

Euler's method is based on a Taylor series approximation of $\mathbf{y}(x)$.

$$\mathbf{y}(x) = \mathbf{y}(a) + \mathbf{y}'(a)(x-a) + \mathbf{y}''(a)\frac{(x-a)^2}{2} + \mathbf{y}'''(a)\frac{(x-a)^3}{6} + \mathbf{y}^{(4)}(a)\frac{(x-a)^4}{24} + \dots \tag{1}$$

$$\mathbf{y}(x+h) = \sum_{n=0}^{\infty} \mathbf{y}^{(n)}(x)\frac{h^n}{n!} \tag{2}$$

```
#y_{i+1}=y_i+h*y'_i=y_i+h*F(x,y)
def euler(F, x, y, xStop, h):
  X = np.array(x)
  Y = np.array(y)
  while x < xStop:
    #Avoid overstepping
    h = min(h, xStop - x)
    #y'=F(x,y)
    y = y + h*F(x,y)#Use equation 6
    #to calculate the next y
    x = x + h
    X = np.append(X, x)
    Y = np.append(Y, y)
  return X,Y
```

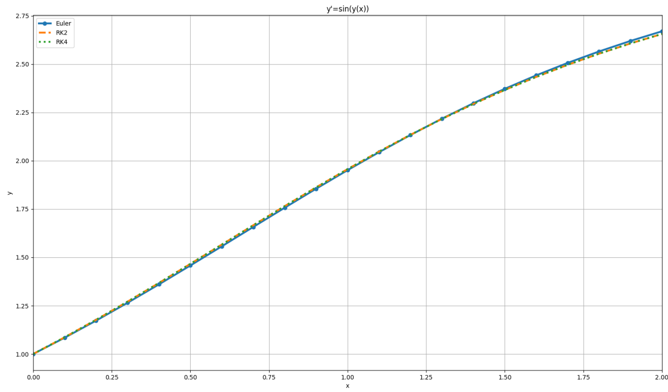

Fig. 1. Various ODE solutions.

$$\mathbf{K}_0 = h\mathbf{F}(x, \mathbf{y}) \tag{3}$$

$$\mathbf{K}_1 = h\mathbf{F}\left(x + \frac{h}{2}, \mathbf{y} + \frac{\mathbf{K}_0}{2}\right) \tag{4}$$

$$\mathbf{K}_2 = h\mathbf{F}\left(x + \frac{h}{2}, \mathbf{y} + \frac{\mathbf{K}_1}{2}\right) \tag{5}$$

$$\mathbf{K}_3 = h\mathbf{F}\left(x + h, \mathbf{y} + \mathbf{K}_2\right) \tag{6}$$

$$\mathbf{y}(x+h) = \mathbf{y}(x) + \frac{1}{6}\left(\mathbf{K}_0 + 2\mathbf{K}_1 + 2\mathbf{K}_2 + \mathbf{K}_3\right) \tag{7}$$

APPENDIX
CODE FOR PYTHON3

```
import numpy as np
#from math import *

a = np.identity(3)
a = a*np.sin(a)


if __name__ == "__main__":
    print("Case Study 4")
    print(a)
```