Double Momentum Correction using Missing Mass^2 ($ep \to ep\pi_0$)

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1 Loss

Loss function:

$$J = \mathbb{E}\left[\left(MM^2_{\pi_0} - MM^2_{pred}\right)^2\right] \tag{1}$$

However, in practice I use: (makes the loss easier to interpret)

$$J = \mathbb{E}\left[\left(1 - \frac{MM^2_{pred}}{MM^2_{\pi_0}}\right)^2\right] \tag{2}$$

$$\pi_0 = beam + targ - e' - p' \tag{3}$$
$$MM^2 = (beam + targ - e' - p')^2 \tag{4}$$

$$eE1' = \sqrt{(c1 \cdot ex')^2 + (c1 \cdot ey')^2 + (c1 \cdot ez')^2 + eM^2)} \tag{5}$$
$$pE2' = \sqrt{(c1 \cdot ex')^2 + (c1 \cdot ey')^2 + (c1 \cdot ez')^2 + eM^2)} \tag{6}$$
$$\tag{7}$$

$$E = eE + pE - eE1' - pE2' \tag{8}$$
$$x = ex + px - c1 \cdot ex' - c2 \cdot px' \tag{9}$$
$$y = ey + py - c1 \cdot ey' - c2 \cdot py' \tag{10}$$
$$z = ez + pz - c1 \cdot ez' - c2 \cdot pz' \tag{11}$$
$$\tag{12}$$

if $c1, c2 = 0, 0$:

$$E = eE + pM - 0 - pM \tag{13}$$
$$x = 0 + 0 - 0 - 0 \tag{14}$$
$$y = 0 + 0 - 0 - 0 \tag{15}$$
$$z = eE + 0 - 0 - 0 \tag{16}$$
$$\tag{17}$$

$$MM^2 = (eE)^2 - (eE)^2 = 0 \tag{18}$$
$$MM^2_{\pi_0} = 0.138^2 = 0.019 \tag{19}$$
$$J = \left(0.138^2 - 0\right)^2 = 0.0004 \tag{20}$$

With the data:

- $c1, c2 = 0, 0$ - MM^2 error: 0.0009 GeV^2 (0.03 GeV)
- $c1, c2 = 1, 1$ - MM^2 error: 0.007 GeV^2 (0.08 GeV)

Loss function thinks $c1, c2 = 0, 0$ ($e'_{corrected} = (eM, 0, 0, 0)$, $p'_{corrected} = (pM, 0, 0, 0)$) is a better solution than $c1, c2 = 1, 1$ ($e'_{corrected} = (eE', ex', ey', ez')$, $p'_{corrected} = (pE', px', py', pz')$, no correction).

2 Regularization

The error should be small so $c1, c2 = 1, 1$ should be very close to the desired minimum. We can penalize predictions that are far from 1.

$$J = \mathbb{E}\left[\left(MM_{\pi_0}^2 - MM_{pred}^2\right)^2\right] - \alpha(1 - c1)^2 - \beta(1 - c2)^2 \tag{21}$$

Introduces two hyper-parameters to tune: α and β. However, cursory testing indicated it does not help.