

MASARYK UNIVERSITY
FACULTY OF INFORMATICS



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BACHELOR'S THESIS

«author»

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1 Introduction

2 Preliminaries

2.1 Büchi Automaton

Büchi automaton is a theoretical finite state machine that decides which infinitely long words (ω -words) belong to its language. Let us define the transition-based Büchi Automaton (TBA).

TBA is a quintuple $A = (\Sigma, Q, q_0, \Delta, \Gamma)$. An alphabet is a set of letters. Σ is a finite alphabet recognized by A . Q is a finite set of states of A , $q_0 \in Q$ is called the initial state of A . Transitions of A directionally connect two states inside Q with a letter from alphabet Σ . We write the set of transitions as $\Delta \subseteq Q \times \Sigma \times Q$. Subset of the transitions $\Gamma \subseteq \Delta$ are accepting transitions.

Now our goal is to define language L_A . It allows us to specify which ω -words are accepted or rejected by the automaton. To achieve our goal, we will need to define run on automaton A .

Run $r \in \Delta^\omega$ on A is an infinite sequence of transitions such that for the first transition $r_0 = (q'_0, a, q''_0) \in r$ holds that q'_0 is the initial state of A and for each subsequent state $r_i = (q'_i, a, q''_i)$ holds that $q'_i = q''_{i-1}$. A run of TBA is accepting iff it contains infinitely many accepting transitions.

Finally, we can define the language $L_A \in \Sigma^\omega$ recognized by the automaton A . An ω -word $w \in \Sigma^\omega$ belongs to L_A if there exists an accepting run r in the automaton A for the word w .

2.2 TGBA

3 To Delete Chapter

3.1 Büchi Automaton

A nondeterministic Büchi automaton (BA) is a tuple $A = (\Sigma, Q, q_0, \Delta, \Gamma)$, where

- Σ is a finite alphabet
- Q is finite set of states
- $q_0 \in Q$ is the initial state
- $\Delta \subseteq Q \times \Sigma \times Q$ are transitions
- $\Gamma \subseteq \Delta$ are accepting transitions

run A run r of A on $w \in \Sigma^\omega$ is an ω -word $r_0, w_0, r_1, w_1, \dots$ in $(Q \times \Sigma)^\omega$ such that $r_0 = q_0 \wedge \forall i > 0, (r_{i-1}, w_{i-1}, r_i) \in \Delta$

inf(r) We write $\text{inf}(r) \subseteq \Delta$ for the set of transitions that appear infinitely often in the run r .

accepting run A run r is accepting if $\text{inf}(r) \cap \Gamma \neq \emptyset$

language The language $L_A \subseteq \Sigma^\omega$ is recognized by A .
 $\forall w \in L_A \exists r$ on w such that r is accepting.

ω -regular language A language is ω -regular if it is accepted by BA.

deterministic automaton $A = (\Sigma, Q, q_0, \Delta, \Gamma)$ is deterministic if
 $(q, \rho, q'), (q, \rho, q'') \in \Delta \implies q' = q''$

complete automaton A is complete if, $\forall w \in \Sigma, \forall q \in Q, \exists (q, w, q') \in \Delta$. A word in Σ^ω has exactly one run in a deterministic, complete automaton.

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zminit v intro

3.2 Markov Decision Processes

A Markov decision process (MDP) M is a tuple (S, A, T, Σ, L) , where

- S is a finite set of states
- A is a finite set of actions
- $T : S \times A \rightarrow D(S)$, where $D(S)$ is set of probability distributions over S , is the probabilistic transition (partial) function
- Σ is an alphabet
- $L : S \times A \times S \rightarrow \Sigma$ is the labeling function of the set of transitions. For a state $s \in S$, $A(s)$ denotes the set of actions available in s .

run A run of M is an ω -word $s_0, a_1, \dots \in A = S \times (A \times S)^\omega$ such that $Pr(s_{i+1}|s_i, a_{i+1}) > 0$ for all $i \geq 0$. A finite run is a finite such sequence.

labeled run We define labeled run as $L(r) = L(s_0, a_1, s_1), L(s_1, a_2, s_2), \dots \in \Sigma^\omega$.

paths We write $\Omega(M)(Paths(M))$ for the set of runs (finite runs) of M and $\Omega_s(M)(Paths_s(M))$ for the set of runs (finite runs) of M starting from state s . When the MDP is clear from the context we drop the argument M .

strategy A strategy in M is a function $\mu : Paths \rightarrow D(A)$ such that $supp(\mu(r)) \subseteq A(last(r))$, where $supp(d)$ is the support of d and $last(r)$ is the last state of r . Let Ω_μ^M denote the subset of runs Ω^M that correspond to strategy μ and initial state s . Let Π_M be the set of all strategies.

pure strategy We say that a strategy μ is pure if $\mu(r)$ is a point distribution for all runs $r \in Paths$.

behavior The behavior of an MDP M under a strategy μ with starting state s is defined on a probability space $(\Omega_s^\mu, F_s^\mu, Pr_s^\mu)$ over the set of infinite runs of μ from s .

3.3 [WIP]Good-for-MDP (GFM) Automata

Given an MDP M and an automaton $A = (\Sigma, Q, q_0, \Delta, \Gamma)$, we want to compute an optimal strategy satisfying the objective that the run of M is in the language of A .

semantic satisfaction probability for given automaton and strategy

We define the semantic satisfaction probability for A and strategy μ from state s as:

$$PSem_A^M(s, \mu) = Pr_s^\mu \{r \in \Omega_s^\mu : L(r) \in L_A\}$$

semantic satisfaction probability for given automaton

$$PSem_A^M(s) = \sup_{\mu \in \Pi_M} PSem_A^M(s, \mu)$$

syntactic variant of the acceptance condition When using automata for given analysis of MDPs, we need a syntactic variant of the acceptance condition.

product of MDP and automaton Given an MDP $M = (S, A, T, \Sigma, L)$ with initial state $s_0 \in S$ and automaton $A = (\Sigma, Q, q_0, \Delta, \Gamma)$, the product $M \times A = (S \times Q, (s_0, q_0), A \times Q, T^\times, \Gamma^\times)$ is an MDP augmented with an initial state $s_0 \in S$ and accepting transitions Γ^\times . The (partial) function $T^\times : (S \times Q) \times (A \times Q) \rightarrow D(S \times Q)$ is defined by

$$T^\times((s, q), (a, q'))((s', q')) = \begin{cases} T(s, a)(s') & \text{if } (q, L(s, a, s'), q^1) \in \Delta \\ \text{undefined} & \text{otherwise} \end{cases}$$

GFM Automata An automaton A is good for MDPs if, for all MDPs M , $PSYN_A^M(s_0) = PSEM_A^M(s_0)$ holds, where s_0 is the initial state of M .

3.4 to be defined

ω -word?, point distribution?, what is F_s^μ in 'pure strategy' paragraph?, TGBA, describe Semi-deterministic as I am going to compare them with SBA

3.4.1 text

GF MDP, model checking

3.5 Algorithms

BP + both slim

4 Implementation

4.1 Technologies

4.2 Implementation inside Seminotor

5 Evaluation

5.1 Alternative Algorithm

5.2 Different Implementation - ePMC

5.3 Semi-deterministic Automata

6 Conclusion