

MASARYK UNIVERSITY  
FACULTY OF INFORMATICS



# **Transformation of Nondeterministic Büchi Automata to Slim Automata**

BACHELOR'S THESIS

**Pavel Šimovec**

Brno, Spring 2021



*Replace this page with a copy of the official signed thesis assignment and a copy of the Statement of an Author.*



## **Declaration**

Hereby I declare that this paper is my original authorial work, which I have worked out on my own. All sources, references, and literature used or excerpted during elaboration of this work are properly cited and listed in complete reference to the due source.

Pavel Šimovec

**Advisor:** doc. RNDr. Jan Strejček, Ph.D.



## **Acknowledgements**

RNDr. František Blahoudek, Ph.D.

# **Abstract**

abstract



## Keywords

keyword1, keyword2, ...



# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Preliminaries</b>	<b>3</b>
2.1	Büchi Automaton . . . . .	3
2.2	Generalized Büchi Automaton . . . . .	3
<b>3</b>	<b>Slim Automata Construction</b>	<b>5</b>
3.1	Breakpoint Automaton . . . . .	5
3.2	Slim automata . . . . .	6
<b>4</b>	<b>Slim Automaton Construction Generalized to TGBA</b>	<b>9</b>
<b>5</b>	<b>Implementation</b>	<b>13</b>
5.1	Technologies/Tools . . . . .	13
5.1.1	Seminator . . . . .	13
5.1.2	Spot . . . . .	13
5.2	Create Slim Automata Using Seminator . . . . .	14
5.3	Implementation of Slim Automata inside Seminator . . . . .	15
5.4	Testing and Verification . . . . .	15
5.5	How to Install Seminator . . . . .	15
5.6	Future of Implementation . . . . .	16
<b>6</b>	<b>Evaluation of automaton size</b>	<b>17</b>
6.1	Slim automata produced by Seminator . . . . .	17
6.1.1	Comparisons among Unoptimized Configurations . . . . .	17
6.1.2	Post-Optimized . . . . .	18
6.2	Slim Automata Produced Seminator versus ePMC . . . . .	21
6.3	Compare with Different Kinds of Automata . . . . .	23
6.4	Note about optimized automata . . . . .	25
<b>7</b>	<b>Mungojerrie benchmarks</b>	<b>27</b>
<b>8</b>	<b>Conclusions</b>	<b>31</b>
	<b>Bibliography</b>	<b>33</b>



## List of Figures

- 3.1 *A Büchi Automaton  $A$  (left) and a breakpoint automaton  $D$  for  $A$  (right).* 6
- 3.2 *Slim automaton (right) and the original Buchi Automaton from Figure 3.1(left)* 7
- 4.1 *The original TGBA (top) and slim automaton with colored states emphasizing different levels (bottom)* 11
- 6.1 *Scatter plot of strong against weak slim automata, equal values excluded. We can see strong slim automaton is smaller in more cases, but there are also cases where weak slim automaton is smaller.* 19
- 6.2 *Scatter plot via-tba against via-tgba, equal values excluded. Plot density is low, as many of results are the same (majority of the automata were already tba). Via-tba has mostly better results, but there are also examples where via-tgba outputs automaton twice smaller automaton.* 20
- 6.3 *On scatter plot comparing sizes we can see that ePMC has some better hits.* 21
- 6.4 *If we compare smallest automata of each tool, Seminotor slim has most of the best results and the difference is even bigger than single option comparison. Combining Seminotor and ePMC for best automata didn't bring much better results. On random dataset total size of automata from Seminotor slim is 7133. If we combine Seminotor slim and ePMC, we get total size 7060, which is not that significant difference.* 22
- 6.5 *Semi-deterministic automata are consistently smaller or just slightly bigger than slim automata.* 23
- 6.6 *Ltl2ldb creates consistently small automata (smaller than 40 states), however its dominance over slim automata is not that consistent. The plot shows we can create reasonably smaller MDP automata on average using both tools and choosing the smallest automaton.* 24
- 7.1 *textit* 29



# 1 Introduction

Büchi automaton is a finite machine over infinite words. It has been a topic of research for almost 60 years. There were discovered various kinds of similar machines with different properties and use cases. Non-deterministic Büchi automata in general are not well suitable for model checking or reinforcement learning, but we can construct non-deterministic Büchi automata with a special property - GFM, that makes the automata suitable. We will focus on slim automata [1]. Slim automata are specially constructed Büchi automata. This kind of automaton was defined by its construction and is good for MDP [1]. We implement the proposed algorithm and its second variant that we call weak [source private conversation]. We introduce the algorithm for generalized Büchi automata. Then we evaluate resulting size of automata and we compare it with different tool to create slim automata and with other kinds of automata.

Preliminary chapter defines Büchi automaton and generalized Büchi automaton. Next chapter defines breakpoint automaton and slim automaton as described in our main source [1] and the weak variant. In fourth chapter we extend the algorithm to generalized Büchi automata. Fifth chapter describes implementation of mentioned slim automata inside Seminotor[**seminator**] tool. In sixth chapter we compare resulting automata size (number of states). First we compare the size internally among implemented options. Then we compare it with ePMC [**epmc**] - another tool implementing slim automata. We finish the chapter by comparisons with different kinds of automata. In the last chapter we use our automata in benchmarks of reinforcement learning tool Mungojerrie[9] and we compare learning speed on it with original automata in provided benchmarks with original benchmark automata and 2 types of automata internally supported by Mungojerrie (slim automata by ePMC and Limit-deterministic Büchi automata by ltl2ldba [8]).





## 2 Preliminaries

This chapter defines a Büchi automaton and a generalized Büchi automaton.

An *alphabet*  $\Sigma$  is a finite set of *letters*, an  $\omega$ -word  $w \in \Sigma^\omega$  is an infinite sequence of letters, and an  $\omega$ -language  $L \subseteq \Sigma^\omega$  is a set of  $\omega$ -words.

### 2.1 Büchi Automaton

A Büchi automaton is a theoretical finite-state machine used to define  $\omega$ -languages. It decides which infinite words ( $\omega$ -words) belong to its language.

A *transition-based Büchi automaton* (TBA) is a tuple  $\mathcal{A} \stackrel{\text{def}}{=} (\Sigma, Q, q_i, \Delta, \Gamma)$ , where

- $\Sigma$  is a non-empty finite *alphabet*,
- $Q$  is a non-empty finite set of *states*,
- $q_i \in Q$  is the initial state of  $\mathcal{A}$ .
- $\Delta \subseteq Q \times \Sigma \times Q$  is a set of *transitions*.
- $\Gamma \subseteq \Delta$  is a set of *accepting transitions*.

Intuitively, a transition  $(s, a, t)$  directionally connects the states  $s$  and  $t$  with the letter  $a$ .

A *run*  $r$  of  $\mathcal{A}$  over an  $\omega$ -word  $w = w_0w_1w_2 \dots$  is an infinite sequence of transitions  $r \stackrel{\text{def}}{=} t_0t_1 \dots \in \Delta^\omega$ , where  $t_k = (q_k, w_k, q_{k+1})$ , such that  $q_0 = q_i$ . A run of  $\mathcal{A}$  is *accepting* if and only if it contains infinitely many accepting transitions from  $\Gamma$ .

Finally, we define the *language*  $L(\mathcal{A}) \subseteq \Sigma^\omega$  recognized by the automaton  $\mathcal{A}$ . An  $\omega$ -word  $w \in \Sigma^\omega$  belongs to  $L(\mathcal{A})$  if and only if there exists an accepting run of  $\mathcal{A}$  over the word  $w$ .

### 2.2 Generalized Büchi Automaton

A *transition-based Generalized Büchi automaton* (TGBA) is a tuple  $\mathcal{A} \stackrel{\text{def}}{=} (\Sigma, Q, q_i, \Delta, G)$ , where  $G \subseteq 2^\Delta$  contains sets of accepting conditions and

## 2. PRELIMINARIES

---

the rest is defined as for TBA. A run of  $\mathcal{A}$  is *accepting* iff it contains infinitely many accepting transitions *for each*  $I \in G$ . TBA can be seen as a special case of TGBA with  $|G| = 1$ .

### 3 Slim Automata Construction

This chapter defines *slim Büchi automaton* (slim automaton) in 2 variants - *strong* and *weak*. Slim automaton is defined through its construction, which is based on breakpoint construction.

#### 3.1 Breakpoint Automaton

BP automata are constructed from BA and are deterministic, but their language is only a subset of the language from original BA.

**Construction** Let us fix a Büchi Automaton  $\mathcal{A} \stackrel{\text{def}}{=} (\Sigma, Q, q_i, \Delta, \Gamma)$ .

We start with some notation. By  $3^Q$  we denote the set  $\{(S, S') \mid S' \subsetneq S \subseteq Q\}$  and by  $3_+^Q$  we denote  $\{(S, S') \mid S' \subseteq S \subseteq Q\}$ .

For convenience we introduce functions by sets of transitions, we define the function  $\delta: 2^Q \times \Sigma \rightarrow 2^Q$  as  $\delta: (S, a) \stackrel{\text{def}}{=} \{q' \in Q \mid (q, a, q') \in \Delta \wedge q \in S\}$ . We define  $\gamma: 2^Q \times \Sigma \rightarrow 2^Q$  analogously from  $\Gamma$  as  $\gamma: (S, a) \stackrel{\text{def}}{=} \{q' \in Q \mid (q, a, q') \in \Gamma \wedge q \in S\}$ .

By definitions,  $\delta$  and  $\gamma$  can be seen as deterministic transition functions on  $2^Q$ .

With  $\delta$  and  $\gamma$ , we define the raw breakpoint transition  $\rho_r: 3^Q \times \Sigma \rightarrow 3_+^Q$  as

$$\rho_r((S, S'), a) \stackrel{\text{def}}{=} (\delta(S, a), \delta(S', a) \cup \gamma(S, a))$$

The first set follows the set of reachable states in the first set and the states that are reachable while passing at least one of the accepting transitions in the second set. The transitions of the breakpoint automaton  $\mathcal{D}$  follow  $\rho$  with an exception: they reset the second set to the empty set when it equals the first; the resetting transitions are accepting. Formally, the breakpoint automaton  $\mathcal{D}$  is  $\stackrel{\text{def}}{=} (\Sigma, 3^Q, (q_i, \emptyset), \Delta_D, \Gamma_D)$  where  $\Delta_D$  and  $\Gamma_D$  are defined as follows.

1.  $((S, S'), a, (R, R')) \in \Delta_D$  if  $\rho_r((S, S'), a) = (R, R')$  where  $R' \subsetneq R$
2.  $((S, S'), a, (R, \emptyset)) \in \Delta_D$  and  $((S, S'), a, (R, \emptyset)) \in \Gamma_D$  if  $\rho_r((S, S'), a) = (R, R)$
3. No other transitions are in  $\Delta_D$  and  $\Gamma_D$

### 3. SLIM AUTOMATA CONSTRUCTION

Figure 3.1 shows application of this construction. The example demonstrates that  $L(\mathcal{D}) \subseteq L(\mathcal{A})$  as the construction did not generate any accepting transition. Therefore original  $L(\mathcal{A}) = \{a^\omega\}$ , but  $L(\mathcal{D})$  is empty.

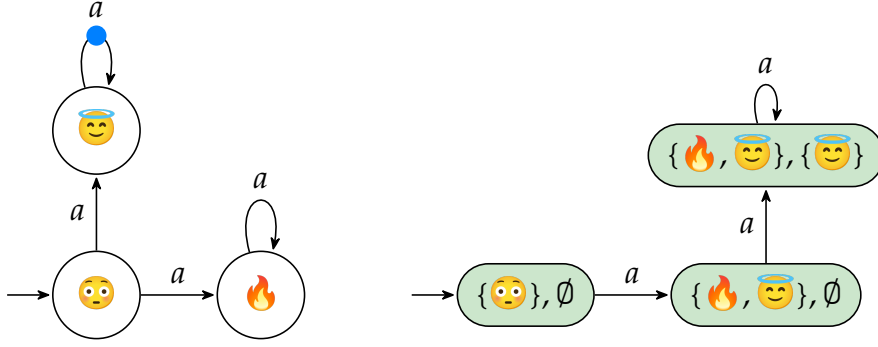


Figure 3.1: A Büchi Automaton  $\mathcal{A}$  (left) and a breakpoint automaton  $\mathcal{D}$  for  $\mathcal{A}$  (right).

### 3.2 Slim automata

Slim automata are BP automata enriched with additional transitions. As a result they are non-deterministic, Good for Markov decision processes [1] and equivalent to the input automaton. In this section we define *Breakpoint automaton* and transitions for *strong slim* ( $\gamma_p$ ) and *weak slim* ( $\gamma_w$ ) automata,  $\gamma_w, \gamma_p : 3^Q \times \Sigma \rightarrow 3^Q$ , that promote the second set of a breakpoint construction to the first set as follows.

1. if  $\delta_S(S', a) = \gamma_S(S, a) = \emptyset$ , then  $\gamma_p((S, S'), a)$  and  $\gamma_w((S, S'), a)$  are undefined, and
2. otherwise  $\gamma_p : ((S, S'), a) = (\delta(S', a) \cup \gamma(S, a), \emptyset)$  and  $\gamma_w : ((S, S'), a) = (\delta(S', a), \emptyset)$

$\mathcal{S} \stackrel{\text{def}}{=} (\Sigma, 3^Q, (q_i, \emptyset), \Delta_S, \Gamma_S)$  is slim, when  $\Delta_S = \Delta_D \cup \Gamma_p$  is set of transitions generated by  $\delta_D$  and  $\gamma_p$ , and  $\Gamma_S = \Gamma_D \cup \Gamma_p$  is set of accepting transitions, that is generated by  $\gamma_D$  and  $\gamma_p$ .  $L(\mathcal{S}) = L(\mathcal{A})$ . The equivalence was proven in [1].

### 3. SLIM AUTOMATA CONSTRUCTION

Alternatively, similarly defined using  $\gamma_w$  instead of  $\gamma_p$ , automaton  $\mathcal{W} \stackrel{\text{def}}{=} (\Sigma, 3^Q, (q_i, \emptyset), \Delta_W, \Gamma_W)$  is slim when  $\Delta_W = \Delta_D \cup \Gamma_w$  is set of transitions generated by  $\delta_D$  and  $\gamma_w$ , and  $\Gamma_W = \Gamma_D \cup \Gamma_w$  is set of accepting transitions, that is generated by  $\gamma_D$  and  $\gamma_w$ .  $L(\dot{\mathcal{S}}) = L(\mathcal{A})$  and  $L(\dot{\mathcal{S}}) = L(\mathcal{A})$ . (proof would go similarly like the one for strong slim)

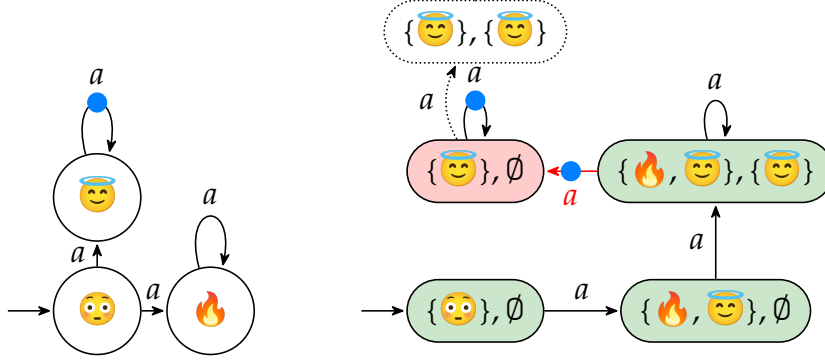


Figure 3.2: Slim automaton (right) and the original Buchi Automaton from Figure 3.1(left)



## 4 Slim Automaton Construction Generalized to TGBA

In this chapter, we discuss slim automata equivalent to a TGBA  $\mathcal{T} \stackrel{\text{def}}{=} (\Sigma, Q, q_i, \Delta, G)$ . One possibility is to *degeneralize*  $\mathcal{T}$  and to use previously mentioned algorithm in section 2.3. In the rest of this chapter we introduce a direct construction of slim TGBA equivalent to  $\mathcal{T}$ .

**Extended slim construction** (We will simulate the original automaton by checking its accepting conditions on by one. In the original automaton have to go through an accepting transition of each accepting condition  $g \in G = \{G_0, G_1, \dots, G_k\}$  infinitely many times. In new automaton we have just one accepting condition and a layer for each original accepting condition. Going through original accepting transitions of layer that we are looking up promotes us to another layer. From the last layer we get back to first layer. Only the transitions that move us layer up are accepting. As we check all accepting conditions of the original automaton, the new automaton will be equivalent to the original one.)

We need to make sure we go infinitely many times through each accepting subset  $g \in G$ . To achieve this, we will go through each subset one by one, using original algorithm. We will keep track of *levels*  $\stackrel{\text{def}}{=} \{0, 1, \dots, |G| - 1\}$  in the names of states. Let  $|G|$  be number of *levels* and  $i \in \mathbb{N}, i < |G|$  the current level. At each level  $i$ , we look at  $i$ th subset of  $G$ . We use same steps as in classic breakpoint construction, but on each accepting transition the new state will be leveled up to  $(i + 1) \bmod |G|$ , otherwise the target state has the same level. Our new automaton simulates  $\mathcal{T}$ , as it accepts a word if it cycles through all levels. If  $|G| = 0$ , we return a trivially accepting automaton

We can use the core of previous construction and just to extend it with levels. Let  $up(x) \stackrel{\text{def}}{=} (x + 1) \bmod |G|$  and let

$$P := 3^Q \times \text{levels}.$$

Let  $(S, S') \in 3^Q$  and let  $i \in \text{levels}$ , by  $P$  we denote a state  $P = (S, S', i)$ .

#### 4. SLIM AUTOMATON CONSTRUCTION GENERALIZED TO TGBA

---

We define  $\gamma_i$  from  $\Gamma_i$  for all  $i \in \text{levels}$  in the same way we did for  $\gamma$  from  $\Gamma$  and it allows us to easily define the raw generalized breakpoint transitions  $\rho_{\Gamma_i}$ : similarly as  $\rho_{\Gamma}$  using  $\gamma_i$  instead of  $\gamma$ .

The generalized breakpoint automaton  $\mathcal{D} = (\Sigma, 3^{\mathcal{Q} \times \mathcal{N}}, (q_i, \emptyset, 0), \delta_B, \gamma_B)$  is defined such that, when  $\delta_R: (P, a) \rightarrow (R, R', j)$ , then there are three cases:

1. if  $R = \emptyset$ , then  $\delta_B(P, a)$  is undefined,
  2. else, if  $R \neq R'$ , then  $\delta_B(P, a) = (R, R', i)$  is a non-accepting transition,
  3. otherwise  $\gamma_B(P, a) = \delta_B(P, a) = (R, \emptyset, up(i))$ .
1. if  $\delta(S', a) = \gamma_i(S, a) = \emptyset$ , then  $\gamma_p(P, a)$  is undefined, and
  2. otherwise  $\gamma_p: (P, a) = (\delta(S', a) \cup \gamma_i(S, a), \emptyset, up(i))$ . (Alternatively, for a weak slim automaton we do not include transitions  $\gamma_i(S, a)$ )

$\mathcal{S} \stackrel{\text{def}}{=} (\Sigma, P, (q_i, \emptyset, 0), \Delta_p, \Gamma_p)$  is slim, when  $\Delta_p$  is set of transitions generated by  $\delta_b$  and  $\gamma_p$ , and  $\Gamma_p$  is set of accepting transitions, that is generated by  $\gamma_b$  and  $\gamma_p$ . We construct weak slim automata



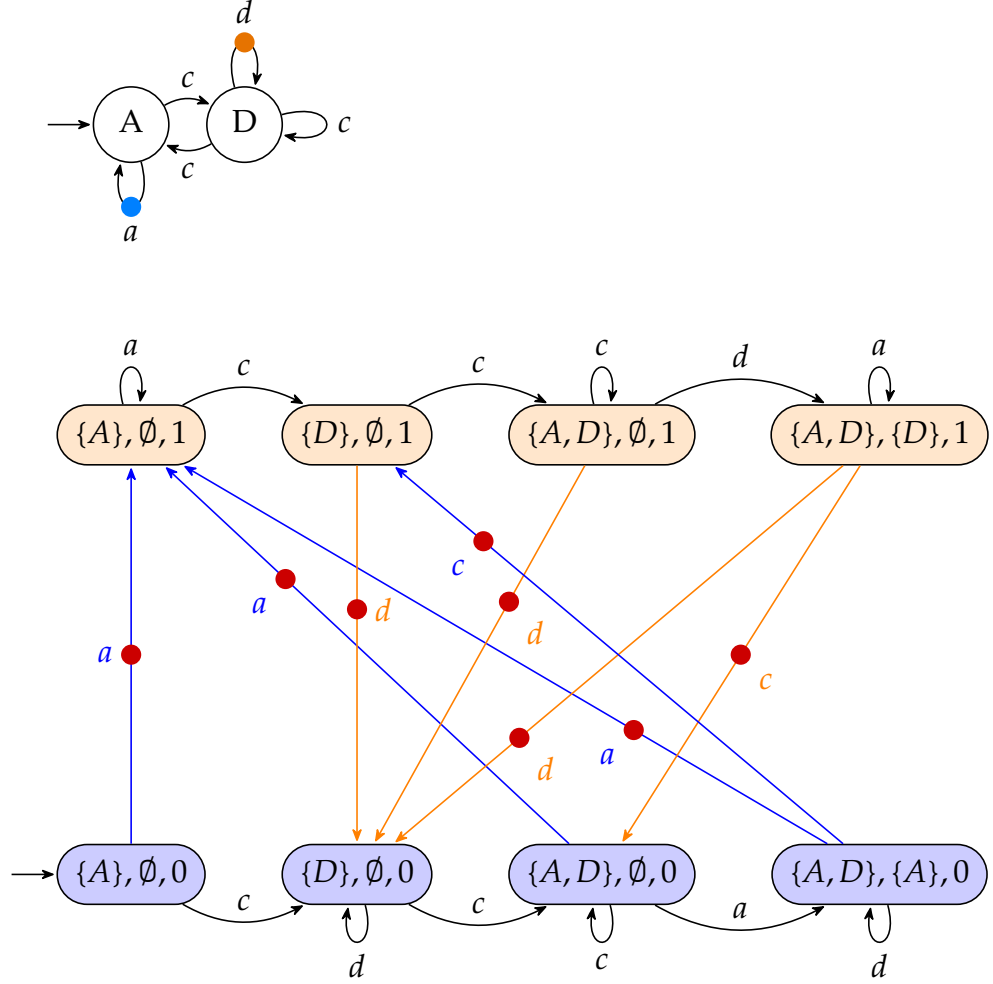


Figure 4.1: The original TGBA (top) and slim automaton with colored states emphasizing different levels (bottom)



## 5 Implementation

I have implemented the generalized construction of slim automata in both weak and strong version (3) (4). I have also added option to create breakpoint automata (3.1)(2.3).

### 5.1 Technologies/Tools

The implementation is inside seminator which is implemented in C++17 builds on Spot library.

#### 5.1.1 Seminator

Seminator is a Linux command-line tool which can be run with the `seminator` command. The tool transforms transition-based generalized Büchi automata (TGBAs) into equivalent semi-deterministic automata. [2]

The tool expects the input automaton in the Hanoi Omega-Automata (HOA) format [3] on the standard input stream, but it can also read the input automaton from a file.

#### 5.1.2 Spot

Spot is a C++ library with Python bindings and an assortment of command-line tools designed to manipulate LTL and  $\omega$ -automata in batch. [4]

Relevant spot tools:

**ltl2tgba** The `ltl2tgba` tool translates LTL or PSL formulas into different types of automata. [5]

**autfilt** The `autfilt` tool can filter, transform, and convert a stream of automata. [6]

**ltlcross** `ltlcross` is a tool for cross-comparing the output of LTL-to-automata translators. [7]

## 5.2 Create Slim Automata Using Seminor

By default, seminor creates sDBA. To create a slim automaton we need to add `--slim` option.

**Options** By default, `--slim` tries all reasonable combinations of options, optimizes the output and chooses an automaton with the smallest number of states.

**Example 1** Transform automaton.hoa to a slim automaton.

```
$/seminor --slim -f automaton.hoa
```

There are several options to specify how we construct the automata.

For example `seminor --slim --strong --optimizations=0 --via-tgba` generates output according to algorithm in 2.5. (Using `--via-tba` converts input to tba first) With automaton

**Example 2** Transform automaton.hoa to unoptimized strong slim automaton

```
$/seminor --slim --strong --via-tgba --  
optimizations=0 -f automaton.hoa
```

`--slim` to generate slim automaton

`--weak` use only weak slim algorithm

`--strong` use only strong slim algorithm

Neither weak or strong option specified - try both options and choose the one with smaller automaton.

`--via-tba` transform input automaton to tba (2.1) first

`--via-tgba` does not modify input automaton to tba.

Neither `--via-tba` nor `--via-tgba`: try both options, choose the smallest automaton

Postprocess optimizations are enabled by default.

### 5.3 Implementation of Slim Automata inside Seminor

I have implemented the generalized slim construction and its options mentioned in previous section 3.2. Furthermore, I have added an option to create breakpoint automata.

There already was basis for breakpoint construction in seminor, inside class `bp_twa`. As we can see in sections 3.1 and 3, slim automata construction builds on breakpoint automata construction.

That allows us to simply extend the `bp_twa` class. We create class `slim` that inherits from `bp_twa`. In the `slim` class we build breakpoint automaton using `compute_successors` method. Then we extend the method by adding accepting transitions  $\gamma_p$ , respectively  $\gamma_w$  according to section 2.4, whenever we receive `--slim` option.

Then we extend main function to recognize our desired CLI options.

As seminor didn't offer a command line option to create a breakpoint automaton, I have added the option `--bp` for comparison.

### 5.4 Testing and Verification

Implemented tests are basic, only language equivalence is checked. `ltlcross` and `ltl2tgba` tools are used. The tests use random LTL formulas that were already generated, the LTL formulas are transformed into automata in HOA format by `ltl2tgba`. Then the tool `ltlcross` cross-compares the automaton with *seminor* `--slim` with all supported additional parameters.

Only `seminor --slim --strong --via-tba --optimizations=0` is proved, as it follows construction from [1] which is proved.

### 5.5 How to Install Seminor

(jeste nevim kde tuto sekci dat, jestli ma mit tento nazev, co vsechno tady bude treba dat... no a jeste ro pak upravim podle toho jak to bude odevzdane v zipu)

To install the tool we need install spot and to run

## 5. IMPLEMENTATION

---

```
autoreconf -i && ./configure && make.
```

### 5.6 Future of Implementation

Implementation: python bindings, optimizations of slim construction (especially from TGBA)

tests/verification: There should be another kind of tests - to check if our slim automata simulate the input automata (so the GFM property is not broken)

Subject of following research, that is out of scope of this thesis, could be to verify if spot optimizations do not break the simulation property.

## 6 Evaluation of automaton size

Evaluation part builds on seminotor-evaluation. We compare amount of states of output automata on 2 datasets. First dataset are 20 literature formulas, second dataset is 500 automata that were randomly generated. First section start by internal comparison of slim automata created by Seminotor. Second section compares these automata against ePMC, which is another tool producing slim automata. Third section compares slim automata against ldba and semi-deterministic automata. Ldba automata are produced by ltl2ldb [8], and semi-deterministic automata are produced by Seminotor. Let us note, that from mentioned types of automata, only semi-deterministic automata do not promise good for Markov decision processes property.

### 6.1 Slim automata produced by Seminotor

In this section we compare automaton size generated by seminotor–slim. We compare weak against slim and via-tba against via-tgba.

#### 6.1.1 Comparisons among Unoptimized Configurations

In this subsection we compare base unoptimized seminotor options.

Table 6.1: Slim automata on both datasets without any post-processing. Strong slim automata have more states than weak ones, as expected, because strong slim automata add more accepting transitions, which can create new states.

seminator	weak		strong	
literature	size	time(s)	size	time(s)
via tba	1095	4	1112	5
via tgba	1122	4	1147	4

seminator	weak		strong	
random	size	time(s)	size	time(s)
via tba	17567	59	18764	57
via tgba	19320	58	20789	58

## 6. EVALUATION OF AUTOMATON SIZE

Transforming automata to TBA first yields smaller automata. This might be caused by Spot having well optimized algorithm for degeneralization. Slim algorithm for TGBA proposed in this paper is naive, without any kind of optimizations, and it degeneralizes the automaton during the process.

Using weak slim algorithm creates smaller slim automata than the strong one.

### 6.1.2 Post-Optimized

In this subsection we post-optimize results using `autfilt` tool.

Table 6.2: In this table we compare all possible post-optimized combinations of parameters. By default `seminator --slim` tries all combination, runs optimizations, and then chooses the smallest automaton (best/best).

seminator	weak		strong		best	
literature	size	time[s]	size	time[s]	size	time[s]
via tba	551	219	370	185	370	404
via tgba	588	210	408	174	402	384
best	551	429	370	359	365	788
seminator	weak		strong		best	
random	size	time[s]	size	time[s]	size	time[s]
via tba	8923	443	7404	476	7219	919
via tgba	10130	654	8500	591	8247	1245
best	8751	1097	7285	1067	7088	2164

From 4 base options; after applying post-optimizations strong slim algorithm surpasses weak one by resulting automaton size, even if it has worse results without the post-optimizations. Degeneralizing the automata as a first step still has smaller results. From 4 base options, strong slim algorithm via-tba creates smallest automata on average. Transforming input automata to tba first creates results which are close to best ones.



## Strong against Weak slim automata

Let us focus on automaton size differences between weak and strong slim automata.

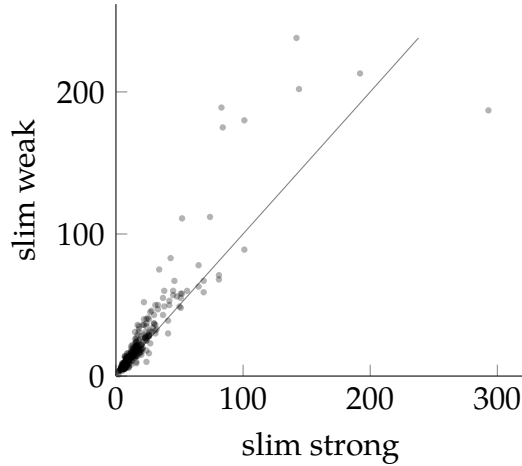


Figure 6.1: Scatter plot of strong against weak slim automata, equal values excluded. We can see strong slim automaton is smaller in more cases, but there are also cases where weak slim automaton is smaller.

## Minimal hits for weak x strong automata

Table 6.3: Strong slim automata lead in unique minimal hits, but weak slim automata have some unique minimal hits too, so it makes sense to try both options and choose better one.

literature	unique minimal hits	minimal hits
weak	4	9
strong	11	16
random	unique minimal hits	minimal hits
weak	68	202
strong	296	430

## Via-tba against via-tgba

Let us note that 13/20 formulas from literature and 391/500 formulas from random dataset create automata that are already TBA.

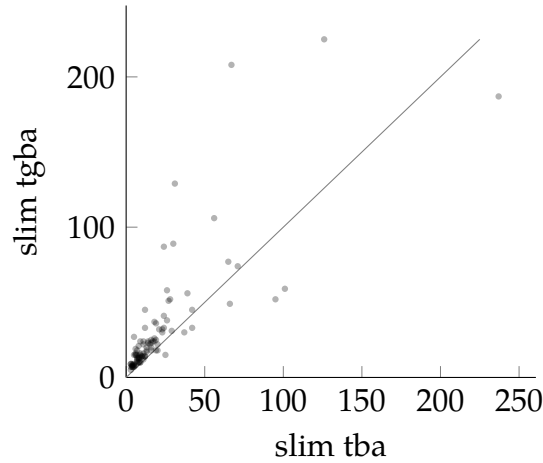


Figure 6.2: Scatter plot via-tba against via-tgba, equal values excluded. Plot density is low, as many of results are the same (majority of the automata were already tba). Via-tba has mostly better results, but there are also examples where via-tgba outputs automaton twice smaller automaton.

Table 6.4: As was visible on scatter plot, via-tba has better results. Via-tgba has only 9 minimal hits compared to 91 from via-tba.

literature	unique minimal hits	minimal hits
via-tba	7	20
via-tgba	0	13
random	unique minimal hits	minimal hits
via-tba	91	489
via-tgba	9	407

## 6.2 Slim Automata Produced Seminator versus ePMC

We can create slim automata using different tool called ePMC.

At first we compare best working basic parameters (parameters that try only 1 option) of each tool to create smallest automata to obtain fair time comparison.

Table 6.5: EPMC acc stands for option, that uses accepting transitions whenever possible, it had slightly better results than other ePMC options. Seminator tba strong

literature	size	time[s]
ePMC acc	650	178
seminator tba strong	436	151
random	size	time[s]
ePMC acc	9643	5146
seminator tba strong	7032	405

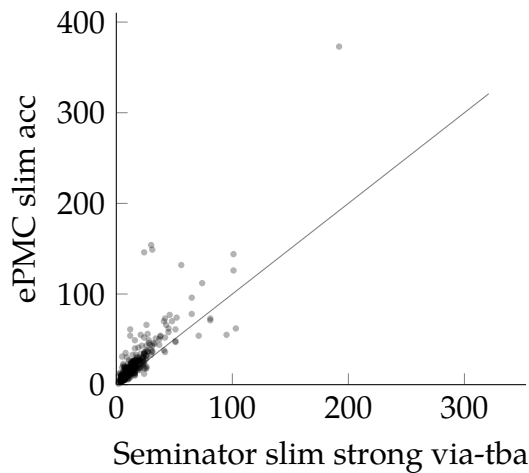


Figure 6.3: On scatter plot comparing sizes we can see that ePMC has some better hits.

The section compares smallest automata of each tool and to see how smaller automata get by combining these tools.

Then this section continues with comparison of minimal hits.

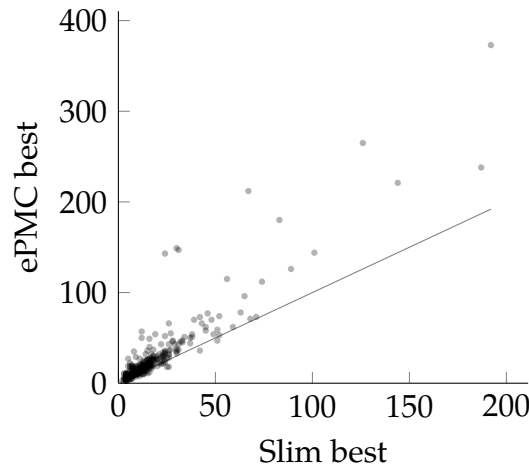


Figure 6.4: If we compare smallest automata of each tool, Seminotor slim has most of the best results and the difference is even bigger than single option comparison. Combining Seminotor and ePMC for best automata didn't bring much better results. On random dataset total size of automata from Seminotor slim is 7133. If we combine Seminotor slim and ePMC, we get total size 7060, which is not that significant difference.

Table 6.6: By unique minimal hits of these tool we can see that ePMC has 35 unique minimal hits compared to 344 of Seminotor. But as we can see from previous scatter plot, the size difference isn't that high.

literature	unique minimal hits	minimal hits
ePMC	1	5
Seminotor	15	19
random	unique minimal hits	minimal hits
ePMC	35	155
Seminotor	344	464

### 6.3 Compare with Different Kinds of Automata

This section brings size comparison with different kinds of automata. It compares slim automata with ltl2ldb [8] and ePMC [epmc].

Table 6.7: Results show that semi-deterministic automata created by Seminor are the smallest on average among the compared tools. Ltl2ldb creates the smallest GFM automata, as semi-deterministic automata do not promise GFM property.

tool	literature	random
ePMC best	602	10570
ltl2ldb	331	4641
Seminor default	263	3896
Seminor slim best	431	7325

Now we compare seminor --slim with other tools using scatter plots.

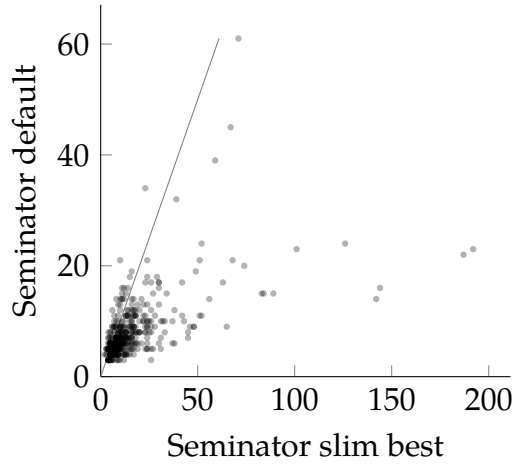


Figure 6.5: Semi-deterministic automata are consistently smaller or just slightly bigger than slim automata.

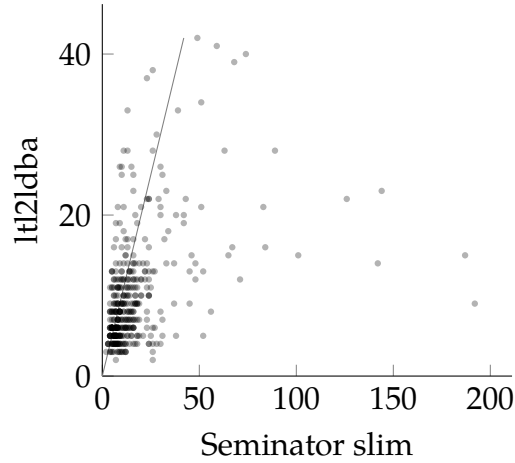


Figure 6.6: Ltl2ldba creates consistently small automata (smaller than 40 states), however its dominance over slim automata is not that consistent. The plot shows we can create reasonably smaller MDP automata on average using both tools and choosing the smallest automaton.

Table 6.8: This table confirms, that combining Seminotor slim and ltl2ldba creates reasonably smaller automata. For literature dataset it even beats semi-deterministic automata from Seminotor.

tool	literature	random
ePMC best	602	10570
ltl2ldba	331	4641
Seminotor default	263	3896
Seminotor slim best	431	7325
Seminotor slim + ltl2ldba best	262	4154

## 6.4 Note about optimized automata

If the automata optimizations by spot's `autfilt` tool break the simulation property, the results in Evaluation chapter are pointless, as they are built on such assumption. But we are confident that spot's `autfilt` does not break the property.





## 7 Mungojerrie benchmarks

This chapter compares seminator's slim automata on reinforcement learning tool mungojerrie [9].

Experiment uses benchmarks provided with the tool - examples/. The examples are built by various ways, some are even handcrafted. Using LTL that is provided we create slim automata for comparison on benchmarks, as Mungojerrie can accept LTL and transform it using internally supported tools ltl2ldba and ePMC, or we can provide automaton that is GFM (the property is not checked by Mungojerrie).

Experiments are searching for lowest necessary amount of episodes needed for reaching probability 1 to hit the goal. Experiments run benchmarks 10 times with pseudo random seeds 0-10. If all runs ends with success, experiment computes median and average of results. A run can fail by timeout (600s) or by not reaching probability 1.

Table 7.1: Table shows that ltl2ldba has the best results most often, but Seminor slim has some best hits too.

	Seminor slim	Examples	ePMC	Ltl2ldba
unique best average	5	7	1	12
unique best median	6	8	0	12
best average	13	12	9	15
second average	9	5	9	6
best median	13	13	7	14
second median	11	4	12	8
failures	4	7	3	4

## 7. MUNGOJERRIE BENCHMARKS

Table 7.2: Table of average episodes needed to reach probability 1.

	slims/	examples/	epmc/	ltl2ldba/
0	718.6	647.0	512.8	167.4
1	1.0	NaN	22.0	6.2
2	NaN	38.9	NaN	NaN
3	3620.4	3408.7	5763.8	1878.0
4	NaN	2171.2	NaN	NaN
5	5193.9	5193.9	NaN	4081.5
6	1.0	NaN	1.0	1.0
7	NaN	NaN	5906.1	1038.6
8	1310.4	1729.6	1156.8	983.8
9	718.6	NaN	512.8	167.4
10	3620.4	NaN	5763.8	1878.0
11	1.0	1.0	1.0	NaN
12	NaN	3236.3	5906.1	1212.8
13	1254.8	206.0	2788.7	5993.1
14	3095.3	4376.1	4376.1	NaN
15	1254.8	685.0	2788.7	6038.7
16	1.0	14.4	22.0	7.2
17	718.6	NaN	512.8	167.4
18	1187.2	320.5	562.4	324.5
19	124.5	124.5	124.5	595.0
20	1.0	12.2	1.0	8.4
21	1254.8	350.1	2788.7	5726.0
22	1250.7	1039.4	1250.7	1250.7
23	1.0	9.2	1.0	7.6
24	3620.4	6206.9	5763.8	1878.0
25	1254.8	299.2	2788.7	5842.1
26	1.0	9.0	22.0	6.4
27	718.6	NaN	512.8	167.4
28	683.8	638.8	618.6	11716.2
29	16559.5	16609.1	16559.5	16559.5
30	718.6	6448.8	512.8	167.4
31	1.0	14.4	22.0	6.4
32	19.3	13.0	84.1	19.5
33	1.0	1.0	1.0	8.8

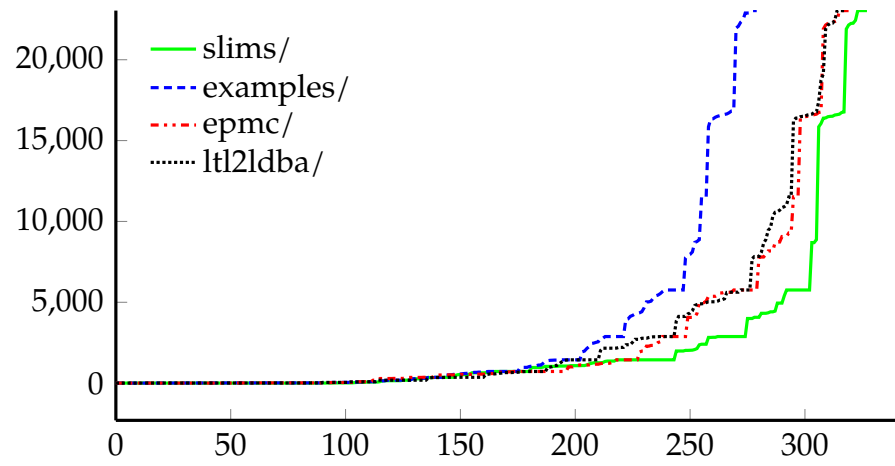


Figure 7.1: Sorted values for each benchmark run for each tool.



## 8 Conclusions

I have described slim automata with its prerequisites and its weak version. I have extended the construction to input TGBA automata. I have also implemented the construction of slim automata inside Seminator, which was the main goal. Then I have compared automaton size of slim automata and different tools. I have ended up with evaluation of performance on Mungojerrie tool.



## Bibliography

1. HAHN, Ernst Moritz; PEREZ, Mateo; SCHEWE, Sven; SOMENZI, Fabio; TRIVEDI, Ashutosh; WOJTCZAK, Dominik. Good-for-MDPs Automata for Probabilistic Analysis and Reinforcement Learning. In: BIERE, Armin; PARKER, David (eds.). *Tools and Algorithms for the Construction and Analysis of Systems - 26th International Conference, TACAS 2020, Held as Part of the European Joint Conferences on Theory and Practice of Software, ETAPS 2020, Dublin, Ireland, April 25-30, 2020, Proceedings, Part I*. Springer, 2020, vol. 12078, pp. 306–323. Lecture Notes in Computer Science. Available from doi: 10.1007/978-3-030-45190-5\_17.
2. BLAHOUEK, František; DURET-LUTZ, Alexandre; STREJČEK, Jan. Seminator 2 Can Complement Generalized Büchi Automata via Improved Semi-Determinization. In: *Proceedings of the 32nd International Conference on Computer-Aided Verification (CAV'20)*. Springer, 2020, vol. 12225, pp. 15–27. Lecture Notes in Computer Science. Available from doi: 10.1007/978-3-030-53291-8\_2.
3. BABIAK, Tomáš; BLAHOUEK, František; DURET-LUTZ, Alexandre; KLEIN, Joachim; KŘETÍNSKÝ, Jan; MÜLLER, David; PARKER, David; STREJČEK, Jan. The Hanoi Omega-Automata Format. In: KROENING, Daniel; PASAREANU, Corina S. (eds.). *Computer Aided Verification - 27th International Conference, CAV 2015, San Francisco, CA, USA, July 18-24, 2015, Proceedings, Part I*. Springer, 2015, vol. 9206, pp. 479–486. Lecture Notes in Computer Science. Available from doi: 10.1007/978-3-319-21690-4\_31.
4. DURET-LUTZ, Alexandre; LEWKOWICZ, Alexandre; FAUCHILLE, Amaury; MICHAUD, Thibaud; RENAULT, Étienne; XU, Laurent. Spot 2.0 — A Framework for LTL and  $\omega$ -Automata Manipulation. In: ARTHO, Cyrille; LEGAY, Axel; PELED, Doron (eds.). *Automated Technology for Verification and Analysis*. Cham: Springer International Publishing, 2016, pp. 122–129. ISBN 978-3-319-46520-3.
5. ROOT. [N.d.]. Available also from: <https://spot.lrde.epita.fr/ltl2tgba.html>.

## BIBLIOGRAPHY

---

6. ROOT. [N.d.]. Available also from: <https://spot.lrde.epita.fr/autfilt.html>.
7. ROOT. [N.d.]. Available also from: <https://spot.lrde.epita.fr/ltlcross.html>.
8. SICKERT, Salomon; ESPARZA, Javier; JAAX, Stefan; KŘETÍNSKÝ, Jan. Limit-Deterministic Büchi Automata for Linear Temporal Logic. In: CHAUDHURI, Swarat; FARZAN, Azadeh (eds.). *Computer Aided Verification*. Cham: Springer International Publishing, 2016, pp. 312–332. ISBN 978-3-319-41540-6.
9. HAHN, Ernst Moritz; PEREZ, Mateo; SCHEWE, Sven; SOMENZI, Fabio; TRIVEDI, Ashutosh; WOJTCZAK, Dominik. Omega-Regular Objectives in Model-Free Reinforcement Learning. In: VOJNAR, Tomáš; ZHANG, Lijun (eds.). *Tools and Algorithms for the Construction and Analysis of Systems*. Cham: Springer International Publishing, 2019, pp. 395–412. ISBN 978-3-030-17462-0.