

MASARYK UNIVERSITY  
FACULTY OF INFORMATICS



# **Transformation of Nondeterministic Büchi Automata to Slim Automata**

BACHELOR'S THESIS

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Brno, Fall 2020



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## Declaration

Hereby I declare that this paper is my original authorial work, which I have worked out on my own. All sources, references, and literature used or excerpted during elaboration of this work are properly cited and listed in complete reference to the due source.

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## **Acknowledgements**

ack

# **Abstract**

abstract



## Keywords

keyword1, keyword2, ...



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# 1 Introduction

Buchi automaton is a finite machine over infinite words. It has been a topic of research for almost 60 years. There were discovered various kinds of similar machines with different properties and use cases. «Some cool guys» have invented slim automata. Slim automata are specially constructed from Buchi automata. It is good for MDP [main source]. Scope of this thesis is to implement algorithm to transform a Buchi automaton to a slim automaton. Another goal is to extend the algorithm to accept Generalized Buchi automata. After our implementation of algorithm as specified in paper, different implementation from `epmc` was found. We can compare some properties of these algorithms and their products.

... slim automata are specially constructed Büchi automata.





## 2 Preliminaries

In this chapter we define a Büchi automaton and its generalized version. Then we continue with breakpoint algorithm. It allows us to introduce slim automata by its construction, which builds on the breakpoint one. Finally we generalize slim automaton construction to work with generalized Büchi automata.

We will need to know that on *alphabet* is a set of letters, an  $\omega$ -word  $w \in \Sigma^\omega$  is an infinite sequence of letters, and a *language*  $L \subseteq \Sigma^\omega$  is a set of  $\omega$ -words.

### 2.1 Büchi Automaton

A Büchi automaton is a theoretical finite-state machine used to define  $\omega$ -languages. It decides which infinitely long words ( $\omega$ -words) belong to its language.

A *transition-based Büchi automaton* (TBA) is a tuple  $\mathcal{A} \stackrel{\text{def}}{=} (\Sigma, Q, q_i, \Delta, \Gamma)$ , where

- $\Sigma$  is a non-empty finite *alphabet*,
- $Q$  is a non-empty finite set of *states*,
- $q_i \in Q$  is the initial state of  $\mathcal{A}$ .
- We write the set of *transitions* as  $\Delta \subseteq Q \times \Sigma \times Q$ . Intuitively, a transition  $(s, a, t)$  directionally connects the states  $s$  and  $t$  with the letter  $a$ .
- $\Gamma \subseteq \Delta$  is a set of *accepting transitions*.

A *run*  $r$  of  $\mathcal{A}$  is an infinite sequence of transitions  $r \stackrel{\text{def}}{=} t_0 t_1 \dots \in \Delta^\omega$ , where  $t_i = (s_i, a_i, s_{i+1})$ , such that  $q_0 = q_i$ . A run of  $\mathcal{A}$  is *accepting* iff it contains infinitely many accepting transitions.

Finally, we define the *language*  $L(\mathcal{A}) \subseteq \Sigma^\omega$  recognized by the automaton  $\mathcal{A}$ . An  $\omega$ -word  $w \in \Sigma^\omega$  belongs to  $L(\mathcal{A})$  iff there exists an accepting run of  $\mathcal{A}$  over the word  $w$ .

## 2.2 Generalized Büchi Automaton

A *transition-based Generalized Büchi automaton* (TGBA) is a tuple  $A \stackrel{\text{def}}{=} (\Sigma, Q, q_i, \Delta, G)$ , where  $\emptyset \subseteq G \subseteq 2^\Delta$  contains sets of accepting conditions and the rest is defined as for TBA. A run of  $A$  is *accepting* iff it contains infinitely many accepting transitions *for each*  $\Gamma \in G$ . TBA can be seen as a special case of TGBA with  $|G| = 1$

## 2.3 Breakpoint Automaton

We want to define *slim GFM<sup>1</sup> Büchi automaton* (slim automaton) through its construction which is based on breakpoint construction.

**Construction** Let us fix a Büchi Automaton  $A \stackrel{\text{def}}{=} (\Sigma, Q, q_i, \Delta, \Gamma)$ . We want to construct a deterministic automaton  $\mathcal{D}$  such that  $L(\mathcal{D}) \subseteq L(A)$ . We denote  $3^Q$  by the set  $\{(S, S') \mid S' \subsetneq S \subseteq Q\}$  and  $3_+^Q$  by  $\{(S, S') \mid S' \subseteq S \subseteq Q\}$ . We define the notation for the transitions and accepting transitions as  $\delta, \gamma : 2^Q \times \Sigma \rightarrow 2^Q$  with

$$\delta : (S, a) \stackrel{\text{def}}{=} \{q' \in Q \mid \exists q \in S. (q, a, q') \in \Delta\} \text{ and}$$

$$\gamma : (S, a) \stackrel{\text{def}}{=} \{q' \in Q \mid \exists q \in S. (q, a, q') \in \Gamma\}$$

(? Let us note that  $\delta$  and  $\gamma$  are deterministic transitions.)

We define the raw breakpoint transition  $\rho_\Gamma : 3^Q \times \Sigma \rightarrow 3_+^Q$  as

$$\rho_\Gamma((S, S'), a) \stackrel{\text{def}}{=} (\delta(S, a), \delta(S', a) \cup \gamma(S, a))$$

We follow the set of reachable states (first set) and the states that are reachable while passing at least one of the accepting transitions (second set). The transitions of the breakpoint automaton  $\mathcal{D}$  follow  $\rho$  with an exception: they reset the second set to the empty set when it equals the first; the resetting transitions are accepting. The breakpoint automaton  $\mathcal{D} \stackrel{\text{def}}{=} (\Sigma, 3^Q, (q_i, \emptyset), \delta_D, \gamma_D)$  is defined such that, when  $\rho : ((S, S'), a) = (R, R')$ , then there are three cases:

1. if  $R = \emptyset$ , then  $\delta_B((S, S'))$  is undefined (or, if a complete automaton is preferred, maps to a rejecting sink),

---

1. Good for Markov decision processes [+zdroj]

2. else, if  $R \neq R'$ , then  $\delta_B((S, S'), a) = (R, R')$  is a non-accepting transition, and  $\gamma_d((S, S'), a)$  is undefined.
3. otherwise  $\delta_D((S, S')) = \gamma_D((S, S'), a) = (R, \emptyset)$  is an accepting transition.

Breakpoint automata recognize/describe/represent subset of original language (example in Figure 2.1). On the other hand, semi-deterministic automata decide !!!superset - spatne!!! asi dat pryc ale nejsou good for mvp of such language.

todle mozna trochu jinak v dalsi sekci We are going to define a few more transitions on top of breakpoint construction which allow us to construct slim automata that decide exactly the class of <GFM languages> spatny pojem. jsou to omega regularni

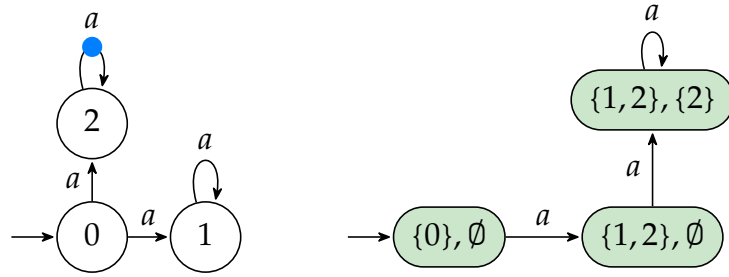


Figure 2.1: Example of breakpoint automaton (right) non-equivalent with the original Buchi Automaton (left)

## 2.4 Slim Automata Construction [separate chapter?]

Breakpoint automata constructed as presented in the previous section are not always equivalent to the input automaton.

In this section we define transitions  $\gamma_w, \gamma_p : 3^Q \times \Sigma \rightarrow 3^Q$  that promote the second set of a breakpoint construction to the first set as follows.

1. if  $\delta_S(S', a) = \gamma_S(S, a) = \emptyset$ , then  $\gamma_p((S, S'), a)$  and  $\gamma_w((S, S'), a)$  are undefined, and

## 2. PRELIMINARIES

2. otherwise  $\gamma_p : ((S, S'), a) = (\delta(S', a) \cup \gamma(S, a), \emptyset)$  and  $\gamma_w : ((S, S'), a) = (\delta(S', a), \emptyset)$

$\mathcal{S} \stackrel{\text{def}}{=} (\Sigma, 3^Q, (q_i, \emptyset), \Delta_S, \Gamma_S)$  is slim, when  $\Delta_S = \Delta_D \cup \Gamma_p$  is set of transitions generated by  $\delta_D$  and  $\gamma_p$ , and  $\Gamma_S = \Gamma_D \cup \Gamma_p$  is set of accepting transitions, that is generated by  $\gamma_D$  and  $\gamma_p$ .  $L(\mathcal{S}) = L(\mathcal{A})$  (proof in text with original definition)

Alternatively, similarly defined using  $\gamma_w$  instead of  $\gamma_p$ , automaton  $\mathcal{W} \stackrel{\text{def}}{=} (\Sigma, 3^Q, (q_i, \emptyset), \Delta_w, \Gamma_w)$  is slim a and  $L(\mathcal{S}) = L(\mathcal{A})$ . (no proof yet)

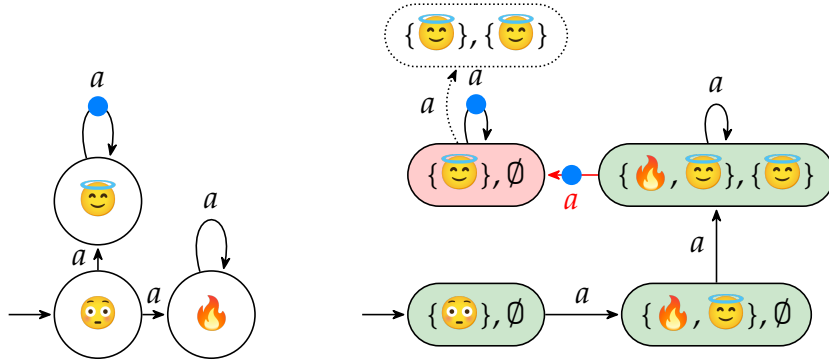


Figure 2.2: Slim automaton (right) and the original Buchi Automaton from Figure 2.1(left)

## 2.5 Slim Automaton Construction Generalized to TGBA

We want to construct a slim automaton from TGBA  $\mathcal{T} \stackrel{\text{def}}{=} (\Sigma, Q, q_i, \Delta, G)$ . One possibility is to *degeneralize*  $\mathcal{T}$  and to use previously mentioned algorithm in section 2.3. Another way is to extend slim automaton construction to TGBA.

**extended slim construction** We need to make sure we go infinitely many times through each accepting subset  $g \in G$ . To achieve this, we will go through each subset one by one, using original algorithm. We will keep track of *levels*  $\stackrel{\text{def}}{=} \{0, 1, \dots, |G| - 1\}$  in the names of states. Let

$|G|$  be number of *levels* and  $i \in \mathbb{N}, i < |G|$  the current level. At each level  $i$ , we look at  $i$ th subset of  $G$ . We use same steps as in classic breakpoint construction, but on each accepting transition the new state will be leveled up to  $(i + 1) \bmod |G|$ , otherwise the target state has the same level. Our new automaton simulates  $\mathcal{T}$ , as it accepts a word iff it cycles through all levels. If  $|G| = 0$ , we return a trivially accepting automaton

We can use the core of previous construction and just to extend it with levels. Let  $up(x) \stackrel{\text{def}}{=} (x + 1) \bmod |G|$

$$P := 3^Q \times \text{levels} \quad (\text{?nepotrebuju and } P_+ := 3_+^Q \times \text{levels})$$

We define  $\gamma_i$  similarly like  $\gamma$ , we just use  $\Gamma_i$  instead of  $\Gamma$  and it allows us to easily define the raw generalized breakpoint transitions  $\rho_{\Gamma_i}$ : similarly as  $\rho_\Gamma$  using  $\gamma_i$  instead of  $\gamma$ .

The generalized breakpoint automaton  $\mathcal{D} = (\Sigma, 3^Q \times \mathbb{N}, (q_i, \emptyset, 0))$  is defined such that, when  $\delta_R: ((S, S', i), a) \rightarrow (R, R', j)$ , then there are three cases:

1. if  $R = \emptyset$ , then  $\delta_B((S, S', i))$  is undefined,
2. else, if  $R \neq R'$ , then  $\delta_B: ((S, S', i), a) = (R, R', i)$  is a non-accepting transition,
3. otherwise  $\delta_B, \gamma_B: \delta_B((S, S', i), a) = (R, \emptyset, up(i))$ .

$$\gamma_p: P \times \Sigma \rightarrow P$$

1. if  $\delta(S', a) = \gamma_i(S, a) = \emptyset$ , then  $\gamma_p((S, S', i), a)$  is undefined, and
2. otherwise  $\gamma_p: ((S, S', i), a) = (\delta(S', a) \cup \gamma_i(S, a), \emptyset, up(i))$ .

$\mathcal{S} \stackrel{\text{def}}{=} (\Sigma, P, (q_i, \emptyset, 0), \Delta_p, \Gamma_p)$  is slim, when  $\Delta_p$  is set of transitions generated by  $\delta_b$  and  $\gamma_p$ , and  $\Gamma_p$  is set of accepting transitions, that is generated by  $\gamma_b$  and  $\gamma_p$ .

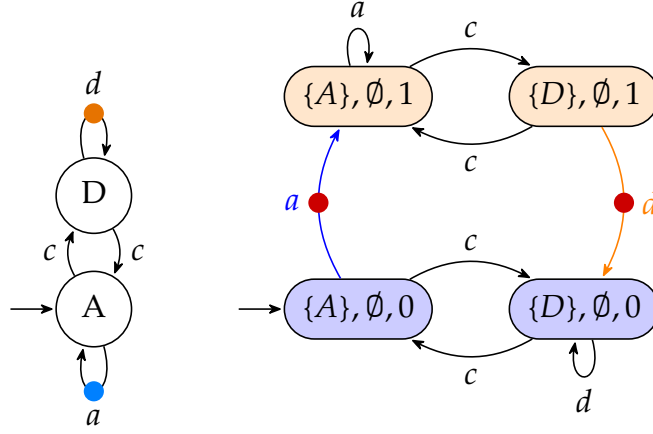


Figure 2.3: Slim automaton (right) and the original TGBA(left)

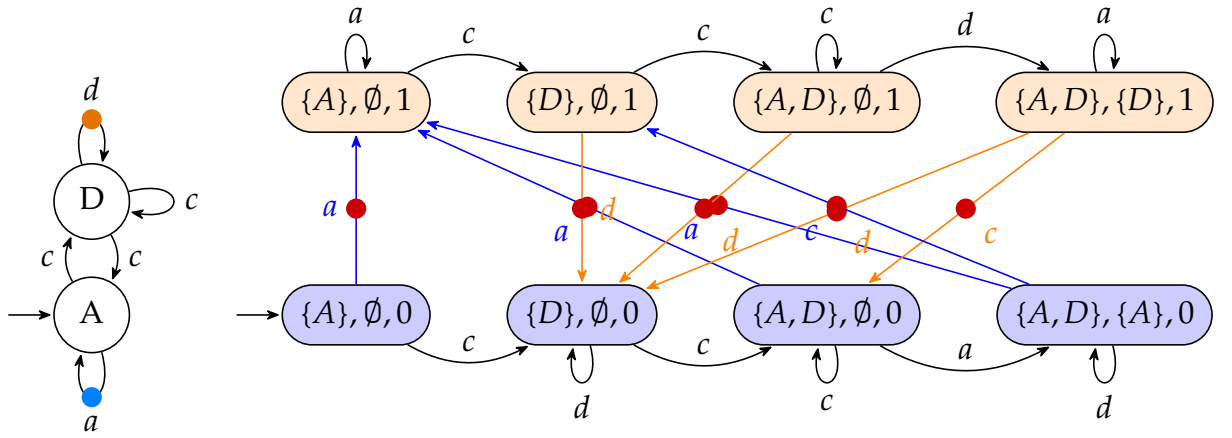


Figure 2.4: Slim automaton (right) and the original TGBA(left)

## 3 Implementation

I have implemented the generalized construction of slim automata in both weak and strong version (2.4,2.5). I have also added option to create breakpoint automata (2.3).

### 3.1 Technologies/Tools

The implementation is inside seminator, which is implemented in C++17 builds on Spot library[24-mklokocka].

#### 3.1.1 Seminator

[zdroj mklokocka thesis] Seminator is a Linux command-line tool which can be run with the seminator command.

The tool expects the input automaton in the Hanoi Omega-Automata(HOA) format [25-mklokocka] on the standard input stream, but it can also read the input automaton from a file.

[EXAMPLE OF sDBA created by seminator?]

#### 3.1.2 Spot

ltl2tgba ...

autfilt ...

ltlcross ...

### 3.2 Create Slim Automata Using Seminator

By default, seminator creates sDBA. To create a slim automaton we need to add `-slim` option.

**Options** By default, `-slim` tries all reasonable combinations of options, optimizes the output and chooses an automaton with the smallest number of states.

### 3. IMPLEMENTATION

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There are several options to specify how we construct the automata.

For example `seminator --slim --strong --optimizations=0 --via-tgba` generates output according to algorithm in 2.5. (Using `--via-tba` converts input to tba first) With automaton

[TBA (or TGBA?) INPUT EXAMPLE]

```
$ cat automaton.hoa | ./seminator --slim --strong
  --via-tgba --optimizations=0
```

we will get output

[SLIM OUTPUT EXAMPLE]

`--slim` to generate slim automata by, defaults to unoptimized, "strong"

slim algorithm

`--weak` use "weak"-slim algorithm instead

`--best` try weak and strong, optimize outputs with spot and choose the one with smaller automaton [delete and use as default]

(add `--strong` to generate just automata just by strong slim algorithm) [not implemented yet] neither `--weak` nor `--strong` specified try both, optimize and choose smaller result

`--via-tba` transform input automaton to tba (2.1) first

`--via-tgba` does not modify input automaton to tba.

neither `--via-tba` nor `--via-tgba`: try both options, choose smallest automaton

`postprocess` optimizations should be used be as a default option, use an option to disable

**Example** Transform automaton.hoa to a slim automaton.

```
$ cat automaton.hoa | ./seminator --slim
```

### 3.3 Implementation of Slim Automata inside Seminor

I have implemented the generalized slim construction and its options mentioned in previous section 3.2. Furthermore, I have added an option to create breakpoint automata.

There already was basis for breakpoint construction in seminor, inside class `bp_twa`. As we can see in sections 2.3 and 2.4, slim automata construction builds on breakpoint automata construction.



That allows us to simply extend the `bp_twa` class. We create class `slim` that inherits from `bp_twa`. In the `slim` class we build breakpoint automaton using `compute_successors` method. Then we extend the method by adding accepting transitions  $\gamma_p$ , respectively  $\gamma_w$  according to section 2.4, whenever we receive `--slim` option.

Then we extend main function to recognize our desired CLI options.

As `seminator` didn't offer a command line option to create a bp automaton, I have added one `,-bp`, for comparison.

### 3.4 Testing and Verification

Implemented tests are basic, only language equivalence is checked. `ltlcross` and `ltl2tgba` tools are used. The tests use random LTL formulas that were already generated, the LTL formulas are transformed into automata in HOA format by `ltl2tgba`. Then the tool `ltlcross` cross-compares the automaton with `seminator -slim` with all supported [not yet] additional parameters.

Only `seminator -slim -strong -via-tba` (and with no optimizations) is proved, as it follows construction from [main source] which is proved.

### 3.5 How to Install Seminor

(jeste nevim kde tuto sekci dat, jestli ma mit tento nazev, co vsechno tady bude treba dat... no a jeste ro pak upravim podle toho jak to bude odevzdane v zipu)

To install the tool we need install `spot` and to run

```
autoreconf -i && ./configure && make.
```

### 3.6 Future of Implementation

Implementation: python bindings, optimizations of `slim` construction (especially from TGBA)

### 3. IMPLEMENTATION

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tests/verification: There should be another kind of tests - to check if our slim automata simulate the input automata (so the GFM property is not broken)

Subject of following research, that is out of scope of this thesis, could be to verify if spot optimizations do not break the simulation property.

## 4 Evaluation

Evaluation part builds on seminator-evaluation. We compare amount of states of output automata. We compare the data on 2 dataset. First dataset are 20 literature formulas, second dataset is 500 automata that were randomly generated.

### 4.1 Overview of Compared Tools

tool	automaton size	running time (s)
ltl2tgba	3086	37
optimized seminator default	3751	67
no.seminator default	4748	57
optimized seminator strongslim viatba	6840	399
optimized seminator strongslim viatgba	7698	571
optimized seminator weakslim viatba	8101	405
optimized seminator weakslim viatgba	9045	632
optimized seminator viatba	6657	804
optimized seminator viatgba	7445	1203
optimized seminator weakslim	7959	1037
optimized seminator strongslim	6721	1070
optimized seminator slim	6526	2107
optimized epmc slim acc	9943	5298
yes.epmc slim best	9081	20798

### 4.2 Seminator –slim

In this section we compare automaton size generated by seminator –slim. We compare weak against slim and via-tba against via-tgba.

#### 4.2.1 Comparisons among Unoptimized options

literature

#### 4. EVALUATION

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seminator	weak		strong	
literature	size	time(s)	size	time(s)
via tba	5x51	21x9	37x0	1x85
via tgba	5x88	21x0	40x8	1x74

500 randomly generated automata

seminator	weak		strong	
literature	size	time(s)	size	time(s)
via tba	551	219	370	185
via tgba	588	210	408	174

Transforming automata to TBA first yields smaller automata. This might be caused by Spot having well optimized algorithm for degeneralization. Slim algorithm for TGBA proposed in this paper is naive, without any kind of optimizations, and it degeneralizes the automaton during the process.

Using weak slim algorithm creates smaller slim automata than the strong one.

##### 4.2.2 Post-Optimized

In this subsection we post-optimize results using autfilt.

literature

seminator	weak		strong		best	
literature	size	time(s)	size	time(s)	size	time(s)
via tba	551	219	370	185	370	404
via tgba	588	210	408	174	402	384
best	551	429	370	359	365	788

500 randomly generated automata

seminator	weak		strong		best	
random	size	time(s)	size	time(s)	size	time(s)
via tba	8923	443	7404	476	7219	919
via tgba	10130	654	8500	591	8247	1245
best	8751	1097	7285	1067	7088	2164

From 4 base options; after applying post-optimizations strong slim algorithm surpasses weak one by resulting automaton size, even if it has worse results without the post-optimizations. Degeneralizing the automata as a first step still has smaller results. From 4 base options, strong slim algorithm via-tba creates smallest automata on average. Transforming input automata to tba first creates results which are close to best ones. If execution time is not a concern

Minimal hits for random automata weak x strong

literature	unique minimal hits	minimal hits
weak	4	9
strong	11	16

random	unique minimal hits	minimal hits
weak	68	202
strong	296	430

Minimal hits for random automata via-tba x via-tgba

Let us note that 13/20 formulas from literature and 391/500 formulas from random dataset create automata that are already TBA.

literature	unique minimal hits	minimal hits
via-tba	7	20
via-tgba	0	13

random	unique minimal hits	minimal hits
via-tba	91	489
via-tgba	9	407

### 4.3 Slim Automata Produced Seminotor x ePMC

popsat co to je ePMC+zdroj prvej vybranej možnosti

At first we compare best working basic parameters (parameters that try only 1 option) of each tool to create smallest automata

#### 4. EVALUATION

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random	size	time(s)
epmc acc	9270	
seminator tba strong	6840	

random	size	time(s)
epmc acc	9270	
seminator tba strong	6840	

Now let us compare smallest automata of each tool and see how smaller automata get by combining these 2 tools.

literature	size	time(s)
epmc best	536	
seminator best	365	
seminator+epmc best	349	

random	size	time(s)
epmc best	10197	
seminator best	7133	
seminator+epmc best	7060	

literature	unique minimal hits	minimal hits
epmc	1	5
seminator	15	19

random	unique minimal hits	minimal hits
epmc	35	155
seminator	344	464

2.4 (2.5) prvne tba zkusiť pro empc

#### 4.4 Compare with semi-deterministic Automata

#### 4.5 To Put Somewhere in This Chapter

If the automata optimizations by spot's `autfilt` tool break the simulation property, the results in following Evaluation chapter are pointless, as they are built on such assumption.

## 5 Conclusion

