# Modelos lineales generalizados

CEPAL

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# Lectura de la base

```
encuesta <- readRDS("../Data/encuesta.rds")
data("BigCity", package = "TeachingSampling")
```

# Definir diseño de la muestra con `srvyr`

```
library(srvyr)

diseno <- encuesta %>%
  as_survey_design(
    strata = Stratum,
    ids = PSU,
    weights = wk,
    nest = T
  )
```

# definir nuevas variables

```
diseno <- diseno %>% mutate(
  pobreza = ifelse(Poverty != "NotPoor", 1, 0),
  desempleo = ifelse(Employment == "Unemployed", 1, 0))
```

# Tablas de doble entrada para el tamaño

```
library(survey)
(tab_pobreza_sexo <- svyby(~factor(pobreza), ~Sex,
      FUN = svytotal, design = as.svrepdesign(diseno),
      se=F, na.rm=T, ci=T, keep.var=TRUE))
```

| | Sex | factor(pobreza)0 | factor(pobreza)1 | se1 | se2 |
|---|---|---|---|---|---|
| Female | Female | 48010 | 31180 | 2206 | 3001 |
| Male | Male | 42310 | 28766 | 2093 | 2239 |

```
(tab <- svytable(~pobreza + Sex, design = diseno))
```

| pobreza/Sex | Female | Male |
|---|---|---|
| 0 | 48010 | 42310 |
| 1 | 31180 | 28766 |

# Tablas de doble entrada para el proporción

```
(tab_pobreza_sexo <- svyby(~factor(pobreza), ~Sex,
      FUN = svymean, design = as.svrepdesign(diseno),
      se=F, na.rm=T, ci=T, keep.var=TRUE))
```

| | Sex | factor(pobreza)0 | factor(pobreza)1 | se1 | se2 |
|---|---|---|---|---|---|
| Female | Female | 0.6063 | 0.3937 | 0.0304 | 0.0304 |
| Male | Male | 0.5953 | 0.4047 | 0.0270 | 0.0270 |

```
prop.table(tab, margin = 2)
```

| pobreza/Sex | Female | Male |
|---|---|---|
| 0 | 0.6063 | 0.5953 |
| 1 | 0.3937 | 0.4047 |

# Prueba de independencia F

$$\hat{\pi}_{rc} = \frac{n_{r+}}{n_{++}} \times \frac{n_{+c}}{n_{++}}$$

$$\chi^2_{pearsom} = n_{++} \times \sum_r \sum_c \left( \frac{(p_{rc} - \hat{\pi}_{rc})^2}{\hat{\pi}_{rc}} \right)$$

$$G^2 = 2 \times n_{++} \times \sum_r \sum_c p_{cr} \times \ln \left( \frac{p_{rc}}{\hat{\pi}_{rc}} \right)$$

donde, $R$ es el número de filas y $C$ representa el número de columnas, la prueba tiene $(R-1) \times (C-1)$ grados de libertad.

# Prueba de independencia F

$$\chi^2_{(R-S)} = \chi^2_{(Pearson)} / GDEFF$$

$$G^2_{(R-S)} = G^2 / GDEFF$$

con $GDEFF$ el efecto generalizado del diseño, esta dado por

$$GDEFF = \frac{\sum_r \sum_c (1 - p_{rc})\, d^2(p_{rc}) - \sum_r (1 - p_{r+})\, d^2(p_{r+}) - \sum_c (1 - p_{+c})\, d^2(p_{+c})}{(R-1)(C-1)}$$

# Prueba de independencia F

$$F_{R-S,Pearson} = \chi^2_{R-S} / \left[ (R-1)(C-1) \right] \sim F_{(R-1)(C-1),(R-1)(C-1)df}$$

$$F_{R-S,LRT} = G^2_{R-S} / (C-1) \sim F_{(C-1),df}$$

donde $C$ es el número de columnas de la tabla cruzada

# Prueba de independencia ChiSq

```
summary(tab, statistic = "Chisq")
```

```
##          Sex
## pobreza Female  Male
##       0  48010 42310
##       1  31180 28766
##
##  Pearson's X^2: Rao & Scott adjustment
##
## data:  NextMethod()
## X-squared = 0.33, df = 1, p-value = 0.6
```

# Prueba de independencia F

```
summary(tab, statistic = "F")
```

```
##          Sex
## pobreza Female  Male
##       0  48010 42310
##       1  31180 28766
##
##  Pearson's X^2: Rao & Scott adjustment
##
## data:  NextMethod()
## F = 0.35, ndf = 1, ddf = 119, p-value = 0.6
```

# Estadístico de Wald

$$Q_{wald} = \hat{\boldsymbol{Y}}^t \left( \boldsymbol{H} \hat{\boldsymbol{V}} \left( \hat{\boldsymbol{N}} \right) \boldsymbol{H}^t \right)^{-1} \hat{\boldsymbol{Y}}$$

donde,

$$\hat{\boldsymbol{Y}} = \left( \hat{N} - E \right)$$

es un vector de $R \times C$ de diferencias entre los recuentos de celdas observadas y esperadas, esto es, $\hat{N}_{rc} - E_{rc}$

La matriz $\boldsymbol{H} \hat{\boldsymbol{V}} \left( \hat{\boldsymbol{N}} \right) \boldsymbol{H}^t$, representa la matriz de varianza-covarianza estimada para el vector de diferencias.

# Estadístico de Wald

La matriz $\boldsymbol{H}$ es la inversa de la matriz $\boldsymbol{J}$ dada por:

$$\boldsymbol{J} = -\left[\frac{\delta^2 \ln PL(\boldsymbol{B})}{\delta^2 \boldsymbol{B}}\right] \mid \boldsymbol{B} = \hat{\boldsymbol{B}}$$

$$\sum_h \sum_a \sum_i x_{hai}^t x_{hai} w_{hai} \hat{\pi}_{hai}(\boldsymbol{B})(1 - \hat{\pi}_{hai}(\boldsymbol{B}))$$

Bajo la hipótesis nula, el estadístico

$$Q_{wald} \sim \chi^2_{(R-1)\times(C-1)}$$

# Estadístico de Wald

$$F_{wald} = Q_{wald} \times \frac{df - (R-1)(C-1) + 1}{(R-1)(C-1)\, df} \sim F_{(R-1)(C-1),\, df-(R-1)(C-1)+1}$$

# Prueba de independencia Wald

```
summary(tab, statistic = "Wald")
```

```
##          Sex
## pobreza Female  Male
##       0  48010 42310
##       1  31180 28766
##
##  Design-based Wald test of association
##
## data:  NextMethod()
## F = 0.35, ndf = 1, ddf = 119, p-value = 0.6
```

# Prueba de independencia adjWald

```
summary(tab, statistic = "adjWald")
```

```
##          Sex
## pobreza Female  Male
##       0  48010 42310
##       1  31180 28766
##
##  Design-based Wald test of association
##
## data:  NextMethod()
## F = 0.35, ndf = 1, ddf = 119, p-value = 0.6
```

# Prueba de independencia lincom

```
summary(tab, statistic = "lincom")
```

```
##          Sex
## pobreza Female  Male
##       0  48010 42310
##       1  31180 28766
##
##  Pearson's X^2: asymptotic exact distribution
##
## data:  NextMethod()
## X-squared = 0.33, p-value = 0.6
```

# Prueba de independencia saddlepoint

```
summary(tab, statistic = "saddlepoint")
```

```
##          Sex
## pobreza Female  Male
##       0  48010 42310
##       1  31180 28766
##
##  Pearson's X^2: saddlepoint approximation
##
## data:  NextMethod()
## X-squared = 0.33, p-value = 0.6
```

# Modelo log lineal para tablas de contingencia

$$\log(p_{ijk}) = \mu + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY},$$

donde:

- $p_{ijk}$ = la proporción esperada en la celda bajo el modelo.
- $\mu = \log(p_0) = \frac{1}{\#\ de\ celdas}$

# Modelo log lineal para tablas de contingencia

```
mod1 <- svyloglin(~pobreza+Sex + pobreza:Sex , diseno)
(s1 <- summary(mod1))
```

```
## Loglinear model: svyloglin(~pobreza + Sex + pobreza:Sex
##                      coef      se          p
## pobreza1          0.20437 0.05676 0.0003172
## Sex1              0.05174 0.02119 0.0146150
## pobreza1:Sex1     0.01144 0.01933 0.5538109
```

# Modelo log lineal para tablas de contingencia

```
mod2 <- svyloglin(~pobreza+Sex, diseno)
(s2 <- summary(mod2))
```

```
## Loglinear model: svyloglin(~pobreza + Sex, diseno)
##                 coef      se          p
## pobreza1 0.20497 0.05690 0.0003157
## Sex1     0.05405 0.01919 0.0048540
```

# Modelo log lineal para tablas de contingencia

```
anova(mod1, mod2)
```

```
## Analysis of Deviance Table
##  Model 1: y ~ pobreza + Sex
## Model 2: y ~ pobreza + Sex + pobreza:Sex
## Deviance= 0.3254 p= 0.555
## Score= 0.3254 p= 0.5549
```

# Modelo de regresión logistica

$$g(\pi(x)) = logit(\pi(x)) = z = \ln\left(\frac{\pi(x)}{1-\pi(x)}\right) = B_0 + B_1 x_1 + \cdots + B_p x_p$$

$$\hat{\pi}(\boldsymbol{x}) = \frac{\exp\left(\boldsymbol{X}\hat{\boldsymbol{B}}\right)}{1-\exp\left(\boldsymbol{X}\hat{\boldsymbol{B}}\right)} = \frac{\exp\left(\hat{B}_0 + \hat{B}_1 x_1 + \cdots + \hat{B} x_p\right)}{1-\exp\left(\hat{B}_0 + \hat{B}_1 x_1 + \cdots + \hat{B} x_p\right)}$$

$$PL(\boldsymbol{B} \mid X) = \prod_{i=1}^{n} \left\{\pi(x_i)^{y_i} \left(1-\pi(x_i)\right)^{1-y_i}\right\}^{w_i}$$

con

$$\pi(x_i) = \frac{\exp(x_i \boldsymbol{B})}{1-\exp(x_i \boldsymbol{B})}$$

$$var\left(\hat{\boldsymbol{B}}\right) = \boldsymbol{J}^{-1} var\left(S\left(\hat{\boldsymbol{B}}\right)\right) \boldsymbol{J}^{-1}$$

$$S(B) = \sum_h \sum_a \sum_i w_{hai} \boldsymbol{D}_{hai}^t \left[\left(\pi_{hai}(\boldsymbol{B})\right)\left(1-\pi_{hai}(\boldsymbol{B})\right)\right]^{-1} \left(y_{hai} - \pi_{hai}(\boldsymbol{B})\right)$$

# Prueba de Wald para los parámetros del modelo

$$G = -2 \ln \left[ \frac{L\left(\hat{\beta}_{MLE}\right)_{reduced}}{L\left(\hat{\beta}_{MLE}\right)_{full}} \right]$$

$$\hat{\psi} = \exp\left(\hat{B}_1\right)$$

$$CI\left(\psi\right) = \exp\left(\hat{B}_j \pm t_{df,1-\frac{\alpha}{2}} se\left(\hat{B}_j\right)\right)$$

# Tablas de contingencia

| Sex | pobreza | se | ci_l | ci_u |
|---|---|---|---|---|
| Female | 0.3937 | 0.0304 | 0.3342 | 0.4533 |
| Male | 0.4047 | 0.0270 | 0.3518 | 0.4576 |

| Zone | pobreza | se | ci_l | ci_u |
|---|---|---|---|---|
| Rural | 0.4206 | 0.0485 | 0.3255 | 0.5157 |
| Urban | 0.3789 | 0.0269 | 0.3261 | 0.4317 |

| Region | pobreza | se | ci_l | ci_u |
|---|---|---|---|---|
| Norte | 0.4213 | 0.0522 | 0.3189 | 0.5236 |
| Sur | 0.3079 | 0.0539 | 0.2023 | 0.4136 |
| Centro | 0.2509 | 0.0461 | 0.1606 | 0.3411 |
| Occidente | 0.3980 | 0.0385 | 0.3226 | 0.4734 |
| Oriente | 0.5453 | 0.0768 | 0.3948 | 0.6957 |

# Prueba de independencia ChiSq

```
## Pearson's X^2: Rao & Scott adjustment
pobreza_sex <-  svychisq(
  formula = ~pobreza + Sex, design = diseno)
pobreza_Zona <- svychisq(
  formula = ~pobreza + Zone, design = diseno)
pobreza_Region <- svychisq(
  formula = ~pobreza + Region, design = diseno)
bind_rows( tidy( pobreza_sex),
           tidy(pobreza_Zona),
           tidy(pobreza_Region)) %>%
  dplyr::select(-method)
```

| ndf | ddf | statistic | p.value |
|---|---|---|---|
| 1.000 | 119.0 | 0.3506 | 0.5549 |
| 1.000 | 119.0 | 0.5730 | 0.4505 |
| 3.194 | 380.1 | 4.2241 | 0.0049 |

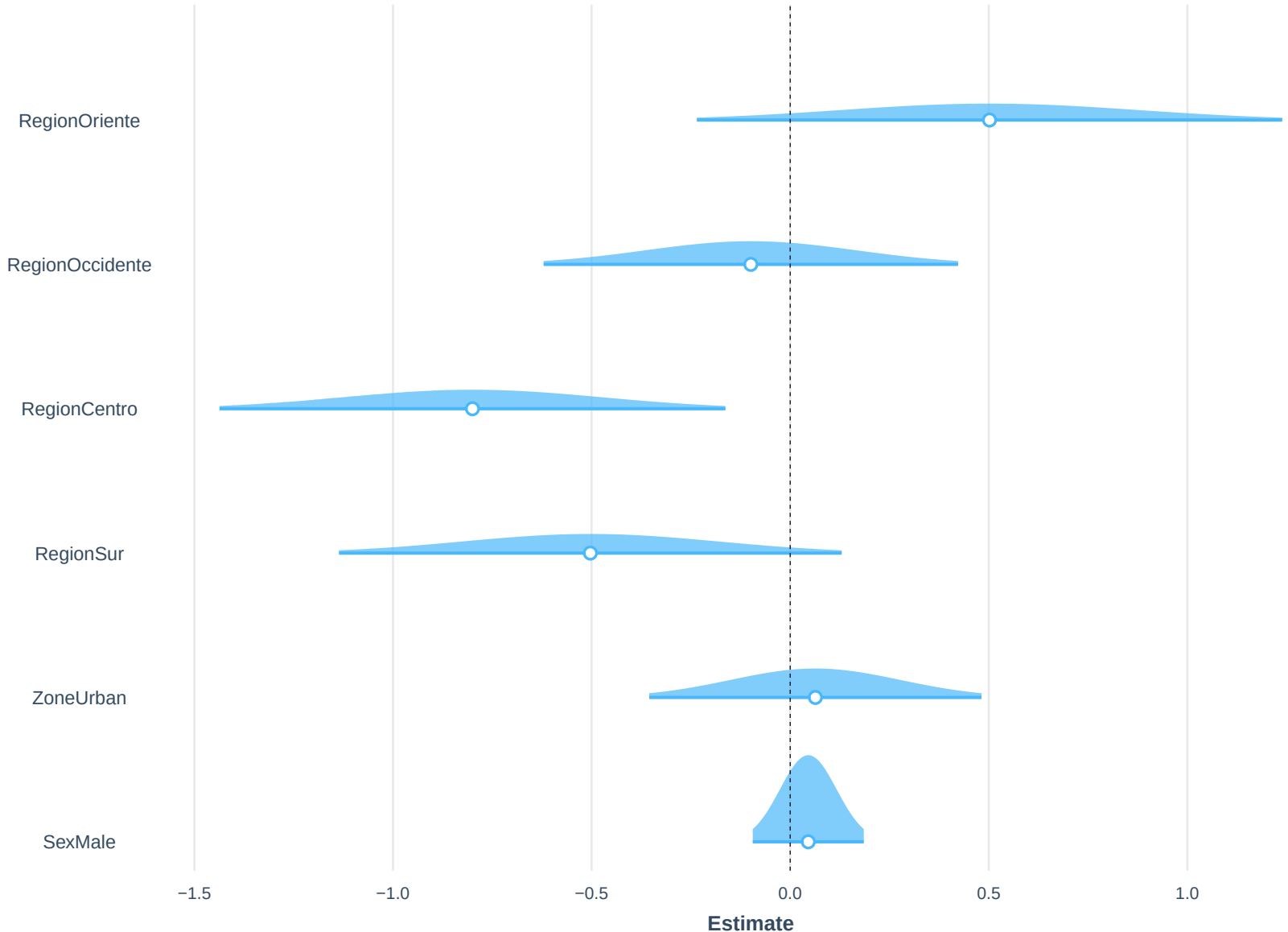
# Modelo log lineal ajustado

```
mod_loglin <- svyglm(pobreza ~ Sex + Zone + Region,
                     family=quasibinomial, design=diseno)
tidy(mod_loglin)
```

| term | estimate | std.error | statistic | p.value |
|---|---|---|---|---|
| (Intercept) | -0.3664 | 0.2486 | -1.4735 | 0.1434 |
| SexMale | 0.0455 | 0.0713 | 0.6384 | 0.5245 |
| ZoneUrban | 0.0634 | 0.2134 | 0.2970 | 0.7670 |
| RegionSur | -0.5031 | 0.3229 | -1.5582 | 0.1220 |
| RegionCentro | -0.7999 | 0.3249 | -2.4619 | 0.0153 |
| RegionOccidente | -0.0990 | 0.2663 | -0.3719 | 0.7107 |
| RegionOriente | 0.5019 | 0.3759 | 1.3349 | 0.1846 |

# Plot de la distribución de los betas

```
plot_summs(mod_loglin,
           scale = TRUE, plot.distributions = TRUE)
```



# Modelo log lineal ajustado

```
bind_cols(
data.frame(exp_estimado = exp(coef(mod_loglin))),
as.data.frame(exp(confint(mod_loglin)))
)
```

| | exp_estimado | 2.5 % | 97.5 % |
|---|---|---|---|
| (Intercept) | 0.6933 | 0.4236 | 1.1345 |
| SexMale | 1.0466 | 0.9087 | 1.2054 |
| ZoneUrban | 1.0654 | 0.6981 | 1.6262 |
| RegionSur | 0.6046 | 0.3189 | 1.1463 |
| RegionCentro | 0.4494 | 0.2361 | 0.8554 |
| RegionOccidente | 0.9057 | 0.5344 | 1.5349 |
| RegionOriente | 1.6518 | 0.7843 | 3.4787 |

# Estadístico de Wald sobre los parámetros

```
regTermTest(model = mod_loglin, ~Sex)
```

```
## Wald test for Sex
##  in svyglm(formula = pobreza ~ Sex + Zone + Region, desi
##      family = quasibinomial)
## F =  0.4076  on  1  and  113  df: p= 0.52
```

```
regTermTest(model = mod_loglin, ~Zone)
```

```
## Wald test for Zone
##  in svyglm(formula = pobreza ~ Sex + Zone + Region, desi
##      family = quasibinomial)
## F =  0.08823  on  1  and  113  df: p= 0.77
```

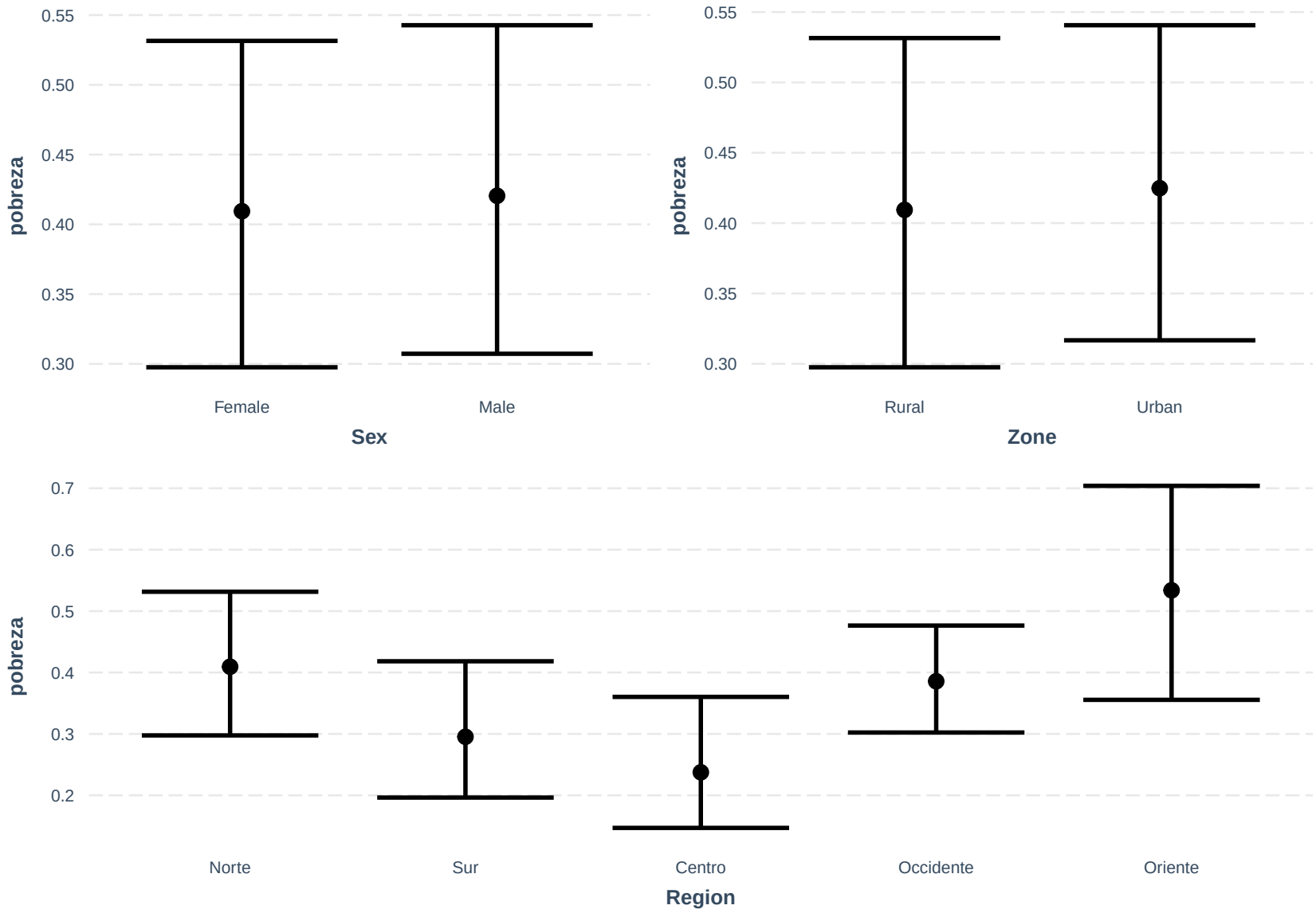
```
regTermTest(model = mod_loglin, ~Region)
```

```
## Wald test for Region
##  in svyglm(formula = pobreza ~ Sex + Zone + Region, desi
##      family = quasibinomial)
## F =  4.33  on  4  and  113  df: p= 0.0027
```

# Efecto del modelo.

```
effe_sex <- effect_plot(mod_loglin, pred = Sex,
                        interval = TRUE)
effe_Zona <-effect_plot(mod_loglin, pred = Zone,
                        interval = TRUE)
effe_Region <- effect_plot(mod_loglin, pred = Region,
                           interval = TRUE)
(effe_sex |effe_Zona)/effe_Region
```

# Efecto del modelo.



# Modelo log lineal ajustado con interacciones

```
mod_loglin_int <- svyglm(pobreza ~ Sex + Zone + Region +
                                   Sex:Zone + Sex:Region,
                         family=quasibinomial, design=diseno)
tab_mod <- tidy(mod_loglin_int) %>% arrange(p.value)
tab_mod %>% slice(1:6)
```

| term | estimate | std.error | statistic | p.value |
|---|---|---|---|---|
| RegionCentro | -0.7863 | 0.3539 | -2.2217 | 0.0284 |
| RegionSur | -0.5432 | 0.3776 | -1.4383 | 0.1532 |
| SexMale:RegionOriente | 0.2616 | 0.1866 | 1.4022 | 0.1637 |
| (Intercept) | -0.3196 | 0.2623 | -1.2183 | 0.2257 |
| RegionOriente | 0.3780 | 0.3854 | 0.9809 | 0.3288 |
| SexMale:RegionOccidente | 0.1648 | 0.1779 | 0.9261 | 0.3564 |

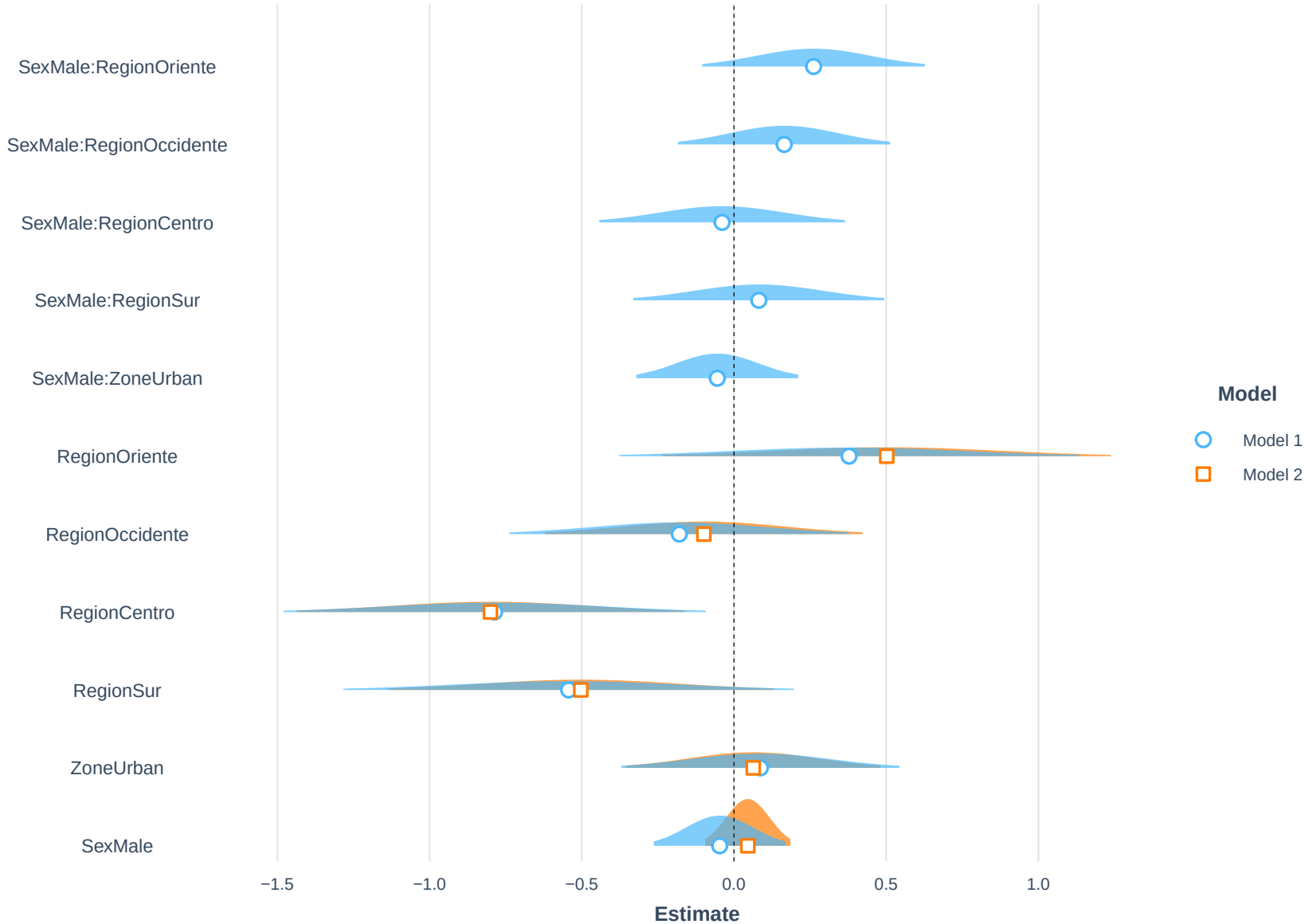
# Modelo log lineal ajustado con interacciones

```
tab_mod %>% slice(7:12)
```

| term | estimate | std.error | statistic | p.value |
|---|---|---|---|---|
| RegionOccidente | -0.1796 | 0.2849 | -0.6304 | 0.5297 |
| SexMale | -0.0470 | 0.1105 | -0.4257 | 0.6712 |
| SexMale:ZoneUrban | -0.0548 | 0.1357 | -0.4037 | 0.6872 |
| SexMale:RegionSur | 0.0817 | 0.2103 | 0.3884 | 0.6985 |
| ZoneUrban | 0.0869 | 0.2330 | 0.3731 | 0.7098 |
| SexMale:RegionCentro | -0.0391 | 0.2059 | -0.1897 | 0.8499 |

# Plot de la distribución de los betas

```
plot_summs(mod_loglin_int, mod_loglin, scale = TRUE, plot.c
```



# Modelo log lineal ajustado

| | exp_estimado | 2.5 % | 97.5 % |
|---|---|---|---|
| (Intercept) | 0.7264 | 0.4319 | 1.2219 |
| SexMale | 0.9541 | 0.7665 | 1.1876 |
| ZoneUrban | 1.0908 | 0.6874 | 1.7312 |
| RegionSur | 0.5809 | 0.2748 | 1.2280 |
| RegionCentro | 0.4555 | 0.2259 | 0.9187 |
| RegionOccidente | 0.8356 | 0.4750 | 1.4698 |
| RegionOriente | 1.4594 | 0.6799 | 3.1328 |
| SexMale:ZoneUrban | 0.9467 | 0.7234 | 1.2389 |
| SexMale:RegionSur | 1.0851 | 0.7152 | 1.6463 |
| SexMale:RegionCentro | 0.9617 | 0.6395 | 1.4463 |
| SexMale:RegionOccidente | 1.1791 | 0.8287 | 1.6778 |
| SexMale:RegionOriente | 1.2990 | 0.8974 | 1.8802 |

# Estadístico de Wald sobre los parámetros

```
regTermTest(model = mod_loglin_int, ~Sex)
```

```
## Wald test for Sex
##  in svyglm(formula = pobreza ~ Sex + Zone + Region + Sex
##      design = diseno, family = quasibinomial)
## F =  0.1812  on  1  and  108  df: p= 0.67
```

```
regTermTest(model = mod_loglin_int, ~Zone)
```

```
## Wald test for Zone
##  in svyglm(formula = pobreza ~ Sex + Zone + Region + Sex
##      design = diseno, family = quasibinomial)
## F =  0.1392  on  1  and  108  df: p= 0.71
```

```
regTermTest(model = mod_loglin_int, ~Region)
```

```
## Wald test for Region
##  in svyglm(formula = pobreza ~ Sex + Zone + Region + Sex
##      design = diseno, family = quasibinomial)
## F =  3.14  on  4  and  108  df: p= 0.017
```

# Estadístico de Wald sobre los parámetros

```
regTermTest(model = mod_loglin_int, ~Sex:Zone)
```

```
## Wald test for Sex:Zone
##  in svyglm(formula = pobreza ~ Sex + Zone + Region + Sex
##     design = diseno, family = quasibinomial)
## F =  0.163  on  1  and  108  df: p= 0.69
```

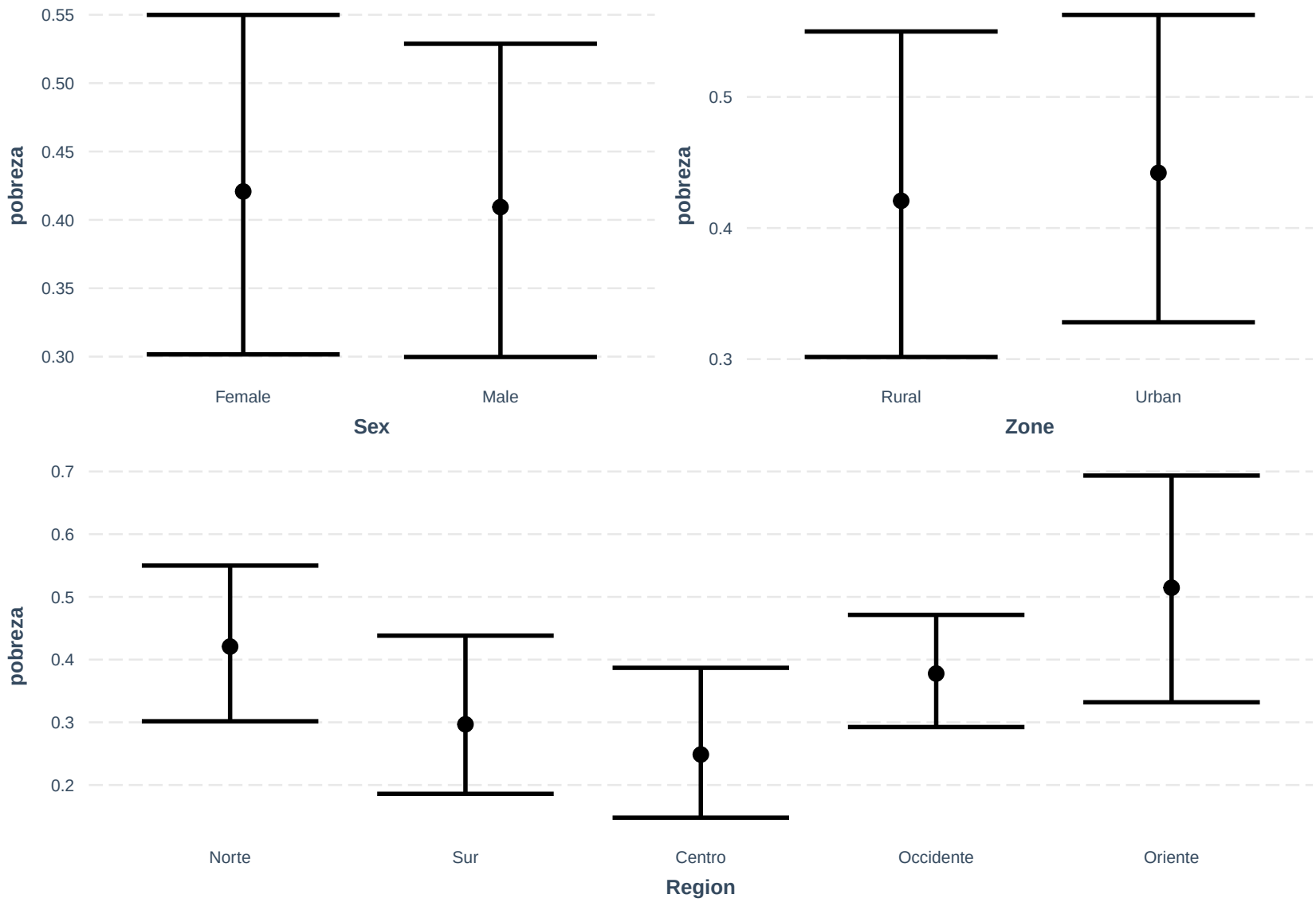
```
regTermTest(model = mod_loglin_int, ~Sex:Region)
```

```
## Wald test for Sex:Region
##  in svyglm(formula = pobreza ~ Sex + Zone + Region + Sex
##     design = diseno, family = quasibinomial)
## F =  0.7353  on  4  and  108  df: p= 0.57
```

# Efecto del modelo.

```
effe_sex <- effect_plot(mod_loglin_int, pred = Sex,
                        interval = TRUE)
effe_Zona <-effect_plot(mod_loglin_int, pred = Zone,
                        interval = TRUE)
effe_Region <- effect_plot(mod_loglin_int, pred = Region,
                           interval = TRUE)
(effe_sex |effe_Zona)/effe_Region
```

# Efecto del modelo.



# Modelo log lineal ajustado con Q_Weighting

```
fit_wgt <- lm(wk ~  Sex + Zone + Region , data = encuesta)
wgt_hat <- predict(fit_wgt)
encuesta %<>% mutate(wk2 = wk/wgt_hat)

diseno_qwgt <- encuesta %>%
  as_survey_design(
    strata = Stratum,
    ids = PSU,
    weights = wk2,
    nest = T
  )
```

# Modelo log lineal ajustado con Q_Weighting

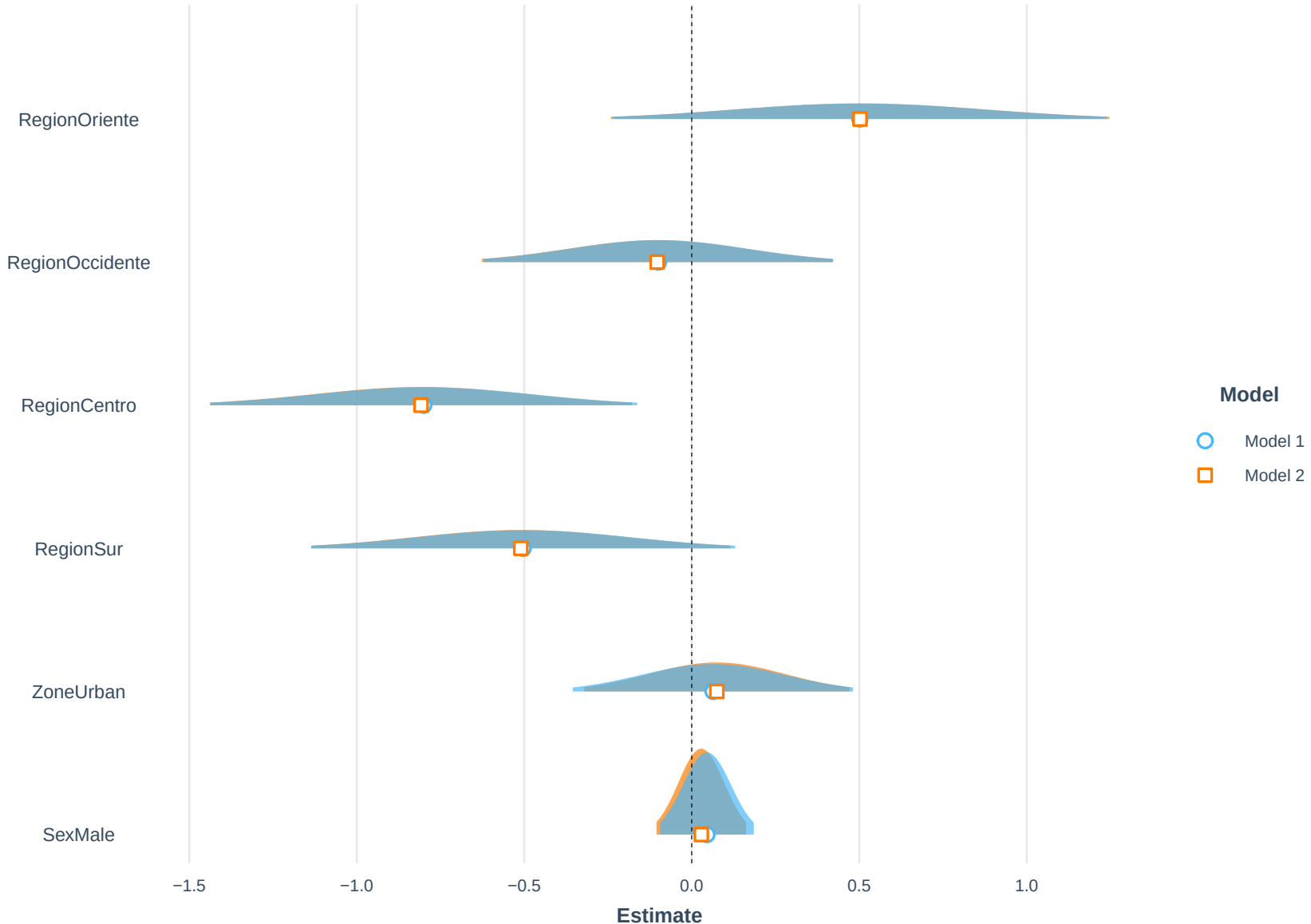
```
diseno_qwgt <- diseno_qwgt %>% mutate(
  pobreza = ifelse(Poverty != "NotPoor", 1, 0))

mod_loglin_qwgt <- svyglm(pobreza ~ Sex + Zone + Region,
                          family=quasibinomial,
                          design=diseno_qwgt)
(tab_mod <- tidy(mod_loglin_qwgt) )
```

# Modelo log lineal ajustado con Q_Weighting

| term | estimate | std.error | statistic | p.value |
|---|---|---|---|---|
| (Intercept) | -0.3611 | 0.2519 | -1.4334 | 0.1545 |
| SexMale | 0.0287 | 0.0683 | 0.4200 | 0.6753 |
| ZoneUrban | 0.0750 | 0.2023 | 0.3706 | 0.7116 |
| RegionSur | -0.5103 | 0.3194 | -1.5976 | 0.1129 |
| RegionCentro | -0.8081 | 0.3215 | -2.5136 | 0.0134 |
| RegionOccidente | -0.1033 | 0.2671 | -0.3867 | 0.6997 |
| RegionOriente | 0.5025 | 0.3796 | 1.3238 | 0.1882 |

# Plot de la distribución de los betas

```
plot_summs(mod_loglin, mod_loglin_qwgt,
           scale = TRUE, plot.distributions = TRUE)
```



# Modelo log lineal ajustado

| | exp_estimado | 2.5 % | 97.5 % |
|---|---|---|---|
| (Intercept) | 0.6969 | 0.4231 | 1.1479 |
| SexMale | 1.0291 | 0.8989 | 1.1782 |
| ZoneUrban | 1.0779 | 0.7220 | 1.6092 |
| RegionSur | 0.6003 | 0.3188 | 1.1303 |
| RegionCentro | 0.4457 | 0.2357 | 0.8427 |
| RegionOccidente | 0.9019 | 0.5312 | 1.5310 |
| RegionOriente | 1.6528 | 0.7792 | 3.5062 |

# Estadístico de Wald sobre los parámetros

```
regTermTest(model = mod_loglin_qwgt, ~Sex)
```

```
## Wald test for Sex
##  in svyglm(formula = pobreza ~ Sex + Zone + Region, desi
##      family = quasibinomial)
## F =  0.1764  on  1  and  113  df: p= 0.68
```

```
regTermTest(model = mod_loglin_qwgt, ~Zone)
```

```
## Wald test for Zone
##  in svyglm(formula = pobreza ~ Sex + Zone + Region, desi
##      family = quasibinomial)
## F =  0.1374  on  1  and  113  df: p= 0.71
```

```
regTermTest(model = mod_loglin_qwgt, ~Region)
```

```
## Wald test for Region
##  in svyglm(formula = pobreza ~ Sex + Zone + Region, desi
##      family = quasibinomial)
## F =  4.225  on  4  and  113  df: p= 0.0032
```

# Efecto del modelo.

```
effe_sex <- effect_plot(mod_loglin_qwgt, pred = Sex,
                        interval = TRUE)
effe_Zona <-effect_plot(mod_loglin_qwgt, pred = Zone,
                        interval = TRUE)
effe_Region <- effect_plot(mod_loglin_qwgt, pred = Region,
                           interval = TRUE)
(effe_sex |effe_Zona)/effe_Region
```

# Efecto del modelo.