Tutorial -2

We have the likelihood function as,

$$\ln p(t|x,w,\beta) = \frac{\beta}{2}\sum_{i=1}^{N}\{y(x_i,w) - t_i\}^2 + \frac{N}{2}\ln\beta - \frac{N}{2}\ln(2\pi)$$

Now sub. $\beta = 1$

$$\Rightarrow \ln p(t|x,w,\beta) = \frac{-1}{2}\sum_{i=1}^{N}\{y(x_i,w) - t_i\}^2 - \frac{N}{2}\ln(2\pi)$$

$\Rightarrow$ Now diff. wrt w to find w_{ML}. } maximize likelihood fn.
or
minimize the error fn.

& Let the line be $y = mx + c$ ($m \to w_1$, $c \to w_0$)

$$\Rightarrow \frac{-1}{2}(2)\sum_{i=1}^{N}\{y(x_i,w) - t_i\}\frac{\partial y(x_i,w)}{\partial w} = 0$$

$$\Rightarrow \sum_{i=1}^{N}\{y(x_i,w) - t_i\} = 0.$$

$$\Rightarrow m(1) + c - 1.2 + m(2) + c - 1.9 + m(3) + c - 3.2 = 0.$$

$$6m + 3c - 6.3 = 0.$$

$$\Rightarrow \boxed{2m + c = 2.1} \quad -(1)$$

Now, maximize wrt β;

$$\Rightarrow c = 2.1 - 2m$$

$$\frac{1}{\beta_{ML}} = \frac{1}{N}\sum_{i=1}^{N}\{y(x_i,w) - t_i\}^2$$

$$\Rightarrow \Sigma = \sum_{i=1}^{N}\{y(x_i,w) - t_i\}^2$$

$$\Rightarrow \Sigma = [m(1) + c - 1.2]^2 + [m(2) + c - 1.9]^2 + [m(3) + c - 3.2]^2$$

$$\Rightarrow \Sigma = [m + 2.1 - 2m - 1.2]^2 + [2m + 2.1 - 2m - 1.9]^2 + [m(3) + 2.1 - 2m - 3.2]^2$$

$$\Rightarrow [-m + 0.9]^2 + [0.2]^2 + [m - 1.1]^2 = \Sigma.$$

$$\Rightarrow 0.81 + m^2 - 1.8m + 0.04 + m^2 + 1.21 - 2.2m = \Sigma$$

$$\Rightarrow 2m^2 - 4m - 0.94 = 0$$

$$\Rightarrow m^2 - 2m - 0.47 = 0. \quad -(2)$$

Now from (2) we get

$$m = \frac{2 \pm \sqrt{5.88}}{2} = \frac{2 \pm 2.425}{2} \Rightarrow$$

$$m = 2.21 \text{ or } m = 0.21$$

(approx)

& $c = -2.32$ or $c = 2.63$

Now, the error value is low only when $m = 2.21$ & $c = -2.32$

$$\Rightarrow \boxed{y = 2.21x - 2.32}$$

Now we plot this graph using matplotlib.