Computer Vision Solution Assignment - 8

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Task-2

Based on the Baker-Matthews.pdf, we let $W(x;p)$ denote the parameterized set of allowed warps; where $p = (p_1, p_2, \ldots, p_n)^T$ is a vector of parameters.

The warp $W(x;p)$ takes the pixel x in the coordinate frame of the template T and maps it to the sub-pixel location $W(x;p)$ in the co-ordinate frame of the image I.

For the case of optical flow, our warp is the following translation:

$$W(x;p) = \begin{pmatrix} x+u \\ y+v \end{pmatrix}, \quad \Delta p = \begin{pmatrix} u \\ v \end{pmatrix} \tag{1}$$

The Jacobian of warp:

$$\frac{\partial W}{\partial p} = \begin{pmatrix} \partial W_x/\partial u & \partial W_x/\partial v \\ \partial W_y/\partial u & \partial W_y/\partial v \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2}$$

$\frac{\partial W}{\partial p}$ actually an identity matrix, we can thus use Equation 10:

$$\Delta p = H^{-1} \sum_x \left[\nabla I \frac{\partial W}{\partial p} \right]^T [T(x) - I(W(x;p))] \tag{3}$$

where

$$\begin{aligned}
H &= \sum_x \left[\nabla I \frac{\partial W}{\partial p} \right]^T \left[\nabla I \frac{\partial W}{\partial p} \right] && (4)\\
&= \sum_x \frac{\partial W^T}{\partial p} \nabla I^T \nabla I \frac{\partial W}{\partial p} && (5)\\
&= \sum_x \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\partial I}{\partial x} \\ \frac{\partial I}{\partial y} \end{pmatrix} \begin{pmatrix} \frac{\partial I}{\partial x} & \frac{\partial I}{\partial y} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} && (6)\\
&= \sum_x \begin{pmatrix} \frac{\partial I}{\partial x}\frac{\partial I}{\partial x} & \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} \\ \frac{\partial I}{\partial y}\frac{\partial I}{\partial x} & \frac{\partial I}{\partial y}\frac{\partial I}{\partial y} \end{pmatrix} && (7)\\
&= \begin{pmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{pmatrix} && (8)
\end{aligned}$$

Therefore:

$$\Delta p = H^{-1} \sum_x \left[\nabla I \frac{\partial W}{\partial p} \right]^T [T(x) - I(W(x;p))] \tag{9}$$

$$H \Delta p = \sum_x \left[\nabla I \frac{\partial W}{\partial p} \right]^T [T(x) - I(W(x;p))] \tag{10}$$

$$= \sum_x \begin{pmatrix} I_x \\ I_y \end{pmatrix} [T(x) - I(W(x;p))] \tag{11}$$

Note that the template $T(x)$ is an extracted sub-region of the image at $t = 1$ and $I(x)$ is the image at $t = 2$. Hence,

$$\begin{pmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \sum \begin{pmatrix} I_x \\ I_y \end{pmatrix} [-I_t] \tag{12}$$

$$\begin{pmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = - \begin{pmatrix} \sum I_x I_t \\ \sum I_y I_t \end{pmatrix} \tag{13}$$