CS-E4850 Computer Vision

Exam 14th of April 2021, Lecturer: Juho Kannala

There are plenty of questions. Possibly many more than can be solved in the given time but answer as many as you can in the available time. The number of points awarded from different parts is shown in parenthesis at the end of each question. The maximum score from the whole exam is 42 points.

The exam must be taken completely alone. Showing or discussing it with anyone is forbidden!

1. Image filtering

   (a) Filter image J with the gaussian filter G using zero padding. (1 p)

$$J = \begin{array}{|c|c|c|}\hline 0 & 1 & 0 \\ \hline 0 & 2 & 0 \\ \hline 0 & 3 & 0 \\ \hline \end{array} \qquad G = \frac{1}{16} \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

   (b) Is it more efficient to filter an image with two 1D filters as opposed to a 2D filter? Why? How does the computational complexity relate to the size of the filter kernel (with K×K pixels) in both cases? (1 p)

   (c) Is the following convolution kernel separable? If so, separate it. (1 p)

$$H = \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 4 \\ \hline 2 & 4 & 6 & 8 \\ \hline 3 & 6 & 9 & 12 \\ \hline 4 & 8 & 12 & 16 \\ \hline \end{array}$$

The bilateral filter consists of a domain kernel $d(i, j, k, l)$ and a range kernel $r(i, j, k, l)$. I is the original image. The coordinates (i, j) represent the pixel to be filtered and (k, l) the neighbouring pixels of the window centered in (i, j). σ_d and σ_r are smoothing parameters and $I(i, j)$ and $I(k, l)$ are the intensity of pixels (i, j) and (k, l) respectively.

$$d(i,j,k,l) = \exp\left(-\frac{(i-k)^2 + (j-l)^2}{2\sigma_d^2}\right), \quad r(i,j,k,l) = \exp\left(-\frac{\|I(i,j) - I(k,l)\|^2}{2\sigma_r^2}\right)$$

$$I = \begin{array}{|c|c|c|c|c|}\hline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & \boxed{255} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline \end{array}$$

   (d) Briefly explain the advantages of a bilateral filter compared to a Gaussian filter. (0.5 p)

   (e) Construct 3×3 range- and domain kernels at the center pixel (i_c, j_c) of I marked with a box. Let $\sigma_d = \sigma_r = 1$. (2 p)

(f) The bilateral weight function is the multiplication of the range- and domain kernels. Construct the 3×3 bilateral weight function $w(i_c, j_c, k, l)$ and briefly discuss what limitation of a bilateral filter your result indicates. (1 p)

Your camera produces 1D images and your task is to detect edges in the images. Consider the example 1D image L below:

$$L = \begin{array}{|c|c|c|c|c|c|c|c|} \hline 133 & 132 & 126 & 115 & 98 & 79 & 64 & 56 \\ \hline \end{array}$$

(g) Propose a suitable kernel for edge detection in 1D images. (1 p)

(h) Using your kernel indicate where an edge would be detected in image L. (0.5 p)

2. Image formation

Consider a camera with a camera projection matrix P and a 3D-point X in homogenous coordinates:

$$P = \begin{bmatrix} 5 & -14 & 2 & 17 \\ -10 & -5 & -10 & 50 \\ 10 & 2 & -11 & 19 \end{bmatrix} \qquad X = \begin{bmatrix} 0 \\ 2 \\ 2 \\ 1 \end{bmatrix}$$

(a) What are the 3D Cartesian coordinates of the point X? (0.5 p)

(b) Compute the Cartesian image coordinates of the projection of X. (0.5 p)

(c) We project point Z and get the following result $PZ = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}^T$. What is the interpretation of the projection of the point Z? (1 p)

(d) Compute the Cartesian coordinates of the camera center. (1 p)

(e) Show that the two cameras $P_1 = K_1[R\,|T]$ and $P_2 = K_2[R\,|T]$ have the same camera center. (0.5 p)

Now we switch to an ideal pinhole camera with the following intrinsic parameters:

- 5 mm focal length
- Each pixel is 0.02 mm × 0.02 mm
- Pixel coordinates start at (0,0) in the upper left corner of the image.
- The image principal point is at pixel (500,500)
- No distortion

The world reference system is the same as the camera's canonical reference system (camera is at the world origin and pointed towards the positive z-axis).

(f) Calculate the intrinsic- and extrinsic matrix. (1 p)

(g) A point **X** has coordinates (100, 150, 800) centimeters in the world reference system. Compute the projection of the point into image coordinates. (0.5 p)

3. Triangulation

Two cameras are looking at the same scene. The projection matrices of the two cameras are $\mathbf{P}_1$ and $\mathbf{P}_2$. They see the same 3D point $\mathbf{X} = (X, Y, Z)^\top$. The observed coordinates for the projections of point $\mathbf{X}$ are $\mathbf{x}_1$ and $\mathbf{x}_2$ in the two images, respectively. The numerical values are as follows:

$$\mathbf{P}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{P}_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}, \quad \mathbf{x}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} \frac{3}{4} \\ 0 \end{bmatrix}.$$

(a) Present a derivation for the linear triangulation method and explain how $\mathbf{X}$ can be solved using that approach in the general case (i.e. no need to compute with numbers in this subtask). (1 p)

(b) Compute the 3D coordinates of the point $\mathbf{X}$ using the given numeric values for the camera projection matrices and image points. It is sufficient to just give the result. (Hint: You can calculate this with a computer or using pen and paper. In the latter case it may be easiest to write the projection equations in homogeneous coordinates by explicitly writing out the unknown scale factors, and to solve X, Y, Z and the scale factors directly from those equations.) (1 p)

(c) A third camera $\mathbf{P}_3$ is added to the scene. Describe how the linear triangulation method above can be extended to use the information from all the three cameras. (1 p)

(d) If there is noise (i.e. measurement errors) in the observed image coordinates of point $\mathbf{X}$, the linear triangulation method above is not the optimal choice but a nonlinear approach can be used instead. What error function is typically minimized in the nonlinear approach? (1 p)

(e) How does the nonlinear triangulation approach differ from the bundle adjustment procedure which is commonly used in structure-from-motion problems (i.e. how is the bundle adjustment problem different)? (1 p)

4. Local feature detection and description

Below we have computed the gradients of an image at each pixel:

$I_x =$

3	2	1	-1	-1
4	3	2	0	-1
4	3	4	2	1
1	1	3	2	2

$I_y =$

2	3	1	1	-1
2	3	2	-1	-1
2	4	4	1	2
-1	0	3	2	3

(a) Compute the second moment matrix M for the coloured 3×3 window W. Assume that the weighting function w is a constant $w(x, y) = 1$

$$M = \begin{bmatrix} \sum_{x,y} w(x,y) I_x^2 & \sum_{x,y} w(x,y) I_x I_y \\ \sum_{x,y} w(x,y) I_x I_y & \sum_{x,y} w(x,y) I_y^2 \end{bmatrix}$$

(1 p)

(b) Compute the value of the corner response function when $\alpha = 0.04$:

$$R = det(M) - \alpha \, trace(M)^2$$

(1 p)

(c) How would you characterise the window W? (1 p)

Let's assume that we detected SIFT regions from two images (i.e. circular regions with assigned orientations) of the same textured plane.

(d) What is the minimum number of SIFT region correspondence pairs needed for computing a similarity transformation between the pair of images? (1 p)

(e) How do we compute a histogram of gradient orientations when generating a SIFT descriptor? (1 p)

5. Epipolar geometry

We have a camera pair with projection matrices $P = \begin{bmatrix} I & 0 \end{bmatrix}$ and $P' = \begin{bmatrix} R & \mathbf{t} \end{bmatrix}$ where R and $\mathbf{t}$ are such that the essential matrix E for the camera pair is the following:

$$E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

(a) Give one possible value for the unit-norm vector $\mathbf{t}$ that points from the second camera center to the first camera center and for the rotation matrix R between the two cameras. Justify your answer briefly. (1 p)

(b) What is the angle between the optical axes of the two cameras? No need to justify your answer, if you are sure about it. (1 p)

(c) A point in the second image has coordinates $\mathbf{b} = \begin{bmatrix} 0.5 & 1 & 1 \end{bmatrix}^T$ in the canonical camera reference system (units are focal distances). Write the equation of the epipolar line of $\mathbf{b}$ in the canonical image reference system of the first camera. Show your derivation. Remember that the canonical (i.e. normalized) image coordinates of a point are the same as its first two canonical camera coordinates. (1 p)

6. Model fitting using RANSAC algorithm

(a) Describe the main stages of the RANSAC algorithm in the general case. (1 p)

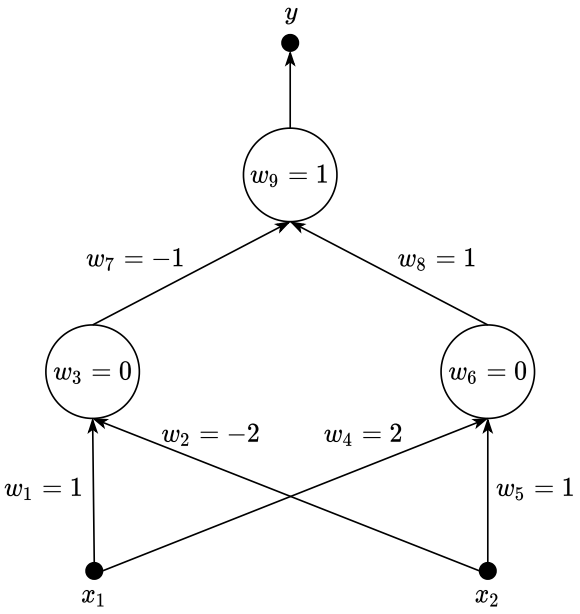
(b) In this context, why is it usually beneficial to sample minimal subsets of data points instead of using more data points? (Minimal subsets have the minimal number of data points required for fitting.) (1 p)

(c) Mention at least two examples of models that can be fitted using RANSAC. Describe how the models are used in computer vision and what is the size of the minimal subset of data points required for fitting in each case. (1 p)

(d) Describe how RANSAC ca be used for panoramic image stitching. Why is RANSAC needed and what is the model fitted in this case? (1 p)

7. Neural networks and object detection

The small neural net in the figure below uses ReLU as the nonlinearity at the output of each neuron. The values specified in the hollow circles are biases, and the values along the edges are gains.



(a) Are all the layers in the network above fully connected? (1 p)

(b) What is the output y from the net above when the input is as follows? (1 p)

$$x_1 = 0 \quad \text{and} \quad x_2 = 3$$

(c) What is the gradient $\mathbf{g}$ of the output y of the network above with respect to the weight vector

$$\mathbf{w} = [w_1,\ w_2,\ w_3,\ w_4,\ w_5,\ w_6,\ w_7,\ w_8,\ w_9]^T$$

when the input has the values given in the previous problem? Just give the result if you are confident of your answer. (2 p)

(d) With image data convolutional neural networks are much more popular than fully connected neural networks. Why is this? (1 p)

(e) Especially deep convolutional neural networks have proven to be effective. What function do the earlier layers (a.k.a. the base network) of a deep convolutional neural network serve and why are they often re-used from pre-existing networks such as VGG16. (1 p)

(f) SSD object detector evaluates only a small set (e.g. 4) of default boxes of different aspect ratios at each location. How can it detect large and small objects if the boxes are of fixed size? (1 p)

8. Feature tracking

Let $I(\mathbf{x})$ and $J(\mathbf{x})$ be two grayscale images of the same scene taken from slightly different viewpoints and possibly slightly different orientations. We'd like to track a point $\mathbf{x}_I$ in image I to it's coordinate $\mathbf{x}_J$ in image J. That is we'd like to know the two dimensional displacement $\mathbf{d}^*$ of point $\mathbf{x}_I$ such that:

$$\mathbf{x}_J = \mathbf{x}_I + \mathbf{d}^*$$

To approximate $\mathbf{d}^*$ we look at a window (small square) $W(\mathbf{x}_I)$ of odd side-length 2h+1 pixels centered around the point $\mathbf{x}_I$ in image I and search for $\mathbf{d}$ that minimizes the dissimilarity between the windows in both images:

$$\mathbf{d}^* = \arg\min_{\mathbf{d}} \epsilon(\mathbf{d})$$

where the dissimilarity $\epsilon(\mathbf{d})$ is defined as a sum over the whole image $\mathbf{x} = (x_1, x_2)$:

$$\epsilon(\mathbf{d}) = \sum_{\mathbf{x}} [J(\mathbf{x} + \mathbf{d}) - I(\mathbf{x})]^2 w(\mathbf{x} - \mathbf{x}_I)$$

$w(\mathbf{x})$ is the indicator function of a $W(\mathbf{x})$:

$$w(\mathbf{x}) = \begin{cases} 1 & \text{if } |x_1| \leq h \text{ and } |x_2| \leq h \\ 0 & \text{otherwise.} \end{cases}$$

We assume that the motion of the camera between the two images is so small that the magnitude of $\mathbf{d}^*$ is much smaller than the diameter of $W(\mathbf{x}_I)$ and use an iterative approach so that we can formulate the problem as follows: find a step displacement $\mathbf{s}_t$ that, when added to $\mathbf{d}_t$, yields a new displacement $\mathbf{d}_{t+1}$ at each iteration t such that $\epsilon(\mathbf{d}_t + \mathbf{s}_t)$ is minimized. We add $\mathbf{d}_t$ into $\mathbf{x}$ as follows $J_t(\mathbf{x}) = J(\mathbf{x} + \mathbf{d}_t)$ and approximate the image function $J_t(\mathbf{x} + \mathbf{s}_t)(= J(\mathbf{x} + \mathbf{d}_t + \mathbf{s}_t))$ with its first-order Taylor expansion:

$$J_t(\mathbf{x} + \mathbf{s}_t) \approx J_t(\mathbf{x}) + [\nabla J_t(\mathbf{x})]^T \mathbf{s}_t$$

Minimizing $\epsilon(\mathbf{d}_t + \mathbf{s}_t)$ leads to a linear system of equations $A\mathbf{s}_t = \mathbf{b}$ where

$$A = \sum_{\mathbf{x}} \nabla J_t(\mathbf{x}) [\nabla J_t(\mathbf{x})]^T w(\mathbf{x} - \mathbf{x}_I) \quad \text{and} \quad \mathbf{b} = \sum_{\mathbf{x}} \nabla J_t(\mathbf{x}) [I(\mathbf{x}) - J_t(\mathbf{x})] w(\mathbf{x} - \mathbf{x}_I)$$

The overall displacement is then the sum of all the steps:

$$\mathbf{d}^* = \sum_t \mathbf{s}_t$$

(a) Show that minimizing $\epsilon(\mathbf{d_t} + \mathbf{s}_t)$ leads to a linear system of equations $A\mathbf{s}_t = \mathbf{b}$. (1 p)

NOTE: the problems (b)-(e) below don't require that you have solved problem (a).

Assuming a window size of 3×3 ($h = 1$) and an initial guess of displacement $\mathbf{d}_0 = [0, 0]^T$. For a particular value of $\mathbf{x}_I$, the two components of $\nabla J_0(\mathbf{x})$ inside the window $W(\mathbf{x}_I)$ are:

$$\frac{\partial J_0}{\partial x_1} = \begin{array}{|c|c|c|} \hline 10 & 10 & 10 \\ \hline 10 & 10 & 10 \\ \hline 10 & 10 & 10 \\ \hline \end{array} \quad \text{and} \quad \frac{\partial J_0}{\partial x_2} = \begin{array}{|c|c|c|} \hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array}$$

and the difference between the two images is:

$$I(\mathbf{x}) - J_0(\mathbf{x}) = \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

(b) Compute A and $\mathbf{b}$. (1 p)

(c) Does the feature at $\mathbf{x}_I$ suffer from the aperture problem? Briefly justify your answer. (1 p)

(d) Give the minimum-norm solution $\mathbf{s}_0$ to the linear system $A\mathbf{s}_0 = \mathbf{b}$ (1 p)

(e) Assume that further iterations of the Lucas-Kanade algorithm do not change the solution $\mathbf{s}_0$ much. Does your answer to the previous question imply that the image motion between I and J at $\mathbf{x}_I$ is approximately horizontal? Briefly justify your answer. (1 p)