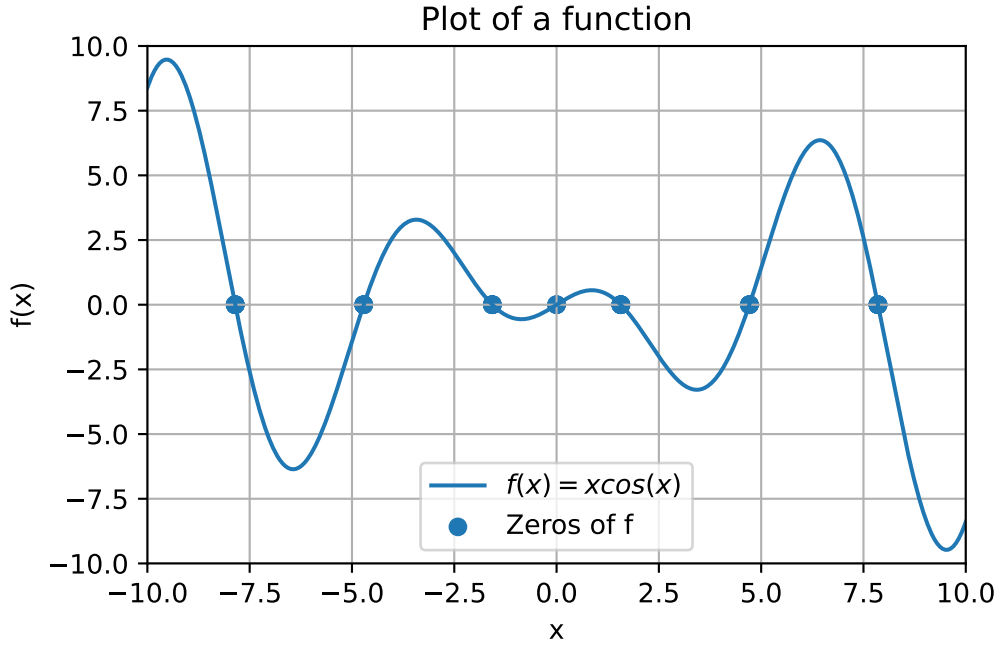
Solutions Lecture 5 (Chapter 7)

Make sure to import Numpy, SciPy and Matplotlib to be able to complete all the exercises.

```
import numpy as np
import scipy.optimize as optimize
import matplotlib.pyplot as plt
```

Question 1

Plot the function $f(x) = x \cdot cos(x)$ on the interval $[-10, 10]$ and indicate all the roots of this function with a dot. To compute all the roots in the interval $[-10, 10]$ use your solution to Question 1 from the Exercises corresponding to Lecture 4. Your plot should look like this.




```
# Define the function f
def f(x):
    return x*np.cos(x)

# Define the x range of x-values
x = np.linspace(-10,10,600)
```

```
# Compute the function values
y = f(x)

# Compute list of roots
guess = np.arange(-10,10.5,0.5)

roots = []
for i in guess:
    x_root = optimize.fsolve(f,x0=i)[0]
    roots.append(x_root)

n = np.size(roots)

# Create list of zeros
zeros = np.zeros(n)

#Create the figure
plt.figure()

# Create the plot
plt.plot(x, y, label='$f(x) = xcos(x)$')

# Create a grid
plt.grid()

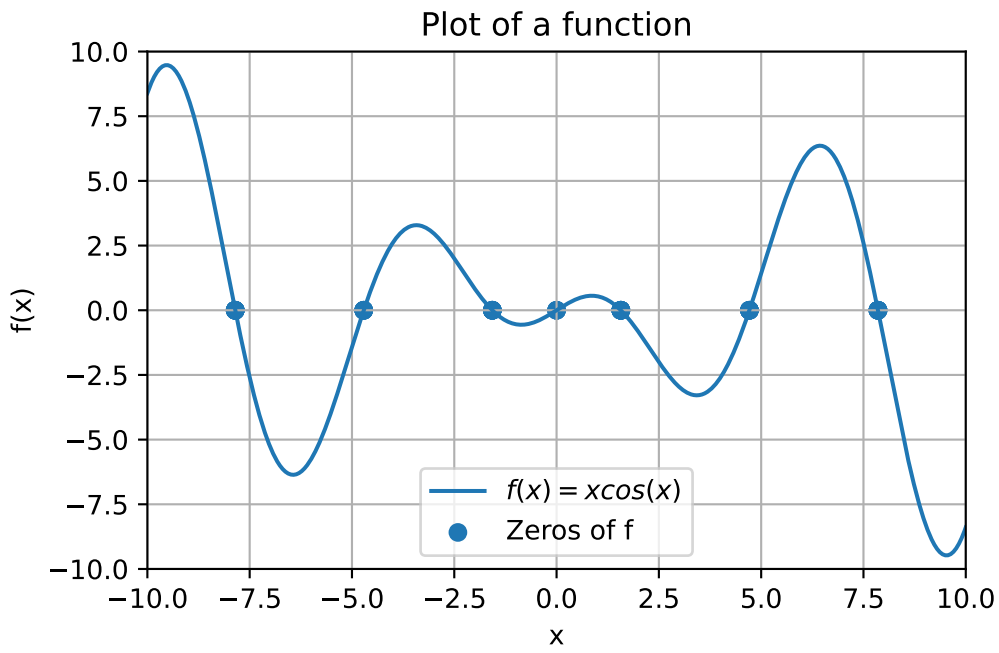
# Create range for axes
plt.xlim(a,b)
plt.ylim(a,b)

# Create legend, axes labels and title
plt.xlabel('x')
plt.ylabel('f(x)')

plt.title('Plot of a function')

# Plot the roots
plt.scatter(roots,zeros,label='Zeros of f')

# Plot the legend with the labels
plt.legend()
```



Question 2

For a one-dimensional array x, we can make a histogram of the values in x using `plt.hist()` as the next example illustrates. We can use the `bins` keyword argument to specify in how many "bins" the values should be divided.

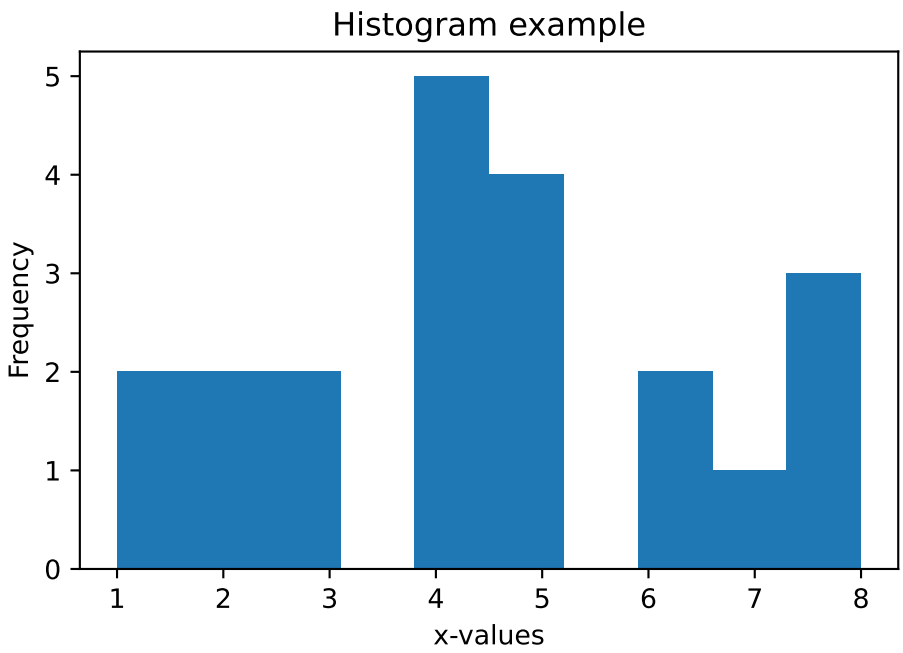
```
x = np.array([1,1,4,2,6,5,4,8,8,8,7,6,5,4,3,4,5,3,4,2,5])

# Create figure
plt.figure()

# Create histogram
plt.hist(x,bins=10)

# Add labels and title
plt.xlabel("x-values")
plt.ylabel("Frequency")
plt.title("Histogram example")

# Show the plot
plt.show()
```



It is well known that if we draw many samples from a probability distribution, then the histogram of these samples should converge to the probability density function of the distribution. We will test this for the normal distribution.

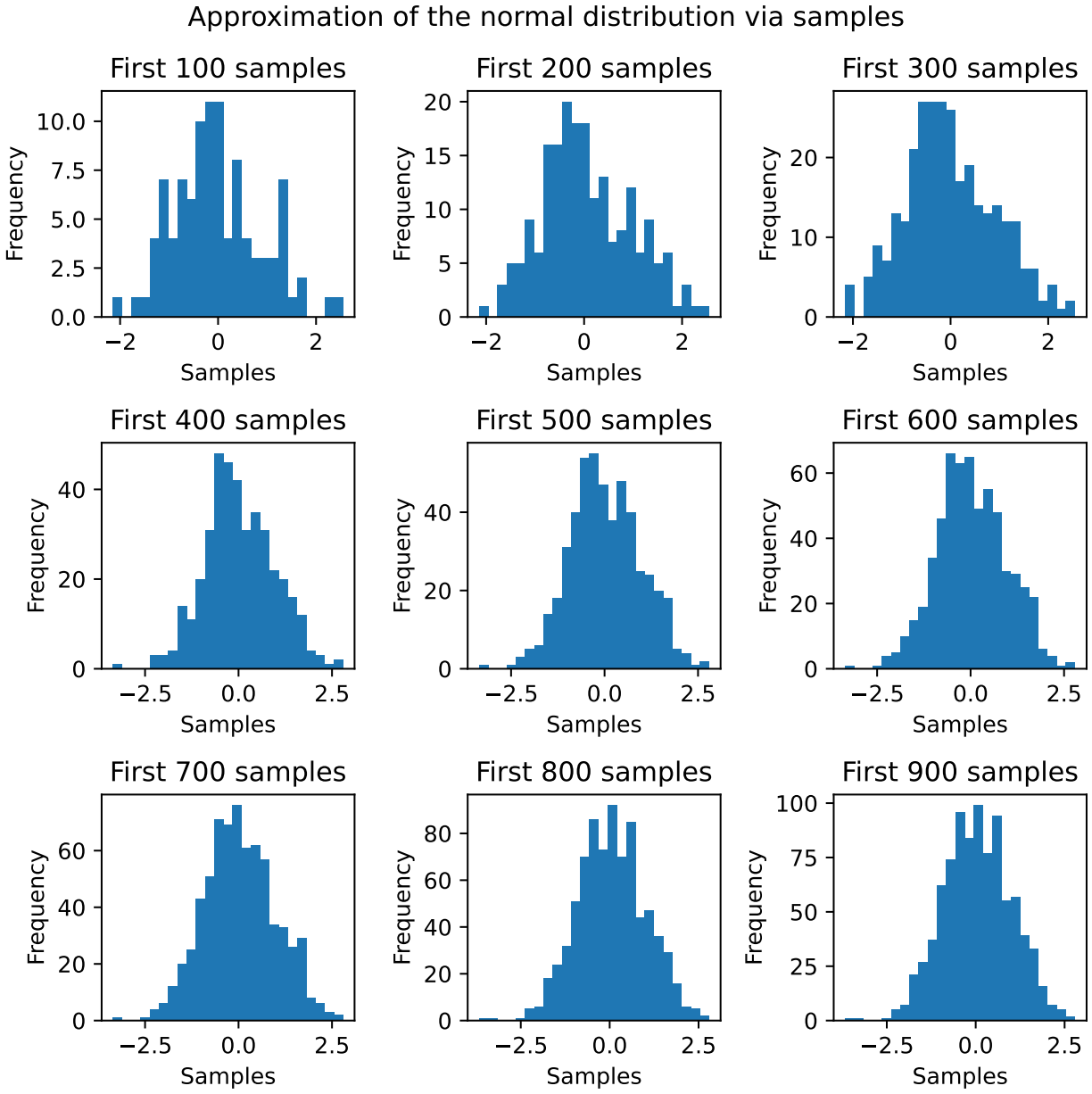
The following code generates an array `samples` with $n = 900$ samples from the normal distribution with mean 0 and standard deviation 1. We will see more on sample generation in a later chapter of this book. For now, we will only need to work with the vector `samples`.

```
# Mean of the distribution
mean = 0

# Standard deviation of the distribution
std_dev = 1

# Generate random samples from normal distribution
n = 900
samples = np.random.normal(mean, std_dev, n)
```

We will generate nine figures that show that the more samples we put in the histogram, the more the histogram will look like a bell-shaped normal distribution curve. To this end, create a figure using `plt.subplots()` with nine subplots where plot i contains the first $100i$ samples of `samples` in a histogram with 25 bins. Avoid unnecessary repetition as much as possible. Your figure should look like the one below. Note that because of the randomness involved in the sample generation, your histograms do not have to look exactly the same. What is important is that the figures (seem to) converge to the bell-shaped curve.




```
# Figure parameters
w = 7
k = 3

# Create figure
fig, ax = plt.subplots(k,k, figsize=(w,w))

# Title of whole figure
fig.suptitle("Approximation of the normal distribution via samples")

# Added samples per plot
```

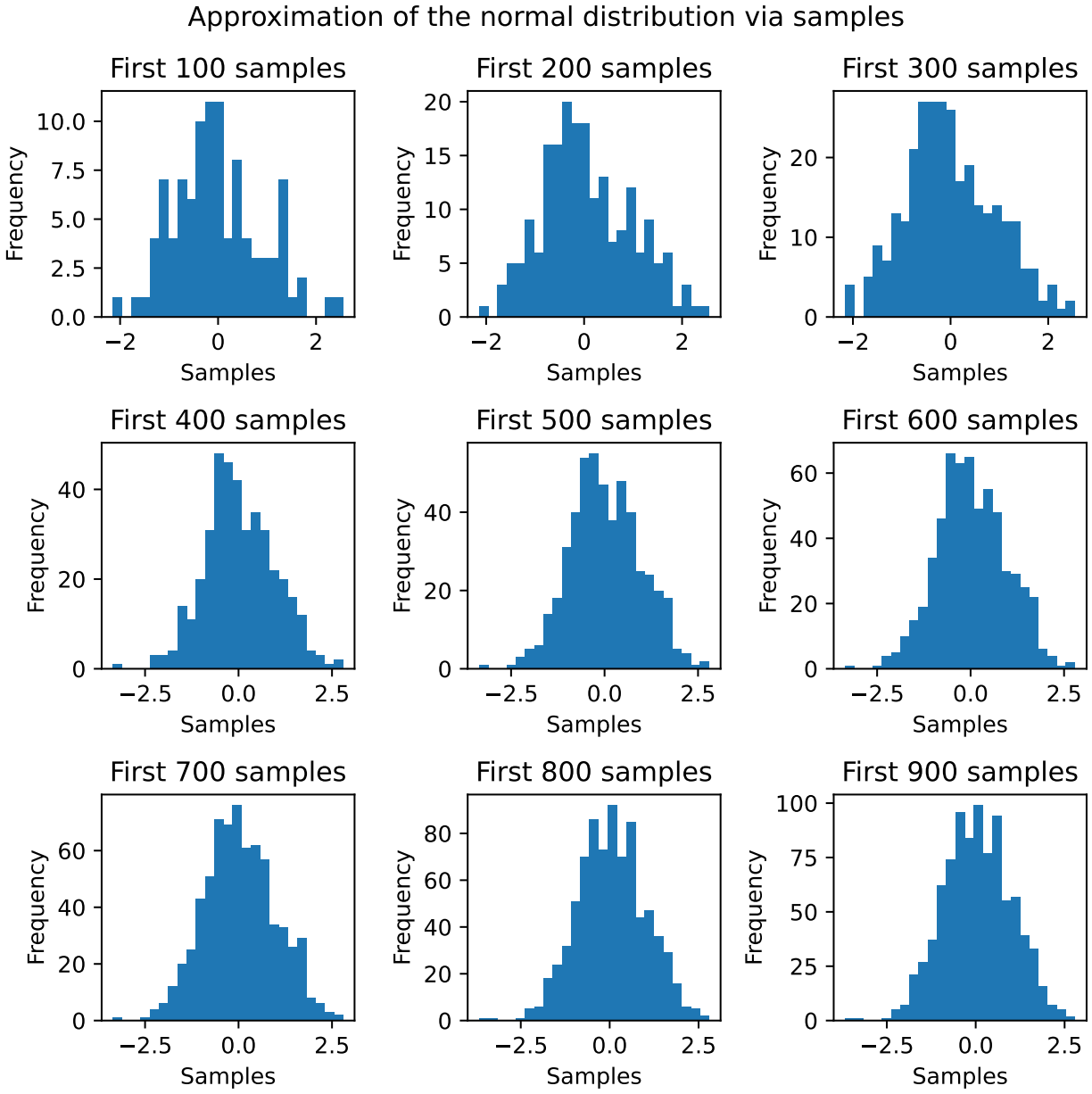
```
m = 100

# Sample numbers
count = m*np.arange(1,10).reshape(k,k)

for i in range(k):
    for j in range(k):
        ax[i,j].hist(samples[:count[i,j]],bins=25)
        ax[i,j].set_xlabel("Samples")
        ax[i,j].set_ylabel("Frequency")
        ax[i,j].set_title(f"First {count[i,j]} samples")

# Tighten layout
plt.tight_layout()

# Show plot
plt.show()
```



Question 3

Write a function `plot_average` that takes as input a two-dimensional array and two numbers a and b. Every row contains function values of n evenly-spaced points $a = x_0, \ldots, x_{n-1} = b$ on the interval $[a, b]$, that is, of points in `x = np.linspace(a,b,n)`. You should think of the elements y_{ij} in row i of the array as function values, i.e., $y_{ij} = f_i(x_j)$ for $j = 0, \ldots, n-1$. (We do not actually need to know the function f_i; we are only interested in the given values for $x_0, \ldots, x_{n-1}$.) The function `plot_average` should output a figure that has subplots of all the functions next to each other on the interval $[a, b]$ (plotted for the points in x). Under these plots, in the same figure, there should be a plot with the average of the functions, which is

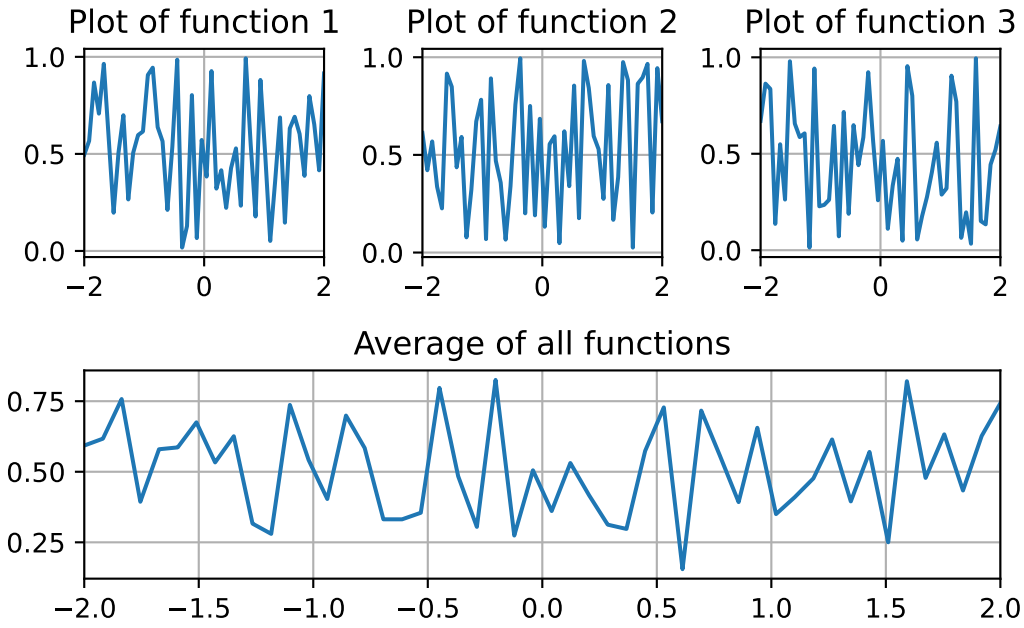
for $j = 0, \ldots, n-1$ the function defined by

$$\bar{f}(x_j) = \frac{1}{m} \sum_i f_i(x_j)$$

where m is the number of rows (or functions f_i) of the input array. Your function should look like this for the given input. Also here, the numerical values in the plots might be different because of the randomness in the data generation.

```
# Generate some random data from [0,1]
y = np.random.rand(3,50)
a = -2
b = 2

plot_average(y,a,b)
```




```
def plot_average(y,a,b):
    # Number of functions and points
    m, n = np.shape(y)

    # x-values
    x = np.linspace(a,b,n)

    # Create figure
    fig = plt.figure()

    # Add function plots
    for i in range(m):
        ax = fig.add_subplot(2,m,i+1)
```

```
            ax.plot(x,y[i,:])
            ax.set_title(f"Plot of function {i+1}")
            ax.set_xlim(a,b)
            ax.grid()

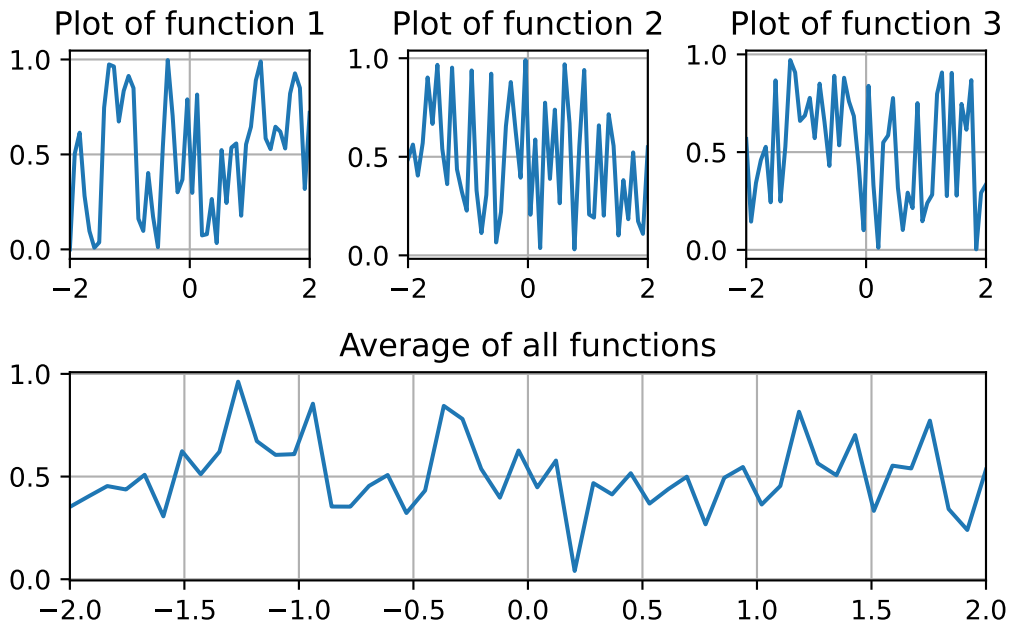
        # Plot the average
        ax = fig.add_subplot(2,m,(m+1,2*m))
        ax.plot(x,np.mean(y,axis=0))
        ax.set_title("Average of all functions")
        ax.set_xlim(a,b)
        ax.grid()

        # Tighten plot
        plt.tight_layout()

        # Show plot
        plt.show()
        return

# Generate some random data from [0,1]
y = np.random.rand(3,50)
a = -2
b = 2

plot_average(y,a,b)
```



Question 4

It is not always possible to use the matrices X and Y generated by `mgrid` directly for visualization of bivariate functions. For example, suppose that the function $f : \mathbb{R}^2 \to \mathbb{R}$ is defined by taking as input an array of shape $(1, 2)$ containing two elements.

```
def f(x):
    return x[:,0]**2 + x[:,1]**2
```

The function f is vectorized in the sense that if we input a $k \times 2$ array, then it computes the function value for every of the k row arrays (with two elements each), but we cannot input the output matrices X and Y of `mgrid` into f. Nevertheless, in this case it is still possible to use `mgrid` for plotting.

a) Write a function `grid_list` that takes as input matrices X and Y. It should output all the two-dimensional points $[X[i, j], Y[i, j]]$ on the rows of an array. Do this by reshaping the matrices X and Y in combination with stacking. Your function should give the following output on the given input.

```
a = 3
b = 5.1
step = 0.2

X, Y = np.mgrid[a:b:step,a:b:step]

print(grid_list(X,Y))
```

```
[[3.  3. ]
 [3.  3.2]
 [3.  3.4]
 [3.  3.6]
 [3.  3.8]
 [3.  4. ]
 [3.  4.2]
 [3.  4.4]
 [3.  4.6]
 [3.  4.8]
 [3.  5. ]
 [3.2 3. ]
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 [3.2 4. ]
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 [3.2 4.6]
 [3.2 4.8]
 [3.2 5. ]
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```

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[4.2 4.2]
```

```
[4.2 4.4]
[4.2 4.6]
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[4.2 5. ]
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[4.6 5. ]
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[5.  4. ]
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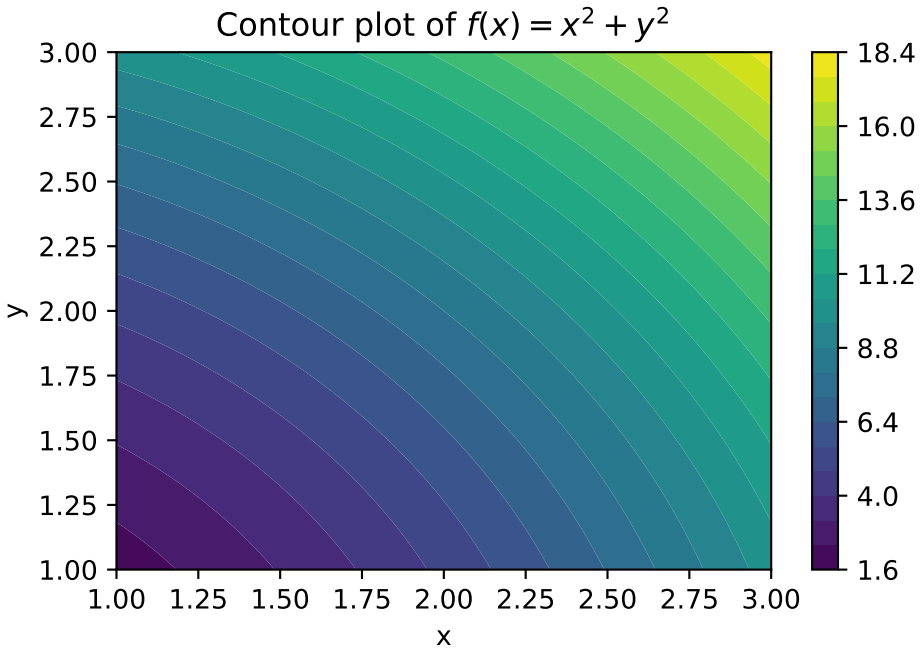
[5.  4.6]
[5.  4.8]
[5.  5. ]]
```

```
def grid_list(X,Y):
    x = X.flatten()
    y = Y.flatten()
    return np.vstack((x,y)).T
```

b) Make a contour plot of the function f (as defined in this exercise) using `plt.contourf()` with 25 levels on the interval $[1,3] \times [1,3]$ with discretization step size 0.01. Do this by computing the function values $Z[i,j]$ of every grid point (i,j) after applying `grid_list` to the desired grid, and then reshape the function value vector into a two-dimensional array again (so that $Z[i,j]$ corresponds to $X[i,j]^2 + Y[i,j]^2$). Your plot should look like this.




```
# Grid parameters
a = 1
b = 3.005
step = 0.01

# Define grid [a,b]^2 with given step size
X,Y = np.mgrid[a:b:step,a:b:step]
coordinates = grid_list(X,Y)

n = np.shape(X)[0]

# Compute function values in all grid points
z = f(coordinates)

# Reshape z back to two-dimensional array
```

```
Z = z.reshape(n,n)

# Create figure
plt.figure()

# Create contour plot with 25 levels
plt.contourf(X, Y, Z, levels=25)

# Add labels and title
plt.xlabel("x")
plt.ylabel("y")
plt.title("Contour plot of $f(x) = x^2 + y^2$")

# Show the plot
plt.colorbar()  # Add a color bar for reference

# Show plot
plt.show()
```