FACULTAD
DE INGENIERIA
Universidad de Buenos Aires

Procesamiento de señales, fundamentos

Maestría en sistemas embebidos
Universidad de Buenos Aires
MSE 5Co2020

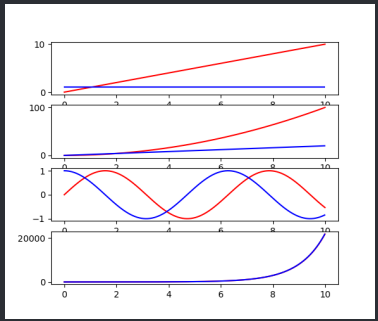
Clase 3 - Euler | Fourier

Ing. Pablo Slavkin
slavkin.pablo@gmail.com
wapp:011-62433453

2.718281828459045090795598298427648842334747314 4

- $f(t) = t$
- $f(t) = t^2$
- $f(t) = sin(t)$

- $f'(t) = 1$
- $f'(t) = 2 * t$
- $f'(t) = cos(t)$



La derivada es igual a la funcion

$$f(t) = e^t \implies f'(t) = e^t$$
$$f(t) = e^{kt} \implies f'(t) = ke^{kt}$$

$e^{j2\pi ft}$

Euler

Pero que pasa con e^{jt}?

La derivada es igual a la funcion

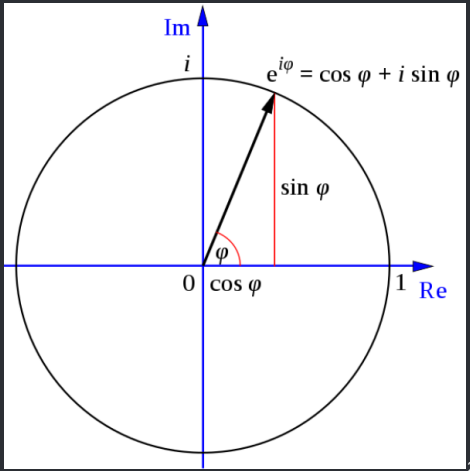
$$f(t) = e^{jt} \implies f'(t) = je^{jt}$$

$$e^{jt} = \cos(t) + j\sin(t)$$

$$e^{j\pi} = -1$$

$$e^{\frac{j\pi}{2}} = j$$

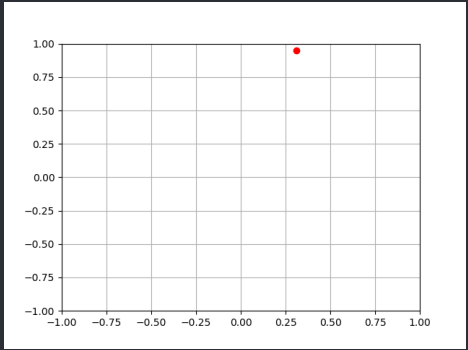
$$e^{\frac{j3\pi}{2}} = -j$$



$e^{j2\pi ft}$

$e^{j2\pi ft}$ animado

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
fig        = plt.figure()
fs         = 20
N          = 20
circleAxe  = fig.add_subplot(1,1,1)
circleLn,  = plt.plot([],[],'ro')
circleAxe.grid(True)
circleAxe.set_xlim(-1,1)
circleAxe.set_ylim(-1,1)
circleFrec = 1
def circle(c,f,n):
    return c*np.exp(-1j*2*np.pi*f*n*1/fs)
def init():
    return circleLn,
def update(n):
    circleLn.set_data(np.real(circle(1,circleFrec,n)),
                      np.imag(circle(1,circleFrec,n)))
    return circleLn,
ani=FuncAnimation(fig,update,N,init,interval=100 ,blit=False,repeat=True
    )
plt.show()
```

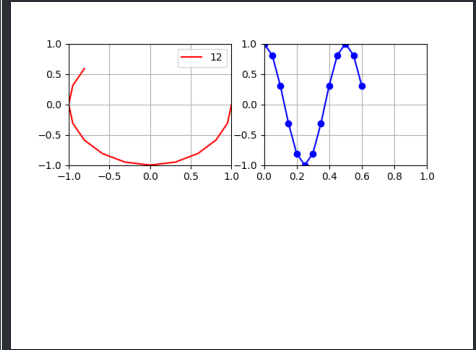


$e^{j2\pi ft}$

$e^{j2\pi ft}$ y $\sin(t)$ animados independientemente

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
#--------------------------------------
fig         = plt.figure()
fs          = 20
N           = 20
#--------------------------------------
circleAxe  = fig.add_subplot(2,2,1)
circleLn,  = plt.plot([],[],'r-')
circleAxe.grid(True)
circleAxe.set_xlim(-1,1)
circleAxe.set_ylim(-1,1)
circleLn.set_label(0)
circleLg=circleAxe.legend()
circleFrec = 1
circleData = []
def circle(f,n):
    return np.exp(-1j*2*np.pi*f*n*1/fs)
#--------------------------------------
signalAxe  = fig.add_subplot(2,2,2)
signalLn,  = plt.plot([],[],'b-o')
signalAxe.grid(True)
signalAxe.set_xlim(0,N/fs)
signalAxe.set_ylim(-1,1)
signalFrec = 2
signalData=[]
```

```
signalData=[]
def signal(f,n):
    return np.cos(2*np.pi*f*n*1/fs)
#--------------------------------------
tData=np.arange(0,N/fs,1/fs)
def init():
    return circleLn,circleLg,signalLn,
def update(n):
    global circleData,signalData
    circleData.append(circle(circleFrec,n))
    circleLn.set_data(np.real(circleData),
                      np.imag(circleData))
    signalData.append(signal(signalFrec,n))
    signalLn.set_data(tData[:n+1],signalData)
    if n==N-1:
        circleData=[]
        signalData=[]
    circleLn.set_label(n)
    circleLg=circleAxe.legend()
    return circleLn,circleLg,signalLn,
ani=FuncAnimation(fig,update,N,init,interval=100 ,
        blit=True,repeat=True)
plt.show()
```

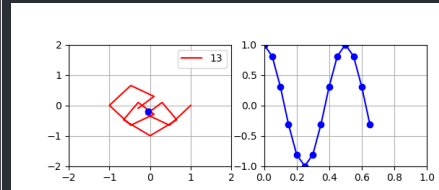


$e^{j2\pi ft}$

$e^{j2\pi ft}$ *modulado por* $\sin(t)$ *y centro de masas*

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
#---------------------------------------
fig         = plt.figure()
fs          = 20
N           = 20
#---------------------------------------
circleAxe  = fig.add_subplot(2,2,1)
circleLn,  = plt.plot([],[],'r-')
circleAxe.grid(True)
circleAxe.set_xlim(-1,1)
circleAxe.set_ylim(-1,1)
circleLn.set_label(0)
circleLg=circleAxe.legend()
circleFrec = 1
circleData = []
def circle(f,n):
    return np.exp(-1j*2*np.pi*f*n*1/fs)
#---------------------------------------
signalAxe  = fig.add_subplot(2,2,2)
signalLn,  = plt.plot([],[],'b-o')
signalAxe.grid(True)
signalAxe.set_xlim(0,N/fs)
signalAxe.set_ylim(-1,1)
signalFrec = 2
signalData=[]
def signal(f,n):
    return np.cos(2*np.pi*f*n*1/fs)
```

```
def signal(f,n):
    return np.cos(2*np.pi*f*n*1/fs)
#---------------------------------------
tData=np.arange(0,N/fs,1/fs)
def init():
    return circleLn,circleLg,signalLn,massLn
def update(n):
    global circleData,signalData
    circleData.append(circle(circleFrec,n)*signal(
            signalFrec,n))
    mass=np.average(circleData)
    massLn.set_data(np.real(mass),
                    np.imag(mass))
    circleLn.set_data(np.real(circleData),
                      np.imag(circleData))
    signalData.append(signal(signalFrec,n))
    signalLn.set_data(tData[:n+1],signalData)

    if n==N-1:
        circleData = []
        signalData = []
    circleLn.set_label(n)
    circleLg=circleAxe.legend()
    return circleLn,circleLg,signalLn,massLn

ani=FuncAnimation(fig,update,N,init,interval=100 ,
        blit=True,repeat=True)
plt.show()
```

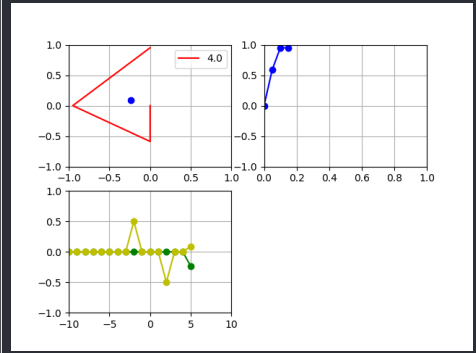


$e^{j2\pi ft}$

$e^{j2\pi ft}$ modulado por $\sin(t)$ y centro de masas en f, DFT?

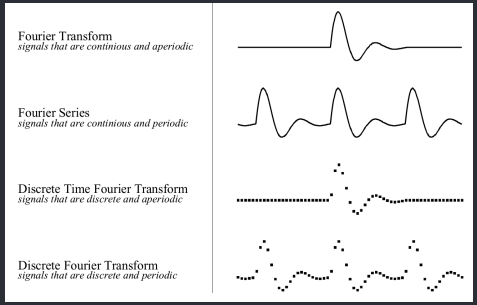
```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
#--------------------------------------
fig        = plt.figure()
fs         = 20
N          = 20
#--------------------------------------
circleAxe  = fig.add_subplot(2,2,1)
circleLn,massLn  = plt.plot([],[],'r-',[],[],'bo')
circleAxe.grid(True)
circleAxe.set_xlim(-1,1)
circleAxe.set_ylim(-1,1)
circleFrec = np.arange(-fs/2,fs/2,fs/N)
circleLn.set_label(circleFrec[0])
circleLg   = circleAxe.legend()
circleData = []
mass       = 0
frecIter   = 0
def circle(f,n):
    return np.exp(-1j*2*np.pi*f*n*1/fs)
#--------------------------------------
signalAxe  = fig.add_subplot(2,2,2)
signalLn,  = plt.plot([],[],'b-o')
signalAxe.grid(True)
signalAxe.set_xlim(0,N/fs)
signalAxe.set_ylim(-1,1)
signalFrec = 2
signalData=[]
def signal(f,n):
    return np.sin(2*np.pi*f*n*1/fs)
#--------------------------------------
promAxe  = fig.add_subplot(2,2,3)
promRLn,promILn,  = plt.plot([],[],'g-o',[],[],'y-o')
promAxe.grid(True)
promAxe.set_xlim(-fs/2,fs/2)
promAxe.set_ylim(-1,1)
promData=np.zeros(N,dtype=complex)
#--------------------------------------
tData=np.arange(0,N/fs,1/fs)
```

```
tData=np.arange(0,N/fs,1/fs)
def init():
    return circleLn,circleLg,signalLn,massLn,promRLn,promILn
def update(n):
    global circleData,signalData,promData,frecIter,circleFrec,
                circleLg
    circleData.append(circle(circleFrec[frecIter],n)*signal(
                signalFrec,n))
    mass=np.average(circleData)
    massLn.set_data(np.real(mass),
                    np.imag(mass))
    circleLn.set_data(np.real(circleData),
                      np.imag(circleData))
    signalData.append(signal(signalFrec,n))
    signalLn.set_data(tData[:n+1],signalData)
    promData[frecIter]=mass
    promRLn.set_data(circleFrec[:frecIter+1],np.real(promData[:
                frecIter+1]))
    promILn.set_data(circleFrec[:frecIter+1],np.imag(promData[:
                frecIter+1]))
    if n==N-1:
        circleData = []
        signalData = []
        circleLn.set_label(circleFrec[frecIter])
        circleLg=circleAxe.legend()
        if frecIter == N-1:
            ani.repeat=False
        else:
            frecIter+=1
    return circleLn,circleLg,signalLn,massLn,promRLn,promILn,
ani=FuncAnimation(fig,update,N,init,interval=100 ,blit=True,
          repeat=True)
plt.show()
```



Transformada de Fourier

Diferentes tipos segun la señal



Time Duration		
Finite	Infinite	
Discrete FT (DFT)	Discrete Time FT (DTFT)	discr.
$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\omega_k n}$	$X(\omega) = \sum_{n=-\infty}^{+\infty} x(n) e^{-j\omega n}$	time
$k = 0, 1, \ldots, N-1$	$\omega \in [-\pi, +\pi)$	n
Fourier Series (FS)	Fourier Transform (FT)	cont.
$X(k) = \frac{1}{P} \int_0^P x(t) e^{-j\omega_k t} dt$	$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$	time
$k = -\infty, \ldots, +\infty$	$\omega \in (-\infty, +\infty)$	t
discrete freq. k	continuous freq. ω	

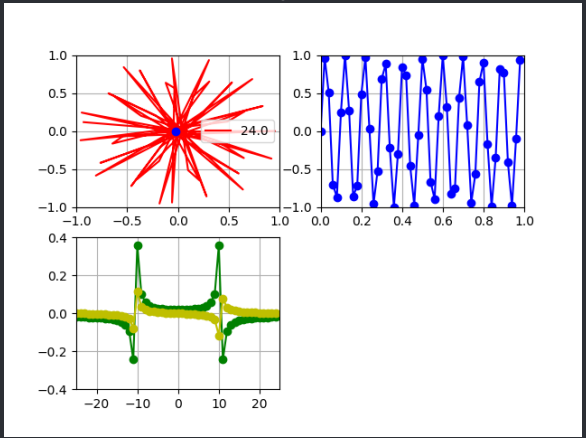
Analisi, Transformada discreta de Fourier

DFT

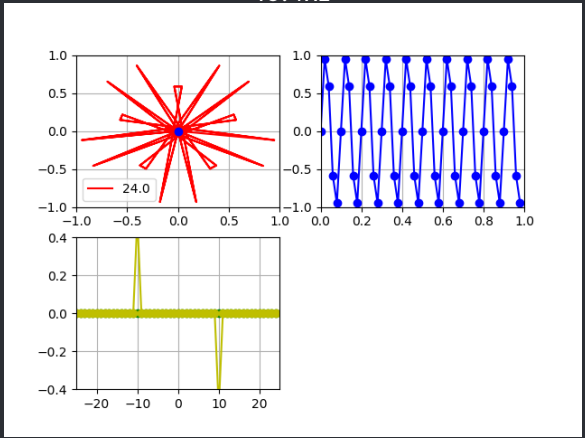
$$X_k = \sum_{n=0}^{N-1} x_n e^{-\frac{2\pi i}{N} kn} \qquad k = 0, \ldots, N-1$$

$e^{j2\pi ft}$ DFT Spectral leakage

10Hz



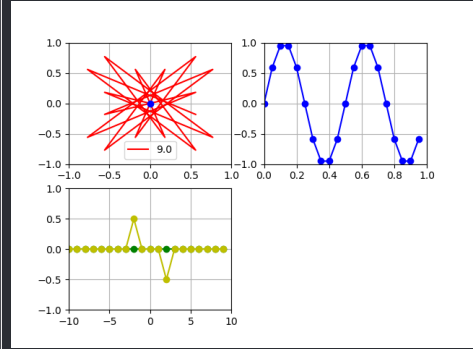
10.4hz



$e^{j2\pi ft}$

$e^{j2\pi ft}$ DFT acelerada

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
#---------------------------------------
fig        = plt.figure()
fs         = 20
N          = 20
#---------------------------------------
circleAxe  = fig.add_subplot(2,2,1)
circleLn,massLn  = plt.plot([],[],'r-',[],[],'bo')
circleAxe.grid(True)
circleAxe.set_xlim(-1,1)
circleAxe.set_ylim(-1,1)
circleFrec = np.arange(-fs/2,fs/2,fs/N)
circleLn.set_label(circleFrec[0])
circleLg   = circleAxe.legend()
circleData = []
mass       = 0
frecIter   = 0
def circle(f,n):
    return np.exp(-1j*2*np.pi*f*n*1/fs)
#---------------------------------------
signalAxe  = fig.add_subplot(2,2,2)
signalLn,  = plt.plot([],[],'b-o')
signalAxe.grid(True)
signalAxe.set_xlim(0,N/fs)
signalAxe.set_ylim(-1,1)
signalFrec = 2
signalData=[]
def signal(f,n):
    return np.sin(2*np.pi*f*n*1/fs)
#---------------------------------------
promAxe  = fig.add_subplot(2,2,3)
promRLn,promILn,  = plt.plot([],[],'g-o',[],[],'y-o')
promAxe.grid(True)
promAxe.set_xlim(-fs/2,fs/2)
promAxe.set_ylim(-0.4,0.4)
promData=np.zeros(N,dtype=complex)
#---------------------------------------
tData=np.arange(0,N/fs,1/fs)

def init():
    return circleLn,circleLg,signalLn,massLn,promRLn,promILn
def update(nn):
    global circleData,signalData,promData,frecIter,circleFrec,circleLg
    circleData = []
    signalData = []
    for n in range(N):
        circleData.append(circle(circleFrec[frecIter],n)*signal(signalFrec,n))
        circleData.append(circle(circleFrec[frecIter],n)*signal(signalFrec,n))
        mass=np.average(circleData)
        signalData.append(signal(signalFrec,n))
        promData[frecIter]=mass
    massLn.set_data(np.real(mass),
                    np.imag(mass))
    circleLn.set_data(np.real(circleData),
                      np.imag(circleData))
    signalLn.set_data(tData[:n+1],signalData)
    promRLn.set_data(circleFrec[:frecIter+1],np.real(promData[:frecIter+1]))
    promILn.set_data(circleFrec[:frecIter+1],np.imag(promData[:frecIter+1]))
    circleLn.set_label(circleFrec[frecIter])
    circleLg=circleAxe.legend()

    if frecIter == N-1:
        ani.repeat=False
    else:
        frecIter+=1
    return circleLn,circleLg,signalLn,massLn,promRLn,promILn,

ani=FuncAnimation(fig,update,N,init,interval=100 ,blit=True,repeat=True)
plt.show()
```



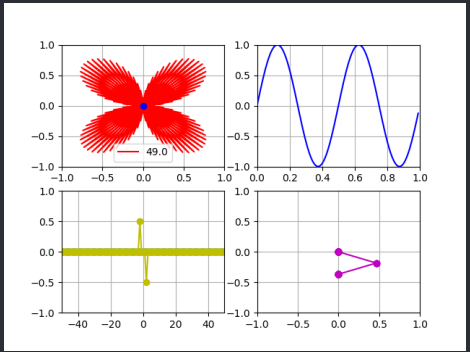
$e^{j2\pi ft}$

Como reconstruyo la señal en el tiempo

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
#--------------------------------------
fig         = plt.figure()
fs          = 100
N           = 100
#--------------------------------------
circleAxe   = fig.add_subplot(2,2,1)
circleLn,massLn,  = plt.plot([],[],'r-',[],[],'bo')
circleAxe.grid(True)
circleAxe.set_xlim(-1,1)
circleAxe.set_ylim(-1,1)
circleFrec = np.arange(-fs/2,fs/2,fs/N)
circleLn.set_label(circleFrec[0])
circleLg=circleAxe.legend()
circleData = []
mass        = 0
frecIter    = 0
def circle(f,n):
    return np.exp(-1j*2*np.pi*f*n*1/fs)
def circleInv(f,n,c):
    return c*np.exp(-1j*2*np.pi*f*n*1/fs)
#--------------------------------------
signalAxe   = fig.add_subplot(2,2,2)
signalLn,   = plt.plot([],[],'b-')
signalAxe.grid(True)
signalAxe.set_xlim(0,N/fs)
signalAxe.set_ylim(-1,1)
signalFrec = 2
signalData=[]
def signal(f,n):
    return np.sin(2*np.pi*f*n*1/fs)
    return np.sin(2*np.pi*f*n*1/fs)
#--------------------------------------
promAxe   = fig.add_subplot(2,2,3)
promRLn,promILn,  = plt.plot([],[],'g-o',[],[],'y-o')
promAxe.grid(True)
promAxe.set_xlim(-fs/2,fs/2)
promAxe.set_ylim(-1,1)
promData=np.zeros(N,dtype=complex)
#--------------------------------------
inversaAxe        = fig.add_subplot(2,2,4)
inversaLn, = plt.plot([],[],'m-o')
inversaAxe.grid(True)
inversaAxe.set_xlim(-1,1)
inversaAxe.set_ylim(-1,1)
inversaData = []
#--------------------------------------
tData=np.arange(0,N/fs,1/fs)
def init():
    return circleLn,circleLg,signalLn,massLn,promRLn,promILn,inversaLn
def updateF(n):
    global promData,fData,inversaData,frecIter
    if aniT.repeat==True:
        return inversaLn,
    inversaData=[0]
    for f in range(N):
        inversaData.append(inversaData[-1]+circleInv(circleFrec[f],frecIter,promData[f]))
    inversaLn.set_data(np.imag(inversaData),np.real(inversaData))
    frecIter+=1
    if frecIter==N:
        frecIter=0
    return inversaLn,
```

Como reconstruyo la señal en el tiempo

```
def updateT(nn):
    global circleData,signalData,promData,frecIter,circleFrec,circleLg

    circleData = []
    signalData = []
    for n in range(N):
        circleData.append(circle(circleFrec[frecIter],n)*signal(signalFrec,n))
        mass=np.average(circleData)
        signalData.append(signal(signalFrec,n))
        promData[frecIter]=mass
    massLn.set_data(np.real(mass),
                    np.imag(mass))
    circleLn.set_data(np.real(circleData),
                      np.imag(circleData))
    signalLn.set_data(tData[:n+1],signalData)
    promRLn.set_data(circleFrec[:frecIter+1],np.real(promData[:frecIter+1]))
    promILn.set_data(circleFrec[:frecIter+1],np.imag(promData[:frecIter+1]))
    circleLn.set_label(circleFrec[frecIter])
    circleLg=circleAxe.legend()

    if frecIter == N-1:
        aniT.repeat=False
    else:
        frecIter+=1
    return circleLn,circleLg,signalLn,massLn,promRLn,promILn,

aniT=FuncAnimation(fig,updateT,N,init,interval=10  ,blit=True,repeat=True)
aniF=FuncAnimation(fig,updateF,N,init,interval=30 ,blit=True,repeat=True)
plt.show()
```



Sintesis, Transformada inversa discreta de Fourier

IDFT

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{\frac{2\pi i}{N} kn} \qquad n = 0, \ldots, N-1$$

$e^{j2\pi ft}$

Señales complejas como entrada?

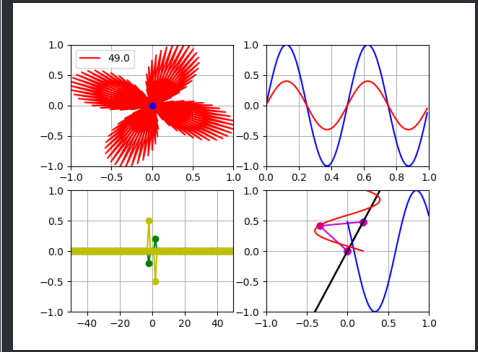
```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
#---------------------------------------
fig         = plt.figure()
fs          = 100
N           = 100
#---------------------------------------
circleAxe  = fig.add_subplot(2,2,1)
circleLn,massLn,  = plt.plot([],[],'r-',[],[],'bo')
circleAxe.grid(True)
circleAxe.set_xlim(-1,1)
circleAxe.set_ylim(-1,1)
circleFrec = np.arange(-fs/2,fs/2,fs/N)
circleLn.set_label(circleFrec[0])
circleLg=circleAxe.legend()
circleData = []
mass       = 0
frecIter   = 0
def circle(f,n):
    return np.exp(-1j*2*np.pi*f*n*1/fs)
def circleInv(f,n,c):
    return c*np.exp(-1j*2*np.pi*f*n*1/fs)
#---------------------------------------
signalAxe  = fig.add_subplot(2,2,2)
signalRLn,signalILn  = plt.plot([],[],'b-',[],[],'r-')
signalAxe.grid(True)
signalAxe.set_xlim(0,N/fs)
signalAxe.set_ylim(-1,1)
signalFrec = 2
signalData=[]
def signal(f,n):
    return np.sin(2*np.pi*f*n*1/fs)+0.4j*np.sin(2*np.pi*f*n*1/fs)
```

```
    return np.sin(2*np.pi*f*n*1/fs)+0.4j*np.sin(2*np.pi*f*n*1/fs)
#---------------------------------------
promAxe  = fig.add_subplot(2,2,3)
promRLn,promILn,  = plt.plot([],[],'g-o',[],[],'y-o')
promAxe.grid(True)
promAxe.set_xlim(-fs/2,fs/2)
promAxe.set_ylim(-1,1)
promData=np.zeros(N,dtype=complex)
#---------------------------------------
inversaAxe        = fig.add_subplot(2,2,4)
inversaLn,penLn,penRLn,penILn = plt.plot([],[],'m-o',[],[],'k-',[],[],'b-',[],[],'r-')
inversaAxe.grid(True)
inversaAxe.set_xlim(-1,1)
inversaAxe.set_ylim(-1,1)
inversaData,penData= [],[]
#---------------------------------------
tData=np.arange(0,N/fs,1/fs)

def init():
    return circleLn,circleLg,signalRLn,signalILn,massLn,promRLn,promILn,inversaLn,penILn,penRLn,

def updateF(n):
    global promData,fData,vectorData,frecIter,penData
    if aniT.repeat==True:
        return inversaLn,
    vectorData=[0]
    for f in range(N):
        vectorData.append(vectorData[-1]+circleInv(circleFrec[f],frecIter,promData[f]))
    inversaLn.set_data(np.imag(vectorData),np.real(vectorData))
    penData.insert(0,vectorData[-1])
    traceData=penData[0:N//2]
    t=np.linspace(0,1,len(traceData))
    penRLn.set_data(t,np.real(traceData))
    penILn.set_data(np.imag(traceData),t)
```

$e^{j2\pi ft}$

Señales complejas como entrada?

```
        penILn.set_data(np.imag(traceData),t)
        penLn.set_data(np.imag(penData),np.real(penData))
        frecIter+=1
        if frecIter==N:
            frecIter=0
        return inversaLn,penLn,penILn,penRLn,

    def updateT(nn):
        global circleData,signalData,promData,frecIter,circleFrec,circleLg

        circleData = []
        signalData = []
        for n in range(N):
            circleData.append(circle(circleFrec[frecIter],n)*signal(signalFrec,n))
            mass=np.average(circleData)
            signalData.append(signal(signalFrec,n))
            promData[frecIter]=mass
        massLn.set_data(np.real(mass),
                        np.imag(mass))
        circleLn.set_data(np.real(circleData),
                          np.imag(circleData))
        signalRLn.set_data(tData[:n+1],np.real(signalData))
        signalILn.set_data(tData[:n+1],np.imag(signalData))
        promRLn.set_data(circleFrec[:frecIter+1],np.real(promData[:frecIter+1]))
        promILn.set_data(circleFrec[:frecIter+1],np.imag(promData[:frecIter+1]))
        circleLn.set_label(circleFrec[frecIter])
        circleLg=circleAxe.legend()

        if frecIter == N-1:
            aniT.repeat=False
        else:
            frecIter+=1
        return circleLn,circleLg,signalRLn,signalILn,massLn,promRLn,promILn,

    aniT=FuncAnimation(fig,updateT,N,init,interval=10  ,blit=True,repeat=True)
    aniF=FuncAnimation(fig,updateF,N,init,interval=30 ,blit=True,repeat=True)
    plt.show()
```



$e^{j2\pi ft}$

Un conejo como entrada?

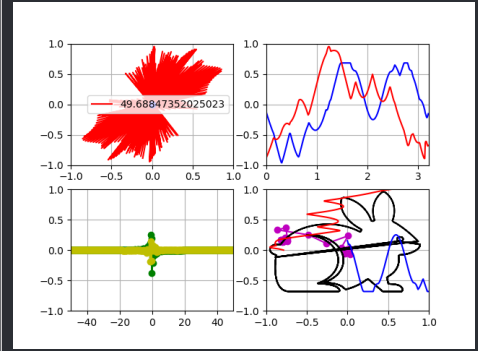
```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
#-------------------------------------
fig         = plt.figure()
fs          = 100
N           = 100
#-------------------------------------
conejo=np.load("conejo.npy")[::1]
N=len(conejo)
def signal(f,n):
    return conejo[n]
#-------------------------------------
circleAxe  = fig.add_subplot(2,2,1)
circleLn,massLn,  = plt.plot([],[],'r-',[],[],'bo')
circleAxe.grid(True)
circleAxe.set_xlim(-1,1)
circleAxe.set_ylim(-1,1)
circleFrec = np.arange(-fs/2,fs/2,fs/N)
circleLn.set_label(circleFrec[0])
circleLg=circleAxe.legend()
circleData = []
mass       = 0
frecIter   = 0
def circle(f,n):
    return np.exp(-1j*2*np.pi*f*n*1/fs)
def circleInv(f,n,c):
    return c*np.exp(-1j*2*np.pi*f*n*1/fs)
#-------------------------------------
signalAxe  = fig.add_subplot(2,2,2)
signalRLn,signalILn  = plt.plot([],[],'b-',[],[],'r-')
signalAxe.grid(True)
signalAxe.set_xlim(0,N/fs)
```

```
signalAxe.set_xlim(0,N/fs)
signalAxe.set_ylim(-1,1)
signalFrec = 2
signalData=[]
#def signal(f,n):
#    return np.sin(2*np.pi*f*n*1/fs)+0.4j*np.sin(2*np.pi*f*n*1/fs)
#-------------------------------------
promAxe  = fig.add_subplot(2,2,3)
promRLn,promILn,  = plt.plot([],[],'g-o',[],[],'y-o')
promAxe.grid(True)
promAxe.set_xlim(-fs/2,fs/2)
promAxe.set_ylim(-1,1)
promData=np.zeros(N,dtype=complex)
#-------------------------------------
inversaAxe          = fig.add_subplot(2,2,4)
inversaLn,penLn,penRLn,penILn = plt.plot([],[],'m-o',[],[],'k-',[],[],'b-',[],[],'r-')
inversaAxe.grid(True)
inversaAxe.set_xlim(-1,1)
inversaAxe.set_ylim(-1,1)
inversaData,penData= [],[]
#-------------------------------------
tData=np.arange(0,N/fs,1/fs)

def init():
    return circleLn,circleLg,signalRLn,signalILn,massLn,promRLn,promILn,inversaLn,penILn,penRLn,

def updateF(n):
    global promData,fData,vectorData,frecIter,penData
    if aniT.repeat==True:
        return inversaLn,
    vectorData=[0]
    for f in range(N):
        vectorData.append(vectorData[-1]+circleInv(circleFrec[f],frecIter,promData[f]))
    inversaLn.set_data(np.imag(vectorData),np.real(vectorData))
```

$e^{j2\pi ft}$

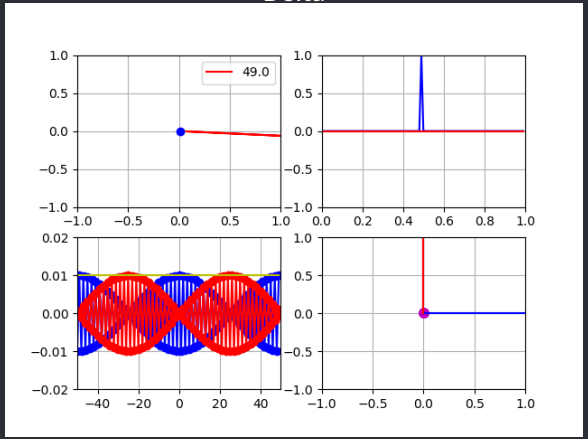
un conejo como entrada?

```
        inversaLn.set_data(np.imag(vectorData),np.real(vectorData))
        penData.insert(0,vectorData[-1])
        traceData=penData[0:N//2]
        t=np.linspace(0,1,len(traceData))
        penRLn.set_data(t,np.real(traceData))
        penILn.set_data(np.imag(traceData),t)
        penLn.set_data(np.imag(penData),np.real(penData))
        frecIter+=1
        if frecIter==N:
            frecIter=0
        return inversaLn,penLn,penILn,penRLn,

    def updateT(nn):
        global circleData,signalData,promData,frecIter,circleFrec,circleLg

        circleData = []
        signalData = []
        for n in range(N):
            circleData.append(circle(circleFrec[frecIter],n)*signal(signalFrec,n))
            mass=np.average(circleData)
            signalData.append(signal(signalFrec,n))
            promData[frecIter]=mass
        massLn.set_data(np.real(mass),
                        np.imag(mass))
        circleLn.set_data(np.real(circleData),
                          np.imag(circleData))
        signalRLn.set_data(tData[:n+1],np.real(signalData))
        signalILn.set_data(tData[:n+1],np.imag(signalData))
        promRLn.set_data(circleFrec[:frecIter+1],np.real(promData[:frecIter+1]))
        promILn.set_data(circleFrec[:frecIter+1],np.imag(promData[:frecIter+1]))
        circleLn.set_label(circleFrec[frecIter])
        circleLg=circleAxe.legend()

        if frecIter == N-1:
            aniT.repeat=False
        else:
            frecIter+=1
        return circleLn,circleLg,signalRLn,signalILn,massLn,promRLn,promILn,

    aniT=FuncAnimation(fig,updateT,N,init,interval=10  ,blit=True,repeat=True)
    aniF=FuncAnimation(fig,updateF,N,init,interval=30 ,blit=True,repeat=True)
    plt.show()
```

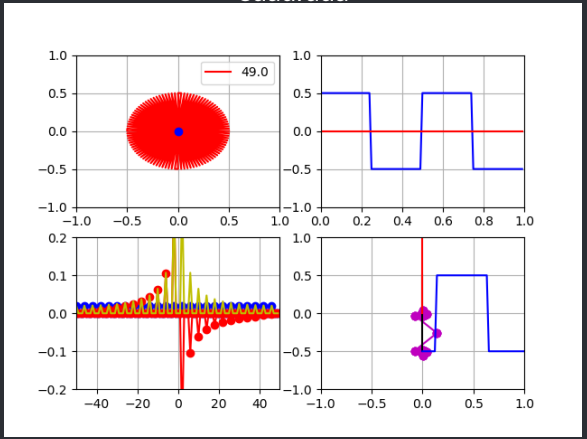


Transformadas relevantes

Delta



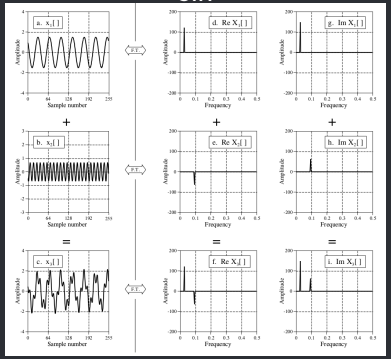
Cuadrada



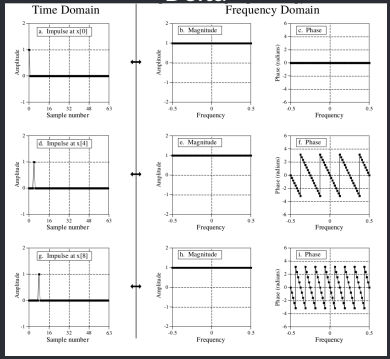
$e^{j2\pi ft}$

Transformadas relevantes

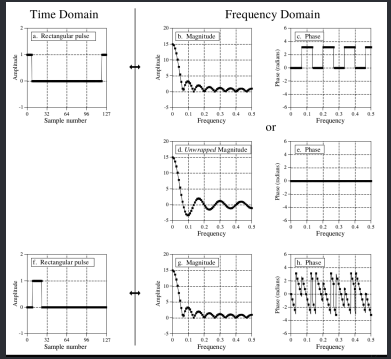
Sin



Delta

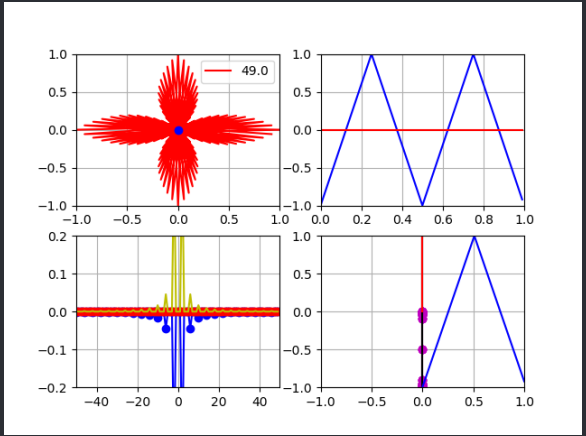


Cuadrada

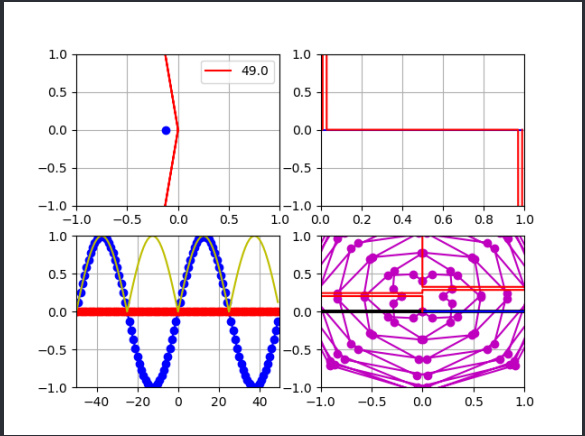


Transformadas relevantes

Triangular

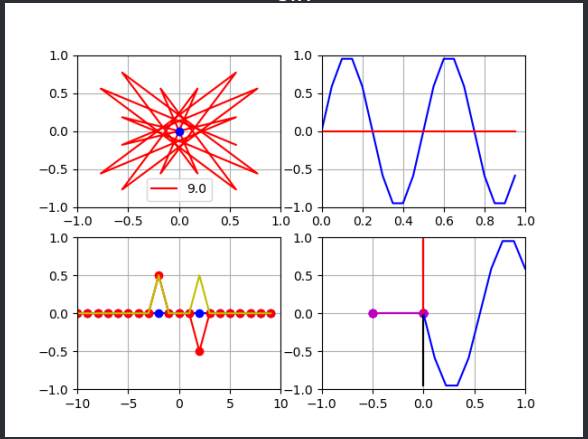


Congujado

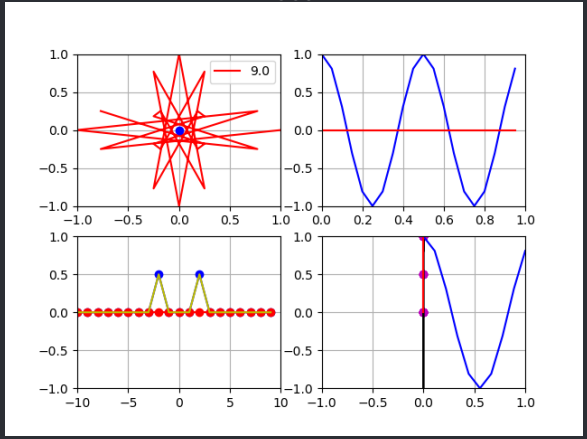


DFT para señales reales RDFT

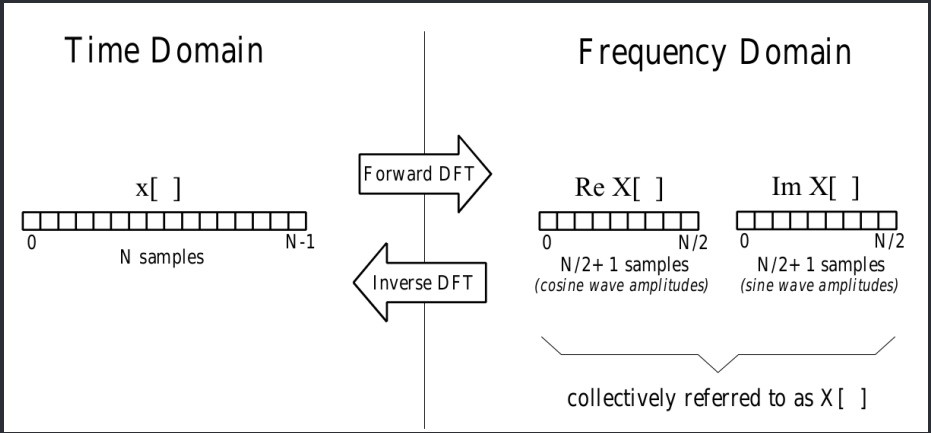
Sin



Cos



DFT para señales reales RDFT



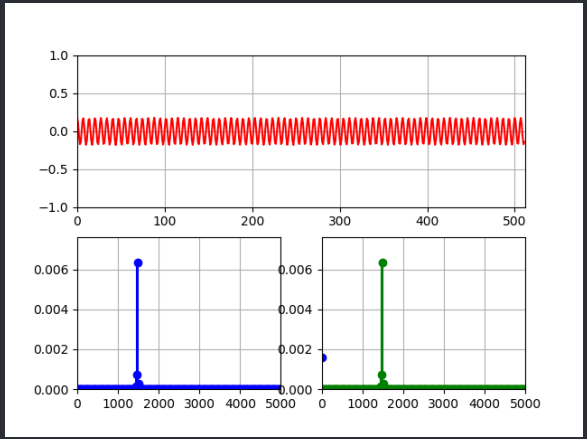
RDFT

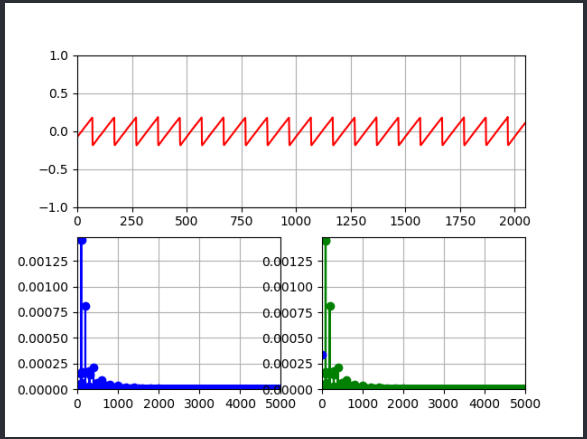
Analisis en la CIAA

```
#include "sapi.h"
#include "arm_math.h"
#include "arm_const_structs.h"
#define MAX_FFT_LENGTH 2048          // maxima longitud para la fft y chunk de samples
#define BITS 10                      // cantidad de bits usado para cuantizar
int16_t fftLength = 32;              // longitud de la fft y samples variable
int16_t adc [ MAX_FFT_LENGTH ];      // guarda los samples
q15_t fftIn [ MAX_FFT_LENGTH ];      // guarda copia de samples en Q15 como in para la fft.La fft corrompe
                                        los datos de la entrada!
q15_t fftOut[ MAX_FFT_LENGTH*2 ];    // salida de la fft
q15_t fftMag[ MAX_FFT_LENGTH/2+1 ]; // magnitud de la FFT
uint32_t maxIndex = 0;               // indexador de maxima energia por cada fft
q15_t maxValue    = 0;               // maximo valor de energia del bin por cada fft
arm_rfft_instance_q15 S;
uint16_t sample   = 0;               // contador para samples
int main ( void ) {
   boardConfig       (                          );
   uartConfig        ( UART_USB, 460800         );
   adcConfig         ( ADC_ENABLE               );
   cyclesCounterInit  ( EDU_CIAA_NXP_CLOCK_SPEED );
   while(1) {
      cyclesCounterReset();                                                    // inicializa el
                conteo de ciclos de reloj
      uartWriteByteArray ( UART_USB ,(uint8_t* )&adc[sample]    ,sizeof(adc[0]) );   // envia el sample
                ANTERIOR
      uartWriteByteArray ( UART_USB ,(uint8_t* )&fftOut[sample] ,sizeof(fftOut[0])); // envia la fft del
                sample ANTERIO
      //TODO hay que mandar fftLength/2 "+1" y solo estoy mandando fftLength/2. revisar
      adc[sample]   =(((int16_t )adcRead(CH1)-512)>>(10-BITS))<<(6+10-BITS);         // PISA el sample que
                se acaba de mandar con una nueva muestra
      fftIn[sample] = adc[sample];                                             // copia del adc
                porque la fft corrompe el arreglo de entrada
      if ( ++sample==fftLength ) {                                             // si es el ultimo
         sample = 0;                                                           // arranca de nuevo
         uartWriteByteArray ( UART_USB ,(uint8_t* )&maxValue ,2);
         uartWriteByteArray ( UART_USB ,(uint8_t* )&maxIndex ,2);
         uartWriteByteArray ( UART_USB ,"header" ,6 );                         // manda el header
                  que casualmente se llama "header" con lo que arranca una nueva trama
         uartWriteByteArray ( UART_USB ,(uint8_t* )&fftLength ,sizeof(fftLength));   // manda el largo de
                  la fft que es variable
         arm_rfft_init_q15 ( &S ,fftLength ,0 ,1 );                            // inicializa una
                  estructira que usa la funcion fft para procesar los datos. Notar el /2 para el largo
         arm_rfft_q15      ( &S ,fftIn     ,fftOut    );                       // por fin.. ejecuta
                  la rfft REAL fft
         arm_cmplx_mag_squared_q15 ( fftOut ,fftMag ,fftLength/2+1 );
         arm_max_q15 ( fftMag ,fftLength/2+1 ,&maxValue ,&maxIndex );
         gpioToggle( LEDR);
         if ( gpioRead(TEC1 )==0) {
            gpioToggle(LEDB);
            if((fftLength<<=1)>MAX_FFT_LENGTH)
               fftLength=32;
            while(gpioRead(TEC1)==0)
               ;
         }
      }
      while(cyclesCounterRead()< 20400) //clk de 204000000 => 10k samples x seg.
         ;
   }
}
```

RDFT

Analisis en la CIAA





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[5] *Interactive Mathematics Site Info*.
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