

Procesamiento de señales, fundamentos

Maestría en sistemas embebidos
Universidad de Buenos Aires
MSE 5Co2020

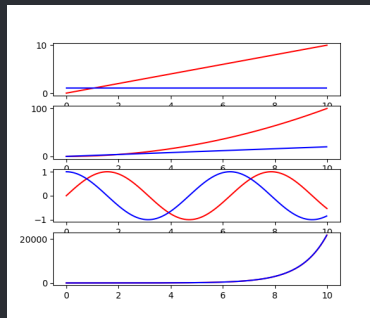
Clase 3 - Euler | Fourier - DFT

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2.7182818284590450907955982984276488423347473144

- $f(t) = t$
- $f(t) = t^2$
- $f(t) = \sin(t)$
- $f'(t) = 1$
- $f'(t) = 2 * t$
- $f'(t) = \cos(t)$



La derivada es igual a la funcion

$$f(t) = e^t \implies f'(t) = e^t$$
$$f(t) = e^{kt} \implies f'(t) = ke^{kt}$$

$$e^{j2\pi ft}$$

Euler

Pero que pasa con e^{jt} ?

La derivada es igual a la funcion

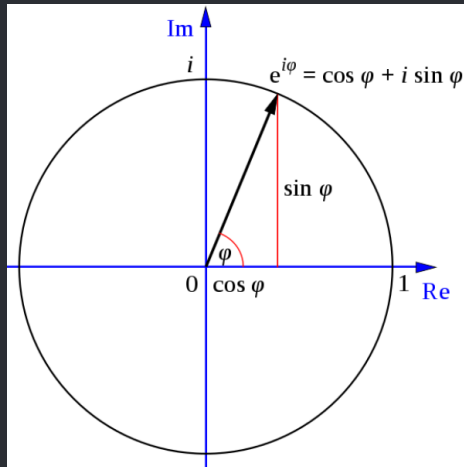
$$f(t) = e^{jt} \implies f'(t) = je^{jt}$$

$$e^{jt} = \cos(t) + j \sin(t)$$

$$e^{j\pi} = -1$$

$$e^{j\frac{\pi}{2}} = j$$

$$e^{j\frac{3\pi}{2}} = -j$$

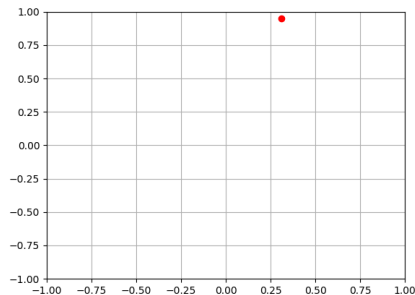


$e^{j2\pi ft}$ $e^{j2\pi ft}$ animado

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation

fig = plt.figure()
fs = 20
N = 20
circleAxe = fig.add_subplot(1,1,1)
circleLn, = plt.plot([],[],'ro')
circleAxe.grid(True)
circleAxe.set_xlim(-1,1)
circleAxe.set_ylim(-1,1)
circleFrec = 1
def circle(c,f,n):
    return c*np.exp(-1j*2*np.pi*f*n*1/fs)
def init():
    return circleLn,
def update(n):
    circleLn.set_data(np.real(circle(1,circleFrec,n)),
                      np.imag(circle(1,circleFrec,n)))

    return circleLn,
ani=FuncAnimation(fig,update,N,init,interval=100 ,blit=False,repeat=True
)
plt.show()
```

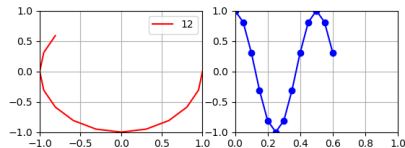


$e^{j2\pi ft}$ $e^{j2\pi ft}$ y $\sin(t)$ animados independientemente

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
from buttons import buttonOnFigure

#-----
fig = plt.figure()
fs = 20
N = 20
#-----
circleAxe = fig.add_subplot(2,2,1)
circleLn = plt.plot([],[],'r-')
circleAxe.grid(True)
circleAxe.set_xlim(-1,1)
circleAxe.set_ylim(-1,1)
circleLn.set_label(0)
circleLg=circleAxe.legend()
circleFrec = 1
circleData = []
def circle(f,n):
    return np.exp(-1j*2*np.pi*f*n*1/fs)
#-----
signalAxe = fig.add_subplot(2,2,2)
signalLn = plt.plot([],[],'b-o')
signalAxe.grid(True)
signalAxe.set_xlim(0,N/fs)
signalAxe.set_ylim(-1,1)
signalFrec = 2
```

```
signalFrec = 2
signalData=[]
def signal(f,n):
    return np.cos(2*np.pi*f*n*1/fs)
#-----
tData=np.arange(0,N/fs,1/fs)
def init():
    return circleLn,circleLg,signalLn,
def update(n):
    global circleData,signalData
    circleData.append(circle(circleFrec,n))
    circleLn.set_data(np.real(circleData),
                      np.imag(circleData))
    signalData.append(signal(signalFrec,n))
    signalLn.set_data(tData[n+1],signalData)
    if n==N-1:
        circleData=[]
        signalData=[]
        circleLn.set_label(n)
        circleLg=circleAxe.legend()
        return circleLn,circleLg,signalLn,
ani=FuncAnimation(fig,update,N,init,interval=500 ,
                  blit=True,repeat=True)
plt.get_current_fig_manager().window.showMaximized()
b=buttonOnFigure(fig,ani)
plt.show()
```



$e^{j2\pi ft}$ $e^{j2\pi ft}$ modulado por $\sin(t)$ y centro de masas

```

import numpy as np
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
from buttons import buttonOnFigure

#-----
fig      = plt.figure()
fs       = 20
N        = 20
#-----
circleAxe = fig.add_subplot(2,2,1)
circleLn, = plt.plot([],[],'r-')
circleAxe.grid(True)
circleAxe.set_xlim(-1,1)
circleAxe.set_ylim(-1,1)
circleLn.set_label(0)
circleLg=circleAxe.legend()
circleFreq = 1
circleData = []
def circle(f,n):
    return np.exp(-1j*2*np.pi*f*n*1/fs)
#-----
signalAxe = fig.add_subplot(2,2,2)
signalLn, = plt.plot([],[],'b-o')
signalAxe.grid(True)
signalAxe.set_xlim(0,N/fs)
signalAxe.set_ylim(-1,1)
signalFreq = 2
signalData=[]
def signal(f,n):

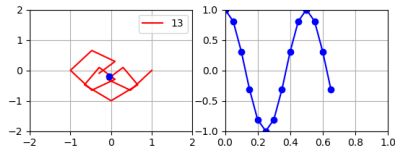
```

```

signalData=[]
def signal(f,n):
    return np.cos(2*np.pi*f*n*1/fs)
#-----
tData=np.arange(0,N/fs,1/fs)

def init():
    return circleLn,circleLg,signalLn,massLn
def update(n):
    global circleData,signalData
    circleData.append(circle(circleFreq,n)*signal(
        signalFreq,n))
    mass=np.average(circleData)
    massLn.set_data(np.real(mass),
                    np.imag(mass))
    circleLn.set_data(np.real(circleData),
                     np.imag(circleData))
    signalData.append(signal(signalFreq,n))
    signalLn.set_data(tData[:n+1],signalData)
    if n==N-1:
        circleData = []
        signalData = []
        circleLn.set_label(n)
        circleLg=circleAxe.legend()
        return circleLn,circleLg,signalLn,massLn
ani=FuncAnimation(fig,update,N,init,interval=10 ,
                  blit=True,repeat=True)
plt.get_current_fig_manager().window.showMaximized()
()
b=buttonOnFigure(fig,ani)
plt.show()

```





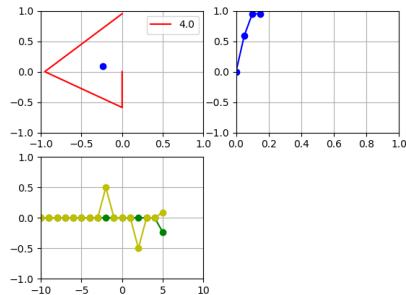
```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
from buttons import buttonOnFigure

#-----
fig = plt.figure()
fs = 50
N = 50
#-----CONJUGADO-----
#conjugado=np.zeros(100,dtype=complex)
#conjugado+=0.2
#w=len(conjugado)
#conjugado[5]=0.5*N
#conjugado[100-6]=0.5*N
#def signal(f,n):
#    return conjugado[n]
#-----
circleAxe = fig.add_subplot(2,2,1)
circleLn, massLn = plt.plot([],[], 'r-', [1,1], 'bo')
circleAxe.grid(True)
circleAxe.set_xlim(-1,1)
circleAxe.set_ylim(-1,1)
circleFrec = np.arange(-fs/2,fs/2,fs/N)
circleLn.set_label(circleFrec[0])
circleLg = circleAxe.legend()
circleData = []
mass = 0
frecIter = 0
def circle(f,n):
    return np.exp(-1j*2*np.pi*f*n*1/fs)
#-----
signalAxe = fig.add_subplot(2,2,2)
signalLn, = plt.plot([],[], 'b-o')
signalAxe.grid(True)
signalAxe.set_xlim(0,N/fs)
signalAxe.set_ylim(-1,1)
signalFrec = 2
signalData=[]
def signal(f,n):
    return np.cos(2*np.pi*f*n*1/fs)
#-----
promAxe = fig.add_subplot(2,2,3)
promRLn, promILn, promMagLn, promPhaseLn = plt.plot([],[], 'b-o', [1,1], 'r-o', [1,1], 'k-o', [1,1], 'y-')
promAxe.grid(True)
```

```
promAxe.set_xlim(-fs/2,fs/2)
promAxe.set_ylim(-1,1)
promData=np.zeros(N,dtype=complex)
#-----
tData=np.arange(0,N/fs,1/fs)

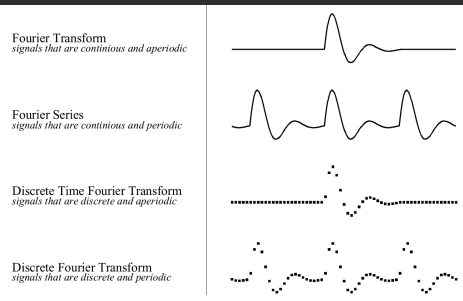
def init():
    return circleLn, circleLg, signalLn, massLn, promRLn, promILn
def update(n):
    global circleData, signalData, promData, frecIter, circleFrec, circleLg
    circleData.append(circle(circleFrec[frecIter],n)*signal(signalFrec,n))
    mass=np.average(circleData)
    massLn.set_data(np.real(mass),
                    np.imag(mass))
    circleLn.set_data(np.real(circleData),
                     np.imag(circleData))
    signalData.append(signal(signalFrec,n))
    signalLn.set_data(tData[n+1], signalData)
    promData[frecIter]=mass
    promRLn.set_data(circleFrec[frecIter+1], np.real(promData[frecIter+1]))
    promILn.set_data(circleFrec[frecIter+1], np.imag(promData[frecIter+1]))
    # promMagLn.set_data(circleFrec[frecIter+1], np.abs(promData[frecIter+1])**2)
    # promPhaseLn.set_data(circleFrec[frecIter+1], np.angle(promData[frecIter+1])/np.
    # pi)
    circleLn.set_label(circleFrec[frecIter])
    circleLg=circleAxe.legend()
    if n==N-1:
        circleData = []
        signalData = []
        if frecIter == N-1:
            ani.repeat=False
        else:
            frecIter+=1
    return circleLn, circleLg, signalLn, massLn, promRLn, promILn, promMagLn, promPhaseLn,

ani=FuncAnimation(fig,update,N,init,interval=10 ,blit=True,repeat=True)
plt.get_current_fig_manager().window.showMaximized()
b=buttonOnFigure(fig,ani)
plt.show()
```



Transformada de Fourier

Diferentes tipos segun la señal



Time Duration		
Finite	Infinite	
Discrete FT (DFT) $X(k) = \sum_{n=0}^{N-1} x(n)e^{-j\omega_k n}$ $k = 0, 1, \dots, N-1$	Discrete Time FT (DTFT) $X(\omega) = \sum_{n=-\infty}^{+\infty} x(n)e^{-j\omega n}$ $\omega \in [-\pi, +\pi)$	discr. time n
Fourier Series (FS) $X(k) = \frac{1}{P} \int_0^P x(t)e^{-j\omega_k t} dt$ $k = -\infty, \dots, +\infty$ discrete freq. k	Fourier Transform (FT) $X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t} dt$ $\omega \in (-\infty, +\infty)$ continuous freq. ω	cont. time t

Analisi, Transformada discreta de Fourier

DFT

$$X_k = \sum_{n=0}^{N-1} x_n e^{-\frac{2\pi i}{N} kn} \quad k = 0, \dots, N - 1$$

DFT

Densidad de potencia espectral



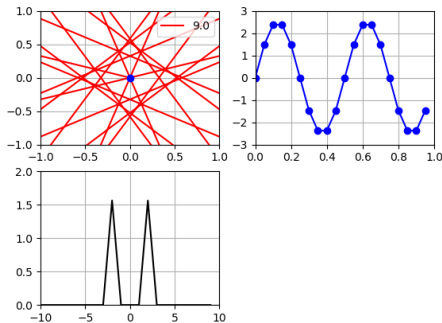
$$P_{sin} = \frac{A^2}{2}$$

$$P_{sin} = \frac{2,5^2}{2}$$

$$P_{sin} = 3,125W$$

$$P = 1,56 + 1,56$$

$$P = 3,125W$$

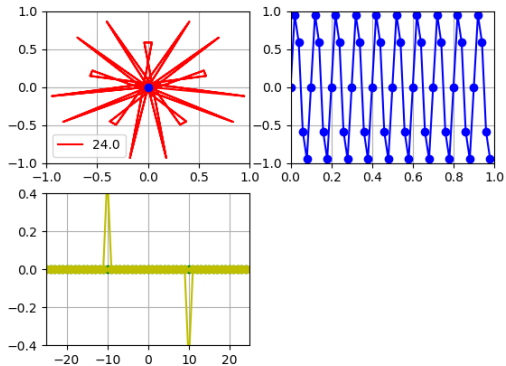


DFT

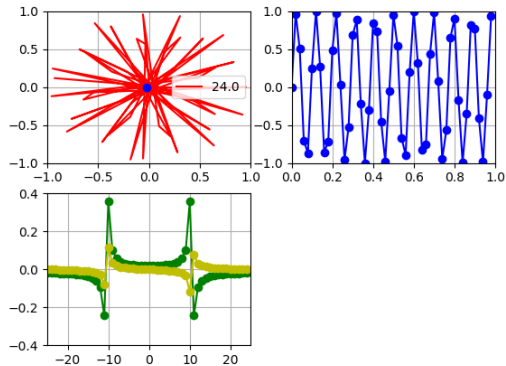
Fuga espectral (Spectral leakage)



10Hz



10.4hz

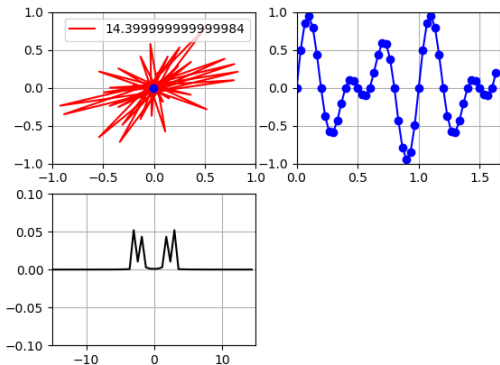


DFT

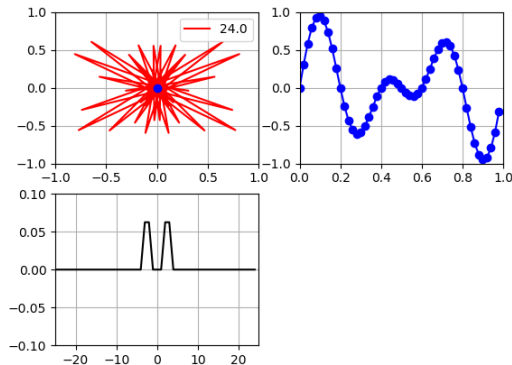
Resolucion espectral



$f_1=2$ $f_2=3$ $f_s=30$ $N=50$



$f_1=2$ $f_2=3$ $f_s=50$ $N=50$



DFT

DFT Zero padding



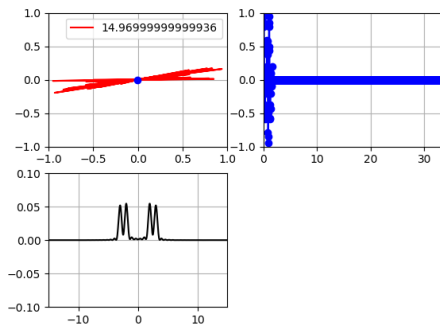
$$f1 = 2$$

$$f2 = 3$$

$$f_s = 10$$

$$N = 50$$

$$Z = 950$$



DFT

Acelerada



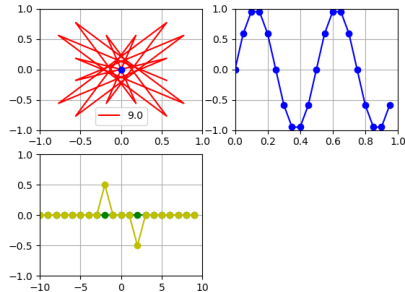
```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
from buttons import buttonOnFigure

#-----
fig = plt.figure()
fs = 20
N = 20
#-----
circleAxe = fig.add_subplot(2,1)
circleLn, massLn = plt.plot([],[], 'r-', [1,1], ['bo'])
circleAxe.grid(True)
circleAxe.set_xlim(-1,1)
circleAxe.set_ylim(-1,1)
circleFreq = np.arange(-fs/2, fs/2, fs/N)
circleLn.set_label(circleFreq[0])
circleLg = circleAxe.legend()
circleData = []
mass = 0
freqIter = 0
def circle(f,n):
    return np.exp(-1j*2*np.pi*f*n*1/fs)
#-----
signalAxe = fig.add_subplot(2,2)
signalLn = plt.plot([],[], 'b-o')
signalAxe.grid(True)
signalAxe.set_xlim(0, N/fs)
signalAxe.set_ylim(-1,1)
signalFreq = 2
signalData = []
def signal(f,n):
    return np.sin(2*np.pi*f*n*1/fs)
# if n<50:
#     return 0.5*np.sin(2*np.pi*f*n*1/fs)+0.5*np.sin(2*np.pi*f*1.5*n*1/fs)
# return 0
#-----
promAxe = fig.add_subplot(2,3)
promRLn, promLn, promMagLn, promPhaseLn = plt.plot([],[], 'b-o', [1,1], 'r-o', [1,1], 'k-', [1,1], 'y-')
promAxe.grid(True)
promAxe.set_xlim(-fs/2, fs/2)
promAxe.set_ylim(-1,1)
promData=np.zeros(N, dtype=complex)
#-----
tData=np.arange(0, N/fs, 1/fs)

def init():
    return circleLn, circleLg, signalLn, massLn, promRLn, promLn
def update(nn):
    global circleData, signalData, promData, freqIter, circleFreq, circleLg
```

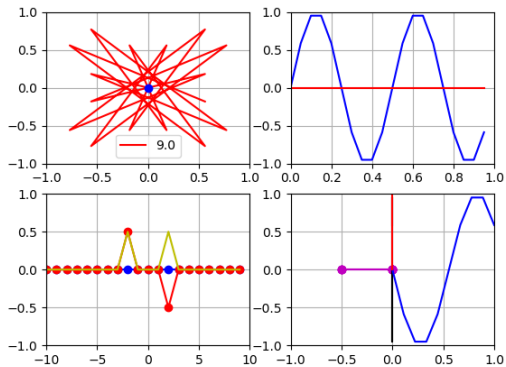
```
global circleData, signalData, promData, freqIter, circleFreq, circleLg
circleData = []
signalData = []
for n in range(N):
    circleData.append(circle(circleFreq[freqIter], n)*signal(signalFreq, n))
    mass=np.average(circleData)
    signalData.append(signal(signalFreq, n))
    promData[freqIter]=mass
    massLn.set_data(np.real(mass),
                    np.imag(mass))
    circleLn.set_data(np.real(circleData),
                     np.imag(circleData))
    signalLn.set_data(tData[:n+1], signalData)
    # promRLn.set_data(circleFreq[:freqIter+1], np.real(promData[:freqIter+1]))
    # promILn.set_data(circleFreq[:freqIter+1], np.imag(promData[:freqIter+1]))
    promMagLn.set_data(circleFreq[:freqIter+1], np.abs(promData[:freqIter+1])**2)
    # promPhaseLn.set_data(circleFreq[:freqIter+1], np.angle(promData[:freqIter+1])/np.
    pi)
    circleLn.set_label(circleFreq[freqIter])
    circleLg=circleAxe.legend()
    if freqIter == N-1:
        ani.repeat=False
    else:
        freqIter+=1
    return circleLn, circleLg, signalLn, massLn, promRLn, promILn, promMagLn, promPhaseLn,

ani=FuncAnimation(fig, update, N, init, interval=100 , blit=True, repeat=True)
plt.get_current_fig_manager().window.showMaximized()
b=buttonOnFigure(fig, ani)
plt.show()
```

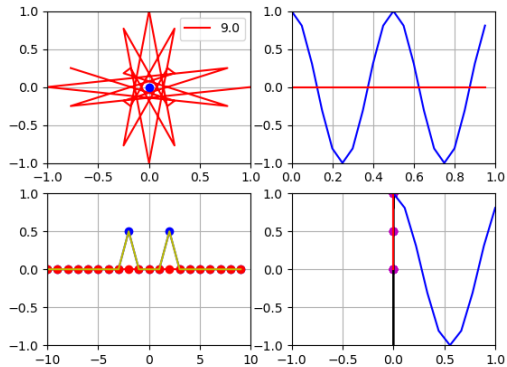


DFT para señales reales RDFT

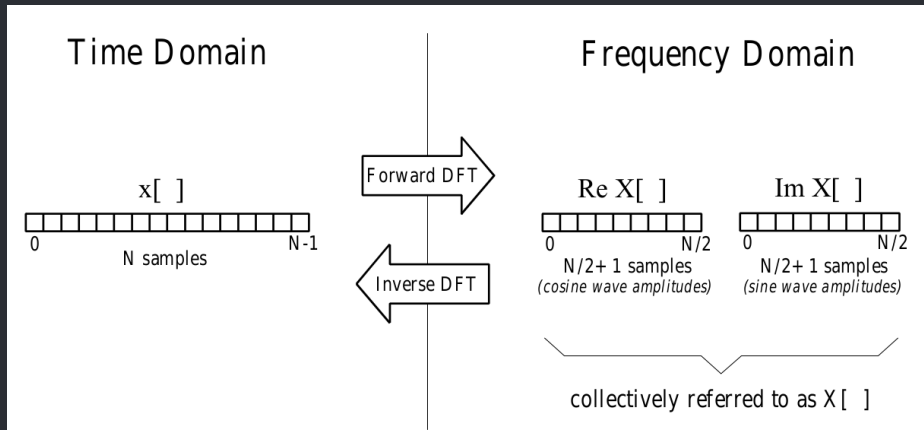
Sin



Cos



DFT para señales reales RDFT





```
#include "sapi.h"
#include "arm_math.h"
#include "arm_const_structs.h"

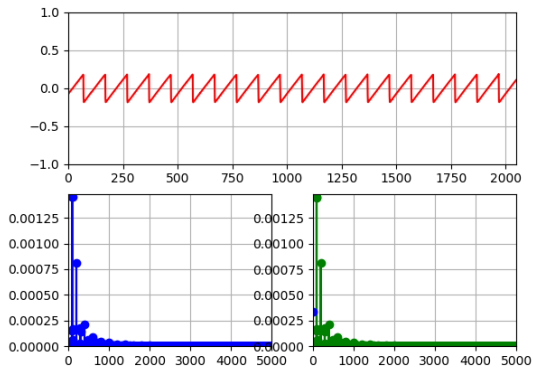
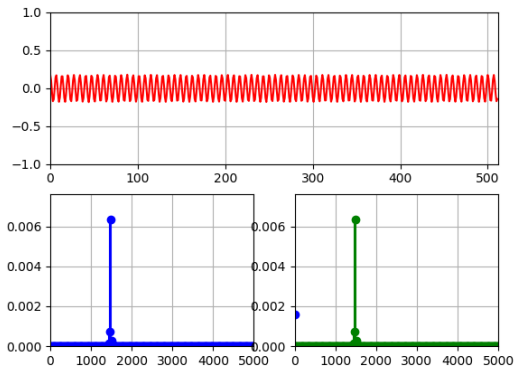
#define MAX_FFT_LENGTH 2048
#define BITS 10
int16_t fftLength = 32; // maxima longitud para la fft y chunk de samples
int16_t adc [ MAX_FFT_LENGTH ]; // cantidad de bits usado para cuantizar
q15_t fftIn [ MAX_FFT_LENGTH ]; // longitud de la fft y samples variable
// guarda los samples
// guarda copia de samples en Q15 como in para la fft. La fft corrompe los datos de la entrada!
q15_t fftOut[ MAX_FFT_LENGTH*2 ]; // salida de la fft
q15_t fftMag[ MAX_FFT_LENGTH/2+1 ]; // magnitud de la FFT
uint32_t maxIndex = 0; // indexador de maxima energia por cada fft
q15_t maxValue = 0; // maximo valor de energia del bin por cada fft
arm_rfft_instance_q15 S;
uint16_t sample = 0; // contador para samples

int main ( void ) {
    boardConfig ( );
    uartConfig ( UART_USB, 460800 );
    adcConfig ( ADC_ENABLE );
    cyclesCounterInit ( EDU_CIAA_NXP_CLOCK_SPEED );
    while(1) {
        cyclesCounterReset(); // inicializa el conteo de ciclos de reloj
        uartWriteByteArray ( UART_USB ,(uint8_t* )&adc[sample] ,sizeof(adc[0]) ); // envia el sample
        ANTERIOR
        uartWriteByteArray ( UART_USB ,(uint8_t* )&fftOut[sample] ,sizeof(fftOut[0])); // envia la fft del sample ANTERIOR
        //TODO hay que mandar fftLength/2 *+1 y solo estoy mandando fftLength/2. revisar
        adc[sample] =(((int16_t )adcRead(CH1)-512)>>(10-BITS))<<(6+10-BITS); // PISA el sample que se acaba de mandar con una nueva muestra
        fftIn[sample] = adc[sample]; // copia del adc
        porque la fft corrompe el arreglo de entrada
        if ( ++sample==fftLength ) { // si es el ultimo
```

```
sample = 0; // arranca de nuevo
uartWriteByteArray ( UART_USB ,(uint8_t* )&maxValue ,2);
uartWriteByteArray ( UART_USB ,(uint8_t* )&maxIndex ,2);
uartWriteByteArray ( UART_USB ,"header" ,6 ); // manda el header
// que casualmente se llama "header" con lo que arranca una nueva trama
uartWriteByteArray ( UART_USB ,(uint8_t* )&fftLength ,sizeof(fftLength)); // manda el largo de la fft que es variable
arm_rfft_init_q15 ( &S ,fftLength ,0 ,1 ); // inicializa una estructura que usa la funcion fft para procesar los datos. Notar el /2 para el largo
arm_rfft_q15 ( &S ,fftIn ,fftOut ); // por fin.. ejecuta la rfft REAL fft
arm_cmplx_mag_squared_q15 ( fftOut ,fftMag ,fftLength/2+1 );
arm_max_q15 ( fftMag ,fftLength/2+1 ,&maxValue ,&maxIndex );
gpioToggle( LEDR);
if ( gpioRead(TEC1 )==0) {
    gpioToggle(LED8);
    if(((fftLength<=1)>MAX_FFT_LENGTH)
        fftLength=32;
        while(gpioRead(TEC1)==0)
        ;
    }
}
while(cyclesCounterRead()< 20400) //clk de 204000000 => 10k samples x seg.
;
}
```

RDFT

Análisis en la CIAA



Bibliografía

Libros, links y otro material

- [1] ARM CMSIS DSP.
https://arm-software.github.io/CMSIS_5/DSP/html/index.html
- [2] Steven W. Smith. *The Scientist and Engineer's Guide to Digital Signal Processing*. Second Edition, 1999.
- [3] Grant Sanderson
<https://youtu.be/spUNpyF58BY>
- [4] *Interactive Mathematics Site Info*.
<https://www.intmath.com/fourier-series/fourier-intro.php>