FACULTAD
DE INGENIERIA
Universidad de Buenos Aires

# Procesamiento de señales, fundamentos

Maestría en sistemas embebidos
Universidad de Buenos Aires
MSE 5Co2020

## Clase 4 - Euler | Fourier - IDFT

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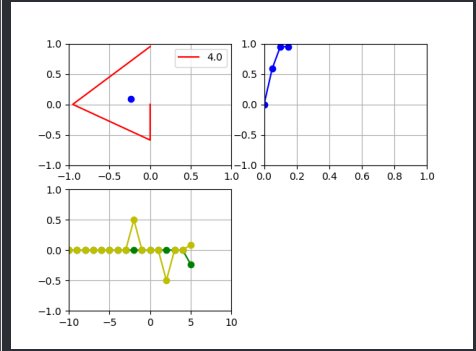
# repaso DFT

## $e^{j2\pi ft}$ *modulado por* $\sin(t)$ *y centro de masas en f, DFT?*

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
from buttons import buttonOnFigure
#--------------------------------------
fig         = plt.figure()
fs          = 50
N           = 50
#----------CONJUGADO---------------------
#conjugado=np.zeros(100,dtype=complex)
#conjugado+=0.2
#N=len(conjugado)
#conjugado[6]=0.5*N
#conjugado[100-6]=0.5*N
#def signal(f,n):
#    return conjugado[n]
#--------------------------------------
circleAxe  = fig.add_subplot(2,2,1)
circleLn,massLn  = plt.plot([],[],'r-',[],[],'bo')
circleAxe.grid(True)
circleAxe.set_xlim(-1,1)
circleAxe.set_ylim(-1,1)
circleFrec = np.arange(-fs/2,fs/2,fs/N)
circleLn.set_label(circleFrec[0])
circleLg   = circleAxe.legend()
circleData = []
mass       = 0
frecIter   = 0
def circle(f,n):
    return np.exp(-1j*2*np.pi*f*n*1/fs)
#--------------------------------------
signalAxe  = fig.add_subplot(2,2,2)
signalLn,  = plt.plot([],[],'b-o')
signalAxe.grid(True)
signalAxe.set_xlim(0,N/fs)
signalAxe.set_ylim(-1,1)
signalFrec=[]
signalData=[]
def signal(f,n):
    return np.cos(2*np.pi*f*n*1/fs)
#--------------------------------------
promAxe  = fig.add_subplot(2,2,3)
promRLn,promILn,promMagLn,promPhaseLn  = plt.plot([],[],'b-o'
        ,[],[],'r-o',[],[],'k-o',[],[],'y-')
promAxe.grid(True)
promAxe.set_xlim(-fs/2,fs/2)
promAxe.set_ylim(-1,1)
promData=np.zeros(N,dtype=complex)
#--------------------------------------
tData=np.arange(0,N/fs,1/fs)

def init():
    return circleLn,circleLg,signalLn,massLn,promRLn,promILn
def update(n):
    global circleData,signalData,promData,frecIter,circleFrec,
            circleLg
    circleData.append(circle(circleFrec[frecIter],n)*signal(
            signalFrec,n))
    mass=np.average(circleData)
    massLn.set_data(np.real(mass),
                    np.imag(mass))
    circleLn.set_data(np.real(circleData),
                      np.imag(circleData))
    signalData.append(signal(signalFrec,n))
    signalLn.set_data(tData[:n+1],signalData)
    promData[frecIter]=mass
    promRLn.set_data(circleFrec[:frecIter+1],np.real(promData[:
            frecIter+1]))
    promILn.set_data(circleFrec[:frecIter+1],np.imag(promData[:
            frecIter+1]))
#     promMagLn.set_data(circleFrec[:frecIter+1],np.abs(promData
#         [:frecIter+1])**2)
#     promPhaseLn.set_data(circleFrec[:frecIter+1],np.angle(
#         promData[:frecIter+1])/np.pi)
    circleLn.set_label(circleFrec[frecIter])
    circleLg=circleAxe.legend()
    if n==N-1:
        circleData = []
        signalData = []
        if frecIter == N-1:
            ani.repeat=False
        else:
            frecIter+=1
    return circleLn,circleLg,signalLn,massLn,promRLn,promILn,
            promMagLn,promPhaseLn,
```



# Repaso DFT

*Analisis, Transformada discreta de Fourier*

$$X_k = \sum_{n=0}^{N-1} x_n e^{-\frac{2\pi i}{N} kn} \qquad k = 0, \ldots, N-1$$

# Síntesis

*Como reconstruyo la señal en el tiempo*

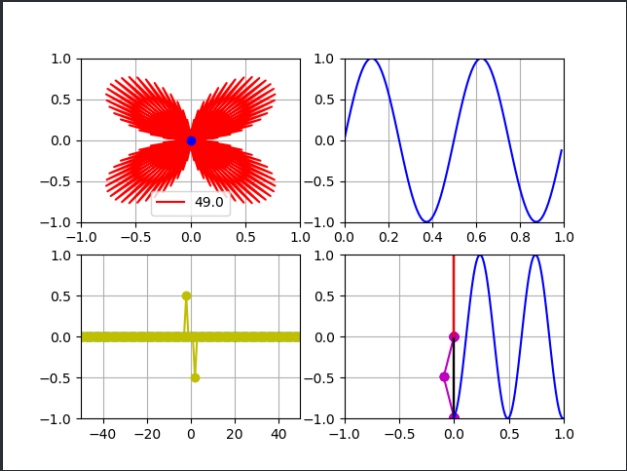
```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
from buttons import buttonOnFigure
#---------------------------------------
fig         = plt.figure()
fs          = 100
N           = 100
#---------------------------------------
circleAxe  = fig.add_subplot(2,2,1)
circleLn,massLn,  = plt.plot([],[],'r-',[],[],'bo')
circleAxe.grid(True)
circleAxe.set_xlim(-1,1)
circleAxe.set_ylim(-1,1)
circleFrec = np.arange(-fs/2,fs/2,fs/N)
circleLn.set_label(circleFrec[0])
circleLg=circleAxe.legend()
circleData = []
mass       = 0
frecIter   = 0
def circle(f,n):
    return np.exp(-1j*2*np.pi*f*n*1/fs)
def circleInv(f,n,c):
    return c*np.exp(1j*2*np.pi*f*n*1/fs)
#---------------------------------------
signalAxe  = fig.add_subplot(2,2,2)
signalLn,  = plt.plot([],[],'b-')
signalAxe.grid(True)
signalAxe.set_xlim(0,N/fs)
signalAxe.set_ylim(-1,1)
signalFrec = 2
signalData=[]
def signal(f,n):
```

```
def signal(f,n):
    return np.sin(2*np.pi*f*n*1/fs)
#---------------------------------------
promAxe  = fig.add_subplot(2,2,3)
promRLn,promILn,  = plt.plot([],[],'g-o',[],[],'y-o')
promAxe.grid(True)
promAxe.set_xlim(-fs/2,fs/2)
promAxe.set_ylim(-1,1)
promData=np.zeros(N,dtype=complex)
#---------------------------------------
inversaAxe         = fig.add_subplot(2,2,4)
inversaLn,penLn,penRLn,penILn = plt.plot([],[],'m-o',[],[],'k-',[],[],'b-',[],[],'r-')
inversaAxe.grid(True)
inversaAxe.set_xlim(-1,1)
inversaAxe.set_ylim(-1,1)
penData= []
#---------------------------------------
tData=np.arange(0,N/fs,1/fs)

def init():
    return circleLn,circleLg,signalLn,massLn,promRLn,promILn,inversaLn

def updateF(n):
    global promData,fData,frecIter,penData
    if anif.repeat==True:
        return inversaLn,
    inversaData=[0]
    for f in range(N):
        inversaData.append(inversaData[-1]+circleInv(circleFrec[f],frecIter,promData[f]))
    inversaLn.set_data(np.imag(inversaData),np.real(inversaData))
    penData.insert(0,inversaData[-1])
    penData=penData[0:N]
    t=np.linspace(0,1,len(penData))
    penRLn.set_data(t,np.real(penData))
```

# Síntesis

*Como reconstruyo la señal en el tiempo*

```
    penRLn.set_data(t,np.real(penData))
    penILn.set_data(np.imag(penData),t)
    penLn.set_data(np.imag(penData),np.real(penData))
    frecIter+=1
    if frecIter==N:
        frecIter=0
    return inversaLn,penLn,penILn,penRLn,circleLn,signalLn,promRLn,promILn,

def updateT(nn):
    global circleData,signalData,promData,frecIter,circleFrec,circleLg

    circleData = []
    signalData = []
    for n in range(N):
        circleData.append(circle(circleFrec[frecIter],n)*signal(signalFrec,n))
        mass=np.average(circleData)
        signalData.append(signal(signalFrec,n))
        promData[frecIter]=mass
    massLn.set_data(np.real(mass),
                    np.imag(mass))
    circleLn.set_data(np.real(circleData),
                      np.imag(circleData))
    signalLn.set_data(tData[:n+1],signalData)
    promRLn.set_data(circleFrec[:frecIter+1],np.real(promData[:frecIter+1]))
    promILn.set_data(circleFrec[:frecIter+1],np.imag(promData[:frecIter+1]))
    circleLn.set_label(circleFrec[frecIter])
    circleLg=circleAxe.legend()

    if frecIter == N-1:
        frecIter=0
        aniT.repeat=False
    else:
        frecIter+=1
    return circleLn,circleLg,signalLn,massLn,promRLn,promILn,

aniT=FuncAnimation(fig,updateT,N,init,interval=10  ,blit=True,repeat=True)
aniF=FuncAnimation(fig,updateF,N,init,interval=20  ,blit=True,repeat=True)
plt.get_current_fig_manager().window.showMaximized()
b=buttonOnFigure(fig,aniT,aniF)
plt.show()
```



# Síntesis, Transformada inversa discreta de Fourier

*IDFT*

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{\frac{2\pi i}{N} kn} \qquad n = 0, \ldots, N-1$$

# IDFT

*Señales complejas como entrada?*

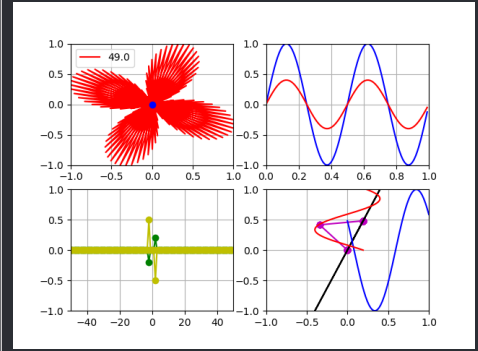
```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
from buttons import buttonOnFigure
#---------------------------------------
fig         = plt.figure()
fs          = 100
N           = 100
#---------------------------------------
circleAxe  = fig.add_subplot(2,2,1)
circleLn,massLn,  = plt.plot([],[],'r-',[],[],'bo')
circleAxe.grid(True)
circleAxe.set_xlim(-1,1)
circleAxe.set_ylim(-1,1)
circleFrec = np.arange(-fs/2,fs/2,fs/N)
circleLn.set_label(circleFrec[0])
circleLg=circleAxe.legend()
circleData = []
mass       = 0
frecIter   = 0
def circle(f,n):
    return np.exp(-1j*2*np.pi*f*n*1/fs)
def circleInv(f,n,c):
    return c*np.exp(1j*2*np.pi*f*n*1/fs)
#---------------------------------------
signalAxe  = fig.add_subplot(2,2,2)
signalRLn,signalILn  = plt.plot([],[],'b-',[],[],'r-')
signalAxe.grid(True)
signalAxe.set_xlim(0,N/fs)
signalAxe.set_ylim(-1,1)
signalFrec = 2
signalData=[]
def signal(f,n):
```

```
def signal(f,n):
    return np.sin(2*np.pi*f*n*1/fs)+0.4j*np.sin(2*np.pi*f*1.5*n*1/fs)
#---------------------------------------
promAxe  = fig.add_subplot(2,2,3)
promRLn,promILn,  = plt.plot([],[],'g-o',[],[],'y-o')
promAxe.grid(True)
promAxe.set_xlim(-fs/2,fs/2)
promAxe.set_ylim(-1,1)
promData=np.zeros(N,dtype=complex)
#---------------------------------------
inversaAxe         = fig.add_subplot(2,2,4)
inversaLn,penLn,penRLn,penILn = plt.plot([],[],'m-o',[],[],'k-',[],[],'b-',[],[],'r-')
inversaAxe.grid(True)
inversaAxe.set_xlim(-1,1)
inversaAxe.set_ylim(-1,1)
penData= []
#---------------------------------------
tData=np.arange(0,N/fs,1/fs)

def init():
    return circleLn,circleLg,signalRLn,signalILn,massLn,promRLn,promILn,inversaLn,penILn,penRLn,

def updateF(n):
    global promData,fData,frecIter,penData
    if aniT.repeat==True:
        return inversaLn,
    inversaData=[0]
    for f in range(N):
        inversaData.append(inversaData[-1]+circleInv(circleFrec[f],frecIter,promData[f]))
    inversaLn.set_data(np.imag(inversaData),np.real(inversaData))
    penData.insert(0,inversaData[-1])
    penData=penData[0:N]
    t=np.linspace(0,1,len(penData))
    penRLn.set_data(t,np.real(penData))
```

# IDFT

*Señales complejas como entrada?*

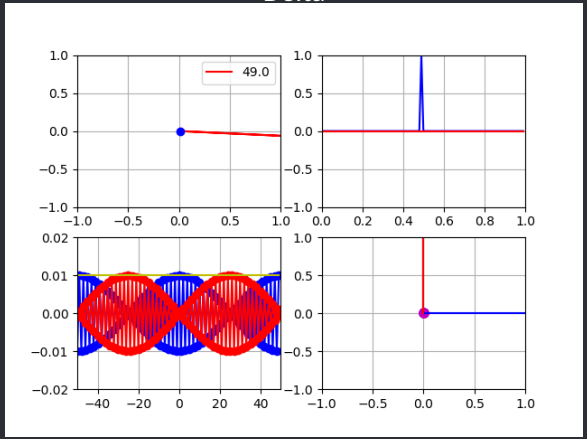
```
        penRLn.set_data(t,np.real(penData))
        penILn.set_data(np.imag(penData),t)
        penLn.set_data(np.imag(penData),np.real(penData))
        frecIter+=1
        if frecIter==N:
            frecIter=0
        return inversaLn,penLn,penILn,penRLn,circleLn,signalRLn,signalILn,promRLn,promILn,

    def updateT(nn):
        global circleData,signalData,promData,frecIter,circleFrec,circleLg

        circleData = []
        signalData = []
        for n in range(N):
            circleData.append(circle(circleFrec[frecIter],n)*signal(signalFrec,n))
            mass=np.average(circleData)
            signalData.append(signal(signalFrec,n))
            promData[frecIter]=mass
        massLn.set_data(np.real(mass),
                        np.imag(mass))
        circleLn.set_data(np.real(circleData),
                          np.imag(circleData))
        signalRLn.set_data(tData[:n+1],np.real(signalData))
        signalILn.set_data(tData[:n+1],np.imag(signalData))
        promRLn.set_data(circleFrec[:frecIter+1],np.real(promData[:frecIter+1]))
        promILn.set_data(circleFrec[:frecIter+1],np.imag(promData[:frecIter+1]))
        circleLn.set_label(circleFrec[frecIter])
        circleLg=circleAxe.legend()

        if frecIter == N-1:
            frecIter=0
            aniT.repeat=False
        else:
            frecIter+=1
        return circleLn,circleLg,signalRLn,signalILn,massLn,promRLn,promILn,

    aniT=FuncAnimation(fig,updateT,N,init,interval=10  ,blit=True,repeat=True)
    aniF=FuncAnimation(fig,updateF,N,init,interval=500 ,blit=True,repeat=True)
    plt.get_current_fig_manager().window.showMaximized()
    b=buttonOnFigure(fig,aniT,aniF)
    plt.show()
```



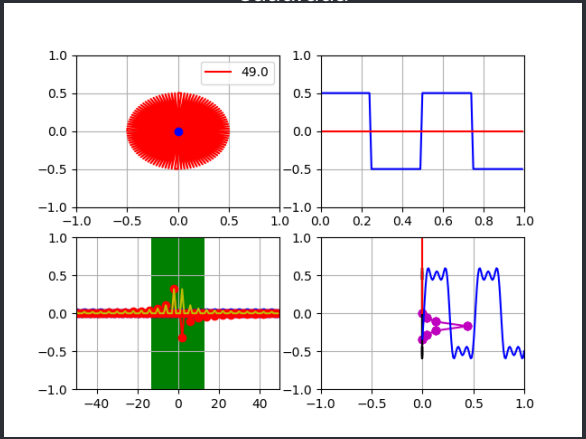
# DFT<>IDFT

*Transformadas relevantes*

Delta



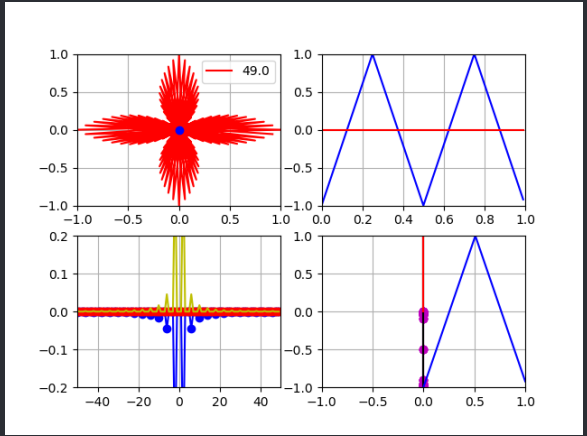

Cuadrada



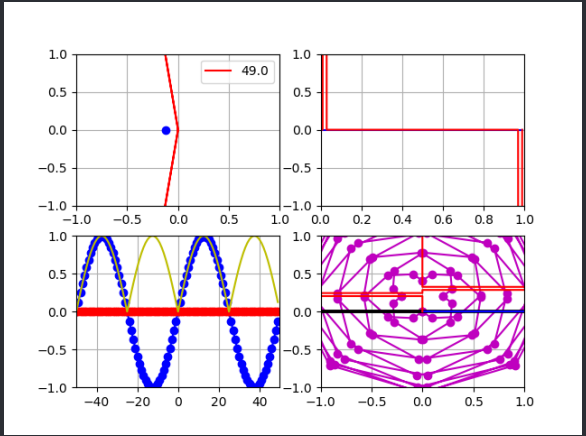
# DFT<>IDFT

*Transformadas relevantes*

## Triangular



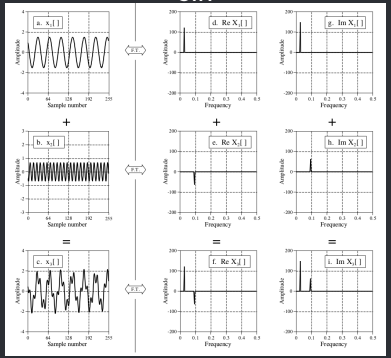

## Conjugado



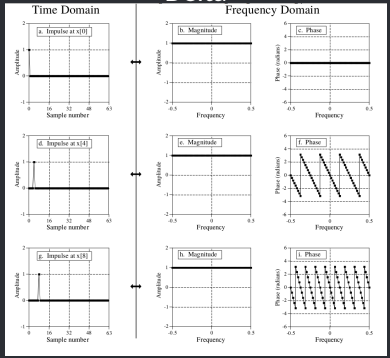
# DFT<>IDFT

*Transformadas relevantes*

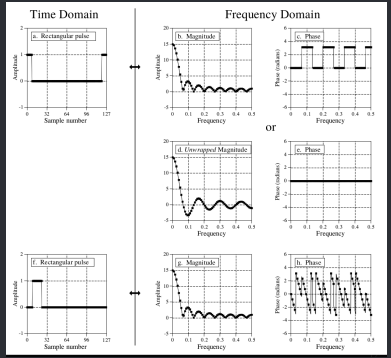
## Sin




## Delta




## Cuadrada



# IDFT

*Un conejo como entrada?*

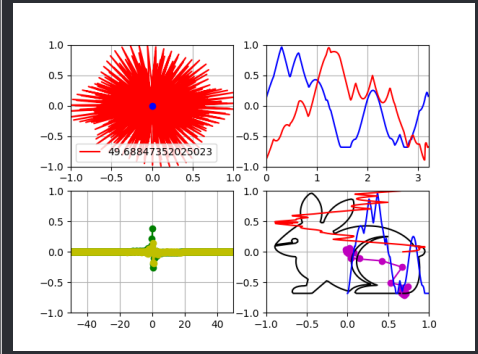
```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
from buttons import buttonOnFigure
#---------------------------------------
fig         = plt.figure()
fs          = 100
#N           = 100
#---------------------------------------
conejo=np.load("4_clase/conejo.npy")[::1]
N=len(conejo)
def signal(f,n):
    return conejo[n]
#---------------------------------------
circleAxe  = fig.add_subplot(2,2,1)
circleLn,massLn,  = plt.plot([],[],'r-',[],[],'bo')
circleAxe.grid(True)
circleAxe.set_xlim(-1,1)
circleAxe.set_ylim(-1,1)
circleFrec = np.arange(-fs/2,fs/2,fs/N)
circleLn.set_label(circleFrec[0])
circleLg=circleAxe.legend()
circleData = []
mass       = 0
frecIter   = 0
def circle(f,n):
    return np.exp(-1j*2*np.pi*f*n*1/fs)
def circleInv(f,n,c):
    return c*np.exp(1j*2*np.pi*f*n*1/fs)
#---------------------------------------
signalAxe  = fig.add_subplot(2,2,2)
signalRLn,signalILn  = plt.plot([],[],'b-',[],[],'r-')
signalAxe.grid(True)
```

```
signalAxe.grid(True)
signalAxe.set_xlim(0,N/fs)
signalAxe.set_ylim(-1,1)
signalFrec = 2
signalData=[]
#def signal(f,n):
#    return np.sin(2*np.pi*f*n*1/fs)+0.4j*np.sin(2*np.pi*f*n*1/fs)
#---------------------------------------
promAxe  = fig.add_subplot(2,2,3)
promRLn,promILn,  = plt.plot([],[],'g-o',[],[],'y-o')
promAxe.grid(True)
promAxe.set_xlim(-fs/2,fs/2)
promAxe.set_ylim(-0.1,0.5)
promData=np.zeros(N,dtype=complex)
#---------------------------------------
inversaAxe          = fig.add_subplot(2,2,4)
inversaLn,penLn,penRLn,penILn = plt.plot([],[],'m-o',[],[],'k-',[],[],'b-',[],[],'r-')
inversaAxe.grid(True)
inversaAxe.set_xlim(-1,1)
inversaAxe.set_ylim(-1,1)
penData= []
harmonics=2
#---------------------------------------
tData=np.arange(0,N/fs,1/fs)

def init():
    return circleLn,circleLg,signalRLn,signalILn,massLn,promRLn,promILn,inversaLn,penILn,penRLn,

def updateF(n):
    global promData,fData,frecIter,penData,harmonics
    if aniT.repeat==True:
        return inversaLn,
    inversaData=[0]
    harmonicRange=range(N//2-harmonics,N//2+1+harmonics,1)
```

# IDFT

*Un conejo como entrada?*

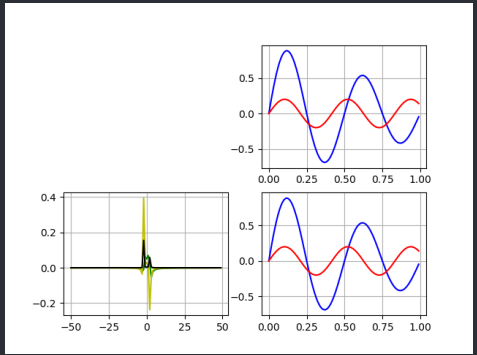
```
        harmonicRange=range(N//2-harmonics,N//2+1+harmonics,1)
        for f in harmonicRange:
            inversaData.append(inversaData[-1]+circleInv(circleFrec[f],frecIter,promData[f]))
        inversaLn.set_data(np.imag(inversaData),np.real(inversaData))
        penData.insert(0,inversaData[-1])
        penData=penData[0:N]
        t=np.linspace(0,1,len(penData))
        penRLn.set_data(t,np.real(penData))
        penILn.set_data(np.imag(penData),t)
        penLn.set_data(np.imag(penData),np.real(penData))
        promHarmomicLn = promAxe.fill_between([circleFrec[harmonicRange[0]],circleFrec[harmonicRange
                [-1]]],1,-1,facecolor="green",alpha=0.1)
        frecIter+=1
        print(harmonics,N)
        if frecIter==N:
            frecIter=0
            harmonics+=1
            if harmonics>=N//2:
                harmonics=1
        return inversaLn,penLn,penILn,penRLn,signalRLn,signalILn,promRLn,promILn,promHarmomicLn

    def updateT(nn):
        global circleData,signalData,promData,frecIter,circleFrec,circleLg

        circleData = []
        signalData = []
        for n in range(N):
            circleData.append(circle(circleFrec[frecIter],n)*signal(signalFrec,n))
            mass=np.average(circleData)
            signalData.append(signal(signalFrec,n))
            promData[frecIter]=mass
        massLn.set_data(np.real(mass),
                        np.imag(mass))
        circleLn.set_data(np.real(circleData),
                          np.imag(circleData))
        signalRLn.set_data(tData[:n+1],np.real(signalData))
        signalILn.set_data(tData[:n+1],np.imag(signalData))
        promRLn.set_data(circleFrec[:frecIter+1],np.real(promData[:frecIter+1]))
        promILn.set_data(circleFrec[:frecIter+1],np.imag(promData[:frecIter+1]))
        circleLn.set_label(circleFrec[frecIter])
        circleLg=circleAxe.legend()

        if frecIter == N-1:
            aniT.repeat=False
        else:
            frecIter+=1
```



# FFT-IFFT

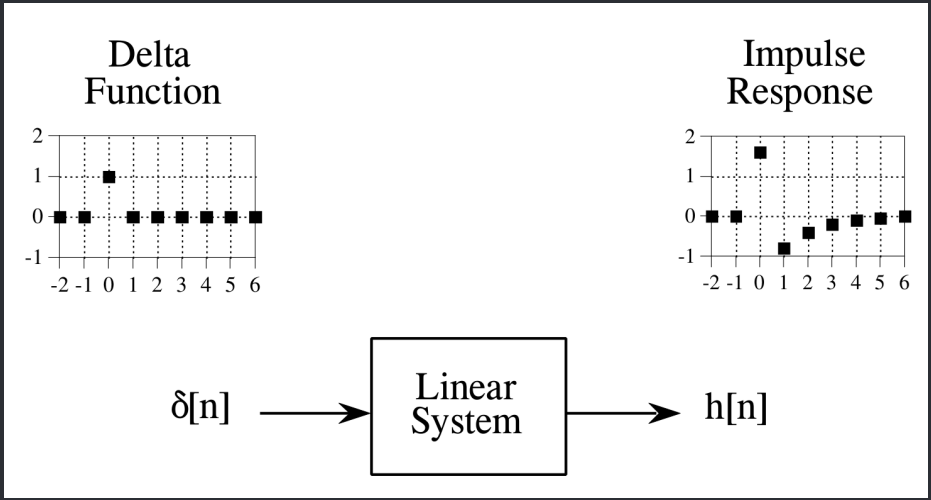
*Transformadas usando FFT en Python*

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
#---------------------------------------
fig         = plt.figure()
fs          = 100
N           = 100
frecIter    = 0
signalFrec = 2
#---------------------------------------
tData       = np.arange(0,N/fs,1/fs)
nData       = np.arange(0,N,1)
circleFrec = np.arange(-fs/2,fs/2,fs/N)
#-----------SIGNAL--------------------------
signalData = np.exp(-nData/fs)*np.sin(2*np.pi*signalFrec*nData*1/fs)+0.2j*np.sin(2*np.pi*1.2*
       signalFrec*nData*1/fs)
signalAxe  = fig.add_subplot(2,2,2)
signalRLn,signalILn,= plt.plot(tData,np.real(signalData),'b-',tData,np.imag(signalData),'r-')
signalAxe.grid(True)
#-----------FFT IFFT-----------------------
fftData  = np.fft.fft(signalData)
ifftData = np.fft.ifft(fftData)
fftData  = np.concatenate((fftData[N//2:N],fftData[0:N//2]))/N
#----------FFT---------------------------
fftAxe              = fig.add_subplot(2,2,3)
fftRLn,fftILn,fftAbsLn = plt.plot(circleFrec,np.real(fftData),'g-' ,circleFrec,np.imag(
       fftData),'y-' ,circleFrec,np.abs(fftData)**2,'k-')
fftAxe.grid(True)
#----------IFFT---------------------------
ifftAxe      = fig.add_subplot(2,2,4)
penRLn,penILn = plt.plot(tData,np.real(ifftData),'b-',tData,np.imag(ifftData),'r-')
ifftAxe.grid(True)
#---------------------------------------
plt.show()
```

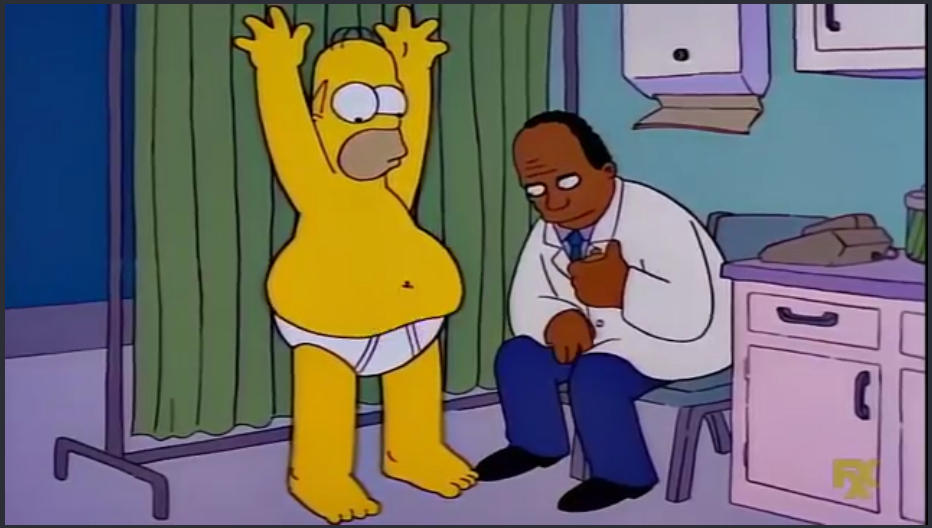


# Función delta

*Respuesta al impulso*



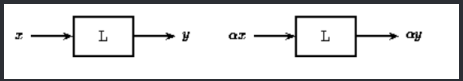
# Todo esta en la respuesta al impulso



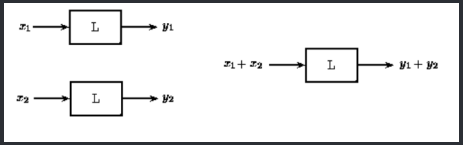
## Linealidad

Un sistema es lineal cuando su salida depende linealmente de la entrada. Satisface el principio de superposición.

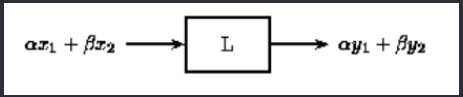
escalado




adición




superposición

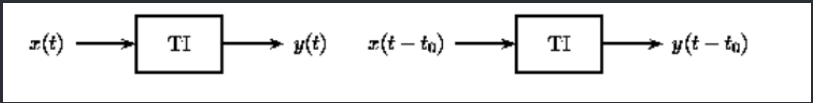



$y(t) = e^{x(t)}$

$y(t) = \frac{1}{2}x(t)$

## Invariantes en el tiempo

Un sistema es invariante en el tiempo cuando la salida para una determinada entrada es la misma sin importar el tiempo en el cual se aplica la entrada

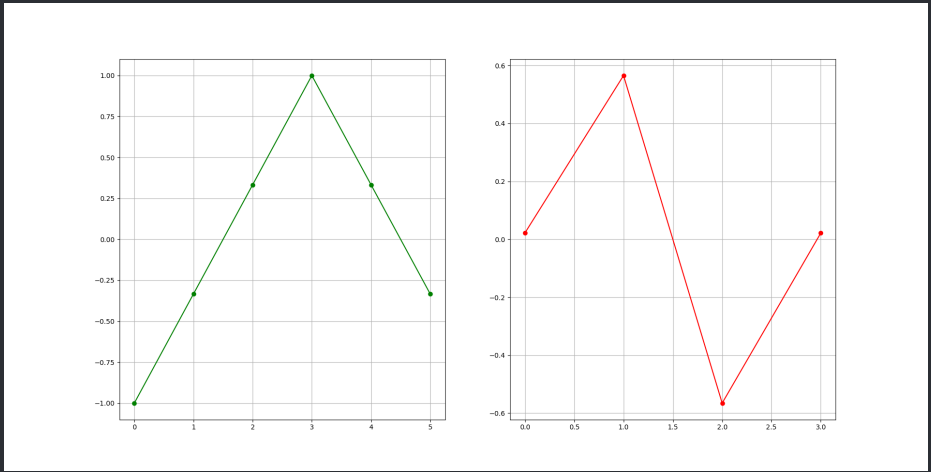



$$y(t) = x(t) * \cos(t)$$

$$y(t) = \cos(x(t))$$

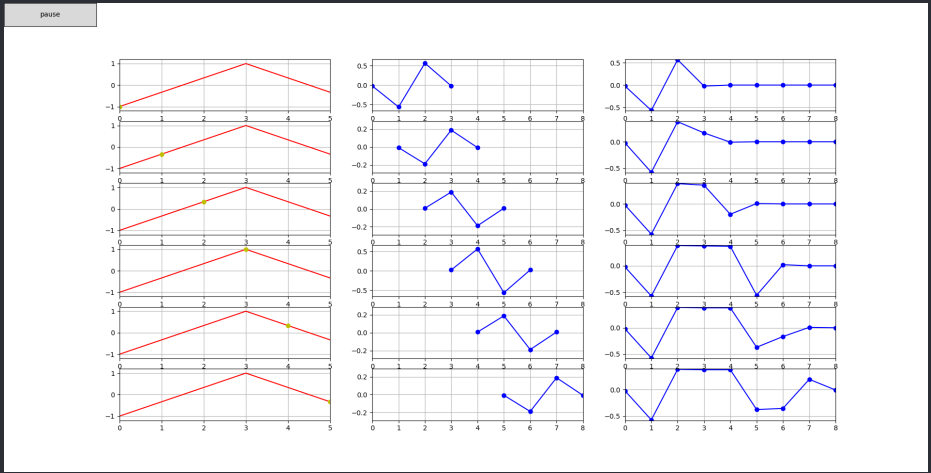
# Convolución

*Señal vs h(n)*



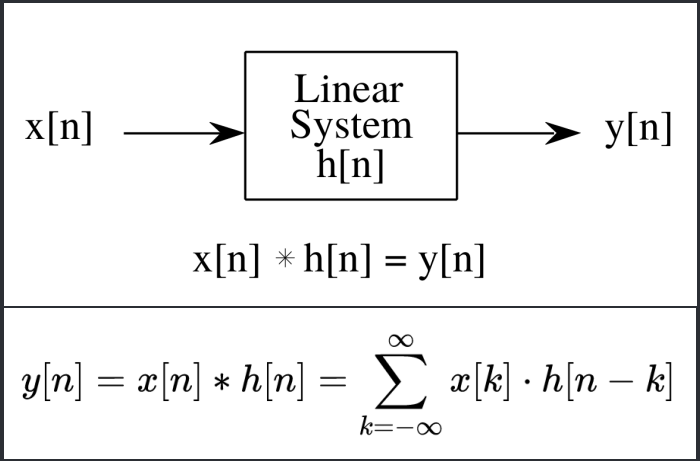
# Convolución

## *Descomposición felta*



# Funcion delta

*Respuesta al impulso*



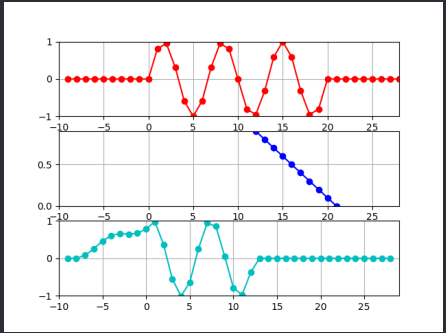

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

# Convolución

## *Respuesta al impulso*

```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib
from matplotlib.animation import FuncAnimation
from buttons import buttonOnFigure
#-------------------------------------
fig = plt.figure()
fs  = 20
N   = 20
M=10
#-------------------------------------
xFrec = 3
def x(f,n):
    return np.sin(2*np.pi*f*n*1/fs)
tData=np.arange(-(M-1),N+(M-1),1)
xData=np.zeros(N+2*(M-1))
xData[M-1:M-1+N]=x(xFrec,tData[M-1:M-1+N])
xAxe  = fig.add_subplot(3,1,1)
xLn,xHighLn  = plt.plot(tData,xData,'r-o',[],[],'y-
        o')
xAxe.grid(True)
xAxe.set_xlim(-M,M+N-2)
xAxe.set_ylim(np.min(xData),np.max(xData))
#-------------------------------------
hData=[0.1*n for n in range(M)]
hAxe  = fig.add_subplot(3,1,2)
```

```
hLn,  = plt.plot([],[],'b-o')
hAxe.grid(True)
hAxe.set_xlim(-M,M+N-2)
hAxe.set_ylim(np.min(hData),np.max(hData))
#-------------------------------------
yAxe  = fig.add_subplot(3,1,3)
yLn,  = plt.plot([],[],'c-o')
yAxe.grid(True)
yAxe.set_xlim(-M,M+N-1)
yAxe.set_ylim(np.min(xData),np.max(xData))
yData=[]
#-------------------------------------
def init():
    global yData
    yData=np.zeros(N+2*(M-1))
    return hLn,xLn,xHighLn,yLn,

def update(i):
    global yData
    t=np.linspace(-(M-1)+i,i,M,endpoint=True)
    yData[i]=np.sum(xData[i:i+M]*hData[::-1])
    xHighLn.set_data(t,xData[i:i+M])
    hLn.set_data(t,hData[::-1])
    yLn.set_data(tData,yData)
    return hLn,xLn,xHighLn,yLn,

ani=FuncAnimation(fig,update,M+N-1,init,interval
        =1000 ,blit=True,repeat=True)
plt.get_current_fig_manager().window.showMaximized
        ()
b=buttonOnFigure(fig,ani)
plt.show()
```

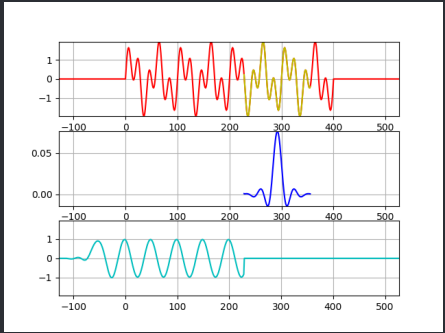


# Filtrado

## *Pasa bajos*

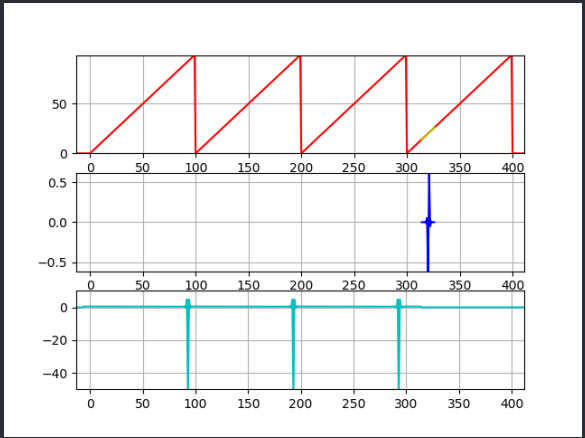
```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib
from matplotlib.animation import FuncAnimation
from buttons import buttonOnFigure
#-------------------------------------
fig = plt.figure()
fs  = 100
N   = 400
fir,=np.load("4_clase/low_pass.npy").astype(float)
M=len(fir)
#fir,=np.load("diferenciador.npy").astype(float)
#M=len(fir)
#-------------------------------------
def x(f,n):
    return np.sin(2*np.pi*2*n*1/fs)+\
           np.sin(2*np.pi*5*n*1/fs)

xFrec = 3
tData=np.arange(-(M-1),N+(M-1),1)
xData=np.zeros(N+2*(M-1))
xData[M:M+N]=x(xFrec,tData[M:M+N])
xAxe  = fig.add_subplot(3,1,1)
xLn,xHighLn  = plt.plot(tData,xData,'r-',[],[],'y-'
       )
xAxe.grid(True)
xAxe.set_xlim(-M,M+N-2)
xAxe.set_ylim(np.min(xData),np.max(xData))
#-------------------------------------
```

```
hData=fir
hAxe  = fig.add_subplot(3,1,2)
hLn,  = plt.plot([],[],'b-')
hAxe.grid(True)
hAxe.set_xlim(-M,M+N-2)
hAxe.set_ylim(np.min(hData),np.max(hData))
#-------------------------------------
yAxe  = fig.add_subplot(3,1,3)
yLn,  = plt.plot([],[],'c-')
yAxe.grid(True)
yAxe.set_xlim(-M,M+N-1)
yAxe.set_ylim(np.min(xData),np.max(xData))
yData=[]
#-------------------------------------
def init():
    global yData
    yData=np.zeros(N+2*(M-1))
    return hLn,xLn,xHighLn,yLn,

def update(i):
    global yData
    t=np.linspace(-(M-1)+i,i,M,endpoint=True)
    yData[i]=np.sum(xData[i:i+M]*hData[::-1])
    xHighLn.set_data(t,xData[i:i+M])
    hLn.set_data(t,hData[::-1])
    yLn.set_data(tData,yData)
    return hLn,xLn,xHighLn,yLn,

ani=FuncAnimation(fig,update,M+N-1,init,interval=10
          ,blit=True,repeat=True)
plt.get_current_fig_manager().window.showMaximized
          ()
b=buttonOnFigure(fig,ani)
plt.show()
```



# Filtrado

*diferenciador*



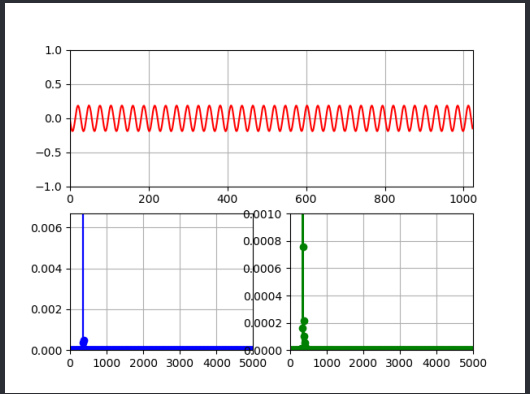
# Convolución

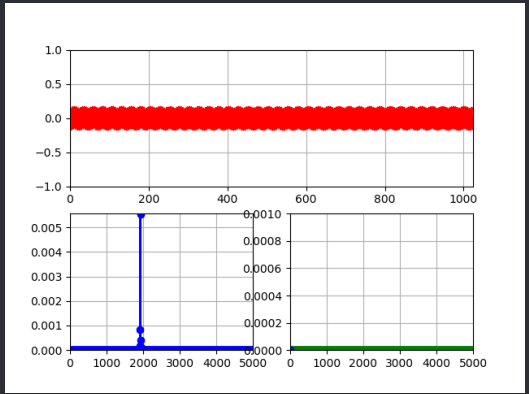
## *Análisis en la CIAA*

```
#include "sapi.h"
#include "arm_math.h"
#include "arm_const_structs.h"
#define MAX_FFT_LENGTH 2048          // maxima longitud para la fft y chunk de samples
#define BITS 10                      // cantidad de bits usado para cuantizar
#define FIR_LENGTH 162
int16_t fftLength = 32;              // longitud de la fft y samples variable
int16_t adc [ MAX_FFT_LENGTH ];      // guarda los samples
q15_t fftIn [ MAX_FFT_LENGTH ];      // guarda copia de samples en Q15 como in para la fft.La fft corrompe
        los datos de la entrada!
q15_t fftOut[ MAX_FFT_LENGTH*2 ];    // salida de la fft
q15_t fftMag[ MAX_FFT_LENGTH/2+1 ]; // magnitud de la FFT
//low_pass 1000hz 127
//q15_t fir[ FIR_LENGTH]={ 22, 21, 24, 19, 6, -14, -40, -66, -83, -86, -73, -43, -3, 36, 65, 73, 56, 16,
          -35, -84, -115, -114, -77, -13, 63, 128, 161, 147, 84, -13, -119, -200, -227, -186, -81, 62,
          204, 299, 310, 224, 56, -152, -341, -445, -422, -259, 11, 320, 575, 687, 593, 287, -176, -683,
          -1083, -1220, -977, -305, 755, 2074, 3450, 4656, 5480, 5772, 5480, 4656, 3450, 2074, 755, -305,
          -977, -1220, -1083, -683, -176, 287, 593, 687, 575, 320, 11, -259, -422, -445, -341, -152, 56,
          224, 310, 299, 204, 62, -81, -186, -227, -200, -119, -13, 84, 147, 161, 128, 63, -13, -77, -114,
           -115, -84, -35, 16, 56, 73, 65, 36, -3, -43, -73, -86, -83, -66, -40, -14, 6, 19, 24, 21, 22};
//bandpass 440hz 162
q15_t fir[ FIR_LENGTH]={ 1, 0, 0, -0, -2, -5, -9, -14, -19, -25, -29, -32, -33, -30, -24, -14, 0, 17, 37,
          57, 76, 91, 101, 104, 98, 83, 60, 31, -1, -36, -68, -95, -113, -121, -117, -103, -81, -54, -26,
          -3, 11, 13, 2, -22, -58, -99, -140, -172, -186, -175, -135, -62, 41, 170, 315, 461, 593, 694,
          748, 742, 667, 522, 312, 46, -254, -566, -864, -1118, -1303, -1396, -1383, -1260, -1028, -704,
          -310, 123, 561, 967, 1307, 1551, 1679, 1679, 1551, 1307, 967, 561, 123, -310, -704, -1028,
          -1260, -1383, -1396, -1303, -1118, -864, -566, -254, 46, 312, 522, 667, 742, 748, 694, 593, 461,
           315, 170, 41, -62, -135, -175, -186, -172, -140, -99, -58, -22, 2, 13, 11, -3, -26, -54, -81,
          -103, -117, -121, -113, -95, -68, -36, -1, 31, 60, 83, 98, 104, 101, 91, 76, 57, 37, 17, 0, -14,
           -24, -30, -33, -32, -29, -25, -19, -14, -9, -5, -2, -0, 0, 0, 1};
q15_t firOut [ MAX_FFT_LENGTH+FIR_LENGTH+1 ];
uint32_t maxIndex = 0;               // indexador de maxima energia por cada fft
q15_t maxValue    = 0;               // maximo valor de energia del bin por cada fft
arm_rfft_instance_q15 S;
uint16_t sample   = 0;               // contador para samples
int main ( void ) {
   boardConfig         (                        );
   uartConfig          ( UART_USB, 460800       );
   adcConfig           ( ADC_ENABLE             );
   cyclesCounterInit  ( EDU_CIAA_NXP_CLOCK_SPEED );
   while(1) {
      sample = 0;                                                    // arranca de nuevo
      uartWriteByteArray ( UART_USB ,(uint8_t* )&maxValue ,2);
      uartWriteByteArray ( UART_USB ,(uint8_t* )&maxIndex ,2);
      uartWriteByteArray ( UART_USB ,"header" ,6 );                  // manda el header
                  que casualmente se llama "header" con lo que arranca una nueva trama
      uartWriteByteArray ( UART_USB ,(uint8_t* )&fftLength ,sizeof(fftLength));   // manda el largo de
               la fft que es variable
      arm_rfft_init_q15 ( &S ,fftLength ,0 ,1 );                     // inicializa una
               estructira que usa la funcion fft para procesar los datos. Notar el /2 para el largo
      arm_rfft_q15      ( &S ,fftIn     ,fftOut    );                // por fin.. ejecuta
               la rfft REAL fft
      arm_cmplx_mag_squared_q15 ( fftOut ,fftMag ,fftLength/2+1 );
      arm_max_q15 ( fftMag ,fftLength/2+1 ,&maxValue ,&maxIndex );
      gpioToggle( LEDR);
      if ( gpioRead(TEC1 )==0) {
         gpioToggle(LEDB);
         if((fftLength<<=1)>MAX_FFT_LENGTH)
            fftLength=32;
         while(gpioRead(TEC1)==0)
            ;
      }
   }
   while(cyclesCounterRead()< 20400) //clk de 204000000 => 10k samples x seg.
      ;
}
```

# Convolución

*Análisis en la CIAA*



# Bibliografía

*Libros, links y otro material*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