CSE 150 Homework 6

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1 Viterbi Algorithm

2 Conditional Independence

$P(S_t|S_{t-1}) = P(S_t|S_{t-1}, O_t)$
False
$P(S_t|S_{t-1}) = P(S_t|S_{t-1}, O_{t-1})$
True
$P(S_t|S_{t-1}) = P(S_t|S_{t-1}, S_{t+1})$
False
$P(S_t|O_{t-1}) = P(S_t|O_1, O_2, ..., O_{t-1})$
False
$P(O_t|S_{t-1}) = P(O_t|S_{t-1}, O_{t-1})$
True
$P(O_t|O_{t-1}) = P(O_t|O_1, O_2, ..., O_{t-1})$
False
$P(O_1, O_2, ..., O_T) = \prod_{t=1}^{T} P(O_t|O_1, O_2, ..., O_{t-1})$
True
$P(S_2, S_3, ..., S_T|S_1) = \prod_{t=2}^{T} P(S_t|S_{t-1})$
True
$P(S_1, S_2, ..., S_{T-1}|S_T) = \prod_{t=1}^{T-1} P(S_t|S_{t+1})$
True
$P(S_1, S_2, ..., S_T|O_1, O_2, ..., O_T) = \prod_{t=1}^{T} P(S_t|O_t)$
False
$P(S_1, S_2, ..., S_T, O_1, O_2, ..., O_T) = \prod_{t=1}^{T} P(S_t, O_t)$
False
$P(O_1, O_2, ..., O_T|S_1, S_2, ..., S_T) = \prod_{t=1}^{T} P(O_t|S_t)$
True

3 More conditional independence

(a)

$$P(S_t|S_{t+1}, S_{t+2}, ..., S_T) = P(S_t|S_{t+1})$$

$$P(S_t|O_t, O_{t-1}, O_{t+1}) = P(S_t|O_t, O_{t-1}, O_{t+1})$$

$$P(S_t|O_t, O_{t+1}, ..., O_T) = (S_t|O_t, O_{t+1}, ..., O_T)$$

$$P(O_t|O_1, O_2, ..., O_{t-1}) = P(O_t|O_1, O_2, ..., O_{t-1})$$

$$P(O_t|S_{t-2}, S_{t-1}, S_{t+1}, S_{t+2}) = P(O_t|S_{t-1}, S_{t+1})$$

$$P(O_t|O_{t-1}, O_{t+1}, S_1, S_T) = P(O_t|O_{t-1}, O_{t+1}, S_1, S_T)$$

(b)

$$P(S_t|O_t, O_{t-1}, O_{t+1}, S_{t-1}, S_{t+1}) = P(S_t|O_t, S_{t-1}, S_{t+1})$$

$$P(S_t|S_1, S_T, O_1, O_t, O_T) = P(S_t|S_1, S_T, O_t)$$

$$P(O_t|O_1, O_2, ..., O_{t-1}, S_{t-1}) = P(O_t|S_{t-1})$$

$$P(O_t|O_1, O_2, ..., O_{t-1}, S_{t-2}) = P(O_t|O_{t-1}, S_{t-2})$$

4 Belief updating

(a)

$$\begin{aligned} P(Y_1|X_1) &= \sum_{x_0} P(Y_1, X_0 = x_0|X_1) \\ &= \sum_{x_0} P(Y_1|X_0 = x_0, X_1) \cdot P(X_0 = x_0|X_1) \\ &= \sum_{x_0} P(Y_1|X_0 = x_0, X_1) \cdot P(X_0 = x_0) \end{aligned}$$

(b)

$$\begin{aligned} P(Y_1) &= \sum_{x_0}\sum_{x_1} P(Y_1, X_0 = x_0, X_1 = x_1) \\ &= \sum_{x_0}\sum_{x_1} P(Y_1|X_0 = x_0, X_1 = x_1) \cdot P(X_0 = x_0, X_1 = x_1) \\ &= \sum_{x_0}\sum_{x_1} P(Y_1|X_0 = x_0, X_1 = x_1) \cdot P(X_0 = x_0) \cdot P(X_1 = x_1) \end{aligned}$$

(c)

$$
\begin{aligned}
&P(X_t|Y_1, Y_2, .., Y_{t-1}) \\
&= \sum_x P(X_t, X_{t-1} = x|Y_1, Y_2, .., Y_{t-1}) && \text{(Marginalization)} \\
&= \sum_x P(X_t|X_{t-1} = x, Y_1, Y_2, .., Y_{t-1}) \cdot P(X_{t-1} = x|Y_1, Y_2, .., Y_{t-1}) && \text{(Prod. rule)} \\
&= \sum_x P(X_t|X_{t-1} = x, Y_{t-1}) \cdot P(X_{t-1} = x|Y_1, Y_2, .., Y_{t-1}) && \text{(C.I.)} \\
&= \sum_x \frac{P(Y_{t-1}|X_t, X_{t-1} = x) \cdot P(X_t|X_{t-1} = x)}{P(Y_{t-1}|X_{t-1} = x)} \cdot P(X_{t-1} = x|Y_1, Y_2, .., Y_{t-1}) && \text{(Bayes' rule)} \\
&= \sum_x \frac{P(Y_{t-1}|X_t, X_{t-1} = x) \cdot P(X_t)}{\sum_{x'} P(Y_{t-1}, X_t = x'|X_{t-1} = x)} \cdot P(X_{t-1} = x|Y_1, Y_2, .., Y_{t-1}) && \text{(C.I. and Marginalization)} \\
&= \sum_x \frac{P(Y_{t-1}|X_t, X_{t-1} = x) \cdot P(X_t)}{\sum_{x'} P(Y_{t-1}|X_t = x', X_{t-1} = x) \cdot P(X_t = x'|X_{t-1} = x)} \cdot P(X_{t-1} = x|Y_1, Y_2, .., Y_{t-1}) && \text{(Prod. Rule)} \\
&= \sum_x \frac{P(Y_{t-1}|X_t, X_{t-1} = x) \cdot P(X_t)}{\sum_{x'} P(Y_{t-1}|X_t = x', X_{t-1} = x) \cdot P(X_t = x')} \cdot P(X_{t-1} = x|Y_1, Y_2, .., Y_{t-1}) && \text{(C.I.)}
\end{aligned}
$$

(d)

$$
\begin{aligned}
P(Y_t|X_t, Y_1, ..., Y_{t-1}) &= \sum_x P(Y_t, X_{t-1} = x|X_t, Y_1, ..., Y_{t-1}) && \text{(Marginalization)} \\
&= \sum_x P(Y_t|X_{t-1} = x, X_t, Y_1, ..., Y_{t-1}) \cdot P(X_{t-1} = x|X_t, Y_1, ..., Y_{t-1}) && \text{(Prod. Rule)} \\
&= \sum_x P(Y_t|X_{t-1} = x, X_t) \cdot P(X_{t-1} = x|Y_1, ..., Y_{t-1}) && \text{(C.I.)}
\end{aligned}
$$

(e)

$$
\begin{aligned}
P(Y_t|Y_1, ..., Y_{t-1}) &= \sum_{x,x'} P(Y_t, X_{t-1} = x, X_t = x'|, Y_1, ..., Y_{t-1}) && \text{(Marginalization)} \\
&= \sum_{x,x'} P(Y_t|X_{t-1} = x, X_t = x', Y_1, ..., Y_{t-1}) \cdot P(X_{t-1} = x, X_t = x'|Y_1, ..., Y_{t-1}) && \text{(Prod. Rule)} \\
&= \sum_{x,x'} P(Y_t|X_{t-1} = x, X_t = x') \cdot P(X_{t-1} = x) \cdot P(X_t = x') && \text{(C.I.)}
\end{aligned}
$$

5 Most likely hidden states