CSE 150 Homework 2

Pedro Sousa Meireles

Fall 2018

1 Variable Elimination Algorithm

(a) **Computing using variable elimination**

Eliminating S:

F	$f'_3(F)$
0	0.01
1	0.9

Eliminating R:

L	$f'_5(L)$
0	0.01
1	0.75

Eliminating L:

$$f_6(A) = \sum_l f'_5(L = l) \cdot f_4(A, L = l)$$

Eliminating L costs 1 addition and 2 products per different A, making 2 additions and 4 products.

A	$f_6(A)$
0	0.01074
1	0.6612

Eliminating A:

$$f_7(T, F) = \sum_a f_2(T, F, A = a) \cdot f_6(A = a)$$

Eliminating A costs 1 addition and 2 products per different (T,F), making 4 additions and 8 products.

T	F	$f_7(T, F)$
0	0	0.010805046
0	1	0.6546954
1	0	0.563631
1	1	0.33597

Eliminating F:

$$f_7(T) = \sum_a f_1(F = f) \cdot f_7(T, F = f) \cdot f_3'(F = f)$$

Eliminating F costs 1 addition and 4 products per different T, making 2 additions and 8 products.

T	$f_7(T)$
0	0.00599922865
1	0.0086036769

Combining T factors:

$$f_8(T) = f_0(T) \cdot f_7(T)$$

Combining T factors costs 0 additions and 1 products per different T, making 0 additions and 2 products.

T	$f_8(T)$
0	0.00587924408
1	0.000172073538

$$P(T = 0|S = 1, R = 1) = \frac{f_8(T = 0)}{f_8(T = 0) + f_8(T = 1)} = 0.97156$$

$$P(T = 1|S = 1, R = 1) = \frac{f_8(T = 1)}{f_8(T = 0) + f_8(T = 1)} = 0.02843$$

(b) **Counting calculations used by the variable elimination algorithm**

Phase of algorithm	# multiplications	# additions	# divisions
Eliminate S (evidence)	0	0	0
Eliminate R (evidence)	0	0	0
Eliminate L	4	2	0
Eliminate A	8	4	0
Eliminate F	8	2	0
Combine T factors	2	0	0
Normalize distribution over T	0	1	2
Total	22	9	2

(c) **Counting calculations used by the enumeration algorithm**

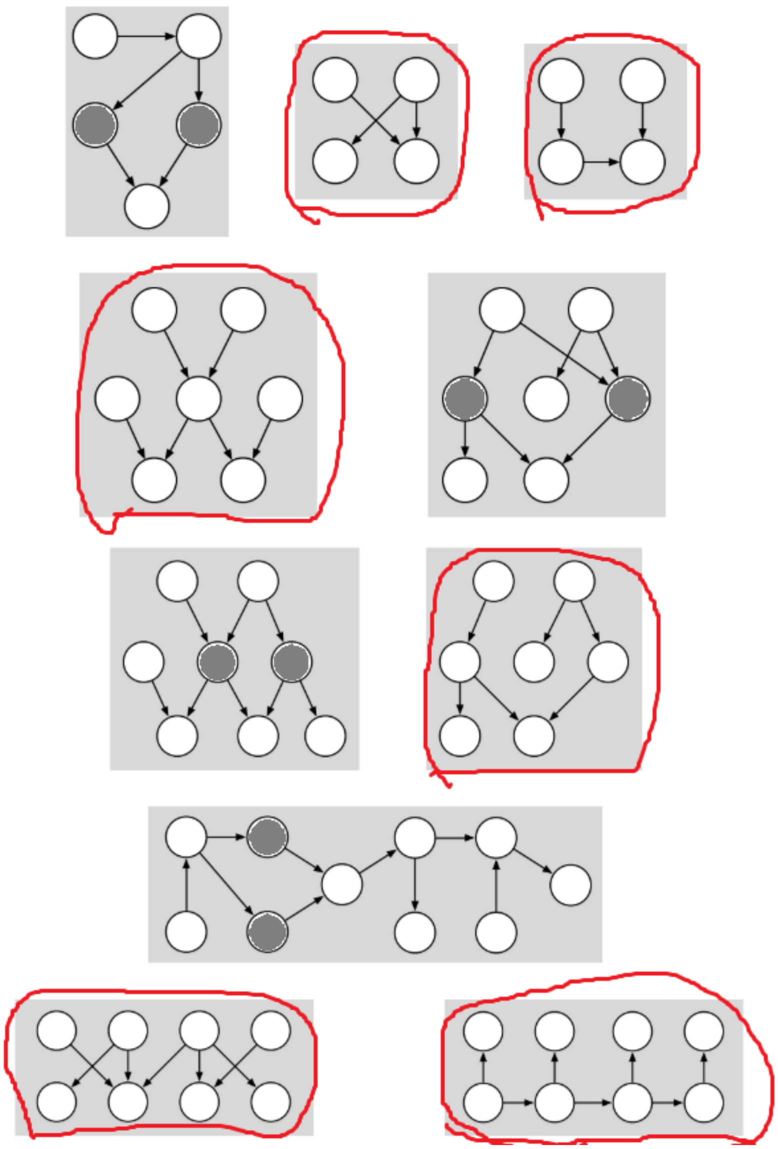
$$P(T=0,S=1,R=1)=\sum_a\sum_f\sum_l P(A=a,F=f,L=l,R=1,S=1,T=0)$$
$$=\sum_a\sum_f\sum_l P(T=0)\cdot P(F=f)\cdot P(S=1|F=f)\cdot P(A=a|T=0,F=f)\cdot P(L=l|A=a)\cdot P(R=1|L=l)$$

$$P(T=1,S=1,R=1)=\sum_a\sum_f\sum_l P(A=a,F=f,L=l,R=1,S=1,T=1)$$
$$=\sum_a\sum_f\sum_l P(T=0)\cdot P(F=f)\cdot P(S=1|F=f)\cdot P(A=a|T=1,F=f)\cdot P(L=l|A=a)\cdot P(R=1|L=l)$$

The sums have 8 terms, which makes 7 additions each, and each term is a product of 6 probabilities, what accounts 5 multiplications per term, which makes 7 additions and 40 multiplications each.

Phase of algorithm	# multiplications	# additions	# divisions
Compute $P(T=0,S=1,R=1)$	40	7	0
Compute $P(T=1,S=1,R=1)$	40	7	0
Normalize distribution over T	0	1	2
Total	80	15	2

2 To be, or not to be, a polytree: that is the question



3 Node clustering

Y_1	Y_2	Y_3	Y	$P(Y\|X=0)$	$P(Y\|X=1)$	$P(Z_1=1\|Y)$	$P(Z_2=1\|Y)$
0	0	0	1	0.0525	0.14625	0.8	0.2
1	0	0	2	0.2975	0.04875	0.7	0.3
0	1	0	3	0.0225	0.07875	0.6	0.4
0	0	1	4	0.525	0.34125	0.5	0.5
1	1	0	5	0.1275	0.02625	0.4	0.6
1	0	1	6	0.2975	0.11375	0.3	0.7
0	1	1	7	0.0225	0.18375	0.2	0.8
1	1	1	8	0.1275	0.06125	0.1	0.9

4 Maximum likelihood estimation for an n-sided die

(a) **Log-likelihood**

$$\begin{aligned} likelihood(p) &= P(x^{(1)}, ..., x^{(T)}) \\ &= \prod_{t=1}^{T} P(X = k) \\ &= p_1^{C_1} \cdot p_2^{C_2} \cdot ... \cdot p_n^{C_n} \end{aligned}$$

Applying log in the equation and separating the log of a productory in a sum of logs:

$$\begin{aligned} L(p) &= C_1 \cdot p_1 + C_2 \cdot p_2 + ... + C_n \cdot p_n \\ &= \sum_{k=1}^{n} C_k \cdot p_k \end{aligned}$$

(b) **KL distance**

$$\begin{aligned} KL(q,p) &= \sum_k q_k \cdot log\left(\frac{q_k}{p_k}\right) \\ &= \sum_k q_k \cdot (log(q_k) - log(p_k)) \\ &= \sum_k q_k \cdot log(q_k) - \sum_k \frac{C_k \cdot log(p_k)}{T} \\ &= \sum_k q_k \cdot log(q_k) - \frac{\sum_k C_k \cdot log(p_k)}{T} \\ &= \sum_k q_k \cdot log(q_k) - \frac{L(p)}{T} \end{aligned}$$

Given a set of tosses, the left term is constant, which means the value of the difference varies only with $L(p)$ variation. So, maximizing $L(p)$ is equivalent to minimizing the KL distance.

(c) **Maximum likelihood estimation**

In the last item we proved that maximizing $L(p)$ is equivalent to minimizing $KL(q,p)$. Also, in homework problem 1.6b, we proved that $KL(p,q) \geq 0$, with equality only when $p = q$. So, to maximize $L(p)$, we have to make $p_k = q_k$. Then, $p_k = \frac{C_k}{T}$.