# CSE 150 Homework 1

Fall 2018

## 1 Probabilistic reasoning

M = children have meltdown
L = I'll be late

Data given by the question:
$P(M = 1) = 0.01$
$P(L = 1|M = 1) = 0.98$
$P(L = 1|M = 0) = 0.03$

Also:

$$\begin{aligned} P(L = 1) &= P(L = 1|M = 1) \cdot P(M = 1) + P(L = 1|M = 0) \cdot P(M = 0) \\ &= 0.98 \cdot 0.01 + 0.03 \cdot (1 - 0.01) \\ &= 0.0297 \end{aligned}$$

The question says that I am late to campus (L = 1). Then what is requested is $P(M = 1|L = 1)$.
Using Bayes rule whe have $P(M = 1|L = 1) = \frac{P(L=1|M=1)*P(M=1)}{P(L=1)}$

$$\begin{aligned} P(M = 1|L = 1) &= \frac{0.98 * 0.01}{0.0297} \\ &= 0.3398 \end{aligned}$$

## 2 Conditioning on background evidence

(a) Product Rule:

$$P(X, Y|E) = P(X|Y, E) \cdot P(Y|E)$$

By isolating the term $P(X|Y, E)$ we get:

$$P(X|Y, E) = \frac{P(X, Y|E)}{P(Y|E)}$$

Then, applying the Product rule again to $P(X, Y|E)$ we have:

$$P(X|Y, E) = \frac{P(Y|X, E) \cdot P(X|E)}{P(Y|E)}$$

(b) Applying the definition of conditional probability to $P(X|E)$ we have:

$$P(X|E) = \frac{P(X, E)}{P(E)}$$

Now applying the marginalization over Y in the numerator:

$$P(X|E) = \frac{\sum_j P(X, E, Y = y_j)}{P(E)} = \sum_j \frac{P(X, E, Y = y_j)}{P(E)}$$

Applying the definition of conditional probability again to each term of the sum we have:

$$P(X|E) = \sum_j P(X, Y = y_j|E)$$

## 3 Conditional independence

(i) <u>Proof that (ii) is true:</u>

Using the product rule for conditional probabilities in $P(X, Y|E)$ we have:

$$P(X, Y|E) = P(X|Y, E) \cdot P(Y|E)$$

Isolating $P(X|Y, E)$ we have:

$$P(X|Y, E) = \frac{P(X, Y|E)}{P(Y|E)}$$

Now using the statement (i):

$$P(X|Y, E) = \frac{P(X|E) \cdot P(Y|E)}{P(Y|E)} = P(X|E)$$

<u>Proof that (iii) is true:</u>

Using the product rule for conditional probabilities in $P(X, Y|E)$ we have:

$$P(X, Y|E) = P(Y|X, E) \cdot P(X|E)$$

Isolating $P(Y|X, E)$ we have:

$$P(Y|X, E) = \frac{P(X, Y|E)}{P(X|E)}$$

Now using the statement (i):

$$P(Y|X, E) = \frac{P(X|E) \cdot P(Y|E)}{P(X|E)} = P(Y|E)$$

(ii) <u>Proof that (i) is true:</u>

Using the product rule for conditional probabilities in $P(X, Y|E)$ we have:

$$P(X, Y|E) = P(X|Y, E) \cdot P(Y|E)$$

Now using the statement (ii) in $P(X|Y, E)$:

$$P(X, Y|E) = P(X|E) \cdot P(Y|E)$$

<u>Proof that (iii) is true:</u>

Using Bayes' rule for conditional probabilities we have:

$$P(Y|X,E) = \frac{P(X|Y,E) \cdot P(Y|E)}{P(X|E)}$$

Now using the statement (ii) in $P(X|Y,E)$:

$$P(Y|X,E) = \frac{P(X|E) \cdot P(Y|E)}{P(X|E)} = P(Y|E)$$

(iii) Proof that (i) is true:

Using the product rule for conditional probabilities in $P(X,Y|E)$ we have:

$$P(X,Y|E) = P(Y|X,E) \cdot P(X|E)$$

Now using the statement (ii) in $P(Y|X,E)$:

$$P(X,Y|E) = P(Y|E) \cdot P(X|E)$$

Proof that (ii) is true:

Using Bayes' rule for conditional probabilities we have:

$$P(X|Y,E) = \frac{P(Y|X,E) \cdot P(X|E)}{P(Y|E)}$$

Now using the statement (ii) in $P(Y|X,E)$:

$$P(X|Y,E) = \frac{P(Y|E) \cdot P(X|E)}{P(Y|E)} = P(X|E)$$

# 4 Creative writing

(a) X = Having skin cancer

Y = Using sunscreen

Z = Going often to the beach

$P(X = 1|Z = 1) > P(X = 1)$: The probability of having skin cancer is bigger if you go often to the beach.

$P(X = 1|Y = 1, Z = 1) < P(X = 1|Z = 1)$: If you go often to the beach, the probability of having skin cancer is smaller if you use sunscreen.

(b) X = Get an A

Y = Do all homeworks

Z = Study frequently

$P(X = 1) < P(X = 1|Y = 1) < P(X = 1|Z = 1, Y = 1)$: The probability to get an A increases if you do all homework and grows even bigger if you also study often.

(c) X = Team A wins

Y = Team B wins

Z = Team A and Team B are not playing agains each other

$P(X, Y|Z = P(X|Z) \cdot P(Y|Z))$: The results of Team A's and Team B's games are independent because they are not playing agains each other.

$P(X = 1, Y = 1) < P(X = 1)P(Y = 1)$: The probability that both teams win is lower than the product of their probabilities of winning, because they may been playing agains each other.