CSE 150 Homework 4

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1 Maximum Likelihood Estimation

$$P_{ML}(X_i = x|pa_i = \pi) = \frac{count(X_i = x, pa_i = \pi)}{\sum_{x'} count(X_i = x', pa_i = \pi)}$$

(a)

$$P(Y = y) = \frac{count(Y = y)}{T}$$
$$P(X = x|Y = y) = \frac{count(X = x, Y = y)}{count(Y = y)}$$
$$P(Z = x|Y = y) = \frac{count(Y = y, Z = z)}{count(Y = y)}$$

(b)

$$P(Z = z) = \frac{count(Z = z)}{T}$$
$$P(Y = y|Z = z) = \frac{count(Y = y, Z = z)}{count(Z = z)}$$
$$P(X = x|Y = y) = \frac{count(X = x, Y = y)}{count(Y = y)}$$

(c) Left DAG:

$$\begin{aligned} P(X = x, Y = y, Z = z) &= P(Y = y) \cdot P(X = x|Y = y) \cdot P(Z = z|Y = y) \\ &= \frac{count(Y = y)}{T} \cdot \frac{count(X = x, Y = y)}{count(Y = y)} \cdot \frac{count(Y = y, Z = z)}{count(Y = y)} \\ &= \frac{count(X = x, Y = y) \cdot count(Y = y, Z = z)}{T \cdot count(Y = y)} \end{aligned}$$

Right DAG:

$$\begin{aligned} P(X = x, Y = y, Z = z) &= P(Z = z) \cdot P(Y = y|Z = z) \cdot P(X = x|Y = y) \\ &= \frac{count(Z = z)}{T} \cdot \frac{count(Y = y, Z = z)}{count(Z = z)} \cdot \frac{count(X = x, Y = y)}{count(Y = y)} \\ &= \frac{count(X = x, Y = y) \cdot count(Y = y, Z = z)}{T \cdot count(Y = y)} \end{aligned}$$

(d) No. Every conditional independence implied by one graph is also present in the other. The only variables that are conditional independent are X and Z given Y. All other combination of variables are conditionally dependent. Given that both graphs gave the same conditional independence relations it is consistent with the result of item C.

2 Survey

Done

3 Statistical language modeling

(a)

w	$P_u(w)$
NINETEEN	0.0028588174836726445
NOT	0.0021457733600835156
NEW	0.001900350279776441
NINE	0.0017613053303564415
NINETY	0.0012920576727483372
NO	0.0010485914217056834
NOW	0.0007126160671875371
N.	0.0006647838218553515
NATIONAL	0.0005965149408928713
NEXT	0.00042879021256620473
NEWS	0.0004290959671387705
NUMBER	0.0003488781974804209
NORTH	0.0002554640604701336
NEVER	0.0002425367571420538
NIGHT	0.00023650727697105726
NEARLY	0.0002124871977502921
NEAR	0.00021042029683974764
NEED	0.00020837785629500844

(b)

w'	$P_u(w' \mid w = HAVE)$
<UNK>	0.4116813942572601
BEEN	0.17246178649415317
TO	0.08213301769351646
A	0.06311299769392462
THE	0.028934103385645192
NO	0.015738454317258833
NOT	0.01344462357911063
SAID	0.013040550192853207
HAD	0.01128956551907104
AN	0.010897736780882022

(c)

$$L_u = -50.562145130091004$$

$$L_b = -43.04832187498447$$

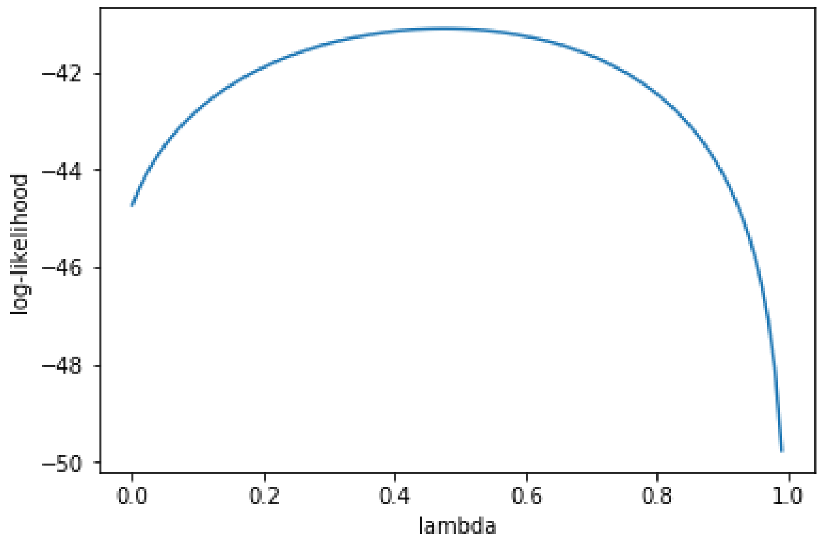
The bigram model yields the highest log-likelihood.

(d)

$$L_u = -44.72653897141283$$

$$L_b = -\infty$$

The pairs of adjacent words that are not observed in the corpus are (RECENT $\rightarrow$ OFFICIALS) and (INCORPORATED $\rightarrow$ PRICES). Since $P(word|evidence) = 0$ when there is no observation in the corpus, the total probability is 0, which makes the log-likelihood approach $-\infty$, because 0 is not in log function domain.



(e) The optimal value of λ is 0.48.

(f)
```
In [1]: words = []
        for line in open("hw4_vocab.txt"):
          words.append(line.split()[0])

        counts = []
        for line in open("hw4_unigram.txt"):
          counts.append(int(line.split()[0]))

        bigram = []
        for line in open("hw4_bigram.txt"):
          bigram.append([int (x) for x in line.split()])

In [2]: unigram = list(zip(words, counts))
```

3.1 (a)

```
In [3]: probabilities = []
        for i in range(0, 499):
            probabilities.append([words[i], counts[i]/sum(counts)])

In [4]: [v for v in probabilities if v[0][0] == 'N' or v[0][0] == 'n']

Out[4]: [['NINETEEN', 0.0028588174836726445],
         ['NOT', 0.0021457733600835156],
         ['NEW', 0.001900350279776441],
         ['NINE', 0.0017613053303564415],
         ['NINETY', 0.0012920576727483372],
         ['NO', 0.0010485914217056834],
         ['NOW', 0.0007126160671875371],
         ['N.', 0.0006647838218553515],
         ['NATIONAL', 0.0005965149408928713],
         ['NEXT', 0.0004287902125662047 3],
         ['NEWS', 0.0004290959671387705],
```

```
 ['NUMBER', 0.0003488781974804209],
 ['NORTH', 0.0002554640604701336],
 ['NEVER', 0.0002425367571420538],
 ['NIGHT', 0.00023650727697105726],
 ['NEARLY', 0.0002124871977502921],
 ['NEAR', 0.00021042029683974764],
 ['NEED', 0.00020837785629500844]]
```

3.2 (b)

```
In [5]: words[35]

Out[5]: 'HAVE'

In [6]: biDist = [[words[v[1]-1],v[2]] for v in bigram if v[0] == 36]

In [7]: s = sum([v[1] for v in biDist])
        biDist = [[v[0],v[1]/s] for v in biDist]

In [8]: biDist = sorted(biDist, key=lambda x: x[1])

In [9]: biDist[-10:]

Out[9]: [['AN', 0.010897736780882022],
         ['HAD', 0.011289565519071004],
         ['SAID', 0.013040550192853207],
         ['NOT', 0.013444623579110063],
         ['NO', 0.015738454317258833],
         ['THE', 0.028934103385645192],
         ['A', 0.06311299769392462],
         ['TO', 0.08213301769351646],
         ['BEEN', 0.17246178649415317],
         ['<UNK>', 0.4116813942572601]]
```

3.3 (c)

```
In [10]: import math

         def probUnigram(word):
             return probabilities[words.index(word)][1]

         def probBigram(word, evidence):
             dist = [[words[v[1]-1],v[2]] for v in bigram if v[0] == words.index(evidence)+1]
             psum = sum([v[1] for v in dist])
             prob = [v[1] for v in dist if v[0] == word]
             if len(prob) == 0:
                 return 0
             else:
                 return prob[0]/psum

In [11]: def logLikelihoods(sentence):
             lu = 1
             lb = 1
             sent = sentence.split()
```

```
        for i in range(0, len(sent)):
            lu = lu * probUnigram(sent[i])
            if i == 0:
                lb = lb * probBigram(sent[i], "<s>")
            else:
                lb = lb * probBigram(sent[i], sent[i-1])
        if lu != 0:
            lu = math.log(lu)
        else:
            lu = "UNDEFINED"

        if lb != 0:
            lb = math.log(lb)
        else:
            lb = "UNDEFINED"
        return lu, lb
```

```
In [12]: lu, lb = logLikelihoods("TEN BILLION DOLLARS DIDN'T LAST VERY LONG")
         print(lu)
         print(lb)
```

```
-50.562145130091004
-43.04832187498447
```

3.4 (d)

```
In [13]: lu, lb = logLikelihoods("THE RECENT OFFICIALS SAID THEY INCORPORATED PRICES")
         print(lu)
         print(lb)
```

```
-44.72653897141283
UNDEFINED
```

```
In [14]: print(probBigram("THE", "<s>"))
         print(probBigram("RECENT", "THE"))
         print(probBigram("OFFICIALS", "RECENT"))
         print(probBigram("SAID", "OFFICIALS"))
         print(probBigram("THEY", "SAID"))
         print(probBigram("INCORPORATED", "THEY"))
         print(probBigram("PRICES", "INCORPORATED"))
```

```
0.15865263383617936
0.0007153649126219887
0
0.22033277138459284
0.020327686493360808
1.1064884482606001e-05
0
```

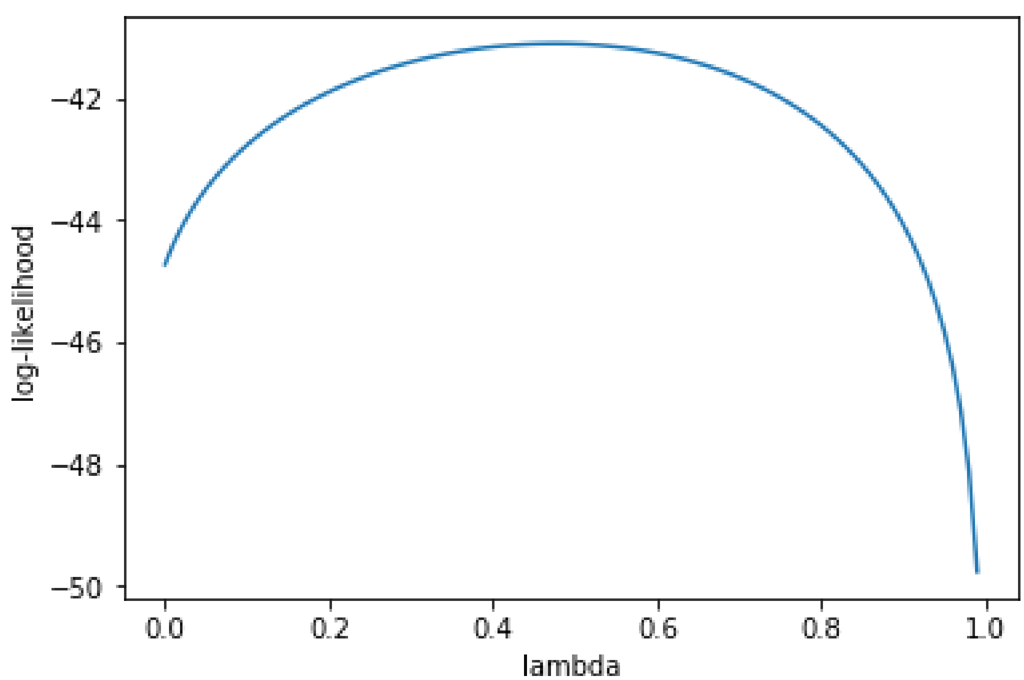
3.5 (e)

In [15]:
```
def mixedProb(word, evidence, l):
    return (1-l)*probUnigram(word) + l*probBigram(word, evidence)

def mixedLL(sentence, l):
    lm = 1
    sent = sentence.split()

    for i in range(0, len(sent)):
        if i == 0:
            lm = lm * mixedProb(sent[i], "<s>", l)
        else:
            lm = lm * mixedProb(sent[i], sent[i-1], l)
    return math.log(lm)
```

In [16]:
```
lambdas = [i/100 for i in range(0,100)]
lls = [mixedLL("THE RECENT OFFICIALS SAID THEY INCORPORATED PRICES", l) for l in lambdas]
```

In [17]:
```
import matplotlib.pyplot as plt
```

In [18]:
```
plt.plot(lambdas, lls)
plt.xlabel('lambda')
plt.ylabel('log-likelihood')
plt.show()
```



In [19]:
```
lambdas[lls.index(max(lls))]
```

Out[19]: 0.48

4 Markov modeling

(a) **Unigram model**

τ	a	b	c	d
$P_1(\tau)$	0.25	0.25	0.25	0.25

(b) **Bigram model**

$P_2(\tau'\|\tau)$	a	b	c	d
a	$\frac{1}{2}$	$\frac{1}{4}$	0	$\frac{1}{4}$
b	0	$\frac{3}{4}$	$\frac{1}{4}$	0
c	0	0	$\frac{2}{3}$	$\frac{1}{3}$
d	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$

(c) **Likelihoods**

$$P_U(S) = P_U(T_1)$$
$$P_U(S) = P_U(T_2)$$
$$P_U(S) = P_U(T_3)$$

$$P_B(T_1) < P_B(S)$$
$$P_B(T_2) < P_B(S)$$
$$P_B(T_3) = P_B(T_2)$$

$$P_U(S) < P_B(S)$$
$$P_U(T_1) = P_B(T_1)$$
$$P_U(T_2) > P_B(T_2)$$
$$P_U(T_3) > P_B(T_3)$$

(d) **Likelihoods**

$$S = D$$
$$T_1 = A$$
$$T_2 = C$$
$$T_3 = B$$