CSE 150 Homework 6

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1 Viterbi Algorithm

```
In [108]: import math
          import matplotlib.pyplot as plt
```

```
In [8]: pi = []
        for l in open('initialStateDistribution.txt'):

            pi.append(float(l.split()[0]))
```

```
In [10]: aij = []
         for l in open('transitionMatrix.txt'):
             numbers = l.split()
             for i in range(0, len(numbers)):
                 numbers[i] = float(numbers[i])
             aij.append(numbers)
```

```
In [13]: bik = []
         for l in open('emissionMatrix.txt'):
             numbers = l.split()
             for i in range(0, len(numbers)):
                 numbers[i] = float(numbers[i])
             bik.append(numbers)
```

```
In [44]: for l in open('observations.txt'):
             obs = l.split()
             for i in range(0, len(obs)):
                 obs[i] = int(obs[i])
```

```
In [25]: li1 = []
         for i in range(0, len(pi)):
             li1.append(math.log(pi[i]*bik[i][1]))
```

```
In [76]: lit = []
         lit.append(li1)
```

```
In [77]: phiit = []
         phiit.append([0]*len(lit[0]))
```

```
In [46]: def computelit(j, t):
             maximize = []
             for x in lit[t-1]:
                 maximize.append(x + math.log(aij[lit[t-1].index(x)][j]))
             maximize = max(maximize)
             return maximize + math.log(bik[j][obs[t]])
```

```
In [78]: for time in range(1, len(obs)):
             actualLit = []
             actualPhiit = []
             for n in range(0, len(li1)):
                 actualLit.append(computelit(n, time))
                 maxlit = []
                 for x in lit[time-1]:
                     maxlit.append(x + math.log(aij[lit[time-1].index(x)][n]))
                 actualPhiit.append(maxlit.index(max(maxlit)))
             lit.append(actualLit)
             phiit.append(actualPhiit)
```

```
In [95]: letters = ['a', 'b', 'c', 'd',
                    'e', 'f', 'g', 'h',
                    'i', 'j', 'k', 'l',
                    'm', 'n', 'o', 'p',
                    'q', 'r', 's', 't',
                    'u', 'v', 'w', 'x',
                    'y', 'z', ' ']
```

```
In [96]: S = ['']*len(phiit)
         S[-1] = lit[-1].index(max(lit[-1]))
         for time in range(len(phiit)-1, 0, -1):
             S[time-1] = phiit[time][S[time]]
```

```
In [97]: for i in range(0, len(S)):
             S[i] = letters[S[i]]
```

```
In [101]: sentence = ""
          sentence += S[0]

          for i in range(0, len(S)):
              if S[i] != sentence[-1]:
                  sentence += S[i]
```

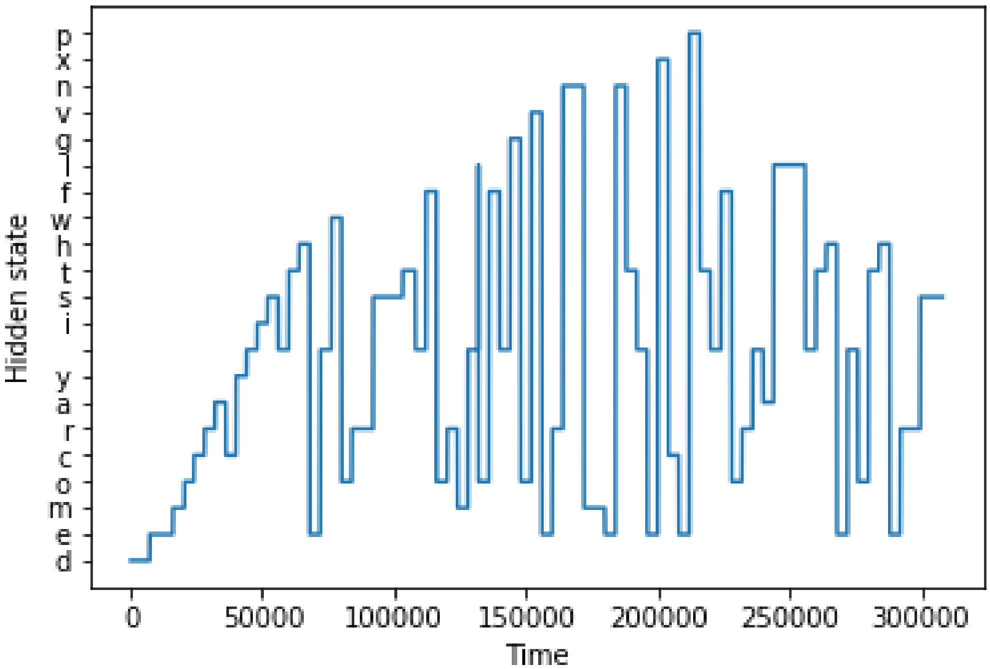
```
In [102]: print(sentence)
```

```
democracy is the worst form lof government except for al the others
```

```
In [104]: times = []
          for i in range(0, len(S)):
              times.append(i)
```

```
In [110]: plt.plot(times, S)
          plt.xlabel('Time')
          plt.ylabel('Hidden state')
```

```
Out[110]: Text(0,0.5,'Hidden state')
```



2 Conditional Independence

$P(S_t|S_{t-1}) = P(S_t|S_{t-1}, O_t)$
False
$P(S_t|S_{t-1}) = P(S_t|S_{t-1}, O_{t-1})$
True
$P(S_t|S_{t-1}) = P(S_t|S_{t-1}, S_{t+1})$
False
$P(S_t|O_{t-1}) = P(S_t|O_1, O_2, ..., O_{t-1})$
False
$P(O_t|S_{t-1}) = P(O_t|S_{t-1}, O_{t-1})$
True
$P(O_t|O_{t-1}) = P(O_t|O_1, O_2, ..., O_{t-1})$
False
$P(O_1, O_2, ..., O_T) = \prod_{t=1}^{T} P(O_t|O_1, O_2, ..., O_{t-1})$
True
$P(S_2, S_3, ..., S_T|S_1) = \prod_{t=2}^{T} P(S_t|S_{t-1})$
True
$P(S_1, S_2, ..., S_{T-1}|S_T) = \prod_{t=1}^{T-1} P(S_t|S_{t+1})$
True
$P(S_1, S_2, ..., S_T|O_1, O_2, ..., O_T) = \prod_{t=1}^{T} P(S_t|O_t)$
False
$P(S_1, S_2, ..., S_T, O_1, O_2, ..., O_T) = \prod_{t=1}^{T} P(S_t, O_t)$
False
$P(O_1, O_2, ..., O_T|S_1, S_2, ..., S_T) = \prod_{t=1}^{T} P(O_t|S_t)$
True

3 More conditional independence

(a)

$$P(S_t|S_{t+1},S_{t+2},...,S_T) = P(S_t|S_{t+1})$$

$$P(S_t|O_t,O_{t-1},O_{t+1}) = P(S_t|O_t,O_{t-1},O_{t+1})$$

$$P(S_t|O_t,O_{t+1},...,O_T) = (S_t|O_t,O_{t+1},...,O_T)$$

$$P(O_t|O_1,O_2,...,O_{t-1}) = P(O_t|O_1,O_2,...,O_{t-1})$$

$$P(O_t|S_{t-2},S_{t-1},S_{t+1},S_{t+2}) = P(O_t|S_{t-1},S_{t+1})$$

$$P(O_t|O_{t-1},O_{t+1},S_1,S_T) = P(O_t|O_{t-1},O_{t+1},S_1,S_T)$$

(b)

$$P(S_t|O_t,O_{t-1},O_{t+1},S_{t-1},S_{t+1}) = P(S_t|O_t,S_{t-1},S_{t+1})$$

$$P(S_t|S_1,S_T,O_1,O_t,O_T) = P(S_t|S_1,S_T,O_t)$$

$$P(O_t|O_1,O_2,...,O_{t-1},S_{t-1}) = P(O_t|S_{t-1})$$

$$P(O_t|O_1,O_2,...,O_{t-1},S_{t-2}) = P(O_t|O_{t-1},S_{t-2})$$

4 Belief updating

(a)

$$\begin{aligned}
P(Y_1|X_1) &= \sum_{x_0} P(Y_1, X_0 = x_0|X_1) \\
&= \sum_{x_0} P(Y_1|X_0 = x_0, X_1) \cdot P(X_0 = x_0|X_1) \\
&= \sum_{x_0} P(Y_1|X_0 = x_0, X_1) \cdot P(X_0 = x_0)
\end{aligned}$$

(b)

$$\begin{aligned}
P(Y_1) &= \sum_{x_0}\sum_{x_1} P(Y_1, X_0 = x_0, X_1 = x_1) \\
&= \sum_{x_0}\sum_{x_1} P(Y_1|X_0 = x_0, X_1 = x_1) \cdot P(X_0 = x_0, X_1 = x_1) \\
&= \sum_{x_0}\sum_{x_1} P(Y_1|X_0 = x_0, X_1 = x_1) \cdot P(X_0 = x_0) \cdot P(X_1 = x_1)
\end{aligned}$$

(c)

$$
\begin{aligned}
&P(X_t|Y_1, Y_2, .., Y_{t-1}) \\
&= \sum_x P(X_t, X_{t-1} = x|Y_1, Y_2, .., Y_{t-1}) && \text{(Marginalization)} \\
&= \sum_x P(X_t|X_{t-1} = x, Y_1, Y_2, .., Y_{t-1}) \cdot P(X_{t-1} = x|Y_1, Y_2, .., Y_{t-1}) && \text{(Prod. rule)} \\
&= \sum_x P(X_t|X_{t-1} = x, Y_{t-1}) \cdot P(X_{t-1} = x|Y_1, Y_2, .., Y_{t-1}) && \text{(C.I.)} \\
&= \sum_x \frac{P(Y_{t-1}|X_t, X_{t-1} = x) \cdot P(X_t|X_{t-1} = x)}{P(Y_{t-1}|X_{t-1} = x)} \cdot P(X_{t-1} = x|Y_1, Y_2, .., Y_{t-1}) && \text{(Bayes' rule)} \\
&= \sum_x \frac{P(Y_{t-1}|X_t, X_{t-1} = x) \cdot P(X_t)}{\sum_{x'} P(Y_{t-1}, X_t = x'|X_{t-1} = x)} \cdot P(X_{t-1} = x|Y_1, Y_2, .., Y_{t-1}) && \text{(C.I. and Marginalization)} \\
&= \sum_x \frac{P(Y_{t-1}|X_t, X_{t-1} = x) \cdot P(X_t)}{\sum_{x'} P(Y_{t-1}|X_t = x', X_{t-1} = x) \cdot P(X_t = x'|X_{t-1} = x)} \cdot P(X_{t-1} = x|Y_1, Y_2, .., Y_{t-1}) && \text{(Prod. Rule)} \\
&= \sum_x \frac{P(Y_{t-1}|X_t, X_{t-1} = x) \cdot P(X_t)}{\sum_{x'} P(Y_{t-1}|X_t = x', X_{t-1} = x) \cdot P(X_t = x')} \cdot P(X_{t-1} = x|Y_1, Y_2, .., Y_{t-1}) && \text{(C.I.)}
\end{aligned}
$$

(d)

$$
\begin{aligned}
P(Y_t|X_t, Y_1, ..., Y_{t-1}) &= \sum_x P(Y_t, X_{t-1} = x|X_t, Y_1, ..., Y_{t-1}) && \text{(Marginalization)} \\
&= \sum_x P(Y_t|X_{t-1} = x, X_t, Y_1, ..., Y_{t-1}) \cdot P(X_{t-1} = x|X_t, Y_1, ..., Y_{t-1}) && \text{(Prod. Rule)} \\
&= \sum_x P(Y_t|X_{t-1} = x, X_t) \cdot P(X_{t-1} = x|Y_1, ..., Y_{t-1}) && \text{(C.I.)}
\end{aligned}
$$

(e)

$$
\begin{aligned}
P(Y_t|Y_1, ..., Y_{t-1}) &= \sum_{x,x'} P(Y_t, X_{t-1} = x, X_t = x'|, Y_1, ..., Y_{t-1}) && \text{(Marginalization)} \\
&= \sum_{x,x'} P(Y_t|X_{t-1} = x, X_t = x', Y_1, ..., Y_{t-1}) \cdot P(X_{t-1} = x, X_t = x'|Y_1, ..., Y_{t-1}) && \text{(Prod. Rule)} \\
&= \sum_{x,x'} P(Y_t|X_{t-1} = x, X_t = x') \cdot P(X_{t-1} = x) \cdot P(X_t = x') && \text{(C.I.)}
\end{aligned}
$$

5 Most likely hidden states

(a) No.

(b) Yes.

(c) No.

(d) No.