$$\frac{dP(t)}{dt} = \mu N - \beta P(t)\frac{S(t)}{N} - \mu P(t)$$

$$\frac{dS(t)}{dt} = \beta P(t)\frac{S(t)}{N} - (\mu+\gamma)S(t)$$

$$\frac{dQ(t)}{dt} = \gamma S(t) - \mu Q(t)$$

Divide by N:

$$\frac{1}{N}\frac{dP(t)}{dt} = \frac{dx(t)}{dt} = \mu - \beta x(t)y(t) - \mu x(t)$$

$$\frac{1}{N}\frac{dS(t)}{dt} = \frac{dy(t)}{dt} = \beta x(t)y(t) - (\mu+\gamma)y(t)$$

$$\frac{1}{N}\frac{dQ(t)}{dt} = \frac{dz(t)}{dt} = \gamma y(t) - \mu z(t)$$

where $x(t) = \frac{P(t)}{N}$, $y(t) = \frac{S(t)}{N}$, $z(t) = \frac{Q(t)}{N}$

Changing time variable: $\tau = \mu t \Rightarrow d\tau = \mu dt$

$\Rightarrow dt = \frac{d\tau}{\mu}$:

$$\frac{dx(\tau)}{d\tau} = 1 - \frac{\beta}{\mu}x(\tau)y(\tau) - x(\tau)$$

$$\frac{dy(\tau)}{d\tau} = \frac{\beta}{\mu}x(\tau)y(\tau) - \left(1+\frac{\gamma}{\mu}\right)y(\tau)$$

$$\frac{dz(\tau)}{d\tau} = \frac{\gamma}{\mu}y(\tau) - z(\tau)$$

$$\frac{dP(t)}{dt} = \mu N - \beta P(t)\frac{S(t)}{N} - \alpha P(t)\frac{L(t)}{N} - \mu_P P(t) + \delta Q(t)$$

$$\frac{dL(t)}{dt} = \alpha P(t)\frac{L(t)}{N} + \beta P(t)\frac{S(t)}{N} - (\mu_L + k + \xi)L(t)$$

$$\frac{dS(t)}{dt} = \xi L(t) - (\mu_S + \gamma)S(t)$$

$$\frac{dQ(t)}{dt} = kL(t) + \gamma S(t) - (\mu_Q + \delta)Q(t)$$

Dividing by N:

$$\frac{dx(t)}{dt} = \mu - \beta x(t)y(t) - \alpha x(t)l(t) - \mu_P x(t) + \delta z(t)$$

$$\frac{dl(t)}{dt} = \alpha x(t)l(t) + \beta x(t)y(t) - (\mu_L + k + \xi)l(t)$$

$$\frac{dz(t)}{dt} = kl(t) + \gamma y(t) - (\mu_Q + \delta)z(t)$$

$$\frac{dy(t)}{dt} = \xi l(t) - (\mu_S + \gamma)y(t)$$