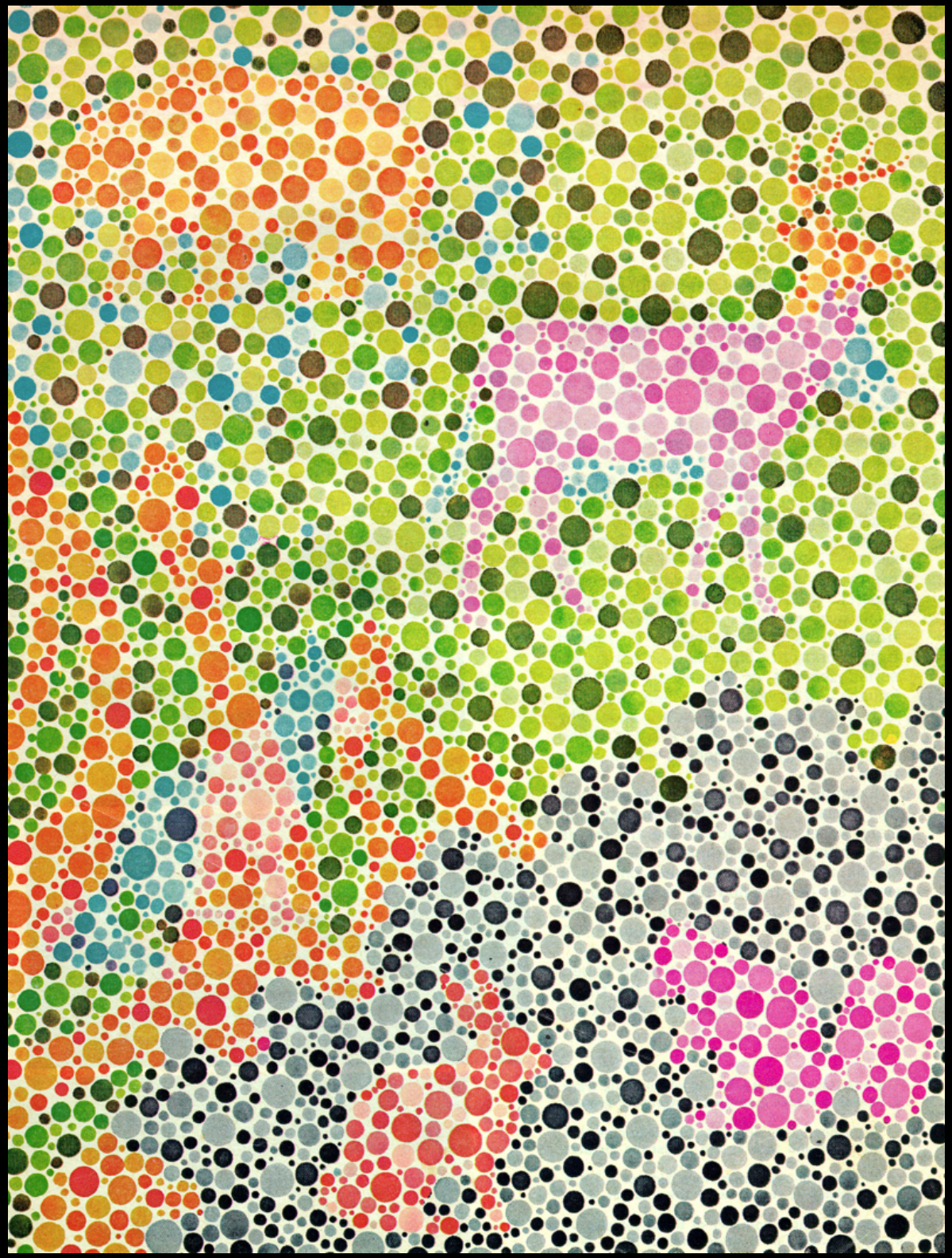
Statistical Rethinking

Week 5: Interactions

Richard McElreath



Fly

Armstrong Whitworth Whitley Mk V

British Medium Bomber

Plastic model kit
Plastik-Modellbausatz
Plastikový model

1/72

Manatees and bombers

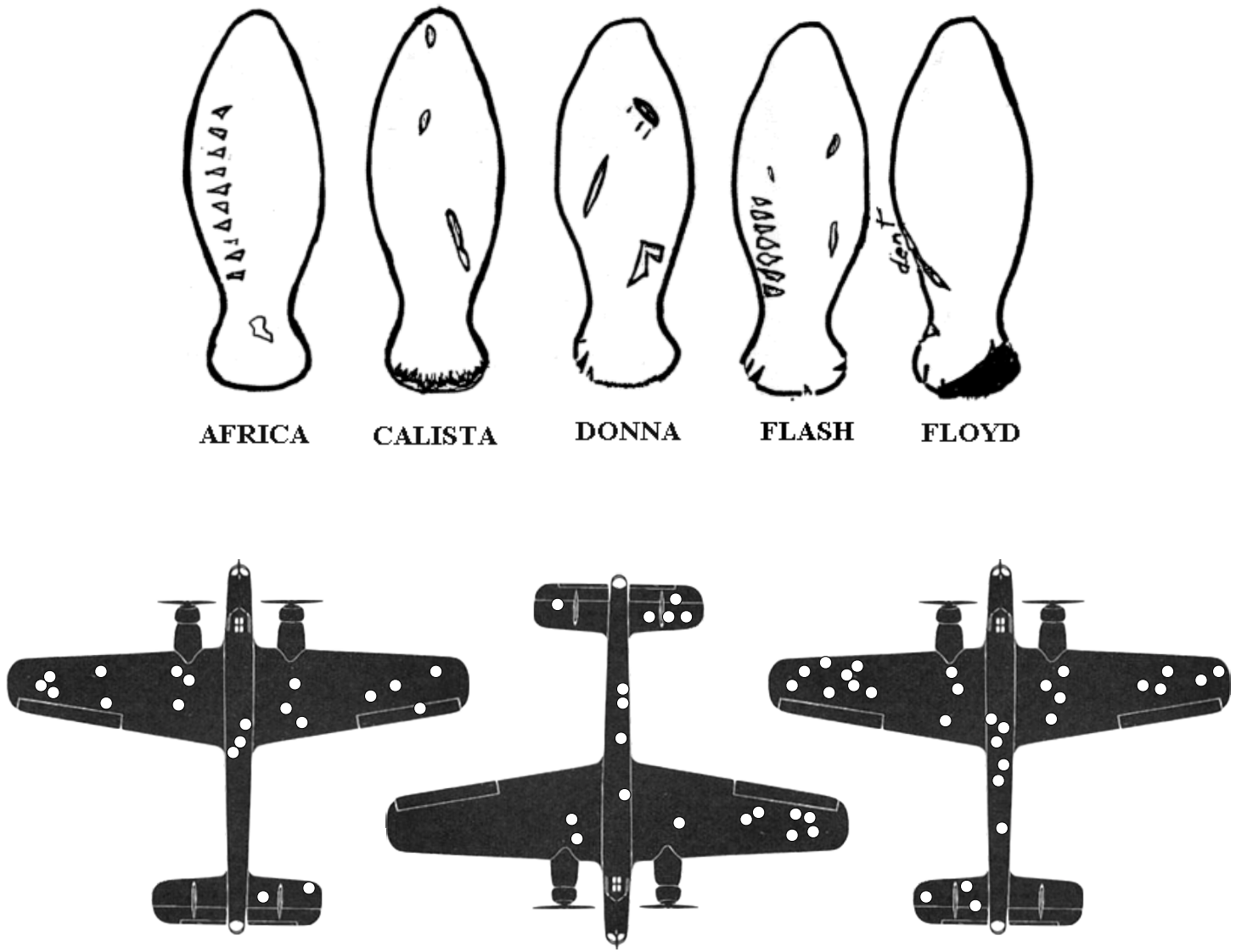


Figure 7.1

Manatees and bombers

- *Conditioning*: Dependence on state
- Everything is conditional
  - On data
  - On model
  - On information state
- *Interactions*: Association of predictor conditional on other predictor(s)

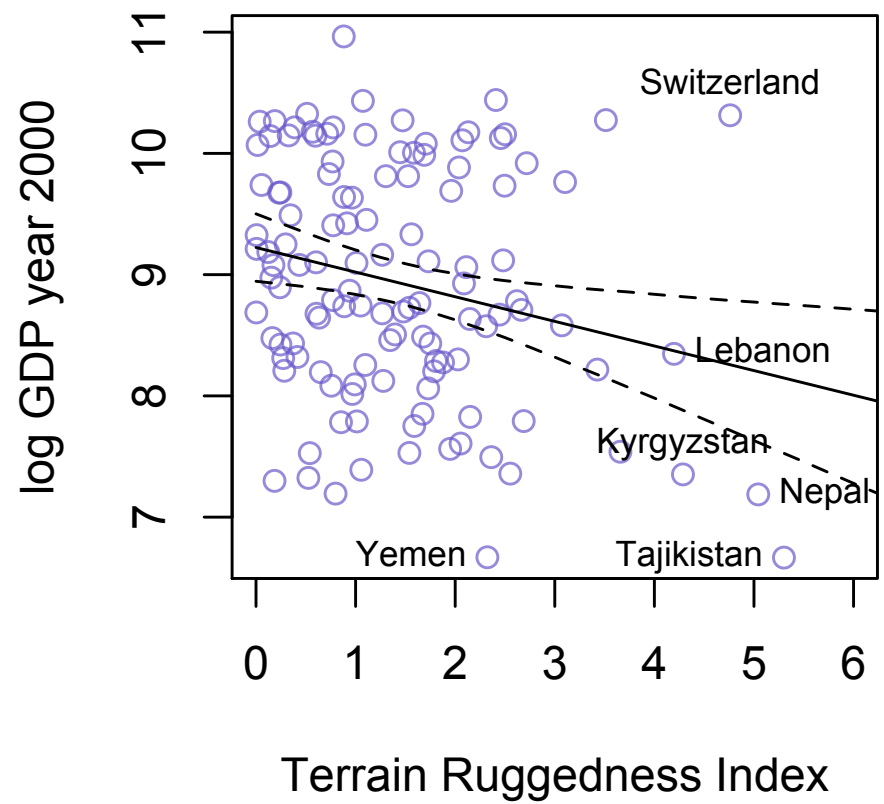
Interaction effects

- *Interactions*: Influence of predictor conditional on other predictor(s)
  - Influence of *sugar* in *coffee* depends on *stirring*
  - Influence of *gene* on *phenotype* depends on *environment*
  - Influence of *skin color* on *cancer* depends on *latitude*
- Generalized linear models (GLMs): All predictors interact to some degree
- Multilevel models: Massive interaction engines

The value of being rugged

```
library(rethinking)
data(rugged)
d <- rugged
```

- Economic indicators and terrain ruggedness for 234 countries



The value of being rugged

- Split data into Africa and non-Africa:

```
# split countries into Africa and not-Africa
d.A1 <- dd[ dd$cont_africa==1 , ] # Africa
d.A0 <- dd[ dd$cont_africa==0 , ] # not Africa
```

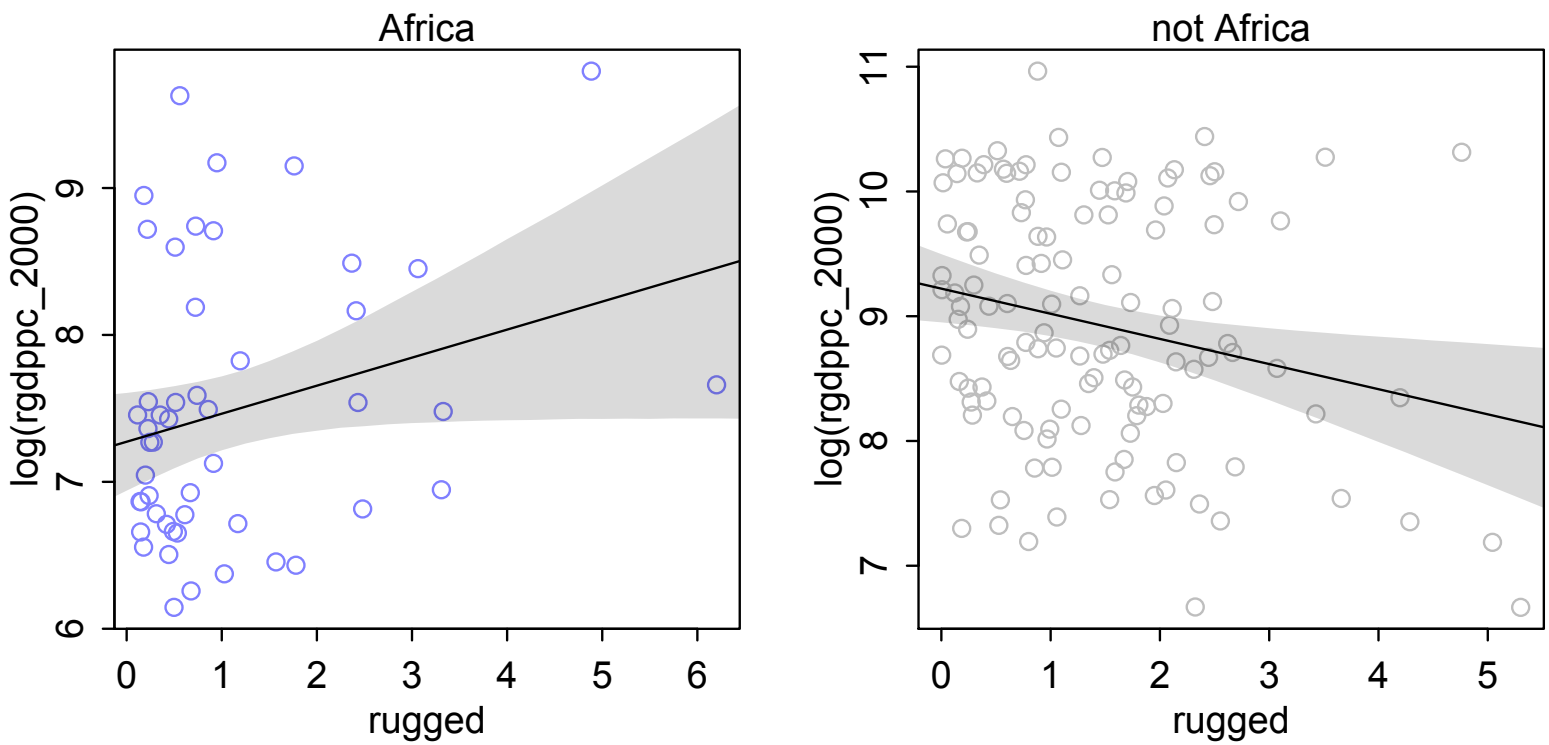
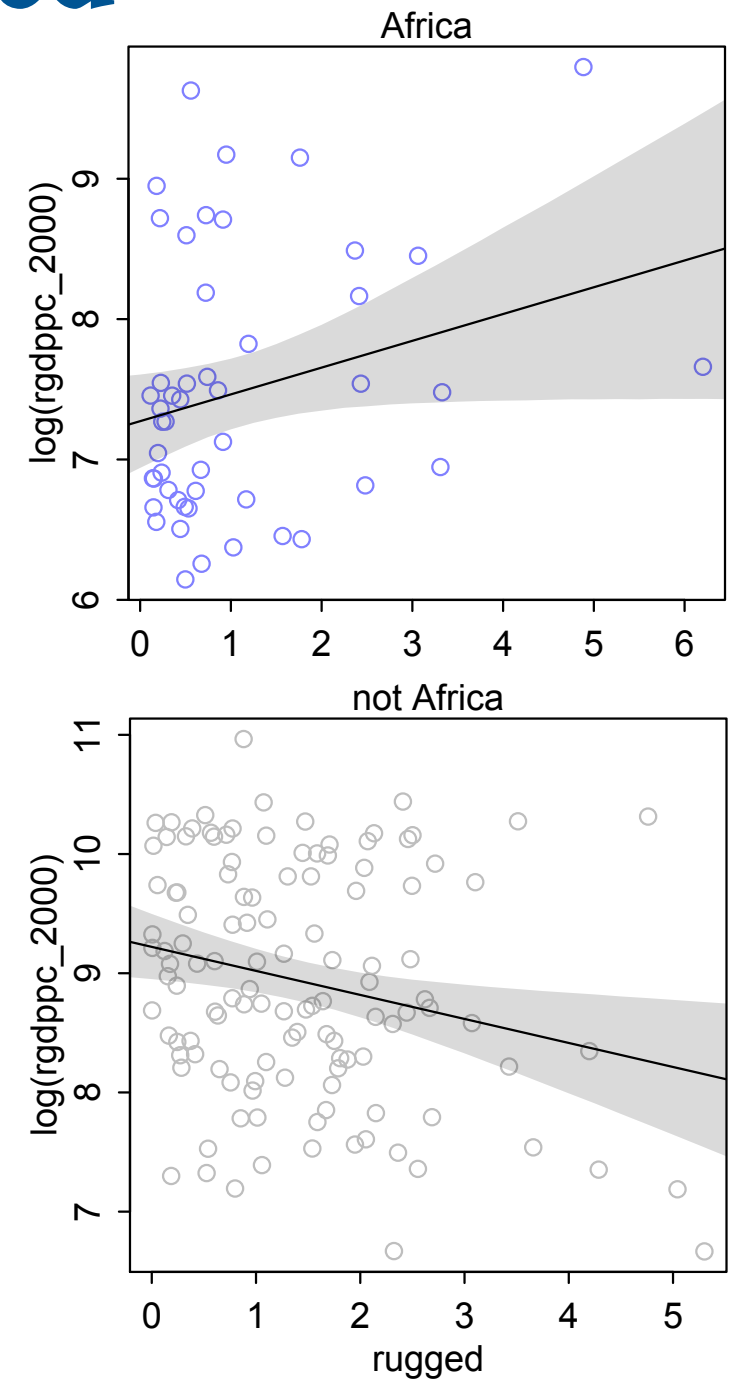


Figure 7.2

The value of being rugged

- Splitting the data is a bad idea:
  - No estimates re how you split the data
  - Does not pool information
- How about adding a categorical variable for Africa?



Dummy doesn't work

- Dummy variable for Africa:

$$Y_i \sim \text{Normal}(\mu_i, \sigma)$$
$$\mu_i = \alpha + \beta_R R_i + \beta_A A_i$$

R code 7.4

```
m7.4 <- map(
    alist(
        log_gdp ~ dnorm( mu , sigma ) ,
        mu <- a + br*rugged + bA*cont_africa ,
        a ~ dnorm( 8 , 100 ) ,
        br ~ dnorm( 0 , 1 ) ,
        bA ~ dnorm( 0 , 1 ) ,
        sigma ~ dunif( 0 , 10 )
    ) ,
    data=dd )
```

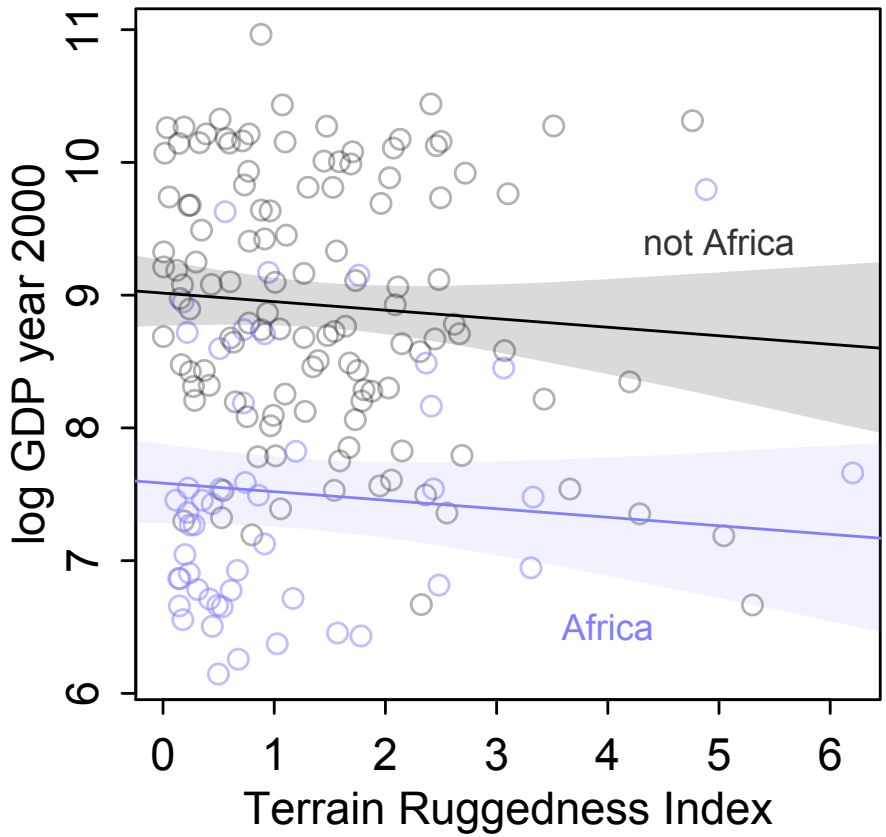


Figure 7.3

Interaction

- Need to allow effect of rugged to depend upon continent

$$Y_i \sim \text{Normal}(\mu_i, \sigma)$$
$$\mu_i = \alpha + \beta_R R_i + \beta_A A_i$$

Interaction

- Need to allow effect of rugged to depend upon continent

$$
\begin{aligned}
Y_i &\sim \text{Normal}(\mu_i, \sigma) \\
\mu_i &= \alpha + \gamma_i R_i + \beta_{\text{A}} A_i \\
\gamma_i &= \beta_R + \beta_{AR} A_i
\end{aligned}
$$

old direct effect of `rugged`

linear effect of Africa on slope

Interaction

- Need to allow effect of rugged to depend upon continent

$$
\begin{aligned}
y_i &\sim \text{Normal}(\mu_i, \sigma) \\
\mu_i &= \alpha + \gamma_i r_i + \beta_{\text{A}} A_i = \alpha + \beta_r r_i + \beta_{Ar} A_i r_i + \beta_{\text{A}} A_i \\
\gamma_i &= \beta_r + \beta_{Ar} A_i
\end{aligned}
$$

Interaction

R code 7.7

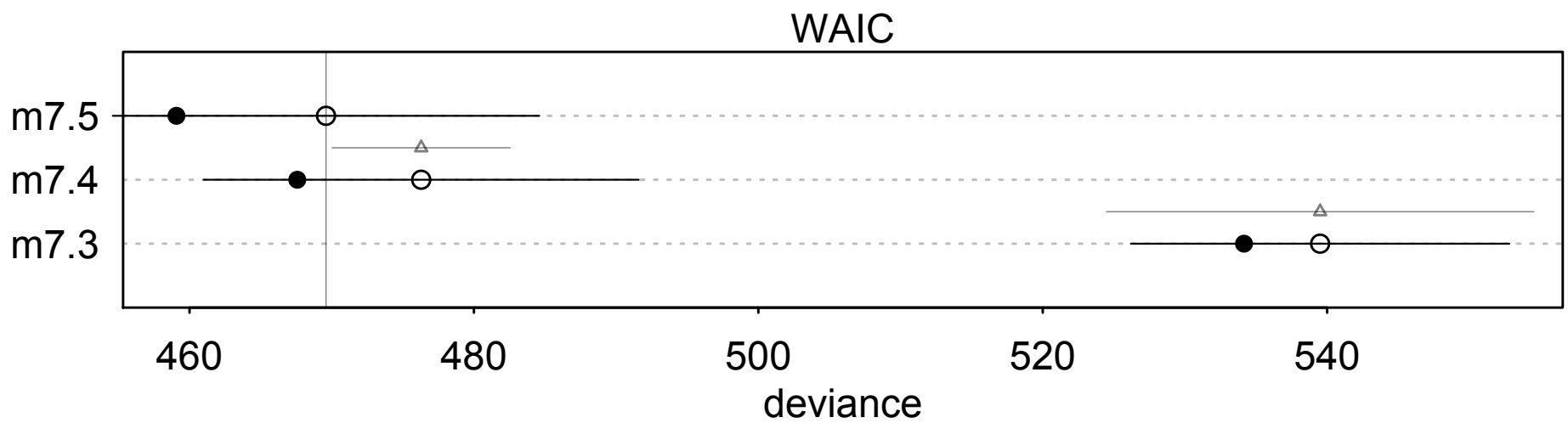
```
m7.5 <- map(
    alist(
        log_gdp ~ dnorm( mu , sigma ) ,
        mu <- a + gamma*rugged + bA*cont_africa ,
        gamma <- br + bAr*cont_africa ,
        a ~ dnorm( 8 , 100 ) ,
        bA ~ dnorm( 0 , 1 ) ,
        br ~ dnorm( 0 , 1 ) ,
        bAr ~ dnorm( 0 , 1 ) ,
        sigma ~ dunif( 0 , 10 )
    ) ,
    data=dd )
```

$$Y_i \sim \text{Normal}(\mu_i, \sigma)$$
$$\mu_i = \alpha + \gamma_i R_i + \beta_A A_i$$
$$\gamma_i = \beta_R + \beta_{AR} A_i$$

```
compare( m7.3 , m7.4 , m7.5 )
```

R code 7.8

```
      WAIC pWAIC dWAIC weight    SE   dSE
m7.5 469.6   5.3   0.0   0.97 15.13    NA
m7.4 476.4   4.4   6.8   0.03 15.35  6.22
m7.3 539.7   2.8  70.1   0.00 13.31 15.22
```



Interpreting interactions

- Is hard
  - Add interaction => other parameters change meaning
  - Influence of predictor depends upon multiple parameters and their covariation

R code 7.12

```
precis(m7.5)
```

```
       Mean StdDev  2.5% 97.5%
a      9.18   0.14  8.92  9.45
br    -0.18   0.08 -0.33 -0.04
bA    -1.85   0.22 -2.27 -1.42
bAr    0.35   0.13  0.10  0.60
sigma  0.93   0.05  0.83  1.03
```

William Thomson's (Lord Kelvin's) tide-prediction engine (1876)

Interpreting interactions

R code 7.12

```
precis(m7.5)
```

```
       Mean StdDev  2.5% 97.5%
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bA    -1.85   0.22 -2.27 -1.42
bAr    0.35   0.13  0.10  0.60
sigma  0.93   0.05  0.83  1.03
```

$$y_i \sim \text{Normal}(\mu_i, \sigma)$$
$$\mu_i = \alpha + \gamma_i r_i + \beta_A A_i$$
$$\gamma_i = \beta_r + \beta_{Ar} A_i$$

Where's gamma?

In Africa: $\gamma = \beta_r + \beta_{Ar}(1) = -0.2 + 0.39 = 0.19$

Outside Africa: $\gamma = \beta_r + \beta_{Ar}(0) = -0.2$

Interpreting interactions

- Need uncertainty as well
  - Sample from posterior
  - Compute posterior distribution of gamma

```
post <- extract.samples( m7.5 )
gamma.Africa <- post$br + post$bAr*1
gamma.notAfrica <- post$br + post$bAr*0
```

R code 7.13

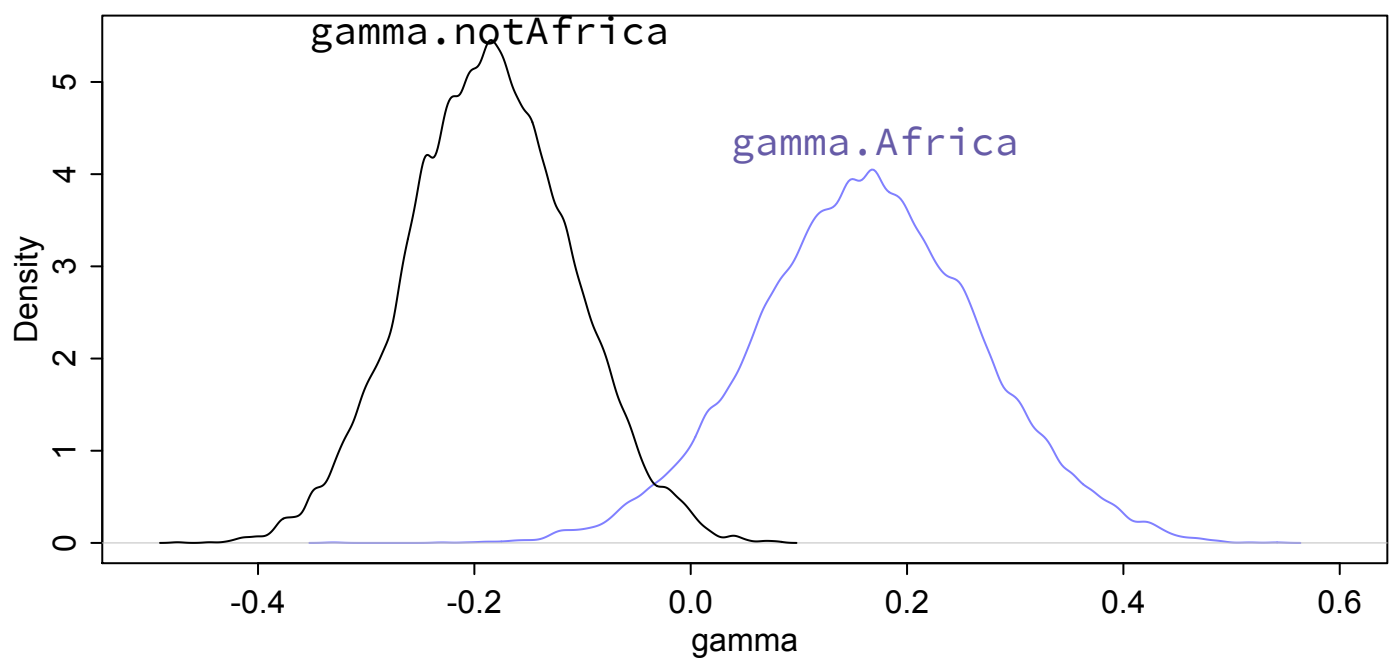


Figure 7.5

Interpreting interactions

R code 7.16

```
diff <- gamma.Africa - gamma.notAfrica
sum( diff < 0 ) / length( diff )
```

```
[1] 0.0036
```

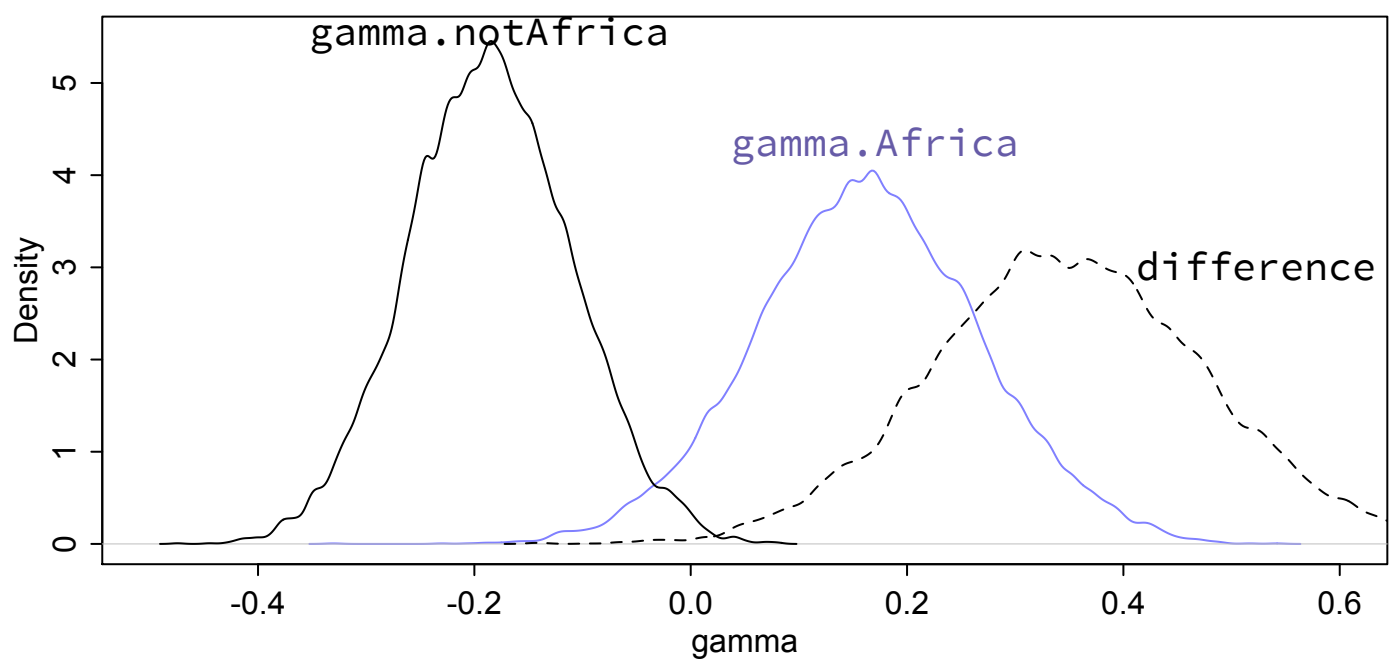


Figure 7.5

Plotting interaction

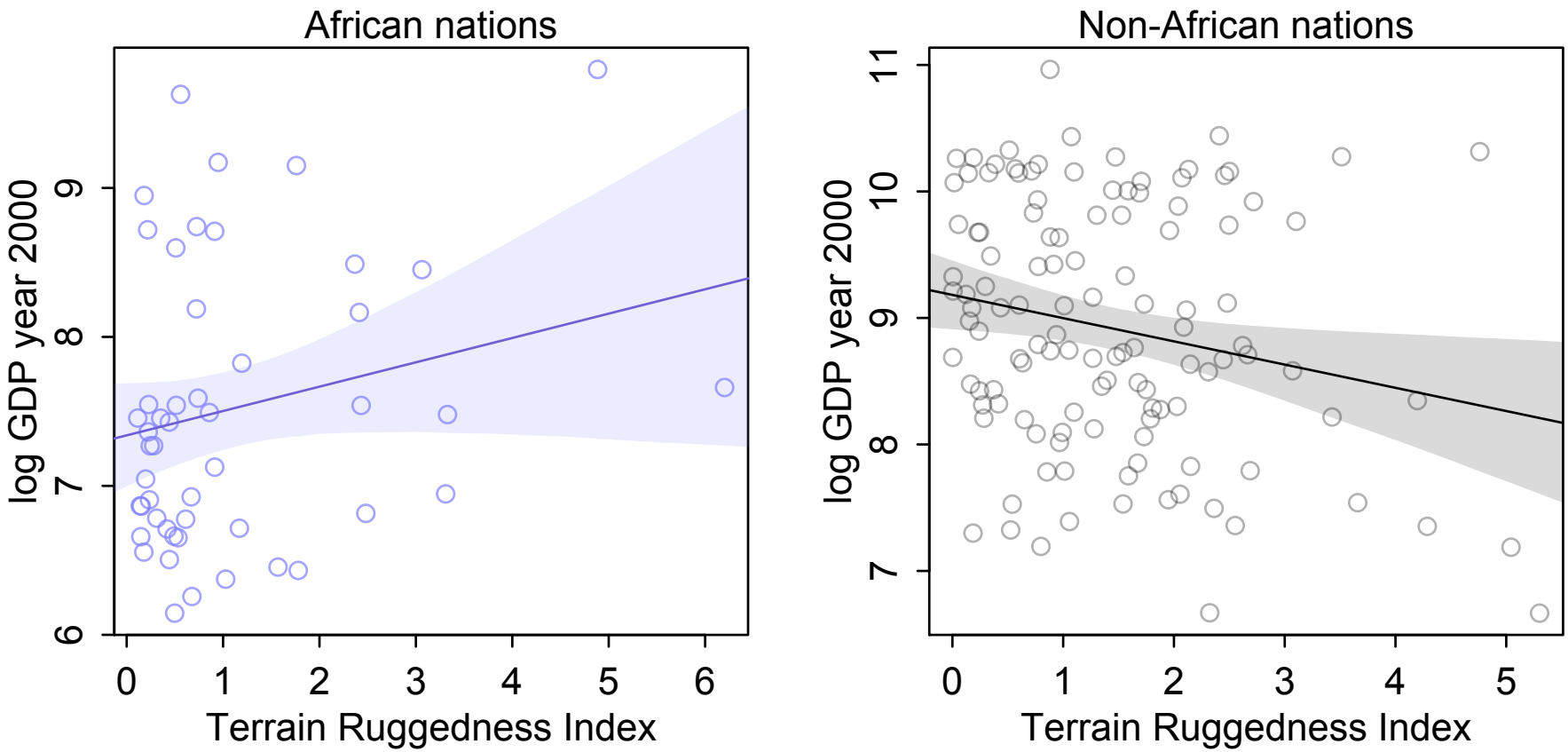
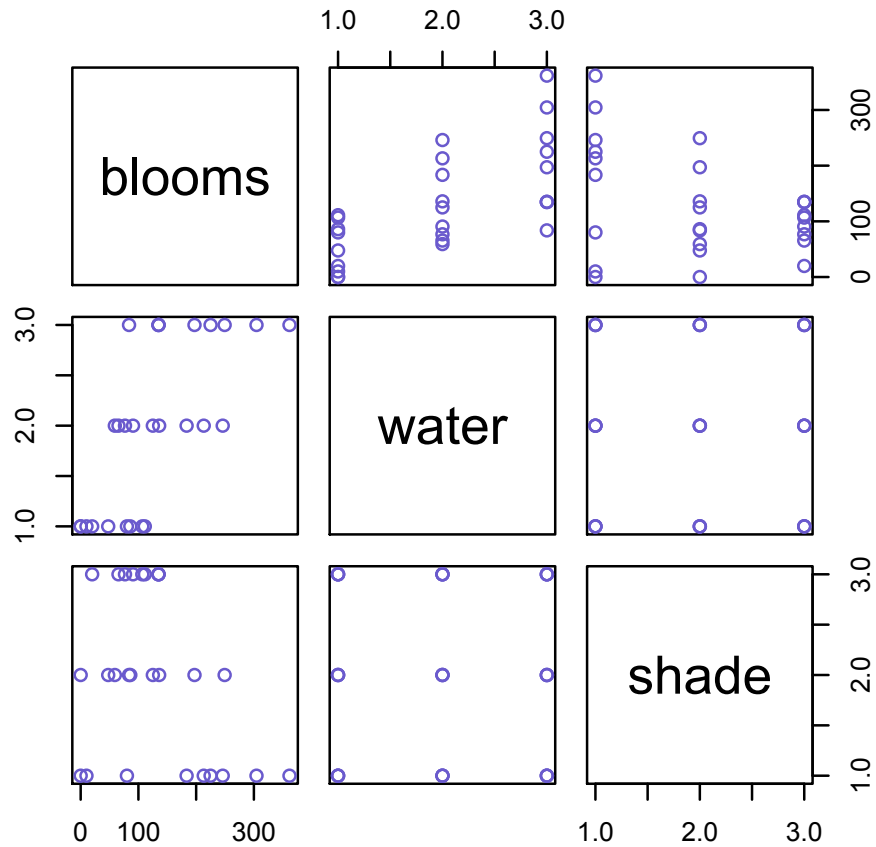


Figure 7.4

Tulip blooms

- 27 replicate blooms across three levels of both water and shade



Tulip blooms

No interaction:
water and shade have independent effects

$$B_i \sim \text{Normal}(\mu_i, \sigma)$$
$$\mu_i = \alpha + \beta_W W_i + \beta_S S_i$$

Interaction:
water and shade have interdependent effects

$$B_i \sim \text{Normal}(\mu_i, \sigma)$$
$$\mu_i = \alpha + \beta_W W_i + \beta_S S_i + \beta_{WS} W_i S_i$$

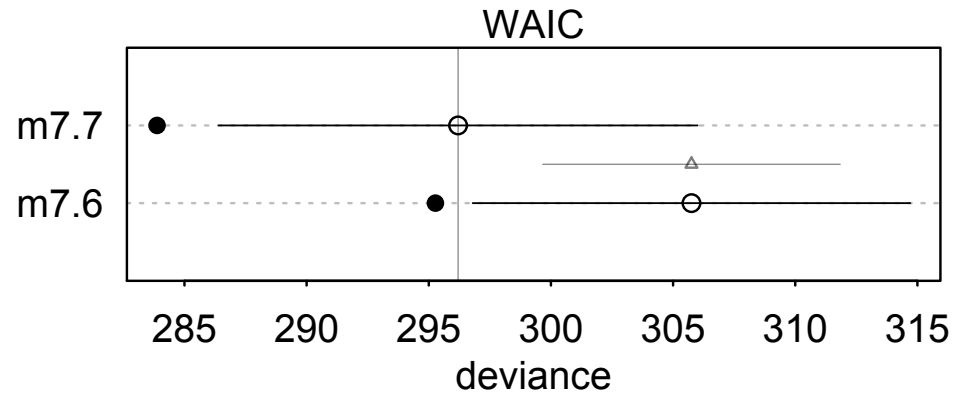
Tulip blooms

- Estimates gone wild!

R code 7.22

```
compare( m7.6 , m7.7 )
```

```
       WAIC pWAIC dWAIC weight   SE  dSE
m7.7 296.3  6.22  0.00   0.99 5.05   NA
m7.6 306.3  5.51 10.01   0.01 4.64 3.02
```



R code 7.21

```
coeftab(m7.6,m7.7)
```

```
       m7.6    m7.7
a       53.46  -84.47
bw      76.36  151.16
bs     -38.92   35.13
sigma   57.40   46.25
bws        NA  -39.67
nobs       27      27
```

Intercept completely different

Influence of shade changes direction?

Interaction negative?

Plotting interaction

- Slope changes with values of other predictor, so use more than one plot
- Here, need three plots, *triptych*

trip•tych | ˈtriptik |

noun

a picture or relief carving on three panels, typically hinged together side by side and used as an altarpiece.

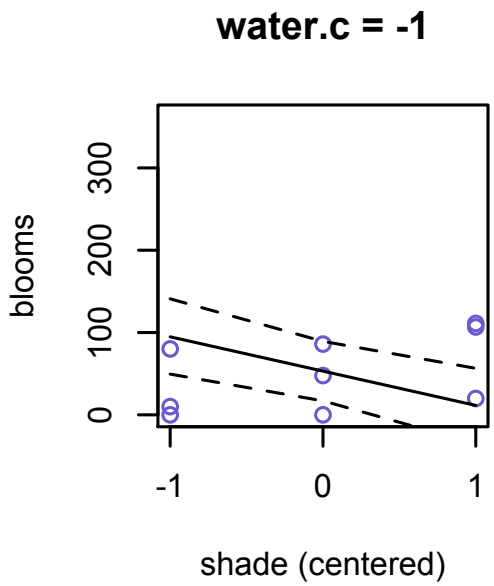
• a set of three associated artistic, literary, or musical works intended to be appreciated together.

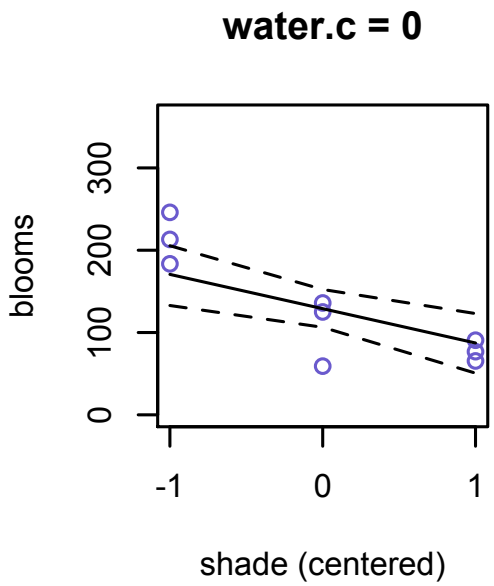
ORIGIN mid 18th cent. (denoting a set of three writing tablets hinged or tied together): from TRI- ***'three,'*** on the pattern of *diptych* .

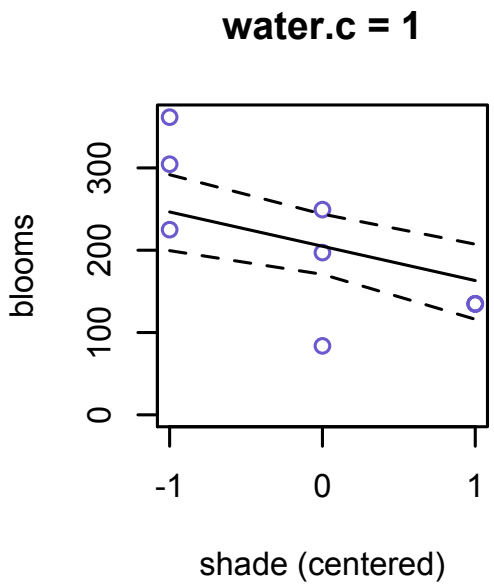
Lewis Powell (1844–1865), before his hanging for conspiracy to assassinate Abraham Lincoln.

No Interaction

slope constant

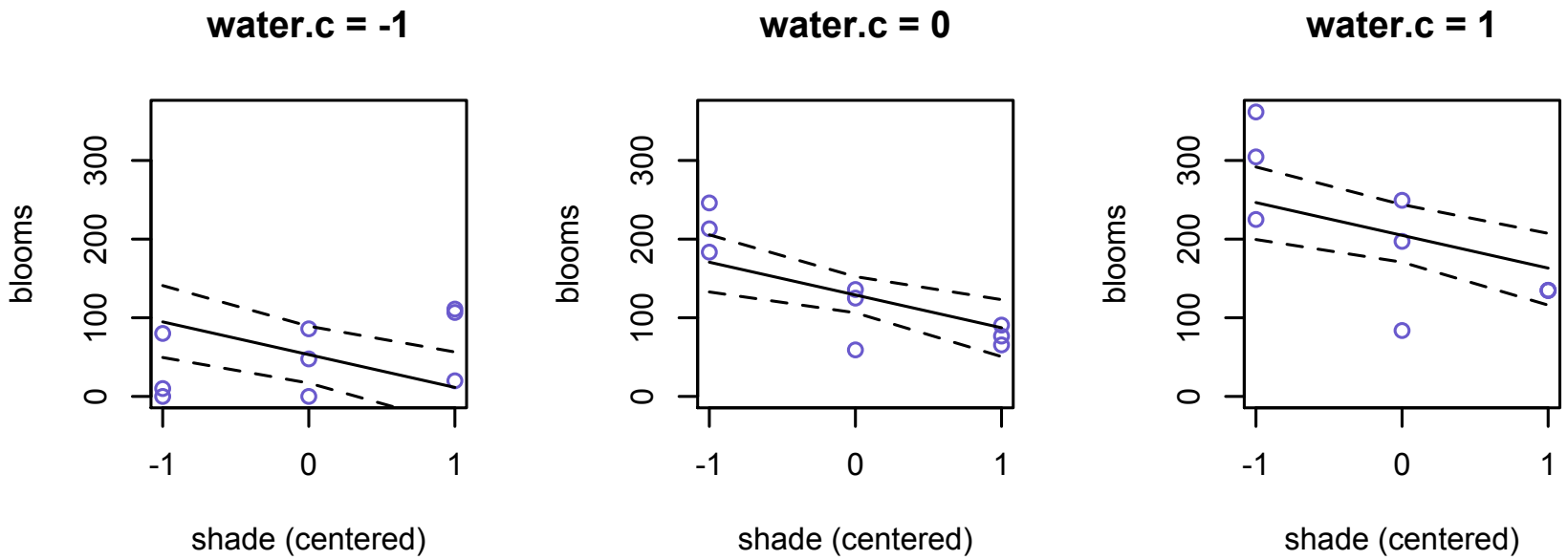






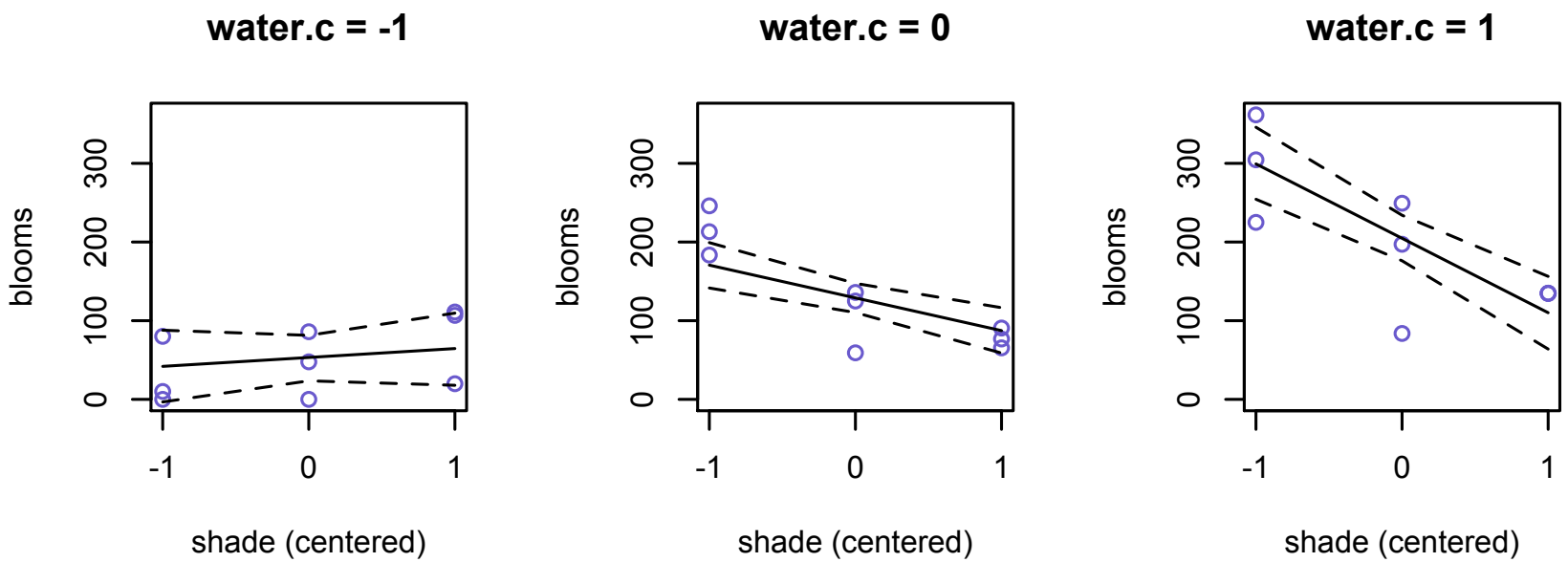
No Interaction

slope constant

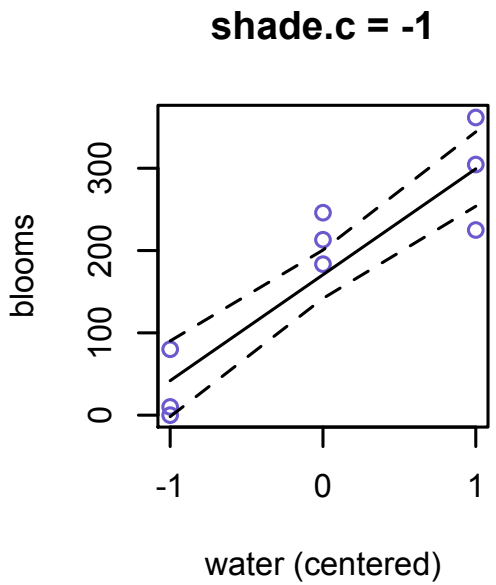


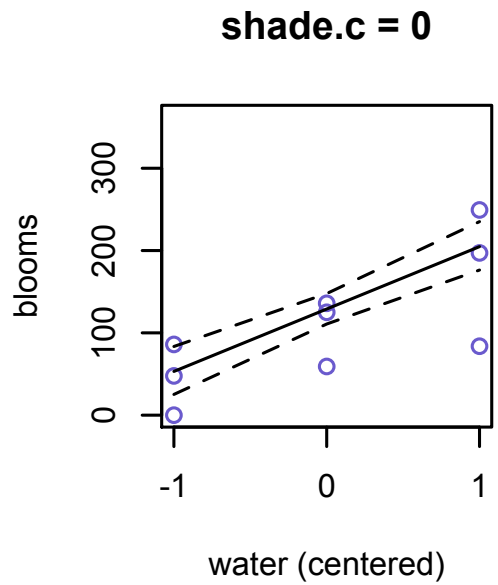
Interaction

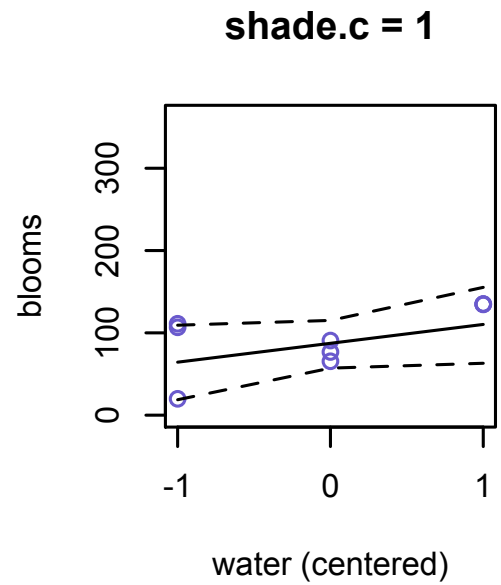
slope varies



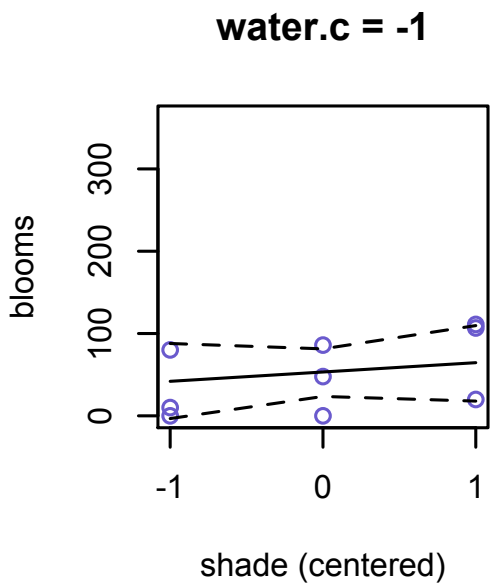
Water depends on Shade

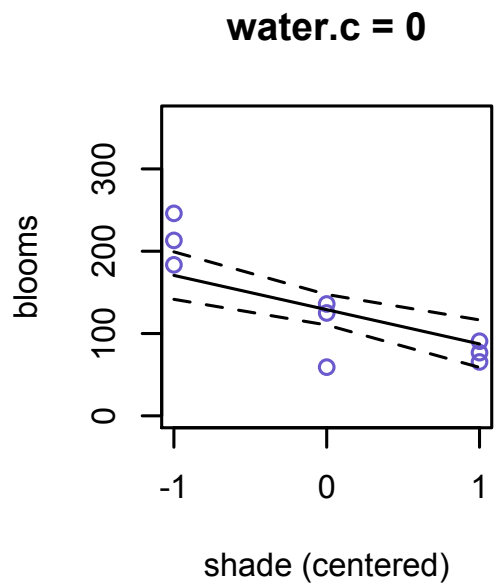


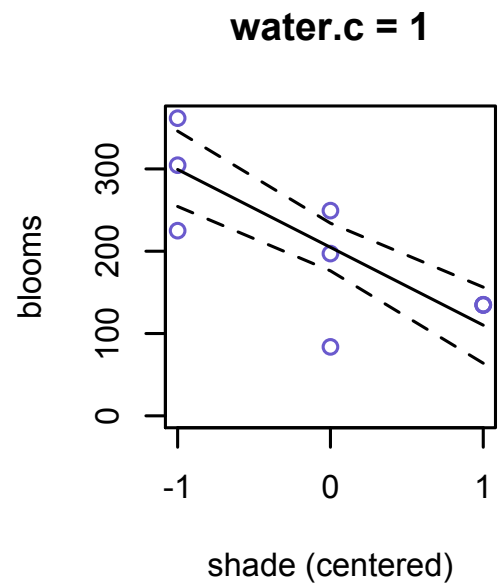




Shade depends on Water







Interactions not always linear

- Suppose all tulip data collected under “cool” temperatures
- Under “hot” temperature, tulips do not bloom
- Interaction, but not a linear one
  - blooms goes to zero at threshold

Higher order interactions

- Just keep multiplying:

$$
\begin{aligned}
y_i &\sim \text{Normal}(\mu_i, \sigma), \\
\mu_i &= \alpha + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} \\
&\quad + \beta_{12} x_{1i} x_{2i} + \beta_{13} x_{1i} x_{3i} + \beta_{23} x_{2i} x_{3i} \\
&\quad + \beta_{123} x_{1i} x_{2i} x_{3i}.
\end{aligned}
$$

Higher order interactions

- Just keep multiplying:

$$
\begin{aligned}
y_i &\sim \text{Normal}(\mu_i, \sigma), \\
\mu_i &= \alpha + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} \\
&\quad + \beta_{12} x_{1i} x_{2i} + \beta_{13} x_{1i} x_{3i} + \beta_{23} x_{2i} x_{3i} \\
&\quad + \beta_{123} x_{1i} x_{2i} x_{3i}.
\end{aligned}
$$

main effects

Higher order interactions

- Just keep multiplying:

$$
\begin{aligned}
y_i &\sim \text{Normal}(\mu_i, \sigma), \\
\mu_i &= \alpha + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} \\
&\quad + \beta_{12} x_{1i} x_{2i} + \beta_{13} x_{1i} x_{3i} + \beta_{23} x_{2i} x_{3i} \\
&\quad + \beta_{123} x_{1i} x_{2i} x_{3i}.
\end{aligned}
$$

main effects

2-way interactions

Higher order interactions

- Just keep multiplying:

$$
\begin{aligned}
y_i &\sim \text{Normal}(\mu_i, \sigma), \\
\mu_i &= \alpha + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} \\
&\quad + \beta_{12} x_{1i} x_{2i} + \beta_{13} x_{1i} x_{3i} + \beta_{23} x_{2i} x_{3i} \\
&\quad + \beta_{123} x_{1i} x_{2i} x_{3i}.
\end{aligned}
$$

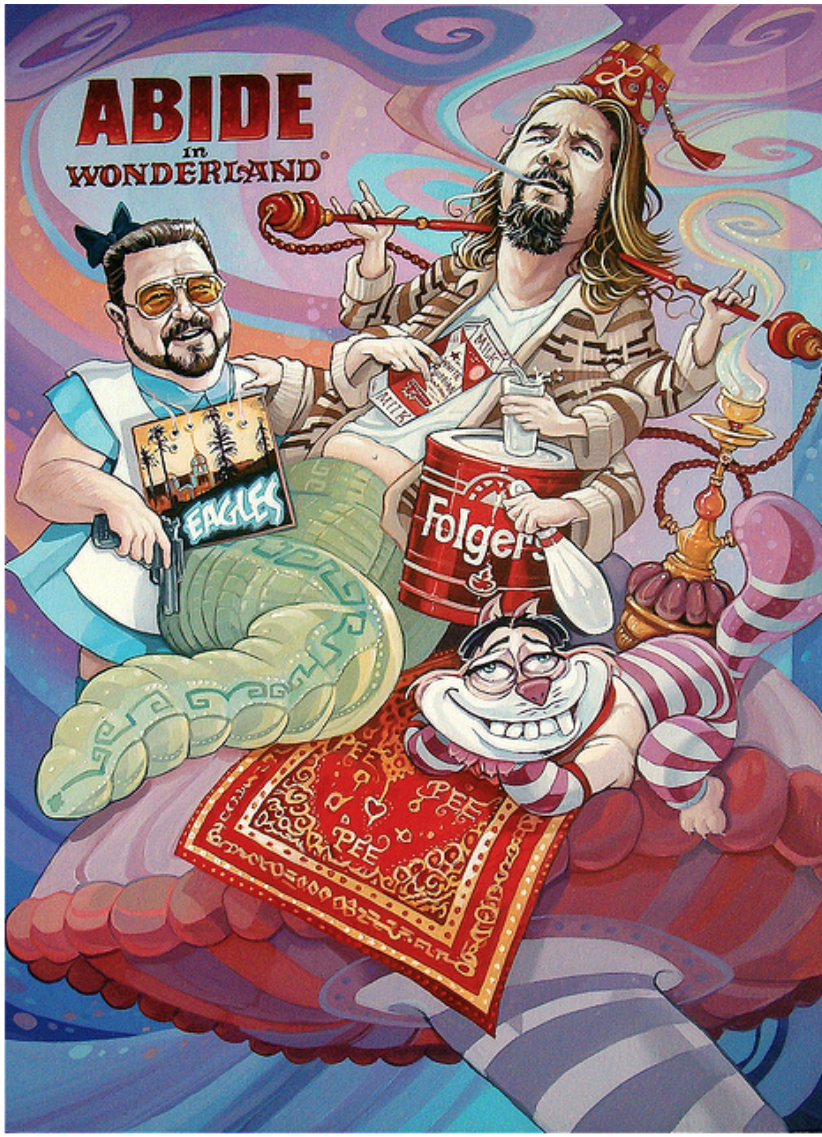
main effects

2-way interactions

3-way interaction

Higher order interactions

- Dangers of high-order interactions
  - Hard to interpret: "The extent to which the effect of x_1 depends upon the value of x_2 depends upon the value of x_3, dude."
  - Hard to estimate: need lots of data, risk multicollinearity --> regularize
  - But you might really need them, because conditionality runs deep



The Dude abides high-order interactions

Higher order interactions

- `data(Wines2012)`
- Judgment of Princeton, 2012
  - New Jersey wines vs fine French wines
- Outcome variable: score
- Predictors:
  - region (NJ/FR)
  - nationality of judge (USA/FR-BE)
  - flight (red/white)

Higher order interactions

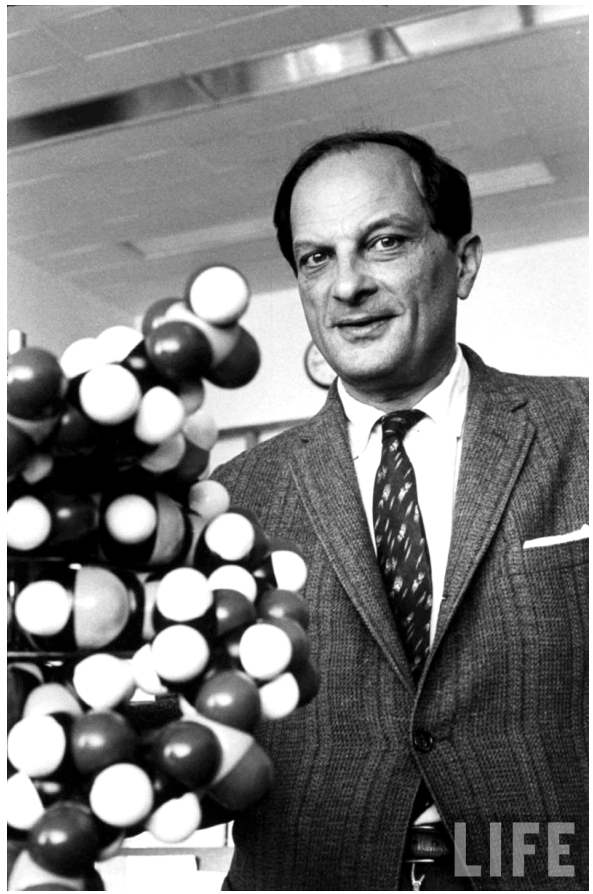
- Predictors: region, nationality of judge, flight
- Consider interactions:
  - Interaction of **region** and **judge** is bias.
    Bias depends upon **flight**.
  - Interaction of **judge** and **flight** is preference.
    Preference depends upon **region**.
  - Interaction of **region** and **flight** is comparative advantage.
    Advantage depends upon **judge**.

Interaction everywhere

- Homework: data(Wines2012)
  - Answer the question: “What predicts score?”

- Next week: Chapters 8, 9, start of 10
- Onward to generalized linear models (GLMs)
  - All predictors interact to some extent
- Onward to multilevel models (GLMMs)
  - Massive interaction engines --> allow parameters to be conditional on group membership
- Need Markov chains

- mc-stan.org
- Install RStan
  1. Get C++ compiler
  2. ???
  3. Profit



Stanislaw Ulam (1909–1984)

Interfaces

ways to run Stan

Stan Interfaces

The Stan modeling language and statistical algorithms are exposed through interfaces into many popular computing environments.

- RStan (R)
- PyStan (Python)
- CmdStan (shell, command-line terminal)
- MatlabStan (MATLAB)
- Stan.jl (Julia)
- StataStan (Stata)
- MathematicaStan (Mathematica)

Programs written in the Stan modeling language are portable across interfaces.