1 Methods

1.1 Species information

1.1.1 Spatial occurrence

1.2 Spatial model of species diversity

Define y_i as the number of species present in grid cell i, where $i = 1, \dots, N$. Also define λ is the inverse-scale or "rate" parameter of the Poisson distribution; α as a constant; and s_i as the effect of spatial autocorrelation on species diversty.

$$
\begin{aligned}
y_i &\sim \text{Poisson}(\lambda) \\
\lambda &= \alpha + s_i.
\end{aligned}
\tag{1}
$$

Individual grid cells, and their relation, are an example of areal data CITATION. Given In order to model the effect of grid cell location, I use a multivariate normal condiational autoregressive (CAR) prior for the spatial term s. A CAR prior has multiple different variables, some of which are defined based on the spatial structure of the data. $\mathbf{W}$ is an adjacency matrix where the off-diagonal terms are 0 or 1, 1 indicating that the two grid cells are adjacent. The diagonal of $\mathbf{W}$ is all zeros as a cell cannot be adjacent to itself. $\mathbf{D_w}$ is a diagonal matrix where the diagonal elements are equal to the total number of cells that cell i is adjacent too CITATIONS.

A CAR prior is defined as

$$
s \sim \text{MVN}(0, \sigma^2(\mathbf{D_w} - \rho \mathbf{W})^{-1})) \tag{2}
$$

where ρ can be weakly interpreted as the "strength" of spatial autocorrelation and σ is a normalizing scale parameter for the covariance matrix CITATION.

1.2.1 Zero-Inflated

Define θ and the probability of observing a cell with 0 species, and a 1 - θ is the probability from observing a cell with Poisson(λ) species. The probability function is then defined as

$$p(y_t|\theta, \lambda_t) = \begin{cases} \theta + (1-\theta) \times \text{Poisson}(0|\lambda_t) & \text{if} y_t = 0, \text{and} \\ (1-\theta) \times \text{Poisson}(y_t|\lambda_t) & \text{if} y_t > 0. \end{cases} \tag{3}$$

1.2.2 Hurdle

Define θ and the probability of observing a cell with 0 species, and a 1 - θ is the probability from observing a cell with Poisson(λ) species. The probability function is then defined as

$$p(y_i|\theta, \lambda_i) = \begin{cases} \theta & \text{if} y_i = 0 \\ (1-\theta)\frac{\text{Poisson}(y_t|\lambda_i)}{1-\text{PoissonCDF}(0|\lambda_i)} & \text{if} y_t > 0, \end{cases} \tag{4}$$

where PoissonCDF is the cummulative density function for the Poisson distribution.