

A bestiary of probability distributions

Peter Smits

A great and terrible bestiary

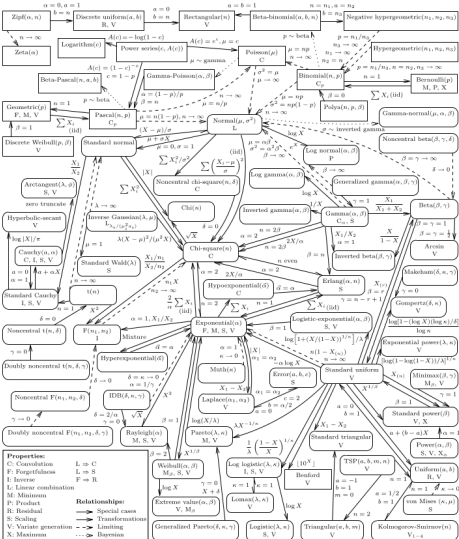


Figure 1. Univariate distribution relationships

The exponential family

Uniform distribution

Discrete probability distributions

Bernoulli distribution

probability of “success” from single trial

Suppose we toss a coin only once. Let $X \in 0, 1$ be a binary random variable, with probability of “success” or heads being θ .

We say that X has a **Bernoulli** distribution, written $X \sim \text{Bern}(\theta)$. The pmf is defined:

$$\text{Bern}(x|\theta) = \begin{cases} \theta & \text{if } x = 1 \\ 1 - \theta & \text{if } x = 0 \end{cases}$$

Binomial distribution part 1

probability of X “successes” in n trials

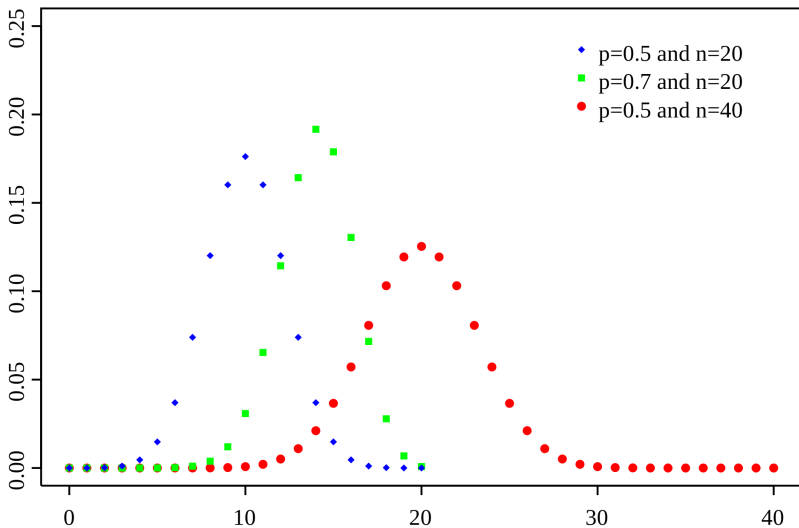
Suppose we toss a coin n times. Let $X \in 0, \dots, n$ be the number of heads, with the probability of heads being θ .

We say X has a **binomial** distribution, written $X \sim \text{Bin}(n, \theta)$. The pmf is defined:

$$\text{Bin}(X|n, \theta) = \binom{n}{X} \theta^X (1 - \theta)^{n-X}$$

Note: Bernoulli distribution is special case of Binomial where $n = 1$

Binomial distribution part 2



Geometric distribution part 1

number of “failures” before a “success”

If each pokeball we throw has probability $1/10$ to catch Mew, the number of failed pokeballs will be distributed $\text{Geom}(1/10)$.

Two definitions:

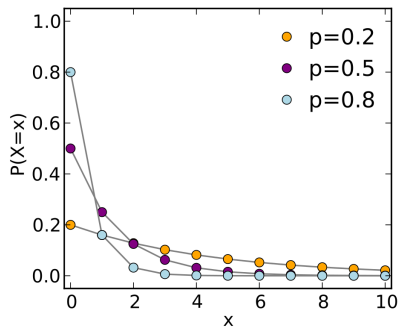
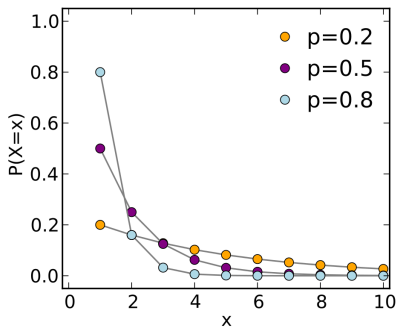
- ▶ The probability distribution of the number X of Bernoulli trials needed to get one success

$$\Pr(X = k) = (1 - \theta)^{k-1} p$$

- ▶ The probability distribution of the number $Y = X - 1$ of failures before the first success

$$\Pr(Y = k) = (1 - \theta)^k p$$

Geometric distribution part 2



Negative Binomial distribution

number of draws until n successes

Hypergeometric distribution

number of “successes” in a fixed number of trials without replacement

Drawing a particular type of card from a deck of cards without replacement.

Poisson distribution

counts of rare events in unit of space or time

Continuous probability distributions

Normal distribution

Exponential distribution

Gamma distribution

Beta distribution

Chi-Square distribution