How predictable is extinction? Forecasting species survival at million-year timescales

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A tenet of conservation palaeobiology is that knowledge of past extinction patterns can help us to better predict future extinctions. Although the future is unobservable, we can test the strength of this proposition by asking how well models conditioned on past observations would have predicted subsequent extinction events at different points in the geological past. To answer this question we analyze the well-sampled fossil record of Cenozoic planktonic microfossil taxa (foramanifera, radiolarians, diatoms, and calcareous nanoplankton.) For each group we examine how extinction probability varies over time as a function of species age, time of observation, current geographic range, change in geographic range, climate state, and change in climate state. Although the best-performing model includes time-varying effects and historical covariates (change in range and change in climate state) the improvement in predictive power over models with constant effects and no historical covariates is very minor. This model has an approximately 78% median probability of correctly ranking the relative extinction risk any two randomly selected species. It performs nearly as well at out-of-sample prediction (70-80%), implying that determinants of extinction risk have varied only modestly through time. An important caveat is that human impacts may substantially disrupt range-risk dynamics so that the future will be less predictable than it has been in the past.

**Keywords:** conservation, palaeobiology, extinction, forecasting

# Introduction

The intensifying biodiversity crisis confronts conservation biologists with the difficult task of trying to predict which species are most threatened with extinction in the nea future. Such predictions are difficult because reliable population and geographic range time series are typically known for only the past few decades in even the best-studied groups, and because few modern extinctions have been adequately documented. This has led to the suggestions that some risk assessments might be improved by incorporating palaeontological data (Finnegan et al. 2015; Kiessling and Kocsis 2016b). The fossil record preserves information about the full histories, include ultimate extinction, of thousands of lineages, and this information can help to augment the shorter-term higher-resolution data used to make risk assessments of extant taxa.

Past palaeobiological studies of extinction have frequently focused on identifying and measuring the effect of various potential predictors on extinction risk (Harnik 2011; Smits 2015; Peters 2008; Payne and Finnegan 2007; Harnik, Simpson, and Payne 2012; Ezard et al. 2011; Foote 2006) or on how to identify or measure these effects (Alroy 2010b; Alroy 2014; Alroy et al. 2001; Alroy, Koch, and Zachos 2000; Alroy 2000; Foote 2001).

This focus means that while we have a good understanding of which factors are strong and general determinants of extinction risk, we have less knowledge of how accurate or strong our predictions about the differences in extinction risk are.

Here we ask how precise risk predictions based on fossil data might be. Because future extinctions are unobservable we cannot directly evaluate the ultimate performance of such predictions. However, we can address the question by taking a given point in the geological past, developing predictive model based on extinction patterns prior to that point, and assessing the predictive performance of this model on unobserved (e.g. “future”, from the point of view of the model) extinction/survival events.

Both intensity (average rate) and selectivity (difference in risk between taxa) (Payne and Finnegan 2007; Payne et al. 2016; Ezard et al. 2011). Given this variation, how accurate are our assessments based on past events likely to be when applied to the future? Putting aside the important question of how human activities will alter the determinants of future extinction risk, we can address this uncertainty by specifically including and modeling the temporal variation in extinction risk across a range of extinction intensities and selectivities.

Species, after all, can go extinct at any “moment” and the relative risk of extinction exhibited by different taxonomic groups and how that risk varies over time is an important dynamic which shapes the rate and structure of extinction. What has not been evaluated is that as extinction intensity and selectivity change over time, how accurate are our assessments based on past events likely to be when applied to the future? By specifically including and modeling the temporal variation in extinction risk, we are able to improve our overall predictions because we incorporate and explicitly model differences between observations from across a range extinction intensities and selectivities.

Numerous studies have established that geographic range is one of the most important determinants of extinction risk in the fossil record, and that geographic range trajectories are highly variable (Foote 2007; Liow et al. 2010; Liow and Stenseth 2007; Kiessling and Kocsis 2016a; Payne and Finnegan 2007; Jablonski and Roy 2003; Jablonski 2008; Jablonski and Hunt 2006). The degree to which the past can help to predict the future fates of species depends in part on the degree to which species’ geographic range trajectories are deterministic versus Markovian. In the former case, knowledge of the specific past trajectory of a species -whether its range has expanded or contracted over a given timespan- might help to improve assessments of its current risk. In the latter case, only the current range of the species conveys useful information about its current risk, although we can still use prior extinction patterns to augment predictions by evaluating the relative extinction risk of species that had similar ranges in the past. Discriminating among these alternative models is thus very important for determining how best to incorporate fossil data in present risk assessments.

For this exercise we chose to analyze one of the best-sampled and studied fossil records – the Cenozoic record of skeletonized marine planktonic microorganisms (Foraminifera, Radiolaria, Diatoms, and Coccolithophores). These data are readily available through the Neptune database, an online repository of species occurrences obtained through the Deep Sea Drilling Program and the Ocean Drilling Project (Lazarus 1994; Spencer-Cervato 1999). This database provides abundant samples in space and time, a high degree of temporal resolution for the entirety of the Cenozoic, and has an internally consistent taxonomic identification strategy – as close to ideal data for this analysis as possible. The Neptune database records multiple phyla-scale taxonomic groups for over 60 million years, with incredible temporal resolution supported by the various age-models of the deep-sea cores from which the occurrences are recorded. Analyzing patterns of extinction and global occurrence at fine temporal scales means we can better elucidate how well we can predict species extinction at human-relevant scales.

# Materials and Methods

## Data Specifications

We analyzed microfossil occurrence information from the Neptune Database <http://www.nsb-mfn-berlin.de/nannotax> (Lazarus 1994; Spencer-Cervato 1999). This dataset of occurrences covered calcareous nannoplankton, diatoms, foraminifera, and radiolarians.These occurrences were then filtered to include only those species who’s first occurrence was no more than 63 Mya. This filtering criterion means that our analysis avoids those taxa which survived the K/Pg boundary, those taxa which arose just after the K/Pg boundary, and ensures that our occurrence histories are congruent with the temperature time-series used as a predictor of extinction (see below).

All fossil occurrences were assigned to 1 My bins based on the estimated age of the fossil occurrence as listed in the Neptune Database. After binning, each taxon’s geographic range was calculated for each of the 1 My bins in which it occurred. Geographic range was calculated as the maximum great circle distance on an ellipsoid (i.e. the Earth) between any two occurrences of that species; this distance was measured in kilometers.

Average global temperature of each 1 My bin was calculated from estimates based on Magnesium/Calcium isotope ratios Cramer et al. (2011). Elemental ratios are considered more accurate estimates of past global temperature when compared to the frequently used Oxygen isotope-based estimates; this is because Mg/Ca based estimates are not effected by ice-volume and fresh-water input (e.g. meteoric water) which can alter Oxygen isotope ratios without reflecting changes to the climate itself. This property is of particular importance for this analysis as polar ice-caps develop midway through the Cenozoic. Our data source, Cramer et al. (2011), estimated temperature for every 0.1 My from 0 to 63 Mya. The temperature estimate for each 1 My interval was calculated as the mean of all estimates within that interval.

See Section [sec:data\_desc] for a further explanation on how observations were temporally binned, and how our covariates were standardized and transformed prior to analysis.

## Model Specifications

We used a discrete-time survival modelling framework in order to estimate how well we can predict extinction risk at one million year time scales. At its core, our model is a multilevel logistic regression with taxon age in millions of years as a varying intercept (Tutz and Schmid 2016). We considered four variations of the predictors included in our model: covariate effects are constant over time and none of our historical covariates are included (Model C), covariate effects are allowed to vary over time but none of our historical covariates are included (Model V), covariate effects are constant over time and our historical covariates are included (Model CP), covariate effects are allowed to vary over time and historical covariates are included (Model VP). We always included species geologic age at time of observation as a non-nested varying intercept term. This factor may or may not contribute to differences in species extinction risk over time (Smits 2015; Finnegan, Payne, and Wang 2008; Ezard et al. 2012; Van Valen 1973; Liow et al. 2011; Crampton et al. 2016), but its inclusion in our model is critical to its nature as a survival model (Tutz and Schmid 2016).

See Table [tab:model\_def] for further explanation of how the four models we considered differ from each other. A complete description of the statistical model used in this analysis is available in Section [sec:model\_desc]. Additionally, the full description of how these models were implemented and coded, including choice of priors, is available Section [sec:model\_est].

Models and their definitions

|  |  |  |  |
| --- | --- | --- | --- |
| Code | Description | Covariates | R Formula Syntax |
| C | Constant effects, no historical cov. |  |  |
| V | Varying effects, no historical cov. |  |  |
| CP | Constant effects, historical cov. |  |  |
| VP | Varying effects, historical cov. |  |  |

See Equation [eq:model] for full statistical model definition.

Species observation where 1 if time of last observation, otherwise 0.

Species geographic range in log km. Mean centered, scaled to sd = 1.

Global temperature in degrees C. Mean centered, scaled to sd = 1.

Species are at observation in millions of years.

Taxonomic group of species (i.e. Foraminifera, Diatoms, Radiolarians, Calcareous nannoplankton).

Change in geographic range since last observation.

Temperature at previous observation.

[tab:model\_def]

## In-sample and out-of-sample forecasting

We are interested in our models’ performance into two contexts: in-sample and out-of-sample predictive performance i.e. forecasting.

In-sample forecasting means we are estimating how well our model predicts extinction probability for observations that that model was fit to. This is a posterior predictive check in that we are comparing the posterior predictive distribution to our observed data. In-sample forecasting measures, however, are not necessarily good estimates of the model’s ability to predict data from the future (Hastie, Tibshirani, and Friedman 2009).

We are particularly interested in understanding how well our model forecasts extinction probability of data from the future that the model was not fit to (out-of-sample data). To quantify our ability to forecast species’ extinction risk, we estimated average out-of-sample forecasting performance using 5-fold time-series cross-validation. For time-series data, the folds (data partitions) are approximately equal segments of time. Each fold represents a sequence of time points. With 63 time points, each of the five folds represents approximately 13 time points. Keep in mind, however, that each time point corresponds to many (100-1000) individual observations.

k-fold cross-validation for time series follows a specific sequence of procedures (Arlot and Celisse 2009; Bergmeir, Hyndman, and Koo 2018). First, the model is fit to the first fold (time segment), and the posterior estimates of that fit are then used to forecast the extinction probability of the second fold (i.e. the future). Then the model is fit to the combined first and second folds, and the posterior estimates of that fit are used to forecast the extinction probability of the third fold. Continuing, the model is then fit to the first three folds combined and is then used to forecast extinction probabilities for the fourth fold. Finally, the model is fit to the first four folds combined and then is used to forecast the fifth fold. The results from these forecasts are then combined to yield our estimate of expected out-of-sample performance.

The relative adequacy of the four model variants was compared using the area under the receiver operating characteristic curve or AUC (Fawcett 2006; Mason and Graham 2002). This measure is commonly used to measure the performance of classification models as it has the desirable characteristic of comparing the model’s true positive rate with its false positive rate, as opposed to accuracy which only considers true positives. AUC ranges between 0.5 and 1, with 0.5 indicating no difference in classification from random and 1 indicating perfect classification. AUC can be interpreted as the probability that our model correctly ranks the relative extinction risks of a randomly selected extinct-extant species pair (Fawcett 2006; Mason and Graham 2002). AUC values of approximately 0.8 or greater can be considered “good” (Tang, He, and Tu 2012), so we interpret values between 0.7 and 0.8 could then be considered “fair,” and values between 0.6 and 0.7 as “poor.”

See our code repository at https://github.com/psmits/trident for full code details. This entire analysis was coded in R and uses tidyverse and tidyverse adjacent tools such as dplyr (Wickham et al. 2018), purr (Henry and Wickham 2018), and tidybayes (Kay 2018). Additionally, all of our models were written using the brms (Bürkner 2017; Bürkner 2018) R package, which implements Stan-based Bayesian models which are fit via Hamiltonian Monte Carlo (Stan Development Team 2017).

# Results

0.45 In-sample ([fig:auc\_hist]) and out-of-sample ([fig:fold\_auc]) AUC estimates for each of our four models. These estimates are calculated from the models posterior predictive distribution or from predictions made to new data, respectively. Models with a higher AUC values indicate better performance over models with lower AUC values. AUC is bounded between 0.5 and 1. See Table [tab:model\_def] for a description of each of the four models.

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## In-sample forecasting adequacy

The in-sample model comparisons are useful comparing the relative ability of our models to represent the data it was fit to. Comparison between the posterior distributions of in-sample AUC for each of the four models demonstrates that all of our models have approximately equal in-sample forecasting performance (Fig. [fig:auc\_hist]). The parameter rich model VP has the greatest median in-sample AUC when compared to the other three models, but there is substantial overlap in their posterior distributions. Additionally, while our parameter rich model VP is possibly the most adequately performing model, the difference or improvement to performance is minimal at best – all four models have approximately equal in-sample AUC posterior distributions. All the in-sample AUC estimates from our models are concentrated on an AUC value of 0.77. It is therefore hard to conclude that there is one “best” model which we can rely upon as they are all nearly functional equivalent.

There are one or more time periods where our posterior distribution of in-sample AUC has a median value less than or equal to 0.5 (Fig. [fig:auc\_taxon\_time]). However, this pattern is absent for the posterior estimates of in-sample AUC for Foraminfera and Radiolaria as fit by the VP model. In contrast, there are fewer periods of low model performance for calcareous nannoplankton and Dinoflagellates as estimated from our VP model than in those estimates from the other three model variations.

Comparison of in-sample forecasting performance measured by AUC for each of the four models, arranged over time and by taxonomic group. These estimates reflect each model’s fit to the various taxonomic groups over time. The black line corresponds to the median AUC value, while the envelopes correspond to multiple credible intervals as indicated in the legend. In all cases, higher AUC values indicate greater predictive performance versus lower AUC values. See Table [tab:model\_def] for a description of each of the four models.

Comparison of in-sample forecasting performance measured by AUC for each of the four models, arranged over time and by taxonomic group. These estimates reflect each model’s fit to the various taxonomic groups over time. The black line corresponds to the median AUC value, while the envelopes correspond to multiple credible intervals as indicated in the legend. In all cases, higher AUC values indicate greater predictive performance versus lower AUC values. See Table [tab:model\_def] for a description of each of the four models.

## Out-of-sample forecasting performance

Expected out-of-sample forecasting performance was estimated using five-fold cross-validation for time series (Arlot and Celisse 2009; Bergmeir, Hyndman, and Koo 2018). The resulting distribution when all folds are combined is very multimodal by their very nature of being fit to and estimated from different data sets and amounts of data (Hastie, Tibshirani, and Friedman 2009). We find that this multimodality increases with model complexity (Fig. [fig:fold\_auc]), most likely because the complex models allow for predictor effects to vary with time which allows for a greater range in possible parameter values which in turn yield a greater range of posterior predictions.

Comparison between the in-sample forecasting performance estimates to the expected out-of-sample AUC estimates reveals a similar range in performance for all models (Fig. [fig:auc\_hist], [fig:fold\_auc]). Interestingly, the differences in posterior predictive distributions of AUC between the four model variations has decreased. For example, the VP model no longer has the greatest median AUC of the four models (Fig. [fig:fold\_auc]). These differences mean that the rank order of median out-of-sample AUC is different from the rank order of median in-sample AUC. However, making interpretations bsaed only incorrect – our estimates are the full posterior, not individual points. Because of this, the four models are effectively indistinguishable in their out-of-sample forecast performance (Fig. [fig:fold\_auc]).

We can also compare the posterior predictive distribution of expected out-of-sample AUC over time and taxonomic group for each of the four models (Fig. [fig:fold\_auc\_taxon\_time]). In all cases, our four models are nearly identical in their out-of-sample forecasting performance over time and between each of the taxonomic groups.

In the analysis of the in-sample forecast performance of the four models, we noted that there were time intervals where our predictions were no better than random (Fig. [fig:auc\_taxon\_time]). This occurrence is generally much rarer for the posterior distribution of AUC from the out-of-sample forecasts. The major exception to this pattern are our estimates for the Dinoflagellates, which has at least one time interval for all four models where the median AUC of the out-of-sample forecasts were no better random. In contrast, the only other group for which median posterior predictive estimate of out-of-sample AUC reaches 0.5 is calcareous nannoplankton, and then only with model V.

Comparison of out-of-sample AUC values over time as aggregated by taxonomic group for each of the four models. The AUC of the individual My intervals within each fold is plotted to highlight the heterogeneity in performance within and between folds. This presentation decomposes each of the 12-million year folds by each of the taxonomic groups into the predictions made for each of the million-year intervals. The black line corresponds to the median AUC estimate, with the envelopes corresponding to multiple credible intervals as indicated in the legend. See Table [tab:model\_def] for a description of each of the four models.

Comparison of out-of-sample AUC values over time as aggregated by taxonomic group for each of the four models. The AUC of the individual My intervals within each fold is plotted to highlight the heterogeneity in performance within and between folds. This presentation decomposes each of the 12-million year folds by each of the taxonomic groups into the predictions made for each of the million-year intervals. The black line corresponds to the median AUC estimate, with the envelopes corresponding to multiple credible intervals as indicated in the legend. See Table [tab:model\_def] for a description of each of the four models.

We compared the difference in our AUC estimates from the out-of-sample forecasts to the AUC estimates from our in-sample forecasts by subtracting the in-sample AUC estimates from the out-of-sample AUC estimates (Fig. [fig:oos\_ins\_diff]). A difference in AUC close to 0 indicates complete congruence between the in-sample and out-of-sample forecasts. A positive difference indicates that our out-of-sample forecasts are actually higher performing than our in-sample forecasts, while negative difference indicates poorer out-of-sample performance than in-sample forecast. Divergences between our out-of-sample and in-sample forecasts are rare and tend to not form multimillion year patters, something that is consistent with the broad visual congruence between the in-sample and out-of-sample forecast performance (Fig. [fig:auc\_taxon\_time], [fig:fold\_auc\_taxon\_time]). The only major multimillion year pattern indicating significantly poorer out-of-sample forecast performance than in-sample forecast performance is for Radiolaria based on the VP model concentrated around 30 Mya (Fig. [fig:oos\_ins\_diff]).

Comparison between our out-of-sample forecasts and in-sample forecasts for all models. This value is calculated as the values presented in Figure [fig:fold\_auc\_taxon\_time] minus those values presented in Figure [fig:auc\_taxon\_time]. A differences close to 0 indicate complete congruence between in-sample and out-of-sample forecasts, while a positive difference indicates that our out-of-sample forecasts are actually higher performing than our in-sample forecasts, and a negative difference indicates poorer out-of-sample performance than in-sample forecast. See Table [tab:model\_def] for a description of each of the four models.

Comparison between our out-of-sample forecasts and in-sample forecasts for all models. This value is calculated as the values presented in Figure [fig:fold\_auc\_taxon\_time] minus those values presented in Figure [fig:auc\_taxon\_time]. A differences close to 0 indicate complete congruence between in-sample and out-of-sample forecasts, while a positive difference indicates that our out-of-sample forecasts are actually higher performing than our in-sample forecasts, and a negative difference indicates poorer out-of-sample performance than in-sample forecast. See Table [tab:model\_def] for a description of each of the four models.

# Discussion

We find that all of our models are expected to correctly forecasting the rank order in extinction risk of two randomly selected future observations between 70% to 80% of the time. One of the most striking aspects of these results is the similarity in forecasting performance for in-sample and out-of-sample observations. A slight decrease in performance when dealing with out-of-sample observations makes sense: each of the models fit during cross-validation is based on fewer data than the model fit on the full data (between 1/5th to 4/5ths of the original). Additionally, a potential decrease in precision when forecasting the future of extinction risk is almost to be expected as the future is not necessarily the same as the past. However, the similarly between the in-sample and out-of-sample results indicates that our model is fairly robust to how extinction intensity has changed over the Cenozoic. An important caveat to our results is that human impacts may substantially alter extinction risk dynamics away from the average Cenozoic “state” so that the future could become less predictable than it has been in the past.

We also find that our four models are practically identical in their ability to make forecasts both in-sample and out-of-sample. Although the in-sample AUC estimates differ between models, all of these estimates are in a narrow range of possible AUC values (Fig. [fig:auc\_hist]). While the “best” model does include the historical covariates and allows all covariate effects to vary over time, its practical difference in performance versus the other models is negligible (Fig. [fig:fold\_auc]). This means that even though model VP has a statistically greater AUC for in-sample forecasts than the other three models, this result is not practically or scientifically significant. The results of the out-of-sample forecasts illustrate this point in full, as all four models have functionally identical out-of-sample performance.

The quality and consistency of our models out-of-sample forecasting performance is surprising given that its estimates are based on rather limited biological and environmental information about the taxa. Our most complex models are only taking into account some aspects of species geographic range and their taxonomic grouping. The principal reason we were not able to include more biological information in our models is that we lack almost any life history or ecological information on most marine micro- and nannoplankton. Forams are an exception to this problem as aspects of life history, ecology, and physiology for many foram species (Ezard et al. 2011). However, comparable information does not exist all foram species, nor does this type of data exist for the other three taxonomic groups studied here. Including this type of information in a predictive model of extinction risk we require analyzing a single, limited taxonomic subset of the fossil occurrences present the Neptune Database. This would mean ignoring the majority of occurrence information present in the Neptune database.

In conclusion, our results provide a promising picture of our ability to predict the relative extinction risk of two randomly selected species. Considering that conservation decisions are made based on a continuum of risk, from most to least, this means that our results are in the same language as how conservation resources are allocated.

# Supplement to Materials and Methods

## Data Specifications

### Binning fossil occurrences

The estimated age of each occurrence is based on the core-specific age-model that observation is from and can be overly precise. To alleviate this overprecision, we coarsened our temporal information in an effort to limit the effects of between-core heterogeneity in age. The occurrence histories of each species was then summarized as a series of binary codes indicating the presence or last occurrence of that species. For every occurrence of a species, except the last, that species existence and survival is recorded as a 0. The last occurrence of that species is considered the bin in which the taxon has gone extinct – and is recorded as 1. This protocol means that we are reading the fossil record “as written,” a practice that is potentially dangerous as it is an overconfident statement of preservation and may be shortening the actual durations of the studied species (Alroy 2010a; Alroy 2000; Alroy 2014; Foote 1997; Foote and Sepkoski 1999; Foote 2001; Foote and Raup 1996; Lloyd et al. 2012; Marshall 1995; Wang and Marshall 2016). However, this practice is common with marine microfossil data due to their exceptional preservation rate (Ezard, Thomas, and Purvis 2013; Ezard and Purvis 2016; Ezard et al. 2011; Liow et al. 2010). In fact, with marine microfossils collected from cores a bigger problem may be over extending the duration of a species due to mixing and smearing within the cores (Mekik and Anderson 2018; Broecker et al. 1999; Mekik 2014; Peng and Broecker 1984).

### Covariate transformation and standardization

Prior to analysis, geographic range was then log-plus-one transformed and standardized by mean-centering the data and then dividing by the standard deviation of the distribution of geographic ranges. This standardization means that a regression coefficient associated with each covariate describes the change in extinction probability per change in standard deviation of that covariate, that coefficients associated with similarly standardized covariates will be directly comparable in magnitude, and that the intercept term corresponds to the expected value of the outcome at when geographic range is its average value (Gelman and Hill 2006). Change in geographic range between observations was measured from the standardized geographic range values and was not standardized separately.

Temperature was also transformed and standardized the in the same manner as geographic range. The change in temperature between an observation and its previous observation was measured from the standardized temperature values and was not standardized separately.

## Model Specifications

In survival analysis, the hazard function describes the instantaneous rate of extinction of a species given its age and covariate information. The hazard function is defined as the conditional probability of a species going extinct by the end of the -th interval given that it survived up until and the relevant covariate information for all 1 My intervals (Tutz and Schmid 2016). For the discrete time intervals , extinction is defined as . The discrete time hazard function is defined as

The hazard function (Eq. [eq:hazard]) is easily reparameterized as a logistic regression by defining that where is a logit inverse-link function and is the probability of a taxon going extinction during interval (Tutz and Schmid 2016). is then modeled as with any regression. In this case, we opted for a hierarchical/mixed-effects model with multiple non-nested varying intercepts and slopes (Gelman and Hill 2006).

Our covariates matrix is a matrix where is the total number of observations and is the total number of covariates. The first column of is entirely 1’s as it corresponds to the intercept term in the regression model. The next two columns of are two aspects of geographic range as continuous covariates: geographic range during interval , and the difference between the geographic range at and . Change in geographic range was calculated from the transformed and standardized geographic range values; this means that change in geographic range is in units of changes in standard deviations. The final two columns are two aspects of global temperature: mean temperature during interval , and the lag of mean temperature (i.e. mean temperature during interval .) As with change to geographic range, the lag of temperature is based on the transformed and standardized temperature estimates.

The matrix of time and phylum varying regression coefficients describing the effects of the covariates on a species’ risk of extinction is called – a by matrix, were is the number of time temporal intervals and is the number of phyla. The elements of this matrix, the regression coefficients, are themselves modeled as being multivariate normally distributed with vector of means describing the average intercept and regression coefficient estimates of each phylum . These phylum averages are themselves modeled as multivariate normally distributed with mean vector describing the overall average regression coefficients, including the intercept. has length and is ordered intercept, range coefficient, change in range coefficient, temperature coefficient, temperature lag coefficient.

The effect of species age on the log-odds of species extinction is modeled as a non-nested random intercept (Tutz and Schmid 2016). This term describes how the log-odds of extinction varies along a species duration, and how this effect can differ between the phyla. is a by matrix, where is the age at observation of a species and is its phylum. is modeled as following a multivariate normal distribution with phylum means being the vector and covariance matrix . The covariation between the elements of vector are modeled as a multivariate normal distribution with a mean vector of all 0s and covariance matrix .

To complete the generative model, we need to assign final priors to the “top-level” parameters. In general, we favored weakly informative priors which help regularize our estimates. In the case of a regression coefficient, this means a Normal distribution with mean 0 and a standard deviation of 3. For our scale parameters (e.g. standard deviations), we used half-Cauchy distributed priors with heavy tails but the majority of probability density near 0.

Our top-level intercept was given a more diffuse prior than our regression coefficients, which reflects our greater degree of uncertainty about its value. Our top-level regression coefficient for the effect of geographic range was given an informative prior reflecting the overwhelming amount of evidence that species with a larger than average geographic range have a lower risk of extinction than species with an average or less than average geographic range. In the context of this analysis, this means that we are again using a weakly informative prior but instead of centering the density around -1 (i.e. larger than average geographic range decreases extinction risk).

Instead of assigning a prior distribution for each of the covariance matrices in the model, we instead decomposed the covariance matrices (e.g. ) which allows us to assign independent priors for the scale and correlation aspects of covariance. The scale parameters were assigned half-Cauchy priors as described above in the context of all other scale parameters. The correlation matrices were assigned LKJ priors each with shape parameter set to 1. This choice of shape parameter produces a uniform distribution over possible correlation matrices. These priors are also slightly more interpretable than other common prior distributions for covariance matrices such as the inverse-Wishart distribution. This approach to assigning priors to a covariance matrix is recommended by the Stan Manual (Stan Development Team 2017).

In total, our model can be expressed as:

with indexing the observation and bracket subscripts referencing the class of the th observation where is the time of the -th observation, is the phylum of the -th observation, and is the age of the -th observation.

## Model Parameter Estimation

We implemented our model (Eq. [eq:model] using the

rstanarm

package for the R programming language (Stan Development Team 2017). This package provides an interface to the Stan probabilistic programming language for writing hierarchical/mixed-effects models in native R. Posterior estimates were obtained through Hamiltonian Monte Carlo, using 2000 steps divided equally between warm-up and sampling. In order to prevent divergent transitions adapt delta was increased to 0.999999; all other HMC/NUTS sampling parameters were kept at the defaults for rstanarm 2.18.2 (Gabry and Goodrich 2018).

To implement our VP model in

rstanarm

, where “data” is a data.frame object of all necessary data (response, covariates), is coded as:

form <- event ~ range + range\_diff + temp + temp\_lag +   
 (1 + range + range\_diff + temp + temp\_lag | mybin/phylum) +   
 (1 | age/phylum),   
stan\_glmer(formula = form,  
 data = data,   
 family = 'binomial',  
 prior = normal(c(-1, 0, 0, 0), rep(1, 4), autoscale = FALSE),   
 prior\_intercept = normal(-2, 5, autoscale = FALSE),   
 prior\_aux = cauchy(0, 1, autoscale = FALSE),   
 chains = 4,  
 thin = 4,  
 adapt\_delta = 0.999999)

Similarly, our VP model can be implemented using the

brms

Stan interface (Bürkner 2017; Bürkner 2018) as:

priors <- c(set\_prior('normal(-2, 5)', class= 'Intercept'),  
 set\_prior('normal(0, 1)', class = 'b'),  
 set\_prior('normal(-1, 1)', class = 'b', coef = 'range'),  
 set\_prior('cauchy(0, 1)', class = 'sd'),  
 set\_prior('lkj(1)', class = 'cor'))  
form <- bf(event ~ range + range\_diff + temp + temp\_lag +  
 (1 + range + range\_diff + temp + temp\_lag | mybin/phylum) +  
 (1 | age/phylum))  
brmfit <- brm(formula = form,  
 data = data,   
 family = bernoulli(),   
 prior = priors,  
 chains = 4,   
 thin = 4,  
 control = list(adapt\_delta = 0.999999)

Posterior convergence was determined using the general and HMC-specific diagnostic criteria: scale reduction factor (; target ), effective sample size (eff; target value eff/steps ), number of samples that saturated the maximum trajectory length for avoiding infinite loops (treedepth; target value 0), sample divergence, and the energy Bayesian Fraction of Mission Information (E-BFMI; target value ). For further explanation of these diagnostic criteria, see the Stan Manual (Stan Development Team 2017).

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